Online Appendix C for "Relational Contracting, Negotiation, and External Enforcement"

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This online appendix contains additional notes on contractual equilibrium and applications. For continuity, it is labeled "Appendix C", to succeed Appendix B in the main article. In the first subsection we give a proof of Lemma 8 from Appendix B1. The next subsection discusses useful variants of the equilibrium definition and provides some technical results, including on how external enforcement and self-enforcement are complementary. The third states and proves an existence result for finite settings. The following two subsections provide an additional illustration and some details for the example in Section II. The final subsection provides details for the application in Section IV.

C.1. Additional analysis for the hybrid model

PROOF OF LEMMA 8:

The proof has two main steps. First we show that for any regime r that is incentive compatible in the action phase and internally bargain-consistent, $\mathcal{V}(r)$ is BSG. We then establish a claim in the opposite direction.

Consider any regime r that is incentive compatible in the action phase and internally bargain-consistent. It is clear from the definitions that if (c, m, α, r') is comparable with r following ψ then $v_i^{\mathrm{a}}(\psi cm, \alpha; r')$ is c-supported relative to $\mathcal{V}(r)$. Note that m plays no role in the definition of "comparable" and is arbitrary. Likewise, if w is c-supported relative to $\mathcal{V}(r)$ then there exists a tuple (c, m, α, r') that is comparable with r following ψ and satisfies $v_i^{\mathrm{a}}(\psi cm, \alpha; r') = w$. This implies that the regime's level is $L(\mathcal{V}(r))$. Because r is internally bargain-consistent and $\underline{v}(\psi; r)$ is $\kappa(\psi)$-supported, a further implication is that $v(\psi; r) \in B(\kappa(\psi), \mathcal{V}(r))$. Taking the union over negotiation-consistent ψ satisfying $\kappa(\psi) = \hat{c}$ for a given $\hat{c}$, we have $V(\hat{c}; r) \subset B(\kappa(\psi), \mathcal{V}(r))$. Recalling that $V(\hat{c}; r)$ is empty for any $\hat{c}$ that does not arise as an inherited contract in any negotiation-consistent history, we conclude that $\mathcal{V}(r)$ is BSG.

We next show that for every BSG collection $\mathcal{W}$ there is a regime r that is incentive compatible in the action phase, internally bargain-consistent, and satisfies $V(c; r) \subset W(c)$ for every $c \in C$. This step follows standard arguments, along the

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lines of the construction detailed in Miller and Watson (2013). We construct the regime by specifying the behavior identified in the self-generation conditions, for histories that will be negotiation-consistent.

Start with the null history ψ^0, note that $\kappa(\psi^0) = c^0$, and pick any element $w \in W(c^0)$ to be the equilibrium continuation value from the beginning of the game. From the self-generation conditions, $w \in B(c^0, \mathcal{W})$ and so we can find an external contract $\overline{c}$, a $\overline{c}$-supported (relative to $\mathcal{W}$) value $\overline{w}$, and a c^0-supported disagreement value $\underline{w}$ such that $L(\mathcal{W}) = \overline{w}_1 + \overline{w}_2$ and $w = \underline{w} + \pi(L(\mathcal{W}) - \underline{w}_1 - \underline{w}_2)$.

Prescribe $r^{\mathrm{c}}(\psi^0) = \overline{c}$ and let $r^{\mathrm{m}}(\psi^0)$ to be the corresponding transfer that achieves w as the continuation value from the beginning of period 1 when $\overline{w}$ is the continuation value from the action phase, so that $w = (1-\delta)r^{\mathrm{m}}(\psi^0) + \overline{w}$. Then prescribe $r^{\mathrm{a}}(\psi^0 c^0 0)$ to be the mixed action α that is identified by self-generation to c^0-support $\underline{w}$. Likewise, prescribe $r^{\mathrm{a}}(\psi^0 r^{\mathrm{c}}(\psi^0) r^{\mathrm{m}}(\psi^0))$ to be the mixed action identified to $\overline{c}$-support $\overline{w}$. For other values of (c^1, m^1), the prescribed action profile $r^{\mathrm{a}}(h^0 c^1 m^1)$ can be arbitrary because such a joint deviation would lead to histories that are not negotiation-consistent with r and thus not subject to the equilibrium conditions.

The construction continues by considering all one-period histories that are negotiation-consistent given the specification of behavior for the first period (the joint actions specified in the previous paragraph, all of the possible action profiles in $A(c^0)$ and $A(\overline{c})$, and every ϕ). For each such history ψ, a specific continuation value from $W(\kappa(\psi))$ is required to provide the incentives and continuation payoffs specified in period 1. We simply repeat the steps in the previous paragraph to specify behavior in period 2 following history ψ. For one-period histories that are not negotiation-consistent, the specification of behavior is arbitrary. The process continues for period 3, 4, and so on, which inductively yields a fully specified regime.

