Online Appendix to "Fiscal Rules, Bailouts, and Reputation in Federal Governments"

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1 Omitted Proofs

1.1 Proof of Lemma 1

Let $\pi' = 0$ and consider two vectors a_1 and a_1' that differ only in transfers. We know that debt issuances $\{b_{i2}\}$ are the unique solution[1] to the system

$$qu'(Y - a_{i1} + qb_{i2}(a_1, 0)) = \frac{\beta}{N}u'\left(Y - \frac{\sum_{j=1}^{N} b_{i2}(a_1, 0)}{N}\right) \quad \text{for all } i$$

We can then see that if $\{b_{i2}(a_1, 0)\}$ solves the system given a_1 then

$$b_{i2}\left(a_1', 0\right) = b_{i2}(a_1, 0) - \frac{1}{q}\left(T_{i1} + T_{i1}'\right) \quad \text{for all } i$$

solves the system given a_1' and leaves public good provisions in period 1 and 2 unchanged. Hence the value is unaffected by transfers in period 1 when $\pi = 0$. A straightforward extension of these arguments implies that this result holds more generally for any two sequences a_1 and a_1' such that $\sum \frac{1}{N} a_{i1} = \sum \frac{1}{N} a_{i1}'$. Q.E.D.

[1] To see this note that the solution satisfies $b_{i2} - b_{12} = 1/q\Delta a_i$ where $a_i = [a_{i1} - a_{11}]$ and

$$qu'(Y - a_{11} + qb_{12}) = \frac{\beta}{N}u'\left(Y - \frac{\sum_{j=1}^{N}\left(b_{12} + \frac{\Delta a_j}{q}\right)}{N}\right)$$

Since u' is strictly increasing there is a unique b_{12} that solves the equation above.

1.2 Preliminary results for proof of Proposition 1-4

For the following proofs it is useful to define the value of enforcing for the optimizing type if the posterior equals π'

$$\omega^{e}\left(b_1,\pi'\right)=\sum_{i=1}^{N}\frac{1}{N}\Big[u\left(Y-b_{i1}-\psi\mathbb{I}_{\{b_{i1}>\bar{b}\}}+q\mathbf{b}_{i2}\left(b_1,\pi'\right)\right) + \beta W_2\left(\mathbf{b}_2\left(b_1+\psi\mathbb{I}_{\{b_{i1}>\bar{b}\}},\pi'\right)\right)\Big] \tag{1}$$

and the the value of non-enforcement

$$\omega^{ne}\left(b_1\right)=\sum_{i=1}^{N}\frac{1}{N}\left[u\left(Y-b_{i1}+q\mathbf{b}_{i2}\left(b_1,0\right)\right)+\beta W_2\left(\mathbf{b}_2\left(b_1,0\right)\right)\right] \tag{2}$$

To prove Proposition 3 we use the following two lemmas:

Lemma 1. *As* $N\to\infty$*, the continuation equilibrium in period 1 given inherited debt* b_1 *and posterior* π *is such that:*

1. *If* $\pi>0$, $\lim_{N\to\infty}\mathbf{b}_{i2}\left(b_1,\pi\right)\to b_{i2}<Y$;
2. *If* $\pi=0$, $\lim_{N\to\infty}\sum_i\frac{\mathbf{b}_{i2}(b_1,0)}{N}\to Y$ *and*

$$\lim_{N\to\infty}\frac{1}{N}u'\left(Y-\sum_i\frac{\mathbf{b}_{i2}\left(b_1,0\right)}{N}\right)=\frac{q}{\beta}u'\left(\left(1+q\right)Y-\sum_i\frac{b_{i1}}{N}\right)>0.$$

Moreover, $\lim_{N\to\infty}V_{i1}\left(b_1,0\right)=u\left(Y\left(1+q\right)-b_1\right)+\beta u\left(0\right)$.

Proof. We know from the text, equation (6), that along a symmetric equilibrium outcome, it must be that

$$qu'\left(Y-b_{i1}+q\mathbf{b}_{i2}\left(b_1,\pi\right)\right)=\beta\pi u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right)+\beta\left(1-\pi\right)\frac{1}{N}u'\left(Y-\frac{\sum_i\mathbf{b}_{i2}\left(b_1,\pi\right)}{N}\right)$$

whenever $\sum_i\mathbf{b}_{i2}\left(b_1,\pi\right)/N<Y$.

Consider part 1 and let $\pi>0$. Clearly, for each finite N, $\mathbf{b}_{i2}<Y$ due to the Inada condition and so the Euler equation above holds. Suppose by way of contradiction that $\mathbf{b}_{i2}\left(b_1,\pi\right)\to Y$ as $N\to\infty$. Then the right side goes to ∞ while the left side goes to $qu'\left(Y-b_1+qY\right)$ which is finite. This is a contradiction.

Consider part 2 and let $\pi = 0$. For all finite N, because of the Inada condition, it must be that $\sum_i \mathbf{b}_{i2}/N < Y$ and so the following Euler equation must hold:

$$qu'\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(b_1, 0\right)\right) = \beta \frac{1}{N} u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}\left(b_1, 0\right)}{N}\right) \tag{3}$$

Suppose by way of contradiction that $\frac{\sum \mathbf{b}_{i2}(b_1,0)}{N} \to B_2 < Y$. Then the left side converges to a positive number, $qu'\left(Y\left(1+q\right) - b_1\right)$, while the right side converges to zero. This is a contradiction. In particular, since the right side is identical for all i,

$$Y - b_{i1} + q\mathbf{b}_{i2}\left(b_1, 0\right) \to \left(1+q\right) Y - \frac{\sum_i b_{i1}}{N}$$

Therefore, it must be that

$$\lim_{N\to\infty} \frac{1}{N} u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}\left(b_1, 0\right)}{N}\right) = \frac{q}{\beta} u'\left(Y - b_{i1} + qY\right).$$

It follows that, if the posterior equals zero, the value of a continuation equilibrium is

$$u\left(Y\left(1+q\right) - b_1\right) + \beta u\left(0\right).$$

□

Lemma 2. *Suppose $\pi = 0$. Then for all i,*

$$\lim_{N\to\infty} \sum_{j\neq i} \frac{\partial \mathbf{b}_{j2}\left(b_1, 0\right)}{\partial b_{i1}} = -\frac{1}{q}$$

Proof. Step 1:$\lim_{N\to\infty} G_{i1}\left(\pi = 0\right) = 0$.
We know from Lemma 1 in the paper that if $\pi = 0$, the equilibrium allocations are identical whether or not there are transfers by the central government in period 1. In the case in which there are transfers $T_{i1} = b_{i1} - \sum_i \frac{1}{N} b_{i1}$, the first order conditions for b_{i1} and b_{i2} respectively are

$$u'\left(G_{i0}\right) q = \beta \left[\frac{1}{N} u'\left(G_{i1}\right) + \frac{\beta}{N} u'\left(G_{i2}\right) \sum_{j\neq i} \frac{\partial b_{j2}^{tr}}{\partial b_{i1}^{tr}}\right] \tag{4}$$

$$u'\left(G_{i1}\right) q = \frac{\beta}{N} u'\left(G_{i2}\right) \tag{5}$$

where the superscript tr denotes outcomes with transfers. Therefore

$$\sum_{j\neq i}\frac{\partial b_{j2}^{tr}}{\partial b_{i1}^{tr}}=\frac{u'\left(G_{i0}\right)\frac{qN}{\beta}-u'\left(G_{i1}\right)}{\beta u'\left(G_{i2}\right)}=\frac{u'\left(G_{i0}\right)\frac{qN}{\beta}-u'\left(G_{i1}\right)}{Nu'\left(G_{i1}\right)q}=\frac{\frac{u'(G_{i0})}{u'(G_{i1})}\frac{q}{\beta}-\frac{1}{N}}{q}\tag{6}$$

