Long Run Growth of Financial Data Technology

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Online Appendix

B Proofs

We start by proving a few preliminary lemmas that are useful in proving the main results. Throughout this appendix, as we will often treat the signal-to-noise ratio in prices as a single variable, we define

$$\xi_t \equiv \frac{C_t}{D_t} \tag{63}$$

Lemma 1 *If $\Omega_{ft} > 0$, then $C_t > 0$.*

Proof. Using equation (58), it suffices to show that $1/(r-G) > 0$ and $(1-\tau_0\hat{V}_t) > 0$. From the setup, we assumed that $r > 1$ and $G < 1$. By transitivity, $r > G$ and $r - G > 0$. For the second term, we need to prove equivalently that $\tau_0\hat{V}_t < 1$ and thus that $\tau_0 < \hat{V}_t^{-1}$. Recall from (35) that $\hat{V}^{-1} = \tau_0 + \Omega_{ft} + \Omega_{pt}$. Since Ω_{ft} and Ω_{pt} are defined as precisions, they must be non-negative. Furthermore, we supposed that $\Omega_{ft} > 0$. Thus, $\tau_0 < \hat{V}_t^{-1}$, which completes the proof. ■

Lemma 2 *$D_t \leq 0$.*

Proof. Start from equation (60) and substitute in (35). Moreover, let $\alpha \equiv \frac{\rho r}{r-G}$. Simplify to get:

$$\xi_t^3(Z_t\tau_x + Z_t\Omega_{xt}) + \xi_t^2(\Omega_{xt}) + \xi_t(\alpha + Z_t\tau_0 + Z_t\Omega_{ft}) + \Omega_{ft} = 0 \tag{64}$$

Then, use the budget constraint to express the first-order conditions as (16). One can solve for both Ω_{xt} and Ω_{ft} in terms of ξ_t:

$$\Omega_f = \Big(\frac{K_t}{\big(1+\frac{1}{\chi_x}\xi_t^4\big)}\Big)^{\frac{1}{2}} \tag{65}$$

$$\Omega_x = \Big(\frac{K_t}{\chi_x}\big(1-\frac{1}{1+\frac{1}{\chi_x}\xi_t^4}\big)\Big)^{\frac{1}{2}} = \Big(\frac{K_t\frac{1}{\chi_x}}{\chi_x\big(1+\frac{1}{\chi_x}\xi_t^4\big)}\Big)^{\frac{1}{2}}\xi_t^2 = \frac{\xi_t^2}{\chi_x}\Big(\frac{K_t}{\big(1+\frac{1}{\chi_x}\xi_t^4\big)}\Big)^{\frac{1}{2}} \tag{66}$$

Substituting these into equation (64) fully determines ξ_t in terms of exogenous variables.

$$\xi_t\Big(\xi_t^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + \xi_t^2\Omega_{xt}(1+\xi_t Z_t) + \Omega_{ft}(1+\xi_t Z_t) = 0 \tag{67}$$

First note that

$$\Omega_{ft} + \xi_t^2\Omega_{xt} = -\frac{\xi_t(\xi_t^2 Z_t\tau_x + \alpha + Z_t\tau_0)}{(1+\xi_t Z_t)} \tag{68}$$

where the left hand side is the objective function. Therefore, we know the maximized value of the objective function solely as a function of $\xi_t = \frac{C}{D}$. Keep in mind that since we already imposed an optimality condition, this latter equation holds only at the optimum.

Substituting in for Ω_{ft} and Ω_{xt} from (65) and (66) yields an equation that implicitly defines ξ_t as a function of primitives, K_t and future equilibrium objects, embedded in Z_t.

$$\xi_t\Big(\xi_t^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + (1+\xi_t Z_t)(1+\frac{1}{\chi_x}\xi_t^4)\Big(\frac{K_t}{(1+\frac{1}{\chi_x}\xi_t^4)}\Big)^{\frac{1}{2}} = 0$$

$$\xi_t^3 Z_t\tau_x + \xi_t(\alpha + Z_t\tau_0) + (1+\xi_t Z_t)(K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} = 0 \tag{69}$$

The left hand side must equal zero for the economy to be in equilibrium. However, all the coefficients $K_t, , \chi_x, \tau_0$, and τ_x are assumed to be positive. Furthermore, Z_t is a variance. Inspection of (37) reveals that it must be strictly positive. Thus, the only way that the equilibrium condition can possibly be equal to zero is if $\xi_t < 0$. Recall that $\xi_t = C_t/D_t$. The previous lemma proved that $C_t > 0$. Therefore, it must be that $D_t < 0$. ∎

The next lemma proves the following: If no one has information about future dividends, then no one's trade is based on information about such dividends, and thus the price cannot contain information about them. Since C_t is the price coefficient on future dividend information, $C_t = 0$ means that the price is uninformative. In short, the price cannot reflect information that no one knows.

Lemma 3 ***When information is scarce, price is uninformative****: As $K_t \to 0$, for any future path of prices (A_{t+j}, B_{t+j}, C_{t+j}, and D_{t+1}, $\forall j > 0$), the unique solution for price coefficient C_t is $C_t = 0$.*

Proof. *Step 1: As $\Omega_{ft} \to 0$, prove $C_t = 0$ is always a solution.*

Start with the equation for D_t (12). Substitute in for Ω using (38) and $1 + B = r/(r - G)$ and rewrite it as

$$D_t = \frac{1}{r-G}\hat{V}_t\left[\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t\hat{V}_t^{-1}\right] \tag{70}$$

Then, express C_t from (58) as $C_t = 1/(r-G)\hat{V}_t(\hat{V}_t^{-1} - \tau_0)$ and divide C_t by D_t, cancelling the $\hat{V}_t/(r-G)$ term in each to get

$$\frac{C_t}{D_t} = \frac{\hat{V}_t^{-1} - \tau_0}{\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t\hat{V}_t^{-1}} \tag{71}$$

If we substitute in $\hat{V}_t^{-1} = \tau_0 + \Omega_{pt} + \Omega_{ft}$ from (35) and then set $\Omega_{ft} = 0$, we get

$$\frac{C_t}{D_t} = \frac{\Omega_{pt}}{\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \Omega_{pt})} \tag{72}$$

Then, we use the solution for price information precision $\Omega_{pt} = (C/D)^2(\tau_x + \Omega_{xt})$ and multiply both sides by the denominator of the fraction to get

$$\frac{C_t}{D_t}\left[\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_{xt}))\right] = \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_{xt}) \tag{73}$$

We can see right away that since both sides are multiplied by C/D, as $\Omega_{ft} \to 0$, for any given future price coefficients C_{t+1} and D_{t+1}, $C = 0$ is always a solution.

Step 2: Prove uniqueness.

Next, we investigate what other solutions are possible by dividing both sides by C/D:

$$\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_{xt})) - \left(\frac{C_t}{D_t}\right)(\tau_x + \Omega_{xt}) = 0 \tag{74}$$

This is a quadratic equation in C/D. Using the quadratic formula, we find

$$\frac{C_t}{D_t} = \frac{\Omega_{xt} \pm \sqrt{\Omega_{xt}^2 - 4Z_t(\tau_x + \Omega_{xt})(\rho r/(r-G) + \tau_0 Z_t)}}{-2Z_t(\tau_x + \Omega_{xt})} \tag{75}$$

If we now take the limit as $\Omega_{xt} \to 0$, the term inside the square root becomes negative, as long as $r - G > 0$. Thus, there are no additional real roots when $\Omega_{xt} = 0$.

Similarly, if Ω_{xt} is not sufficiently large, (75) has no real roots, which proves that: as $\Omega_{ft} \to 0$, if we take C_{t+1} and D_{t+1} as given and Ω_{xt} is sufficiently small, then the unique solution for price coefficient C is $C = 0$. ■

Proof of Result 1. From lemma 3, we know that as $C_t = 0$. From the first-order condition for information (16), we see that the marginal utility of demand information relative to fundamental information (the marginal rate of substitution) is a positive constant times $(C_t/D_t)^2$. If $C_t = 0$, then $\partial U_{it}/\partial \Omega_{xit}$ is a positive constant times zero, which is zero. ■

Proof of Result 2.

(2a)

Part 1: $\frac{dC/|D|}{dK} > 0$. In the model where $\pi = 0$, a simpler set of equations characterize a solution. In this environment, we can show exactly how changes in parameters affect information choices and price coefficients. These static forces are also at play in the dynamic model. But there are additional dynamic forces that govern the model's long-run behavior.

Let $\xi = \frac{C}{D}$. With $\pi = 0$,

$$C = \frac{1}{r}(1 - \tau_0 \hat{V})$$
$$D = \frac{1}{r}(\tau_x \frac{C}{D} - \rho)\hat{V}$$

Divide and rearrange to get

$$\tau_x \xi^2 - \rho\xi = \hat{V}^{-1} - \tau_0$$

Substitute for $\hat{V}^{-1} = \tau_0 + \Omega_f + \xi^2(\tau_x + \Omega_x)$ and cancel terms on both sides. The following equations characterize the equilibrium of the static ($\pi = 0$) model:

$$\xi^2\Omega_x + \xi\rho + \Omega_f = 0 \tag{76}$$

which has two solutions

$$\xi = \frac{-\rho \pm \sqrt{\rho^2 - 4\Omega_f\Omega_x}}{2\Omega_x}. \tag{77}$$

We pick the larger solution (with +), because, when there is no demand information (for instance $\chi_x \to \infty$), the solution converges to the unique solution in the models where there is only fundamental information acquisition, $-\frac{\Omega_f}{\rho}$.

Thus,

$$\xi = \frac{-\rho + \sqrt{\rho^2 - 4\Omega_f\Omega_x}}{2\Omega_x} \tag{78}$$

Now there are two extra equations to complete the model, budget constraint and investor FOC

$$\Omega_f^2 + \chi_x \Omega_x^2 = K$$
$$\frac{\Omega_x}{\Omega_f} = \frac{1}{\chi_x}\xi^2$$

which, using equation (78), implies

$$\Omega_f = \sqrt{\frac{K}{(1+\frac{\xi^4}{\chi_x})}} \tag{79}$$

$$\Omega_x = \frac{\xi^2}{\chi_x}\sqrt{\frac{K}{(1+\frac{\xi^4}{\chi_x})}} \tag{80}$$

Put this back into equation (78) to get the signal-to-noise ratio

$$\xi = -\frac{1}{\sqrt{2}}\sqrt{\frac{\rho^2\chi_x}{K} \pm \frac{\sqrt{-\chi_x\left(4K^2-\rho^4\chi_x\right)}}{K}}$$

Again, we pick the solution that is consistent with the limit $\chi_x \to \infty$, $\xi = -\frac{\sqrt{K}}{\rho}$

$$\xi = -\frac{1}{\sqrt{2}}\sqrt{\frac{\rho^2\chi_x}{K} - \frac{\sqrt{-\chi_x\left(4K^2-\rho^4\chi_x\right)}}{K}} = -\rho\sqrt{\frac{\chi_x}{2K}}\sqrt{1-\sqrt{1-4K^2/(\rho^4\chi_x)}}$$

which implies

$$\frac{d(\frac{C}{D})}{dK} = \frac{d\xi}{dK} = -\frac{\rho\sqrt{\chi_x}}{(2K)^{3/2}}\frac{\sqrt{1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}}}{\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}} < 0 \tag{81}$$

which means $\frac{C}{|D|}$ is increasing, i.e, the signal-to-noise ratio improves as more information becomes available.

Part 2: $\frac{\partial C/|D|}{\partial \Omega_f}$ *and* $\frac{\partial C/|D|}{\partial \Omega_x}$. Let $\xi = \frac{C}{D}$ denote the equilibrium signal-to-noise ratio associated with total information capacity K_t, and for brevity suppress subscript t. We have

$$\begin{aligned}\frac{d\xi}{dK} &= \frac{\partial\xi}{\partial\Omega_f}\left(\frac{d\Omega_f}{dK} + \frac{\partial\Omega_f}{\partial\xi}\frac{d\xi}{dK}\right) + \frac{\partial\xi}{\partial\Omega_x}\left(\frac{d\Omega_x}{dK} + \frac{\partial\Omega_x}{\partial\xi}\frac{d\xi}{dK}\right) \\ &= \frac{\partial\xi}{\partial\Omega_f}\frac{d\Omega_f}{dK} + \frac{\partial\xi}{\partial\Omega_x}\frac{d\Omega_x}{dK} + \left(\frac{\partial\xi}{\partial\Omega_f}\frac{\partial\Omega_f}{\partial\xi} + \frac{\partial\xi}{\partial\Omega_x}\frac{\partial\Omega_x}{\partial\xi}\right)\frac{d\xi}{dK}\end{aligned}$$

The first term is the direct effect of the change in K on ξ through the change in fundamental analysis, the second term is the direct effect through the change in demand analysis, and the third term (in parentheses) is the indirect effect. We have

$$\frac{\partial\xi}{\partial\Omega_f}\frac{d\Omega_f}{dK} = -\frac{\Omega_f}{2K(2\xi\Omega_x+\rho)}$$
$$\frac{\partial\xi}{\partial\Omega_x}\frac{d\Omega_x}{dK} = \frac{\xi\rho+\Omega_f}{2K(2\xi\Omega_x+\rho)}$$
$$\left(\frac{\partial\xi}{\partial\Omega_f}\frac{\partial\Omega_f}{\partial\xi} + \frac{\partial\xi}{\partial\Omega_x}\frac{\partial\Omega_x}{\partial\xi}\right)\frac{d\xi}{dK} = \frac{\Omega_f\left(\xi^4+\chi_x\right)+\xi\rho\chi_x}{\frac{K\Omega_x\left(\xi^4+\chi_x\right)(2\xi\Omega_x+\rho)\left(\rho^2\chi_x-2K\xi^2\right)}{\rho^2\chi_x}}$$

Note that

$$\Omega_f\left(\xi^4+\chi_x\right)+\xi\rho\chi_x=\xi^4\Omega_f+\chi_x(\xi\rho+\Omega_f)=\xi^4\Omega_f-\xi^2\chi_x\Omega_x=\xi^2\left(\xi^2\Omega_f-\Omega_x\right)=0$$

that is, the indirect effect is zero, consistent with what the envelope theorem implies. Thus we have the following decomposition

$$\frac{d\xi}{dK}=\frac{\partial\xi}{\partial\Omega_f}\frac{d\Omega_f}{dK}+\frac{\partial\xi}{\partial\Omega_x}\frac{d\Omega_x}{dK}=\frac{-\Omega_f}{2K(2\xi\Omega_x+\rho)}+\frac{\xi\rho+\Omega_f}{2K(2\xi\Omega_x+\rho)}.$$

From equation (76), $\xi\rho+\Omega_f<0$, thus both effects have the same sign. Moreover, we have already proven in result 2 that $\frac{d\xi}{dK}<0$, which in turn implies that both effects must be negative and $2\xi\Omega_{xt}+\rho>0$. Thus the increase in either type of information acquisition, following an increase in capacity, improves the signal-to-noise ratio (i.e, $\frac{C}{D}$ increases in absolute value).

(2b)

Recall that with $\pi=0$

$$C=\frac{1}{r}\left(1-\frac{\tau_0}{\tau_0+\Omega_f+\xi^2(\tau_x+\Omega_x)}\right)=\frac{1}{r}\left(1-\tau_0\hat{V}\right)$$

Thus to prove $\frac{dC}{dK}>0$, it is sufficient to show that $\frac{d\hat{V}}{dK}<0$. Using the first-order condition, along with definition of $\hat{V}$ and that $\frac{d\xi^2}{dK}>0$, we have that a sufficient condition for $\frac{d\hat{V}}{dK}<0$ is $d\sqrt{\left(1+\frac{\xi^4}{\chi_x}\right)K}/dK>0$, which is true. Thus $\frac{dC}{dK}>0$.

(2c)

Recall that with $\pi=0$,

$$D=\frac{1}{r}\frac{\tau_x\xi-\rho}{\tau_0+\Omega_f+\xi^2(\tau_x+\Omega_x)}=\frac{1}{r}(\tau_x\xi-\rho)\hat{V}.$$

Thus,

$$\frac{dD}{dK}=\frac{1}{r}\left[\tau_x\hat{V}\frac{d\xi}{dK}+(\tau_x\xi-\rho)\frac{d\hat{V}}{dK}\right].$$

The derivative is the sum of two terms. The first term is negative since $\frac{d\xi}{dK}<0$, while the second term is positive since $\frac{d\hat{V}}{dK}<0$, as argued in part (2b), and $(\tau_x\xi-\rho)<0$. So we have to determine which one is larger.

To do so, substitute the closed form solutions into the above expression, and solve for $\bar{K}_D=\min\{\bar{K},K_D\}$ such that

$$\frac{1}{r}\left[\tau_x\hat{V}\frac{d\xi}{dK}+(\tau_x\xi-\rho)\frac{d\hat{V}}{dK}\right]_{K=K_D}=0. \tag{82}$$

The algebra is cumbersome, but it is straightforward to show that $0<\bar{K}_D<\bar{K}$ is unique, and that $\frac{dD}{dK}<0$ if and only if $K<\bar{K}_D$. To observe the latter point, note that when $K\to 0$, $\xi\to 0$, thus

$$\frac{dD}{dK}\Leftrightarrow\frac{d\xi}{dK}+\frac{\rho}{2\tau_0\sqrt{K}}<0.$$

Substitute for $\frac{d\xi}{dK}$ from equation (81) and use L'Hopital rule to get that as $K\to 0$, the latter inequality holds.

■

Proof of Result 3. From the individual first-order condition (16), the only channel where aggregate information choices affect the individual choice is through the signal-to-noise ratio. More specifically for $\pi=0$, one can solve for both the signal-to-noise ratio and individual information choices in closed form, as we did in the proof of result 2.

As $\xi_t < 0$, from equation (78) it is immediate that $\frac{\xi_t}{\Omega_{ft}} < 0$. Next, equation (80) implies

$$\frac{d\mathbf{\Omega}_{xit}}{d\xi_t} = \frac{2\xi_t\sqrt{\frac{k\chi_x}{\xi_t^4+\chi_x}}}{\xi_t^4+\chi_x} < 0,$$

which together implies $\frac{d\mathbf{\Omega}_{xit}}{d\Omega_{ft}} > 0$.

■

Proof of Result 4.

(4a)

Substitute the closed form for ξ into Ω_f and take the derivative to get[20]

$$\frac{d\Omega_f}{dK} = \frac{2\left(8K^4 + 3K^2\rho^4\chi_x\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}-1\right)\right)}{\rho^8\chi_x^2\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}-1\right)^2\sqrt{K\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}+1\right)}\sqrt{2-\frac{8K^2}{\rho^4\chi_x}}}$$

Each term in the denominator is positive. Thus for $\frac{d\Omega_x}{dK}$ to be positive, it must be that

$$8K^2 + 3\rho^4\chi_x\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}-1\right) > 0.$$

Manipulating the latter equation, the necessary and sufficient condition is

$$K < \frac{\sqrt{3}}{4}\rho^2\sqrt{\chi_x} = \frac{\sqrt{3}}{2}\bar{K},$$

where $\bar{K} = \frac{\rho^2\sqrt{\chi_x}}{2}$, as is defined in the main text.

(4b)

From equation (80)

$$\Omega_x = \frac{1}{\chi_x}\sqrt{\frac{K}{\left(\frac{1}{\xi^4}+\frac{1}{\chi_x}\right)}}$$

Therefore, as $K\uparrow$, the numerator increases and the denominator falls (part a), thus $\frac{d\Omega_x}{dK} > 0$. ■

Proof of Result 5.