By construction from the self-generation conditions, the regime's continuation values have the desired properties and the regime is incentive compatible in the action phase and internally bargain-consistent. For every negotiation-consistent history ψ the continuation value $v(\psi; r)$ is an element of $W(\kappa(\psi))$. Thus, $V(c; r) \subset W(c)$ for every $c \in C$. (We are using the fact that $V(c; r) = \emptyset$ for every c for which no negotiation-consistent history ψ has $\kappa(\psi) = c$.)

To finish the proof, take any contractual equilibrium regime r and let ℓ be its level. We have shown that $\mathcal{V}(r)$ is BSG. We have shown also that every BSG collection corresponds to a regime that is incentive compatible in the action phase, is internally bargain-consistent, and has the same level as does the BSG collection. Therefore, if there were a BSG collection with a level $\ell' > \ell$, there would exist a corresponding incentive compatible, internally bargain-consistent regime with level ℓ', contradicting that r is a contractual equilibrium. Thus, $\mathcal{V}(r)$ is a CEV collection. The same argument works in reverse to establish the second claim of the lemma.

C.2. A CE variant and general enforcement complementarity

Our definition of contractual equilibrium (CEV collection in Section II and the corresponding CE regime in Appendix B.1) generalizes that of Miller and Watson (2013) to relationships with external enforcement, so it coincides if Γ is a singleton. We describe here a variant of BSG that, by being more permissive, helps establish additional results regarding general complementarity of external enforcement and self-enforcement, existence, and computation of a CEV collection. It is straightforward to write the corresponding definition for regimes.

The variant BSG$'$ expresses self-generation in reference to two collections: a collection $\mathcal{W}$ of continuation values from the negotiation phase and a "paired collection" $\underline{\mathcal{W}} = \{\underline{W}(c)\}_{c\in C}$ of continuation values from the action phase. With inherited contract $\hat{c}$ in a period, the disagreement point must be in $W(\hat{c})$ and the players negotiate over the values in $\cup_{c\in C} W(c)$. The difference between BSG$'$ and BSG is that, with the former, $\underline{W}(c)$ need not contain all values that are c-supported relative to $\mathcal{W}$.

For any $\underline{\mathcal{W}}$, define $M(\underline{\mathcal{W}}) \equiv \max_{c\in C,\, w\in \underline{W}(c)}(w_1 + w_2)$ if this maximum exists. Let us say that $\underline{\mathcal{W}}$ *is supported relative to* $\mathcal{W}$ if, for all $c \in C$, every element of $\underline{W}(c)$ is c-supported relative to $\mathcal{W}$. A collection $\mathcal{W}$ is a *BSG$'$ collection* if there is a collection $\underline{\mathcal{W}}$ that is supported relative to $\mathcal{W}$ and has the following property: For every $\hat{c} \in C$ and $w \in W(\hat{c})$, there exists a value $\underline{w} \in \underline{W}(\hat{c})$ such that $w = \underline{w} + \pi\big(M(\underline{\mathcal{W}}) - \underline{w}_1 - \underline{w}_2\big)$. The level is $M(\underline{\mathcal{W}})$, which equals $M(\mathcal{W})$. Clearly every BSG collection is a BSG$'$ collection, and the latter may exist when the former does not. Let us call a collection $\mathcal{W}$ a *CEV$'$ collection* if it is BSG$'$ and its level is maximal among the set of BSG$'$ collections.

It is easy to show that the union of CEV$'$ collections is also a CEV$'$ collection; the same is true for BSG$'$. Additionally, we have a general version of Theorem 3 in Section III.D, regarding the complementarity of self-enforcement and external enforcement:

Theorem 3$'$. *If contractual setting $(\tilde{\Gamma}, c^0, \pi)$ is stronger than (Γ, c^0, π), and if a CEV$'$ collection exists under both technologies, then the contractual-equilibrium welfare level is weakly higher under $(\tilde{\Gamma}, c^0, \pi)$.*

PROOF OF THEOREM 3$'$:

Suppose $\Gamma \subset \tilde{\Gamma}$, let C and $\tilde{C}$ be the sets of contracts for Γ and $\tilde{\Gamma}$, and take any collection $\mathcal{W}$ that is BSG$'$ in setting (Γ, c^0). The collection $\underline{\mathcal{W}}$ that is used to establish that $\mathcal{W}$ is BSG$'$ can be extended by specifying $\underline{W}(c) = \emptyset$ for $c \in \tilde{C} \setminus C$, and this makes $\mathcal{W}$ a BSG$'$ collection in setting $(\tilde{\Gamma}, \tilde{c}^0)$. So, if $(\tilde{\Gamma}, \tilde{c}^0)$ is stronger than (Γ, c^0) and if a CEV$'$ collection exists under both technologies, then the welfare level is weakly higher under $(\tilde{\Gamma}, \tilde{c}^0)$.