We know from Lemma 1 that $\lim_{N\to\infty}G_{i2}\left(0\right)=0$. Now suppose by way of contradiction that $\lim_{N\to\infty}G_{i1}\left(0\right)>0$. Then from (6) we see that

$$\lim_{N\to\infty}\sum_{j\neq i}\frac{\partial b_{j2}^{tr}}{\partial b_{i1}^{tr}}=\frac{u'\left(G_{i0}\right)}{\beta u'\left(G_{i1}\right)}>0$$

Next, we can combine (4) and (5) to obtain

$$u'\left(G_{i0}\right)q=\beta\frac{u'\left(G_{i1}\right)}{N}\left[1+q\sum_{j\neq i}\frac{\partial b_{j2}^{tr}}{\partial b_{i1}^{tr}}\right]\tag{7}$$

If $G_{i1}>0$ then the term $\frac{u'(G_{i1})}{N}$ converges to zero as $N\to\infty$, while the argument above establishes that the limit of $q\sum_{j\neq i}\frac{\partial b_{j2}^{tr}}{\partial b_{i1}^{tr}}$ is finite. Therefore, as $N\to\infty$, the right side of (7) converges to zero while the left side is finite. This is a contradiction. Since the equilibrium outcome with transfers in period 1 and the one without are equivalent when $\pi=0$ then $\lim_{N\to\infty}G_{i1}\left(\pi=0\right)=0$.
Step 2: $\lim_{N\to\infty}\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}(b_1,0)}{\partial b_{i1}}=-\frac{1}{q}$.
Now consider the case in which there are no transfers in period 1. In this case the first order conditions imply that

$$\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}\left(b_1,0,N\right)}{\partial b_{i1}}=\frac{u'\left(G_{i0}\right)\frac{qN}{\beta}-u'\left(G_{i1}\right)N}{\beta u'\left(G_{i2}\right)}=N\left(\frac{u'\left(G_{i0}\right)\frac{q}{\beta}-u'\left(G_{i1}\right)}{Nu'\left(G_{i1}\right)q}\right)=\frac{\frac{u'(G_{i0})}{u'(G_{i1})}\frac{q}{\beta}-1}{q}$$

Since we just established that $\lim_{N\to\infty}G_{i1}=0$ and by the Inada condition $\lim_{G\to 0}u'\left(G\right)=\infty$, taking limits on both sides of the above equation yields the result since $\lim_{N\to\infty}\frac{u'(G_{i0})}{u'(G_{i1})}\frac{q}{\beta}=0$. □

Lemma 3. *If* $b_1=\{b_{i1}\}$ *is degenerate in that* $b_{i1}=b_{j1}$ *for all* i,j *then* $\lim_{N\to\infty}\frac{1}{N}\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial\pi}<\infty$.

Proof. By applying the implicit function theorem to (6) in the paper we obtain

$$\frac{\partial \mathbf{b}_{i2}\left(b_1,0\right)}{\partial\pi}=\frac{\beta\frac{N-1}{N}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{\left[q^2u''\left(Y-b_1+q\mathbf{b}_{i2}\left(b_1,0\right)\right)+\frac{\beta}{N}u''\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right)\right]}$$

so

$$\frac{1}{N}\frac{\partial \mathbf{b}_{i2}\left(b_1,0\right)}{\partial \pi}=\left(1-\frac{1}{N}\right)\frac{\beta\frac{1}{N}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{\left[q^2u''\left(Y-b_1+q\mathbf{b}_{i2}\left(b_1,0\right)\right)+\frac{\beta}{N}u''\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right)\right]}$$

As $N\to\infty$, the above converges to

$$\frac{\beta\frac{1}{N}\sum_{j\neq i}u'\left(0\right)}{\left[q^2u''\left(Y-b_1+qY\right)+\beta\frac{u''(0)}{N}\right]}$$

We know from Lemma 1 that the numerator $\beta\frac{1}{N}\sum_{j\neq i}u'\left(G_{i2}\right)$ converges to a finite number. If $\beta\frac{u''(G_{i2})}{N}$ converges to a finite constant or zero then the denominator converges to a finite number and this the fraction converges to a finite number. If it converges to ∞ then the above converges to zero. In both cases, as $N\to\infty$, $\frac{1}{N}\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial\pi}$ converges to a finite number. □

Lemma 4. *i) For all π, $\omega^e\left(\cdot,\pi\right)$ is continuous and differentiable.*

ii) For all b, for π small enough, $\omega^e\left(b,\cdot\right)$ is increasing in π.

Proof. For convenience, rewrite (1):

$$\omega^e\left(b,\pi\right)=\sum_i\frac{1}{N}\left[u\left(Y-b_i+q\mathbf{b}_{i2}\left(b,\pi\right)\right)+\beta u\left(Y-\frac{\sum_i\mathbf{b}_{i2}\left(b,\pi\right)}{N}\right)\right]$$

Part i). The fact that ω_1^e is continuous and differentiable in b follows from continuity and differentiability of u and $\mathbf{b_2}$.

Part ii). Consider the derivative with respect to π:

$$\frac{\partial\omega^e\left(b,\pi\right)}{\partial\pi}=\sum_i\frac{1}{N}\left[qu'\left(G_{i1}\right)\frac{\partial\mathbf{b}_{i2}}{\partial\pi}-\beta\frac{u'\left(G_{i2}\right)}{N}\frac{\partial\sum_i\mathbf{b}_{i2}\left(b,\pi\right)}{\partial\pi}\right]$$

While we cannot sign this term in general, at $\pi=0$, since $qu'\left(G_{i1}\right)=\frac{\beta}{N}u'\left(G_{i2}\right)$, we have

$$\frac{\partial\omega^e\left(b,\pi\right)}{\partial\pi}=-\beta\sum_i\frac{u'\left(G_{i2}\right)}{N^2}\sum_{j\neq i}\frac{\partial\mathbf{b}_{-i2}}{\partial\pi}=-\beta\frac{u'\left(G_{i2}\right)}{N}\frac{\left(N-1\right)}{N}\frac{\partial\mathbf{B}_2}{\partial\pi}$$

where $\mathbf{B}_2\equiv\sum_i\mathbf{b}_{i2}$ and so if $\frac{\partial\mathbf{B}_2}{\partial\pi}<0$, then $\frac{\partial\omega^e(b,\pi)}{\partial\pi}>0$.

