Here we compare the physical climate model of LR17 with the model we derived from Joos et al. (2013) and Geoffroy et al. (2013) and which was employed by IPCC (2013, ch. 8).

Let us first look at the decay of atmospheric CO$_2$, then temperature inertia. LR17 model the decay of atmospheric CO$_2$ as

$$\dot{M}_t = E - \delta M_t,$$

where $M_t$ is the increase in the atmospheric CO$_2$ concentration from the pre-industrial level, $\delta$ the decay rate and $E$ is the baseline flow of CO$_2$ emissions into the atmosphere. The difficulty facing this simple representation of the decay of atmospheric CO$_2$ is that the global carbon cycle has multiple timescales and a significant fraction of CO$_2$ emissions will remain in the atmosphere for many thousands of years. This can be represented by

$$\dot{M}_t = \sum_{i=0}^{3} \dot{M}_t^i = \sum_{i=0}^{3} a_i E - \delta_i M_t^i$$

(1)

with $\sum_{i=0}^{3} a_i = 1$ and $\delta_0 = 0$ and $M_t = \sum_{i=0}^{3} M_t^i$. Following the use of this specification in IPCC (2013), we use the best fit of Equation (1) to 16 independent, more sophisticated models of the carbon cycle (Joos et al., 2013). This allows us to compare LR17’s climate dynamics with a set of more physically realistic
carbon-cycle models.

Figure 3: Increase in temperature for a constant increase in CO$_2$ concentration by 47 ppm

Black lines represent the climate representation in LR17 for their high, medium (bold) and low inertia scenarios. The green lines represent calibration to 16 independent, more sophisticated models of the temperature response in Geoffroy et al. (2013).

Second, consider the treatment of temperature inertia in response to the atmospheric concentration of CO$_2$ in LR17. This is modelled as an exponential process towards a steady-state temperature,

$$\dot{T}_t = \phi(sF(M_t) - T),$$

with $T$ being global mean surface warming above the pre-industrial level, $F$
the radiative forcing ($W/m^2$) resulting from elevated atmospheric CO$_2$, and $s$ a transformation of the parameter known as climate sensitivity, i.e. the long-run equilibrium warming that would result from a doubling of the CO$_2$ concentration.$^{11}$ $\phi$ is the crucial thermal inertia parameter.

A single response timescale is insufficient to characterize the response of the surface climate system to radiative forcing, as shown in Held et al. (2010) and see Figure 3. A more representative model comprises two heat reservoirs, one for the warming of the atmosphere and the upper ocean $T$, and one for the warming of the deep ocean $T^o$.$^{12}$

$$\dot{T}_t = \frac{1}{c}(F(M_t) - bT_t) - \frac{\gamma}{c}(T_t - T^o_t) \quad (2)$$

$$\dot{T}^o_t = \frac{\gamma}{c_0}(T_t - T^o_t). \quad (3)$$

IPCC (2013, ch. 8) employs this simple model, calibrated on the outputs of 16 independent, more sophisticated climate models by Geoffroy et al. (2013), and we do likewise. The calibrations were based on behaviour of the more sophisticated models under an instantaneous quadrupling of atmospheric CO$_2$ concentrations, which are then held fixed.$^{13}$

$^{11}$Here, $sF$, for doubled CO$_2$ concentration, corresponds to the equilibrium climate sensitivity.

$^{12}$Here $c$ and $c_0$ are effective heat capacities per unit area, $\lambda$ is a radiative feedback parameter per unit area for an additional degree of warming and $\gamma$ is a heat exchange coefficient representing the transfer of heat for a difference of 1 degree between upper and lower ocean, see Geoffroy et al. (2013).

$^{13}$Further, we assume the same formula for radiative forcing as LR17: $F(M) = \alpha \ln((M + M_{pre})/M_{pre})$. Defining climate sensitivity $cs$ as steady state warming for a doubling of atmospheric carbon emissions, allows to easily compare our formulation of temperature response $\dot{T} = b/c(cs/\ln 2 \ast \ln((M + M_{pre})/M_{pre}) - T) - \gamma/c(T - T)$ with LR17’s expression $\dot{T} = \phi(cs/\ln 2 \ast \ln((M + M_{pre})/M_{pre}) - T)$, with $M_{pre}$ the pre-industrial concentration level. These formulas were used to set different climate sensitivities in Geoffroy et al. (2013) to $3^\circ$C for Figures 1 and 3.
Comparison to Golosov carbon cycle  We examine the alternative specification of the carbon cycle due to Golosov et al. (2014), which was employed by LR17 in their Online Appendix D. Figure 4 shows the results from substituting in the carbon decay model of Golosov et al. (2014). When the model of Golosov et al. (2014) is put in, the disparity with the IPCC models is even greater.
Figure 4: The effect of a CO$_2$ emission pulse, including Golosov et al. decay

In addition to Figure 1 and 3, red lines represent the climate model in Online Appendix D of LR17, based on Golosov et al. (2014), for their high, medium and low temperature inertia scenarios.