We continue by describing another variant of BSG that helps us establish a connection between CEV and CEV$'$. For a collection $\mathcal{W}$, any number K, and a contract $\hat{c}$, define

$$B^K(\hat{c},\mathcal{W}) \equiv \left\{\underline{w} + \pi\big(K - \underline{w}_1 - \underline{w}_2\big) \mid \underline{w} \text{ is } \hat{c}\text{-supported relative to } \mathcal{W}\right\}.$$

This normalizes to level K and ignores whether $K \geq \underline{w}_1 + \underline{w}_2$ (the opposite inequality would be nonsensical for a bargaining solution) but it is no matter. Clearly B^K is monotone in $\mathcal{W}$ and $B^K(\hat{c},\mathcal{W}) \equiv B^0(\hat{c},\mathcal{W}) + \pi K$. Therefore, $\mathcal{W}$ is a fixed point of B^K, meaning that $W(c) \subset B^K(c,\mathcal{W})$ for every $c \in C$ (that is, it is self-generating), if and only if $\mathcal{W} - \pi K$ is a fixed point of B^0. Because B^0 is monotone, the component-wise union of fixed points, which we call $\mathcal{W}^0$, is also a fixed point.

Note that $\mathcal{W}$ is BSG$'$ with paired collection $\underline{\mathcal{W}}$ if and only if $\mathcal{W}$ is a fixed point of $B^{M(\underline{\mathcal{W}})}$ and $\underline{\mathcal{W}}$ is supported relative to $\mathcal{W}$, and in this case $M(\mathcal{W}) = M(\underline{\mathcal{W}})$. Further, $\mathcal{W}$ is BSG if and only of it is a fixed point of $B^{L(\mathcal{W})}$ and $L(\mathcal{W})$ exists.

LEMMA C1: *If* $\max\{w_1 + w_2 \mid c \in C,\ w \text{ is } c\text{-supported relative to } \mathcal{W}^0\} \equiv \theta$ *exists then* $\mathcal{W}^0 + \pi\theta/(1-\delta)$ *is both a CEV$'$ collection and a CEV collection, and it equals* $\mathcal{W}^*$.

PROOF:

To prove this result, first note that because $\mathcal{W}^0$ is a fixed point of B^0, we know that $\mathcal{W}^0 + \pi\theta/(1-\delta)$ is a fixed point of $B^{\theta/(1-\delta)}$. By definition of θ, the maximum joint value that can be c-supported relative to $\mathcal{W}^0 + \pi\theta/(1-\delta)$ (maximizing over $c \in C$) is $\theta + \delta\theta/(1-\delta) = \theta/(1-\delta)$. Therefore we have $L(\mathcal{W}^0 + \pi\theta/(1-\delta)) = \theta/(1-\delta)$ and so $\mathcal{W}^0 + \pi\theta/(1-\delta)$ is BSG.

Now presume that there is a BSG$'$ collection $\tilde{\mathcal{W}}$ with level $K > \theta/(1-\delta)$ and we will find a contradiction. Note that $\tilde{\mathcal{W}}$ is a fixed point of B^K, and there exists $\tilde{c} \in C$ and a value $\tilde{w}$ that is $\tilde{c}$-supported relative to $\tilde{\mathcal{W}}$ such that $\tilde{w}_1 + \tilde{w}_2 = K$. Importantly, $\tilde{w} = (1-\delta)\tilde{u} + \delta\tilde{y}$, where $\tilde{u}$ is the expected current-period payoff and $\tilde{y}$ is the expected continuation value from the next period. Because $\tilde{y}_1 + \tilde{y}_2 = K$, we know that $\tilde{u}_1 + \tilde{u}_2 = K$ as well. Shifting the collection by πK, we likewise have that $\tilde{\mathcal{W}} - \pi K$ is a fixed point of B^0. By definition of $\mathcal{W}^0$, we know $\tilde{W}(c) - \pi K \subset W^0(c)$ for every $c \in C$. Therefore, $(1-\delta)\tilde{u} + \delta(\tilde{y} - \pi K)$ is $\tilde{c}$-supported relative to $\tilde{\mathcal{W}} - \pi K$. Noting that $(1-\delta)\tilde{u} + \delta(\tilde{y} - \pi K) = \tilde{w} - \delta\pi K$, the joint value achieved is $(1-\delta)K$, which strictly exceeds θ, contradicting the definition of θ.