(5a)

Prove: $\frac{d\left(C_{t+1}^2\tau_0^{-1}+D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} > 0$, *keeping* K_{t+j}, $\forall\, j>1$ *constant.* By differentiating (58) and (128), we can show that

$$\frac{dC_{t+1}}{dK_{t+1}} = -\frac{\tau_0}{r-G}\frac{d\hat{V}_{t+1}}{dK_{t+1}}$$
$$\frac{dD_{t+1}}{dK_{t+1}} = \frac{1}{r-G}\left[\tau_x\hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}} + (\tau_x\xi_{t+1} - \frac{r\rho}{r-G})\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right]$$

[20] For brevity, we suppress the t subscripts.

Also, recall that $\frac{d\xi_{t+1}}{dK_{t+1}} < 0$ and $\frac{d|\xi_{t+1}|}{dK_{t+1}} > 0$.

$$
\begin{aligned}
\frac{d\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} &= 2\left(\tau_0^{-1}C_{t+1}\frac{dC_{t+1}}{dK_{t+1}} + \tau_x^{-1}D_{t+1}\frac{dD_{t+1}}{dK_{t+1}}\right) \\
&= 2\left(-\tau_0^{-1}C_{t+1}\frac{\tau_0}{r-G}\frac{d\hat{V}_{t+1}}{dK_{t+1}} + \tau_x^{-1}D_{t+1}\frac{1}{r-G}\left[\tau_x\hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}} + (\tau_x\xi_{t+1} - \frac{r\rho}{r-G})\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right]\right) \\
&= \frac{2D_{t+1}}{r-G}\left(-\xi_{t+1}\frac{d\hat{V}_{t+1}}{dK_{t+1}} + \hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}} + (\xi_{t+1} - \frac{r\rho\tau_x^{-1}}{r-G})\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right) \\
&= \frac{2D_{t+1}}{r-G}\left(\hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}} - \frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right) \qquad (83)
\end{aligned}
$$

The term outside the parentheses is negative. Inside the parentheses, the first term is negative, while the second term (with the minus sign) is positive. Therefore, we need to show that the first term is larger in magnitude. Next, move to computing $\frac{d\hat{V}_{t+1}}{dK_{t+1}}$. Rewrite

$$
\begin{aligned}
\hat{V}_{t+1}^{-1} &= \tau_0 + \Omega_{ft+1} + \xi_{t+1}^2(\tau_x + \Omega_{xt+1}) = \tau_0 + \xi_{t+1}^2\tau_x + (\Omega_{ft+1} + \xi_{t+1}^2\Omega_{xt+1}) \\
&= \tau_0 + \xi_{t+1}^2\tau_x - \frac{\xi_{t+1}(\xi_{t+1}^2 Z_{t+1}\tau_x + \frac{\rho r}{r-G} + Z_{t+1}\tau_0)}{(1+\xi_{t+1}Z_{t+1})} \\
&= \tau_0 + \frac{\xi_{t+1}(\xi_{t+1}\tau_x - \frac{\rho r}{r-G} - Z_{t+1}\tau_0)}{(1+\xi_{t+1}Z_{t+1})}
\end{aligned}
$$

where the second line follows from equation (68) in the main text.

This also implies

$$
\hat{V}_{t+1} = \frac{(1+\xi_{t+1}Z_{t+1})}{\tau_0(1+\xi_{t+1}Z_{t+1}) + \xi_{t+1}(\xi_{t+1}\tau_x - \frac{\rho r}{r-G} - Z_{t+1}\tau_0)} = \frac{(1+\xi_{t+1}Z_{t+1})}{\tau_0 - \xi_{t+1}\frac{\rho r}{r-G} + \xi_{t+1}^2\tau_x} \qquad (84)
$$

Thus, we have

$$
\frac{d\hat{V}_{t+1}}{dK_{t+1}} = \frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}
$$

which reduces equation (83) to

$$
\frac{d\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} = \frac{2D_{t+1}}{(r-G)}\frac{d\xi_{t+1}}{dK_{t+1}}\left(\hat{V}_{t+1} - \frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\right) \qquad (85)
$$

Next, we compute $\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}$:

$$
\frac{d\hat{V}_{t+1}}{d\xi_{t+1}} = -\frac{d\hat{V}_{t+1}}{d|\xi_{t+1}|} = \hat{V}_{t+1}^2\frac{-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0}{(1+\xi_{t+1}Z_{t+1})^2} > 0 \qquad (86)
$$

We use that to rewrite equation (85) as

$$
\frac{d\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} = \frac{2\hat{V}_{t+1}D_{t+1}}{(r-G)}\frac{d\xi_{t+1}}{dK_{t+1}}\left(1 + \frac{r\rho\tau_x^{-1}}{r-G}\hat{V}_{t+1}\frac{\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) - \frac{\rho r}{r-G} - Z_{t+1}\tau_0}{(1+\xi_{t+1}Z_{t+1})^2}\right)
$$

The term outside the parentheses on the rhs is positive, so for the lhs to be positive, we need the term inside the parentheses to also be positive.

Since $D_{t+1} < 0$ and $r - G > 0$, it is sufficient that

$$\frac{d\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} > 0 \iff \frac{d\xi_{t+1}}{dK_{t+1}}\Big(\hat{V}_{t+1} - \frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\Big) < 0$$
$$\iff \frac{d\xi_{t+1}}{dK_{t+1}}\hat{V}_{t+1} < \frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}$$
$$\overset{\frac{d\xi_{t+1}}{dK_{t+1}}<0,\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}>0}{\iff} \hat{V}_{t+1}|\frac{d\xi_{t+1}}{dK_{t+1}}| > \frac{r\rho\tau_x^{-1}}{r-G}|\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}| = \frac{r\rho\tau_x^{-1}}{r-G}|\frac{d\hat{V}_{t+1}}{dK_{t+1}}|.$$

Therefore, for the future information risk to be increasing in K_{t+1}, we need

$$\frac{\hat{V}_{t+1}\tau_x(r-G)}{r\rho}|\frac{d\xi_{t+1}}{dK_{t+1}}| > |\frac{d\hat{V}_{t+1}}{dK_{t+1}}|. \tag{87}$$

Notice that $\frac{d\hat{V}_{t+1}}{dK_{t+1}} = \frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}$, with $\frac{d\hat{V}_{t+1}}{d\xi_{t+1}} > 0$ and $\frac{d\xi_{t+1}}{dK_{t+1}} < 0$. Plug them into equation (87) and the $\frac{d\xi_{t+1}}{dK_{t+1}}$ terms cancel, thus

$$\frac{\hat{V}_{t+1}\tau_x(r-G)}{r\rho} > \frac{d\hat{V}_{t+1}}{d\xi_{t+1}} = \hat{V}_{t+1}^2\frac{-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0}{(1+\xi_{t+1}Z_{t+1})^2}.$$

Cancelling $\hat{V}_{t+1}$ and rearranging, we have

$$\tau_x(r-G)(1+\xi_{t+1}Z_{t+1})^2 > \hat{V}_{t+1}r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big) \tag{88}$$

We showed that $\hat{V}_{t+1} = \frac{1+\xi_{t+1}Z_{t+1}}{\tau_0 - \xi_{t+1}\frac{\rho r}{r-G} + \xi_{t+1}^2\tau_x}$. Substituting it in equation (88) leads to

$$\tau_x(r-G)(1+\xi_{t+1}Z_{t+1})^2(\tau_0-\xi_{t+1}\frac{\rho r}{r-G}+\xi_{t+1}^2\tau_x) > (1+\xi_{t+1}Z_{t+1})r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1})+\frac{\rho r}{r-G}+Z_{t+1}\tau_0\Big). \tag{89}$$

From equation (80) in the online appendix, we can write $\Omega_f + \xi^2\Omega_x = -\frac{\xi(\xi^2 Z_{t+1}\tau_x + \alpha + Z_{t+1}\tau_0)}{1+\xi Z_{t+1}}$. The LHS is positive. For the RHS, we know that $\xi < 0$, and thus $-\xi$ is positive. The remaining term in the parentheses must be positive because it is a sum of variances, precisions, and squares. To make the signs of the LHS and the RHS match, we must have $1 + \xi_{t+1}Z_{t+1} > 0$. This helps reduce inequality (89):

$$\tau_x(r-G)(1+\xi_{t+1}Z_{t+1})(\tau_0 - \xi_{t+1}\frac{\rho r}{r-G} + \xi_{t+1}^2\tau_x) > r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big)$$
$$\iff \tau_x(1+\xi_{t+1}Z_{t+1})((\tau_0+\xi_{t+1}^2\tau_x)(r-G) - \xi_{t+1}\rho r) > r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big) \tag{90}$$
$$\iff \tau_x(1+\xi_{t+1}Z_{t+1})(\tau_0+\xi_{t+1}^2\tau_x)(r-G) > r\rho\Big(-\tau_x\xi_{t+1} + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big). \tag{91}$$

Both the LHS and RHS are positive. For a sufficiently small ρ, this inequality will hold.

There are other ways to arrive at this result. From equation (90), we can find a sufficient condition, that is

$$\tau_x(1+\xi_{t+1}Z_{t+1})(\tau_0+\xi_{t+1}^2\tau_x)(r-G) > r\rho\Big(-\tau_x\xi_{t+1}(1+\xi_{t+1}Z_{t+1}) - \tau_x\xi_{t+1} + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big)$$
$$= \underbrace{r\rho Z_{t+1}\Big(-\tau_x\xi_{t+1}^2 + \tau_0\Big)}_{\gtreqless 0} + \underbrace{r\rho\Big(-2\tau_x\xi_{t+1} + \frac{\rho r}{r-G}\Big)}_{>0}.$$

A sufficient condition for the above inequality to hold is

$$\tau_x(1+\xi_{t+1}Z_{t+1})(\tau_0+\xi_{t+1}^2\tau_x)(r-G) > r\rho\Big(-\tau_x\xi_{t+1}(1+\xi_{t+1}Z_{t+1}) - \tau_x\xi_{t+1} + \tfrac{\rho r}{r-G} + Z_{t+1}\tau_0\Big)$$
$$\iff \tau_x(1+\xi_{t+1}Z_{t+1})\Big[\underbrace{(\tau_0+\xi_{t+1}^2\tau_x)(r-G)}_{>0} + \underbrace{r\rho\xi_{t+1}}_{<0}\Big] > \underbrace{r\rho\Big(-\tau_x\xi_{t+1} + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big)}_{>0}.$$

For a sufficiently small ρ, the right side will be small and the negative term on the left will also be small, ensuring that the inequality will hold.

The conclusion is that, if the risk aversion is not too high ($\rho < \bar{\rho}$), then future information risk is increasing in K_{t+1}. The economic force is this: As K_{t+1} increases, $Var[y_{t+2} \mid \mathcal{I}_{t+1}]$ decreases, but C_{t+1} increases. The effect of decreasing $Var[y_{t+2} \mid \mathcal{I}_{t+1}]$ is mediated by risk aversion. If that's not too large, then the future information risk also increases.

(5b)

Prove that $\partial(C_t/|D|_t)/\partial K_{t+1} < 0$.

Future information K_{t+1} shows up through the variance term Z_t. Therefore, we begin by differentiating equation (69) with respect to Z_t.

$$\frac{\partial \xi_t}{\partial Z_t} = -\frac{\left(\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right) + \chi_x \sqrt{\frac{\xi_t^4+\chi_x}{\chi_x}}\left(\alpha + Z_t\left(3\xi_t^2\tau_x + \tau_0\right)\right)\right)}{\xi_t \chi_x \sqrt{\frac{\xi_t^4+\chi_x}{\chi_x}}\left(\sqrt{K_t}\sqrt{\frac{\xi_t^4+\chi_x}{\chi_x}} + \xi_t^2\tau_x + \tau_0\right)} \tag{92}$$

First, we argue that the numerator is positive. Consider the first term,

$$\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right).$$

We will argue that this term is also always positive. As $K_t \to 0$ from above, this term converges to zero since ξ_t, C_{t+1} and D_{t+1} are all bounded. As $K_T \to \infty$, we have already shown that $\xi_t \to -\frac{1}{Z_t}$, thus this term converges to $\sqrt{K_t}\left(-\xi_t^3 - \frac{\chi_x}{\xi_t}\right) > 0$. Next, take the derivative of the above expression with respect to K_t (keeping Z_t constant):

$$\frac{\partial\left(\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right)\right)}{\partial K_t} = \frac{1}{2\sqrt{K_t}}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right)$$

Thus this expression and its derivative always have the same sign. Now assume this expression is negative for some K_t. The derivative then has to be negative as well, which means that as K_t grows, the expression can never become positive again. However, we showed that as $K_t \to \infty$, this expression is positive, a contradiction.

Next, the denominator is negative, because all terms are positive, except ξ, which is negative. Thus $\frac{\partial \xi_t}{\partial Z_t} > 0$. In other words, price informativeness (signal-to-noise ratio $\frac{C_t}{|D_t|}$ falls) $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}$ increases. Moreover, result 5 proves that $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}$ is increasing in K_{t+1} if ρ is not too high, which completes the proof. ■

Re-proving $\partial(C_t/|D|_t)/\partial K_t > 0$ with long-lived assets. We begin by differentiating equation (69) with respect to K_t.

$$\frac{\partial \xi_t}{\partial K_t} = -\frac{2\sqrt{K_t}}{(\xi_t^4 + \chi_x)(1 + \xi_t Z_t)}\left(\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right) + \chi_x\sqrt{\frac{\xi_t^4 + \chi_x}{\chi_x}}\left(\alpha + Z_t\left(3\xi_t^2\tau_x + \tau_0\right)\right)\right) \tag{93}$$

Consider the first term, in front of the large parentheses. From Lemma 4, we know that $1 + \xi_t Z_t > 0$. Thus the ratio outside these parentheses is positive. Inside the parentheses, this is the same term as in the $\frac{\partial \xi_t}{\partial Z_t}$ expression above. We signed that positive. If the term in parentheses in (93) is positive and the term in front is also positive, then $\frac{\partial \xi_t}{\partial K_t} < 0$. In other words, price informativeness (the signal-to-noise ratio, $\frac{C_t}{|D_t|}$) rises as information becomes more abundant.

Lemma 4 ***Balanced data processing growth depends on future information risk and long-lived assets.*** *$|D_t| \geq \frac{\rho(r-G)}{r}\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)C_t$, with strict inequality if $K_t > 0$.*

Proof. Use equation (69) to write

$$(1+\xi Z_t)(1+\frac{1}{\chi_x}\xi^4)^{\frac{1}{2}} = -(\frac{1}{K_t})^{\frac{1}{2}}\xi(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0), \tag{94}$$

since we've proven that $\xi \leq 0$ (lemma 2). And we know from the structure of the optimization problem (linear objective subject to convex cost) that for any $K_t > 0$, $\Omega_{ft} > 0$, which implies that $C_t > 0$, and thus $\xi < 0$ with strict inequality. The other terms on the right side are strictly positive squares or positive constants, with a negative sign in front. Thus, the right hand side of equation (94) is positive. On the left, since $(1+\frac{1}{\chi_x}\xi^4)^{\frac{1}{2}}$ is a square root, and therefore positive, $(1+\xi Z_t)$ must be also positive for the equality to hold. $(1+\xi Z_t) > 0$ implies that $Z_t < -1/\xi$. Substitute for Z_t to get the result. This result puts a bound on how liquid the price can be. The liquidity is bounded by the product of price informativeness and un-learnable, future risk. ■

Proof of Result 6.

(6a) Ω_{ft}/Ω_{xt} does not converge to 0.

If Ω_{ft}/Ω_{xt} converges to 0, then by the first-order condition, it must be that $\xi_t \to \infty$. It is sufficient to show that $\xi_t \to \infty$ violates equation (69). Rearrange (69) to get

$$\left[\xi_t Z_t\left(\xi_t^2\tau_x + (K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} + \tau_0\right) + \xi_t\alpha\right] + (K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} = 0 \tag{95}$$

The term in square brackets is negative and the term outside is positive. Assume $\xi_t \to \infty$. If Z_t does not go to zero, then the negative term grows faster and the equality cannot hold. So it must be that $Z_t \to 0$. That requires that both $C_{t+1} \to 0$ and $D_{t+1} \to 0$ (see (37)). In order for C_{t+1} to go to zero, $\hat{V} \to \tau_0^{-1}$. But since $\xi_t \to \infty$, from equation (35), $\hat{V} \to 0$, which is a contradiction.

(6b) As $K_t \to \infty$, Ω_{ft}/Ω_{xt} does not converge to ∞.

If Ω_{ft}/Ω_{xt} did converge to ∞ as $K_t \to \infty$, then by first-order condition (16), it would have to be that $\xi_t \to 0$. So it suffices to show that $\Omega_{ft}/\Omega_{xt} = \infty$ is inconsistent with $\xi_t = 0$, in equilibrium.

Start from the equilibrium condition (67), which must be zero in equilibrium. If $\xi_t \to 0$, then the first term goes to zero. The proof of lemma 4 proves, along the way, that $(1+\xi_t Z_t) > 0$. (Otherwise, (67) can never be zero because it is always negative.) Thus the second term $\Omega_{xt}\xi_t^2(1+\xi_t Z_t)$ must be non-negative.

The third term $\Omega_{ft}(1+\xi_t Z_t)$ also converges to ∞ because $\Omega_{ft} \to \infty$ and $(1+\xi_t Z_t) > 0$. How do we know that $\Omega_{ft} \to \infty$? In principle, Ω_{ft}/Ω_{xt} could become infinite either because Ω_{ft} became infinite or because Ω_{xt} goes to zero. But if Ω_{xt} goes to zero and Ω_{ft} is finite, then information processing constraint (3), which requires that the weighted sum of Ω_{ft} and Ω_{xt} be K_t, cannot be satisfied as $K_t \to \infty$.

Since one term of (67) becomes large and positive and the other two are non-negative in the limit, the sum of these three terms cannot equal zero. Therefore, $\Omega_{ft}/\Omega_{xt} \to \infty$ cannot be an equilibrium.

(6c) there exists an equilibrium where Ω_{ft}/Ω_{xt} converges to a constant.

By first-order condition (16), we know that Ω_{ft}/Ω_{xt} converges to a constant, if and only if ξ_t also converges to a constant. Thus, it suffices to show that there exists a constant ξ_t that is consistent with equilibrium, in the high-K limit.

Suppose ξ_t and Z_t are constant in the high-K limit. In equation (69) as $K_t \to \infty$, the last term goes to infinity, unless $\xi_t \to -\frac{1}{Z_t}$. If the last term goes to infinity and the others remain finite, this cannot be an equilibrium because equilibrium requires the left side of (69) to be zero. Therefore, for a constant solution to ξ_t, and thus for Ω_{ft}/Ω_{xt} and Z_t to exist, it must be that $\xi_t \to \frac{-1}{Z_t}$, at the correct rate

$$\xi_t^3 Z_t\tau_x + \xi_t(\alpha + Z_t\tau_0) + (1+\xi_t Z_t)(K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} = 0$$

$$\lim_{K_t\to\infty}\left[\xi_t-(-\frac{1}{Z_t})\right]=\frac{\frac{\alpha}{Z_t^2}\tau_x+\frac{\alpha}{Z_t}+\tau_0}{Z_t(\frac{1}{})^{\frac{1}{2}}(1+\frac{1}{\chi_x}\frac{1}{Z_t^4})^{\frac{1}{2}}}\frac{1}{\sqrt{K_t}}\to 0$$

The question that remains is whether ξ_t and Z_t are finite constants in the high-K limit, or whether one explodes and the other converges to zero.

