Online Appendix to "Financial Crises, Dollarization, and Lending of Last Resort in Open Economies"

by Luigi Bocola and Guido Lorenzoni

A Proofs for Sections II and III

A.1 Derivation of Equation (7)

The consumer optimality conditions in a continuation equilibrium are

$$\omega\left(c_1^T\right)^{\omega(1-\gamma)-1}\left(c_1^N\right)^{(1-\omega)(1-\gamma)} = \lambda \tag{A.1}$$

$$(1-\omega)\left(c_1^T\right)^{\omega(1-\gamma)}\left(c_1^N\right)^{(1-\omega)(1-\gamma)-1} = \lambda p_1 \tag{A.2}$$

$$\beta\omega\left(c_2^T\right)^{\omega(1-\gamma)-1}\left(c_2^N\right)^{(1-\omega)(1-\gamma)} = \lambda\beta \tag{A.3}$$

$$\beta(1-\omega)\left(c_2^T\right)^{\omega(1-\gamma)}\left(c_2^N\right)^{(1-\omega)(1-\gamma)-1} = \lambda\beta p_2, \tag{A.4}$$

where λ is the Lagrange multiplier on the intertemporal budget constraint. Combining (A.1), (A.3), and the market clearing condition $c_t^N = e^N$, we get $c_1^T = c_2^T$. Combining (A.2) and (A.4) we then get $p_1 = p_2 = p$. Combining (A.1) and (A.2), we also get $c_t^T/(p_1 c_1^N) = \omega/(1-\omega)$, and similarly we get the same relation at $t = 2$. Substituting these conditions in the intertemporal budget constraint, we obtain

$$c_1^T = c_2^T = \frac{\omega}{1+\beta}W, \quad c_1^N = c_2^N = \frac{1}{p}\frac{1-\omega}{1+\beta}W,$$

where

$$W = a_1^T + pa_1^N + w_1 + \beta w_2 + p\left(e_{c,1}^N + \beta e_{c,2}^N\right)$$

is the total wealth of the consumers at date 1. Setting $c_1^N = e^N$ gives equation (7) in the text.

A.2 Proof of Proposition 1

If $\{a_1^T, a_1^N, b_1^T, b_1^N, K_1\}$ satisfy (6), we have that $\mathcal{P}(.)$ is increasing in K and concave. Let $p^* = \mathcal{P}(K^*)$ and $\underline{p} = \mathcal{P}(\underline{K})$. We can have three types of equilibria: either we have

$$\underline{K} \geq \frac{1}{1-\beta\theta}\left[\alpha K_1^\alpha - b_1^T + \underline{p}(e_{b,1}^N - b_1^N)\right],$$

and we have an equilibrium with $\underline{K} = \mathcal{K}(\underline{p})$ and $\underline{p} = \mathcal{P}(\underline{K})$; or we have

$$K^* \leq \frac{1}{1-\beta\theta}\left[\alpha K_1^\alpha - b_1^T + p^*(e_{b,1}^N - b_1^N)\right],$$

and we have an equilibrium with $K^* = \mathcal{K}(p^*)$ and $p^* = \mathcal{P}(K^*)$; or we have

$$K = \frac{1}{1-\beta\theta}\left[\alpha K_1^\alpha - b_1^T + \mathcal{P}(K)(e_{b,1}^N - b_1^N)\right],$$

for some $(\underline{K}, K^*)$, and we have an interior equilibrium. If the first two types of equilibria do not exist, then the third type of equilibrium must exist by a continuity argument. This establishes existence.

Let us now show that if there are multiple equilibria, one of them must be of the first type. By contradiction, we show that if a first type equilibrium does not exist, then the equilibrium is unique. The function $h(K) = \frac{1}{1-\beta\theta}\left[\alpha K_1^\alpha - b_1^T + \mathcal{P}(K)(e_{b,1}^N - b_1^N)\right]$ is concave, from the concavity of $\mathcal{P}$ and $b_1^N \leq e_{b,1}^N$. If an equilibrium of the first type does not exist, then $h(\underline{K}) > \underline{K}$ and two cases are possible: either the function h crosses the 45^o line from above at some $K \in (\underline{K}, K^*)$, in which case that is the unique equilibrium, or the function remains above the 45^o line on the whole interval, in which case $h(K^*) \geq K^*$ and there is a unique equilibrium at K^*. □

A.3 Proof of Proposition 2

Points (i)-(iii) and the consumers' part of point (iv) follow immediately from the fact that p_1 and K_2 are lower in the crisis equilibrium. For the bankers, we have that their consumption in period 2 is

$$\frac{r_2 - \theta}{1-\beta\theta} n_1.$$

If $K_2 \in (\underline{K}, K^*)$, this expression becomes

$$\frac{r_2 - \theta}{1-\beta\theta} n_1 = (r_2 - \theta)K_2 = \alpha K_2^\alpha - \theta K_2,$$

where the first equality uses the banks' budget constraint and the fact that the banks are constrained when K_2 is in that interval, and the second equality uses the definition of the rental rate of capital.

If there is an equilibrium at $\underline{K}$, then

$$\frac{r_2 - \theta}{1-\beta\theta} n_1 \leq (\alpha \underline{K}^{\alpha-1} - \theta)\underline{K} = \alpha \underline{K}^\alpha - \theta \underline{K},$$

whereas if there is an equilibrium at K^*,

$$\frac{r_2 - \theta}{1 - \beta\theta} n_1 \geq (\alpha (K^*)^{\alpha-1} - \theta) K^* = \alpha (K^*)^\alpha - \theta K^*.$$

These derivations and the concavity of $\alpha K^\alpha - \theta K$ imply that a sufficient condition for the banker to be better off at any equilibrium with $K_2 > \underline{K}$, than at the crisis equilibrium with $\underline{K}$, is

$$\alpha (K^*)^\alpha - \theta K^* > \alpha \underline{K}^\alpha - \theta \underline{K}.$$

Given the definitions of K^* and $\underline{K}$, this inequality is equivalent to condition (10) in the statement of the proposition. □

A.4 Proof of Proposition 3

The proof is by construction. Fix the wages and rental rates that prevail in the good and the bad equilibria under $\{a_1^T, a_1^N, b_1^T, b_1^N, K_1\}$. If condition (15) is satisfied, there exists a $\gamma > 0$ such that

$$\left(\frac{w_1 + \beta w_2^B + a_1^T}{w_1 + \beta w_2^G + a_1^T} \right)^{\omega(1-\gamma)-1} = \frac{r_2^B - \theta}{r_2^G - \theta}.$$

The above condition guarantees that the asset positions are consistent with consumers' and banks' optimality. We now show that we can select initial positions $\{a_0^T, a_0^N, b_0^T, b_0^N, K_0\}$ such that all the remaining equilibrium conditions of the model at date $t = 0$ are satisfied. As we have some degree of freedom, we set $a_0^N = b_0^N = 0$.

