

Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Online Appendix

Decomposition of Equity Premium into Price and Quantity of Risk

We can write the standard relationship $E_t m_{t+1} R_{t+1}^e = 1$ as

$$\text{Cov}_t(m_{t+1}, R_{t+1}^e) + E_t m_{t+1} E_t R_{t+1}^e = 1$$

and rearrange terms, using $E_t m_{t+1} = 1/R_{t+1}^f$, to get

$$\begin{aligned} E_t R_{t+1}^e - R_{t+1}^f &= -\frac{\text{Cov}_t(m_{t+1}, R_{t+1}^e)}{E_t m_{t+1}} \\ &= -\frac{\text{Cov}_t(m_{t+1}, R_{t+1}^e)}{\text{Var}_t m_{t+1}} \frac{\text{Var}_t m_{t+1}}{E_t m_{t+1}}. \end{aligned} \quad (*)$$

The left-hand side of (*) is the equity premium, and the two terms on the right-hand side can be interpreted as the quantity of risk and the price of risk, respectively, as discussed in Cochrane (2005) and elsewhere.

In Figure OA-1, below, I decompose the equity premium results from Figure 1 in the paper, for the Epstein-Zin preference specification of the model, into its price of risk and quantity of risk components. (Numerical solutions for these figures use a third-order rather than a fifth-order perturbation solution to reduce the computation time, but the results for the fifth-order solution are close to these.)

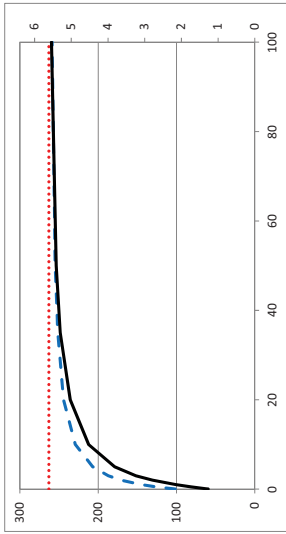
The results for the price of risk are given by the purple solid lines in the middle column, while the results for the quantity of risk are given by the orange solid lines in the rightmost column. In these figures, the variation in the equity premium is almost entirely due to variation in the price of risk (the purple lines in the middle column), rather than the quantity of risk. Similarly, variation in the coefficient of relative risk aversion R^c , as defined in the paper, matches the variation in the price of risk closely.

The quantity of risk varies the most in the middle-right-hand panel, in an intuitive way: as γ decreases, the household's IES increases and household consumption becomes more volatile. This increases the volatility of the consumption claim payouts and thus the quantity of risk—the covariance of these payouts with the SDF. In contrast, the quantity of risk in the upper- and lower-right-hand panels of the figure varies only slightly as χ and $f(\Theta)$ are varied. In all three right-hand panels, the variation in the quantity of risk does not help to explain the variation in the equity premium; indeed, if anything, it moves in the opposite direction.

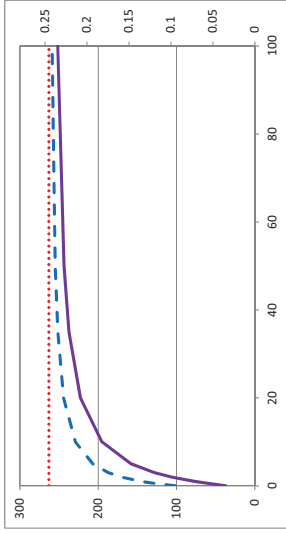
Accuracy of Closed-Form Steady-State Approximation to Risk Aversion

In Swanson (2012, 2018), I showed that the closed-form solutions at the nonstochastic steady state were very good approximations to the “true” risk aversion measure that depends on the economic