Suppose $\xi_t=-\frac{1}{Z_t}$, which is constant ($\xi_t=\bar{\xi}$). $Z_t=\bar{Z}$ is then also constant. The rest of the proof checks to see if such a proposed constant- $\bar{\xi}$ solution is consistent with equilibrium. We do this by showing that ξ_t does not explode or contract as K_t increases. In other words, for $\xi_t=\frac{-1}{Z_t}$ to be stable and thus the ratio of fundamental to technical analyses to be stable, we need it to be that $\partial\xi_t/\partial K_t\to 0$, in other words, ξ_t and therefore Ω_{ft}/Ω_{xt} converges to a constant as $K_t\to\infty$.

Step 1: Derive $d\xi_t/dK_t$: Start from the equilibrium condition for ξ_t (69) and apply the implicit function theorem:

$$\Big(3Z_t\tau_x\xi_t^2+A+Z_t\tau_0\Big)d\xi_t+\frac{1}{2}(\frac{1}{K_t})^{\frac{1}{2}}(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}dK_t$$
$$+\left[\frac{1}{2}(\frac{K_t}{\chi_f})^{\frac{1}{2}}(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{-\frac{1}{2}}(4\frac{1}{\chi_x}\xi_t^3)+Z_t(K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}\right]d\xi_t=0$$

So we have

$$\frac{d\xi_t}{dK_t}=\frac{1}{2}(\frac{1}{K_t})^{\frac{1}{2}}\frac{-(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}}{3Z_t\tau_x\xi_t^2+A+Z_t\tau_0+2\frac{1}{\chi_x}K_t^{\frac{1}{2}}(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{-\frac{1}{2}}\xi_t^3+Z_tK_t^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}}$$

Use equation (69) to write the numerator as

$$(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}=-(\frac{1}{K_t})^{\frac{1}{2}}\xi_t(\xi_t^2Z_t\tau_x+A+Z_t\tau_0)\tag{96}$$

Now use this to rewrite $\frac{d\xi_t}{dK_t}$ as

$$\frac{d\xi_t}{dK_t}=\frac{1}{2K_t}\frac{1}{\frac{3Z_t\tau_x\xi_t^2+A+Z_t\tau_0}{\xi_t(\xi_t^2Z_t\tau_x+A+Z_t\tau_0)}-2\frac{1}{\chi_x}(1+\frac{1}{\chi_x}\xi_t^4)^{-1}\xi_t^3-\frac{Z_t}{(1+\xi_tZ_t)}}\tag{97}$$

Step 2: Show that $d\xi_t/dK_t\to 0$ as $K_t\to\infty$, as long as $X(\cdot)\not\to 0$

As $K_t\to\infty$, it is clear that $1/2K_t\to 0$. As long as the term that multiplies $1/2K_t$ stays finite, the product will converge to zero. Since the numerator is just 1, the second term will be finite, as long as the denominator does not go to zero. Define

$$X(\xi_t,Z_t)=\frac{3Z_t\tau_x\xi_t^2+A+Z_t\tau_0}{\xi_t(\xi_t^2Z_t\tau_x+A+Z_t\tau_0)}-2\frac{1}{\chi_x}(1+\frac{1}{\chi_x}\xi_t^4)^{-1}\xi_t^3-\frac{Z_t}{(1+\xi_tZ_t)}\tag{98}$$

which is the denominator of the second fraction on the rhs of equation (97). Then if $X\not\to 0$, $1/X$ is finite, then $1/2K_t*1/X$ goes to zero as K_t gets large. Thus, we get that $\partial\xi_t/\partial K_t\to 0$ as $K_t\to\infty$.

Step 3: $X(\cdot)\not\to 0$.

To complete the proof, we need to show that $\bar{\xi}=-\frac{1}{\bar{Z}}$, which satisfies equilibrium condition (103) as $K_t\to\infty$, does not cause $X(\cdot)=0$. We can check this directly: in equation (98), if $\xi_t=-\frac{1}{Z_t}$, the denominator of the last term becomes zero; so the last term becomes infinite. The only term in (98) with the opposite sign is the middle term, which is finite if $\xi=\frac{C}{D}$ is finite (the running assumption). If the last term of X tends to infinity and the only term of the opposite sign is finite, the sum cannot be 0. Thus, for $\bar{\xi}=-\frac{1}{\bar{Z}}$, which is the limit attained in the limit as $K_t\to\infty$, we have that $X(\bar{\xi})\neq 0$.

Step 4: As $K_t \to \infty$, if (104) holds, a real-valued, finite-ξ solution exists.

From equations (35-38), as $K_t \to \infty$ at least one of the two information choices goes to ∞, so with finite, non-zero $\frac{C}{D}$:

$$\lim_{K_t\to\infty} \hat{V}_t = 0 \tag{99}$$

$$\lim_{K_t\to\infty} \Omega_t^{-1} = \frac{r}{\rho(r-G)} Z_t = D_{t+1}^2(\xi_{t+1}^2\tau_0^{-1} + \tau_x^{-1}) \tag{100}$$

$$\lim_{K_t\to\infty} D_t = -\frac{\rho}{r}\Omega_t^{-1} = -\frac{1}{(r-G)} Z_t \tag{101}$$

A word of interpretation here: Equation (38), which defines Ω^{-1}, is the total future payoff risk. As $\hat{V} \to 0$, it means that the predictable part of this variance vanishes as information capacity gets large. Z_t, which is the unpredictable part, remains and governs liquidity, D_t.

Next, we solve (100) for D_{t+1}, then backdate the solution 1 period to get an expression for D_t. And we equate it to the expression for D_t in (101). This implies that $\lim_{K_t\to\infty} D = \bar{D}$ is constant and equal to both of the following expressions

$$\bar{D}^2 = \frac{-rZ_t}{\rho(r-G)\bar{\xi}(\bar{\xi}^2\tau_0^{-1} + \tau_x^{-1})} = \frac{Z_t}{(r-G)^2\bar{\xi}^2} \tag{102}$$

We can cancel Z_t on both sides, which delivers a quadratic equation in one unknown in $\bar{\xi}$:

$$\bar{\xi}^2\tau_0^{-1} + \frac{r(r-G)}{\rho}\bar{\xi} + \tau_x^{-1} = 0. \tag{103}$$

In order for $\bar{\xi}$ to exist, equation (103) requires the expression inside the square root term of the quadratic formula (often written as $(b^2 - 4ac)$) to not be negative. This imposes the parametric restriction

$$\left(\frac{r(r-G)}{\rho}\right)^2 - 4\tau_0^{-1}\tau_x^{-1} \geq 0,$$

or equivalently,

$$\tau_0\tau_x \geq \left(\frac{4\rho}{r(r-G)}\right)^2. \tag{104}$$

Rearranging this to put τ_0 on the left delivers $\tau_0 \geq \underline{\tau}$, where $\underline{\tau} = 4\tau_x^{-1}\rho^2(r(r-G))^{-2}$. If we instead rearrange this to put τ_x on the left, we get $\tau_x \geq \underline{\tau}$, where $\underline{\tau} = 4\tau_0^{-1}\rho^2(r(r-G))^{-2}$.

Thus if (104) holds, we have

$$\bar{\xi} = (r-G)\frac{-r \pm \sqrt{r^2 - 4(\frac{\rho}{r-G})^2\tau_0^{-1}\tau_x^{-1}}}{2\rho\tau_0^{-1}} \tag{105}$$

$$\bar{C} = \frac{1}{r-G} \tag{106}$$

$$\bar{D} = -\frac{r \pm \sqrt{r^2 - 4(\frac{\rho}{r-G})^2\tau_0^{-1}\tau_x^{-1}}}{2\rho\tau_x^{-1}} \tag{107}$$

Step 5: Balanced growth. Finally, use lemma 4 to prove the existence of balanced growth. The lemma shows that $C_t/|D_t| < \left(\rho\left((r-G)/r\right)(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1})\right)^{-1}$. The first term is just fixed parameters. The second term, $(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1})$, is the variance of the part of tomorrow's price that depends on future shocks, x_{t+1} and y_{t+1}. This is the future information risk. It converges to a large, positive number as K_t grows. When information is abundant, high future information risk pushes $C_t/|D_t|$ down toward a constant.

In contrast, if demand analysis were to keep growing faster than fundamental analysis (Ω_{ft}/Ω_{xt} were to fall to zero), by first-order condition (16), $(C_t/D_t)^2$ would keep rising to infinity. But if $(C_t/D_t)^2$ is converging to infinity, then at some point, it must violate the inequality above because the right side of the inequality is decreasing over time. Thus, demand analysis cannot grow faster than fundamental analysis forever.

The only solution that reconciles the first-order condition with the equilibrium price coefficients is one where $(\Omega_{ft}/\Omega_{xt})$ stabilizes and converges to a constant. If fundamental analysis grows proportionately with demand analysis, then the rise in the amount of fundamental analysis makes prices more informative about dividends: C_t increases. Proportional growth in fundamental and demand analyses allows C_t to keep up with the rise in D_t, described above. Therefore, as information technology grows ($K_t \to \infty$), a stable C_t/D_t rationalizes information choices (Ω_{xt}, Ω_{ft}) that grow proportionately, so that Ω_{xt}/Ω_{ft} converges to a constant.

(6d) No perfect liquidity equilibrium, $D_t \neq 0$, $\forall\, t$.

Lemmas 1 and 2 prove that for any $\Omega_{ft}, \Omega_{xt} \geq 0$, $C \geq 0$, and $D_t \leq 0$. Moreover, from the structure of the optimization problem (the linear objective subject to convex cost), for any $K_t > 0$, $\Omega_{ft} > 0$, which implies $C_t > 0$. Since $C_t > 0$, if $D_t \to 0$, the first-order condition implies that Ω_{ft}/Ω_{xt} has to converge to zero. This directly violates equation (95) for any finite K_t, and part (6a) of the result shows that the same contradiction happens in the limit as $K_t \to \infty$. Thus there is no level of technological progress for which the market becomes perfectly liquid, $D_t = 0$.

■

Proof of Result 7.

For the static model, we want to evaluate the effect on price informativeness of reallocating attention from the supply shock to fundamental. Because we have to respect the budget constraint on attention allocation, we have that

$$\Omega_f = \sqrt{K - \chi_x \Omega_x^2}.$$

Thus

$$\frac{d\Omega_f}{d\Omega_x} = -\chi_x \frac{\Omega_x}{\Omega_f}$$

using, the F.O.C $\frac{\Omega_x}{\Omega_f} = \frac{\xi^2}{\chi_x}$, we get that in equilibrium $\frac{d\Omega_f}{d\Omega_x} = -\xi^2$.

We are going to calculate the effect of increasing one unit of Ω_x on $\xi \equiv C/D$, but while considering the decrease in Ω_f needed to achieve the increase. Again, our starting point is

$$\xi^2 \Omega_x + \xi \rho + \Omega_f = 0$$

Differentiating with respect to Ω_x, we have

$$2\xi \frac{d\xi}{d\Omega_x} \Omega_x + \xi^2 + \frac{d\xi}{d\Omega_x} \rho + \frac{d\Omega_f}{d\Omega_x} = 0$$

Replacing $\frac{d\Omega_f}{d\Omega_x} = -\xi^2$, we finally obtain

$$\frac{d\xi}{d\Omega_x} [2\xi \Omega_x + \rho] = 0$$

The term in brackets is 0 only if $\xi = \frac{-\rho}{2\Omega_x}$. In fact, we know that $\xi > \frac{-\rho}{2\Omega_x}$ because the solution of $\xi^2\Omega_x + \xi\rho + \Omega_f = 0$ that behaves as expected when $\chi_x \to \infty$ is

$$\xi = \frac{-\rho}{2\Omega_x} + \frac{\sqrt{\rho^2 - 4\Omega_f \Omega_x}}{2\Omega_x}$$

Thus, the only solution to the equation $\frac{d\xi}{d\Omega_x}[2\xi\Omega_x + \rho] = 0$ is $\frac{d\xi}{d\Omega_x} = 0$.

Second order condition: Of course, it could be that the equilibrium allocation minimizes the price informativeness. To show that this is a maximum, we also need to show that the second-order condition is negative.

Thus starting from

$$2\xi \frac{d\xi}{d\Omega_x}\Omega_x + \xi^2 + \frac{d\xi}{d\Omega_x}\rho + \frac{d\Omega_f}{d\Omega_x} = 0,$$

we group the terms, use $\frac{d\Omega_f}{d\Omega_x} = -\chi_x \frac{\Omega_x}{\Omega_f}$, and then differentiate a second time to get

$$\begin{aligned}
\frac{d\xi}{d\Omega_x}(2\xi\Omega_x + \rho) &= -\xi^2 + \chi_x \frac{\Omega_x}{\Omega_f} \\
\frac{d^2\xi}{d\Omega_x^2}(2\xi\Omega_x + \rho) + \frac{d\xi}{d\Omega_x}\frac{d(2\xi\Omega_x + \rho)}{d\Omega_x} &= -2\xi\frac{d\xi}{d\Omega_x} + \chi_x\left[\frac{1}{\Omega_f} - \frac{\Omega_x}{\Omega_f^2}\frac{d\Omega_f}{d\Omega_x}\right]
\end{aligned}$$

Now we use $\frac{d\xi}{d\Omega_x} = 0$, and $\frac{\Omega_x}{\Omega_f} = \frac{\xi^2}{\chi_x}$ to get

$$\begin{aligned}
\frac{d^2\xi}{d\Omega_x^2}(2\xi\Omega_x + \rho) &= \frac{\chi_x}{\Omega_f}\left[1 + \chi_x\left(\frac{\Omega_x}{\Omega_f}\right)^2\right] \\
\frac{d^2\xi}{d\Omega_x^2} &= \frac{\chi_x}{\Omega_f(2\xi\Omega_x + \rho)}\left(1 + \frac{1}{\chi_x}\xi^4\right) > 0
\end{aligned}$$

While this is positive, it is positive in ξ, which is (C/D). Since $D < 0$, this implies that the second derivative with respect to $C/|D|$ is positive. In other words, the efficient allocation minimizes (C/D), the negative signal-to-noise ratio. Since C/D is a negative number, minimizing it is maximizing the absolute value. Thus, the equilibrium information processing allocation maximizes the measure of price informativeness $C/|D|$.

■

Result 8 ***Information response to technological growth (dynamic).*** *For* $\pi = 1$,

(a) If $\Omega_x < \tau_0 + \Omega_f$ *and* $Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t] < \max\{\sqrt{3}, \frac{1}{2}|C_t/D_t|\}$ *, then* $\frac{\partial C/D}{\partial \Omega_f} < 0$ *and* $\frac{\partial C/D}{\partial \Omega_x} \leq 0$.

(b) Both fundamental and demand analyses increase the price information sensitivity. If $r - G > 0$ *and* $(\tau_x + \Omega_{xt})$ *is sufficiently small, then* $\partial C_t/\partial\Omega_{ft} > 0$ *and* $\partial C_t/\partial\Omega_{xt} > 0$.

(c) If demand is not too volatile, then both fundamental and demand analyses improve concurrent liquidity. If $\tau_x > \rho r/(r - G)$ *and* $D_t < 0$*, then* $\partial D_t/\partial\Omega_{ft} > 0$ *and* $\partial D_t/\partial\Omega_{xt} > 0$.

Proof.

(8a)

The strategy for proving this result is to apply the implicit function theorem to the price coefficients that come from coefficient matching in the market-clearing equation. After equating supply and demand and matching all the coefficients on x_{t+1}, we arrive at (12). Rearranging that equation gives us the expression for C_t/D_t in (71). If we subtract the right side of (71) from the left, we are left with an expression that is equal to zero in equilibrium. We will name this expression F:

$$F = \frac{C_t}{D_t} - \frac{\hat{V}_t^{-1} - \tau_0}{\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t \hat{V}_t^{-1}}$$

We compute $\frac{\partial C/D}{\partial \Omega_x} = -\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial \Omega_x}$ and $\frac{\partial C/D}{\partial \Omega_f} = -\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial \Omega_f}$. In particular, we have:

$$\begin{aligned}
\frac{\partial F}{\partial C/D} &= 1-\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-1} \\
&\quad +(\hat{V}^{-1}-\tau_0)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2}\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right) \\
&= 1-\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2} \\
&\quad \left[\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)-(\hat{V}^{-1}-\tau_0)\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right)\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F}{\partial \Omega_f} &= -(1)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-1}+(\hat{V}^{-1}-\tau_0)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2}(-Z_t) \\
&= -\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2}\left[\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)\right]
\end{aligned}$$

We notice that $\frac{\partial F}{\partial \Omega_x}=\left(\frac{C_t}{D_t}\right)^2\frac{\partial F}{\partial \Omega_f}$ since

$$\frac{\partial F}{\partial \Omega_x}=\frac{\partial F}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial \Omega_x}=\frac{\partial F}{\partial \hat{V}^{-1}}\left(\frac{C_t}{D_t}\right)^2\frac{\partial \hat{V}^{-1}}{\partial \Omega_f}=\left(\frac{C_t}{D_t}\right)^2\frac{\partial F}{\partial \Omega_f}$$

.

then:

$$\frac{\partial C/D}{\partial \Omega_f}=\frac{\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)}{\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^2-\left[\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)-(\hat{V}^{-1}-\tau_0)\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right)\right]} \tag{108}$$

Part 1: If $\Omega_x<\tau_0+\Omega_f$ and $C/D>-Z_t/2$, then $\frac{\partial C/D}{\partial \Omega_f}<0$ and $\frac{\partial C/D}{\partial \Omega_x}\leq 0$.

The numerator of (108) is

$$\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)=\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\tau_0<0$$

The inequality holds since we have proven that $C_t/D_t<0$ and $r>G$.

In the denominator, however, not all the terms are negative. The denominator of (108), divided by $\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)$ is:

$$\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)-\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)+(\hat{V}^{-1}-\tau_0)\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-1} \tag{109}$$

The only positive term is $-2\frac{C_t}{D_t}\Omega_x$. As a result, it is easy to see that if C/D is sufficiently close to zero, then $-2\frac{C_t}{D_t}\Omega_x<\frac{\rho r}{r-G}+Z_t(\tau_0+\Omega_f)$, so (109) is negative.

The numerator is thus negative. And if C/D is sufficiently close to zero, the denominator is positive, so $\frac{\partial C/D}{\partial \Omega_f}<0$ and $\frac{\partial C/D}{\partial \Omega_x}=\left(\frac{C_t}{D_t}\right)^2\frac{\partial C/D}{\partial \Omega_f}<0$ if $C/D<0$ and $\frac{\partial C/D}{\partial \Omega_x}=0$ if $C/D=0$.

Part 2: If $C/D<-\frac{2Z_t^{-1}}{3}$, then $\frac{\partial C/D}{\partial \Omega_f}<0$ and $\frac{\partial C/D}{\partial \Omega_x}\leq 0$.