We now show that $\mathbf{B}_2\left(b_1,\pi\right)$ is decreasing in π for π small enough. Recall the first order condition in period 1, equation (6) in the paper, rewritten here for convenience:

$$qu'\left(Y-b_{i1}+q\mathbf{b}_{i2}\right)=\beta\pi u'\left(Y-\mathbf{b}_{i2}\right)+\beta\left(1-\pi\right)\frac{u'\left(Y-\frac{\sum_j\mathbf{b}_{i2}}{N}\right)}{N}\tag{8}$$

First define

$$\Delta MU_i \equiv \beta \left[u'\left(Y - b_{i2}\right) - \frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}}{N}\right)}{N} \right]$$

$$A_i \equiv \left[-\beta\pi u''\left(G_{i2}^c\right) - \frac{\beta\left(1-\pi\right)}{2N} u''\left(G_{i2}\right) - q u''\left(G_{i1}\right) \right] > 0$$
$$a_i \equiv \frac{2N}{\beta\left(1-\pi\right)} A_i > 0$$

where $G_{i2}^c = Y - \mathbf{b}_{i2}$. Using the implicit function theorem we have

$$A_i d\mathbf{b}_{i2} = \frac{\beta\left(1-\pi\right)}{2N} u''\left(G_{i2}\right) d\mathbf{b}_{-i2} - \Delta MU_i d\pi$$

and so

$$\frac{\partial \mathbf{b}_{i2}}{\partial \pi} = \frac{1}{1 - \frac{u''\left(G_{i2}\right)}{a_i}\frac{u''\left(G_{-i2}\right)}{a_{-i}}} \frac{-\Delta MU_i}{A_i} + \frac{u''\left(G_{i2}\right)}{a_i}\frac{-\Delta MU_{-i}}{A_i}.$$

Next, we have

$$\begin{aligned}\frac{\partial \mathbf{B}_2}{\partial \pi} &= \frac{1}{1 - \frac{u''\left(G_{s2}\right)}{a_s}\frac{u''\left(G_{n2}\right)}{a_n}} \frac{-\Delta MU_s}{A_s} + \frac{u''\left(G_{s2}\right)}{a_s}\frac{-\Delta MU_n}{A_s} \\ &+ \frac{1}{1 - \frac{u''\left(G_{s2}\right)}{a_s}\frac{u''\left(G_{n2}\right)}{a_n}} \frac{-\Delta MU_n}{A_n} + \frac{u''\left(G_{n2}\right)}{a_n}\frac{-\Delta MU_s}{A_n}\end{aligned}$$

At $\pi = 0$,

$$A_i = \left[-\frac{\beta}{4} u''\left(G_{i2}\right) - q u''\left(G_{i1}\right) \right] = A > 0$$
$$a_i = \frac{4}{\beta} A_i = a > 0$$

Therefore evaluating $\frac{\partial \mathbf{B}_2}{\partial \pi}$ at $\pi = 0$, we obtain

$$\frac{d\mathbf{B}_2}{d\pi} = \left[-\frac{1}{1 - \frac{u''\left(G_{s2}\right)}{a}\frac{u''\left(G_{s2}\right)}{a}} - \frac{u''\left(G_{n2}\right)}{a} \right] \frac{1}{A} \left[\Delta MU_s + \Delta MU_n\right] \tag{9}$$

We know that

$$\frac{1}{1 - \frac{u''\left(G_{s2}\right)}{a}\frac{u''\left(G_{n2}\right)}{a}} > 1$$

and

$$\frac{u''\left(G_{n2}\right)}{a}=\frac{u''\left(G_{n2}\right)}{\left[-u''\left(G_{n2}\right)-q\frac{4}{\beta}u''\left(G_{n1}\right)\right]}>-1$$

Therefore

$$-\frac{1}{1-\frac{u''(G_{s2})}{a}\frac{u''(G_{n2})}{a}}-\frac{u''\left(G_{n2}\right)}{a}<-1+1=0$$

Next, notice that

$$\Delta MU_s+\Delta MU_n=\beta\left[u'\left(Y-\mathbf{b}_{s2}\right)+u'\left(Y-\mathbf{b}_{n2}\right)-u'\left(Y-\frac{\mathbf{b}_{s2}+\mathbf{b}_{n2}}{2}\right)\right]$$

Clearly, if $\Delta=0$ then $\Delta MU_s+\Delta MU_n=\beta u'\left(Y-\mathbf{b}_{s2}\right)>0$. Thus, by continuity, $\Delta MU_s+\Delta MU_n>0$ if Δ is small enough.[2] Therefore, for π close to zero, $\frac{\partial \mathbf{B}_2}{\partial \pi}\leqslant 0$ because all three terms in (9) are positive. □

1.3 Proof of Proposition 1

Assume first that the local governments expect that the central government will not make any transfers in period 1 and will mutualize debt in period 2 with probability $1-\pi$. We will denote the proposed equilibrium outcome with a superscript "no-rules." The optimality condition of problem (10) in the paper and the envelope condition from problem (5) in the paper imply that debt issuance in period 0 satisfies (13) in the paper and the debt issuance in period 1 is $b_2^{\text{no-rules}}=\mathbf{b}_{i2}\left(b_1^{\text{no-rules}},\pi\right)$.

We now study the incentives for the central government to implement positive transfers in period 1 on-path. First we show that it is optimal not to make transfers if Δ small enough. Fix some $\pi>0$. Clearly, for $\Delta=0$, the central government strictly prefers to not transfer due the reputational benefits because the inherited debt distribution is degenerate. By continuity, for Δ small but positive, it will also strictly prefer to implement zero transfers and enforce the constitution.

Next we show that it is optimal not make transfers if π is small enough. Fix some $\Delta>0$. We now show that even though the central government faces a non-degenerate distribution of debt $\left\{b_{i1}^{\text{no-rules}}\right\}$ in period 1, it does not have incentives to implement positive transfers if π is small enough. Define the difference between the value of enforcement if $\pi'=\pi$ and not for a central government that inherits debts $b_1^{\text{no-rules}}\left(\pi\right)=\left\{b_{i1}^{\text{no-rules}}\right\}$ as

$$\mathcal{W}\left(\pi\right)\equiv\omega^e\left(b_1^{\text{no-rules}}\left(\pi\right),\pi\right)-\omega^{ne}\left(b_1^{\text{no-rules}}\left(\pi\right)\right)$$

where since there are no fiscal rules we set $\psi=0$ in the definition of ω^e in (1). Note that

[2] One can prove the same result for arbitrary Δ if $u'''>0$.

for an equilibrium with enforcement to exist, it must be that $\mathcal{W}(\pi) \geqslant 0$. Since the utility and policy functions are continuous in π, $\mathcal{W}$ is continuous in π. Moreover $\mathcal{W}(0) = 0$ so it is enough to show that $\mathcal{W}'(0) > 0$. Differentiating $\mathcal{W}$ we obtain:

$$\mathcal{W}'(\pi) = \sum_i \left(\left[\frac{\partial \omega^e \left(b_1^{\text{no-rules}}(\pi), \pi \right)}{\partial b_{i1}} - \frac{\partial \omega^{ne} \left(b_1^{\text{no-rules}}(\pi) \right)}{\partial b_{i1}} \right] \frac{\partial b_{i1}^{\text{no-rules}}(\pi)}{\partial \pi} \right) + \frac{\partial \omega^e \left(b_1^{\text{no-rules}}(\pi), \pi \right)}{\partial \pi}$$

Evaluating the expression above at $\pi = 0$, using that $\omega^{ne}(\cdot) = \omega^e(\cdot, \pi = 0)$ when $\psi = 0$ and so $\partial \omega^e \left(b_1^{\text{no-rules}}(0), 0 \right) / \partial b_{1i} = \partial \omega^{ne} \left(b_1^{\text{no-rules}}(0) \right) / \partial b_{1i}$, we obtain

$$\mathcal{W}'(0) = \frac{\partial \omega^e \left(b_1^{\text{no-rules}}(0), 0 \right)}{\partial \pi} > 0$$

as desired. That ω^e is increasing in π for π close to zero is established in Lemma 4 part ii).