(a) Equity Premium



(b) Price of Risk



(c) Quantity of Risk

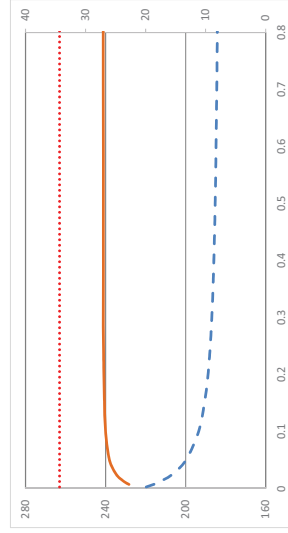
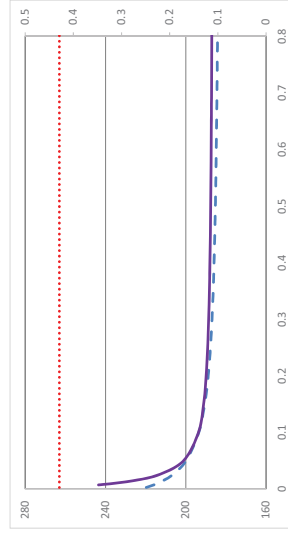
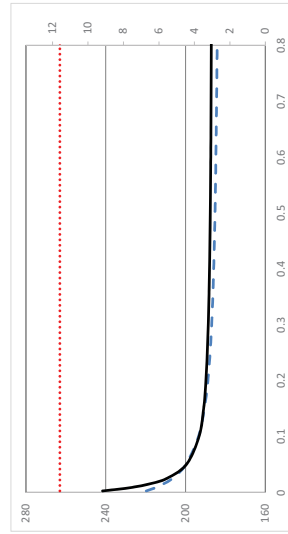
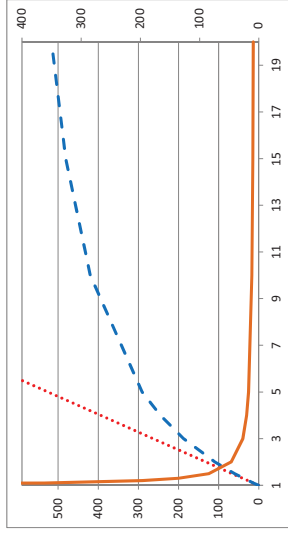
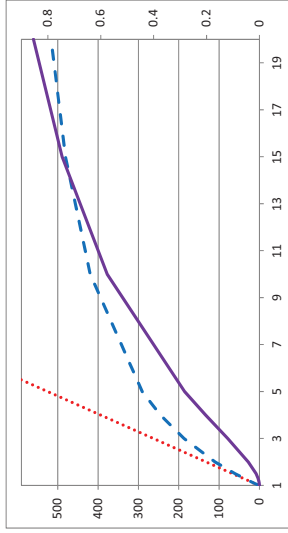
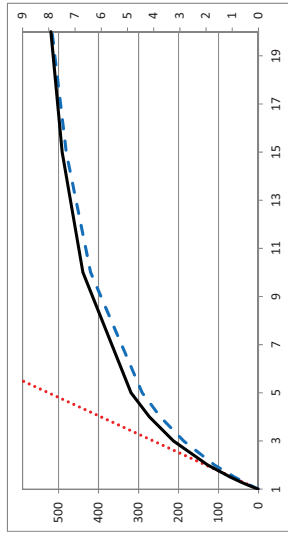
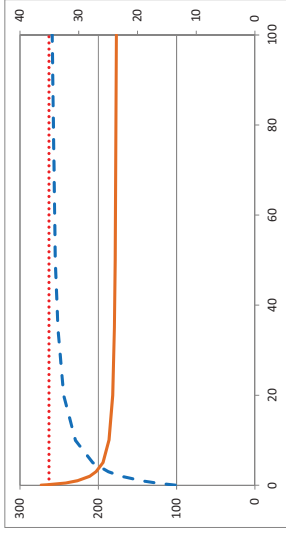


Figure OA-1. Equity premium $E_t R_{t+1}^e - R_{t+1}^f$ is defined as in Figure 1 of the paper, for the model with Epstein-Zin preferences. Price of risk is $\frac{\text{Var}_t m_{t+1}}{E_t m_{t+1}}$; quantity of risk is $-\frac{\text{Cov}_t(m_{t+1}, R_{t+1}^e)}{\text{Var}_t m_{t+1}}$. Returns R_{t+1}^e are at an annual rate. Numerical results are for a third-order perturbation solution to the model.