To see this, we analyze whether, under these new conditions, inequality (109) holds. We have:

$$
\begin{aligned}
& -\frac{\rho r}{r-G} - Z_t(\tau_0+\Omega_f) - 2\frac{C_t}{D_t}\Omega_x - 3Z_t\left(\frac{C_t}{D_t}\right)^2(\tau_x+\Omega_x) \\
= \quad & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2-3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x
\end{aligned}
$$

So if $C/D < -\frac{2Z_t^{-1}}{3}$, we can prove the above claim:

$$
\begin{aligned}
= \quad & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2-3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\
< \quad & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\
< \quad & 0
\end{aligned}
$$

Now, by combining the two previous claims, if $\Omega_x < \tau_0 + \Omega_f$ and $Z_t > \frac{1}{\sqrt{3}}$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$. The condition $Z_t > \frac{1}{\sqrt{3}}$ implies that $\frac{-Z_t}{2} < -\frac{2Z_t^{-1}}{3}$, which in turn implies the result for the entire support of C/D.

(8b)

From (58), $C_t = \frac{1}{r-G}(1-\tau_0\hat{V}_t)$.

From (35), $\hat{V}_t$ is defined as

$$
\hat{V} = [\tau_0 + \Omega_{ft} + \left(\frac{C_t}{D_t}\right)^2(\tau_x+\Omega_{xt})]^{-1} \tag{110}
$$

Notice that C_t shows up twice, once on the left side and once in $\hat{V}$. Therefore, we use the implicit function theorem to differentiate. If we define $F \equiv C_t - \frac{1}{r-G}(1-\tau_0\hat{V})$, then $\partial F/\partial C_t = 1 + \frac{1}{r-G}\tau_0\partial\hat{V}/\partial C_t$. Since τ_x and Ω_{xt} are both precisions, both are positive. Therefore, $\partial\hat{V}^{-1}/\partial C_t = 2C_t/D_t^2(\tau_x+\Omega_{xt})$. This is positive, since we know that $C_t > 0$. That implies that the derivative of the inverse is $\partial\hat{V}/\partial C_t = -\hat{V}^2 2C_t/D_t^2(\tau_x+\Omega_{xt})$, which is negative. The $\partial F/\partial C_t$ term is therefore one plus a negative term. The result is positive, as long as the negative term is sufficiently small: $\frac{2}{r-G}\tau_0\hat{V}^2C_t/D_t^2(\tau_x+\Omega_{xt}) < 1$. We can express this as an upper bound on $\tau_x+\Omega_{xt}$ by rearranging the inequality to read: $(\tau_x+\Omega_{xt}) < 1/2(r-G)\tau_0^{-2}\hat{V}^{-2}D_t^2/C_t$.

Next, we see that $\partial\hat{V}^{-1}/\partial\Omega_{ft} = 1$. Thus, $\partial\hat{V}/\partial\Omega_{ft} < 0$. Since $\partial F/\partial\hat{V} > 0$, this guarantees that $\partial F/\partial\Omega_{ft} < 0$.

Likewise, $\partial\hat{V}^{-1}/\partial\Omega_{xt} = (C_t/D_t)^2$. Since the square is always positive, $\partial\hat{V}/\partial\Omega_{xt} < 0$. Since $\partial F/\partial\hat{V} > 0$, this guarantees that $\partial F/\partial\Omega_{xt} < 0$.

Finally, the implicit function theorem states that $\partial C_t/\partial\Omega_{ft} = -(\partial F/\partial\Omega_{ft})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial\Omega_{ft} > 0$. Likewise, $\partial C_t/\partial\Omega_{xt} = -(\partial F/\partial\Omega_{xt})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial\Omega_{xt} > 0$.

(8c)

Part 1: $\partial D_t/\partial\Omega_{ft} > 0$.

From market clearing:

$$
D_t = [r - (1+B)\hat{V} + \Omega_p\frac{1}{C}]^{-1}[-\rho\Omega_t^{-1} - (1+B)\frac{C}{D}\hat{V}\Omega_x] \tag{111}
$$

Use $\Omega_p = (\frac{C}{D})^2(\Omega_x+\tau_x)$ to get $D_t r - (1+B)\hat{V}_t\frac{C}{D}(\tau_x) = -\rho\Omega_t^{-1}$. Then, use the stationary solution for B :

$1 + B = \frac{r}{r-G}$:

$$D_t - \frac{1}{r-G}\hat{V}_t \frac{C}{D}\tau_x = -\frac{\rho}{r}\Omega_t^{-1} \tag{112}$$

Then use (38) to substitute in for Ω_t^{-1}:

$$D_t = -\frac{1}{r-G}Z_t - \frac{r\rho}{(r-G)^2}\hat{V} + \frac{1}{r-G}\hat{V}_t \frac{C_t}{D_t}\tau_x \tag{113}$$

In the above, the RHS, less the last term, is the loading on X_{t+1}. The last term represents price feedback. We then define $F \equiv$ lhs of (113) $-$ rhs of (113). So that we can apply the implicit function theorem as $\partial D_t / \partial \Omega_f = -\frac{\partial F}{\partial \Omega_f} / \frac{\partial F}{\partial D_t}$. We begin by working out the denominator.

$$\frac{\partial F}{\partial D_t} = 1 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial D_t} - \frac{1}{r-G}\frac{\partial \hat{V} + \frac{C_t}{D_t}}{\partial D_t}\tau_x \tag{114}$$

$$\frac{\partial \hat{V}}{\partial D_t} = \frac{\partial \hat{V}}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial D_t} = -\hat{V}^2[-\frac{2C_t^2}{D_t^3}(\tau_x + \Omega_x)] = 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) \tag{115}$$

$$\frac{\partial \hat{V}\frac{C_t}{D_t}}{\partial D_t} = \frac{C_t}{D_t}\frac{\partial \hat{V}_t}{\partial D_t} + \hat{V}(-\frac{C}{D^2}) \tag{116}$$

$$= \frac{C}{D^2}\hat{V}[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{117}$$

$$\frac{\partial F}{\partial D_t} = 1 + \frac{r\rho}{(r-G)^2} \cdot 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{118}$$

$$\frac{\partial F}{\partial \Omega_f} = 0 - 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial \Omega_t} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x \frac{\partial \hat{V}}{\partial \Omega_t} \tag{119}$$

Recall the definition $\hat{V}_t \equiv [\tau_0 + \Omega_{ft} + \frac{C_t}{D_t}^2(\tau_x + \Omega_x)]^{-1}$. Differentiating $\hat{V}$, we get

$$\frac{\partial \hat{V}}{\partial \Omega_f} = \frac{\partial \hat{V}_t}{\partial \hat{V}_t^{-1}} \cdot \frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2 \frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2 \tag{120}$$

Substituting this in to (119) yields

$$\frac{\partial F}{\partial \Omega_f} = \frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}] \tag{121}$$

Substituting in the derivative of $\hat{V}$, we get

$$\frac{\partial D_t}{\partial \Omega_f} = -\frac{\frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}]}{1\frac{2r\rho}{(r-G)^2}\frac{C^2}{D^3}\hat{V}_t^2(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C}{\rho}(\tau_x + \Omega_x) - 1]} \tag{122}$$

We observe that if $\frac{C_t}{D_t} < 0$ and $r > G$, then the numerator is positive (including the leading negative sign).

The denominator is also positive if the following expression is positive:

$$\frac{r-G}{\frac{C}{D^2}\hat{V}} + 2\rho\frac{r}{r-G}\frac{C_t}{D_t}\hat{V}_t(\tau_x+\Omega_x) - \tau_x\hat{V}_t[\frac{2C}{D}(\tau_x+\Omega_x-1)] > 0 \tag{123}$$

This is equivalent to

$$\frac{r-G}{\hat{V}_t}\frac{D^2}{C} + 2\hat{V}_t\frac{C_t}{D_t}(\tau_x+\Omega_x)[\frac{r\rho}{r-G} - \tau_x] + \tau_x\hat{V}_t > 0. \tag{124}$$

Lemma 2 proves that $D < 0$. That makes the middle term potentially negative. However, if $[\frac{r\rho}{r-G} - \tau_x] < 0$ as well, the product of this and D is positive, which means that the middle term is positive. That inequality can be rearranged as $\tau_x > \frac{r\rho}{r-G}$. Since the rest of the terms are squares and precisions, the rest of the expression is positive as well.

Thus if $\tau_x > \frac{r\rho}{r-G}$, then $\frac{\partial D_t}{\partial \Omega_t} > 0$.

Part 2: $\partial D_t/\partial\Omega_{xt} > 0$.

Begin with the implicit function theorem: $\partial D_t/\partial\Omega_x = -\frac{\partial F}{\partial\Omega_x}/\frac{\partial F}{\partial D_t}$. The previous proof already proved that if $\tau_x > \frac{r\rho}{r-G}$, the denominator is positive. All that remains is to sign the numerator.

$$\frac{\partial F}{\partial\Omega_x} = 0 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial\hat{V}}{\partial\Omega_x} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\frac{\partial\hat{V}}{\partial\Omega_x}$$

where $\partial\hat{V}/\partial\Omega_x = -\hat{V}^2(C^2)/(D^2)$. Substituting the partial of $\hat{V}$ into the partial of F yields

$$\frac{\partial F}{\partial\Omega_x} = \hat{V}^2\frac{C^2}{D^2}(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x).$$

Combining terms,

$$\frac{\partial D_t}{\partial\Omega_x} = -\frac{\hat{V}^2\frac{C^2}{D^2}(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x)}{\frac{\partial F}{\partial D_t}}$$

We know from lemmas 1 and 2 that $\frac{C_t}{D_t} < 0$. Since $r > G$, by assumption, $\partial F/\partial\Omega_x$ is negative (i.e., the $\frac{C^2}{D^2}$ factor does not change the sign). Applying the implicit function theorem tells us that $\partial D_t/\partial\Omega_{xt} > 0$.

■

Corollary 1 ***Complementarity in demand analysis (dynamic).*** *For $\pi = 1$, if $\Omega_{xt} < \tau_0 + \Omega_{ft}$, then $\frac{\partial\boldsymbol{\Omega}_{xit}}{\partial\Omega_{xt}} \geq 0$.*

Proof. With the exact same argument that we use in our proof of result 1, complementarity follows from the individual first condition whenever $|\frac{C}{D}|$ is increasing. ■

C Additional Results and Features of the Model

C.1 Comparative Statics: Risk Aversion and Demand Data Relative Cost

To explore the role of risk aversion and the possibility of unbalanced technological change, we take the comparative statics of the static ($\pi = 0$) version of the model with respect to absolute risk aversion ρ and the relative cost of demand data χ_x. We begin with risk aversion.

$$\frac{d(\frac{C}{D})}{d\rho} = \frac{\rho^4\chi_x\left(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\right)-2K^2}{2K^2\rho^3\left(\frac{\chi_x}{L}\right)^{3/2}\sqrt{2(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}})}\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}}$$

The sign is the same as the sign of the numerator. Manipulate the numerator to get

$$\begin{aligned}
&\rho^4\chi_x\left(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\right)-2K^2>0\\
&1-\frac{2K^2}{\rho^4\chi_x}>\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\\
&1+\frac{4K^4}{\rho^8\chi_x^2}-\frac{4K^2}{\rho^4\chi_x}>1-\frac{4K^2}{\rho^4\chi_x}\\
&\frac{4K^4}{\rho^8\chi_x^2}>0
\end{aligned}$$

The interpretation is that, as agents become more risk averse, the signal-to-noise ratio of prices deteriorates.

Next, we explore changes in the shadow cost of demand data, without changing the ability to process fundamental data.

$$\frac{d(\frac{C}{D})}{d\chi_x} = \frac{\sqrt{\frac{\chi_x}{K}}\left(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\right)}{\sqrt{2(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}})}\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}}>0$$

which means the signal-to-noise ratio of prices deteriorates (falls in absolute value) as the marginal cost of demand data increases.

C.2 Extension: Informed and Uninformed Investors

C.2.1 Bayesian Updating

Throughout this section, we denote informed investors by i and uninformed investors by i'. We use the same notation for any relevant aggregates. The analysis of an informed individual investor is identical to the baseline model. For uninformed investor i', the optimal quantity of asset demand has the same form, except $\Omega_{fi't}=\Omega_{xi't}=0$.

Next, we turn to the aggregation.

Average expectations and precisions: The price information content for an informed investor i is $\Omega_{pit}\equiv(C_t/D_t)^2(\tau_x+\Omega_{xit})$, and for an uninformed investor i' is $\Omega_{pi't}\equiv(C_t/D_t)^2\tau_x$. Since all investors within the same group are ex-ante identical, they make identical information decisions. Thus, $\Omega_{pit}=\Omega_{pt}$

($\Omega_{pi't}=\Omega'_{pt}$) for all informed (uninformed) investors i (i'). The realized price signal still differs because the signal realizations are heterogeneous. Thus for informed investors

$$\int\eta_{pit}di=\frac{1}{C_t}(p_t-A_t-B(d_t-\mu))-\frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}x_{t+1}$$

And for uninformed investors

$$\int\eta_{pi't}di=\frac{1}{C_t}(p_t-A_t-B(d_t-\mu))$$

Next, we add equivalent definitions of the conditional variance / precision terms that simplify notation for uninformed investors.

$$\begin{aligned}
\hat{V}'_t &= (\tau_0 + (C_t/D_t)^2\tau_x)^{-1} \\
\Omega_t'^{-1} &= \pi C_{t+1}^2\tau_0^{-1} + \pi D_{t+1}^2\tau_x^{-1} + (1+\pi B_{t+1})^2\hat{V}'_t \\
Z'_t &= \frac{\pi\rho}{r}(r-\pi G)(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) = Z_t \\
\Omega_t'^{-1} &= \frac{r}{\rho(r-\pi G)}Z'_t + (\frac{r}{r-\pi G})^2\hat{V}'_t
\end{aligned}$$

Note that future information risk is the same for the two types of investors, since it is by definition unlearnable today.

Next, we can compute the average expectations

$$\begin{aligned}
&\int E[y_{t+1}|\bar{\mathcal{I}}_{i't}]\,di' = \hat{V}'_t\Omega'_{pt}\frac{1}{C_t}(p_t - A_t - B(d_t-\mu)) = (1-\tau_0\hat{V}'_t)\frac{1}{C_t}(p_t - A_t - B(d_t-\mu)) \\
&\int E[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_{i't}]\,di' = A_t + (1+\pi B)E[d_{t+1}|\bar{\mathcal{I}}'_t] = A_t + (1+\pi B)\left(\mu + G(d_t-\mu) + E[y_{t+1}|\bar{\mathcal{I}}'_t]\right).
\end{aligned}$$

C.2.2 Solving for Equilibrium Prices

The price conjecture is again

$$p_t = A_t + B_t(d_t-\mu) + C_t y_{t+1} + D_t x_{t+1} \tag{125}$$

We will solve for the prices for general supply of asset, $\bar{x}$, although in the main text it is normalized to one unit.

The average price signal in the economy is

$$\lambda\int \eta_{pi}di + (1-\lambda)\int \eta_{pi'}di = \frac{1}{C_t}(p_t - A_t - B_t(d_t-\mu)) - \lambda\frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}x_{t+1}$$

where $\Omega_{pt}^{-1} = (D_t/C_t)^2 Var(x_{t+1}|\bar{\mathcal{I}}_t)$.

Solving for non-stationary equilibrium prices. To solve for equilibrium prices, we start from the portfolio first-order condition for investors (7) and equate total demand with total supply. The total risky asset demand (excluding noisy demand) is

$$\begin{aligned}
&\lambda\int q_{it}di + (1-\lambda)\int q_{i't}di' \\
&= \frac{\lambda}{\rho}\Omega_t\left[\pi A_{t+1} + (1+\pi B_{t+1})\left(\mu + G(d_t-\mu) + \hat{V}_t\left[\Omega_{ft}y_{t+1} + \Omega_{pt}\frac{1}{C_t}(p_t - A_t - B_t(d_t-\mu)) - \frac{C_t}{D_t}\Omega_{xt}x_{t+1}\right]\right) - \pi B_{t+1}\mu - p_t r\right] \\
&\quad + \frac{1-\lambda}{\rho}\Omega'_t\left[\pi A_{t+1} + (1+\pi B_{t+1})\left(\mu + G(d_t-\mu) + \hat{V}'_t\Omega'_{pt}\frac{1}{C_t}(p_t - A_t - B_t(d_t-\mu))\right) - \pi B_{t+1}\mu - p_t r\right].
\end{aligned}$$

The market clearing condition equates the expression above to the residual asset supply $\bar{x} + x_{t+1}$. To simplify notation, let

$$\begin{aligned}
\bar{\Omega}_t &= \lambda\Omega_t + (1-\lambda)\Omega'_t \\
\lambda_{It} &= \frac{\lambda\Omega_t}{\bar{\Omega}_t}, \quad \lambda_{Ut} = \frac{(1-\lambda)\Omega'_t}{\bar{\Omega}_t} = 1 - \lambda_{It}.
\end{aligned}$$

Matching the coefficients on $(d_t - \mu)$ yields:

$$B_t = \left[r - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}'_t\Omega'_{pt}\right)\frac{1}{C_t}\right]^{-1}\left[(1+\pi B_{t+1})G - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}'_t\Omega'_{pt}\right)\frac{B_t}{C_t}\right]$$

Multiplying on both sides by the inverse term

$$rB_t - (1+\pi B_{t+1})(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}')\frac{1}{C_t}B_t = (1+\pi B_{t+1})G - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}'\right)\frac{B_t}{C_t}$$

and canceling the last term on both sides yields

$$B_t = \frac{1}{r}(1+\pi B_{t+1})G \tag{126}$$

As long as r and G do not vary over time, a stationary solution for B exists. That stationary solution would be (10).