We are left to show that an individual government has no incentives to increase its debt and force the central government to make a transfer. Suppose local government i chooses $b_{i1} > b_1^{\text{no-rules}}$ to induce the central government to make a transfer to region i in period 1 with some positive probability. The value for the best deviation for such local government is:

$$V_i^{dev} = \max_{b_{i1}} u(Y_{i0} + q b_{i1}) + \beta \left[\pi + (1-\pi)\, \sigma \left(\pi, b_{i1}, b_{-i1}^{\text{no-rules}} \right) \right] V_{i1} \left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi' \right) + \beta (1-\pi) \left[1 - \sigma \left(\pi, b_{i1}, b_{-i1}^{\text{no-rules}} \right) \right] V_{i1} \left(b_{i1}, b_{-i1}^{\text{no-rules}}, 0 \right)$$

subject to

$$\omega^e \left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi \right) \leqslant \omega^{ne} \left(b_{i1}, b_{-i1}^{\text{no-rules}} \right) \tag{10}$$

Let V_i be the value along the conjectured equilibrium and $\Delta V_i = V_i - V_i^{dev}$. At $\pi = 0$ Note that by construction, $b_1^{\text{no-rules}}$ solves (10) in the paper or

$$V_i = \max_{b_{i1}} u(Y + q b_{i1}) + {} + \beta\pi \left[u \left(Y - b_{i1} + q \mathbf{b}_{i2} \left(\left(b_{i1}, b_{-i1}^{\text{no-rules}} \right), \pi \right) \right) + \beta u \left(Y - \mathbf{b}_{i2} \left(\left(b_{i1}, b_{-i1}^{\text{no-rules}} \right), \pi \right) \right) \right] + \beta (1-\pi) \left[u \left(Y - b_{i1} + q \mathbf{b}_{i2} \left(\left(b_{i1}, b_{-i1}^{\text{no-rules}} \right), \pi \right) \right) + \beta u \left(Y - \frac{\sum_j \mathbf{b}_{j2} \left(\left(b_{i1}, b_{-i1}^{\text{no-rules}} \right), \pi \right)}{N} \right) \right]$$

Note that for $\pi = 0$, $\Delta V_i = 0$. Now suppose that $\pi > 0$. Notice that as N gets large, b_{i1} needs to increase in order to induce the central government to make a transfer. In particular, for any finite b_{i1}, as $N \to \infty$ then, eventually, $\omega^e \left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi \right) > \omega^{ne} \left(b_{i1}, b_{-i1}^{\text{no-rules}} \right)$.

This is because

$$
\begin{aligned}
&\lim_{N\to\infty}\left(\omega^{e}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right) - \omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right)\right) \\
&= u\left(Y - b_{1}^{\text{no-rules}} + q\mathbf{b}_{-i2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{j2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)\right) \\
&- \left[u\left(Y - b_{1}^{\text{no-rules}} + q\mathbf{b}_{i2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), 0\right)\right) + \beta u\left(Y - \mathbf{b}_{j2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), 0\right)\right)\right] \\
&> 0
\end{aligned}
$$

As a result, a necessary condition for $\omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right) \geqslant \omega^{e}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right)$ as $N \to \infty$ is that $b_{i1} \to \infty$ which violates feasibility when facing the commitment type. For each π there exists $N(\pi)$ such that for $N > N(\pi)$, the deviation is infeasible. And so for $N > \max_{\pi} N(\pi)$, the constructed outcome is an equilibrium outcome.

We are left to show that such an equilibrium is unique (among symmetric pure strategy equilibria). First, fix some $\Delta > 0$. Suppose there exists an interval $(0, \pi_1)$ such that for all $\pi \in (0, \pi_1)$, there exists an equilibrium in which the optimizing type implements positive transfers with strictly positive probability. Then, it must be that $\mathcal{W}(\pi) \leqslant 0$. However, this contradicts our earlier argument that $\mathcal{W}(\pi) > 0$ for π sufficiently close to zero. As a result, an equilibrium in which $\sigma > 0$ cannot exist for π sufficiently small.

Next, fix some $\pi > 0$. We know that for $\Delta = 0$, in any symmetric equilibrium, $\mathcal{W}(\pi) > 0$. Therefore, by continuity this inequality will continue to hold for Δ sufficiently small by positive. As a result, an equilibrium in which $\sigma > 0$ cannot exist for Δ sufficiently small. Q.E.D.

1.4 Proof of Proposition 3

We first show that under our assumptions, there exists a unique equilibrium with no enforcement if π is sufficiently small.

To this end consider first the problem a local government i that expects that i) other local governments are going to violate the fiscal rule, ii) the optimizing type central government is not going to enforce the fiscal rule punishment in period 1. Consequently, local government i expects to learn the type of the central government in period 1. We will denote the proposed equilibrium outcome with a superscript "rules." The problem for the local government at time 0 is then:

$$
\Omega(\pi) = \max_{b_{i1}} u\left(Y_{i0} + qb_{i1}\right) + \beta\pi V_{i1}\left(\left(b_{i1} + \psi, b_{-i1}^{\text{rules}} + \psi\right), 1\right) + \beta(1-\pi) V_{i1}\left(\left(b_{i1}, b_{-i1}^{\text{rules}}\right), 0\right)
$$

where $b_{-i1}^{\text{rules}} > \bar{b}$ is the debt chosen by the other local governments and b_{i1}^{rules} is the solution to the problem above and $b_1^{rules} = \left(b_{i1}^{rules}, b_{-i1}^{rules}\right)$. The optimality condition is:

$$qu'\left(Y_{i0} - qb_{i1}^{rules}\right) = \beta\pi \frac{\partial V_{i1}\left(b_1^{rules} + \psi, 1\right)}{\partial b_{i1}} - \beta\left(1-\pi\right)\frac{\partial V_{i1}\left(b_1^{rules}, 0\right)}{\partial b_{i1}}$$

and using the envelope conditions for $V_{i1}\left(b_1^{rules} + \psi, 1\right)$ and $V_{i1}\left(b_1^{rules}, 0\right)$ we obtain

$$\begin{aligned} qu'\left(Y_{i0} + qb_{i1}^{rules}\right) = {} & \beta\pi u'\left(Y - \left(b_{i1}^{rules} + \psi\right) + q\mathbf{b}_{i2}\left(b_1^{rules} + \psi, 1\right)\right) \\ & + \beta\left(1-\pi\right)u'\left(Y - b_{i1}^{rules} + q\mathbf{b}_{i2}\left(b_1^{rules}, 0\right)\right) \\ & + \beta^2\left(1-\pi\right)u'\left(Y - \frac{\sum_{j=1}^{N}\mathbf{b}_{j2}\left(b_1^{rules}, 0\right)}{N}\right)\sum_{j=1, j\neq i}^{N}\frac{1}{N}\frac{\partial\mathbf{b}_{j2}\left(b_1^{rules}, 0\right)}{\partial b_{i1}}, \end{aligned} \tag{11}$$

which is equation (14) in the paper. Note that for Δ small enough, $b_{i1}^{rules} > \bar{b}$ for all i.