Pick any probabilities π^G, π^B and set c_0^T to satisfy the consumers' Euler equation

$$\left(c_0^T \right)^{\omega(1-\gamma)-1} = \sum_{s=G,B} \pi^s \left(c_1^{T,s} \right)^{\omega(1-\gamma)-1}.$$

We can then choose a_0^T to satisfy the consumers' budget constraint at $t = 0$:

$$c_0^T + p_0 e^N + \beta a_1^T = p_0 e_{c,0}^N + w_0 + a_0^T$$

where

$$p_0 = \frac{1-\omega}{\omega} \frac{c_0^T}{e^N}$$

guarantees that the market for non-tradable goods clears. So, households are optimizing, and their budget constraint is satisfied.

Assuming that the collateral constraint of the banks is slack, we have that optimal capital choice at date $t = 0$ solves

$$\lambda_{b,0} = \mathbb{E}[\lambda_{b,1}] r_1.$$

Combining the above equation with the banks' optimality condition for tradable denominated bonds, we obtain $K_1 = K^*$. Thus, the budget constraint of the banks is satisfied if

$$K^* = \alpha K_0^{\alpha} + p_0 e_{b,0}^N - b_0^T + \beta b_1^T.$$

For any K_0, there always exists a b_0^T that guarantees that the above constraint holds. Moreover, the financial constraint $b_1^T \leq K_1$ is satisfied by construction, verifying the assumption that the banks' collateral constraint is slack. Thus, the bankers are also optimizing, and their constraint is satisfied. □

A.5 Proof of Proposition 4

Having stable multiple equilibria ex ante requires that we have a pair of capital stocks $(\underline{K}, K_2)$ such that

$$\left(\frac{w_1 + \beta w_2^B + a_1^T}{w_1 + \beta w_2^G + a_1^T}\right)^{\omega(1-\gamma)-1} = \frac{r_2^B - \theta}{r_2^G - \theta},$$

where wages and rental rates are derived from the two capital stocks. This is impossible if

$$\left(\frac{w_1 + \beta w_2^B + a_1^T}{w_1 + \beta w_2^G + a_1^T}\right)^{\omega(1-\gamma)-1} < \frac{r_2^B - \theta}{r_2^G - \theta}$$

for all pairs $(K_2, \underline{K})$ with $K_2 \in [\underline{K}, K^*]$. To find sufficient conditions for this inequality, let us study the function

$$f(k) = \ln\left(\alpha k^{\alpha-1} - \theta\right) + (\omega(\gamma - 1) + 1) \ln\left(w_1 + \beta(1-\alpha) k^{\alpha} + a_1^T\right).$$

If $f'(k) < 0$ for all $k \in [\underline{K}, K^*]$, then the above inequality holds. The derivative of f is

$$f'(k) = \frac{\alpha(\alpha - 1) k^{\alpha-2}}{\alpha k^{\alpha-1} - \theta} + (1 + \omega(\gamma - 1)) \frac{\beta(1-\alpha) \alpha k^{\alpha-1}}{w_1 + \beta(1-\alpha) k + a_1^T}$$

and has the same sign as

$$(\beta \alpha k^{\alpha-1} - \beta\theta)(1 + \omega(\gamma - 1)) k - w_1 - \beta(1-\alpha) k - a_1^T.$$

Since $w_1 \geq 0$ and $\beta \alpha k^{\alpha-1} \leq \phi$ and we are assuming $a_1^T \geq 0$, the last expression is bounded above by

$$(\phi - \beta\theta)\left(1 + \omega\left(\gamma - 1\right)\right)k - \beta\left(1 - \alpha\right)k,$$

which is negative by assumption. $\square$

A.6 Proof of Proposition 5

Some algebra shows that consumption of tradables in a fragile equilibrium satisfies

$$c_0^T = \psi_0\left(w_0 + a_0^T - \beta a_1^T\right),$$

$$c_1^{T,s} = \psi_1\left(w_1 + \beta w_2^s + a_1^T\right),$$

for $s = \{B, G\}$, where

$$\psi_0 = \frac{\omega e^N}{\omega e^N + (1-\omega)\, e_{b,0}^N},$$

$$\psi_1 = \frac{\omega e^N}{(1+\beta)\,\omega e^N + (1-\omega)\left(e_{b,1}^N + \beta e_{b,2}^N\right)}.$$

The Euler equation for consumers

$$\left(c_0^T\right)^{\omega(1-\gamma)-1} = \sum_s \pi^s \left(c_1^{T,s}\right)^{\omega(1-\gamma)-1}$$

implies

$$c_0^T < \mathbb{E}_0\left[c_1^{T,s}\right] < c_1^{T,G},$$

which implies

$$\left(\frac{1}{\psi_0} + \beta\frac{1}{\psi_1}\right)c_0^T < \frac{c_0^T}{\psi_0} + \beta\frac{c_1^{T,G}}{\psi_1} = \frac{w_0 + w_1 + \beta w_2^G + a_0^T}{1 + \beta + \beta^2}. \tag{A.5}$$

In a safe equilibrium, consumption of tradables must be constant $c_0^T = c_1^T = c_2^T = \hat{c}^T$. Therefore, to construct a safe equilibrium, we look for a vector $\hat{c}^T, \hat{p}, \hat{n}_1, \hat{K}_2$ that satisfies the following four equations:

$$\left(\frac{1}{\psi_0} + \beta\frac{1}{\psi_1}\right)\hat{c}^T = w_0 + \beta w_1 + \beta^2\left(1-\alpha\right)\hat{K}_2^\alpha + a_0^T,$$

$$\hat{p} = \frac{1-\omega}{\omega}\frac{\hat{c}^T}{e^N},$$

$$\hat{n}_1 = \frac{1}{\beta}\left[\hat{p}\left(e^N_{b,0} + \beta e^N_{b,1}\right) + \alpha K_0^\alpha - b_0^T\right],$$

$$\hat{K}_2 = \min\left\{\frac{1}{1-\beta\theta}\hat{n}_1, K^*\right\}.$$

The equation for banks' net worth follows from the fact that we constructed fragile equilibria with $K_1 = K^*$, so the rate of return on banks' net worth is $1/\beta$ between periods 0 and 1.