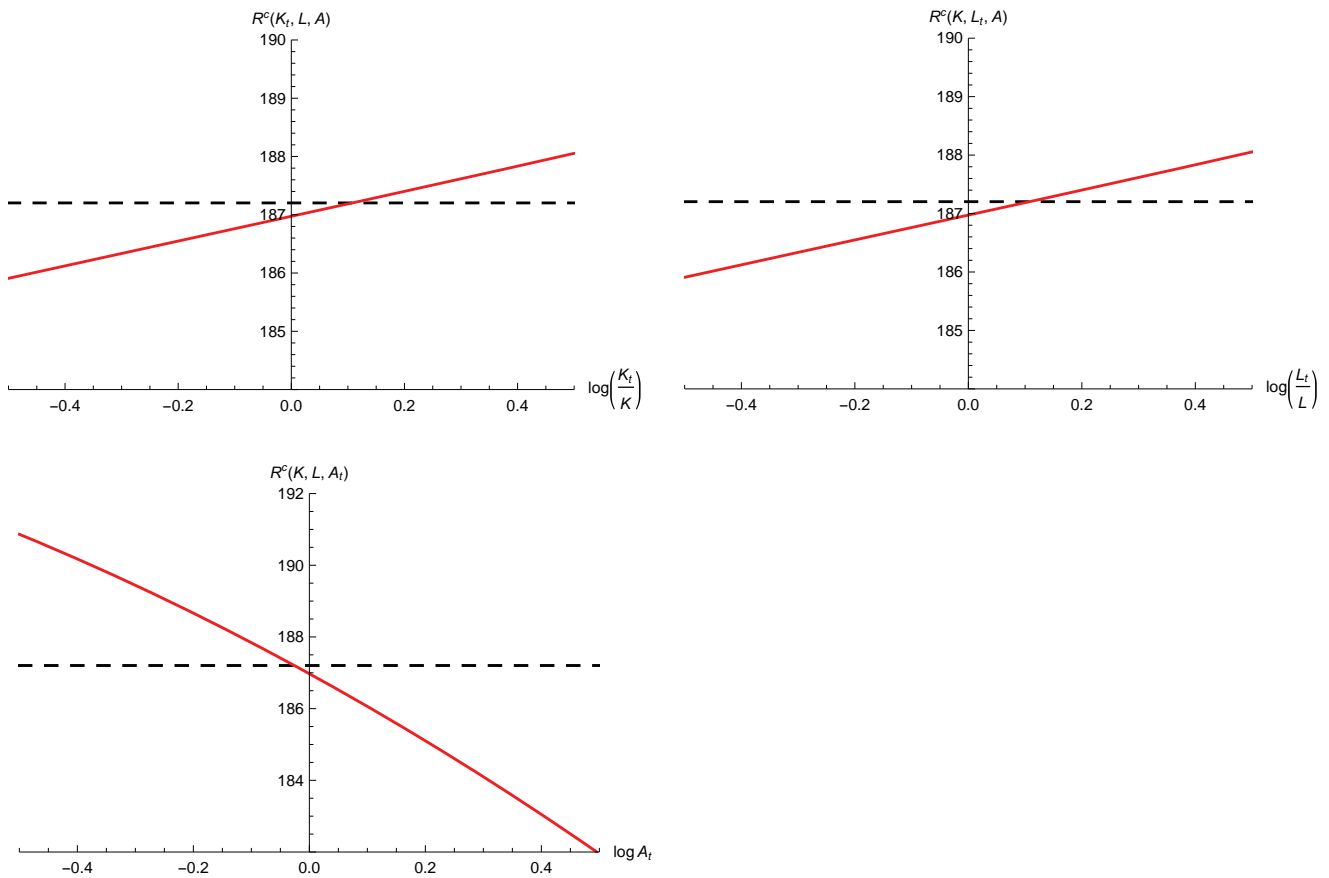


Figure OA-2. Solid red lines: numerical computation of relative risk aversion coefficient $R^c(K_t, L_t, A_t)$ for $\pm 50\%$ range of values for each state variable, K_t , L_t , and A_t , holding the other state variables fixed at their steady-state values. Dashed black lines: closed-form approximation to $R^c(K_t, L_t, A_t)$ computed at the nonstochastic steady state, (K, L, A) .

state, even for values quite far away from the steady state. In the interest of space, I didn't repeat that analysis in the paper, but I report those results in Figure OA-2 for a fifth-order perturbation solution to the Epstein-Zin parameterization of the model. In that figure, the dashed black lines report the closed-form approximation to the risk aversion coefficient R^c of 187.2, as reported in the paper. For comparison, the solid red lines in each panel report the "true" value of risk aversion $R^c(K_t, L_t, A_t)$ as each of the state variables K_t , L_t , and A_t is varied in turn over a $\pm 50\%$ range around its nonstochastic steady state value. As can be seen in the figure, even over this very wide range of values for the state variables, the closed-form approximate value for R^c is quite close to the true value.

Note that the mean capital stock is about 20% higher than its nonstochastic steady-state value, and the mean employment level is about 0.3% higher than its nonstochastic steady-state value, so they are well within the ranges covered in Figure OA-2. (The mean technology level is equal to its nonstochastic steady-state value.)