Next, collecting all the terms in y_{t+1}

$$\begin{aligned}
&\frac{\lambda}{\rho}\Omega_t\left[(1+\pi B_{t+1})\left(\hat{V}_t\left[\Omega_{ft}y_{t+1} + \Omega_{pt}y_{t+1}\right]\right) - C_t y_{t+1} r\right] + \frac{1-\lambda}{\rho}\Omega_t'\left[(1+\pi B_{t+1})\left(\hat{V}_t'\Omega_{pt}'y_{t+1}\right) - C_t y_{t+1} r\right] = 0\\
&\lambda\Omega_t(1+\pi B_{t+1})\hat{V}_t\left[\Omega_{ft} + \Omega_{pt}\right] + (1-\lambda)\Omega_t'(1+\pi B_{t+1})\hat{V}_t'\Omega_{pt}' = rC_t\bar{\Omega}_t\\
&\lambda_{It}\bar{\Omega}_t(1+\pi B_{t+1})(1-\tau 0\hat{V}_t) + \lambda_{Ut}\bar{\Omega}_t(1+\pi B_{t+1})(1-\tau_0\hat{V}_t') = rC_t\bar{\Omega}_t.
\end{aligned}$$

Thus, C_t simplifies to

$$C_t = \frac{1}{r-\pi G}\left(1 - \tau_0(\lambda_{It}\hat{V}_t + (1-\lambda_{It})\hat{V}_t')\right).$$

Similar to $\bar{\Omega}_t$, let

$$\bar{\hat{V}}_t = (\lambda_{It}\hat{V}_t + (1-\lambda_{It})\hat{V}_t'),$$

which in turn implies that

$$C_t = \frac{1}{r-\pi G}\left(1 - \tau_0\bar{\hat{V}}_t\right). \tag{127}$$

Finally, we collect the terms in x_{t+1}.

$$D_t = [r - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}'\right)\frac{1}{C_t}]^{-1}[-\rho\Omega_t^{-1} - (1+\pi B_{t+1})\lambda_{It}\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}]$$

We multiply by the inverse term, and then use $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt})$ and $\Omega_{pt}' = (C_t/D_t)^2\tau_x$ to get

$$rD_t - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\frac{C_t}{D_t}(\tau_x + \Omega_{xt}) + (1-\lambda_{It})\hat{V}_t'\frac{C_t}{D_t}\tau_x\right) = -\rho\Omega_t^{-1} - (1+\pi B_{t+1})\lambda_{It}\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}$$

By substituting in B in the stationary solution and using the $\bar{\hat{V}}_t$, we get

$$\begin{aligned}
D_t &= \frac{1}{r-\pi G}\bar{\hat{V}}_t\tau_x\frac{C_t}{D_t} - \frac{\rho}{r}\bar{\Omega}_t^{-1}\\
D_t &= \frac{1}{r-\pi G}\left[\left(\tau_x\frac{C_t}{D_t} - \frac{r\rho}{r-\pi G}\right)\bar{\hat{V}}_t - Z_t\right]
\end{aligned} \tag{128}$$

Next we compute the expression for informed trader demand, q_t. Since A_{t+1}, B, C_{t+1} and D_{t+1} are non-random (conditional on $\bar{\mathcal{I}}_t$), y_{t+2} and x_{t+2} are independent of the elements of $\bar{\mathcal{I}}_t$ (and so $\mathbb{E}[z_{t+2}|\bar{\mathcal{I}}_t] = \mathbb{E}[z_{t+2}] = 0$ for

$z \in \{x, y\}$) it follows that:

$$
\begin{aligned}
\mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] &= \pi A_{t+1} + (1+\pi B)\mathbb{E}[d_{t+1}|\mathcal{I}_{it}] - \pi B\mu + \pi C_{t+1}\mathbb{E}[y_{t+2}|\mathcal{I}_{it}] + \pi D_{t+1}\mathbb{E}[x_{t+2}|\mathcal{I}_{it}] \\
&= \pi A_{t+1} + (1+\pi B)\mathbb{E}[\mu + G(d_t - \mu) + y_{t+1}|\mathcal{I}_{it}] - \pi B\mu \\
&= \pi A_{t+1} + \mu + \pi B\mu + (1+\pi B)G(d_t - \mu) + (1+\pi B)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}] - \pi B\mu \\
&= \pi A_{t+1} + \mu + (1+\pi B)G(d_t - \mu) + (1+\pi B)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}].
\end{aligned}
$$

which implies that

$$
\begin{aligned}
\mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] - rp_t &= \mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] - r\left(A_t + B(d_t - \mu) + C_t y_{t+1} + D_t x_{t+1}\right) \\
&= \big(\pi A_{t+1} + \mu - rA_t\big) + \big((1+\pi B)G - rB\big)(d_t - \mu) + \big(1+\pi B\big)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}] - rC_t y_{t+1} - rD_t x_{t+1}
\end{aligned}
$$

As a result, we obtain

$$
\begin{aligned}
\mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] &= \pi A_{t+1} + \mu + (1+\pi B)G(d_t - \mu) + (1+\pi B)\frac{\Omega_{pit}\eta_{pt} + \Omega_{fit}\eta_{ft}}{\tau_0 + \Omega_{pit} + \Omega_{fit}} \\
&= \pi A_{t+1} + \mu + \frac{rG}{r - \pi G}(d_t - \mu) + \frac{r}{r - \pi G}\frac{\Omega_{pt}\eta_{pit} + \Omega_{ft}\eta_{fit}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}
\end{aligned}
$$

where the second line uses symmetry in information choices. Since by Bayes' rule,

$$
\mathbb{E}[x_{t+1}|\eta_{xit}] = \frac{\Omega_{xt}\eta_{xt}}{\tau_x + \Omega_{xt}}.
$$

And

$$
\begin{aligned}
\Omega_t^{-1} := \mathrm{Var}[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t] &= \pi(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) + (1+\pi B)^2\left(\tau_0 + \Omega_{ft} + \Omega_{pt}\right)^{-1} \\
&= \pi(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) + (1+\pi B)^2\left(\tau_0 + \Omega_{ft} + (C/D)^2(\tau_x + \Omega_{xt})\right)^{-1}.
\end{aligned}
$$

Next, we substitute the above expressions in q_t to obtain:

$$
\begin{aligned}
q_t =& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big) + \big((1+\pi B)G - rB\big)(d_t - \mu) + \big(1+\pi B\big)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}] - rC_t y_{t+1} - rD_t x_{t+1}\right] \\
=& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big) + \frac{r}{r - \pi G}\frac{\Omega_{ft}(y_{t+1} + \tilde{\epsilon}_{fit}) + \Omega_{pt}\left(y_{t+1} + (D_t/C_t)(x_{t+1} - \mathbb{E}[x_{t+1}|\eta_{xit}])\right)}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - rC_t y_{t+1} - rD_t x_{t+1}\right] \\
=& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big)\right] + \frac{r\Omega_t}{\rho}\left[\frac{1}{r - \pi G}\frac{\Omega_{ft} + \Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - C_t\right]y_{t+1} + \frac{\Omega_t}{\rho}\frac{r}{r - \pi G}\left[\frac{\Omega_{ft}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\right]\tilde{\epsilon}_{fit} \\
&+ \frac{r\Omega_t}{\rho}\left[\frac{1}{(r - \pi G)}\frac{D_t}{C_t}\frac{\Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\frac{\tau_x x_{t+1} - \Omega_x\tilde{\epsilon}_{xit}}{\tau_x + \Omega_{xt}} - D_t x_{t+1}\right] \\
=& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big)\right] + \frac{r\Omega_t}{\rho}\left[\frac{1}{r - \pi G}\frac{\Omega_{ft} + \Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - C_t\right]y_{t+1} + \frac{\Omega_t}{\rho}\frac{r}{r - \pi G}\left[\frac{\Omega_{ft}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\right]\tilde{\epsilon}_{fit} \\
&+ \frac{r\Omega_t}{\rho}\left[\frac{1}{(r - \pi G)}\frac{C_t}{D_t}\frac{\tau_x}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - D_t\right]x_{t+1} - \frac{r\Omega_t}{\rho}\left[\frac{1}{(r - \pi G)}\frac{C_t}{D_t}\frac{\Omega_{xt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\right]\tilde{\epsilon}_{xit}
\end{aligned}
$$

where the last equality substitutes $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt})$.

Covariance between q_t and x_{t+1}. Note that the first term in q_t is a constant and does not appear in any covariance. Moreover, $y_{t+1} \sim \mathcal{N}(0, \tau_0^{-1})$ and iid, and $|G| < 1$, thus d_{t+1} is a (weakly) stationary AR(1) process and so $\mathbb{E}[d_{t+1}] = \mu < \infty$. Thus with $x_{t+1} \sim \mathcal{N}(0, \tau_x^{-1})$, we have $\mathbb{E}[x_{t+1}] = 0$ and so $\mathrm{Cov}(q_t, x_{t+1}) = \mathbb{E}[q_t x_{t+1}]$. Lastly, because y_{t+1}, $\tilde{\epsilon}_{xit}$ and $\tilde{\epsilon}_{fit}$ are iid, they are independent of x_{t+1}, thus $\mathbb{E}[x_{t+1}y_{t+1}] = \mathbb{E}[x_{t+1}\tilde{\epsilon}_{xit}] = \mathbb{E}[x_{t+1}\tilde{\epsilon}_{fit}] = 0$.

Therefore,

$$\begin{aligned}
\mathrm{Cov}(q_t, x_{t+1}) = \mathbb{E}[q_t x_{t+1}] &= \frac{r\Omega_t}{\rho}\left[\frac{1}{(r-\pi G)}\frac{C_t}{D_t}\tau_x \hat{V}_t - D_t\right]\tau_x^{-1} \\
&= \frac{r\Omega_t}{\rho}\left[\frac{1}{(r-\pi G)}\frac{C_t}{D_t}\tau_x \hat{V}_t - \frac{1}{r-\pi G}\bar{\hat{V}}_t \tau_x \frac{C_t}{D_t} + \frac{\rho}{r}\bar{\Omega}_t^{-1}\right]\tau_x^{-1} \\
&= \frac{r\Omega_t}{\rho}\left[\frac{\rho}{r}\bar{\Omega}_t^{-1} - \frac{1}{(r-\pi G)}\frac{C_t}{D_t}\tau_x(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t)\right]\tau_x^{-1} \\
&= \frac{\Omega_t}{\lambda\Omega_t + (1-\lambda)\Omega_t'}\tau_x^{-1} - \frac{r\Omega_t}{\rho(r-\pi G)}\frac{C_t}{D_t}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t),
\end{aligned}$$

which is equation (138) in the main text.

Covariance between q_t and y_{t+1}. Since $\mathbb{E}[y_{t+1}] = 0$, $\mathrm{Cov}(q_t, y_{t+1}) = \mathbb{E}[q_t y_{t+1}]$. Additionally, as y_{t+1} is independent of x_{t+1}, ϵ_{xt+1}, and ϵ_{ft+1}, then using the same expression for q_t, we have:

$$\begin{aligned}
\mathrm{Cov}(q_t, y_{t+1}) = \mathbb{E}[q_t y_{t+1}] &= \frac{r\Omega_t}{\rho}\left[\frac{1}{r-\pi G}\frac{\Omega_{ft} + \Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - C_t\right]\tau_0^{-1} \\
&= \frac{r\Omega_t}{\rho}\left[\frac{1}{r-\pi G}(1-\tau_0\hat{V}_t) - \frac{1}{r-\pi G}\left(1-\tau_0\bar{\hat{V}}_t\right)\right]\tau_0^{-1} \\
&= \frac{r\Omega_t}{\rho(r-\pi G)}(\bar{\hat{V}}_t - \hat{V}_t) = \frac{r\Omega_t}{\rho(r-\pi G)}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t) \\
&= \frac{r}{\rho(r-\pi G)}(\pi C_{t+1}^2\tau_0^{-1} + \pi D_{t+1}^2\tau_x^{-1} + (\frac{r}{r-\pi G})^2\hat{V}_t)^{-1}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t),
\end{aligned}$$

which is equation (139) in the main text.

C.2.3 Static Economy, $\pi = 0$

In the static economy, where $\pi = 0$, we can further simplify equations (138) and (139). First, note that with $\pi = 0$, $\bar{\Omega}_t = \lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}$. Thus we have

$$\begin{aligned}
\mathrm{Cov}(q_t, x_{t+1}) &= \frac{\hat{V}_t^{-1}}{\lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}}\tau_x^{-1} - \frac{C_t}{D_t}\frac{1}{\rho}\hat{V}_t^{-1}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t) \\
&= \frac{\tau_x^{-1}}{\lambda + (1-\lambda)\frac{\hat{V}_t'}{\hat{V}_t}} - \frac{C_t}{D_t}\mathrm{Cov}(q_t, y_{t+1}) \\
&= \frac{\tau_x^{-1}}{\lambda + (1-\lambda)\frac{\tau_0 + (C/D)^2\tau_x}{\tau_0 + \Omega_{ft} + (C/D)^2(\tau_x + \Omega_{xt})}} - \frac{C_t}{D_t}\mathrm{Cov}(q_t, y_{t+1}) \qquad (129)
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{Cov}(q_t, y_{t+1}) &= \frac{1}{\rho}\hat{V}_t^{-1}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t) = \frac{1}{\rho}\hat{V}_t^{-1}(1-\lambda)\frac{\hat{V}_t'^{-1}}{\lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}}(\hat{V}_t' - \hat{V}_t) \\
&= \frac{1-\lambda}{\rho}\frac{1}{\lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}}(\hat{V}_t^{-1} - \hat{V}_t'^{-1}) = \frac{1-\lambda_{It}}{\rho}\frac{\Omega_{ft} + (C/D)^2\Omega_{xt}}{\tau_0 + (C/D)^2\tau_x} \\
&= \frac{1-\lambda}{\rho}\frac{\Omega_{ft} + (C/D)^2\Omega_{xt}}{\tau_0 + (C/D)^2\tau_x + \lambda\left(\Omega_{ft} + (C/D)^2\Omega_{xt}\right)} = \frac{1-\lambda}{\rho}\left(\lambda + \frac{\tau_0 + (C/D)^2\tau_x}{\Omega_{ft} + (C/D)^2\Omega_{xt}}\right)^{-1} \qquad (130)
\end{aligned}$$

We use equation (130) to compute the total effective precision acquired about innovation in dividends:

$$\Omega_{ft} + (\frac{C_t}{D_t})^2\Omega_{xt} = \frac{\rho \text{Cov}(q_t, y_{t+1})}{1 - \lambda\left(1 + \rho \text{Cov}(q_t, y_{t+1})\right)} \left(\tau_0 + (\frac{C_t}{D_t})^2 \tau_x\right), \tag{131}$$

and then we use that in equation (129) to infer the size of informed trading:

$$\begin{aligned} \lambda &= \frac{\left(1 - \tau_x \text{Cov}(q_t, x_{t+1})\right)\left(\tau_0 + (\frac{C_t}{D_t})^2\tau_x\right) + \text{Cov}(q_t, y_{t+1})\left(1 - \tau_x \frac{C_t}{D_t}\left(\tau_0 + (\frac{C_t}{D_t})^2\tau_x\right)\right)}{\tau_x \text{Cov}(q_t, y_{t+1})\left(\text{Cov}(q_t, x_{t+1}) + \frac{C_t}{D_t}\text{Cov}(q_t, y_{t+1})\right)} \\ &= \frac{\text{Cov}(q_t, y_{t+1}) + \left(\tau_0 + (\frac{C_t}{D_t})^2\tau_x\right)\left[1 - \tau_x\left(\text{Cov}(q_t, x_{t+1}) + \frac{C_t}{D_t}\text{Cov}(q_t, y_{t+1})\right)\right]}{\text{Cov}(q_t, y_{t+1})\tau_x\left(\text{Cov}(q_t, x_{t+1}) + \frac{C_t}{D_t}\text{Cov}(q_t, y_{t+1})\right)}. \end{aligned} \tag{132}$$

Thus equations (131), (132), and (76) can be used to make the same inference for the static economy.

C.3 CRRA utility and heterogeneous risk aversion

Solving a CRRA portfolio problem with information choice is challenging because the equilibrium prices are no longer linear functions of the shocks. With two sources of information, this non-linearity implies that one of the signals no longer has normally-distributed signal noise about the asset fundamental. That makes combining the two sources of information analytically intractable.

At the same time, we can come very close to CRRA with state-dependent risk aversion in exponential utility. For example, suppose we set the absolute risk aversion to $\rho_{it} = [(\gamma-1)ln(C_{it}) + ln(\gamma-1)]/C_{it}$, where C_{it} denotes consumption at time t. In this case, the two utility functions would be identical: $exp(\rho_{it}C_{it}) = (\gamma-1)C_{it}^{\gamma-1}$. The problem with doing so is that risk aversion becomes a random variable that depends on asset payoffs, through C_{it}. But suppose we do a close approximation to this. Suppose we allow ρ_{it} to be a function of $E_t[C_{it}]$, where t denotes the beginning of the period t information set, prior to any information processing. This approximation implies that the utility is

$$U(C_{it}) \approx -exp\left[-\left((\gamma-1)ln(E_t[C_{it}]) + ln(\gamma-1)\right)\frac{C_{it}}{E_t[C_{it}]}\right]$$

We can then rewrite this log-linear approximation in a form that is like exponential utility $-\exp(-\rho_{it}C_{it})$, with a coefficient of absolute risk aversion

$$\rho_{it} \equiv [(\gamma-1)ln(E_t[C_{it}]) + ln(\gamma-1)]/E_t[C_{it}]. \tag{133}$$

This form of risk aversion introduces wealth effects on portfolio choices but preserves linearity in prices.

Each investor chooses a number of shares q of the risky asset to maximize (C.3) subject to budget constraint (3). The first-order condition of that problem is

$$q_{it} = \frac{E[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] - rp_t}{\rho_{it}Var[f_{it}|\mathcal{I}_{it}]} - h_{it}$$

Given this optimal investment choice, we can impose market clearing (6) and obtain a price function that is linear in asset payoffs and noisy demand shocks:

$$p^{CRRA} = A + B(d_t - \mu) + Cy + Dx$$

where A, B, C, and D are the same as before, except that in place of each homogeneous ρ is $\bar{\rho} \equiv (\int 1/\rho_i di)^{-1}$, which is the harmonic mean of investors' risk aversions, and captures aggregate wealth effects.

Of course, in this formulation, if investors' wealth grows over time, asset prices trend up. In that sense, the solution changes. However, it is still the case that the decision to learn about fundamental or demand data depends on $(C/D)^2$. But now wealth is an additional force that moves D_t over time. Because ρ^2 shows up in the numerator once and ρ shows up in the denominator, these effects largely cancel each other out. Quantitatively, the effect on D is small. But large changes in wealth can now have an effect on data choices.

C.4 A Linear or Entropy-Based Information Constraint

The reason we use a constraint that is convex in signal precision is that it produces an interior optimum. A constraint that is linear in signal precision or that takes the form of an entropy reduction produces information choices that are corner solutions. Such corner solutions have the same forces as those at work in our version of the model. The reason is that the main results – the substitutability of fundamental data, the complementarity of demand data and the interactions between the two information types – all arise from the marginal utility for information, not from the cost formulation. However, keeping track of corner solutions introduces some additional complexity. This subsection describes how one can work out that version of the one-period asset model ($\pi = 0$).

Step 1. Individual objective. This is still the same as before:

$$\max\ \Omega_f + (\frac{C}{D})^2\Omega_x$$

subject to an mutual information (entropy - reduction) constraint, $H(x,\eta_x) + H(y,\eta_y) \leq K$ or a linear constraint,

$$\Omega_f + \chi_x\Omega_x = K$$

Step 2. Information aggregation and price coefficients given information choices. We follow very the derivations of Appendix A.4, which solves the model for two types of agents. There, one set of agents has both types of information. Here, one group specializes in fundamental data, and another in demand data (denoted by $'$ agents).

Let λ denote the fraction of agents specializing in fundamental data and $1-\lambda$ in demand data. Moreover, let $\xi = \frac{C}{D}$. Using the same notation as before, in the static model, we have that for an individual fundamental specialist versus demand specialist:

$$\Omega_i = \hat{V}_i^{-1} = \tau_0 + \Omega_{fi} + \xi^2\tau_x$$
$$\Omega_i' = \hat{V}_i'^{-1} = \tau_0 + \xi^2(\tau_x + \Omega_{xi})$$

where the first equality in each line is true because the model is static. And since all fundamental agents are identical, and all demand agents are identical, we have $\Omega = \Omega_i$ and $\Omega' = \Omega_i'$. Thus, to aggregate,

$$\bar{\Omega} = \lambda\Omega + (1-\lambda)\Omega'$$
$$\bar{\hat{V}}^{-1} = \frac{\lambda\Omega}{\bar{\Omega}}\hat{V}^{-1} + \frac{(1-\lambda)\Omega'}{\bar{\Omega}}\hat{V}'^{-1} = \frac{1}{\bar{\Omega}}$$

The information aggregation is therefore very simple

$$\bar{\Omega} = \bar{\hat{V}}^{-1} = \tau_0 + \Omega_f + \xi^2(\tau_x + \Omega_x)$$

where $\Omega_x = \frac{(1-\lambda)K}{\chi_x}$, and $\Omega_f = \lambda K = K - \chi_x\Omega_x$.