It is possible to use equation (A.5) together with monotonicity and concavity properties of the functions involved to show that the four equations above have a unique solution with

$$\hat{K}_2 \geq K_2^G, \hat{n}_1 \geq n_1^G, \hat{c}^T > c_0^T.$$

We can then construct a safe equilibrium based on the allocation just derived and set the positions in non-tradables to $\hat{a}_1^N = \hat{b}_1^N = e_b^N$. □

B Proofs for Section IV and V

B.1 Microfoundations for limited fiscal capacity

Suppose that firms in the tradable sector can operate in an informal sector that cannot be taxed. Capital and labor can freely move to the informal sector, but labor in the informal sector is less efficient, by a factor $1-\xi$. That is, the production function in the informal sector is

$$\tilde{y} = \tilde{K}^\alpha \left(\left(1-\xi\right)\tilde{L}\right)^{1-\alpha}.$$

The wage in the informal sector is

$$\tilde{w} = (1-\alpha)(1-\xi)\left(\frac{\tilde{K}}{(1-\xi)\tilde{L}}\right)^\alpha.$$

Optimality for capital in the informal sector requires

$$\alpha\left(\frac{\tilde{K}}{(1-\xi)\tilde{L}}\right)^{\alpha-1} = r = \alpha\left(\frac{K}{L}\right)^{\alpha-1},$$

which implies

$$\frac{\tilde{K}}{(1-\xi)\tilde{L}} = \frac{K}{L}.$$

Workers weakly prefer the formal sector if

$$(1-\tau)(1-\alpha)\left(\frac{K}{L}\right)^{\alpha} \geq (1-\alpha)(1-\xi)\left(\frac{\tilde{K}}{(1-\xi)\tilde{L}}\right)^{\alpha} = (1-\xi)(1-\alpha)\left(\frac{K}{L}\right)^{\alpha},$$

that is, if

$$\tau \leq \xi.$$

If $\tau > \xi$, all labor and capital will shift to the informal sector and tax revenues are zero. If $\tau = \xi$, agents are indifferent and we assume that they choose to work in the formal sector. If $\tau < \xi$, there is no activity in the informal sector and taxes are non-distortionary. Thus, this economy is equivalent to our baseline model where labor taxes are non-distortionary but are bounded above by ξ.

B.2 Definition and properties of the mapping f

First, we prove a lemma that fully characterizes the mapping σ_P that gives the equilibrium allocation $\{c_1^T, c_2^T, c_b^T, k_2, K_2\}$ given the government's choice of the vector $(\tau_1, b_{g,2}^T, T_b)$.

Lemma A-1. *Given a pair (p_1, B) suppose the government chooses a vector $(\tau_1, b_{g,2}^T, T_b)$ that satisfies*

$$0 \leq T_b = \tau_1 w_1 + \beta b_{g,2}^T \leq \xi w_1 + \beta B.$$

Then the equilibrium allocation $\sigma_P(\tau_1, b_{g,2}^T, T_b)$ is uniquely determined as follows:

1. *The banks' post-transfer net worth is:*

$$N = \alpha K_1^{\alpha} - b_1^T + p_1(e_{b,1}^N - b_1^N) + T_b;$$

2. *The banks' investment is:*

$$k_2 = \min\left\{\frac{N}{1-\beta\theta_2}, K^*\right\};$$

3. *Total capital is:*

$$K_2 = \max\left\{k_2, \underline{K}\right\};$$

4. *Bankers' consumption is:*

$$c_b^T = \frac{\alpha K_2^{\alpha-1} - \theta_2}{1-\beta\theta_2} N;$$

5. *Consumers' consumption in periods $t = 1, 2$ is:*

$$c^T = \frac{1}{1+\beta}\left[a_1^T - b_1^T + K_1^{\alpha} + \beta K_2^{\alpha} - k_2 - \phi\left(K_2 - k_2\right) - \beta c_b^T\right].$$

Proof. The arguments for points (1) to (4) closely follow analogous arguments derived for the model with no government intervention. To derive point (5) consolidate the intertemporal budget constraints of consumers, bankers and of the government and use market clearing for non-tradable goods, for labor and capital, and for N-denominated bonds, to obtain the intertemporal resource constraint

$$c_1^T + \beta c_2^T + \beta c_b^T \le a_1^T - b_1^T + K_1^\alpha + \beta K_2^\alpha - k_2 - \phi(K_2 - k_2).$$

Given any government policy, the consumers' Euler equation yields $c_1^T = c_2^T = c^T$, which, substituting in last equation, yields the desired result. □

Define $V(N)$ to be the value of the welfare function $(1+\beta)U(c^T, e^N) + \Phi c_b^T$ at the allocation defined by points (2)-(5) in Lemma A-1, which only depends on N. Notice that the government by choosing a feasible vector $(\tau_1, b_{g,2}^T, T_b)$ can reach any N that satisfies $n_1 \le N \le n_1 + \xi w_1 + \beta B$. This proves the following lemma.

Lemma A-2. *The equilibrium allocation of the subgame that begins with the government's choice of* $(\tau_1, b_{g,2}^T, T_b)$ *can be found solving*

$$\max_N \{V(N) \quad s.t. \quad n_1 \le N \le n_1 + \xi w_1 + \beta B\}, \tag{A.6}$$

where $n_1 = \alpha K_1^\alpha - b_1^T + p_1(e_{b,1}^N - b_1^N)$*, and using Lemma A-1.*

We are now ready to define the mapping f as follows. For a given pair (p_1, B), solve the optimization problem (A.6) and compute the allocation $\{c^T, c_b^T, k_2, K_2\}$ following points (2)-(5) in Lemma A-1. Let

$$p_1' = \frac{1-\omega}{\omega}\frac{c^T}{e^N}. \tag{A.7}$$

and

$$B' = \beta\xi(1-\alpha)K_2^\alpha. \tag{A.8}$$

Define

$$f(p_1, B) = (p_1', B').$$

The construction above implies the following proposition.

Proposition A-1. *If* $(p_1, B) = f(p_1, B)$*, then there is a continuation equilibrium of the model with government intervention in which the non-tradable price and the debt limit are* (p_1, B)*. For any continuation equilibrium the pair* (p_1, B) *satisfies* $(p_1, B) = f(p_1, B)$*.*

B.3 Properties of the function V and government best response

First, we want to show that there is a value of N lower than n_1^{Good}, denoted by $\tilde{N}$, such that the economy with government intervention can have no equilibrium with $N \in [\tilde{N}, n_1^{Good})$. To derive the value of $\tilde{N}$ consider the mapping from N to the allocation $\{c^T, c_b^T, k_2, K_2\}$ defined by points (2)-(5) in Lemma A-1 and focus on the relation between N and c^T. First, notice that the relation is continuous. Next, notice that for values of N that satisfy $N \geq (1 - \beta\theta_2)K^*$ the relation is decreasing, because once $K > K^*$ increasing N is a pure transfer from consumers to bankers with no other effect on the allocation. In particular, for $N \geq (1 - \beta\theta_2)K^*$ we have

$$dc^T = -\frac{\beta}{1+\beta} dc_b^T = -\frac{1}{1+\beta} dN.$$

Finally, notice at $N = n_1^{Bad}$ and at $N = n_1^{Good}$, the we obtain, respectively, $c^T = c^{T,Bad}$ and $c^T = c^{T,Good}$, where $c^{T,Good} > c^{T,Bad}$. These observations imply that the relation must be increasing over some range between n_1^{Bad} and n_1^{Good} and that once N crosses $(1 - \beta\theta_2)K^*$ it must be decreasing. Given the properties just derived, there must exists an $\tilde{N} < (1 - \beta\theta_2)K^*$ such that: (i) at $N = \tilde{N}$ the mapping delivers $c^T = c^{T,Good}$ and (ii) $c^T \geq c^{T,Good}$ for all $N \geq \tilde{N}$.