We now show that for N large enough and π small enough no individual local government has an incentive to deviate from b_{i1}^{rules} and choose $b_{i1} = \bar{b}$ to attain value

$$\begin{aligned} \bar{\Omega}\left(\pi\right) = {} & u\left(Y_{i0} + q\bar{b}\right) + \beta\left[\pi + \left(1-\pi\right)\sigma\left(\pi, \bar{b}, b_{-i1}^{rules}\right)\right]V_{i1}\left(\bar{b}, b_{-i1}^{rules}, \pi'\right) \\ & + \beta\left(1-\pi\right)\left[1 - \sigma\left(\pi, \bar{b}, b_{-i1}^{rules}\right)\right]V_{i1}\left(\bar{b}, b_{-i1}^{rules}, 0\right) \end{aligned}$$

First notice that as $N \to \infty$,

$$\omega^e\left(\bar{b}, b_{-i1}^{rules}\left(\pi\right), 1\right) - \omega^{ne}\left(\bar{b}, b_{-i1}^{rules}\left(\pi\right)\right) \to \omega^e\left(b_1^{rules}\left(\pi\right), 1\right) - \omega^{ne}\left(b_1^{rules}\left(\pi\right)\right)$$

This is because as $N \to \infty$, the value for the central government is independent of the debt issued by an individual local government. Further

$$\omega^e\left(b_1^{rules}\left(\pi\right), 1\right) - \omega^{ne}\left(b_1^{rules}\left(\pi\right)\right) < 0$$

since we are constructing an equilibrium in which the central government finds it optimal not to enforce. Therefore there exists $\tilde{N}_1$ such that for $N \geqslant \tilde{N}_1$, $\sigma\left(\pi, \bar{b}, b_{-i1}^{rules}\right) = 0$. Next, we have that

$$\begin{aligned} \Omega\left(\pi\right) - \bar{\Omega}\left(\pi\right) = {} & \left[u\left(Y_{i0} + qb_{i1}^{rules}\left(\pi\right)\right) - u\left(Y_{i0} + q\bar{b}\right)\right] \\ & + \beta\pi\left[V_{i1}\left(\left(b_{i1}^{rules}\left(\pi\right) + \psi, b_{-i1}^{rules}\left(\pi\right) + \psi\right), 1\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}\left(\pi\right) + \psi\right), 1\right)\right] \\ & + \beta\left(1-\pi\right)\left[V_{i1}\left(\left(b_{i1}^{rules}\left(\pi\right), b_{-i1}^{rules}\left(\pi\right)\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}\left(\pi\right)\right), 0\right)\right] \end{aligned}$$

Clearly, since $b_{i1}^{rules}(\pi) > \bar{b}$ we know that

$$\left[u\left(Y_{i0} + q b_{i1}^{rules}(\pi)\right) - u\left(Y_{i0} + q\bar{b}\right)\right] > 0,$$
$$\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi) + \psi, b_{-i1}^{rules}(\pi) + \psi\right), 1\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi) + \psi\right), 1\right)\right] < 0.$$

Notice that as $N \to \infty$, $\left[V_{i1}\left(\left(b_{i1}^{rules}, b_{-i1}^{rules}\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}\right), 0\right)\right] \to 0$. Let $\tilde{N}_2^*$ be the threshold, such that for $N \geqslant \tilde{N}_2^*$,

$$\left[u\left(Y_{i0} + q b_1\right) - u\left(Y_{i0} + q\bar{b}\right)\right] + \beta\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi), b_{-i1}^{rules}(\pi)\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi)\right), 0\right)\right] > 0$$

for all π. Notice that

$$\begin{aligned}
\Omega(\pi) - \bar{\Omega}(\pi) = &\left[u\left(Y_{i0} + q b_{i1}^{rules}(\pi)\right) - u\left(Y_{i0} + q\bar{b}\right)\right] \\
&+ \beta(1-\pi)\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi), b_{-i1}^{rules}(\pi)\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi)\right), 0\right)\right] \\
&+ \beta\pi\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi) + \psi, b_{-i1}^{rules}(\pi) + \psi\right), 1\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi) + \psi\right), 1\right)\right] \\
\geqslant &\left[u\left(Y_{i0} + q b_{i1}^{rules}(\pi)\right) - u\left(Y_{i0} + q\bar{b}\right)\right] \\
&+ \beta\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi), b_{-i1}^{rules}(\pi)\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi)\right), 0\right)\right] \\
&+ \beta\pi\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi) + \psi, b_{-i1}^{rules}(\pi) + \psi\right), 1\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi) + \psi\right), 1\right)\right]
\end{aligned}$$

Since for $N \geqslant \tilde{N}_2$ the first two terms are positive, there exists a $\tilde{\pi}_1$ such that for $\pi \leqslant \tilde{\pi}_1$, $\Omega(\pi) - \bar{\Omega}(\pi) > 0$, and thus a local government has no incentives to satisfy the rule in the conjectured equilibrium.

The next step in establishing that the conjectured equilibrium exists is to show that the optimizing type central government when faced with debt $b_1 = b_1^{rules}$ for all i prefers to not enforce the punishment ψ and reveal its type ($\pi' = 0$ thereafter) than enforce the punishment and have the posterior jump to one (as the local governments expect only the commitment type to enforce the fiscal rule). That is, it must be that

$$\omega^e\left(b_1^{rules}(\pi) + \psi, 1\right) \leqslant \omega^{ne}\left(b_1^{rules}(\pi)\right)$$

which is true if π and β is sufficiently small. In particular, this is true for $\beta \leqslant \bar{\beta}(\pi, N)$ where $\bar{\beta}(\pi, N) \equiv$

$$\frac{\sum_{i=1}^{N} \frac{1}{N}\left[u\left(Y - b_{i1}^{rules}(\pi) + q\mathbf{b}_{i2}\left(b_1^{rules}(\pi), 0\right)\right) - u\left(Y - \left(b_{i1}^{rules}(\pi) + \psi\right) + q\mathbf{b}_{i2}\left(b_1^{rules}(\pi) + \psi, 1\right)\right)\right]}{u\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b_1^{rules}(\pi) + \psi, 1\right)}{N}\right) - u\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b_{i1}^{rules}(\pi), 0\right)}{N}\right)}.$$

The right side of the expression above implicitly defines the maximal discount factor under which it is optimal not to enforce. Therefore, if $\beta < \bar{\beta}(\pi, N)$, $\omega^{e}\left(b_1^{rules}(\pi) + \psi, 1\right) \leqslant \omega^{ne}\left(b_1^{rules}(\pi)\right)$. Therefore, we have shown that under our assumptions an equilibrium in which fiscal rules are violated and not enforced exists.