We have thus shown that there is no BSG$'$ collection with level higher than $\theta/(1-\delta)$, proving that $\mathcal{W}^0 + \pi\theta/(1-\delta)$ is CEV$'$. Because $\mathcal{W}^0 + \pi\theta/(1-\delta)$ is also BSG, and every BSG collection is also BSG$'$, we conclude that $\mathcal{W}^0 + \pi\theta/(1-\delta)$ is CEV. Because $\mathcal{W}^0$ is maximal and every BSG collection corresponds, by subtracting $\pi\theta/(1-\delta)$, to a fixed point of B^0, we conclude that $\mathcal{W} = \mathcal{W}^0 + \pi\theta/(1-\delta)$.

Thus, under the maximum existence condition of Lemma C1 we have existence of a CEV collection, one can calculate the maximal CEV collection by determining $\mathcal{W}^0$, and the complementarity result holds. The maximum exists under the conditions of Theorems 1 and 2, and we show in the next subsection that the same is true in another wide class of settings.

C.3. Existence in finite settings

In this subsection, we provide an existence result for settings with finite stage games and a finite set C. Here the other aspects of the relational contracting games are fully general. We can drop the assumption that c^0 specifies the same stage game for every history and we make no assumptions regarding verifiability or external enforcement.

THEOREM C1: *For any relational-contract setting in which C is finite and every game in Γ is finite, the maximization problem described in Lemma C1 has a solution and therefore a contractual equilibrium exists.*

PROOF:

We start by proving that B^0 has a (nonempty) fixed point (a self-generating collection), so the feasible set in Lemma C1's optimization problem is nonempty. For any point $\nu = (w^c)_{c\in C} \in \mathbb{R}^{|C|}$, let $\mathcal{W}(\nu)$ be defined as the collection given by $W(c) = \{w^c\}$ for all $c \in C$. Note that $w_1^c + w_2^c = 0$ for all $c \in C$. Also, let $f(\nu) \equiv \prod_{c\in c} \operatorname{co} B^0(c, \mathcal{W}(\nu))$. Because (i) the stage games are finite, (ii) the bargaining solution maps supported values to the zero-value line along the ray π, and (iii) continuation values are discounted, we can find a bound ξ such that $w^c \in [-\xi, \xi]^2$ for all $c \in C$ implies that $B^0(c, \mathcal{W}(\nu)) \subset [-\xi, \xi]^2$. Further, because each stage game is finite and the Nash correspondence is nonempty and upper hemi-continuous in payoff vectors, $B^0(c, \mathcal{W}(\nu))$ is nonempty valued and upper hemi-continuous as a function of ν. Thus, f is a correspondence from a compact set to itself, it is nonempty and convex-valued, and it is upper-hemicontinuous. By Kakutani's theorem, f has a fixed point $\nu^* = (w^{c*})_{c\in C}$.

The fixed point property for f means that $w^{c*} \subset \operatorname{co} B^0(c, \mathcal{W}(\nu^*))$ for all $c \in C$, but it is not necessarily the case that $w^{c*} \subset B^0(c, \mathcal{W}(\nu^*))$ for all $c \in C$, as is required to have a fixed point of B^0. However, if this latter condition fails, then we can find two points $w^{c*\prime}, w^{c*\prime\prime} \in B^0(c, \mathcal{W}(\nu^*))$ such that w^{c*} is on the line between $w^{c*\prime}$ and $w^{c*\prime\prime}$. We then redefine $\mathcal{W}(\nu^*)$ so that $W(c) = \{w^{c*\prime}, w^{c*\prime\prime}\}$, which weakly enlarges $B^0(c, \mathcal{W}(\nu^*))$ because, in the definition of c-support, continuation values are allowed to be in the convex hull of the value collection. We thus have that $\mathcal{W}(\nu^*)$ is a fixed point of B^0.

We complete the proof by establishing that the maximization problem described in Lemma C1 has a solution. Because B^0 is upper hemi-continuous and $\mathcal{W}^0$ is a fixed point, we know that the closure of $\mathcal{W}^0$ is also a fixed point and so $\mathcal{W}^0$ must be a collection of closed sets. Thus, for each $c \in C$, the problem of maximizing $u_1(\alpha; c) + u_2(\alpha; c)$ over all c-enforced action profiles $\alpha \in \Delta A(c)$, relative to $\mathcal{W}$, has a solution. Because there are a finite number of external contracts, the overall maximum exists.