Can there be an equilibrium in which the government chooses $N \in [\tilde{N}, n_1^{Good})$? The answer is no because if such an equilibrium existed it would deliver $c^T \geq c^{T,Good}$ and hence, from (A.7), $p^T \geq p^{T,Good}$. In this case, the banks would have net worth $n_1 \geq n_1^{Good}$ under a zero transfer so N would violate the constraint in problem (A.6), yielding a contradiction.

We have just proved the followig.

Lemma A-3. *There is no equilibrium in which the bank's net worth with government transfer is $N \in [\tilde{N}, n_1^{Good})$.*

The following lemma derives properties of the function V that will be used to characterize the government optimization problem (A.6). These properties are illustrated in Figure A-1. Let $\overline{N}$ denote the highest feasible value of N, which corresponds to the value that delivers $c^T = 0$. Define

$$N^o = (1 - \beta\theta_2)\underline{K}.$$

Lemma A-4. *The function V:*

i. is continuous on $[0, \overline{N})$;

ii. has a unique local maximum at $N = n_1^{Bad}$ on the interval $[0, N^o]$;

iii. is strictly increasing on $[N^o, \tilde{N}]$

iv. has a global maximum at $N = n_1^{Good}$;

Proof. Part (i): Continuity follows immediately from the definition of V.

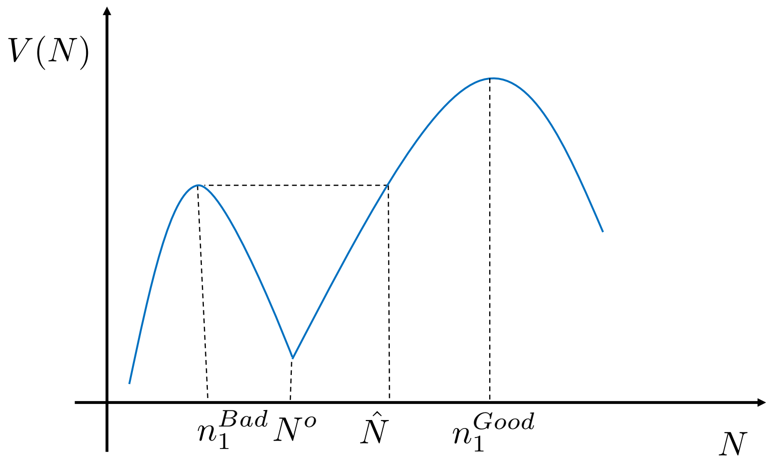

Figure A-1: Properties of the function V

Part (ii): The definition of V implies that the function is strictly concave on the interval $[0, N^o]$. Let us prove that there is a local maximum at $N = n_1^{Bad}$, strict concavity then implies that this local maximum is unique. Suppose we start at the bad equilibrium and the government transfers dN dollars from consumers to banks. Since the collateral constraint is binding, banks increase k_2 by $dk_2 = 1/(1-\beta\theta)dN$ and the effect on bankers' consumption is

$$dc_b^T = \frac{\alpha \underline{K}^{\alpha-1} - \theta_2}{1-\beta\theta_2} dN.$$

Total investment $K_2 = \underline{K}$ is unaffected and the present value of total resources increases by $(\phi - 1)dk_2$ because the production technology of the banks is more efficient than the inferior technology. From the intertemporal budget constraint, the effect on consumers' consumption is then

$$dc^T = \frac{1}{1+\beta}\left[(\phi-1)dk_2 - \beta dc_b^T\right] = \frac{1}{1+\beta}\left[(\phi-1)\frac{1}{1-\beta\theta_2}dN - \beta\frac{\alpha\underline{K}^{\alpha-1}-\theta_2}{1-\beta\theta_2}dN\right].$$

Since $\phi = \beta\alpha\underline{K}^{\alpha-1}$ the last equation becomes

$$dc^T = -\frac{1}{1+\beta}dN.$$

Substituting in the social welfare function and using $\alpha\underline{K}^{\alpha-1} = r_2^{Bad}$ yields

$$V'(N) = -U_{c^T}(c^T, e^N) + \Phi\frac{r_2^{Bad} - \theta_2}{1-\beta\theta_2}.$$

We can then check that $V'(n_1^{Bad}) = 0$ from the equilibrium risk sharing condition (**??**) and our choice of Φ. This shows that, in a neighborhood of the bad continuation equilibrium, the government has no incentive to support the banks.

Part (iii): The function V is strictly concave on $[N^o, N^*]$ because in that interval

$$c_b^T = \alpha K_2^\alpha - \theta K_2,$$

$$c^T = \frac{1}{1+\beta}\left[a_1^T - b_1^T + K_1^\alpha + \beta(1-\alpha)K_2^\alpha - (1-\beta\theta_2)K_2\right],$$

and K_2 is linear in N. A marginal change dN in the interior of $[N^o, N^*]$ has the following effects

$$dc_b^T = \frac{\alpha^2 K_2^{\alpha-1} - \theta_2}{1-\beta\theta_2} dN,$$

$$dc^T = \frac{1}{1+\beta}\left[\frac{\alpha\beta K_2^{\alpha-1} - 1}{1-\beta\theta_2} dN - \beta\frac{\alpha^2 K_2^{\alpha-1} - \theta_2}{1-\beta\theta_2} dN\right],$$

and we obtain

$$V'(N) = U_{c^T}(c^T, e^N)\frac{\alpha\beta K_2^{\alpha-1} - 1}{1-\beta\theta_2} + \left[\Phi - \beta U_{c^T}(c^T, e^N)\right]\frac{\alpha^2 K_2^{\alpha-1} - \theta_2}{1-\beta\theta_2}.$$

The first term captures the gain in productive efficiency associated to increased investment. The second term captures the reallocation from consumers to bankers due to the transfer (net of the endogenous increase in wages). Recall that at $N = \tilde{N}$ we have $c^T = c^{T,Good}$, that the Pareto weight Φ satisfies $\Phi = \beta U_{c^T}(c^{T,Good}, e^N)$, that $\tilde{N} < (1-\beta\theta_2)K^*$ so at $N = \tilde{N}$ we have $\alpha\beta K_2^{\alpha-1} > 1$. Combining these observations, we obtain

$$V'(\tilde{N}) = U_{c^T}(c^T, e^N)\frac{\alpha\beta K_2^{\alpha-1} - 1}{1-\beta\theta_2} > 0. \tag{A.9}$$

The concavity of V implies that $V'(N) > 0$ on the whole interval $[N^o, \tilde{N}]$.

