Online Appendix: "Acquiring Information Through Peers"

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In Sections A.1, A.2 and A.3, we prove Proposition 1, Lemma 1, and Proposition 2, respectively. We prove Proposition 3 in Section B.1, and, in Section B.2, we show an example in which Properties 1 and 2 do not hold in an out-of-equilibrium network. In Sections B.3, B.4, and B.5, we prove Theorem 1, Proposition 4, and Theorem 2. Finally, in Section B.6, we prove Proposition 5.

A. Proof of Propositions 1 and 2

We prove Proposition 1 in Section A.1 and Proposition 2 in Section A.3.

1. Proof of Proposition 1

We prove Proposition 1 in two steps. First, we fully characterize the linear equilibrium in Section A.1. Second, we show that there is a unique equilibrium in the first stage of the game in Section A.1.

Linear Equilibrium

Let $\overline{a} \equiv \frac{1}{n}\sum_{j=1}^{n} a_j$ be the average action, and let $\overline{a}_{-i} \equiv \frac{1}{n-1}\sum_{j\neq i} a_j = \frac{n}{n-1}\overline{a} - \frac{1}{n-1}a_i$ be the average action without agent i. We will verify the following guess:

$$\overline{a} = \sum_{j=0}^{n} \beta_j e_j. \tag{A.1}$$

From the first order condition, agent i's optimal action satisfies: $a_i = (1-r)\mathbb{E}\left[\theta|\mathbb{I}_i\right] + rE\left[\overline{a}_{-i}|\mathbb{I}_i\right] = (1-\tilde{r})\mathbb{E}\left[\theta|\mathbb{I}_i\right] + \tilde{r}E\left[\overline{a}|\mathbb{I}_i\right]$, where $\tilde{r} = \frac{rn}{r+n-1}$. Using Bayes updating, the expected value of the state of the world given agent i's informational set is given by $\mathbb{E}\left[\theta|\mathbb{I}_i\right] = \sum_{j=0}^{n} \tilde{g}_{ij} e_j \equiv \overline{e}_i$, where $\tilde{g}_{i0} = \frac{1}{1+\sum_{s=1}^{n} g_{is}\sigma^{-2}} = \frac{\sigma^2}{\mathcal{K}_i+1+\sigma^2}$, $\tilde{g}_{ij} = \frac{g_{ij}\sigma^{-2}}{1+\sum_{s=1}^{n} g_{is}\sigma^{-2}} = \frac{g_{ij}}{\mathcal{K}_i+1+\sigma^2}$ for $j \geq 1$, and $e_0 = 0$ is the prior's mean. The expected value of the average action given i's information is given by

$$E\left[\overline{a}|\mathbb{I}_i\right] = \sum_{j=0}^{n} \beta_j E\left[e_j|\mathbb{I}_i\right] = \sum_{j=0}^{n} \beta_j g_{ij} e_j + \sum_{j=0}^{n} \beta_j (1-g_{ij})\overline{e}_i.$$

Thus, player i's action is simplified to

$$a_i = (1-\tilde{r})\overline{e}_i + \tilde{r}\sum_{j=0}^{n} \beta_j g_{ij} e_j + \tilde{r}\sum_{j=0}^{n} \beta_j (1-g_{ij})\overline{e}_i. \tag{A.2}$$

In order to verify our initial guess, we sum over i,

$$n\overline{a} = \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} \overline{e}_i - \tilde{r}\sum_{i=1}^{n} \overline{e}_i + \tilde{r}\sum_{j=0}^{n} \beta_j \sum_{i=1}^{n} g_{ij} e_j + \tilde{r}\sum_{j=0}^{n} \beta_j \sum_{i=1}^{n} \overline{e}_i - \tilde{r}\sum_{j=0}^{n} \beta_j \sum_{i=1}^{n} g_{ij}\overline{e}_i.$$

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Using matrix notation let

$$\overline{e}=\begin{bmatrix}\overline{e}_1\\ \vdots\\ \overline{e}_n\end{bmatrix}_{n\times 1},\quad e=\begin{bmatrix}e_1\\ \vdots\\ e_n\end{bmatrix}_{n\times 1},\quad G=\begin{bmatrix}g_{10} & g_{11} & \cdots & g_{1n}\\ \vdots & & \ddots & \\ g_{n0} & g_{n1} & \cdots & g_{nn}\end{bmatrix}_{n\times n+1}.$$

Hence, the sum of all action becomes

$$n\overline{a}=\mathbf{1}'\overline{e}-\tilde{r}\mathbf{1}'\overline{e}+\tilde{r}\beta'\text{diag}(G'\mathbf{1})\begin{bmatrix}0\\ e\end{bmatrix}+\tilde{r}\beta'\mathbf{1}\mathbf{1}'\overline{e}-\tilde{r}\beta'G'\overline{e}$$

where $\mathbf{1}$ is a column vector of ones with the appropriate dimension, and diag$(\cdot)$ creates a diagonal matrix. Let

$$\tilde{G}=\begin{bmatrix}\tilde{g}_{10} & \tilde{g}_{11} & \cdots & \tilde{g}_{1n}\\ \vdots & & \ddots & \\ \tilde{g}_{n0} & \tilde{g}_{n1} & \cdots & \tilde{g}_{nn}\end{bmatrix}_{n\times n+1},$$

and we have that $\overline{e}=\tilde{G}\begin{bmatrix}0\\ e\end{bmatrix}$. The sum of actions becomes:

$$\begin{aligned}n\overline{a}&=\mathbf{1}'\tilde{G}\begin{bmatrix}0\\ e\end{bmatrix}-\tilde{r}\mathbf{1}'\tilde{G}\begin{bmatrix}0\\ e\end{bmatrix}+\tilde{r}\beta'\text{diag}(G'\mathbf{1})\begin{bmatrix}0\\ e\end{bmatrix}+\tilde{r}\beta'\mathbf{1}\mathbf{1}'\tilde{G}\begin{bmatrix}0\\ e\end{bmatrix}-\tilde{r}\beta'G'\tilde{G}\begin{bmatrix}0\\ e\end{bmatrix}\\ \overline{a}&=\frac{1}{n}\left\{(1-\tilde{r})\,\mathbf{1}'\tilde{G}+\tilde{r}\beta'\left(\text{diag}(G'\mathbf{1})+\mathbf{1}\mathbf{1}'\tilde{G}-G'\tilde{G}\right)\right\}\begin{bmatrix}0\\ e\end{bmatrix}\end{aligned}$$

Next, we use method of undetermined coefficient to solve for the vector of loadings β based on our initial guess, $\overline{a}=\beta'\begin{bmatrix}0\\ e\end{bmatrix}$:

$$\begin{aligned}\beta'&=\frac{1}{n}\left\{(1-\tilde{r})\,\mathbf{1}'\tilde{G}+\tilde{r}\beta'\left(\text{diag}(G'\mathbf{1})+\mathbf{1}\mathbf{1}'\tilde{G}-G'\tilde{G}\right)\right\}\\ &=\frac{1}{n}(1-\tilde{r})\,\mathbf{1}'\tilde{G}\left[\text{I}-\frac{1}{n}\tilde{r}\left(\text{diag}(G'\mathbf{1})+\mathbf{1}\mathbf{1}'\tilde{G}-G'\tilde{G}\right)\right]^{-1}.\end{aligned}\tag{A.3}$$

We can verify that the average action loadings sum to 1. Starting from the equation above and post-multiplying by a vector of ones (remember that $\tilde{G}\mathbf{1}=\mathbf{1}$):

$$\begin{aligned}n\beta'\mathbf{1}&=(1-\tilde{r})\,\mathbf{1}'\tilde{G}\mathbf{1}+\tilde{r}\beta'\left(\text{diag}(G'\mathbf{1})\mathbf{1}+\mathbf{1}\mathbf{1}'\tilde{G}\mathbf{1}-G'\tilde{G}\mathbf{1}\right)\\ n\beta'\mathbf{1}&=n-\tilde{r}n+\tilde{r}\beta'G'\mathbf{1}+\tilde{r}\beta'\mathbf{1}\mathbf{1}'\mathbf{1}-\tilde{r}\beta'G'\mathbf{1}\\ \beta'\mathbf{1}(n-\tilde{r}n)&=n-\tilde{r}n\\ \beta'\mathbf{1}&=1.\end{aligned}\tag{A.4}$$

From Equation A.2, the action of each agent in vector notation is given by

$$a=\overline{e}-\tilde{r}\overline{e}+\tilde{r}G\text{diag}(\beta)\begin{bmatrix}0\\ e\end{bmatrix}+\tilde{r}\overline{e}-\tilde{r}\text{diag}(\beta'G')\overline{e}=\Lambda\begin{bmatrix}0\\ e\end{bmatrix}$$

where Λ is a $n\times n+1$ matrix of loadings

$$\Lambda=\tilde{G}-\tilde{r}\tilde{G}+\tilde{r}G\text{diag}(\beta)+\tilde{r}\tilde{G}-\tilde{r}\text{diag}(\beta'G')\tilde{G},\tag{A.5}$$

Solving for λ_{ij} and β_j using sum notation

Let λ_{ij} be the element (i,j) in the matrix Λ. Following Equation (A.5), the λs in sum notation are given by:

$$\lambda_{ij} = (1-\tilde{r})\,\tilde{g}_{ij} + \tilde{r}\beta_j g_{ij} + \tilde{r}\tilde{g}_{ij} - \tilde{r}\left(\sum_{s=0}^{n}\beta_s g_{is}\right)\tilde{g}_{ij} \tag{A.6}$$

for $i = 1,\dots,n$, and $j = 0,\dots,n$. Notice that $\sum_{j=0}^{n}\lambda_{ij} = 1$, since $\sum_{j=0}^{n}\tilde{g}_{ij} = 1$. Also notice that $\lambda_{ij} = 0$ whenever $g_{ij} = 0$, and $\lambda_{ij} > 0$ whenever $g_{ij} = 1$. By substituting $\tilde{g}_{ij}$ in, we get for every $j = 1,\dots,n$:

$$\lambda_{ij} = (1-\tilde{r})\,\frac{g_{ij}}{\mathcal{K}_i + 1 + \sigma^2} + \tilde{r}\beta_j g_{ij} + \tilde{r}\frac{g_{ij}}{\mathcal{K}_i + 1 + \sigma^2} - \tilde{r}\left(\sum_{s=0}^{n}\beta_s g_{is}\right)\frac{g_{ij}}{\mathcal{K}_i + 1 + \sigma^2} \tag{A.7}$$

Next, we use Equation (A.3) to derive βs in sum notation:

$$\begin{aligned} n\beta' &= (1-\tilde{r})\,\mathbf{1}'\tilde{G} + \tilde{r}\beta'\text{diag}(G'\mathbf{1}) + \tilde{r}\beta'\mathbf{1}\mathbf{1}'\tilde{G} - \tilde{r}\beta' G'\tilde{G} \\ n\beta_j &= (1-\tilde{r})\textstyle\sum_{i=1}^{n}\tilde{g}_{ij} + \tilde{r}\beta_j\sum_{i=1}^{n} g_{ij} + \tilde{r}\sum_{i=1}^{n}\tilde{g}_{ij} - \tilde{r}\sum_{i=1}^{n}\sum_{s=0}^{n}\beta_s g_{is}\tilde{g}_{ij} \end{aligned}$$

Given that $\sum_{j=0}^{n}\beta_j = 1$ (Equation A.4), we have:

$$n\beta_j = \sum_{i=1}^{n}\tilde{g}_{ij} + \tilde{r}\beta_j\sum_{i=1}^{n} g_{ij} - \tilde{r}\sum_{i=1}^{n}\sum_{s=0}^{n}\beta_s g_{is}\tilde{g}_{ij} \tag{A.8}$$

Also, notice that by definition we have:

$$\beta_j = \frac{1}{n}\sum_{i=1}^{n}\lambda_{ij}. \tag{A.9}$$

By substituting $\tilde{g}_{ij}$ in, we get:

$$\beta_0 = \frac{1}{n}\sum_{i=1}^{n}\frac{\sigma^2}{\sigma^2 + \mathcal{K}_i + 1} + \frac{\tilde{r}}{n}\left[\beta_0 n - \sum_{i=1}^{n}\sum_{s=0}^{n}\frac{\beta_s g_{is}\sigma^2}{\sigma^2 + \mathcal{K}_i + 1}\right] \tag{A.10}$$

$$\beta_j = \frac{1}{n}\sum_{i=1}^{n}\frac{g_{ij}}{\sigma^2 + \mathcal{K}_i + 1} + \frac{\tilde{r}}{n}\left[\beta_j\left(\overline{\mathcal{K}}_j + 1\right) - \sum_{i=1}^{n}\sum_{s=0}^{n}\frac{\beta_s g_{is} g_{ij}}{\sigma^2 + \mathcal{K}_i + 1}\right] \quad \forall\ j = 1,\dots,n \tag{A.11}$$

where $\overline{\mathcal{K}}_j = \sum_{s=1, s\neq j}^{n} g_{sj}$, and $\mathcal{K}_i = \sum_{s=1, s\neq i}^{n} g_{is}$.

Since $\sum_{s=0}^{n}\beta_s = 1$, we can rearrange the expression above as follows:

$$\begin{aligned} \left[n - \tilde{r}\left(\overline{\mathcal{K}}_j + 1\right)\right]\beta_j &= (1-\tilde{r})\sum_{i=1}^{n}\frac{g_{ij}}{\sigma^2 + \mathcal{K}_i + 1} + \tilde{r}\sum_{i=1}^{n}\frac{g_{ij}}{\mathcal{K}_i + \sigma^2 + 1}\left[\sum_{s=0}^{n}\beta_s\left(1 - g_{is}\right)\right] \quad \forall\ j = 1,\dots,n, \\ n\left[1 - \tilde{r}\right]\beta_0 &= (1-\tilde{r})\sum_{i=1}^{n}\frac{\sigma^2}{\sigma^2 + \mathcal{K}_i + 1} + \tilde{r}\sum_{i=1}^{n}\frac{\sigma^2}{\mathcal{K}_i + \sigma^2 + 1}\left[\sum_{s=0}^{n}\beta_s\left(1 - g_{is}\right)\right] \end{aligned}$$

which are Equations (6) and (7) in the main text.

Finally, the loading $\beta_{-i,j}$ is by definition given by

$$\beta_{-i,j} = \frac{1}{n-1}\sum_{k\neq i}\lambda_{kj}, \tag{A.12}$$

which can be written as:

$$\beta_{-i,j} = \frac{n}{n-1}\beta_j - \frac{1}{n-1}\lambda_{ij}. \tag{A.13}$$

Notice that

$$\sum_{j=0}^{n} \beta_{-i,j} = 1 \tag{A.14}$$

because $\sum_{j=0}^{n} \beta_j = 1$ and $\sum_{j=0}^{n} \lambda_{i,j} = 1$, as shown earlier.