C.4. Graphical depiction of the example with verifiable signal

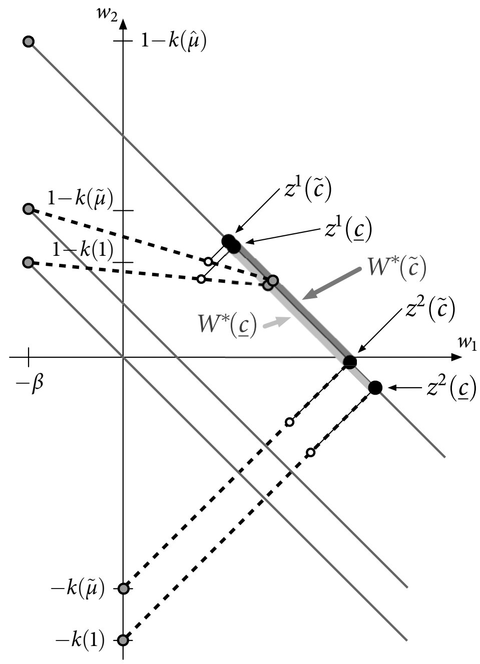

Figure C1. Contractual equilibrium with verifiable signal, but no contingent transfers.

Note: This figure illustrates a non-semi-stationary contract of the kind described in Section II.C, drawn to scale using the same parameters as the figures in Section II. Contract $\underline{c}$ (with monitoring level $\underline{\mu} = 1$) supports a higher maximum utility $z_1^2(\underline{c}) > z_1^2(\tilde{c})$ for the worker, while contract $\tilde{c}$ (with monitoring level $\tilde{\mu} \approx 0.82$) supports a higher maximum utility $z_2^1(\tilde{c}) > z_2^1(\underline{c})$ for the manager. On the equilibrium path, the parties agree on a contract c (with monitoring level $\hat{\mu}$) that specifies continuing with contract $\underline{c}$ and continuation utility $z^2(\underline{c})$ if $x = 1$, but contract $\tilde{c}$ and continuation utility $z^1(\tilde{c})$ if $x = 0$. While this arrangement is better than using semi-stationary contract c^* (described in Section II.C), we do not claim that it is optimal, which is why we do not place the figure in the main text.

C.5. Notes on the example with contingent transfers

Here are a few analytical details for the example in Section II.D.

First, averaging over x for a given action profile a, we compute the expected payoff function:

$$u(a) = (-\beta a_1, a_1 - k(\mu)) + \\ (1, -1)\left[(1 - a_2)\varepsilon b_1(0) + (1 - (1 - a_2)\varepsilon)(b_1(1) - \mu(1 - a_1)(b_1(1) - b_1(0)))\right]$$

Next, the logic of using current-period contingent transfers to substitute for differences in continuation contracts implies that we can focus on stationary contracts to determine $\underline{c}$. To enforce $a = (1, 1)$ in the disagreement point associated with $z^1(\underline{c})$, the worker's incentive constraint is

$$(1 - \delta)\beta \le \mu(1 - \delta)r + \mu\delta\rho \tag{C1}$$

and the manager's incentive constraint is

$$(1 - \delta)\varepsilon r \le \delta(d^* - \rho), \tag{C2}$$

where $r = b_1(1) - b_1(0)$, ρ is the bonus in continuation value to the worker for $x_1 = 1$, and d^* is the maximal span. The span appears here because after play of $a_2 = 0$ the manager is punished by having the players coordinate on the continuation value that most favors the worker. As in the initial example, the worker's incentive condition should bind. Using this condition to substitute for ρ, the manager's incentive condition becomes $\beta/\mu \le \delta d^*/(1 - \delta) + r(1 - \varepsilon)$. In the expression for z^1 that we derive, the terms with r cancel and we find that z^1 and z^2 are characterized as in the initial example. The requirement $\rho \ge 0$ simplifies to $\beta \ge r\mu$, so we optimally set $r = \beta/\mu$ and $\rho = 0$. Finally, raising μ both relaxes the incentive condition and increases the span, so $\mu = 1$ is best in contract $\underline{c}$.

Regarding the manager's incentive to not jam the signal when the worker is supposed to choose high effort, recall that the players coordinate on the manager's favorite continuation value $z^1(\underline{c})$ if $x_2 = 1$ (no jamming), and they coordinate on $z^2(\underline{c})$ if $x_2 = 0$. Both incentive conditions C1 and C2 must bind to minimize the monitoring cost while achieving high effort. Combining them yields $(1 - \delta)\beta\varepsilon = \mu\delta d^*$ and the conclusions described in the text follow. The sufficient condition for cooperation is weaker than in the initial example, implying that L^* is higher since μ can be set lower.