Step 3. Solving for optimal information choices. Let Ω_f (Ω_x) denote the total information of agents specializing in fundamental (demand) analysis. We have

$$\xi = \frac{C}{D} = \frac{-\rho + \sqrt{\rho^2 - 4\Omega_f \Omega_x}}{2\Omega_x}$$

and, from the aggregation step,

$$\Omega_f = K - \chi_x \Omega_x$$

We know that when $K \to 0$, $\Omega_x \to 0$ and $\xi = -\frac{\Omega_f}{\rho}$ while $\xi \geq -1$. Thus if $K < \rho$, $\Omega_f = K$ and $\Omega_x = 0$.

Next assume $K > \rho$. For both types of information to be processed, it must be that $\xi = -1$. Let $\Omega_f = K - \chi_x \Omega_x$ and substitute into ξ equation above and solve

$$-1 = \frac{-\rho + \sqrt{\rho^2 - 4\Omega_x (K - \chi_x \Omega_x)}}{2\Omega_x}$$

This yields

$$\Omega_x = \frac{K - \rho}{\chi_x - 1}$$

Since $K > \rho$, it must be that $\chi_x > 1$ for $\Omega_x > 0$, a valid solution. Moreover, $\Omega_x < \frac{K}{\chi_x}$. Thus we must have

$$K < \rho \chi_x$$

As before, when information becomes abundant, no solution exists.

To summarize:

1. $K < \rho$: $\Omega_f = K$ and $\Omega_x = 0$.
2. $K > \rho$ while $K < \rho\chi_x$ and $\chi_x > 1$: $\Omega_f = \frac{\rho\chi_x - K}{\chi_x - 1}$ and $\Omega_x = \frac{K-\rho}{\chi_x - 1}$.
3. Otherwise there is no solution. Once $K > \rho$, it must be that $\chi_x > 1$, as otherwise there is no solution. Even when $\chi_x > 1$, as K becomes sufficiently large, $K > \rho\chi_x$, the solution ceases to exist.

C.5 The Real Economic Benefits of Price Information Sensitivity

We have argued that growth in financial technology has transformed the financial sector and affected financial market efficiency in unexpected ways. But why should we care about financial market efficiency? What are the consequences for real economic activity? There are many possible linkages between the financial and real sectors. In this section, we illustrate two possible channels through which changes in information sensitivity and price impact can alter the efficiency of real business investment.

Manager Incentive Effects The key friction in the first spillover model is that the manager's effort choice is unobserved by equity investors. The manager makes a costly effort only because he or she is compensated with equity. Managers only have an incentive to exert themselves if the value of their equity is responsive to their efforts. Because of this, the efficiency of a manager's effort choice depends on the asset price information sensitivity.

Of course, this friction reflects the fact that the wage is not an unconstrained optimal contract. The optimal compensation for the managers is to pay them for their effort directly, or to give them all the equity in their firm. We do not model the reasons why this contract is not feasible because it would distract from our main point. Our stylized sketch of a model is designed to show how commonly-used compensation contracts that tie wages to firm equity prices (e.g., options packages) also tie price information sensitivity to optimal effort.

Time is discrete and infinite. A single firm with profits d_{t+1} depends on a firm manager's labor choice l_t. Specifically, instead of the dividend process specified in Section 1, asset payoffs take the static form: $d_{t+1} = g(l_t) + y_{t+1}$, where g is increasing and concave and $y_{t+1} \sim N(0, \tau_0^{-1})$ is unknown at t. Because effort is unobserved, the manager's pay, w_t, is tied to the firm's equity price p_t: $w_t = \bar{w} + p_t$. However, effort is costly. We normalize the units of effort so that a unit of effort corresponds to a unit of utility cost. Insider trading laws prevent the manager from participating in the equity market. Thus the manager's objective is

$$U_m(l_t) = \bar{w} + p_t - l_t \tag{134}$$

The firm pays out all its profits each period as dividends to its shareholders. Firm equity purchased at time t is a claim to the present discounted stream of future profits $\{d_{t+1}, d_{t+2} \dots\}$.

Investors' preferences, endowments, budget constraint, and information choice sets are the same as they were before. The demand data signals are defined as before. Fundamental analysis now generates signals of the form $\eta_{fit} = g(l_t) + y_{t+1} + \tilde{\epsilon}_{fit}$, where the signal noise is $\tilde{\epsilon}_{fit} \sim N(0, \Omega_{ft})$. Investors choose the precision Ω_{ft} of this signal, as well as their demand signal Ω_{xt}. Equilibrium is defined as before, with the additional condition that the manager effort decision maximizes (134).

Solution As before, the asset market equilibrium has a linear equilibrium price:

$$p_t = A_t + C_t(g(l_t) + y_{t+1}) + D_t x_{t+1} \tag{135}$$

Notice that since dividends are not persistent, d_t is no longer relevant for the t price.

The firm manager chooses his effort to maximize (134). The first-order condition is $C_t g'(l_t) = 1$, which yields an equilibrium effort level $l_t = (g')^{-1}(1/C_t)$. Notice that the socially optimal level would set the marginal utility cost of effort equal to the marginal product, $g'(l_t) = 1$. When C_t is below one, managers under-provide effort relative to the social optimum because their stock compensation moves less than one-to-one with the true value of their firm.

Similar to before, the equilibrium level of price information sensitivity C is

$$C_t = \frac{1}{r}\left(1 - \tau_0 Var[g(l_t) + y_{t+1}|\bar{\mathcal{I}}_t]\right). \tag{136}$$

Thus, as more information is analyzed, dividend uncertainty ($Var[g(l_t) + y_{t+1}|\bar{\mathcal{I}}_t]$) falls, C_t rises and managers are better incentivized to exert optimal effort. While the model is stylized and the solution presented here is only a sketch, it is designed to clarify why trends in financial analysis matter for the real economy.

The most obvious limitation of the model is its single asset. One might wonder whether the effect would disappear if the asset's return was largely determined by aggregate risk, which is beyond the manager's control. However, if there were many assets, one would want to rewrite the compensation contract so that the manager gets rewarded for high firm-specific returns. This would look like benchmarked performance pay. If the contract focused on firm-specific performance, the resulting model would look similar to the single asset case here.

In short, this mechanism suggests that recent financial sector trends boost real economic efficiency. More data analysis – of either type – improves price information sensitivity, and thereby incentives. But this is only one possible mechanism that offers one possible conclusion. Our next example presents an alternative line of thought.

C.6 Real Economic Benefits of Liquidity

The second real spillover highlights a downside of financial technology growth. More information technology creates future information risk, which raises the risk of holding equity, raising the equity premium, and making capital more costly for firms. This enormously simplified mechanism is meant as a stand-in for a more nuanced relationship, such as that in Bigio (2015).

Suppose that a firm has a profitable investment opportunity and wants to issue new equity to raise capital for that investment. For every dollar of capital invested, the firm can produce an infinite stream of dividends, d_t. Dividends follow the same stochastic process as described in the original model. However, the firm needs funds to invest, which it raises those funds by issuing equity. The firm chooses the number of shares, $\bar{x}$, to maximize the total revenue raised (maximize output). Each share sells at price p, which is determined by the investment market equilibrium, minus the investment or issuance cost:

$$E[\bar{x}p - c(\bar{x})|\mathcal{I}_{it}]$$

The firm makes its choice conditional on the same prior information that all the investors have. But the firm does not condition on p. It does not take the price as given. Rather, the firm chooses $\bar{x}$, taking into account its impact on the equilibrium price. The change in issuance is permanent and unanticipated. The rest of the model is identical to the dynamic model in section 1.

Solution Given the new asset supply, $\bar{x}$, the asset market and information choice solutions to the problem are the same as before. But how the firm chooses $\bar{x}$ depends on how new issuances affect the asset price. When the firm issues new equity, all asset market participants are aware that new shares are coming online. Equity issuance permanently changes the known supply of the asset $\bar{x}$. Supply $\bar{x}$ enters the asset price in only one place in the equilibrium pricing formula, through A_t. Recall from (9) that

$$A_t = \frac{1}{r}\left[A_{t+1} + \frac{r\mu}{r-G} - \rho Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]\bar{x}\right]. \tag{137}$$

Taking A_{t+1} as given for the moment, $dA_t/d\bar{x} = -\rho Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]/r$.[21] In other words, the impact of a one-period change in the asset supply depends on the conditional variance (the uncertainty about) the future asset payoff, $p_{t+1} + d_{t+1}$. Recall from the discussion of the price impact of trades in Section 3.4 that in a dynamic model, more information analysis reduces dividend uncertainty but it can result in more uncertainty about future prices. These two effects largely offset each other.

When we simulate the calibrated model, we find a modest change in the payoff risk from these competing effects on the price impact of issuing new equity. To give the units of the price impact some meaning, the issuance cost is scaled by the average dividend payment so that it can be interpreted as the change in the price-dividend ratio from a one-unit change in equity supply. Thus a one-unit increase in issuance reduces the asset price by an amount equal to 4 months of dividends, on average.

We learn that technological progress in information analysis – of either type – initially makes asset payoffs slightly more uncertain, making it more costly to issue new equity. When we now take into account the fact that the increase in asset supply is permanent, the effect of issuance is amplified, relative to the one-period (fixed A_{t+1}) case. But when analysis becomes sufficiently productive, issuance costs decrease again, as the risk-reducing power of more precise information dominates.

Again, a key limitation of the model is its single asset. With multiple assets, one firm's issuance is a tiny change in the aggregate risk supply. But the change in the supply of firm-specific risk looks similar to this problem. If one were to evaluate this mechanism quantitatively, the magnitude would depend on how much the newly issued equity loads on the idiosyncratic versus the aggregate risk.

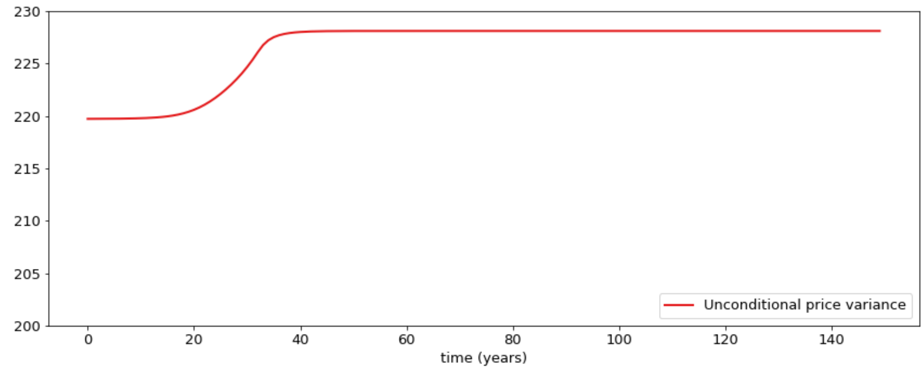
C.7 Price Volatility

One concern with the model is that the future information risk might manifest itself as an implausible rise in price volatility. Price volatility is $Var[p_t]$. Taking the unconditional variance of the model's pricing equation, we get

[21] In principle, a change in issuance $\bar{x}$ could change the payoff variance, $Var[p_{t+1}+d_{t+1}|\bar{\mathcal{I}}_t]$. However, in this setting, the conditional variance does not change because the information choices do not change. Information does not change because the marginal rate of transformation of fundamental and demand information depends on $(C_t/D_t)^2$, which is not dependent on $\bar{x}$. If there were multiple assets, issuance would affect information choices, as in Begenau, Farboodi and Veldkamp (2017).

$B_t^2 Var[d_t] + C_t^2 Var[y_{t+1}] + D_t^2 Var[x_{t+1}] = B_t^2 \tau_0^{-1}/(1-G^2) + C_t^2 \tau_0^{-1} + D_t^2 \tau_x^{-1}$. Figure 4 shows that the price volatility time series exhibits a modest increase. Because price is larger in magnitude than dividends are, a small increase in price volatility can offset a large decrease in dividend uncertainty.

Figure 4: **Price Volatility (model)** Price volatility is $Var[p_t] = \left(B^2/(1-G^2) + C_t^2\right)\tau_0^{-1} + D_t^2 \tau_x^{-1}$.



D Decomposing the Numerical Results

In this section, we explore what part of the results are attributable to the growth in fundamental information, what part to the growth in demand information, and what role future payoff risk plays.

D.1 Turning off Demand Data

For this next set of results, we turn off demand data by setting $\Omega_{xt} = 0$. We keep Ω_{ft}, on the same sequence that it was in the unconstrained model. Obviously, that is not an optimal choice for fundamental data in this setting because it leaves some data capacity unused. But it does allow for a clear comparison of results because it does not conflate the effects of less demand data with more or less fundamental data. In Figure 5, the amount of fundamental data analysis is exactly the same as in Figure 2. The only difference is that the results here have zero demand data analysis. In other words, we substitute $\Omega_{xt} = 0$ into pricing coefficient equations 11 and 12.

To highlight the differences between this no-demand-data version of the model and the original results, we plot each price coefficient as the difference from the level in the original model. Figure 6 reveals that the lack of demand data has only a tiny effect on C_t but a sizeable effect on D_t. Specifically, removing demand data makes the market significantly more illiquid. At high levels of data K_t, the price impact of a trade without demand data is nearly double what it would be with demand data. Note that the jump at the end of the plot is a relic of our calibration procedure. It arises because period 150 is assumed to be the steady state. Since the steady state is the same in both cases, the difference appears as zero. Nothing economically interesting occurs there.

Figure 7 plots how much of the precision in the investors' forecast of $p_{t+1} + d_{t+1}$ comes from their demand data. When K_t is low (left side), there is almost no demand data processing. So, readding the equilibrium amount of demand data adds almost nothing to forecast precision. When K_t gets high (on the right), almost 100% of the forecast precision comes from demand data. The forecasts without demand data have almost no precision.

Alternatively, we turn off demand data by setting $\Omega_{xt} = 0$ and re-optimize over Ω_{ft} given a path of K_t. Here, data capacity is used to its fullest. Trivially, we have $\Omega_{ft} = \sqrt{K_t}$. In the long-run, we see clearly that the shift in demand analysis has no effects on market efficiency, whether it is measured by the steady state value of C, D, or

Figure 5: **Price Information Sensitivity (C_t) and the Price Impact of Trades ($|D_t|$) without Demand Data.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal value of demand information relative to fundamental information. The x-axis is time in years.

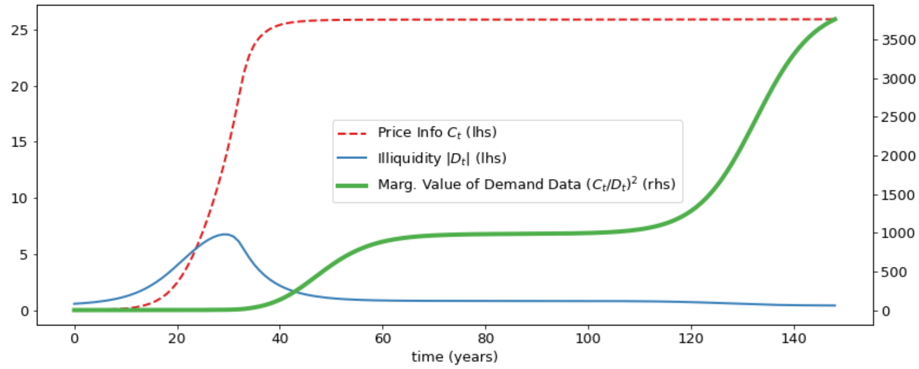
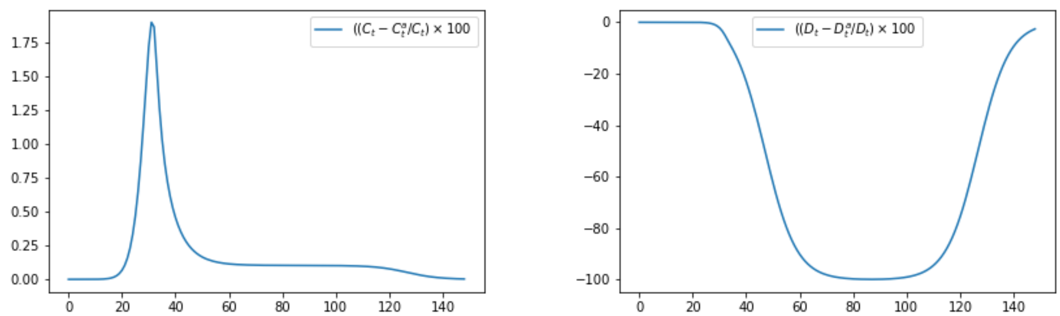


Figure 6: **Change in C_t and $|D_t|$ from Removing Demand Data.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. These plots report the percentage change in the coefficient that would result from changing $\Omega_{xt} = 0$ back to its optimal level. The x-axis is time.



C/D. This exercise also illustrates the effects of demand analysis on the transition path. Allowing demand data to adjust endogenously smooths out the bumps in the marginal value of demand information ($C/|D|$).

Similar to the no-demand-data version with the same sequence for Ω_{ft} as in the unconstrained model, we plot each price coefficient as a difference from the level in the original model. Figure 9 reveals that the lack of demand data has only a tiny effect on C_t, but it has a sizeable effect on D_t.

In this scenario, we repeat the exercise of computing and plotting how much of the precision in investors' forecasts of $p_{t+1} + d_{t+1}$ comes from their demand data. The result looks indistinguishable from Figure 7. When K_t is low (left side), there is almost no demand data processing. When K_t becomes high (on the right), the forecast precision comes almost entirely from demand data processing.

D.2 Turning off Fundamental Data Growth

Next, we perform the opposite exercise to see what effect fundamental information has on the results. We turn off fundamental data by setting $\Omega_{ft} = 0.01$. Unlike before, when we set the demand data precision exactly to zero, we cannot set the fundamental precision to zero. Doing so would trivially give us $C_t = 0$ no matter what. Instead, to see

Figure 7: **Additional Payoff Forecast Precision from Demand Data.** V^{-1} is $Var[d_t|\bar{\mathcal{I}}_t]^{-1}$ in the main model. V_a^{-1} is $Var[d_t|\bar{\mathcal{I}}_t]^{-1}$ in the model without demand analysis ($\Omega_{xt} = 0$). The vertical axis, $(V^{-1} - V_a^{-1})/V^{-1}$ represents the fraction of forecast precision due to demand analysis. The x-axis is time in years.

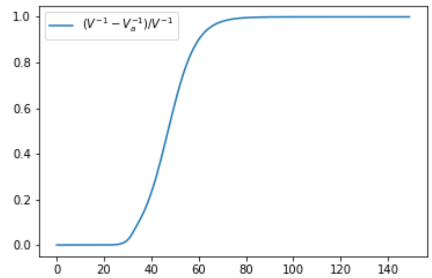
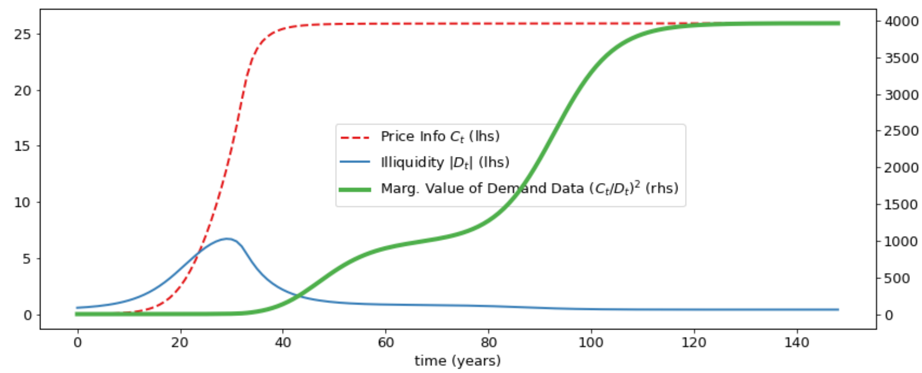


Figure 8: **Price Information Sensitivity (C_t) and the Price Impact of Trades ($|D_t|$) without Demand Data, Ω_{ft} Optimized.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal value of demand information relative to fundamental information. The x-axis is time in years.



the role of demand analysis, we hold the fundamental data precision at a small, exogenous amount. For Ω_{xt}, we keep it on the same sequence as it was in the unconstrained model. In Figure 10, the amount of demand data analysis is exactly the same as in Figure 2. The only difference is that the results here have zero fundamental data analysis. In other words, we substitute $\Omega_{ft} = 0$ into pricing coefficient equations 11 and 12.