Next, we show that an equilibrium with enforcement cannot exist for π small. For this to be an equilibrium, it must be that if all other regions are following the rule, no single region has an incentive to deviate and violate it. The value of such a deviation is given by

$$\begin{aligned} V_i^{dev}(\pi) = \max_{b_{i1} > \bar{b}} u\left(Y_{i0} + q b_{i1}\right) + \beta\left[\pi + (1-\pi)\sigma\left(\pi, b_{i1}, \bar{b}_{-i}\right)\right] V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, \pi'\right) \\ + \beta(1-\pi)\left[1 - \sigma\left(\pi, b_{i1}, \bar{b}_{-i}\right)\right] V_{i1}\left(b_{i1}, \bar{b}_{-i}, 0\right) \end{aligned}$$

First, notice that because the reputational benefit shrinks to zero as π goes to zero,

$$\lim_{\pi \to 0}\left(\omega^{e}\left(b_{i1}^{rules}, \bar{b}, \pi\right) - \omega^{ne}\left(b_{i1}^{rules}, \bar{b}\right)\right) < 0$$

so that $\lim_{\pi \to 0} \sigma\left(\pi, b_{i1}, \bar{b}_{-i}\right) = \sigma_0 < 1$. But then

$$\lim_{\pi \to 0} V_i^{dev}(\pi) = u\left(Y_{i0} + q b_{i1}\right) + \beta\sigma_0 V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right) + \beta\left[1 - \sigma_0\right] V_{i1}\left(b_{i1}, \bar{b}_{-i}, 0\right)$$

where we used that $V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, \pi'\right) = V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right)$ since

$$\lim_{\pi \to 0} \pi' = \lim_{\pi \to 0} \frac{\pi}{\pi + (1-\pi)\sigma} = 0.$$

Next, recall from Lemma 1, that the value $V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right)$ depends on the average level of debt $\frac{1}{N}\left(b_{i1} + \psi\right) + \frac{(N-1)}{N}\bar{b}$. Therefore, as $N \to \infty$, $V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right) \to V_{i1}\left(\bar{b}, 0\right)$ which implies that value of punishment for the deviating local government shrinks to zero. Therefore, this deviation is strictly profitable. And so there exists some $\tilde{N}_3$ such that for $N \geqslant \tilde{N}_3$ there exists $\tilde{\pi}_2$ such that for $\pi \leqslant \tilde{\pi}_2$, this deviation is strictly profitable.

We can then conclude that if $N \geqslant \max\left\{\tilde{N}_1, \tilde{N}_2, \tilde{N}_3\right\}$ and $\pi \leqslant \min\{\tilde{\pi}_1, \tilde{\pi}_2\}$ there exists a unique equilibrium with non-enforcement.

To compare the debt levels in period 0 with and without binding fiscal rules, it is useful to rewrite conditions (13) and (14) in the paper to make them more comparable. For the case without fiscal rules, we can combine (13) with (6) in the paper to obtain a condition that characterizes the debt issuance in period 0:

$$u'(Y+qb_1)\,q = \frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1,\pi)) + \frac{\beta^2(1-\pi)}{qN}u'(Y-\mathbf{b}_{i2}(b_1,\pi)) + \frac{\beta^2(1-\pi)}{N}u'(Y-\mathbf{b}_{i2}(b_1,\pi))\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}(b_1,\pi)}{\partial b_{i1}}. \tag{12}$$

For the case with fiscal rules, we can combine (14) with (6) in the paper to obtain

$$u'(Y+qb_1)\,q = \frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1+\psi,1)) + \frac{\beta^2(1-\pi)}{qN}u'(Y-\mathbf{b}_{i2}(b_1,0)) \quad \frac{\beta^2(1-\pi)}{N}u'(Y-\mathbf{b}_{i2}(b_1,0))\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}(b_1,0)}{\partial b_{i1}}. \tag{13}$$

Taking the limit as N goes to infinity for $\pi > 0$ but small, since $\lim_{N\to\infty} u'(Y-\mathbf{b}_{i2}(b_1,\pi)) < \infty$, as shown in Lemma 1, condition (12) reduces to

$$u'(Y+qb_1)\,q = \frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1,\pi)) \tag{14}$$

as the sum of the second and third terms on the right side converge to zero. Condition (13) instead reduces to

$$u'(Y+qb_1)\,q = \frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1+\psi Y,1)), \tag{15}$$

because, as shown in Lemma 1 and 2,

$$\lim_{N\to\infty}\frac{\beta u'(Y-\mathbf{b}_{i2}(b_1,0))}{N}\frac{1}{q} = -\lim_{N\to\infty}\frac{u'(Y-\mathbf{b}_{i2}(b_1,0))}{N}\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}(b_1,0)}{\partial b_{i1}}.$$

We can then compare the right hand side of (14) and (15). We know that for π small enough, $\mathbf{b}_{i2}(b_1,\pi) > \mathbf{b}_{i2}(b_1+\psi Y,1)$, because as $\pi\to 0$, $\mathbf{b}_{i2}(b_1,\pi)\to Y$ but $\mathbf{b}_{i2}(b_1+\psi Y,1)$ is bounded away from Y (see Lemma 1 for details). This observation along with the concavity of u implies that

$$\frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1+\psi Y,1)) < \frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1,\pi)).$$

Therefore, from (14) and (15) we see that the expected marginal cost of issuing debt in period 0 is lower when there is early revelation of the central government's type. Hence, local governments will issue more debt in period 0 because of the lower expected marginal cost. Q.E.D.

1.5 Proof of Proposition 4

We first show that for π close to 1, there exists an equilibrium with enforcement. At $\pi = 1$, the value for a local government of respecting the fiscal rule is $u\left(Y_{i0} + q\bar{b}\right) + \beta V_{i1}\left(\bar{b}, \pi\right)$ while the value of violating is $\max_{b_i > \bar{b}} u\left(Y_{i0} + qb_i\right) + \beta V_{i1}\left(b_i + \psi, \bar{b}_{-i}, \pi\right)$. That the latter is larger than the former follows directly from Assumption 2. By continuity, there exists some $\tilde{\pi}_1 < 1$ such that for $\pi \geqslant \tilde{\pi}_1$, the inequality continues to hold.