C.6. Options and allocation of decision rights

PROOF OF PROPOSITION 1:

In optimization problem $\Lambda(d)$, the objective is to be maximized is $\omega_1(\gamma, a^2, y^2) - \omega_1(\gamma, a^1, y^1)$, where

$$\omega_1(\gamma, a^j, y^j) = (1 - \delta)\Big(u_1(a^j) - \pi_1\big(u_1(a^j) + u_2(a^j)\big)\Big) + \delta\bar{y}_1^j(a^j)$$

by choice of the game γ, action profiles a^1 and a^2, and normalized continuation value mappings y^1 and y^2, subject to incentive compatibility constraints. Recall that that $\bar{y}^j(a)$ is the expectation of the normalized continuation value when the parties choose action profile a, and all normalized continuation values must lie within the normalized span: $y^1, y^2 : X \to \mathbb{R}_0^2(d) = \big\{m \in \mathbb{R}^2 \mid m_1 + m_2 = 0 \text{ and } m_1 \in [0, d]\big\}$. The largest fixed point of Λ, written as d^*, will be the span of the optimal contract.

The worker's effort is enforced by the following IC constraints: for $j = 1, 2$:

$$(1-\delta)(p^j - \beta a_1^j) + \delta \bar{y}_1^j(a^j) \geq (1-\delta)(p^j - \beta a_1') + \delta \bar{y}_1^j(a_1', a_2^j),$$

for any $a_1' \in \{0, 1\}$.[1]

The manager must have incentives to select the appropriate option. If he complies and selects the intended option $a_2^j = (\mu^j, p^j)$, the worker will choose a_1^j. If the manager deviates, he can be maximally punished by having the worker shirk and then continuing with his worst continuation payoff (here $-d$) for any outcome.[2] Thus the manager's option selection is enforced by the following IC constraints: for $j, j' \in \{1, 2\}$ and $j \neq j'$,

$$\text{(C3)} \qquad (1-\delta)\big(a_1^j - p^j - k(\mu^j)\big) + \delta \bar{y}_2^j(a^j) \geq (1-\delta)(-p^{j'} - k(\mu^{j'})) - \delta d.$$

We will prove the proposition by showing the following:

- For $\delta d < (1-\delta)\beta$ only low effort can be enforced, and we have

$$\text{(C4)} \qquad \Lambda(d) = (1-\delta)\pi_2\big(k(1) - k(0)\big) + \delta d.$$

- For $\delta d \geq (1-\delta)\beta$ we have

$$\text{(C5)} \qquad \Lambda(d) = (1-\delta)\bigg(1 - \beta + \pi_2\Big(k(1) - k\big(\tfrac{(1-\delta)\beta}{\delta d}\big)\Big)\bigg) + \delta d.$$

The value in Equation C5 is attained using a stage game with menu items featuring monitoring levels $\mu^1 = 1$ and $\mu^2 = \frac{(1-\delta)\beta}{\delta d} \leq 1$ and payments p^1 and p^2 that satisfy Equation 14, and by directing the worker to exert high effort ($a_1^1 = a_1^2 = 1$) if the manager does not deviate.

It follows from this that if $\delta \geq \beta$, then Λ has a largest fixed point d^* satisfying $\delta d^* \geq (1-\delta)\beta$, and given by the largest solution to Equation 13.[3] The proof is thus complete if we verify Equation C4 and Equation C5. We do so in two steps.

Step 1

The objective $\omega_1(\gamma, a^2, y^2) - \omega_1(\gamma, a^1, y^1)$ to be maximized is no larger than

$$(1-\delta)(a_1^2 + \pi_1(a_1^1 - a_1^2)(1-\beta) + (k(\mu^1) - k(\mu^2))\pi_2) + \delta d - \delta \bar{y}_1^1(a_1^2, a_2^1)$$

[1] We abuse notation here by using a_1^j to indicate the worker's equilibrium effort if the manager does not deviate, with the understanding that the worker should simply exert low effort if the manager does deviate, as explained below.

[2] Shirking is optimal for the worker when the continuation payoff is independent of the signal outcome.

[3] If instead $\delta < \beta$, then the unique fixed point of Λ is $d = \pi_2\big(k(1) - k(0)\big)$, which is obtained by using contractual payments to induce the manager to choose different monitoring levels in different states, even though the worker always exerts low effort.

This upper bound is attained when two enforcement constraints bind: (i) the worker's IC for preferring a_1^1 to a_1^2 when the manager has complied and chosen (μ^1, p^1), and (ii) the manager's IC for preferring (μ^2, p^2) to (μ^1, p^1).