To highlight the differences between this no-demand-data version of the model and the original results, we plot each price coefficient as the difference from the level in the original model. Figure 11 reveals that the lack of fundamental data has only a tiny effect on D_t but a sizeable effect on C_t. This is the reverse of the previous exercise. Specifically, removing fundamental data makes the market significantly less sensitive to dividend innovations. At low levels of data (K_t low), the price dividend sensitivities, with and nearly-without fundamental data are very different. At high levels of K_t, since most of the information comes from demand data anyway, the levels with and nearly-without fundamental data are almost the same.

D.3 Turning Off Dynamics

To see the role that long-lived assets play in the results, it is useful to remove all dynamic effects by setting $\pi = 0$ and seeing how the results change. We can only report results from the first few periods because, after that the

Figure 9: **Change in C_t and $|D_t|$ from Removing Demand Data and Optimizing over Ω_{ft}.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. These plots report the percentage change in the coefficient that would result from changing $\Omega_{xt} = 0$ and $\Omega_{ft} = \sqrt{K_t}$ back to its optimal level. The x-axis is time.

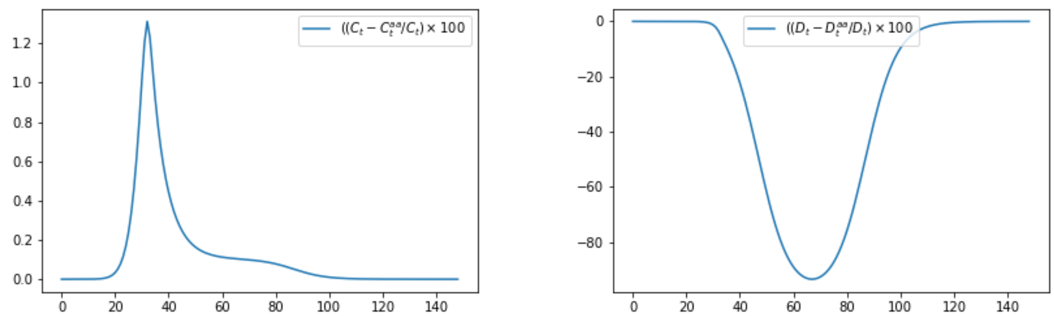
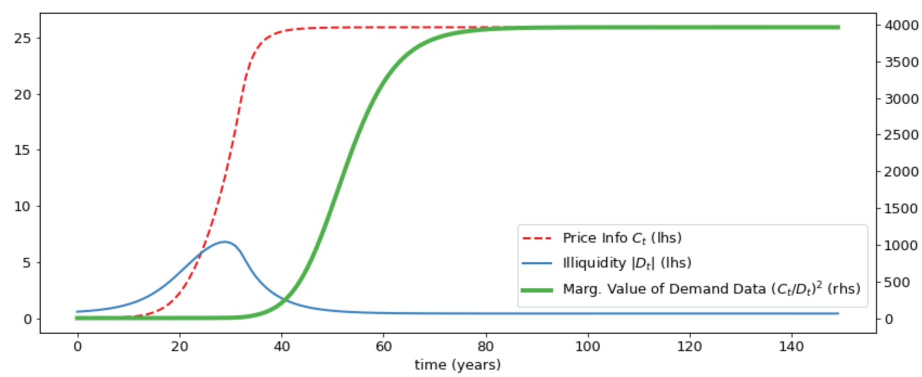


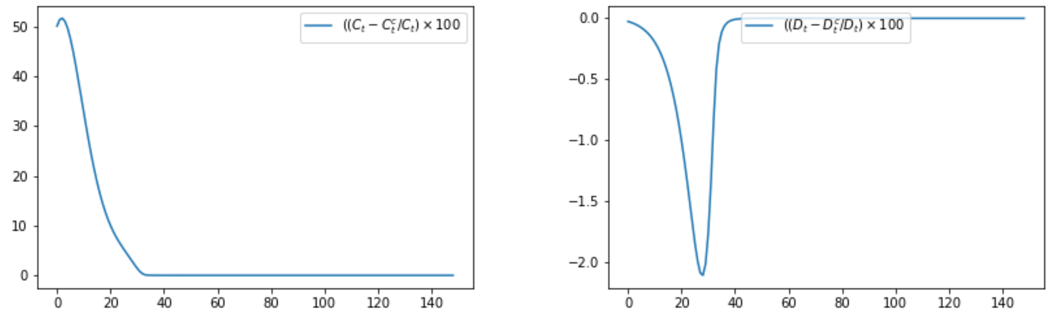
Figure 10: **Price Information Sensitivity (C_t) and the Price Impact of Trades ($|D_t|$) without Fundamental Data.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal value of demand information relative to fundamental information. The x-axis is time in years.



equilibrium no longer exists.

The main difference between the static and dynamic models is that the magnitudes are quite different. The static model features price sensitivity to fundamentals and demand shocks that are between four and ten times less than the same coefficients in the dynamic model. In the dynamic model, a whole stream of payoffs is affected by the dividend information observed today. A small change in a signal affects not only today's dividend estimate, but also tomorrow's and every future date's dividend. That cumulative effect moves the price by more. It also raises the effect of demand shocks because these shocks affect the price, which is used as a signal about future dividends. Because any signal about dividends, including a price signal, has more impact on price, and because demand shocks affect the price signal, demand shocks also have a larger impact on price.

Figure 11: **Change in C_t and $|D_t|$ from Removing Fundamental Data.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. These plots report the percentage change in the coefficient that would result from changing $\Omega_{xt} = 0$ back to its optimal level.



E Robustness of the Numerical Results

We want to investigate the effect of changing parameters on the predictions of the numerical model. First, we show how re-calibrating the model with different risk aversion affects the values of other calibrated parameters. Then we show how changes in risk aversion and other parameters have modest effects on the results. We first consider changes to the exogenous parameters: time preference, risk aversion, and the growth rate of the data technology. Then we consider altering the endogenous, calibrated parameters: dividend innovation variance, noise trade variance, and the relative cost of demand information.

Changes to fixed parameters

We consider a lower/higher time preference and risk aversion. Whenever a parameter is changed, all other parameters are re-calibrated to match that new value and the numerical model is simulated again.

Table 2 shows the original calibration alongside a higher and lower-risk aversion calibration to show how the other parameters adjust when risk aversion changes.

Figure 12 shows the model outcomes for various levels of risk aversion. Data demands are almost identical and market outcomes are qualitatively similar, particularly in the first 20 periods, which correspond to observed, past data. The reason that not much changes is that the other parameters adjust to the change in risk aversion. Figures 13 and 17 repeat the analogous robustness exercises for the rate of the time preference and the technological growth rate.

Table 2: Parameters

	lower risk av	original calibration	high risk av
G	0.98	0.98	0.98
μ	0.04	0.04	0.04
τ_0	80.08	80.08	80.08
τ_x	19.75	19.75	19.75
χ_x	21.13	21.12	21.12
r	1.02	1.02	1.02
ρ	0.0425	0.05	0.0575

Figure 12: Results with Different Risk Aversion. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top plots and measures C_t and $|D_t|$ on the bottom plots. The remaining lines are measured on the right side axis. Column 1 corresponds to $\rho = 0.0425$. Column 2 is the baseline calibration used in the paper, $\rho = 0.05$, and column 3 displays the paths with $\rho = 0.0575$.

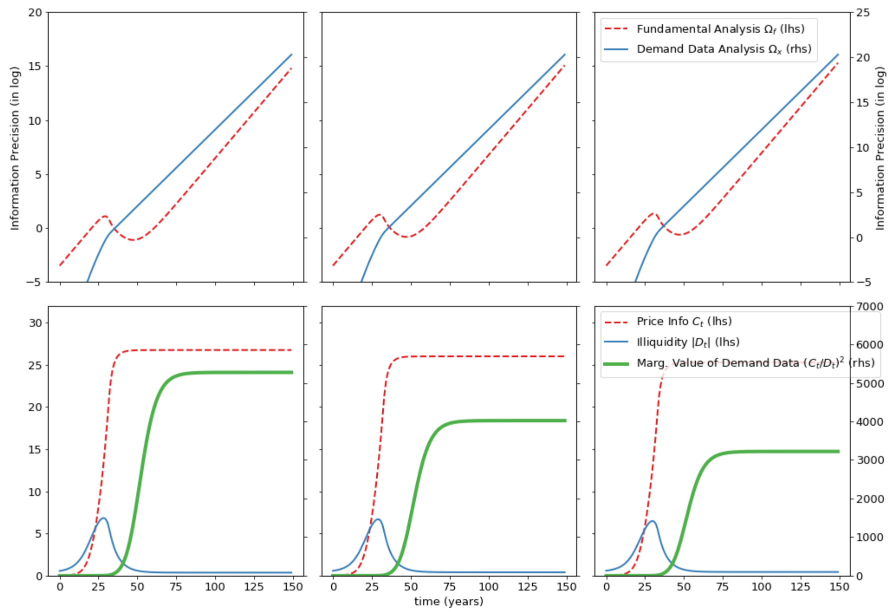
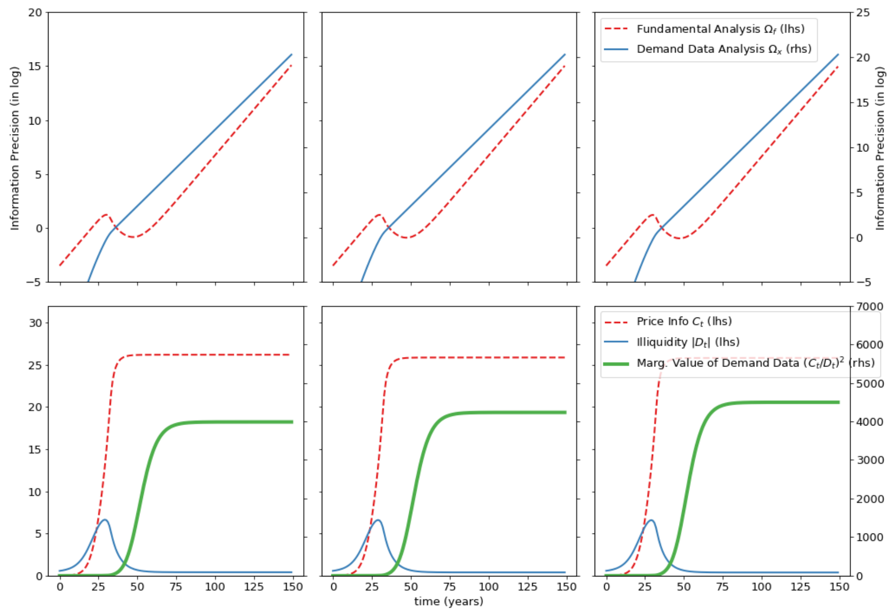


Figure 13: Results with Different Rates of Time Preference. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top plots and C_t and $|D_t|$ on the bottom plots. The remaining lines are measured on the right side axis. Column 1 displays the path with $r = 1.02$, column 2 is the baseline calibration used in the paper corresponding to $r = 1.05$, and column 3 displays the path with $r = 1.08$.



Changes to calibrated parameters

We consider lower/higher dividend shock variance, noise trade variance, the growth rate of the data capacity constraint, and the relative cost of demand information. As these parameters are determined jointly by the calibration, we cannot simply change them and re-calibrate as above. Rather, we calibrate to the baseline then change the parameter of interest for the experiment and then recover the model's terminal values associated with that new parameter of interest. It is important to note that when we make changes here, we do not re-calibrate the other parameters.

Figure 14: Results with Different Values of τ_0. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 2 is the baseline calibration used in the paper. Column 1 displays the path for $\tau_0^* = 0.75 \times \tau_0$ and column 3 for $\tau_0^* = 1.25 \times \tau_0$.

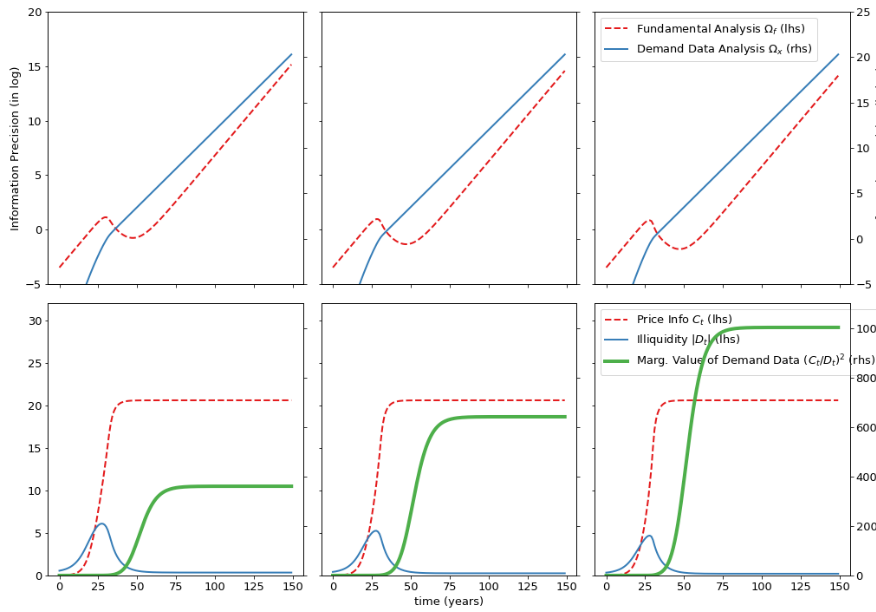


Figure 15: Results with Different Values of τ_x. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 2 is the baseline calibration used in the paper. Column 1 displays the path for $\tau_x^* = 0.75 \times \tau_x$ and column 3 for $\tau_x^* = 1.25 \times \tau_x$.

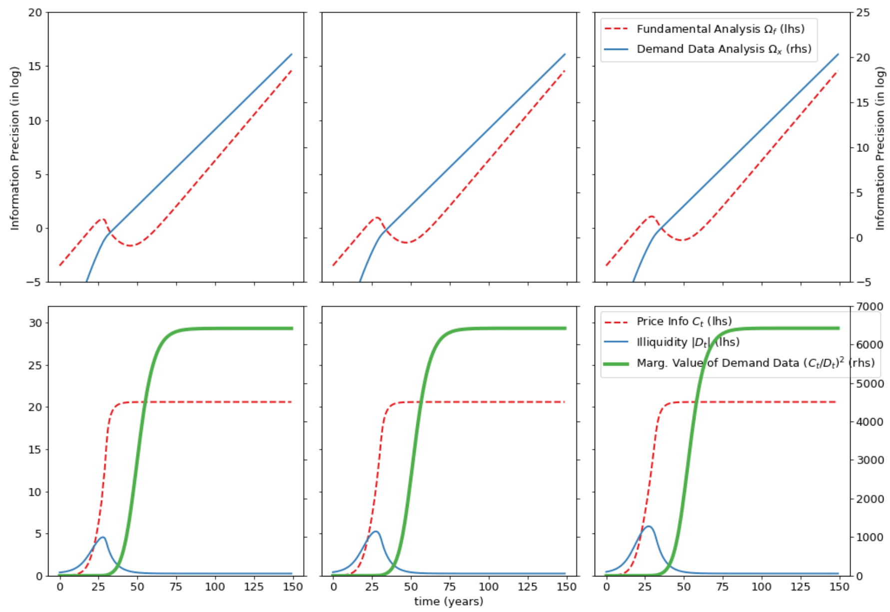


Figure 16: Results with Different Values of χ_x. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 1 is the baseline calibration used in the paper. Columns 2 and 3 display paths for $\chi_x^* = 0.5 \times \chi_x$ and $\chi_x^* = 2 \times \chi_x$, respectively.

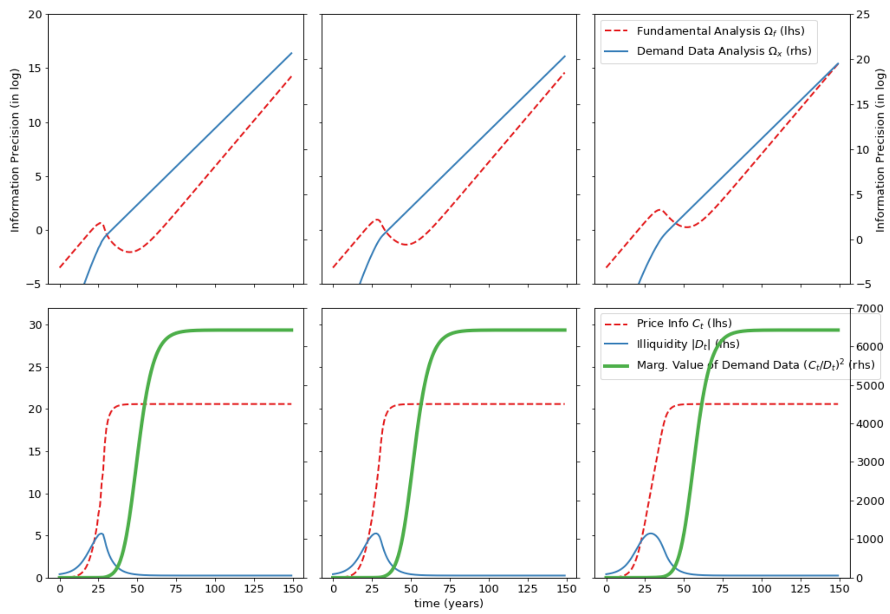


Figure 17: Results with Different Growth Rates of K_t. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 1 has $\alpha = 0.47$, column 2 is the baseline specification in the paper with $\alpha = 0.49$, and column 3 has $\alpha = 0.51$.