Part (iv): Consider the problem of choosing $c_1^T, c_2^T, c_b^T, K_2, k_2$ to maximize the social welfare function subject only to the intertemporal resource constraint

$$c_1^T + \beta c_2^T + \beta c_b^T \leq a_1^T - b_1^T + K_1^\alpha + \beta K_2^\alpha - k_2 - \phi(K_2 - k_2).$$

It is easy to show that the good equilibrium allocation satisfy the sufficient conditions for optimality of this problem, given that $\Phi = \beta U_{c^T}(c^{T,Good}, e^N)$. This result and the definition of V give the desired result. □

We can now define the cutoff $\hat{N}$. Part (ii) of Lemma A-4 implies that $V(n_1^{Bad}) > V(N^o)$.

Since the good equilibrium Pareto dominates the bad equilibrium, we have $V(n_1^{Good}) > V(n_1^{Bad})$. By continuity of V there must exist an $N \in (N^o, n_1^{Good})$ such that $V(N) = V(n_1^{Bad})$. Such value is unique since V is increasing on $[N^o, n_1^{Good}]$. Let $\hat{N}$ be that unique value. The construction of $\hat{N}$ is illustrated in Figure A-1.

The properties of V and the definition of $\hat{N}$ just given imply that we can characterize the government best response as follows.

Proposition A-2. *(Government best response) If $n_1^{Bad} \in [n_1, n_1 + \xi w_1 + \beta]$ and $n_1 + \xi w_1 + \beta B < \hat{N}$, the solution of problem (A.6) is to set*

$$N = n_1^{Bad}.$$

If $n_1 + \xi w_1 + \beta B > \hat{N}$ two possibilities arise: either $n_1 + \xi w_1 + \beta B \leq \tilde{N}$ and the government best response is

$$N = n_1 + \xi w_1 + \beta B,$$

or $n_1 + \xi w_1 + \beta B > \tilde{N}$ and the government best response is some $N \geq \tilde{N}$.

B.4 Proof of Proposition 6

The case of multiplicity is shown in the discussion of Figure 4, so here we focus on proving uniqueness when (23) holds.

Consider the following algorithm to check if N is part of an equilibrium. Given a candidate value for N, use the characterization in Lemma A-1 to obtain c^T and K_2, and use (A.7) and (A.8) to compute p_1' and B'. This means that for every N we can compute a lower bound

$$\mathcal{N}_L(N) = \alpha K_1^{\alpha} - b_1^T + p_1'(e_{b,1}^N - b_1^N)$$

and an upper bound

$$\mathcal{N}_U(N) = \mathcal{N}_L(N) + \xi w_1 + \beta B'.$$

We then have a necessary and sufficient condition for an equilibrium: N is part of an equilibrium iff

$$N \in \arg\max_n \{V(n) : n \in [\mathcal{N}_L(N), \mathcal{N}_U(N)]\}.$$

If condition (23) holds, we have $\mathcal{N}_U(N) > \hat{N}$ for all $N \geq n_1^{Bad}$. The characterization of the government best response in Proposition A-2 then implies that either the government chooses $N \geq \tilde{N}$ or it chooses $N = n_1 + \xi w_1 + \beta B$. In the first case, we cannot have an equilibrium, due to Lemma A-3. This means that any equilibrium in the interval $[n_1^{Bad}, \tilde{N}]$

must satisfy the fixed point condition

$$N = \mathcal{N}_U(N).$$

Given the characterization in Lemma A-1 it is easy to show that $\mathcal{N}_U(N)$ is concave. Moreover, we have

$$\mathcal{N}_U(n_1^{Bad}) > \hat{N} > n_1^{Bad},$$

and

$$\mathcal{N}_U(\tilde{N}) > n_1^{Good} > \tilde{N},$$

So there can be no N in $.[n_1^{Bad}, \tilde{N}]$ that satisfies $N = \mathcal{N}_U(N)$. Equilibria with $N < n_1^{Bad}$ and $N > n_1^{Good}$ can also be ruled out, so the equilibrium at n_1^{Good} is unique. □

C A Case Study: the Ecuador Financial Crisis of 1999

In this section, we discuss the experience of Ecuador during the second half of the 1990s, a financial crisis that illustrates well the key economic mechanisms discussed in our paper. We first provide a timeline of the main events, and then discuss how our theory helps in understanding important aspects of this crisis. The sources for our account are the detailed analysis of the Ecuadorian case provided in Jacome (2004), Beckerman (2001) and De la Torre, Garcia, and Mascaro (2002).

Timeline: Over the course of 1999, Ecuador experienced one of the deepest crises of its history. The crisis was accompanied by a large devaluation of the sucre, until that point in a crawling peg with the U.S. dollar, widespread withdrawals from banks' deposits that resulted in the bankruptcy of major financial institutions, and a fiscal crisis that culminated in the default on the Ecuadorian government on its external debt.

The Ecuadorian crisis had its roots in the large unhedged foreign currency positions of firms and financial institutions that formed during the mid-1990s, and it was triggered by a sequence of adverse domestic and external shocks that took place over the course of 1998. The costal floods associated with El Nino phenomena destroyed vast agricultural areas impairing banks' assets, while the reduction in the price of crude oil following the East Asian crises of 1997 hurt public finances substantially.[1] These events contributed to a decline in economic activity that impaired banks' assets, and a deterioration of market sentiments.

The initial stage of the crisis can be traced back to the failure of *Solbanco*, a small bank, in April 1998. This event triggered a financial panic which took the form of deposit with-

[1] At the time, revenues from oil exports represented roughly 30% of total government revenues.

drawals for other banks in the system. The response of the *Banco Central de Ecuador* (BCE) was to provide liquidity to financial institutions, with total emergency loans reaching close to 30% of the money base by end-September 1998. As a result of this monetary expansion, the BCE widened the bands of the crawling peg and was subsequently forced to abandon it completely in February 1999 as speculative pressures on the sucre kept mounting. The rapid depreciation of the sucre that followed hit the banks' unhedged foreign currency debtors hard, eroding banks' equity, and further impairing their solvency. In March 1999, the Ecuadorian government declared a week-long bank holiday, and later implemented a freeze on most of the domestic and off-shore bank deposits in order to prevent further runs on its financial institutions. As the government gradually unfroze the deposits, more withdrawals from the private sector occurred. By the end of 1999, major Ecuadorian banks were bankrupt, the government was in default on its external debt, and the economy was experiencing a de-facto dollarization as inflation spiraled out of control due to the monetization of the financial sector losses. Eventually, Ecuador adopted the dollar as its legal tender in January 2000.