Uniqueness

In this section, we prove that there is a unique equilibrium in the second stage of the game. We follow closely the uniqueness proof in Hellwig and Veldkamp (2009), but adapted to our setting. From the first-order condition, in any equilibrium agent i's optimal action satisfies:

$$a_i = (1-\tilde{r})\mathbb{E}\left[\theta|\mathbb{I}_i\right] + \tilde{r}E\left[\overline{a}|\mathbb{I}_i\right] \tag{A.15}$$

To prove uniqueness, we proceed in two steps. First, we show in Lemma A.1 that the following expression constitutes the unique solution to agents' first-order conditions:

$$a_i = a(\mathbb{I}_i) = (1-\tilde{r})\sum_{t=1}^{\infty} \tilde{r}^t \mathbb{E}_i\left[\overline{\mathbb{E}}^t(\theta)\right] \tag{A.16}$$

where $\mathbb{E}_i(\cdot) = \mathbb{E}_i[\cdot|\mathbb{I}_i]$, $\overline{\mathbb{E}}(\cdot) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}_i(\cdot)$, $\overline{\mathbb{E}}^0(\theta) = \theta$, and $\overline{\mathbb{E}}^t(\theta) = \overline{\mathbb{E}}\left[\overline{\mathbb{E}}^{t-1}(\theta)\right]$. We write the action of agent i as a function of her information set, i.e., $a_i = a(\mathbb{I}_i)$, based on the equilibrium definition—see Definition 1. The second step is in Lemma A.2, where we show that Equation (A.16) is a unique linear combination of the available signals in the economy.

LEMMA A.1: *There is a unique equilibrium in the second stage of the game, in which agent i's action a given by Equation (A.16).*

PROOF:
We follow closely the proof of Proposition 1 in Hellwig and Veldkamp (2009), but adapting to our framework. The main difference is that in our setting there is a finite number of signals. Let $\hat{a}$ be the proposed equilibrium from Equation (A.16) and let $\mathbb{A}$ be the set of functions $a_i = a(\mathbb{I}_i)$ such that

$$a(\mathbb{I}_i) = \int_\omega \left[(1-\tilde{r})b'\omega + \tilde{r}\frac{1}{n}\sum_{j=1}^{n} a(\mathbb{I}_j)\right] dF\left(\omega|\mathbb{I}_i\right),$$

where $\omega = (\theta, \varepsilon_1, \dots, \varepsilon_n)'$ is a vector of i.i.d. standard normal random variables, $b = (1, 0, \dots, 0)'$, and $F(\omega|\mathbb{I}_i)$ characterizes the distribution of ω given $\mathbb{I}_i$ as information set. Notice that the set $\mathbb{A}$ is the set of functions that satisfies the first-order conditions from Equation A.15. We will show that $\tilde{a} \in \mathbb{A}$ if and only if $\tilde{a} = \hat{a}$ almost everywhere.

Let us define the functional $\mathcal{L}(\dots)$ from L^2 to the real line:

$$\mathcal{L}(a) = \int_\omega \frac{1}{n}\sum_{i=1}^{n}\left[a(\mathbb{I}_i) - b'\omega\right]^2 dF(\omega) - \tilde{r}\int_\omega \left(\frac{1}{n}\sum_{i=1}^{n} a(\mathbb{I}_i) - b'\omega\right)^2 dF(\omega),$$

We proceed in two steps. First, we show that $\mathcal{L}(a)$ is strictly convex, and therefore if $\tilde{a}_1, \tilde{a}_2 \in \arg\min_a \mathcal{L}(a)$ then $\tilde{a}_1 = \tilde{a}_2$ almost everywhere. Second, we show that $\mathbb{A} = \arg\min_a \mathcal{L}(a)$, that is, $\tilde{a} \in \mathbb{A}$ if and only if $\tilde{a} \in \arg\min_a \mathcal{L}(a)$. Since $\hat{a} \in \mathbb{A}$, then $\hat{a}$ is unique except for measure zero perturbations.

First, we show that the functional $\mathcal{L}(a)$ is strictly convex. For any distinct functions $a_1(\mathbb{I}_i)$ and $a_2(\mathbb{I}_i)$, scalar $\alpha \in (0,1)$, and $\Delta(\mathbb{I}_i) \equiv a_2(\mathbb{I}_i) - a_1(\mathbb{I}_i)$, we have:

$$\begin{aligned}
&\mathcal{L}(\alpha a_1 + (1-\alpha)a_2) - \alpha\mathcal{L}(a_1) - (1-\alpha)\mathcal{L}(a_2) = \alpha\left[\mathcal{L}(a_1 + (1-\alpha)\Delta) - \mathcal{L}(a_1)\right] + (1-\alpha)\left[\mathcal{L}(a_2 - \alpha\Delta) - \mathcal{L}(a_2)\right] \\
&= \alpha\int_\omega \Bigg[(1-\alpha)^2\frac{1}{n}\sum_{i=1}^{n}\Delta(\mathbb{I}_i)^2 + 2(1-\alpha)\frac{1}{n}\sum_{i=1}^{n}\Delta(\mathbb{I}_i)\left[a_1(\mathbb{I}_i) - b'\omega\right] \\
&\quad -\tilde{r}(1-\alpha)^2\left(\frac{1}{n}\sum_{i=1}^{n}\Delta(\mathbb{I}_i)\right)^2 - 2\tilde{r}(1-\alpha)\left(\frac{1}{n}\sum_{i=1}^{n}a_1(\mathbb{I}_i) - b'\omega\right)\left(\frac{1}{n}\sum_{i=1}^{n}\Delta(\mathbb{I}_i)\right)\Bigg] dF(\omega)
\end{aligned}$$

$$+(1-\alpha)\int_\omega \left[\alpha^2 \frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)^2 - 2\alpha\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\left[a_2(\mathbb{I}_i)-b'\omega\right]\right.$$

$$\left. -\tilde{r}\alpha^2\left(\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)^2 + 2\tilde{r}\alpha\left(\frac{1}{n}\sum_{i=1}^n a_2(\mathbb{I}_i)-b'\omega\right)\left(\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)\right] dF(\omega)$$

$$= \int_\omega \left[\left(\alpha(1-\alpha)^2+\alpha^2(1-\alpha)-2\alpha(1-\alpha)\right)\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)^2 - \tilde{r}\left(\alpha(1-\alpha)^2+\alpha^2(1-\alpha)-2\alpha(1-\alpha)\right)\left(\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)^2\right] dF(\omega)$$

$$= -\alpha(1-\alpha)\int_\omega \left[\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)^2 - \tilde{r}\left(\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)^2\right] dF(\omega)$$

$$= -\alpha(1-\alpha)\int_\omega \left[\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)^2 - \left(\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)^2 + (1-\tilde{r})\left(\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)^2\right] dF(\omega)$$

$$= -\alpha(1-\alpha)\int_\omega \left[\frac{1}{n}\sum_{i=1}^n \left(\Delta(\mathbb{I}_i) - \frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)^2 + (1-\tilde{r})\left(\frac{1}{n}\sum_{i=1}^n \Delta(\mathbb{I}_i)\right)^2\right] dF(\omega) \leq 0$$

The last inequality is strict if $\Delta(\mathbb{I}_i)$ is different from zero for a positive measure of events. Since $\mathcal{L}(a)$ is strictly convex, if $\tilde{a}_1, \tilde{a}_2 \in \arg\min_a \mathcal{L}(a)$ then $\tilde{a}_1 = \tilde{a}_2$ almost everywhere.

Next, we show that $\mathbb{A} = \arg\min_a \mathcal{L}(a)$. For any functions $a(\mathbb{I}_i)$ and $\delta(\mathbb{I}_i)$, and a scalar t, we have:

$$\mathcal{L}(a+t\delta) - \mathcal{L}(a) = t^2 A(\delta) + 2tB(a,\delta),$$

where

$$A(\delta) = \int_\omega \frac{1}{n}\sum_{i=1}^n \delta(\mathbb{I}_i)^2 - \tilde{r}\left(\frac{1}{n}\sum_{i=1}^n \delta(\mathbb{I}_i)\right)^2 dF(\omega)$$

$$B(a,\delta) = \int_\omega \frac{1}{n}\sum_{i=1}^n \delta(\mathbb{I}_i)\left[a(\mathbb{I}_i)-b'\omega\right] - \tilde{r}\left(\frac{1}{n}\sum_{i=1}^n a(\mathbb{I}_i)-b'\omega\right)\left(\frac{1}{n}\sum_{i=1}^n \delta(\mathbb{I}_i)\right) dF(\omega)$$

We have that $A(\delta) > 0$ whenever $\delta(\cdot)$ is different from zero for a positive measure. Therefore, $\mathcal{L}(a+t\delta)$ is minimized at $t^* = -\frac{B(a,\delta)}{A(\delta)}$ and $\mathcal{L}(a+t^*\delta) = \mathcal{L}(a) - \frac{B(a,\delta)^2}{A(\delta)}$. If $\tilde{a} \in \arg\min_a \mathcal{L}(a)$, then by the convexity of $\mathcal{L}(a)$ we have that $\tilde{a}$ is unique. Thus for any $\delta(\cdot)$, we have $B(\tilde{a},\delta) = 0$ since t^* minimizes $\mathcal{L}(a+t\delta)$ for any a and δ. If $\tilde{a} \in \arg\min_a \mathcal{L}(a)$ and $\tilde{a}' \notin \arg\min_a \mathcal{L}(a)$, then for $\delta = \tilde{a} - \tilde{a}'$ we have $B(\tilde{a}',\delta) \neq 0$. Therefore, $\tilde{a} \in \arg\min_a \mathcal{L}(a)$ if and only if $B(\tilde{a},\delta) = 0$ for every $\delta(\cdot)$.

We can write $B(a,\delta)$ as follows:

$$B(a,\delta) = \int_\omega \frac{1}{n}\sum_{i=1}^n \delta(\mathbb{I}_i)\left[a(\mathbb{I}_i)-b'\omega\right] - \tilde{r}\left(\frac{1}{n}\sum_{i=1}^n a(\mathbb{I}_i)-b'\omega\right)\left(\frac{1}{n}\sum_{i=1}^n \delta(\mathbb{I}_i)\right) dF(\omega)$$

$$= \frac{1}{n}\sum_{i=1}^n \int_\omega \delta(\mathbb{I}_i)\left[a(\mathbb{I}_i) - (1-\tilde{r})b'\omega - \tilde{r}\frac{1}{n}\sum_{j=1}^n a(\mathbb{I}_j)\right] dF(\omega)$$

using law of iterated expectations,

$$= \frac{1}{n}\sum_{i=1}^n \int_{\mathbb{I}_i}\int_\omega \delta(\mathbb{I}_i)\left[a(\mathbb{I}_i) - (1-\tilde{r})b'\omega - \tilde{r}\frac{1}{n}\sum_{j=1}^n a(\mathbb{I}_j)\right] dF(\omega|\mathbb{I}_i)\, dF(\mathbb{I}_i)$$

$$= \frac{1}{n}\sum_{i=1}^n \int_{\mathbb{I}_i} \delta(\mathbb{I}_i)\left[a(\mathbb{I}_i) - \int_\omega (1-\tilde{r})b'\omega + \tilde{r}\frac{1}{n}\sum_{j=1}^n a(\mathbb{I}_j) dF(\omega|\mathbb{I}_i)\right] dF(\mathbb{I}_i)$$

Notice that if $\tilde{a} \in \mathbb{A}$, then $B(\tilde{a},\delta) = 0$ for any $\delta(\cdot)$, using the definition of the set $\mathbb{A}$. This implies that $\tilde{a} \in \arg\min_a \mathcal{L}(a)$.

Finally, if $\tilde{a} \notin \mathbb{A}$, then by setting $\delta(\mathbb{I}_i) = \tilde{a}(\mathbb{I}_i) - \int_\omega (1-\tilde{r})b'\omega + \tilde{r}\frac{1}{n}\sum_{j=1}^n \tilde{a}(\mathbb{I}_j)dF(\omega|\mathbb{I}_i)$, we have $\delta(\mathbb{I}_i) \neq 0$ and thus $B(a,\delta) = \frac{1}{n}\sum_{i=1}^n \int_{\mathbb{I}_i} \delta(\mathbb{I}_i)^2 dF(\mathbb{I}_i) > 0$. As a result, $\tilde{a} \notin \arg\min_a \mathcal{L}(a)$. □

LEMMA A.2: *Equation (A.16) is a unique linear combination of the available signals in the economy.*

PROOF:

In this proof, we use the same notation as in the previous lemma: $\omega = (\theta, \varepsilon_1, \ldots, \varepsilon_n)'$ and $b = (1, 0, \ldots, 0)'$. Notice that we can write the signal structure of game in matrix notation as follows:

$$\text{(A.17)} \qquad \underbrace{\begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix}_{n\times 1}}_{\equiv e} = \underbrace{\begin{bmatrix} 1 & \sigma & 0 & \cdots & 0 \\ 1 & 0 & \sigma & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 1 & 0 & \cdots & 0 & \sigma \end{bmatrix}_{n\times n+1}}_{\equiv \Gamma} \underbrace{\begin{bmatrix} \theta \\ \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}_{n+1\times 1}}_{\equiv \omega}$$

or simply $e = \Gamma\omega$, where the vector ω is a vector independent standard normal random variables.

However, player i only observes the signal e_j of players he is connected to, in addition to his own signal e_i. Thus, let the $\mathcal{K}_i + 1$ by n matrix X_i be the matrix that selects the signals observed by player i:

$$\text{(A.18)} \qquad \mathbb{I}_i = \{e_j\}_{j=0:g_{ij}=1}^{n} = \begin{bmatrix} 0 \\ X_i e \end{bmatrix} = \begin{bmatrix} 0 \\ X_i \Gamma\omega \end{bmatrix}$$

Using Bayes'updating rules,[1] we have that $\mathbb{E}[\omega|\mathbb{I}_i] = \text{Cov}(\omega, X_i\Gamma\omega)' \, \text{Var}(X_i\Gamma\omega)^{-1} X_i\Gamma\omega$, where $\text{Var}(X_i\Gamma\omega) = X_i\Gamma\Gamma' X_i'$ and $\text{Cov}(\omega, X_i\Gamma\omega) = X_i\Gamma$.