The displayed formula in Step 1 follows by substituting directly from the two IC constraints in the objective. To see this, let $l(a_1^j, \mu^j) = (1-\beta)a_1^j - k(\mu^j)$. The objective $\omega_1(\gamma, a^2, y^2) - \omega_1(\gamma, a^1, y^1)$ can then be written as

$$(1-\delta)(p^2 - \beta a_1^2 - \pi_1 l(a_1^2, \mu^2) - p^1 + \beta a_1^1 + \pi_1 l(a_1^1, \mu^1)) + \delta \bar{y}_1^2(a^2) - \delta \bar{y}_1^1(a^1) \tag{C6}$$

From the worker's IC for preferring a_1^1 to a_1^2 when the manager has complied and chosen (μ^1, p^1) we see that the above expression is no larger than

$$(1-\delta)(-p^1 + p^2 + \pi_1(l(a_1^1, \mu^1) - l(a_1^2, \mu^2))) + \delta \bar{y}_1^2(a^2) - \delta \bar{y}_1^1(a_1^2, a_2^1)$$

Using the manager's IC constraint for selecting Option 2, ie.

$$(1-\delta)(a_1^2 - p^2 - k(\mu^2) + \delta \bar{y}_2^2(a^2) \geq (1-\delta)(-p^1 - k(\mu^1)) - \delta d,$$

plus the fact that $\bar{y}_1^2(a^2) + \bar{y}_2^2(a^2) = 0$, then verifies Step 1.

Observe that it follows from the formula in Step 1 that if $\delta d < (1-\delta)\beta$ and thus no effort can be implemented, then the objective is maximal for $\mu^1 = 1$, $\mu^2 = 0$ and $\bar{y}_1^1(0, a_2^1) = 0$; proving Equation C4.

In the following assume $\delta d \geq (1-\delta)\beta$. Consider $\bar{y}_1^1(a_1, a_2^1) = E(y_1^1(x) \big| \, a_1, a_2^1)$, where $y_1^1(x) \in [0, d]$ is the continuation value depending on monitor signal $x \in \{0, 1\}$.

Step 2

The objective $\omega_1(\gamma, a^2, y^2) - \omega_1(\gamma, a^1, y^1)$ to be maximized is no larger than

$$(1-\delta)(a_1^2 + (a_1^1 - a_1^2)\left[\pi_1(1-\beta) + \beta\right] + (k(1) - k(\mu^2))\pi_2) + \delta d - \delta \rho a_1^1,$$

where $\rho = y_1^1(1) \in [0, d]$. This upper bound is attained when $y_1^1(0) = 0$, $\mu^1 = 1$, and the enforcement constraints (i) and (ii) stated in Step 1 bind.

To verify Step 2, observe first that we have

$$\bar{y}_1^1(a_1^2, a_2^1) = y_1^1(1) - \mu^1 \left[y_1^1(1) - y_1^1(0)\right](1 - a_1^2)$$

It follows that the expression in Step 1 is decreasing in $y_1^1(0)$ and increasing in μ^1, hence it is no larger than

$$(1-\delta)(a_1^2 + \pi_1(a_1^1 - a_1^2)(1-\beta) + (k(1) - k(\mu^2))\pi_2) + \delta d - \delta \rho a_1^2,$$

where $\rho = y_1^1(1) \in [0, d]$ and we have set $y_1^1(0) = 0$ and $\mu^1 = 1$.

Now consider the binding worker's IC for preferring a_1^1 to a_1^2 when the manager has complied and chosen (μ^1, p^1), with $\mu^1 = 1$ (perfect monitoring):

$$(1-\delta)(p^1 - \beta a_1^1) + \delta\rho a_1^1 = (1-\delta)(p^1 - \beta a_1^2) + \delta\rho a_1^2$$

Substituting this into the previous displayed expression we obtain the formula displayed in Step 2.

Finally we show that the expression in Step 2 is maximal for $a_1^1 = a_1^2 = 1$. First, it is larger for $a_1^1 = 1$ than for $a_1^1 = 0$. For $a_1^1 = 1$ the minimal required bonus ρ is (when $\mu^1 = 1$) given by $\delta\rho = (1-\delta)\beta$. The terms involving a_1^1 then yield $(1-\delta)\pi_1(1-\beta)a_1^1$, hence $a_1^1 = 1$ strictly dominates $a_1^1 = 0$.

Secondly, the expression in Step 2 is larger for $a_1^2 = 1$ than for $a_1^2 = 0$. The terms involving a_1^2 can be written as

$$(1-\delta)(a_1^2\pi_2(1-\beta) - k(\mu^2)a_1^2\pi_2),$$

where the required monitor level μ^2 to implement $a_1^2 = 1$ is given by $\delta d\mu^2 = (1-\delta)\beta$. This expression is maximal for $a_1^2 = 1$, since by assumption $1-\beta-k(1) > -k(0)$.

Substituting these values for a_1^1, a_1^2, ρ and μ^2 in the displayed expression in Step 2 yields the expression for $\Lambda(d)$ given in Equation C5. It can be checked that no enforcement constraints are violated by this solution, hence it is indeed optimal.

This completes the verification of Equation C5, and hence the proof of Proposition 1.