The process of dollarization: Consistent with the continuation equilibrium of our model, a key aspect of the Ecuadorian crisis was the negative effects that the exchange rate depreciations had on the balance sheets of banks. These spillovers were due to the fact that loans that banks extended to domestic firms in the pre-crisis period were denominated in U.S. dollars: as the sucre depreciated, a large fraction of these dollar-denominated loans went into arrears, impairing banks' assets (Beckerman, 2001).

An important question is whether our framework can help explain the process of liability dollarization that took place in Ecuador during the mid-1990s. In what follows, we document two facts: i) the process of liability dollarization of private sector loans coincided with the process of deposit dollarization; ii) after correcting for expected depreciation, interest rates on dollar-denominated loans were substantially below those on sucre-denominated loans during the second half of the 1990s.

Starting with the first fact, the left panel of Figure A-2 plots the percentage of deposits denominated in U.S. dollars while the right panel plots the percentage of dollar-denominated loans held by Ecuadorian banks. Financial dollarization was already present in the early 1990s, but accelerated during the second half of the 1990s: by 1999, the year of the crisis, more than half of domestic deposits and roughly 2/3 of private sector loans were denominated in U.S. dollars. These dynamics did not exclusively reflect valuation effects due to the progressive depreciation of the sucre after 1995, see the comparison between the solid and dotted line in the figures.[2] In addition, they understate the upward trend of financial dollar-

[2] Let $d_t^{\$}$ be dollar denominated deposits and d_t the sucre denominated one. Let s_t be the exchange rate

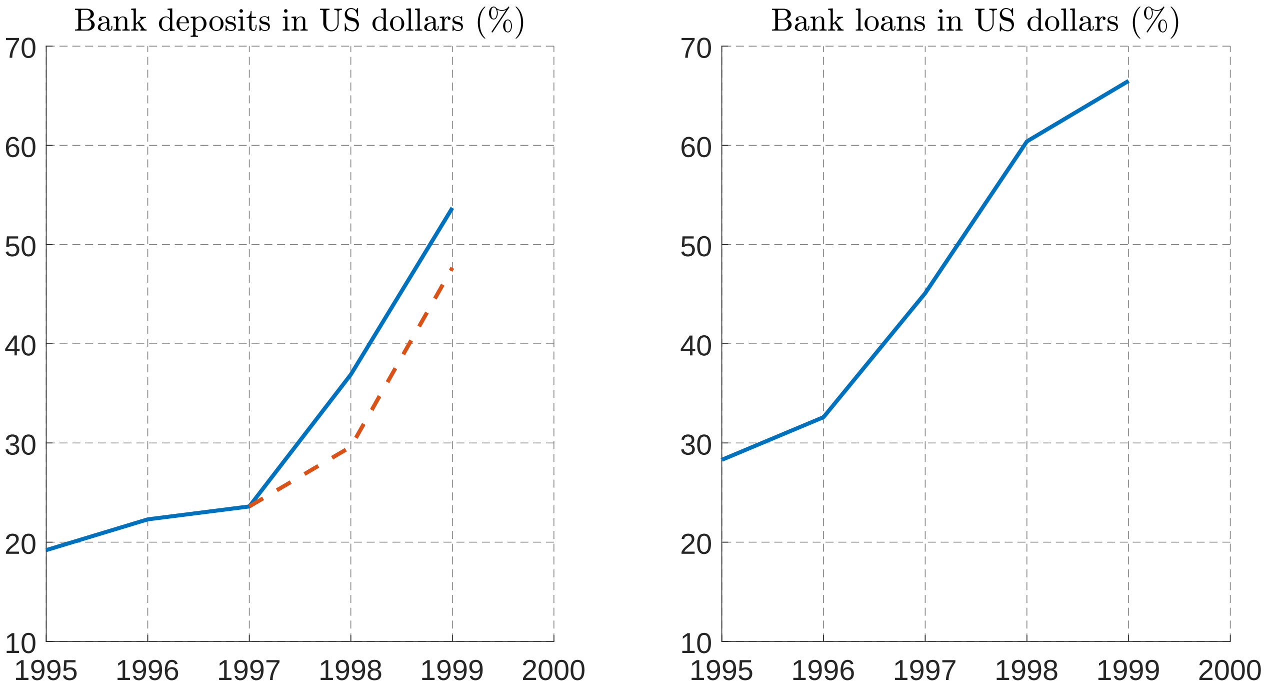

Figure A-2: US dollars bank deposits and bank loans in Ecuador: 1995-1999

Notes: *The solid line in the left panel reports the percentage of onshore bank deposits denominated in US dollars over total onshore bank deposits. The data source is Levy-Yeyati (2006). The dotted line reports the "counterfactual" described in footnote 2. The solid line in the right panel reports the percentage of dollar denominated bank loans over total bank loans. The data source is Beckerman (2001), Table 2.*

ization for the Ecuadorian economy because the data excludes the balance sheet positions of off-shore Ecuadorian banks.[3]

The first two columns of Table A-1 report the average interest rates on short-term loans to Ecuadorian firms in sucre (i_t^{sucre}) and US dollars ($i_t^{\$}$). The third column reports the ex post deviations from UIP, defined as

$$\text{uip}_t^{\text{ex post}} = i_t^{\text{sucre}} - i_t^{\$} - \log(s_{t+1}/s_t) \times 100, \tag{A.10}$$

with s_t being the nominal exchange rate (sucre per US dollar) at the beginning of time t. The fourth column reports the expression above with the exception that $\log(s_{t+1}/s_t)$ is replaced by $\mathbb{E}_t[\log(s_{t+1}/s_t)]$, computed by fitting an AR(1) with a time trend on the exchange rate series. We can see that interest rates in sucre were on average 41 percentage points higher than those in US dollars during the 1995-1999 period, while the average depreciation of the

(sucre per US dollar). The dotted line in the left panel of Figure A-2 reports $d_{1997}^{\$}/(d_{1997}^{\$} + s_t d_{1997})$. That is, we fix the positions at their 1997 level, and construct the "counterfactual" dollarization index that results only through changes in the exchange rate.