Let $\Delta_i = \Gamma' X_i' \left(X_i\Gamma\Gamma' X_i'\right)^{-1} X_i\Gamma$, we can write $\mathbb{E}[\omega|\mathbb{I}_i] = \Delta_i\omega$. Thus, we have $\overline{\mathbb{E}}(\omega) = \frac{1}{n}\sum_{i=1}^{n}\Delta_i\omega = \overline{\Delta}\omega$, where $\overline{\Delta} = \frac{1}{n}\sum_{i=1}^{n}\Delta_i$. Notice that Δ_i is idempotent and thus its eigenvalues are either zero or one. Since $\frac{1}{n}\Delta_i$ is symmetric with eigenvalues between zero and $\frac{1}{n}$, then the eigenvalues of $\overline{\Delta}$ are between zero and one.[2] Furthermore, we have that $\overline{\mathbb{E}}^0(\omega) = \omega$, $\overline{\mathbb{E}}^1(\omega) = \overline{\Delta}\omega$, $\overline{\mathbb{E}}^2(\omega) = \overline{\Delta}^2\omega$, and, more generally, $\overline{\mathbb{E}}^t(\omega) = \overline{\Delta}^t\omega$. Hence, using $b = (1, 0, \ldots, 0)'$, we can write Equation (A.16) as follows:

$$a(\mathbb{I}_i) = (1-\tilde{r})\sum_{t=1}^{\infty}\tilde{r}^t\mathbb{E}_i\left[\overline{\mathbb{E}}^t(\theta)\right] = (1-\tilde{r})\sum_{t=1}^{\infty}\tilde{r}^t\mathbb{E}_i\left[\overline{\mathbb{E}}^t(b'\omega)\right] = (1-\tilde{r})b'\sum_{t=1}^{\infty}\tilde{r}^t\overline{\Delta}^t\mathbb{E}_i[\omega] = (1-\tilde{r})b'\sum_{t=1}^{\infty}\tilde{r}^t\overline{\Delta}^t\Delta_i\omega.$$

Since the largest eigenvalue of $\overline{\Delta}$ in absolute value is less than or equal to one, the eigenvalues of $\tilde{r}\overline{\Delta}$ are stricly less than one in absolute value. Thus the limit is unique and given by: $a(\mathbb{I}_i) = (1-\tilde{r})\tilde{r}b'\left[I - \tilde{r}\overline{\Delta}\right]^{-1}\overline{\Delta}\Delta_i\omega$. □

2. Proof of Lemma 1

Item (a) follows directly from taking the limit in Equations (6) and (7) as $r \to 0$. For item (b), we start from Equations (6) and (7) for a signal that is observed by everyone, i.e. $j \in \text{PU}$ or $j = 0$:

$$\text{(A.19)} \qquad n\left(1-\tilde{r}\right)\beta_j = (1-\tilde{r})\sum_{i=1}^{n}\tilde{g}_{ij} + \tilde{r}\sum_{i=1}^{n}\tilde{g}_{ij}\sum_{s=0}^{n}\beta_s\left(1-g_{is}\right)\ \forall\, j \in \text{PU } \textit{or } j = 0.$$

If we take the limit as $r \to 1$, we have: $\lim_{r\to 1}\sum_{i=1}^{n}\tilde{g}_{ij}\sum_{s=0}^{n}\beta_s\left(1-g_{is}\right) = 0$. We know that $\beta_s > 0$ for every s and that $g_{ij} \in \{0,1\}$ from Equations (6) and (7). As result, $\beta_s\left(1-g_{is}\right) \geq 0$ for every i and s, and hence $\lim_{r\to 1}\beta_s\left(1-g_{is}\right) = 0$ for every i and s. For every $s \notin \text{PU}$ with $s \geq 1$, there exists an i such that $g_{is} = 0$. Therefore, $\lim_{r\to 1}\beta_s = 0\ \forall\, s \notin \text{PU},\ s \geq 1$. From Equation (A.19) for $j \in \text{PU}$, notice that the right-hand-side of the equation is the same for every $j \geq 1$ because $\tilde{g}_{ij} = \frac{1}{\sigma^2+\mathcal{K}_i+1}$, which implies that $\beta_j = \beta_s \equiv \beta_{PU}$ for every r and for every $s, j \in \text{PU}$. Similarly, for the common prior, we have $\beta_0 = \sigma^2\beta_{PU}$ for every r. Given that $\sum_{j=0}^{n}\beta_j = 1$, we have that $\left(\sigma^2 + \mathcal{K}_{PU}\right)\beta_{PU} + \sum_{j\notin PU}^{n}\beta_j = 1$ and therefore $\beta_{PU} = \frac{1-\sum_{j\notin PU}^{n}\beta_j}{\sigma^2+\mathcal{K}_{PU}} \implies \lim_{r\to 1}\beta_{PU} = \frac{1}{\sigma^2+\mathcal{K}_{PU}}$, which implies $\lim_{r\to 1}\beta_0 = \frac{\sigma^2}{\sigma^2+\mathcal{K}_{PU}}$ and $\lim_{r\to 1}\beta_j = \frac{1}{\sigma^2+\mathcal{K}_{PU}}\ j \in \text{PU}$.

[1]See equations 2 and Appendix A.1 in Hellwig and Veldkamp (2009).

[2]See Theorem 1 in Thompson and Freede (1971).

3. *Proof of Proposition 2*

The payoff function of player i net of link formation costs is given by

$$U_i = -\left(a_i - a_i^*\right)^2,$$

where $a_i^* = (1-r)\theta + r\overline{a}_{-i}$ is player i's bliss action and $\overline{a}_{-i} = \frac{1}{n-1}\sum_{j=1 j\neq i}^{n} a_j = \sum_{j=1}^{n} \beta_{-i,j} e_j$ is the average action without i's own action.[3] Notice that player i takes all $\beta_{-i,j}$'s as given. Using the notation from Equations (A.17) and (A.18), notice that the optimal action of player i is $a_i = \mathbb{E}\left[a_i^* | \mathbb{I}_i\right]$, and the expected payoff net of link formation costs for a given network G conditional on the common prior is given by

$$\mathbb{E}\left[U_i|G\right] = -\mathbb{E}\left[\left(a_i - a_i^*\right)^2|G\right] = -\mathbb{E}\left[\mathbb{E}\left[\left(a_i - a_i^*\right)^2|\mathbb{I}_i\right]|G\right]$$

where the last equality hold based on law of iterated expectations. Using the optimal action choice, the expected value conditional on player i's informational set is a conditional variance. Hence, the payoff function net of link formation costs is further simplified to

$$\mathbb{E}\left[U_i|G\right] = -\mathbb{E}\left[\mathrm{Var}\left(a_i^*|\mathbb{I}_i\right)|G\right]$$

Next, let's write the bliss action in matrix notation:

$$a_i^* = (1-r)\theta + r\overline{a}_{-i} = (1-r)\theta + r\sum_{j=1}^{n}\beta_{-i,j}e_j = \underbrace{\left[1 - r\beta_{-i,0},\ r\sigma\beta_{-i,1},\ r\sigma\beta_{-i,2},\ \cdots,\ r\sigma\beta_{-i,n}\right]}_{\equiv F_i'}\omega = F_i'\omega \tag{A.20}$$

where $\beta_{-i,0} = 1 - \sum_{j=1}^{n}\beta_{-i,j}$ and ω is defined in Equation (A.17). The vector F_i does not depend on player i observed signal, it only depends on the network itself.

Additionally, we can use Bayes updating rule to represent the optimal action as follows:

$$a_i = \mathbb{E}\left[a_i^*|\mathbb{I}_i\right] = \mathbb{E}\left[F_i'\omega|\mathbb{I}_i\right] = F_i'\mathbb{E}\left[\omega|\mathbb{I}_i\right], \tag{A.21}$$

where $\mathbb{E}\left[\omega|\mathbb{I}_i\right] = \mathrm{Cov}\left(\omega, X_i\Gamma\omega\right)'\,\mathrm{Var}\left(X_i\Gamma\omega\right)^{-1}X_i\Gamma\omega$.[4] Player i's expected payoff net of link formation costs becomes:

$$\mathbb{E}\left[U_i|G\right] = -\mathbb{E}\left[\mathrm{Var}\left(a_i^*|\mathbb{I}_i\right)|G\right] = -F_i'\mathrm{Var}\left(\omega|\mathbb{I}_i\right)F_i \tag{A.22}$$

and we can use Bayes' updating rule to compute the variance covariance term:[5]

$$\mathrm{Var}\left(\omega|\mathbb{I}_i\right) = \mathrm{Var}\left(\omega\right) - \mathrm{Cov}\left(\omega, X_i\Gamma\omega\right)'\,\mathrm{Var}\left(X_i\Gamma\omega\right)^{-1}\mathrm{Cov}\left(\omega, X_i\Gamma\omega\right), \tag{A.23}$$

where $\mathrm{Var}\left(\omega\right) = \mathrm{I}$, $\mathrm{Var}\left(X_i\Gamma\omega\right) = X_i\Gamma\Gamma'X_i'$, and $\mathrm{Cov}\left(\omega, X_i\Gamma\omega\right) = X_i\Gamma$. In order to successfully invert the variance-covariance matrix $\mathrm{Var}\left(X_i\Gamma\omega\right)$, let's rewrite Γ from Equation (A.17) as $\Gamma = [\mathbf{1} \quad \Phi]$ where $\mathbf{1}$ is a column vector of ones and

$$\Phi = \begin{bmatrix} \sigma & 0 & \cdots & 0 \\ 0 & \sigma & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \sigma \end{bmatrix}_{n\times n} \tag{A.24}$$

Using the above notation, we can simplify $\mathrm{Var}\left(X_i\Gamma\omega\right)$ as follows: $\mathrm{Var}\left(X_i\Gamma\omega\right) = X_i\Gamma\Gamma'X_i' = X_i\Phi\Phi'X_i' + \mathbf{11}'$. Notice that $X_i\Phi\Phi'X_i'$ is a diagonal matrix variance of signals that player i observes. This simplification is useful because $X_i\Phi\Phi'X_i'$ is easy

[3] Remember that $e_0 = 0$, so we could have defined $\overline{a}_{-i} = \sum_{j=0}^{n}\beta_{-i,j}e_j$ instead.
[4] See equations 2 and Appendix A.1 in Hellwig and Veldkamp (2009).
[5] See equations 3 and Appendix A.1 in Hellwig and Veldkamp (2009).

to invert and we can apply Sherman-Morrison theorem:[6]

$$\text{(A.25)}\qquad \text{Var}\left(X_i\Gamma\omega\right)^{-1} = (X_i\Phi\Phi' X_i')^{-1} - \frac{1}{\phi_i}(X_i\Phi\Phi' X_i')^{-1}\mathbf{1}\mathbf{1}'(X_i\Phi\Phi' X_i')^{-1}$$

where $\phi_i = 1 + \mathbf{1}'(X_i\Phi\Phi' X_i')^{-1}\mathbf{1} = 1 + \sum_{j=1}^n g_{ij}\sigma^{-2}$.

After some algebraic manipulation, we can use the simplified inverse of the variance to compute the following:

$$\text{Cov}\left(\omega, X_i\Gamma\omega\right)' \text{Var}\left(X_i\Gamma\omega\right)^{-1} \text{Cov}\left(\omega, X_i\Gamma\omega\right) = \begin{bmatrix} \mathbf{1}' \\ \Phi' X_i' \end{bmatrix} \text{Var}\left(X_i\Gamma\omega\right)^{-1} [\mathbf{1} \quad X_i\Phi]$$

$$= \frac{1}{\phi_i}\begin{bmatrix} \phi_i - 1, & \frac{g_{i1}}{\sigma}, & \frac{g_{i2}}{\sigma}, & \cdots & \frac{g_{in}}{\sigma} \\ \frac{g_{i1}}{\sigma}, & \phi_i g_{i1} - \frac{g_{i1}g_{i1}}{\sigma^2}, & -\frac{g_{i1}g_{i2}}{\sigma^2}, & \cdots & -\frac{g_{i1}g_{in}}{\sigma^2} \\ \frac{g_{i2}}{\sigma}, & -\frac{g_{i2}g_{i1}}{\sigma^2}, & \phi_i g_{i2} - \frac{g_{i2}g_{i2}}{\sigma^2}, & \cdots & -\frac{g_{i2}g_{in}}{\sigma^2} \\ \vdots & \vdots & & \ddots & \vdots \\ \frac{g_{in}}{\sigma}, & -\frac{g_{in}g_{i1}}{\sigma^2}, & -\frac{g_{in}g_{i2}}{\sigma^2}, & \cdots & \phi_i g_{in} - \frac{g_{in}g_{in}}{\sigma^2} \end{bmatrix}$$

and player i's expected payoff net of link formation costs becomes

$$\mathbb{E}\left[U_i|G\right] = -F_i'\text{Var}\left(\omega|\mathbb{I}_i\right)F_i$$

$$= -\frac{1}{\phi_i}F_i'\begin{bmatrix} 1, & -\frac{g_{i1}}{\sigma}, & -\frac{g_{i2}}{\sigma}, & \cdots & -\frac{g_{in}}{\sigma} \\ -\frac{g_{i1}}{\sigma}, & (1-g_{i1})\phi_i + \frac{g_{i1}}{\sigma^2}, & \frac{g_{i1}g_{i2}}{\sigma^2}, & \cdots & \frac{g_{i1}g_{in}}{\sigma^2} \\ -\frac{g_{i2}}{\sigma}, & \frac{g_{i2}g_{i1}}{\sigma^2}, & (1-g_{i2})\phi_i + \frac{g_{i2}}{\sigma^2}, & \cdots & \frac{g_{i2}g_{in}}{\sigma^2} \\ \vdots & \vdots & & \ddots & \vdots \\ -\frac{g_{in}}{\sigma}, & \frac{g_{in}g_{i1}}{\sigma^2}, & \frac{g_{in}g_{i2}}{\sigma^2}, & \cdots & (1-g_{in})\phi_i + \frac{g_{in}}{\sigma^2} \end{bmatrix}F_i$$

$$\text{(A.26)}\qquad = -\frac{1}{\phi_i}(1 - r\sum_{j=0}^n g_{ij}\beta_{-i,j})^2 - r^2\sum_{j=0}^n(1-g_{ij})\beta_{-i,j}^2\sigma^2$$

where $\phi_i = 1 + \sum_{j=1}^n g_{ij}\sigma^{-2} = \frac{\sigma^2+\mathcal{K}_i+1}{\sigma^2}$ and $\mathcal{K}_i = \sum_{j=1,j\neq i}^n g_{ij}$.

By substituting ϕ_i, the expected payoff including link formation costs becomes:

$$-\frac{\sigma^2}{\sigma^2+\mathcal{K}_i+1}\left(1 - r\sum_{j=0}^n g_{ij}\beta_{-i,j}\right)^2 - r^2\sigma^2\sum_{j=0}^n(1-g_{ij})\beta_{-i,j}^2 - C\left(\mathcal{K}_i\right),$$

which is exactly the payoff expression in Proposition 2.

B. Equilibrium Properties

In Section B.1, we prove Proposition 3 by showing that any equilibrium satisfies properties 1 and 2. In Section B.2, we show an example of an out-of-equilibrium violation of these properties. In Section B.3, we prove our first main result, Theorem 1. In Section B.4, we prove Proposition 4. In Section B.5, we prove our second main result, Theorem 2. Finally, Section B.6 contains the proof of Proposition 5.

[6] For any non-singular matrix A, column vectors u and v, and a scalar α, Sherman-Morrison theorem states that $(A+\alpha uv')^{-1} = A^{-1} - \frac{\alpha}{\phi}A^{-1}uv'A^{-1}$, where $\phi = 1 + \alpha v'A^{-1}u$. We apply this result by setting $A = X_i\Phi\Phi' X_i'$, $\alpha = 1$, $u = v = \mathbf{1}$. See Golub and Van Loan (2012) for more details.

1. *Proof of Proposition 3*

Property 1

We start by showing that given Assumption 1, any strict Nash equilibrium of the game above satisfies Property 1.

