Online Appendix: The Cost-effectiveness Implications of Carbon Price Certainty

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January 2020

1 Details about Model Calibration

1.1 Details about Price Calibration

We assume prices follow Geometric Brownian Motion and estimate the corresponding drift and volatility parameters by maximum likelihood estimation with data on historical EU ETS prices.

Prices following GBM will evolve according to the following (stochastic) law of motion:

$$P_t = P_0 \exp\left((\alpha - \frac{\sigma^2}{2})t + \sigma\sqrt{t}W_t\right)$$

. where $W_t \sim N(0,1)$.

Note that this set-up can also be written as:

$$\ln\left(\frac{P_t}{P_0}\right) \sim N((\alpha - \frac{\sigma^2}{2}), \sigma^2 t)$$

We estimate the drift and volatility coefficients using maximum likelihood estimation. Recall that the maximum likelihood estimator for the mean of a normal random variable is $\hat{a} = \frac{1}{n}\sum_{j=1}^{n} x_j$ and the maximum likelihood estimator for the variance of a normal random variable is $\hat{s}^2 = \frac{1}{n}\sum_{j=1}^{n}(x_j - \bar{x})^2$.

Therefore, we have:

$$\hat{a} = (\hat{\alpha} - \frac{\hat{\sigma}^2}{2})t = \frac{1}{n}\sum_{i=1}^{T-1} \ln(\frac{P_{i+1}}{P_i})$$

$$\hat{s}^2 = \hat{\sigma}^2 t = \frac{1}{n}\sum_{j=1}^{n}(\ln(\frac{P_{i+1}}{P_i}) - \hat{a})^2$$

In this case, we have weekly price data, but it is more reasonable to assume that the relevant decision period is quarterly or annually. Therefore, we set $t = \frac{1}{52}$ (to reflect 52 weeks/year) when estimating $\hat{\alpha}$ and $\hat{\sigma}$. Using this procedure with EU ETS prices from 2008 through 2018 yields $\hat{\alpha} = 0.0508$ and $\hat{\sigma} = 0.3925$.

From the set-up above, we have:

$$
\begin{aligned}
\mathrm{E}[P_{t+1}] &= \mathrm{E}[P_0 \exp((\alpha - \frac{\sigma^2}{2}) \cdot 1 + \sigma\sqrt{1}W_t)] \\
&= P_t \exp(\alpha - \frac{\sigma^2}{2})\mathrm{E}[\exp(\sigma W_t)] = P_t \exp(\alpha - \frac{\sigma^2}{2}) \exp(\frac{\sigma^2}{2}) \\
&= P_t \exp(\alpha)
\end{aligned}
$$

For estimated drift parameters around 0.0508, this yields expected price increases of $\exp(0.0508) = 1.0522$, or 5.22%. Data on historical EU ETS allowance prices is taken from Sandbag - Smarter Climate Policy (2018); we convert to real allowance prices using inflation data data from European Central Bank: Statistical Data Warehouse (2020).

1.2 Details about Abatement Function Calibration

We obtained data for the abatement function calibration from Barron et al. (2018b). From reviewing the results of this modeling exercise, it seems reasonable to assume that the estimated emissions reductions in each period relative to the baseline scenario depends on a) expectations of future allowance prices; b) the existing stock of abatement, insofar as the "low hanging fruit" is addressed first; and c) technology improvements over time. With the exception of expected allowance prices, we do not observe these components of the underlying model. Furthermore, the observed emissions reductions likely include both variable abatement (e.g., behavioral responses to reduce energy consumption, carbon capture, etc.) and fixed abatement investment (e.g., retrofitting plant to reduce energy consumption, installing carbon capture equipment, etc.), yet we do not observe the relative contribution of these two types of emissions reductions. As a consequence, we make certain assumptions about the abatement cost function, which we describe below.

As a first step, note that the Stanford EMF data includes estimated emissions at different years over the period 2010 to 2020 for the 10 different models. We extract emissions data for 2015, 2020, 2025, and 2030 specifically, since these years are included for (almost) all models; then we average over all 10 models for each price scenario, to obtain the values underlying the red lines from Figure 1 in Barron et al. (2018a). (See McFarland et al. (2018) for a technical discussion of the models underlying this data.) We calculate net emissions reductions over these periods for each price scenario, averaged across all models and adjusted for any changes in baseline emissions.