Next, we want show that there is an interval around $\pi = 1$ for which the enforcement equilibrium is unique. For an equilibrium with non-enforcement $\left(b_1 = b_1^{rules}\right)$ to exist, it must be that it is optimal for a local government to violate the fiscal rule rather than obeying the rule when all other local governments are violating the rule. That is, $\Omega\left(\pi\right) \geqslant \bar{\Omega}\left(\pi\right)$ where these objects were defined in the proof of Proposition 3. Note that

$$
\begin{aligned}
\bar{\Omega}\left(1\right) &= u\left(Y_{i0} + q\bar{b}\right) + \beta V_{i1}\left(\bar{b}, 1\right) \\
&> \max_{b_i > \bar{b}} u\left(Y_{i0} + qb_i\right) + \beta V_{i1}\left(b_i + \psi, \bar{b}_{-i}, 1\right) \\
&= \max_{b_i > \bar{b}} u\left(Y_{i0} + qb_i\right) + \beta V_{i1}\left(b_i + \psi, b_{-1}^{rules} + \psi, 1\right) \\
&= \Omega\left(1\right)
\end{aligned}
$$

where the first line is the definition of $\bar{\Omega}\left(1\right)$, the second line follows from Assumption 2, the third line follows from the fact that the debt holdings of other regions are irrelevant if the central government is the commitment type for sure ($\pi = 1$), and the last line is the definition of $\Omega\left(1\right)$. Hence, by continuity, if π is sufficiently close to 1, $\bar{\Omega}\left(\pi\right) > \Omega\left(\pi\right)$, and the local government i will prefer to deviate from b_{i1}^{rules} and not violate the fiscal rule. Therefore there exists some $\tilde{\pi}_2$ such that $\pi \geqslant \tilde{\pi}_2$, an equilibrium with non-enforcement cannot exist. Thus, for $\pi \geqslant \max\{\tilde{\pi}_1, \tilde{\pi}_2\}$ there exists a unique equilibrium with enforcement. Q.E.D.

1.6 Proof of Proposition 5

The proof proceeds as follows. We first show that there exists $\underline{\beta}$ such that for $\beta \geqslant \underline{\beta}$, the commitment type chooses $\psi = \bar{\psi}$ to separate in period 1. In our construction we assume (and later verify) that the optimizing type chooses the same fiscal constitution as the commitment type in period 0 and does not enforce the fiscal rule if $\psi = \bar{\psi}$ in period 1. We showed in Proposition 3 that the latter is true if $\beta \leqslant \bar{\beta}$. Next, we show that if $\Delta > 0$ then $\underline{\beta} < \bar{\beta}$.

Recall that $b_{i1}^{er}\left(\pi, \alpha\right)$ denotes the debt issued in period 0 when the local governments expect to learn the central government type in period 1 defined in (14) in the paper given

$\alpha = (\bar{b}, \psi)$; $b_{i1}^{lr}(\pi, \alpha)$ denotes the debt issued in period 0 when the local governments do not expect to learn the central government type in period 1 defined in (13) in the paper given $\alpha = (\bar{b}, \psi)$.

If the commitment type chooses $\psi = \bar{\psi}$ and $\beta \leqslant \bar{\beta}$ where $\bar{\beta}$ is defined in the proof of Proposition 3, since $\bar{b}$ is binding, we know that for π small enough there exists a unique equilibrium with separation in period 1 and early resolution of uncertainty. Thus we can write $W_0^{c,sep}$ as

$$W_0^{c,sep} = \sum_i \frac{1}{N} u\left(Y_{i0} + q b_{i1}^{er}(\pi, \alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \left(b_{i1}^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1} > \bar{b}}\right) + q \mathbf{b}_{i2}\left(b_1^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) \end{array} \right]$$

If instead the commitment type chooses $\psi = 0$, for π close to zero, there is no separation in period 1 and so its value $W_0^{c,pool}$ is

$$W_0^{c,pool} = \sum_i \frac{1}{N} u\left(Y_{i0} + q b_i^{lr}(\pi, \alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - b_{i1}^{lr}(\pi,\alpha) + q \mathbf{b}_{i2}\left(b_1^{lr}(\pi,\alpha), \pi\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{lr}(\pi,\alpha), \pi\right)\right) \end{array} \right].$$

The commitment type will then impose a binding rule if and only if $W_0^{c,sep} \geqslant W_0^{c,pool}$. Let $\Gamma(\pi) = W_0^{c,sep} - W_0^{c,pool}$. As $\pi \to 0$, $\Gamma(\pi) \to$

$$\beta \sum_i \frac{1}{N} \left[u\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right] \\ -\beta \sum_i \frac{1}{N} \left[u\left(Y - b_{i1} + q \mathbf{b}_{i2}(b_1, 0)\right) + \beta u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]$$

since $b_{i1}^{er}(0,\alpha) = b_{i1}^{lr}(0,\alpha) = b_{i1}$. (From now on we use $b_{i1} = b_{i1}^{er}(0,\alpha) = b_{i1}^{lr}(0,\alpha)$.) Rearranging the expression above we obtain

$$\frac{\beta^2}{N} \sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) - u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right] \\ -\frac{\beta}{N} \sum_i \left[u\left(Y - b_{i1} + q \mathbf{b}_{i2}(b_1, 0)\right) - u\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right].$$

Note that both terms in square brackets are positive, thus we can define the cutoff β such that the expression above equals zero:

$$\underline{\beta}(\pi,N) \equiv \frac{\sum_i \left[u\left(Y-b_{i1}+q\mathbf{b}_{i2}(b_1,0)\right)-u\left(Y-\psi\mathbb{I}_{b_{i1}>\bar{b}}-b_{i1}+q\mathbf{b}_{i2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)\right)\right]}{\sum_i \left[u\left(Y-\mathbf{b}_{i2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)\right)-u\left(Y-\mathbf{b}_{i2}(b_1,0)\right)\right]}$$

Then for $\beta < \underline{\beta}(\pi,N)$, $\Gamma(0) < 0$. Thus, for π small, $W_0^{c,sep} < W_0^{c,pool}$ and the unique constitution will feature no fiscal rules. Conversely, for $\beta > \underline{\beta}(\pi,N)$, $\Gamma(0) > 0$. Thus, for π small, $W_0^{c,sep} > W_0^{c,pool}$ and the unique constitution will feature fiscal rules.

To show that this is an equilibrium for $\beta > \underline{\beta}(\pi,N)$, we need to show that the optimizing type does indeed not want to enforce the constitution in period 1 (and induce separation). We know from the proof of Proposition 3 that if $\beta < \bar{\beta}(\pi,N)$, where $\bar{\beta}(\pi,N) \equiv$

$$\frac{\sum_{i=1}^{N}\frac{1}{N}\left[u\left(Y-b_{i1}^{rules}(\pi)+q\mathbf{b}_{i2}\left(b_1^{rules}(\pi),0\right)\right)-u\left(Y-\left(b_{i1}^{rules}(\pi)+\psi\right)+q\mathbf{b}_{i2}\left(b_1^{rules}(\pi)+\psi,1\right)\right)\right]}{u\left(Y-\frac{\sum \mathbf{b}_{i2}\left(b_1^{rules}(\pi)+\psi,1\right)}{N}\right)-u\left(Y-\frac{\sum \mathbf{b}_{i2}\left(b_{i1}^{rules}(\pi),0\right)}{N}\right)},$$

then for π close to zero, the optimizing will strictly prefer to not enforce the rule at $t=1$. Thus we have our desired result for $\beta \in \left[\underline{\beta}(\pi,N), \bar{\beta}(\pi,N)\right]$. To show that this a well defined interval, we need to show that $\bar{\beta}(0,N) > \underline{\beta}(0,N)$. This is true if

$$\begin{aligned}0 > &\left[Nu\left(Y-\frac{\sum \mathbf{b}_{i2}\left(b+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)}{N}\right)-\sum_i u\left(Y-\mathbf{b}_{i2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)\right)\right]\\ &-\left[Nu\left(Y-\frac{\sum \mathbf{b}_{i2}(b_1,0)}{N}\right)-\sum_i u\left(Y-\mathbf{b}_{i2}(b_1,0)\right)\right]\end{aligned}$$