The argument works in two main steps. First, we show that an agent's best response to other agents' choices of connections is to observe the signal of the most influential agent. In Lemma A.3, we show that agent i's best response to other agents' choices of connections is to observe the signal of the most influential agent, where agents are ranked by a centrality measure that is specific to agent i, $\beta_{-i,\cdot}$. While in Lemma A.4 we show that, in equilibrium, all agents rank which signal to observe in the same way. That is, we show that the agent specific ranking $\beta_{-i,\cdot}$ coincides in equilibrium for all agents, and is captured by the vector of influences over the average action β. Second, we show that an agent whose signal is more observed is also the one that has a more influential signal. In Lemma A.6 we show that more people observe agent m's signal than agent l's signal if, and only if, agent m's signal is more influential for the average action than agent l's signal. This guarantees that, in equilibrium, the ranking implied by influence is the same ranking implied by the number of agents observing a signal.

Let us start by showing the monotonicity of best responses. We show that for any agent i, set of connections of i g_i, set of connections of other agents, and other agents strategies, agent i's best response dictates that if she finds optimal to observe another player's signal, then she observes any other more influential signal as well.

LEMMA A.3: *For any strategy played by the other agents, in any best response by agent i, if $i \neq l$ and $g_{i,l} = 1$, then $g_{i,m} = 1$ for any signal m such that $\beta_{-i,m} > \beta_{-i,l}$.*

Furthermore, if the best response is strict, then if $i \neq l$ and $g_{i,l} = 1$, then $g_{i,m} = 1$ for any signal m such that $\beta_{-i,m} \geq \beta_{-i,l}$.

PROOF:

Observe agent i's expected payoff formula from Proposition 2. First, note that player i's connection decisions or action decisions do not influence the vector of influences given by $\beta_{-i,\cdot}$. Thus, an agent connections only affect her payoff through the g_i. Finally, observe that player's i payoff derivative with respect to $\beta_{-i,l}$ is strictly positive for any $g_{i,l} = 1$, and strictly negative for $g_{i,l} = 0$. This completes the proof. □

We now proceed to the second part of our argument. We establish that the agent, j, whose signal is more observed is also the agent j with higher $\beta_{-i,j}$, for all i.

The first step is to establish a relationship between $\beta_{-i,j}$ and β_j in any strict equilibrium.

LEMMA A.4: *For any agent i, given a strategy played by the other agents, in any strict best response by i, $\beta_m \geq \beta_l$ implies that $\beta_{-i,m} \geq \beta_{-i,l}$. Furthermore, if $\beta_m > \beta_l$, then we have $\beta_{-i,m} > \beta_{-i,l}$*

PROOF:

We proceed by exhaustion. Agent i's connections must satisfy one of the following situations: (i) $g_{im} = 1$ and $g_{il} = 0$; (ii) $g_{im} = g_{il} = 0$; (iii) $g_{im} = g_{il} = 1$; or (iv) $g_{im} = 0$ and $g_{il} = 1$. We start with the first case. If $g_{if} = 1$ and $g_{ih} = 0$, it must be that $\beta_{-i,f} > \beta_{-i,h}$ by Lemma A.3. For the other 3 cases, using Equations (A.13) and (A.7), we have:

$$\beta_{-i,j} = \frac{n}{n-1}\beta_j - \frac{1}{n-1}\lambda_{ij} = \frac{n}{n-1}\beta_j - \frac{g_{ij}}{n-1}\left[\frac{1}{\mathcal{K}_i + 1 + \sigma^2} + \tilde{r}\left[\beta_j - \frac{\sum_{s=0}^{n}\beta_s g_{is}}{\mathcal{K}_i + 1 + \sigma^2}\right]\right]$$

Applying the above to our three remaining cases gives us the following: (i) If $g_{im} = g_{il} = 0$, then $\beta_{-i,m} - \beta_{-i,l} = \frac{n}{n-1}(\beta_m - \beta_l) \geq 0$. (ii) If $g_{im} = g_{il} = 1$, then $\beta_{-i,m} - \beta_{-i,l} = \left(\frac{n}{n-1} - \frac{\tilde{r}}{n-1}\right)(\beta_m - \beta_l) \geq 0$. (iii) If $g_{im} = 0$ and $g_{il} = 1$, then $\beta_{-i,m} - \beta_{-i,l} = \frac{n}{n-1}(\beta_m - \beta_l) + \frac{1 - \tilde{r}\sum_{s=0}^{n}\beta_s g_{is}}{(n-1)(\mathcal{K}_i + 1 + \sigma^2)} + \frac{\tilde{r}}{n-1}\beta_l > 0$. Thus, we have that $\beta_m \geq \beta_l \implies \beta_{-i,m} \geq \beta_{-i,l}$ and $\beta_m > \beta_l \implies \beta_{-i,m} > \beta_{-i,l}$. □

To show that in any strict Nash equilibrium Property 1 is satisfied, all what's left to show is that an agent whose signal is more observed has a higher influence on the average action in any strict equilibrium, that is $\overline{\mathcal{K}}_m \geq \overline{\mathcal{K}}_l \implies \beta_m \geq \beta_l$. Before that, we show a simple property about cross-looks: If a higher ranked agent observes the signal of a lower ranked agent, then in equilibrium the lower ranked agent also observes the higher ranked agent signal. Although in content the following Lemma is closer to Property 2, its proof is a lot simpler. To prove the lemma, we present a revealed preference argument. It suffices in this case (and not for Property 2) as we only have to compare one player's deviation at a time. We use this result in the proof of Lemma A.6.

LEMMA A.5: *Let agents h and f be ranked by their influence over the average action, such that $\beta_f < \beta_h$. If in equilibrium $g_{h,f} = 1$, then $g_{f,h} = 1$.*

PROOF:

The proof proceeds by contradiction. Assume that in a proposed equilibrium, $g_{h,f} = 1$ and $g_{f,h} = 0$.

Let Π_f and Π_h be the players' equilibrium payoffs. Since $g_{h,f} = 1$, agent f can simply copy agent h's connections if she wanted to—and also copy the action weights—which implies that $\Pi_f \geq \Pi_h$ by revealed preference. It is worth noting that if agent f copied agent h's connections they would both have the same information set. This guarantees that they could have the same payoff, as well as same action coefficients.

Let $\hat{\Pi}_f$ be the payoff of agent f if agent f stops observing her own signal and observes player h signal for free. By the monotonicity of agent i's expected payoff formula, given in Proposition 2, and Lemma A.4, $\beta_{-f,h} > \beta_{-f,f}$, and thus $\hat{\Pi}_f > \Pi_f$.

Finally, if agent h observes the same set of signals as agent f does in the proposed equilibrium, (except that she observes her own signal and does not observe agent f's signal), her payoff cannot be less than $\hat{\Pi}_f$. Agent h could simply copy the connections and action weights used by agent f to obtain the payoff $\hat{\Pi}_f$. By revealed preference, this gives us $\hat{\Pi}_f \leq \Pi_h$. A contradiction. □

LEMMA A.6: *In a strict equilibrium, agent m's signal has a weakly higher impact in the average action than agent l's has if, and only if, agent m is weakly more observed than agent l.*

$$\beta_m \geq \beta_l \iff \overline{\mathcal{K}}_m \geq \overline{\mathcal{K}}_l$$

PROOF:

First of all, note that lemmata A.3 and A.4 guarantee one side of the argument, that if $\beta_m \geq \beta_l$ then $\overline{\mathcal{K}}_m \geq \overline{\mathcal{K}}_l$. However, we need to show the other direction to guarantee Property 1. Thus, assume not. That is, assume that $\beta_m < \beta_l$ and at the same time $\overline{\mathcal{K}}_m \geq \overline{\mathcal{K}}_l$.

From the fact that all players use the beta ranking to decide who to look, we have that $\beta_l > \beta_m$ implies $\beta_{-i,l} > \beta_{-i,m}$ $\forall i$, and thus $g_{i,l} \geq g_{i,m}$ $\forall i \neq m$. Let us now proceed in cases. There are two possible cases, (i) $g_{l,m} = 1$, which gives us $g_{m,l} = 1$ by Lemma A.5, or (ii) $g_{l,m} = 0$, which gives us $g_{m,l} = 0$ otherwise $\overline{\mathcal{K}}_l > \overline{\mathcal{K}}_m$. So, in both cases, we have that $g_{l,m} = g_{m,l}$. Since $g_{i,l} \geq g_{i,m}$ $\forall i \neq m$ and $\overline{\mathcal{K}}_m \geq \overline{\mathcal{K}}_l$, it must be that $\overline{\mathcal{K}}_m = \overline{\mathcal{K}}_l$, which implies that $g_{i,l} = g_{i,m}$ $\forall i$.

Let's now use the formula for the influence of a signal j, β_j, from Equation (A.11), and apply it to both signals m and l. The difference between β_m and β_l is given by:

$$\beta_m - \beta_l = \frac{1}{n}\sum_{i=1}^{n}\frac{g_{im}-g_{il}}{\sigma^2+\mathcal{K}_i+1} + \frac{\tilde{r}}{n}\left[\beta_m(\overline{\mathcal{K}}_m+1) - \beta_l(\overline{\mathcal{K}}_l+1) - \sum_{i=1}^{n}\sum_{s=0}^{n}\frac{\beta_s g_{is}(g_{im}-g_{il})}{\sigma^2+\mathcal{K}_i+1}\right] = \frac{\tilde{r}}{n}\left[(\beta_m-\beta_l)(\overline{\mathcal{K}}_m+1)\right]$$

Given that $\tilde{r}\left(\overline{\mathcal{K}}_m+1\right) < n$, we must have $\beta_m - \beta_l = 0$, a contradiction. □

Finally, to show that any strict Nash equilibrium of the game satisfies Property 1 all what is left is to use the Lemmas above. In any strict equilibrium, by Lemma A.6, if $\overline{\mathcal{K}}_m \geq \overline{\mathcal{K}}_l$ we have that $\beta_m \geq \beta_l$. Lemma A.4, shows that $\beta_m \geq \beta_l$ guarantees $\beta_{-i,m} \geq \beta_{-i,l}$, and finally by Lemma A.3 $\beta_{-i,m} \geq \beta_{-i,l}$ implies that for any $l \neq i$, $g_{i,l} = 1 \implies g_{i,m} = 1$.

Property 2

The proof that Property 2 holds in equilibrium is a little more evolved. Before discussing the details of the proof, let us define two sets, D_M and D_L, for two players m and l with

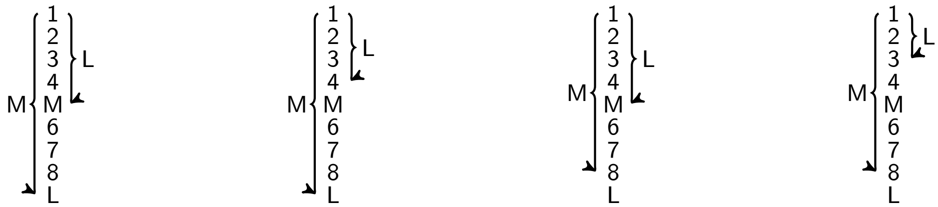
$$\mathcal{K}_m > \mathcal{K}_l.$$

By lemmata A.3 and A.4, all players are ranked according to a common list and thus all signals that player l pays to observe, player m also observes.

Let there be $d = \mathcal{K}_m - \mathcal{K}_l \geq 1$ signals. Abusing notation, we call the corresponding set of signals, D_M and D_L defined as follows: D_M is the set of d signals that agent m is currently observing but would stop observing if agent m were to observe d fewer signals. Similarly, D_L is the set of d signals that agent l is not currently observing but would start observing if agent l were to observe d additional signals. Accordingly, if l were to form connections to D_L she would be obtaining the same number of signals as m. Note that we define D_M and D_L to be the set of signals that give the best possible information set for players m and l that satisfy the above.

The fact that player m receives signal m for free disciplines the sets D_M and D_L. They are not equal, as agent m cannot deviate and stop observing her own signal. Let us also define the set S_M, to be the set of signals that agent m observes and l does not observe, while the set S_L is the set of signals that agent l observes and m does not.

EXAMPLE A.1: *Consider the following example, in which player m is the 5^{th} most attractive signal to be tapped into while player l is the 9^{th}. In each of the four situations above, we contemplate a different configuration between the two players. In*



(a)	(b)	(c)	(d)
$M \to L$	$M \to L$	$M \not\to L$	$M \not\to L$
$L \to M$	$L \not\to M$	$L \to M$	$L \not\to M$
$D_M = \{7, 8, L\}$	$D_M = \{6, 7, 8, L\}$	$D_M = \{7, 8\}$	$D_M = \{4, 6, 7, 8\}$
$D_L = \{6, 7, 8\}$	$D_L = \{M, 6, 7, 8\}$	$D_L = \{6, 7\}$	$D_L = \{4, M, 6, 7\}$
$d = 3$	$d = 4$	$d = 2$	$d = 4$

Figure A.1. : Four different configurations concerning players m and l choices of connections.

the first, they both tap into each other signals, while on the last neither does so. In the second, even though m taps into $l's$ signal, l does not correspond, and finally, the third situation presents the inverse.

The four examples pictured above show different possibilities for the sets D_M and D_L, depending on the connections formed. It is also interesting to understand what is the information set of players in each situation. In the first situation, the players information sets are $I_M = \{e_0, e_1, e_2, e_3, e_4, e_M, e_6, e_7, e_8, e_L\}$ and $I_L = \{e_0, e_1, e_2, e_3, e_4, e_M, e_L\}$, and thus $S_M = \{e_6, e_7, e_8\}$ and $S_L = \{\emptyset\}$. In the second situation, they are $I_M = \{e_0, e_1, e_2, e_3, e_4, e_M, e_6, e_7, e_8, e_L\}$ and $I_L = \{e_0, e_1, e_2, e_3, e_4, e_L\}$ (and thus $S_M = \{e_M, e_6, e_7, e_8\}$ and $S_L = \{\emptyset\}$), while on the third they are $I_M = \{e_0, e_1, e_2, e_3, e_4, e_M, e_6, e_7, e_8\}$ and $I_L = \{e_0, e_1, e_2, e_3, e_4, e_M, e_L\}$, with $S_M = \{e_6, e_7, e_8\}$ and $S_L = \{e_L\}$. Finally, in the fourth configuration, $I_M = \{e_0, e_1, e_2, e_3, e_4, e_M, e_6, e_7, e_8\}$ and $I_L = \{e_0, e_1, e_2, e_3, e_L\}$, and thus $S_M = \{e_4, e_M, e_6, e_7, e_8\}$ and $S_L = \{e_L\}$.

The example above highlights comparative properties of the sets D_M and D_L. The signals listed in D_L weakly dominate the ones listed in D_M. This is a direct result of the fact that player m receives the signal m for free (and m is more attractive than l) and cannot stop observing it, while player l receives signal l.

Comparing Deviations

We will prove Property 2 by contradiction. The general structure of the argument is to assume that in equilibrium an agent m is at the same time strictly observing more signals and has her signal observed more than another player l, that is:

$$\overline{\mathcal{K}}_m \geq \overline{\mathcal{K}}_l \text{ and } \mathcal{K}_m > \mathcal{K}_l$$

The contradiction will be constructed in the following way: If it is worth for agent m to pay and observe more signals than l does (even though agent $m's$ free signal is more observed than agent l's free signal), then agent l's deviation to look at those signals is profitable. The argument of the proof is to show that if player m is not willing to deviate and stop observing signals in D_M, then it is optimal for agent l to deviate and start observing signals in D_L.