[3]Deposits in off-shore entities linked to Ecuadorian banks were dollar denominated, and they grew substantially during the second half of the 1990s. In 1999, they totaled 2.9b US dollar, while on-shore deposits totaled 3.5b US dollars. See Beckerman (2001).

Table A-1: Lending rates in sucre and US dollar and deviations from UIP: 1995-1999

	i_t^{sucre}	$i_t^{\$}$	Ex post UIP	Ex ante UIP
1995	57.76	13.29	28.99	18.70
1996	46.28	16.23	8.24	4.28
1997	43.58	10.59	10.41	7.25
1998	59.79	16.15	12.72	17.87
1999	70.84	13.69	-20.00	31.38
Average	55.65	13.98	8.01	15.90

Notes: i_t^{sucre} are the average annualized lending rates to non-financial corporations for loans denominated in sucre with a maturity of up to 1 year, while $i_t^{\$}$ are the corresponding lending rate for loans denominated in US dollars. The data are from Banco Central de Ecuador. The third column reports the expression in equation (A.10), where s_t is obtained from the International Financial Statistics of the IMF. The fourth column substitutes $\log(s_{t+1}/s_t)$ in equation (A.10) with $\mathbb{E}_t[\log(s_{t+1}/s_t)]$, obtained after fitting an AR(1) with a deterministic time trend to s_t.

sucre over this period was of the order of 33% per year. This implies an average ex post UIP deviations of 8%. These observations suggest that borrowing in dollars was effectively cheaper than borrowing in sucre over this period.

Both observations are consistent with the key mechanism in our model: precautionary demand for US dollars by domestic savers makes dollar borrowing cheap and incentivizes domestic borrowers to take up loans in foreign currency. Thus, one way of interpreting the process of financial dollarization in Ecuador through the lens of our theory would be to think about a shift from the safe to the fragile equilibrium. While we do not have a theory of why such a switch might occur, it is interesting to note that this process took place after the Mexican crisis of 1995, in response to which the BCE widened substantially the crawling bands on the peg with the US dollar.

D An Extension: Limited Participation of Foreign Investors

In this section we modify the benchmark model to allow foreign investors to participate in the market for claims denominated in non-tradable goods. We will show that this version of the model can still feature fragile equilibria.

The main ingredients of the model are those described in Section I, with the exception of the decision problem of foreign investors. As in the paper, we assume that there are risk-neutral foreign intermediaries with discount factor β who only take positions in bonds denominated in tradable goods. In addition, there are now foreign investors that are specialized in bonds denominated in non-tradable goods. These specialists have an initial endowment, $\hat{w}_0^{T*}$, and we denote by $\hat{a}_1^{T*}$ and $\hat{a}_1^{N*}$ their financial position chosen at date 0. Thus, their date 0 budget constraint is

$$\hat{c}_0^{T*} + q_0^T \hat{a}_1^{T*} + q_0^N p_0 \hat{a}_1^{N*} \leq \hat{w}_0^{T*}, \tag{A.11}$$

where $\hat{c}_0^{T*}$ is their consumption.

From date $t = 1$ onward, the interest rate for bonds denominated in tradable goods is still determined by the risk-neutral foreign investors and it equals $1/\beta$. This will also be the interest rate for bonds denominated in non-tradable goods because there is no more exchange rate risk from date $t = 1$ onward. Therefore, the financial positions that foreign specialists choose at date $t = 1$ do not affect equilibrium prices and quantities in the continuation equilibrium. Without loss of generality, we assume that the specialists liquidate their positions at $t = 1$ and consume tradable goods,

$$\hat{c}_1^{T*} \leq \hat{a}_1^{T*} + p_1 \hat{a}_1^{N*}. \tag{A.12}$$

The objective of the specialists is to choose $\{\hat{a}_1^{T*}, \hat{a}_1^{N*}\}$ to maximize

$$\mathbf{E}_0 \left[\sum_{t=0}^{1} \beta^t \frac{(\hat{c}_t^{T*})^{1-\sigma}}{1-\sigma} \right],$$

subject to (A.11) and (A.11).

An equilibrium of the economy with specialists is defined as in Section II.B, with the exception that the market clearing conditions for bonds at $t = 0$ are now

$$a_1^T + a_1^{T*} + \hat{a}_1^{T*} = b_1^T \qquad a_1^N + \hat{a}_1^{N*} = b_1^N.$$

Differently from the baseline model, bankers can issue claims denominated in non-tradable goods not only to domestic consumers, but also to foreign specialists.

D.1 Fragile equilibria

Continuation equilibria in this model can be analyzed following the same steps of Section II. Specifically, and given $(a_1^T, a_1^N, b_1^T, b_1^N, \hat{a}_1^{T*}, \hat{a}_1^{N*}, K_1)$, a continuation equilibrium is fully characterized by a pair of $(\tilde{K}_2, \tilde{p})$ such that $\tilde{p} = \mathcal{P}(\tilde{K}_2)$ and $\tilde{K}_2 = \mathcal{K}_2(\tilde{p})$, where $\mathcal{P}(.)$ and $\mathcal{K}(.)$ are defined in equation (8) and equation (9) in the paper. The only difference with the analysis of Section II is that now b_1^N is decoupled from a_1^N. For a given a_1^N, this implies that the conditions for equilibrium multiplicity are more stringent than in the benchmark model, as long as $\hat{a}_1^{N*} > 0$.

The analysis of the agents' portfolio choices at $t = 0$ is also close to the one in Section III. Take a vector $(a_1^T, a_1^N, b_1^T, b_1^N, \hat{a}_1^{T*}, \hat{a}_1^{N*}, K_1)$ such that there are multiple continuation equilibria. Because financial markets are complete, date $t = 0$ optimality implies that the portfolio choices must satisfy the risk-sharing condition

$$\left(\frac{c_1^{T,B}}{c_1^{T,G}}\right)^{\omega(1-\gamma)-1} = \frac{r_2^B - \theta}{r_2^G - \theta} = \left(\frac{\hat{a}_1^T + p^B \hat{a}_1^N}{\hat{a}_1^{T*} + p^G \hat{a}_1^{N*}}\right)^{-\sigma}. \tag{A.13}$$

That is, in any equilibrium we must have that the marginal rate of substitutions of consumers, entrepreneurs and foreign specialists are equalized.