We adopt the simplifying assumption that all abatement is long-lived and thus abatement in the current period persists into the next period, adjusting for depreciation; this assumption matches our theoretical modeling of abatement as durable capital stock. Likewise, we also assume that abatement investment in a given year becomes available for compliance in the following year. Based on these assumptions, we calculate the discounted value of the tax payment avoided through abatement investment in each period. Following the Stanford EMF price scenarios, we assume that the emissions price increases at either 1% or 5% annually from 2020 to 2050, after which the price levels off (indefinitely); we also assume that firms correctly anticipate this price trajectory beginning in 2015. We set depreciation $\delta = 0.10$ and the firm's discount factor $\beta = 0.9505$.[1]

As illustration, a firm reducing emissions by A_{2020} in the year 2020 avoids the following tax payment:

$$\begin{aligned}\text{Avoided Tax} = \beta \cdot A_{2020} \cdot P_{2021} \cdot & \frac{1 - [(1+g)(1-\delta)\beta]^{2050-2021+1}}{1-(1+g)(1-\delta)\beta} \\ & + \beta^{2051-2020} \cdot (1-\delta)^{2051-2021-1} \cdot A_{2020} \cdot P_{2050} \cdot \frac{1}{1-(1-\delta)\beta}\end{aligned}$$

where the first term refers to the avoided tax payment up to 2050, while the price is growing at rate g, and the second term refers to the avoided tax payment for all periods thereafter.

After computing the avoided tax payment from abatement investments in each year 2015 to 2030, we then rewrite each of these expressions to solve for A_t explicitly:

$$\begin{aligned}\text{Avoided Tax}_{2020}/\{\beta \cdot P_{2021} \cdot & \frac{1 - [(1+g)(1-\delta)\beta]^{2050-2021+1}}{1-(1+g)(1-\delta)\beta} \\ & + \beta^{2051-2020} \cdot (1-\delta)^{2051-2021-1} \cdot P_{2050} \cdot \frac{1}{1-(1-\delta)\beta}\} = A_{2020}\end{aligned}$$

By assuming that the investment cost function takes the form $\psi(A) = \phi A^2$, we use the firm's first-order conditions to set the marginal investment cost equal to the discounted stream of avoided

[1] This discount factor corresponds to $1/\exp(\alpha)$.

tax payments. We calculate the depreciated sum of abatement and compare that to average emissions reductions observed in the modeling scenarios (relative to the baseline scenario). Setting these two values equal then allows use to estimate the abatement cost parameter ϕ.

To illustrate, the total accumulated abatement stock in 2030 is given by:

$$K_{2030} = A_{2015} \cdot (1-\delta)^{14} + A_{2016} \cdot (1-\delta)^{13} + ... + A_{2028} \cdot (1-\delta) + A_{2029}$$

Emissions reductions relative to baseline are then given by:

$$\Delta E_{2030} = \bar{E}_{2030} - K_{2030}$$

Substituting my expressions for each A_t into this equation then allows me to solve for ϕ. For A denoted in metric tons of CO_2, estimated $\hat{\phi}$ values are provided in the table below:

	Modeling Scenario			
Years	**\$25, 5%**	**\$25, 1%**	**\$50, 5%**	**\$50, 1%**
2015-2030	$8.30 \cdot 10^{-07}$	$6.74 \cdot 10^{-07}$	$1.19 \cdot 10^{-06}$	$8.15 \cdot 10^{-07}$

Table 1: Estimated Abatement Cost Function Parameter from Stanford EMF-32 Modeling Scenarios

We use the parameter associated with a \$25 tax growing at 5% annually. Figure 1 shows the annual emissions reduction predicted by our calibrated model versus each of the ten modeling scenarios from the Stanford EMF-32 exercise.

2 Details about Model Simulations

To model the representative firm's response to simulated price trajectories, we first performed backward induction to determine the firm's optimal abatement policy as a function of the accumulated abatement cost stock, the realized allowance price in the previous compliance period, and the number of elapsed compliance periods. Given computational limitations and the need to discretize the state space, the representative firm is able to accumulate abatement capital stock in multiples of 1 million metric tons of avoided annual CO_2 emissions; the upper bound on permitted abatement