(i) First, we define an artificial deviation for player m. That is, a deviation not available to player m, but which will be used as a hypothetical tool in this proof. That deviation is for player m to not observe the signals in the set D_L, saving the cost of not paying for d signals. This is not a proper deviation available to player m for two reasons: (1) it might involve player m to not observe her own signal; and (2) for the purpose of costs, in the hypothetical deviation, when player m stops observing her own signal she saves on costs as if she had had to pay for it. This hypothetical deviation is very useful as now we can compare the benefit of agent m to stop observing signals in D_L, with the benefit of agent l to start observing signals in D_L. The set of signals and the cost difference is the same in both cases.

(ii) Second, by the fact that the set D_L weakly dominates the set D_M, and by Lemma A.3 and Lemma A.4, we know that the hypothetical deviation for agent m to stop observing signals in D_L is weakly dominated by the original (and available to player m) deviation to stop observing the signals in D_M. Hence, if agent m is not willing to stop observing the signals in D_M, then agent m is not willing to take the hypothetical deviation described above (i.e. to stop observing the signals in D_L).

To simplify the notation, from now on we call the set D_L as D, for deviation.

(iii) The rest of the proof consists of showing that, if player m is not willing to deviate and stop observing signals in D_L, then it would be optimal for agent l to deviate and start observing signals in D_L. We do this is in a few steps:

1) Lemmata A.7 and A.8 show that the influence of signals inside (outside) of the set D is higher (lower) if the average action excludes player l than if it excludes player m, as agent l does not observe those signals.

2) We construct the ex-ante payoff gain for player m to take the hypothetical deviation of stop observing the set D of signals. We call this $\Delta\Pi_m$, which is formally the payoff of player m under the hypothetical deviation minus the payoff without the hypothetical deviation which is to keep observing the signals in the set D. Similarly, we construct the ex-ante payoff gain for player l to take the deviation of start observing the set D of signals. We call this $\Delta\Pi_l$, which is formally the payoff of player l under the deviation minus the payoff without the deviation which is to keep not observing the signals in the set D.

3) By assumption, player m is not willing to take the deviation, thus $\Delta\Pi_m$ must be weakly negative. That is, $\Delta\Pi_m \leq 0$. Also by the contradiction assumptions, player l is not willing to accept the deviation, thus $\Delta\Pi_l$ must be weakly negative as well. That is, $\Delta\Pi_l \leq 0$. To formally show our contradiction, we show that $\Delta\Pi_m + \Delta\Pi_l > 0$. To show this last inequality, we use the results in Lemmata A.10 and A.9 to simplify the expression for the payoff gains.

Next, we proceed by showing these three step above, starting with Lemmata A.7 and A.8.

LEMMA A.7: *For a given non-empty set of signals D and two distinct agents l and m such that m observe all signals in D while agent l does not observe signals in D, the summed influence of the signals in the set D is higher if the average action excludes agent l's action than if it excludes agent m's action:*

$$\beta_{-l,j} > \beta_{-m,j} \text{ for every } j \in D.$$

PROOF:

This is a direct result of the fact that l is not observing any signal in D while m is observing signals of D. This implies that l's action cannot respond to signals in D, i.e., $\lambda_{lj} = 0 \,\forall\, j \in D$. Using the expressions for $\beta_{-m,j}$ and $\beta_{-l,j}$ from Equation (A.13), we have that:

$$\beta_j = \frac{n-1}{n}\beta_{-m,j} + \frac{1}{n}\lambda_{m,j} = \frac{n-1}{n}\beta_{-l,j} + \frac{1}{n}\lambda_{l,j} = \frac{n-1}{n}\beta_{-l,j}$$

Hence,

$$\beta_{-m,j} = \beta_{-l,j} - \frac{\lambda_{m,j}}{n-1} < \beta_{-l,j},$$

since $\lambda_{mj} > 0$ whenever $g_{mj} = 1$, i.e., for every $j \in D$, as discussed in Appendix A.1 (Equation A.6). □

LEMMA A.8: *For a given non-empty set of signals D and two distinct agents l and m such that m observe all signals in D while agent l does not observe signals in D such that $\sum_{j=0}^{n}(g_{mj} - g_{lj})\beta_j \geq \sum_{j\in D}\beta_j$, then:*

$$\sum_{j\notin D} g_{m,j}\beta_{-m,j} - \sum_{j\notin D} g_{l,j}\beta_{-l,j} \geq \frac{1}{n-1}\sum_{j\in D}\lambda_{m,j} > 0$$

PROOF:

Observe that l is not tapping into any signal in D, i.e., $g_{lj} = 0 \,\forall\, j \in D$, thus we have:

$$\sum_{j\notin D} g_{l,j}\beta_{-l,j} = \sum_{j=0}^{n} g_{l,j}\beta_{-l,j} = 1 - \sum_{j=0}^{n}(1-g_{l,j})\beta_{-l,j}.$$

Player m is tapping into all signals in the set D, i.e., $g_{mj} = 1 \,\forall\, j \in D$, and thus:

$$\sum_{j\notin D} g_{m,j}\beta_{-m,j} = \sum_{j=0}^{n} g_{m,j}\beta_{-m,j} - \sum_{j\in D}\beta_{-m,j} = 1 - \sum_{j=0}^{n}(1-g_{m,j})\beta_{-m,j} - \sum_{j\in D}\beta_{-m,j}.$$

Subtracting the first from the second, we have:

$$\sum_{j\notin D} g_{m,j}\beta_{-m,j} - \sum_{j\notin D} g_{l,j}\beta_{-l,j} = \sum_{j=0}^{n}(1-g_{l,j})\beta_{-l,j} - \sum_{j=0}^{n}(1-g_{m,j})\beta_{-m,j} - \sum_{j\in D}\beta_{-m,j}.$$

We can partition all signals in the economy into four groups. The set of signals they are both observing S_B; the set of signals neither is observing S_N; the set of signals m is observing and l is not, S_M; and the set of signals l is observing and m is not,

S_L.

$$\sum_{j\notin D} g_{m,j}\beta_{-m,j} - \sum_{j\notin D} g_{l,j}\beta_{-l,j} = \sum_{j\in S_N}\left(\beta_{-l,j}-\beta_{-m,j}\right) + \sum_{j\in S_M}\beta_{-l,j} - \sum_{j\in S_L}\beta_{-m,j} - \sum_{j\in D}\beta_{-m,j}$$

From Equation (A.13), $\beta_j = \frac{n-1}{n}\beta_{-m,j} + \frac{1}{n}\lambda_{m,j}$. According to Equation (A.6) in Appendix A.1, we know that $\lambda_{ij} = 0$ whenever $g_{ij} = 0$. Hence, for a signal j in S_N, both $\lambda_{m,j}$ and $\lambda_{l,j}$ are zero. For a signal in S_M, $\lambda_{l,j} = 0$ and for a signal in S_L, $\lambda_{m,j} = 0$. Thus, using these relationships, we can rewrite the difference above as

$$\begin{aligned}\sum_{j\notin D} g_{m,j}\beta_{-m,j} - \sum_{j\notin D} g_{l,j}\beta_{-l,j} &= \frac{n}{n-1}\left[\sum_{j\in S_M}\beta_j - \sum_{j\in S_L}\beta_j\right] - \sum_{j\in D}\beta_{-m,j}\\ &= \frac{n}{n-1}\left[\underbrace{\sum_{j\in S_M}\beta_j - \sum_{j\in S_L}\beta_j - \sum_{j\in D}\beta_j}_{\geq 0} + \sum_{j\in D}\frac{\lambda_{m,j}}{n}\right] \geq \frac{1}{n-1}\underbrace{\sum_{j\in D}\lambda_{m,j}}_{>0} > 0.\end{aligned}$$

where the first inequality holds because $\sum_{j\in S_M}\beta_j - \sum_{j\in S_L}\beta_j = \sum_{j=0}^{n}(g_{mj}-g_{lj})\beta_j \geq \sum_{j\in D}\beta_j$, using the condition of this lemma. □

Given that all players are ranked according to a common list (lemmata A.3 and A.4), for the players l and m defined in this proof, the condition in the lemma above is satisfied by definition.

Using the lemmata proven above, we now proceed to look at agent's payoffs if they deviated and chose to observe different signals. Let $\Delta\Pi_m$ be the difference of ex-ante expected payoff of player m between breaking those d extra links in D or maintaining them. Notice that if player m unilaterally deviates and breaks those links, no other player changes her action. Thus the influence of signals to other players action will not change, and $\beta_{-m,j}$ must be constant. Similarly, let $\Delta\Pi_l$ be the difference of ex-ante expected payoff of player l between observing those d signals or not forming those links. Given that by assumption we are at strict equilibrium, both expected payoff differences should be strictly smaller than zero.

Regarding notation, we keep $g_{i,j}$ to be the original linking strategy of the proposed equilibrium, in which m observes the signals from D and agent l does not observe them. Thus, we have to add or subtract elements to the expression accordingly. Formally, in both payoff expressions used in $\Delta\Pi_m$—one where player m observes the signals in D, and one where she does not—we consider the same $g_{m,j}$ with $g_{m,j} = 1\ \forall j\in D$. Analogously, we consider for both elements of $\Delta\Pi_l$ that $g_{l,j} = 0\ \forall j\in D$. Using the payoff function from Proposition 2, we compute $\Delta\Pi_m$ as follows:

$$\begin{aligned}\Delta\Pi_m = &-\underbrace{\frac{\sigma^2\left(1-r\sum_{j\notin D} g_{mj}\beta_{-m,j}\right)^2}{\sigma^2+\mathcal{K}_m-d+1}}_{\text{Without observing}} + \underbrace{\frac{\sigma^2\left(1-r\sum_{j=0}^{n} g_{mj}\beta_{-m,j}\right)^2}{\sigma^2+\mathcal{K}_m+1}}_{\text{Observing D}} - \underbrace{r^2\sigma^2\left(\sum_{j=0}^{n}(1-g_{mj})\beta_{-m,j}^2 + \sum_{j\in D}\beta_{-m,j}^2\right)}_{\text{Without observing}}\\ &+\underbrace{r^2\sigma^2\sum_{j=0}^{n}(1-g_{mj})\beta_{-m,j}^2}_{\text{Observing D}} + \underbrace{\Delta C}_{\substack{\text{Cost difference of}\\ \text{not observing signals}}}\end{aligned}$$

where the first element sums over $j\notin D$, as player m is not observing the signals in D, while $g_{m,j} = 1\ \forall j\in D$. For the same reason, the third term includes the additional sum over $j\in D$, $\sum_{j\in D}\beta_{-m,j}^2$. Simplifying the expression above,

$$\begin{aligned}\Delta\Pi_m = &-\left(1-r\sum_{j\notin D} g_{mj}\beta_{-m,j}\right)^2\left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_m-d+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_m+1}\right)\\ &-2\frac{r\sigma^2}{\sigma^2+\mathcal{K}_m+1}\left(1-r\sum_{j\notin D} g_{mj}\beta_{-m,j}\right)\sum_{j\in D}\beta_{-m,j} + \frac{r^2\sigma^2}{\sigma^2+\mathcal{K}_m+1}\left(\sum_{j\in D}\beta_{-m,j}\right)^2 - r^2\sigma^2\sum_{j\in D}\beta_{-m,j}^2 + \Delta C\end{aligned}$$

If we analyze the ex-ante expected payoff difference for player l, we have:

$$\Delta\Pi_l = -\underbrace{\frac{\sigma^2\left(1-r\sum_{j=0}^{n} g_{lj}\beta_{-l,j} - r\sum_{j\in D}\beta_{-l,j}\right)^2}{\sigma^2+\mathcal{K}_l+d+1}}_{\text{Observing D}} + \underbrace{\frac{\sigma^2\left(1-r\sum_{j=0}^{n} g_{lj}\beta_{-l,j}\right)^2}{\sigma^2+\mathcal{K}_l+1}}_{\text{Without observing}} - \underbrace{r^2\sigma^2\left(\sum_{j=0}^{n}(1-g_{lj})\beta_{-l,j}^2 - \sum_{j\in D}\beta_{-l,j}^2\right)}_{\text{Observing D}}$$

$$+ \underbrace{r^2\sigma^2\sum_{j=0}^{n}(1-g_{lj})\beta_{-l,j}^2}_{\text{Without observing}} - \underbrace{\Delta C}_{\text{Cost difference}}$$

where the first element includes an additional term, a sum over $j \in D$, as player l is observing the signals in D, while $g_{l,j} = 0\ \forall j \in D$. For the same reason, the third element subtracts the additional term, $\sum_{j\in D}\beta_{-l,j}^2$.