Importantly, because $p^B < p^G$, the consumption of foreign specialists is lower in the bad continuation equilibrium than in the good one when $\hat{a}_1^{N*} > 0$. Thus, if $\sigma > 0$, their marginal utility will be higher in the bad continuation equilibrium. This feature makes it possible to construct examples of fragile equilibria, following the same logic of Section III.[4]

Proposition A-3. *Fix all the model parameters except γ, σ and the initial asset positions at $t = 0$. Take a vector of date 1 initial positions $\{a_1^T, a_1^N, b_1^T, b_1^N, \hat{a}_1^{T*}, \hat{a}_1^{N*}, K_1\}$, with*

$$a_1^N = 0, \quad \hat{a}_1^{N*} = b_1^N > 0 \quad K_1 = K^*, \quad b_1^T \leq K_1.$$

Suppose that, given these positions, there are two continuation equilibria that satisfy

$$\left(\frac{w_1 + \beta w_2^B + a_1^T}{w_1 + \beta w_2^G + a_1^T}\right)^{\omega-1} < \frac{r_2^B - \theta_2}{r_2^G - \theta_2}. \tag{A.14}$$

Then there exist a coefficient of relative risk aversion for the consumers, γ, a coefficient of relative risk aversion for the foreign specialists, σ, and date 0 initial positions $\{a_0^T, a_0^N, b_0^T, b_0^N, \hat{w}_0^{T*}, K_0\}$ that generate a fragile equilibrium in which the two continuation equilibria above are realized with positive probability.

Proof. The proof is by construction, and it follows the same steps of the proof of Proposition 3. Fix the wages, rental rates and exchange rates that prevail in the good and the bad equilibria under $\{a_1^T, a_1^N, b_1^T, b_1^N, \hat{a}_1^T, \hat{a}_1^{N*}, K_1\}$. If condition (A.14) is satisfied, there exists a $\gamma > 0$ such that

$$\left(\frac{w_1 + \beta w_2^B + a_1^T}{w_1 + \beta w_2^G + a_1^T}\right)^{\omega(1-\gamma)-1} = \frac{r_2^B - \theta_2}{r_2^G - \theta_2},$$

[4] As in the benchmark model discussed in the paper, an economy that admits a fragile equilibrium also admits a safe one. We omit the proof of this result because the steps mirror closely those of Proposition 5.

a condition that guarantees that the asset positions are consistent with consumers' and banks' optimality.

Consider now the foreign specialists. Because $p^B < p^G$ and $\hat{a}_1^{N*} > 0$, we have that $\hat{a}_1^{T*} + p^B \hat{a}_1^{N*} < \hat{a}_1^{T*} + p^G \hat{a}_1^{N*}$. Thus, there exists a $\sigma > 0$ such that

$$\left(\frac{\hat{a}_1^T + p^B \hat{a}_1^N}{\hat{a}_1^{T*} + p^G \hat{a}_1^{N*}} \right)^{-\sigma} = \frac{r_2^B - \theta}{r_2^G - \theta}.$$

This implies that the asset positions of foreign specialists are also consistent with optimality.

We can then verify that there exists initial conditions $\{a_0^T, a_0^N, b_0^T, b_0^N, \hat{w}_0^{T*}, K_0\}$ such that the budget constraints of the consumers, banks and foreign specialists are satisfied. As we have some degrees of freedom, we set $a_0^N = b_0^N = 0$.

Pick any probabilities π^G, π^B and set c_0^T to satisfy the consumers' Euler equation for T-denominated bonds

$$\left(c_0^T\right)^{\omega(1-\gamma)-1} = \sum_{s=G,B} \pi^s \left(c_1^{T,s}\right)^{\omega(1-\gamma)-1}.$$

We can then choose a_0^T to satisfy the consumers' budget constraint at $t = 0$:

$$c_0^T + p_0 e^N + \beta a_1^T = p_0 e_{c,0}^N + w_0 + a_0^T$$

where

$$p_0 = \frac{1-\omega}{\omega} \frac{c_0^T}{e^N}$$

guarantees that the market for non-tradable goods clears. So, consumers are optimizing, and their budget constraint is satisfied. Note that from the consumers' Euler equation for NT-denominated bonds we can obtain q_0^N,

$$q_0^N = \frac{1}{p_0 \left(c_0^T\right)^{\omega(1-\gamma)-1}} \beta \sum_{s=G,B} \pi^s \left(c_1^T\right)^{\omega(1-\gamma)-1} p^s.$$

Similarly, we can obtain the date $t = 0$ consumption of the specialists from their Euler equation for tradable bonds,

$$\left(\hat{c}_0^{T*}\right)^{-\sigma} = \sum_{s=G,B} \pi^s \left(\hat{c}_1^{T*,s}\right)^{-\sigma},$$

and set $\hat{w}_0^{T*}$ such that their time $t = 0$ budget constraint is satisfied, given $\{\hat{c}_0^T, \hat{a}_1^T, \hat{a}_1^{N*}\}$

$$\hat{w}_0^{T*} = \hat{c}_0^{T*} + \beta \hat{a}_1^{T*} + p_0 q_0^N \hat{a}_1^{N*}.$$

Therefore, foreign specialists are optimizing and their budget constraint is satisfied.

Finally, we can follow the same steps of the proof of Proposition **??** and solve for the initial conditions guaranteeing that the banks' budget constraint is satisfied at $\{b_1^T, b_1^N, K_1\}$. $\square$

D.2 Remarks

Proposition A-3 shows that the stark form of segmentation assumed in the paper is not necessary for the existence of fragile equilibria, as we can obtain them in environments where foreigners can lend to domestic banks in non-tradables. The main intuition of our construction is that, as it was the case for domestic consumers, also foreign specialists have a higher marginal utility in the bad continuation equilibrium when $\hat{a}_1^{N*} > 0$. Thus, they are willing to hold these assets only at a premium. If this premium is large enough, it can be optimal for the domestic banks to choose a mismatched balance sheet that exposes the economy to the possibility of financial crises.

This argument does require, however, some form of segmentation. To see why, suppose that foreign specialists did not consume at $t = 0$. Using their $t = 0$ budget constraint, we can express the ratio of their marginal utilities across the two equilibria as

$$\left[\frac{1 + \frac{\hat{a}_1^{N*}}{\hat{w}_0^{T*}}\left(p_1^B - p_0 q_0^N\right)}{1 + \frac{\hat{a}_1^{N*}}{\hat{w}_0^{T*}}\left(p_1^G - p_0 q_0^N\right)}\right]^{-\sigma}.$$

From Section II we know that $\hat{a}_1^{N*} = e_{b,1}^N$ would guarantee a unique continuation equilibrium. Thus, a sufficient condition that would eliminate fragile equilibria is that the wealth of foreign specialists is large compared to the endowment of the small open economy,

$$\frac{e^N}{\hat{w}_0^{T*}} \to 0.$$

In this case, the foreign specialists would price bonds denominated in non-tradables *as if* they were risk-neutral, and the domestic banks would have no incentives to choose a risky balance sheet ex ante. Hence, a necessary conditions for fragile equilibria in this model is that the foreign specialists are not too "large" compared to the small open economy, an implicit form of market segmentation.

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