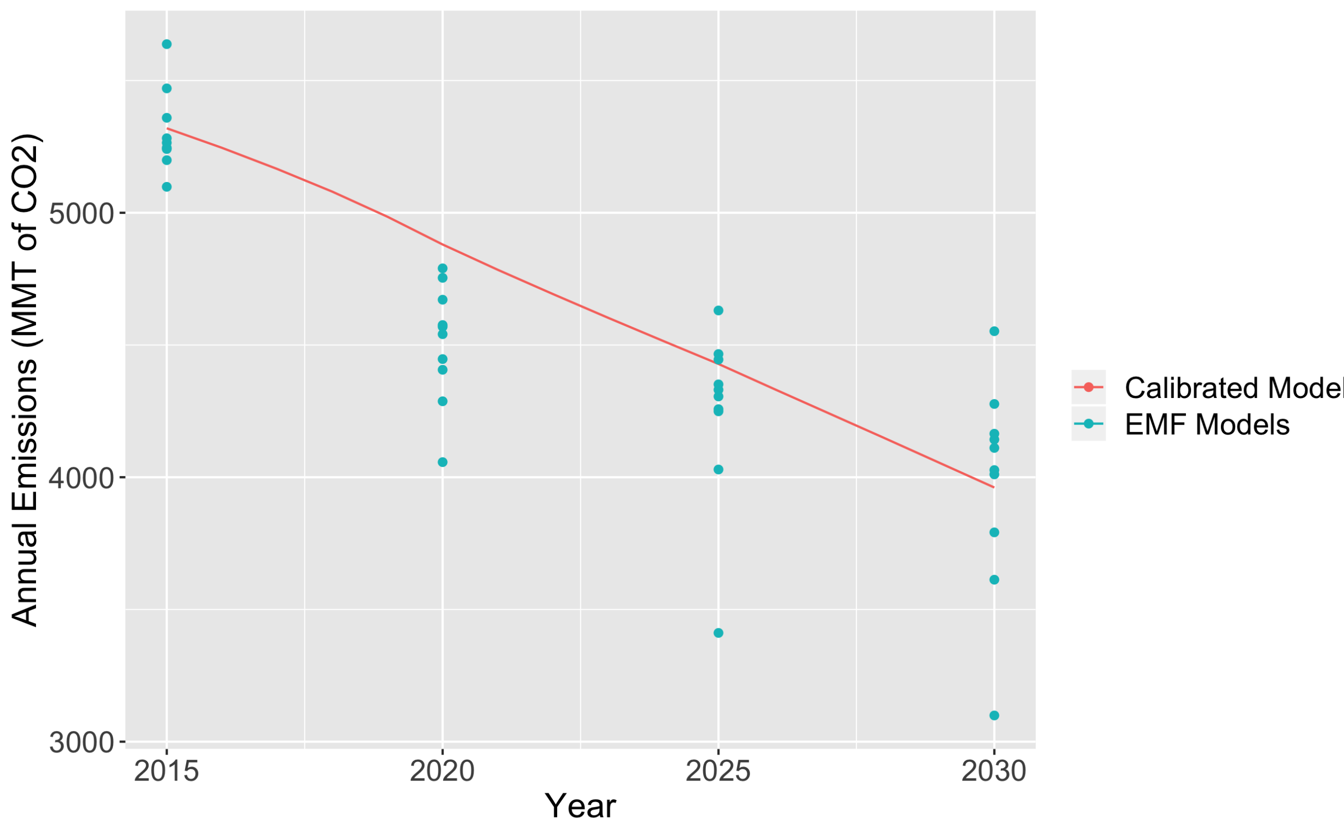

Figure 1: Annual Emissions from Calibrated Model versus EMF 32 Results

The points refer to the projected U.S. CO_2 emissions levels from each of the ten models in EMF 32, assuming an initial price of \$25/ton of CO_2 in 2020, rising at 5% annually until 2050 and then leveling off. The line reflects implied emissions reduction from our calibrated abatement investment cost function, when applying the same carbon price trajectory.

capital stock is total annual U.S. emissions in 2020, as modeled in EMF 32 baseline scenarios. We construct the price transition matrix by simulating 10 million evolutions of a stochastic process with our calibrated drift and volatility parameters and then calculating the probability that the next period allowance price will fall into each "price bin," conditional on the current period price. Each price bin is defined as a particular integer dollar value. Note that we set an effective price ceiling at $1000 per ton when discretizing the state space to perform backwards induction; however, given our drift and volatility parameters and the number of periods considered, this upper bound affects fewer than 0.1% of simulated price trajectories.

After constructing the representative firm's optimal policy matrix, we perform forward simulation to model abatement investment paths for 100,000 simulated stochastic price trajectories. To be consistent with the Stanford EMF modeling exercise, we assume that the price levels off indefinitely after period T. We then sum the total avoided emissions from each year of abatement investment and the firm's total current value cost of that investment. To compare the representative firm's response under each of these stochastic trajectories to responses under "tax trajectories," we calculate the initial price P_0 that would yield the same total emissions reduction if that initial price were to increase smoothly each period at the rate of interest. In this scenario, we assume that firms have perfect information about the price path. We then calculate the difference in total abatement investment costs between the "stochastic" and "tax" scenarios, having constrained total emissions reductions to be the same in both cases.[2]

References

A. R. Barron, A. A. Fawcett, M. A. C. Hafstead, J. R. McFarland, and A. C. Morris. Policy insights from the EMF 32 study on U.S. carbon tax scenarios. *Climate Change Economics*, 9(1): 1840003–1 – 1840003–47, 2018a.

A. R. Barron, A. A. Fawcett, M. A. C. Hafstead, J. R. McFarland, and A. C. Morris. Data from:

[2]Because of the discretization of the state space, we cannot always achieve a given level of emissions reduction *exactly* following this approach. In practice, therefore, we calculate the total emissions reduction and total abatement investment cost from a sequence of smoothly increasing initial prices and then plot a curve from these emissions-cost pairs.

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