Notice that $\sum_{j\notin D} g_{lj}\beta_{-lj} = \sum_{j=0}^{n} g_{lj}\beta_{-lj}$ because $g_{lj} = 0\,\forall\, j \in D$. Hence, payoff difference can be written as:

$$\Delta\Pi_l = \left(1 - r\sum_{j\notin D} g_{lj}\beta_{-l,j}\right)^2\left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_l+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_l+d+1}\right) + 2\frac{r\sigma^2}{\sigma^2+\mathcal{K}_l+d+1}\left(1 - r\sum_{j\notin D} g_{lj}\beta_{-l,j}\right)\sum_{j\in D}\beta_{-l,j}$$
$$- \frac{r^2\sigma^2}{\sigma^2+\mathcal{K}_l+d+1}\left(\sum_{j\in D}\beta_{-l,j}\right)^2 + r^2\sigma^2\sum_{j\in D}\beta_{-l,j}^2 - \Delta C$$

Before we proceed, note that the cost difference is the same in both cases, and also that $\mathcal{K}_l + d = \mathcal{K}_m$. Thus, as we sum the two differences, we write as follows:

$$\Delta\Pi_l + \Delta\Pi_m = \left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_l+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_l+d+1}\right)\left[\left(1 - r\sum_{j\notin D} g_{lj}\beta_{-l,j}\right)^2 - \left(1 - r\sum_{j\notin D} g_{mj}\beta_{-m,j}\right)^2\right]$$
$$+ 2\frac{r\sigma^2}{\sigma^2+\mathcal{K}_l+d+1}\left[\left(1 - r\sum_{j\notin D} g_{lj}\beta_{-l,j}\right)\sum_{j\in D}\beta_{-l,j} - \left(1 - r\sum_{j\notin D} g_{mj}\beta_{-m,j}\right)\sum_{j\in D}\beta_{-m,j}\right]$$
$$+ r^2\sigma^2\left[\left(\sum_{j\in D}\beta_{-l,j}^2 - \sum_{j\in D}\beta_{-m,j}^2\right) - \frac{\left(\sum_{j\in D}\beta_{-l,j}\right)^2 - \left(\sum_{j\in D}\beta_{-m,j}\right)^2}{\sigma^2+K_l+d+1}\right]$$

We are now ready to sign the first two lines of the expression above. First, $d > 0$ and Lemma A.8 give us that the first line is the product of two strictly positive terms. For the second line, by Lemma A.7 and Lemma A.8, we have that the second line is also strictly positive. In what follows, we show that the third line is non-negative, characterizing the contradiction. To sign the third line is equivalent to sign the following expression:

$$\frac{\left(\sum_{j\in D}\beta_{-l,j}^2 - \sum_{j\in D}\beta_{-m,j}^2\right)}{d} - \frac{d}{\sigma^2+K_l+d+1}\frac{\left(\sum_{j\in D}\beta_{-l,j}\right)^2 - \left(\sum_{j\in D}\beta_{-m,j}\right)^2}{d^2}$$
$$\geq \frac{\sum_{j\in D}\left(\beta_{-l,j}^2 - \beta_{-m,j}^2\right)}{d} - \frac{\left(\sum_{j\in D}\beta_{-l,j}\right)^2 - \left(\sum_{j\in D}\beta_{-m,j}\right)^2}{d^2}$$
$$= \frac{1}{d}\sum_{j\in D}\left(\beta_{-l,j} - \frac{1}{d}\sum_{s\in D}\beta_{-l,s}\right)^2 - \frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j} - \frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)^2$$

From Equation (A.13) along with the fact that $\lambda_{lj} = 0$, we know that $\beta_{-l,j} = \beta_{-m,j} + \frac{1}{n-1}\lambda_{m,j}$, and therefore the expression simplifies to:

$$
\begin{aligned}
&= \frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}+\frac{1}{n-1}\lambda_{m,j}-\frac{1}{d}\sum_{s\in D}\left(\beta_{-m,s}+\frac{1}{n-1}\lambda_{m,s}\right)\right)^2-\frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)^2\\
&= \left(\frac{1}{n-1}\right)^2\frac{1}{d}\sum_{j\in D}\left(\lambda_{m,j}-\frac{1}{d}\sum_{s\in D}\lambda_{m,s}\right)^2+\left(\frac{2}{n-1}\right)\frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)\left(\lambda_{m,j}-\frac{1}{d}\sum_{s\in D}\lambda_{m,s}\right)\\
&\geq \left(\frac{2}{n-1}\right)\frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)\left(\lambda_{m,j}-\frac{1}{d}\sum_{s\in D}\lambda_{m,s}\right).
\end{aligned}
$$

We still have to show that this term is positive. It amounts to show that a player considers more, and thus gives it a higher weight (higher $\lambda_{-m,j}$) to more influential signals (higher $\beta_{-m,j}$). The next lemma rewrite λ_{ij} as a function of $\{\beta_{-i,j}\}_{j=0}^n$, and then Lemma A.10 shows that this relation holds.

LEMMA A.9: *For every agent i, λ_{ij} can be expressed as a function of $\beta_{-i,j}$ as follows:*

$$
\lambda_{i,j}=g_{i,j}\left[\frac{1-r\beta_{-i,0}}{\sigma^2+\mathcal{K}_i+1}+r\beta_{-i,j}-\frac{r}{\sigma^2+\mathcal{K}_i+1}\sum_{s=1}^{n}\beta_{-i,s}g_{i,s}\right]\ \textit{for } j=1,\dots,n. \tag{A.27}
$$

PROOF:

We start by computing the best action, as a function of signals observed. From Appendix A.3, Equation (A.20) and (A.21), we know that $a_i^*=F_i'\omega$, and that $a_i=\mathbb{E}\left[F_i'\omega|\mathbb{I}_i\right]=F_i'\mathrm{Cov}\left(\omega,X_i\Gamma\omega\right)'\mathrm{Var}\left(X_i\Gamma\omega\right)^{-1}X_i\Gamma\omega$, where F_i, X_i, Γ, and ω were all defined in Appendix A.3, Equations (A.17), (A.18), and (A.20). Let us start by computing the following term: $\mathrm{Cov}\left(\omega,X_i\Gamma\omega\right)'\mathrm{Var}\left(X_i\Gamma\omega\right)^{-1}X_i=\begin{bmatrix}B_1\\B_2\end{bmatrix}_{(n+1)\times n}$, where B_1 and B_2 are given by:

$$
\begin{aligned}
B_1 &= \mathbf{1}'(X_i\Phi\Phi'X_i')^{-1}X_i-\frac{1}{\phi_i}\mathbf{1}'(X_i\Phi\Phi'X_i')^{-1}\mathbf{1}\mathbf{1}'(X_i\Phi\Phi'X_i')^{-1}X_i\\
B_2 &= \Phi'X_i'(X_i\Phi\Phi'X_i')^{-1}X_i-\frac{1}{\phi_i}\Phi'X_i'(X_i\Phi\Phi'X_i')^{-1}\mathbf{1}\mathbf{1}'(X_i\Phi\Phi'X_i')^{-1}X_i.
\end{aligned}
$$

The matrix Φ is a diagonal matrix of σ's, and $\phi_i=1+\sum_{j=1}^n g_{ij}\sigma^{-2}$, both terms are defined in Appendix A.3, and in Equations (A.24) and (A.25). We can further simplify some expressions as follows:

$$
\begin{aligned}
X_i\Phi\Phi'X_i' &= \sigma^2\mathbf{I}_{\mathcal{K}_i+1}\\
\mathbf{1}'(X_i\Phi\Phi'X_i')^{-1}X_i &= \sigma^{-2}[g_{i,1},g_{i,2},...g_{i,n}]_{1\times n}\\
\mathbf{1}'(X_i\Phi\Phi'X_i')^{-1}\mathbf{1} &= \sigma^{-2}(\mathcal{K}_i+1)_{1\times 1}\\
\phi_i &= 1+(\mathcal{K}_i+1)\sigma^{-2},
\end{aligned}
$$

which gives us a simplified expressions for B_1 and B_2:

$$
B_1=\frac{1}{\sigma^2+\mathcal{K}_i+1}[g_{i,1},g_{i,2},...g_{i,n}]_{1\times n},\ \text{and}\ B_2=\sigma^{-1}\mathrm{diag}([g_{i,1},g_{i,2},...g_{i,n}])-\frac{\sigma^{-1}}{\sigma^2+\mathcal{K}_i+1}\begin{bmatrix}g_{i,1}\\g_{i,2}\\\cdots\\g_{i,n}\end{bmatrix}_{n\times 1}[g_{i,1},g_{i,2},...g_{i,n}].
$$

Next, we combine the simplified expressions for B_1 and B_2 along with the definition of F_i. For any signal e_j, $j\in\{1,2,...,n\}$, we can compute the linear coefficient λ_{ij} of that particular signal over the action of agent i: $[\lambda_{i1},\lambda_{i2},...,\lambda_{in}]=F_i'\begin{bmatrix}B_1\\B_2\end{bmatrix}$, where $F_i'=[1-r\beta_{-i,0},\ r\sigma\beta_{-i,1},\ r\sigma\beta_{-i,2},\ \cdots,\ r\sigma\beta_{-i,n}]$. Hence: $\lambda_{i,j}=g_{i,j}\left[\frac{1-r\beta_{-i,0}}{\sigma^2+\mathcal{K}_i+1}+r\beta_{-i,j}-\frac{r}{\sigma^2+\mathcal{K}_i+1}\sum_{s=1}^n\beta_{-i,s}g_{i,s}\right]$. □

LEMMA A.10: *Consider a subset D of an agent m's information set, i.e., $g_{mj}=1$ for every $j\in D$. The covariance between the influence a signal has over the average action not including agent m's action and how influential that particular signal is* *to agent m's action is non-negative.*

$$\frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)\left(\lambda_{m,j}-\frac{1}{d}\sum_{s\in D}\lambda_{m,s}\right)\geq 0$$

PROOF:

Using Lemma A.9, when agents are ex-ante identical we have that:

$$\lambda_{i,j}=g_{i,j}\left[\frac{1-r\beta_{-i,0}}{\sigma^2+\mathcal{K}_i+1}+r\beta_{-i,j}-\frac{r}{\sigma^2+\mathcal{K}_i+1}\sum_{s=1}^{n}\beta_{-i,s}g_{i,s}\right].$$

For $i=m$ and $j\in D$, we have that $g_{mj}=1$. Hence, $\lambda_{m,j}=\frac{1-r\beta_{-m,0}}{\sigma^2+\mathcal{K}_m+1}+r\beta_{-m,j}-\frac{r}{\sigma^2+\mathcal{K}_m+1}\sum_{s=1}^{n}\beta_{-m,s}g_{m,s}$ for every $j\in D$.

Now that we have written $\lambda_{m,j}$ for $j\in D$ as a function of $\{\beta_{-i,s}\}_{s=1}$, we conclude the proof of this lemma as follows:

$$\begin{aligned}\frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)\left(\lambda_{m,j}-\frac{1}{d}\sum_{s\in D}\lambda_{m,s}\right)&=\frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)\left(r\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}r\beta_{-m,s}\right)\\&=r\frac{1}{d}\sum_{j\in D}\left(\beta_{-m,j}-\frac{1}{d}\sum_{s\in D}\beta_{-m,s}\right)^2\geq 0.\end{aligned}$$

□

This gives us that $\Delta\Pi_m+\Delta\Pi_l>0$, even though by assumption both elements were smaller than zero, characterizing our contradiction.

2. Example of out-of-equilibrium violation of Proprerties 1, 2, and 3

In this section, we provide a numerical example highlighting that Properties 1, 2, and 3 do not hold in out of equilibrium networks. Let us assume an economy with 10 agents with identical preference parameters given by $r=0.5$ and $\sigma^2=1$. Also, we assume a linear cost function given by $C(\mathcal{K})=0.12\mathcal{K}$. Agents are connected as described in Figure A.2. In this informational structure, agents 1, 2, and 8 observe agent 3's signal in additional to their own respective signal and the common prior. Agents 3, 9, and 10 do not observe any additional signal. Finally, agents 4, 5, 6, and 7 observe agent 6's, 7's, 8's, 9's, and 10's signal in addition to their own respective signal and the common prior.

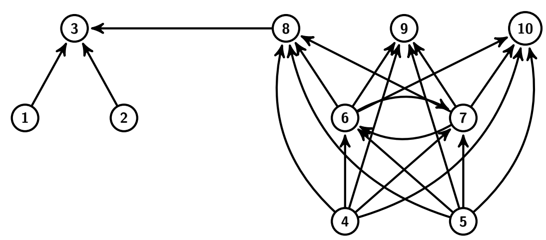


Figure A.2. : Example showing that Properties 1 and 2 do not hold in out of equilibrium networks.

Although this network is not an equilibrium, we can compare all possible deviations using the payoff deviation formula from Proposition 2, and numerically verify that agents 3 and 8 do not want to deviate, taking all other agents actions and connections as given. That is, all agents choose optimal actions given their connections and agents 3 and 8 do not want to form or break connections. This example violates Property 1 because agent 8 wants to keep observing agent 3 instead of, for instance, agent 10. However, agent 10's signal is observed by 4 other agents in addition to agent 10 herself, i.e. $\overline{\mathcal{K}}_{10}=4$, while 3's signal is observed by 3 other agents in addition to agent 3 herself, i.e. $\overline{\mathcal{K}}_3=3$. Hence, $\mathcal{K}_{10}>\mathcal{K}_3$. This is a clear violation of Property 1. Property 2 does not hold in this example because agent 3 wants to keep observing no additional signal, i.e., $\mathcal{K}_3=0$, while agent 8 wants to keep observing agent 3's signal, i.e. $\mathcal{K}_8=1$. However, agent 8's signal is observed by 4 other agents in additional to agent 8 herself, i.e. $\overline{\mathcal{K}}_8=4$, while 3's signal is observed by 3 other agents in additional to agent 3 herself, i.e. $\overline{\mathcal{K}}_3=3$. This violates Property 2 since $\overline{\mathcal{K}}_8>\overline{\mathcal{K}}_3$ and $\mathcal{K}_8>\mathcal{K}_3$. Finally, this example violates Property 3 as while player 6 observes the signals of players 8 and 9, player 8 does not find it optimal to observe player 9's signal.

3. *Proof of Theorem 1*

We present the proof below. We inductively construct the sets from Definition 2, A_1,...,A_n. To characterize the set A_1, the key step is to show that all agents not in A_1 must be observing any agent in it. We do so by using one more inductive argument. Consider a directed network G that satisfies Property 1 and Property 2. Let us define the following sets, B_1, B_2, ..., B_N. The first set, B_1, is such that $i \in B_1 \iff \overline{\mathcal{K}}_i \geq \overline{\mathcal{K}}_k\ \forall k$. To construct the second set, B_2, consider the auxiliary network G_2 obtained from G by omitting all agents from B_1 and their respective connections. We can compute $\overline{\mathcal{K}}$'s and $\mathcal{K}$'s using the network G_2, and the set B_2 is defined in a similar fashion as the set B_1, but using G_2 instead of G. Inductively, we can construct the sets $B_3, B_4, \ldots, B_N$. The proof consists in showing that the set B_s satisfies the definition for the set A_s. The argument proceeds inductively, and we first show that the set B_1 above is, indeed, the set A_1 of the definition of hierarchical directed network.

If for every agent i, $g_{i,j} = 0$ for every $j \notin \{i, 0\}$, then the proof is done. The empty network is a hierarchical network. Let us focus on the interesting case: The network is not empty. Let k_1 be an element of the non-empty set B_1.

(i) First, we need to show that if $k_2 \notin B_1$, then it must be the case that $g_{k_2,k_1} = 1$. That is, if k_2 is not among the set of most-connected agents, it must be observing k_1. The structure of the argument will be based on contradiction. By contradiction, assume that there exists $k_2 \notin B_1$, such that $g_{k_2,k_1} = 0$. There are two cases to be considered, depending on whether or not agent k_2 observes any signal, j.

If agent k_2 observes some agent j, $g_{k_2,j} = 1$ for an agent j, the contradiction is immediate. Agent k_2 violates Property 1, as $\overline{\mathcal{K}}_{k_1} \geq \overline{\mathcal{K}}_j$ should imply $g_{k_2,j} = 1 \implies g_{k_2,k_1} = 1$. Agent k_2 should be observing agent k_1. Observe that this holds true even if $j \in B_1$.

If agent k_2 does not observe any other agent j, $g_{k_2,j} = 0$ for any j, then we proceed with the following steps to reach a contradiction:

1. We know that, in particular, $g_{k_2,s} = 0$ for any s such that $\overline{\mathcal{K}}_s \geq \overline{\mathcal{K}}_{k_1}$.

2. Let B_N be the group of least-observed agents. It must be that $k_2 \notin B_N$; otherwise, by Property 2, all other agents would also not be observing any additional signal, and we would have an empty network.

3. Given that $k_2 \notin B_1$, we know that $\overline{\mathcal{K}}_{k_2} < \overline{\mathcal{K}}_{k_1}$, and, thus, there is an agent k_3 such that $g_{k_3,k_2} = 0$ and $g_{k_3,k_1} = 1$. By Property 1, it also holds that $g_{k_3,s} = 0$ for all s such that $\overline{\mathcal{K}}_s \leq \overline{\mathcal{K}}_{k_2}$.