PROOF OF PROPOSITION 2:

As noted at the beginning of Section IV.B, the problem with worker decision rights is similar to the case with manager decision rights. Specifically, the objective function and the worker's effort incentive constraints are the same, but in place of the manager's incentive constraint, the worker now has an additional incentive constraint for choosing the appropriate option from the menu. If she selects the appropriate option, her effort incentive constraint ensures that she will exert the intended effort. If she deviates and selects the other option, however, she can be maximally punished by receiving her worst continuation value regardless of the monitoring signal, and then she will be willing to exert only low effort. Accordingly, her option incentive constraints are, for $j, j' \in \{1,2\}$ and $j \neq j'$:

$$(1-\delta)(-\beta a_1^j + p^j) + \delta\bar{y}_1^j(a^j) \geq (1-\delta)p^{j'}.$$

We will now show the following: For $\delta d < (1-\delta)\beta$, where only low effort can be enforced, we have

$$\Lambda^{\mathrm{W}}(d) = (1-\delta)\pi_1\Big(k(1) - k(0)\Big) + \delta d, \tag{C7}$$

where the "W" superscript signifies that decision rights are allocated to the worker. For $\delta d \geq (1-\delta)\beta$ we have

$$\Lambda^{\mathrm{W}}(d) = (1-\delta)\pi_1\Big(1-\beta+k(1)-k\left(\frac{(1-\delta)\beta}{\delta d}\right)\Big)+\delta d, \tag{C8}$$

Moreover, the latter value is attained using a stage game with menu items featuring monitoring levels $\mu^1 = \frac{(1-\delta)\beta}{\delta d} \leq 1$ and $\mu^2 = 1$; payments p^1 and p^2 that satisfy Equation 16; and by directing the worker to exert high effort ($a_1^1 = 1$) if Option 1 is correctly selected, but low effort if Option 2 is correctly selected or if the wrong option is selected.

These facts imply that if $\delta \geq \frac{\beta}{\pi_1(1-\beta)+\beta}$, then Λ has a largest fixed point d^{W} satisfying $\delta d^{\mathrm{W}} \geq (1-\delta)\beta$, given by the largest solution to Equation 15 in the text; as asserted there.

It thus remains to verify Equation C7 and Equation C8 stated here, plus Equation 16 given in the text. To this we now turn. The objective $\omega_1(\gamma, a^2, y^2) - \omega_1(\gamma, a^1, y^1)$ to be maximized is again given by Equation C6. Using the worker's IC constraint for preferring Option 1 to Option 2,

$$(1-\delta)(-\beta a_1^1 + p^1) + \delta \bar{y}_1^1(a^1) \geq (1-\delta)p^2,$$

we see that the objective Equation C6 is no larger than

$$\begin{aligned} & (1-\delta)(-\beta a_1^2 + \pi_1(l(a_1^1,\mu^1) - l(a_1^2,\mu^2)) + \delta\bar{y}_1^2(a^2) \\ = \; & (1-\delta)(-\beta a_1^2 + \pi_1(a_1^1 - a_1^2)(1-\beta) - (k(\mu^1)-k(\mu^2))\pi_1) + \delta\bar{y}_1^2(a^2) \end{aligned}$$

This uppper bound is attained when the constraint binds.

For the terms involving Option 2 in the last expression, we obtain a maximal value by setting $a_1^2 = 0$, $\mu^2 = 1$ and $y_1^2(1) = y_1^2(0) = d$ so that $\bar{y}_1^2(a^2) = d$.

The terms involving a_1^1 are $\pi_1(a_1^1(1-\beta) - k(\mu^1))$. If $\delta d < (1-\delta)\beta$ and therefore only $a_1^1 = 0$ is feasible, the expression is maximal for $\mu^1 = 0$, proving Equation C7. If $\delta d \geq (1-\delta)\beta$, then the expression is maximal for $a_1^1 = 1$ and μ^1 being the minimal monitor level that induces effort, i.e. μ^1 given by $\delta d \mu^1 = (1-\delta)\beta$. This yields value

$$(1-\delta)\left(1-\beta+k(1)-k\left(\frac{(1-\delta)\beta}{\delta d}\right)\right)\pi_1 + \delta d,$$

and thus verifies Equation C8. The option payments in the latter case are given by the binding option constraint with $a_1^1 = 1$, thus

$$(1-\delta)(-p^1+p^2) = (1-\delta)(-\beta) + \delta\bar{y}_1^1(1, a_2^1) = -(1-\delta)\beta + \delta d,$$

where by definition of μ^1 we have $\delta d = (1-\delta)\beta/\mu^1$. This verifies Equation 16 given in the text.

REFERENCES

Miller, David A., and Joel Watson. 2013. "A Theory of Disagreement in Repeated Games With Bargaining." *Econometrica*, 81(6): 2303–2350.