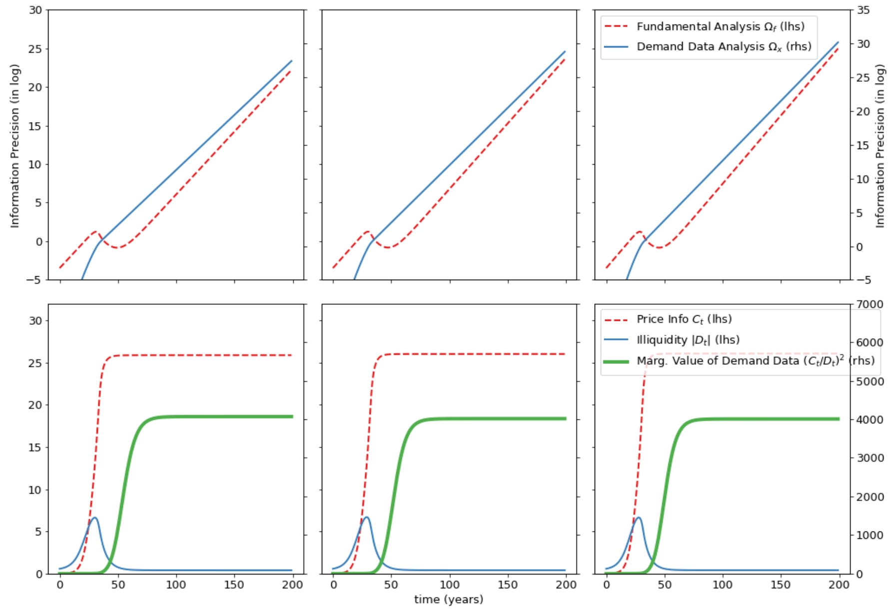
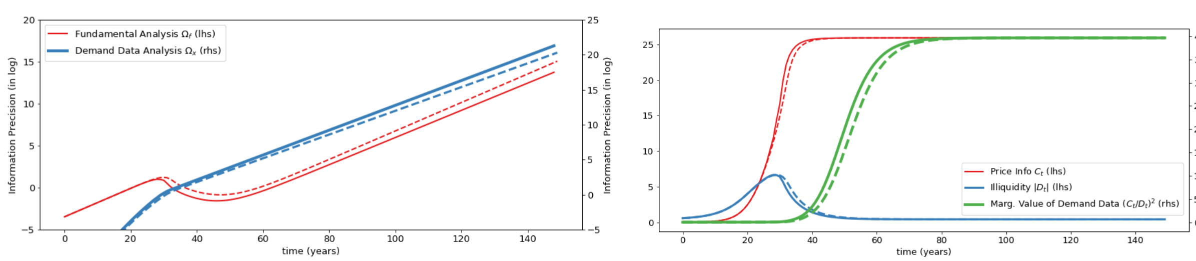


Figure 18: **Unbalanced Technological Progress: χ_x falls.** This figure shows the information choices (left plot) and market efficiency (right plot) with faster productivity growth in demand analysis. The left axis measures Ω_{ft} on left plot and C_t and $|D_t|$ on the right plots. The remaining lines are measured on the right side axis. The path for total information K_t is the same as before. But the marginal cost of demand analysis χ_x follows a path that is log linear: The points $ln(\chi_{xt})$ are evenly spaced between 0 and $ln(\chi_{xt}/10)$. The $x-$axis is time.



(a) Information Acquisition

(b) Price Coefficients

Generated Regressor Problem. In our numerical results, we run time series regression (2) asset by asset and take the regression residual as the innovation term in dividends (y_{t+1}). We then use this innovation term as a regressor in cross sectional regression (8) to get the price coefficients as well as the variance of the residual for each point of time. The variance of the residual corresponds to $D_t^2 \tau_x^{-1}$ in our model, and we use it as one of our moment conditions to calibrate our parameters. Since one of the regressors in the regression of (8) is generated inside the model, this can potentially lead to the generated regressor problem and contaminate the variance of the residual in regression (8).

This problem is unlikely to be quantitatively important in our setting because G_i is estimated quite precisely. Of the 171 firms we consider, 103 of them have G_i estimates that are significantly different from zero, at the 5% level, and 80 of them are significant at the 1% level. With such small standard errors and such large variance of the residuals, the generated regressor is clearly not the main source of the variance of the AR(1) residuals.

F Data Appendix

Asset Data: Compustat Database The Compustat database provides more than 500 company-level fundamentals, including items such as income statements, balance sheets, and flow of funds. Our variables for analysis includes earnings before interest and taxes (EBIT), total assets and market capitalization. The Compustat database is owned by S&P Global. Researchers can subscribe to the data. Many universities and research institutions have such a subscription. More info on data access for academia can be found at https://www.spglobal.com/marketintelligence/en/client-segments/academia.

Hedge Fund Data: Lipper TASS Database The figure showing the shift over time in investment strategies is based on hedge fund data from Lipper. Lipper TASS provides performance data on over 7,500 actively reporting hedge funds and funds of hedge funds and also provides historical performance data on over 11,000 graveyard funds that have liquidated or that stopped reporting. In addition to performance data, data are also available on certain fund characteristics, such as the investment approach, management fees, redemption periods, minimum investment amounts and geographical focus. The Lipper TASS database is accessible via Thomson Reuters underneath "Current Subscriptions" on the Wharton Research Data Services homepage https://wrds-web.wharton.upenn.edu/.

Though the database provides a comprehensive window into the hedge fund industry, data reporting standards are low. A large portion of the industry (representing about 42% of assets) simply does not report anything (Edelman, Fund and Hsieh, 2013). Reporting funds regularly report only performing assets (Bali, Brown and Caglayan, 2014). While any empirical analysis must be considered with caution, some interesting stylized facts about the current state and evolution of the hedge fund industry do exist in these data.

All hedge fund data is monthly and come from Lipper TASS. In total, the database reports on 17,534 live and defunct funds. Data are from 1994-2015, as no data was kept on defunct funds before 1994. A significant portion of this total consists of the same fund reported in different currencies and thus are not representative of independent fund strategies (Bali, Brown and Caglayan, 2014). Therefore, we limit the sample to only U.S.-based hedge funds and remove funds of funds. This limits the sample size to 10,305 funds. As the focus is to gain insight into the division between fundamental and quantitative strategies in the market, we further limit the sample to the 7093 funds that explicitly possess these characteristics, described below. Throughout the sample, funds are born and die regularly. At any point in time, there are never more than 3000 existing, qualifying funds. By the end of 2015, there were just over 1000 qualifying funds.

Lipper TASS records data on each fund's investment strategy. In total, there are 18 different classifications, most of which have qualities of both fundamental and quantitative analyses. As an example of a strategy that could be considered both, "Macro: Active Trading strategies utilize active trading methods, typically with high frequency position turnover or leverage; these may employ components of both Discretionary and Systematic Macro strategies." However, 4 strategy classifications explicitly denote a fund's strategy as being either fundamental or quantitative. They are:

- Fundamental: This denotes that the fund's strategy is explicitly based on fundamental analysis.
- Discretionary: This denotes that the fund's strategy is based upon the discretion of the fund's manager(s).
- Technical: This denotes that the fund deploys a technical strategy.
- Systematic Quant: This denotes that funds deploy a technical/algorithmic strategy.

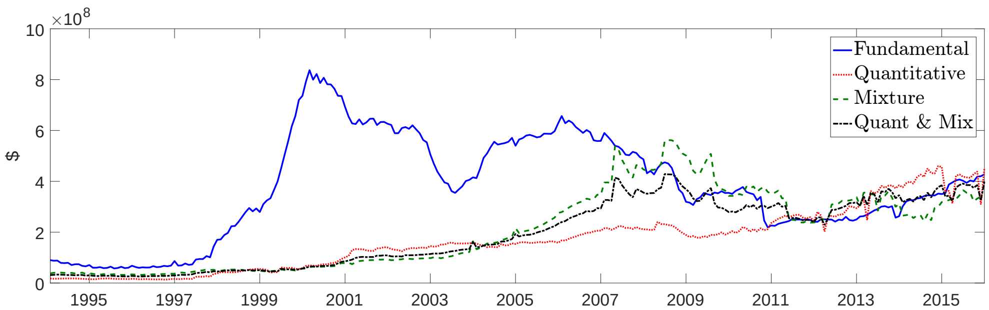
Using these classifications, it is possible to divide hedge fund strategy into three broad groups:

- Fundamental: Those funds with a strategy that is classified as fundamental and/or discretionary, and *not* technical and/or sytematic quant.
- Quantitative: Those funds with a strategy that is classified as technical and/or systematic quant, and *not* fundamental and/or discretionary.

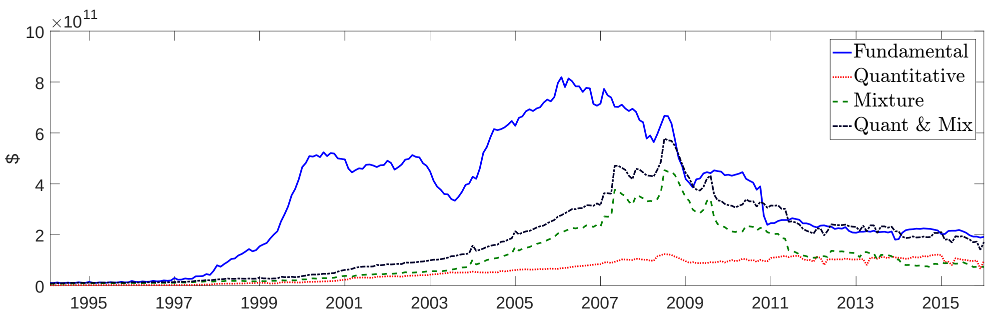
- Mixture: Those funds with a strategy that is classified as having at least one of fundamental or discretionary and at least one of technical or systematic quant.

From 2000-2015, the assets under management (AUM) systematically shifted away from fundamental funds and towards those that deploy some sort of quantitative analysis in their investment approach. In mid-2000, the assets under management per fundamental fund was roughly 8 times the size of those in a quantitative or mixture fund, but by 2011 this had equalized, representing a true shift away from fundamental and towards quantitative analysis in the hedge fund industry.

Figure 19: **Hedge Funds are Shifting away from Fundamental Analysis.**
Source: Lipper TASS. The data are monthly from 1994-2015. The database reports on 17,534 live and defunct funds.



(a) Assets Under Management per Fund



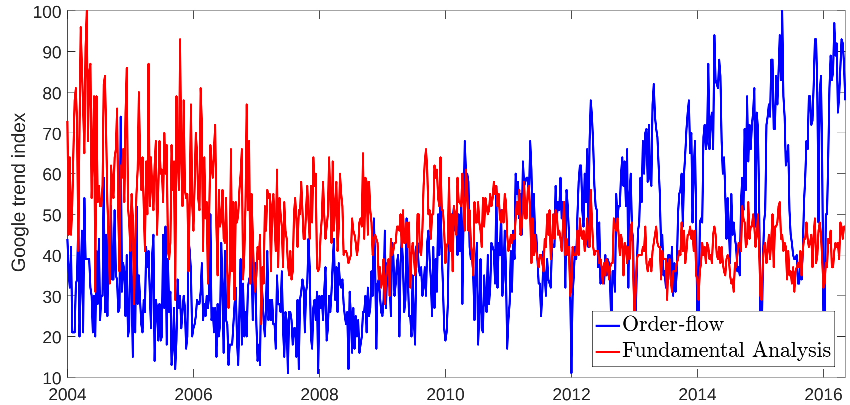
(b) Total Assets Under management

Evidence for Growth of Algorithmic Trade Some suggestive evidence supports the notion that funds are shifting away from fundamental analysis. Figure 19 plots the share of funds that report their own style to fundamental. This share is falling, both as a share of funds and as a share of assets managed. Figure 20 presents very different sort of evidence that points in the same direction. It shows that the fraction of google searches for the term "fundamental analysis" has been falling, while searches for the term "order flow" have been rising.

G How To Test This Model

One of the benefits of a framework like this is that it can generate testable predictions to isolate technology's effects. This section lays out a new measurement strategy for using the model to infer information choices. The end of the section describes how to use these new information measures to test this model, or other related theories.

Figure 20: **Google Trends:** Fraction of Google searches involving "order flow" or "fundamental analysis." Source: Google trends. The data are the weekly fraction of searches involving these search terms. The series is normalized to make the highest data point equal to 100.



G.1 Testable Predictions

Two predictions are central to the main point of this paper. We first lay out these predictions and then describe how one might infer information choices in order to test them.

Prediction 1 ***Demand Data Grew, Relative to Fundamental Data***

The implied measure of Ω_{xt} *has grown at a faster rate than the measure of* Ω_{ft}*.*

The model calibration points to the current regime as one in which demand data is rising relative to fundamental data (Figure 1). With the implied data measures, this would be simple to test by constructing growth rates and testing for differences in means. One could also examine whether demand data growth is speeding up, suggesting complementarity.

Prediction 2 ***Price Informativeness Predicts Demand Data Usage***

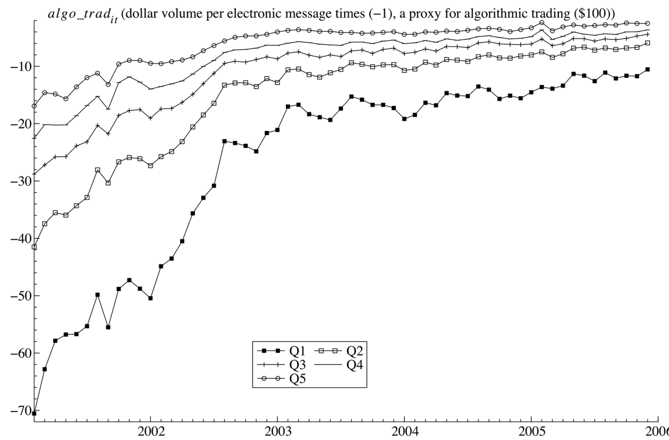
When prices are highly informative (large $C_t/|D_t|$*), investors use more demand data (high* Ω_{xt}*).*

The key insight of the information choice part of the model is that the marginal rate of substitution of demand for fundamental data is proportional to $(C_t/D_t)^2$. One could test whether, controlling for other factors, highly informative prices coincide with, or predict, demand data increases.

G.2 Extending the Model to Facilitate Empirical Testing

The key barrier to testing the predictions above is that one cannot observe investors' data choices. However, data choices do show up in portfolio choice. Data is valuable because it allows investors to trade in a way that is correlated with what they observe. They can buy when dividends are likely to be high or sell when the price appears high for non-fundamental reasons. These strategies are not feasible – not measurable in theory parlance – without observing the relevant data. If many investors systematically buy when payouts are going to be high, this would be conclusive evidence of information. But not all investors can buy at one time, as doing so violates the market clearing condition. In order to test this hypothesis, we need to consider a simple extension of the model to incorporate informed and uninformed traders.

Figure 21: **Algorithmic Trading Growth** 2001-2006. Source: Hendershott, Jones and Menkveld (2011). Their proxy for algorithmic trading is the dollar volume of trade per electronic message. The rise is more pronounced for the largest market cap (Q1) stocks. Q1-Q5 are the 5 quintiles of NYSE stocks, ordered by size (market capitalization).



We extend the model to include a measure λ of investors, who are endowed with capacity K to acquire information, and the complementary measure of investors who do not acquire information but who submit demand optimally based on their priors. Priors are common to all investors.

This extension facilitates testing because it allows informed investors' portfolios to react more to shocks about which they have data, and still have the market clear. That is crucial because our measures of fundamental information and demand information are based on the covariance of an informed investor's portfolio q_t with shocks $\tilde{x}_t$ and $\tilde{y}_t$. This extension resolves the tension because uninformed investors can hold less of an asset that informed investors demand more of.

The solution to this model, derived in Online Appendix C.2, is a simple variant of the original solution. We denote all variables corresponding to uninformed investors with a prime ($'$). The uninformed agents' portfolio takes the same form as (7), except that the mean and variance are conditional on all information revealed in the last period and today's price.

The equilibrium price coefficients, adjusted for heterogeneous agents, are given by (127) and (128). To construct the portfolio covariances with shocks, we take the portfolio first-order condition (7) and then substitute in the definition of signals, equilibrium price (8), and conditional expectations and variances (32) and (33). Expressing q as a function of the shocks x_{t+1}, y_{t+1} and signal noise ($\tilde{\epsilon}_{fit}$, $\tilde{\epsilon}_{xit}$) allows us to compute

$$\begin{aligned}\mathrm{Cov}(q_{it}, x_{t+1}) = & \frac{Var[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}]}{\lambda Var[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] + (1-\lambda)Var[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{i,t-1}, p_t]}\tau_x^{-1} \\ & - \frac{r\Omega_{it}}{\rho(r-\pi G)}\frac{C_t}{D_t}(1-\lambda_{It})(Var[y_{t+1}|\mathcal{I}_{i,t-1}, p_t] - Var[y_{t+1}|\mathcal{I}_{it}]), \quad (138)\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(q_{it}, y_{t+1}) = & \frac{r}{\rho(r-\pi G)}(\pi C_{t+1}^2\tau_0^{-1} + \pi D_{t+1}^2\tau_x^{-1} \\ & + (\frac{r}{r-\pi G})^2 Var[y_{t+1}|\mathcal{I}_{it}])^{-1}(1-\lambda_{It})(Var[y_{t+1}|\mathcal{I}_{i,t-1}, p_t] - Var[y_{t+1}|\mathcal{I}_{it}]). \quad (139)\end{aligned}$$

These covariances depend on aggregate terms like C_t, D_t, as well as Ω_{it} and conditional variances, which depend on individual i's data precision, Ω_{fit} and Ω_{xit}.

G.3 Measuring Information

To test the model, one needs to measure the pricing coefficients in (127) and (128), as well as the covariances in (138) and (139), and combine them, in order to back out Ω_{fit} and Ω_{xit}. Then, determine whether data processing of each type is increasing or not.

To construct these measures, we first need to estimate the variance and persistence of dividends (τ_0^{-1} and G), riskless rate (r), variance of demand shocks (τ_x^{-1}), and a sequence of pricing equation coefficients. Section 3.1 details how we estimated these objects from publicly available financial data. Given these estimates and a decision about whether to use a static ($\pi = 0$) or dynamic ($\pi = 1$) framework, we can construct Z_t from (25), $Var[y_{t+1}|\bar{\mathcal{I}}_t]$ from (13), and $Var[y_{t+1}|\mathbb{I}_t^-, p_t] = (\tau_0 + \tau_x(C_t/D_t)^2)^{-1}$.

To compute the portfolio covariance with shocks requires a time series of the portfolio holdings of some informed investors. Mutual fund or hedge fund portfolio holdings might make a good informed data set.Then for each fund, we compute the covariance over the window of a year, or over the first and second halves of the sample. Backing out Ω_{ft} and Ω_{xt} then requires solving two equations, (138) and (139), for the two unknowns, Ω_{ft} and Ω_{xt}.

With multiple assets, a simple principal components analysis would allow a researcher to construct linear combinations of assets that are independent. For each independent asset or risk factor, one could follow the above procedure, to recover Ω_{ft} and Ω_{xt} data for that asset or risk (as in Kacperczyk, Van Nieuwerburgh and Veldkamp (2016)). One could use these measures and the model structure to answer many questions. For example, one could infer a series for χ_x, the relative shadow cost of processing demand versus fundamental data. That would inform the debate about the role of technological change in high frequency trading.

Cross-fund implied information might be interesting in relation to questions pertaining to the distribution of skill or financial income inequality. But for questions about the long-run trend, averaging the implied precisions (Ω_{ft}, Ω_{xt}) of various investors is consistent with the model: with heterogeneous information quality, the aggregates in the model are the same as they are for a representative agent who has information precision that is the average of all investors' precisions.

Of course, these measures depend on a model that is never entirely correct. However, the parts of the model used to derive these measures are the most standard parts. Specifically, (138) and (139) depend on the form of the first-order condition, which has a very standard form in portfolio problems. They also depend on the way in which the model assumes that agents form expectations and conditional variances, using Bayes law and extracting information from linear prices. But these measures do not depend on the information choice portion of the model. They do not assume that agents optimally allocate data. These measures infer what data must be present in order for agents to be making the portfolio choices that they make and for prices to reflect the information they contain. As such, they offer meaningful ways of testing this model, as well as others.