4. Given that $\mathcal{K}_{k_3} > \mathcal{K}_{k_2}$, by Property 2 it must be that $\overline{\mathcal{K}}_{k_3} < \overline{\mathcal{K}}_{k_2}$.

5. By the same argument as in (3) above, it must be that there is an agent k_4 such that $g_{k_4,k_3} = 0$ and $g_{k_4,k_2} = 1$.

6. Note that $g_{k_3,k_4} = 0$. If $g_{k_3,k_4} = 1$, then we would have $\overline{\mathcal{K}}_{k_4} > \overline{\mathcal{K}}_{k_2} > \overline{\mathcal{K}}_{k_3}$. By Property 2, this would imply $\mathcal{K}_{k_4} < \mathcal{K}_{k_3}$. From Property 1 we would have that $g_{k_4,s} = 1$ implies $g_{k_3,s} = 1$ for any $s \neq k_4, k_3$, which contradicts $g_{k_3,k_2} = 0$ but $g_{k_4,k_2} = 1$.

7. Given that $g_{k_3,k_2} = 0$ (step 3) and $g_{k_4,k_2} = 1$ (step 5), we have from Property 1 that $g_{k_3,s} = 1$ implies $g_{k_4,s} = 1$ for any $s \neq k_4, k_3$. Combined with the fact that $g_{k_3,k_2} = 0$ (step 3), $g_{k_4,k_3} = 0$ (step 5), $g_{k_4,k_2} = 1$ (step 5) and $g_{k_3,k_4} = 0$ (step 6), we have that $\mathcal{K}_{k_4} > \mathcal{K}_{k_3}$. By Property 2, we have that $\mathcal{K}_{k_4} > \mathcal{K}_{k_3}$ implies $\overline{\mathcal{K}}_{k_4} < \overline{\mathcal{K}}_{k_3}$.

8. As the number of agents is finite, this induction must end. There exists a final agent $k_{\bar{N}}$ such that $g_{k_{\bar{N}},k_{\bar{N}-1}} = 0$ and $g_{k_{\bar{N}},k_{\bar{N}-2}} = 1$. Since this is the last step of the induction, there is no agent, s, outside of the sequence, $s \notin \{k_1, ..., k_{N-2}\}$, such that $g_{s,k_{\bar{N}-1}} = 1$ and $g_{s,k_{\bar{N}}} = 0$. Also, from the penultimate step of the induction, we have $g_{k_{\bar{N}-1},k_{\bar{N}-2}} = 0$ and $g_{k_{\bar{N}-1},k_{\bar{N}-3}} = 1$.

9. Given that $g_{k_{\bar{N}-1},k_{\bar{N}-2}} = 0$ and $g_{k_{\bar{N}},k_{\bar{N}-2}} = 1$, we have from Property 1 that $g_{k_{\bar{N}-1},s} = 1$ implies $g_{k_{\bar{N}},s} = 1$ for any $s \neq k_{\bar{N}}, k_{\bar{N}-1}$. Also, remember that $g_{k_{\bar{N}},k_{\bar{N}-1}} = g_{k_{\bar{N}-1},k_{\bar{N}-2}} = g_{k_{\bar{N}-1},k_{\bar{N}}} = 0$ and $g_{k_{\bar{N}},k_{\bar{N}-2}} = 1$. Hence, $\mathcal{K}_{k_{\bar{N}-1}} < \mathcal{K}_{k_{\bar{N}}}$. By Property 2, $\mathcal{K}_{k_{\bar{N}-1}} < \mathcal{K}_{k_{\bar{N}}}$ implies that $\overline{\mathcal{K}}_{k_{\bar{N}-1}} > \overline{\mathcal{K}}_{k_{\bar{N}}}$. If $\overline{\mathcal{K}}_{k_{\bar{N}-1}} > \overline{\mathcal{K}}_{k_{\bar{N}}}$, then there is an agent $k_{\bar{N}+1}$ such that $g_{k_{\bar{N}+1},k_{\bar{N}}} = 0$ and $g_{k_{\bar{N}+1},k_{\bar{N}-1}} = 1$. This contradicts agent $k_{\bar{N}}$ being the final agent in the sequence.

(ii) Second, we need to show that if an agent in B_1 is observed by any other member of B_1, then she must observe and be observed by all agents in B_1.

1. If $k_1 \in B_1$ is observed by $m \in \{1, 2, ..., n\}$ other agents in B_1, then all agents in B_1 are observed by m other members of B_1. As all agents in B_1 are observed by the same number of agents from outside B_1, and by the same number of agents in total, they must also be observed by the same number of agents in B_1.

2. If an agent $k_1 \in B_1$ observes another agent in B_1, then, by Property 1, she observes all other agents in B_1. Thus, if all agents observe someone, from Property 1, we already have that all agents observe everyone else in B_1.

3. The last step is to show that if only a subset of agents from B_1 observe other agents in B_1, then we reach a contradiction. We proceed by considering a partition of the set B_1: the subset of agents who observe at least one other agent in B_1, $S_l \subset B_1$, and its complement $S_b = B_1 \backslash S_l$. Let $C(S_l)$ be the cardinality of the set S_l. By Property 1, every member of S_l observes everyone else in B_1.

   The contradiction to be reached regards the number of agents observing the signal from an agent in S_l and the number of agents observing the signal from an agent in S_b. Since they are all in B_1, this number should be the same. However, we show that the number of agents observing a signal of an agent in S_l is strictly smaller than the number of agents observing a signal of an agent in S_b. An agent in S_b is observed by other agents in B_1—in particular, by all agents in S_l; thus, her signal is observed by $C(S_l) + 1$ agents in B_1—the plus one refers to herself. However, an agent in S_l's signal is observed by only S_l agents in B_1. Contradiction.

(*iii*) To complete the construction of B_1, we need to show that if an agent j in B_1 observes a signal from an agent s not in B_1, then: (a) j observes the signals of all agents in B_1; (b) all other agents in B_1 observe agent s's signal; (c) agent s is in B_2; and (d) j observes no signal from an agent not in B_1, except for s.

The proofs of items (a) and (c) are direct applications of Property 1. The proof of item (b) proceeds by contradiction. Consider any two agents $i, j \in B_1$, and suppose, by contradiction, that agent j observes the signal of agent s, but agent i does not, $g_{i,s} = 0$ and $g_{j,s} = 1$. By definition of the set B_1, agents $i, j \in B_1$ have the same number of agents observing their signal, $\overline{\mathcal{K}}_i = \overline{\mathcal{K}}_j$. By part (ii) above, $i, j \in B_1$ observe the same number of players in B_1. Thus, by Property 2, they must observe the same number of signals, $\mathcal{K}_i = \mathcal{K}_j$. Thus, there exists an agent k, $k \notin B_1$, such that $g_{i,k} = 1$ and $g_{j,k} = 0$. If $\overline{\mathcal{K}}_k \geq \overline{\mathcal{K}}_s$, then agent j violates Property 1; and, if $\overline{\mathcal{K}}_k \leq \overline{\mathcal{K}}_s$, then agent i violates Property 1. Finally, the proof of item (d) also proceeds by contradiction. Suppose not—that is, suppose that $g_{i,j} = 1$ and $g_{i,l} = 1$ with $i \in B_1$ and $j, l \in B_2$. It cannot be that all agents in B_2 observe each others' signal; otherwise, $\overline{\mathcal{K}}_j = \overline{\mathcal{K}}_l = \overline{\mathcal{K}}_i$, but agent i is a member of B_1, and, thus, j and l would not be members of the second tier, but of the first. Thus, members of B_2 do not observe signals from other members of B_2. The final step is to count the number of signals each agent observes. Agent $j \in B_2$ observes only all members of B_1; thus, $\mathcal{K}_j = C(B_1)$. However, agent $i \in B_1$ observes $\mathcal{K}_i = C(B_1) - 1 + 2$, where the minus one refers to herself, and plus two refers to agents j and l. This contradicts Property 2.

Items (*i*), (*ii*), and (*iii*) of the proof show that the set B_1 satisfies the conditions (*i*), (*ii*), and (*iii*) of the definition of a hierarchical directed network, respectively. Thus, the set B_1 constructed satisfies the definition of the set A_1.

The next step is to show that B_2 satisfies the definition of the set A_2. Follow the same steps (*i*), (*ii*), and (*iii*) detailed above using the network G_2. By induction, this guarantees that the sets $B_2, B_3, \dots, B_N$ are the sets $A_2, A_3, \dots, A_N$ from the definition of hierarchical directed networks.

4. *Proof of Proposition 4*

Given Assumption 1, Properties 1 and 2 hold in equilibrium (Proposition 3), and any strict Nash equilibrium is a hierarchical directed network (Theorem 1 and Corollary 1). Hence, we have to show that Property 3 holds in equilibrium given Assumptions 1 and 2.

We will prove Proposition 4 by contradiction. Let us assume that there are three agents, a, b, and c, such that c is connected to both a and b, i.e., $g_{ca} = 1$ and $g_{cb} = 1$, but a and b are not connected to each other, i.e., $g_{ab} = 0$. We know that the equilibrium features a hierarchical network as information structure. Hence, the connections between these three agents characterize different hierarchical networks, and there are only three possibilities:[7] (i) networks in which members of the top tier do not observe each other (a and b in the top tier); and (ii) networks in which members of the bottom tier observe each other (b and c in the bottom tier); or (iii) networks with more than two tiers (a, b, and c each in a different tier).

Due to the hierarchical structure, agent c observes at least as many signals as agent a in addition to the signal from b and her own. Thus, if agent c stopped observing agent b's signal, her information set would still be a strict super set of agent a's.

We will compare two deviations, for agent c to break the link with all players she is observing and player a is not, call such set D, and for player a to form links with such set. Note that the set D is not empty, and has $d \geq 1$ elements. Note also that, even after player a forms links with all players in D, player a's information set will still be a strict subset of player c's. The reason is that c is observing a, but the converse is not true. This follows from the definition of hierarchical network. If a and b are in same tier (case i above), than it must be that $g_{ac} = 0$ because $g_{ab} = 0$. If a and b are not in the same tier, there are two possibilities. One possibility is that b and c are in the same tier (case ii above), then $g_{cb} = 1$ implies $g_{bc} = 1$ (full tier). Also, $g_{ca} = 1$ implies $g_{ba} = 1$ and thus a is in a tier above b and c's tier; and, since $g_{ab} = 0$ and the tier of agents b and c is full, then $g_{ac} = 0$ because otherwise c would be in a's tier and a would have to observe b's signal as well. Finally, the other possibility is that a, b, and c are in different tiers (case iii above), then we know that b is in a tier above c's, and thus $g_{ab} = 0$ implies $g_{ac} = 0$.

Notice that $\mathcal{K}_a + d = \mathcal{K}_c - 1 < \mathcal{K}_c$ because c is connected to a but a is not connected to c, which makes $d = \mathcal{K}_c - \mathcal{K}_a - 1$. Regarding notation, we keep $g_{i,j}$ to be the original linking strategy of the proposed equilibrium, in which c observes the signals from D and agent a does not observe them. Thus, we have to add or subtract elements to the expression accordingly. Formally,

[7] Notice that these are the only three possibilities: (i) $g_{ba} = 0$; (ii) $g_{ba} = 1$ and $g_{bc} = 1$; or (iii) $g_{ba} = 1$ and $g_{bc} = 0$.

in both payoff expressions used in $\Delta\Pi_c$—one where player c observes the signals in D, and one where she does not—we consider the same $g_{c,j}$ with $g_{c,j} = 1\ \forall j \in D$. Analogously, we consider for both elements of $\Delta\Pi_a$ that $g_{a,j} = 0\ \forall j \in D$. First we use Proposition 2 to compute the payoff difference for player c, comparing the payoff of deviating with the payoff of maintaining the links.

$$\Delta\Pi_c = -\underbrace{\frac{\sigma^2\left(1 - r\sum_{j\notin D} g_{cj}\beta_{-c,j}\right)^2}{\sigma^2 + \mathcal{K}_c - d + 1}}_{\text{Without observing}} + \underbrace{\frac{\sigma^2\left(1 - r\sum_{j=0}^{n} g_{cj}\beta_{-c,j}\right)^2}{\sigma^2 + \mathcal{K}_c + 1}}_{\text{Observing D}} - \underbrace{r^2\sigma^2\left(\sum_{j=0}^{n}(1-g_{cj})\beta^2_{-c,j} + \sum_{j\in D}\beta^2_{-c,j}\right)}_{\text{Without observing}}$$
$$+ \underbrace{r^2\sigma^2\sum_{j=0}^{n}(1-g_{cj})\beta^2_{-c,j}}_{\text{Observing D}} + \underbrace{C(\mathcal{K}_c) - C(\mathcal{K}_c - d)}_{\text{Cost difference of not observing signals}}$$

where the first element sums over $j \notin D$, as player c is not observing the signals in D, while $g_{c,j} = 1\ \forall j \in D$. For the same reason, the third term includes the additional sum over $j \in D$, $\sum_{j\in D}\beta^2_{-c,j}$. Simplifying the expression above,

$$\Delta\Pi_c = -\left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right)^2\left(\frac{\sigma^2}{\sigma^2 + \mathcal{K}_c - d + 1} - \frac{\sigma^2}{\sigma^2 + \mathcal{K}_c + 1}\right) - 2\frac{r\sigma^2}{\sigma^2 + \mathcal{K}_c + 1}\left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right)\sum_{j\in D}\beta_{-c,j}$$
$$- r^2\sigma^2\left[\sum_{j\in D}\beta^2_{-c,j} - \frac{1}{\sigma^2 + \mathcal{K}_c + 1}\left(\sum_{j\in D}\beta_{-c,j}\right)^2\right] + C(\mathcal{K}_c) - C(\mathcal{K}_c - d)$$

For this not be a profitable deviation for agent c, it must be that $\Pi_c < 0$, which implies that:

$$C(\mathcal{K}_c) - C(\mathcal{K}_c - d) < \left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right)^2\left(\frac{\sigma^2}{\sigma^2 + \mathcal{K}_c - d + 1} - \frac{\sigma^2}{\sigma^2 + \mathcal{K}_c + 1}\right)$$
$$+ 2\frac{r\sigma^2}{\sigma^2 + \mathcal{K}_c + 1}\left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right)\sum_{j\in D}\beta_{-c,j}$$
$$+ r^2\sigma^2\left[\sum_{j\in D}\beta^2_{-c,j} - \frac{1}{\sigma^2 + \mathcal{K}_c + 1}\left(\sum_{j\in D}\beta_{-c,j}\right)^2\right]$$

Following similar steps for agent a, we compute a's payoff gain, i.e., $\Delta\Pi_a$, from deviating and observing the signals from D. We have that:

$$\Delta\Pi_a = -\underbrace{\frac{\sigma^2\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j} - r\sum_{j\in D}\beta_{-a,j}\right)^2}{\sigma^2 + \mathcal{K}_a + d + 1}}_{\text{Observing D}} + \underbrace{\frac{\sigma^2\left(1 - r\sum_{j=0}^{n} g_{a,j}\beta_{-a,j}\right)^2}{\sigma^2 + \mathcal{K}_a + 1}}_{\text{Without observing}}$$
$$-\underbrace{r^2\sigma^2\left(\sum_{j=0}^{n}(1-g_{aj})\beta^2_{-a,j} - \sum_{j\in D}\beta^2_{-a,j}\right)}_{\text{Observing D}} + \underbrace{r^2\sigma^2\left(\sum_{j=0}^{n}(1-g_{aj})\beta^2_{-a,j}\right)}_{\text{Without observing}} - \underbrace{[C(\mathcal{K}_a + d) - C(\mathcal{K}_a)]}_{\text{Cost difference}}$$

where the first element includes an additional term, a sum over $j \in D$, as player a is observing the signals in D, while $g_{a,j} = 0\ \forall j \in D$. For the same reason, the third element subtracts the additional term, $\sum_{j\in D} \beta^2_{-a,j}$.

$$\begin{aligned}\Delta\Pi_a = &-\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_a+d+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_a+1}\right)\\ &+ 2\frac{r\sigma^2}{\sigma^2+\mathcal{K}_a+d+1}\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right)\sum_{j\in D}\beta_{-a,j}\\ &+ r^2\sigma^2\left[\sum_{j\in D}\beta^2_{-a,j} - \frac{1}{\sigma^2+\mathcal{K}_a+d+1}\left(\sum_{j\in D}\beta_{-a,j}\right)^2\right] - \left[C(\mathcal{K}_a+d) - C(\mathcal{K}_a)\right]\end{aligned}$$

For this not to be a profitable deviation for agent a, it must be that $\Pi_a < 0$, which implies that:

$$\begin{aligned}C(\mathcal{K}_a+d) - C(\mathcal{K}_a) > &\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_a+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_a+d+1}\right)\\ &+ 2\frac{r\sigma^2}{\sigma^2+\mathcal{K}_a+d+1}\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right)\sum_{j\in D}\beta_{-a,j}\\ &+ r^2\sigma^2\left[\sum_{j\in D}\beta^2_{-a,j} - \frac{1}{\sigma^2+\mathcal{K}_a+d+1}\left(\sum_{j\in D}\beta_{-a,j}\right)^2\right]\end{aligned}$$

Given that $\mathcal{K}_c = K_a + d + 1 > K_a + d$, by convexity of the cost curve (Assumption 2), we have that $C(\mathcal{K}_c) - C(\mathcal{K}_c - d) \geq C(\mathcal{K}_a + d) - C(\mathcal{K}_a)$, which combined with $\Pi_c < 0$ and $\Pi_a < 0$ gives us:

$$\begin{aligned}&\left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_c-d+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_c+1}\right) + 2\frac{r\sigma^2}{\sigma^2+\mathcal{K}_c+1}\left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right)\sum_{j\in D}\beta_{-c,j}\\ &+ r^2\sigma^2\left[\sum_{j\in D}\beta^2_{-c,j} - \frac{1}{\sigma^2+\mathcal{K}_c+1}\left(\sum_{j\in D}\beta_{-c,j}\right)^2\right]\\ > &\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_a+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_a+d+1}\right) + 2\frac{r\sigma^2}{\sigma^2+\mathcal{K}_a+d+1}\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right)\sum_{j\in D}\beta_{-a,j}\\ &+ r^2\sigma^2\left[\sum_{j\in D}\beta^2_{-a,j} - \frac{1}{\sigma^2+\mathcal{K}_a+d+1}\left(\sum_{j\in D}\beta_{-a,j}\right)^2\right]\end{aligned}$$

and can be simplified to:

$$\begin{aligned}&\left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_c-d+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_c+1}\right) + r^2\sigma^2\sum_{j\in D}\beta^2_{-c,j}\\ &+ \frac{r\sigma^2}{\sigma^2+\mathcal{K}_c+1}\left(\sum_{j\in D}\beta_{-c,j}\right)\left[2\left(1 - r\sum_{j\notin D} g_{c,j}\beta_{-c,j}\right) - r\sum_{j\in D}\beta_{-c,j}\right]\\ > &\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2+\mathcal{K}_a+1} - \frac{\sigma^2}{\sigma^2+\mathcal{K}_a+d+1}\right) + r^2\sigma^2\sum_{j\in D}\beta^2_{-a,j}\\ &+ \frac{r\sigma^2}{\sigma^2+\mathcal{K}_a+d+1}\left(\sum_{j\in D}\beta_{-a,j}\right)\left[2\left(1 - r\sum_{j\notin D} g_{a,j}\beta_{-a,j}\right) - r\sum_{j\in D}\beta_{-a,j}\right].\end{aligned}$$

We will show that the above inequality never holds, characterizing a contradiction. Observe that the following inequalities hold: (i) $\mathcal{K}_c - d > \mathcal{K}_a$ because $d = \mathcal{K}_c - \mathcal{K}_a - 1$; (ii) $\beta_{-a,j} > \beta_{-c,j}$ for every $j \in D$ from Lemma A.7; and (iii) $\sum_{j \notin D} g_{c,j}\beta_{-c,j} \geq \sum_{j \notin D} g_{a,j}\beta_{-a,j}$ from Lemma A.8.[8] These three inequalities combined imply:

$$\left(1 - r\sum_{j \notin D} g_{c,j}\beta_{-c,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2 + \mathcal{K}_c - d + 1} - \frac{\sigma^2}{\sigma^2 + \mathcal{K}_c + 1}\right) < \left(1 - r\sum_{j \notin D} g_{a,j}\beta_{-a,j}\right)^2 \left(\frac{\sigma^2}{\sigma^2 + \mathcal{K}_a + 1} - \frac{\sigma^2}{\sigma^2 + \mathcal{K}_a + d + 1}\right)$$

and $r^2\sigma^2 \sum_{j \in D} \beta_{-c,j}^2 < r^2\sigma^2 \sum_{j \in D} \beta_{-a,j}^2$. Finally, we can work with the last term so that:

$$\begin{aligned} &\frac{r\sigma^2}{\sigma^2 + \mathcal{K}_c + 1}\left(\sum_{j \in D} \beta_{-c,j}\right)\left[2\left(1 - r\sum_{j \notin D} g_{c,j}\beta_{-c,j}\right) - r\sum_{j \in D} \beta_{-c,j}\right] \\ &\qquad < \frac{r\sigma^2}{\sigma^2 + \mathcal{K}_a + d + 1}\left(\sum_{j \in D} \beta_{-a,j}\right)\left[2\left(1 - r\sum_{j \notin D} g_{a,j}\beta_{-a,j}\right) - r\sum_{j \in D} \beta_{-a,j}\right] \end{aligned}$$

because (i) $\mathcal{K}_c > \mathcal{K}_a + d$, which makes $\frac{r\sigma^2}{\sigma^2 + \mathcal{K}_a + d + 1} > \frac{r\sigma^2}{\sigma^2 + \mathcal{K}_c + 1}$; (ii) from Lemma A.7 we have that $\sum_{j \in D} \beta_{-a,j} > \sum_{j \in D} \beta_{-c,j}$; and (iii) using Lemma A.8 along with the definitions of $\beta_{-a,j}$ and $\beta_{-c,j}$ (Equation A.13) and the fact that $\lambda_{aj} = 0$ for every $j \in D$, we have that

$$\begin{aligned} &2\left(1 - r\sum_{j \notin D} g_{a,j}\beta_{-a,j}\right) - r\sum_{j \in D} \beta_{-a,j} - \left[2\left(1 - r\sum_{j \notin D} g_{c,j}\beta_{-c,j}\right) - r\sum_{j \in D} \beta_{-c,j}\right] \\ &= 2r\left(\sum_{j \notin D} g_{c,j}\beta_{-c,j} - \sum_{j \notin D} g_{a,j}\beta_{-a,j}\right) - r\sum_{j \in D} \beta_{-a,j} + r\sum_{j \in D} \beta_{-c,j} \\ &\geq 2r\left(\frac{1}{n-1}\sum_{j \in D} \lambda_{c,j}\right) - r\sum_{j \in D} \beta_{-a,j} + r\sum_{j \in D} \beta_{-c,j} = r\left(2\frac{1}{n-1}\sum_{j \in D} \lambda_{c,j} - \frac{1}{n-1}\sum_{j \in D} \lambda_{c,j}\right) = r\frac{1}{n-1}\sum_{j \in D} \lambda_{c,j} > 0. \end{aligned}$$

This characterizes a contradiction.

5. *Proof of Theorem 2*

We know, by Theorem 1, that any directed network satisfying Properties 1 and 2 is a hierarchical directed network. Next, we show in three steps that Property 3 restricts the set of possible networks to core-periphery. First, we show that a hierarchical network satisfying Property 3 has, at most, two tiers. Second, we show that all agents in the top tier must be connected to each other. Third, we show that agents in the bottom tier are not connected to each other.

(*i*) Suppose that there are more than two tiers. Thus, there exists an agent in the bottom tier connected to all agents in the top tier and all agents in a medium tier. There also exists an agent in a medium tier not being observed by an agent in the top tier—otherwise, there would not exist a medium tier. This violates property 3.

(*ii*) Suppose that there is more than one agent in the top tier and at least one agent in the bottom tier. From the definition of hierarchical networks, all agents in the top tier are either connected to each other or to no one at all. If the top-tier agents are not connected to each other, then this would violate Property 3 because an agent in the bottom tier is connected to those in the top tier, but they are not connected to each other.

(*iii*) Suppose that there is at least one agent in the top tier and more than one in the bottom tier. From the definition of hierarchical networks, all agents in the bottom tier are either connected to each other or to no one at all. If the bottom-tier agents are connected to each other, then this would violate Property 3 because an agent in the bottom tier is connected to those in the top tier and those in the bottom tier, but they are not connected to each other.

6. *Proof of Proposition 5*

For item (a), as r approaches zero, according to Proposition 2, the expected payoff of agent i only depends on $\mathcal{K}_i$. In fact, agent i chooses $\mathcal{K}_i$ to maximize $-\frac{\sigma^2}{\sigma^2 + \mathcal{K}_i + 1} - c(\mathcal{K}_i)$. Given Assumptions 1 and 2, all agents choose exactly the same number

[8] The condition in Lemma A.8 holds with equality by the definition of the set D.

of links, which implies a core-periphery network with members of the core also observing one signal from a peripheral agent (core-periphery observing down network). Hence, the equilibrium is unique.

For item (b), as r approaches one, all betas of non-public signal converge to zero (Lemma 1). When costs of forming an additional link is strictly positive, we have that when β_j is sufficiently close to zero no agent is willing form a link with agent j. Thus, for r sufficiently close to one, agents only connect to public signals, which delivers a simple core-periphery network structure.

From Equation (A.7), under the simple-core-periphery network, we have that $\lambda_{ij} \to 0$ for every $j \geq 1, j \notin$ PU:

$$\lim_{r\to 1^-} \lambda_{ij} = \lim_{r\to 1^-} g_{ij}\beta_j + \tilde{g}_{ij}\left(1 - \lim_{r\to 1^-} \beta_0 - \sum_{s\in \text{ PU}} \lim_{r\to 1^-} \beta_s\right) = \lim_{r\to 1^-} g_{ij}\beta_j = 0, \quad \forall j \notin \text{ PU}.$$

As a result, from Equation (A.13), we have that $\lim_{r\to 1^-} \beta_{-i,j} = 0$, $\forall j \geq 1, j \notin$ PU. Thus agents do not want to observe non-public signals as r goes to one. Formally, in any simple core periphery network, agents never want to form additional links because it leads to a close-to-zero benefit in expected payoff as r approaches one, but they still have to pay the link formation costs.

A first implication of the result above is that the empty network is always an equilibrium as players do not want to form links. Next, we look at players incentives to break links. For $j \in$ PU, under the simple core periphery network, we have from Equation (A.7):

$$\lim_{r\to 1^-} \lambda_{ij} = \lim_{r\to 1^-} \beta_j + \tilde{g}_{ij}\left(1 - \lim_{r\to 1^-} \beta_0 - \sum_{s\in \text{ PU}} \lim_{r\to 1^-} \beta_s\right) = \lim_{r\to 1^-} \beta_j = \begin{cases} \frac{\sigma^2}{\sigma^2+n_c} & \text{if } j = 0 \\ \frac{1}{\sigma^2+n_c} & \text{if } j \geq 1, \ j \in \text{ PU} \end{cases}$$

Finally, based on Equation (A.13), we have that $\lim_{r\to 1^-} \beta_{-i,j} = \lim_{r\to 1^-} \beta_j$.

When deciding whether to break a link or not in a simple core-periphery network with n_c core players, according to Proposition 2, an agent compares the benefit and costs. We know that i will not want to observe non-public signals. Thus, agent i will observe up to n_c public signals and her optimization problem is simplified to choose the number of core players signals to observe.

Core players observe one core signal for free (their own signal), thus their optimization problem is given by:

$$\text{(A.28)} \qquad \max_{\mathcal{K}_i\in\{0,\ldots,n_c-1\}} -\frac{\sigma^2}{\sigma^2+\mathcal{K}_i+1}\left(1-\frac{\sigma^2}{\sigma^2+n_c}-\frac{\mathcal{K}_i+1}{\sigma^2+n_c}\right)^2 - \sigma^2\left(n_c-\mathcal{K}_i-1\right)\left(\frac{1}{\sigma^2+n_c}\right)^2 - C\left(\mathcal{K}_i\right).$$

while, for peripheral agents, the optimization is given by:

$$\text{(A.29)} \qquad \max_{\mathcal{K}_i\in\{0,\ldots,n_c\}} -\frac{\sigma^2}{\sigma^2+\mathcal{K}_i+1}\left(1-\frac{\sigma^2}{\sigma^2+n_c}-\frac{\mathcal{K}_i}{\sigma^2+n_c}\right)^2 - \sigma^2\left(n_c-\mathcal{K}_i\right)\left(\frac{1}{\sigma^2+n_c}\right)^2 - C\left(\mathcal{K}_i\right).$$

Both objective functions are strictly concave and thus feature a unique solution. Also, we focus on the link-breaking incentives of peripheral agents, as the marginal benefit of peripheral agents is higher than of core players. If the optimal solution implies $\mathcal{K}_i \neq n_c$, then a simple core periphery network with n_c core players would not be equilibrium.

The marginal benefit of breaking a link increases with the number of core players. Hence, if a player does not want to break a link under a simple core periphery network with n_c core players, then she will not break the link in a simple core periphery network with $n < n_c$ core players. Thus, if a simple core periphery network with n_c core player holds in equilibrium, then any simple core periphery network with fewer than n_c core player also holds in equilibrium. Formally, there is a upper bound $n_c^* \leq n$, such that any simple core periphery network with $n_c \in \{0, \ldots, n_c^*\}$ core players is an equilibrium, and these $n_c^* + 1$ network structure fully characterize the set of possible equilibria. The value of n_c^* is the highest number of core players such that peripheral agent are not willing to break a link. Formally, a peripheral agent observing n_c core players prefers not to break a link if, and only if, the payoff from observing all n_c core signals is greater than the payoff from observing $n_c - 1$ core players (and saving on the link formation cost).

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