Given the concavity of u, this is true if $\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right) < \mathbf{b}_{s2}(b_1,0)-\mathbf{b}_{n2}(b_1,0)$. From the first order conditions for $\mathbf{b}_{i2}(b_1,0)$ we have

$$u'\left(Y-b_{i1}+q\mathbf{b}_{i2}(b_1,0)\right)q = \frac{\beta}{N}u'\left(Y-\frac{\sum \mathbf{b}_{i2}(b_1,0)}{N}\right)$$

This implies that

$$\mathbf{b}_{s2}(b_1,0)-\mathbf{b}_{n2}(b_1,0) = \frac{b_{s1}-b_{n1}}{q} \tag{16}$$

Next from the first order conditions for $\mathbf{b}_{i2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)$ we have

$$u'\left(Y-\psi\mathbb{I}_{b_{i1}>\bar{b}}-b_{i1}+q\mathbf{b}_{i2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)\right)q = \beta u'\left(Y-q\mathbf{b}_{i2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)\right)$$

Then, if the rule is not binding for the North:

$$\begin{aligned}
&u'(Y-\psi-b_{s1}+qb_{s2})-u'(Y-b_{n1}+qb_{n2})\\
&=\beta u'(Y-qb_{s2})-\beta u'(Y-qb_{n2})>0
\end{aligned}$$

and so

$$\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}(b_1,1)<\frac{\psi+b_{s1}-b_{n1}}{q} \tag{17}$$

If instead the rule is binding for the North as well we have

$$\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}(b_1,1)<\frac{b_{s1}-b_{n1}}{q} \tag{18}$$

So from (16) and (17)-(18) it follows that for ψ small enough, $\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)<\mathbf{b}_{s2}(b_1,0)-\mathbf{b}_{n2}(b_1,0)$ and so $\bar{\beta}(0,N)>\underline{\beta}(0,N)$. Therefore, for β in this range and π small enough, we have an equilibrium in which $\psi=\bar{\psi}$ and the rules are not enforced in period 1 by the optimizing type.

Finally, we need to show that the optimizing type will mimic the commitment type in period 0 and announce the same rule anticipating it will not enforce it in period 1. The value of choosing the same constitution as the commitment type in period 0 is given by

$$\begin{aligned}
W_0^m(\pi,\alpha)&=\sum_i u\left(Y_{i0}+qb_{i1}^{er}(\pi,\alpha)\right)+\beta W_1^{er}\left(b_1^{er}(\pi,\alpha)\right)\\
&=\sum_i \left[u\left(Y_{i0}+qb_{i1}^{er}(\pi,\alpha)\right)+\beta u\left(Y-\psi-b_{i1}^{er}(\pi)+q\mathbf{b}_{i2}\left(b_{i1}^{er}(\pi,\alpha),0\right)\right)\right.\\
&\quad\left.+\beta^2 u\left(Y-\frac{\sum_j \mathbf{b}_{j2}\left(b_{i1}^{er}(\pi,\alpha),0\right)}{N}\right)\right]
\end{aligned}$$

while the value of choosing a different constitution is $W_0^m(0,\alpha)$ because the local governments learn that they are facing the optimizing type. We will establish that $\frac{\partial}{\partial\pi}W_0^m(\pi,\alpha)>0$, at $\pi=0$ which in turn implies that if π is close to 0, the optimizing type will always find it optimal to mimic. Differentiating $W_0^m(\pi,\alpha)$ with respect to π and evaluating at $\pi=0$ yields

$$\begin{aligned}
\frac{\partial}{\partial\pi}W_0^m(0,\alpha)=\sum_i&\left[u'(G_{i0})\,q\frac{\partial b_{i1}^{er}(0)}{\partial\pi}-\beta u(G_{i1})\frac{\partial b_{i1}^{er}(0)}{\partial\pi}+\right.\\
&\left.+u'(G_{i1})\,q\frac{\partial\mathbf{b}_{i2}}{\partial b_{j1}}\frac{\partial b_{j1}^{er}(0)}{\partial\pi}-\frac{\beta^2}{N}u'(G_{i2})\frac{\partial\mathbf{B}_2}{\partial b_{j1}}\frac{\partial b_{j1}^{er}(0)}{\partial\pi}\right]
\end{aligned}$$

Recall the first order conditions for the local government in periods 1 and 2

$$
\begin{aligned}
u'(G_{i0})\,q &= \beta u'(G_{i1}) + \frac{\beta^2}{N} u'(G_{i2}) \sum_{j \neq i} \frac{\partial \mathbf{b}_{j2}}{\partial b_{i1}} \\
u'(G_{i1})\,q &= \frac{\beta}{N} u'(G_{i2})
\end{aligned}
$$

Substituting these into the previous equation yields

$$
\begin{aligned}
\frac{\partial}{\partial \pi} W_0^m(0,\alpha) = & \sum_i u'(G_{i1})\,q \frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}} \frac{\partial b_{-i1}^{er}(0)}{\partial \pi} \\
& = u(G_{i1})\,q \frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}} \frac{\partial B_1^{er}(0)}{\partial \pi} > 0
\end{aligned}
$$

since at $\pi = 0$, $\frac{\partial}{\partial b_{N1}} \mathbf{b}_{S2}(b_1, 0) = \frac{\partial}{\partial b_{S1}} \mathbf{b}_{N2}(b_1, 0) < 0$ and $\partial B_1^{er}(0)/\partial \pi < 0$. Q.E.D.

2 Data underlying Figure 1

We use two datasets:

1. Dataset used in Kotia and Lledó (2016). They construct an index for the strength of subnational fiscal rules using a database from the European Commission (EC), measuring the strength of all the fiscal rules present in each EU country. The EC dataset includes all types of numerical fiscal rules—budget balance rules, debt rules, expenditure rules, and revenue rules—covering different levels of government—central, regional, and local—in force since 1990 across EU countries. They then weight the scores for the components applicable at the subnational level: regional and local. See Appendix B in Kotia and Lledó (2016) for details about the construction of the index.

   The dataset also contains information on

   (a) subnational primary balances—based on authors' own consolidation of total revenue and expenditures across local and (when applicable) state or regional governments using non-consolidated fiscal data from Eurostat;

   (b) output gap from the World Economic Outlook;

   (c) population above 65 years of age from the World Development Indicators;

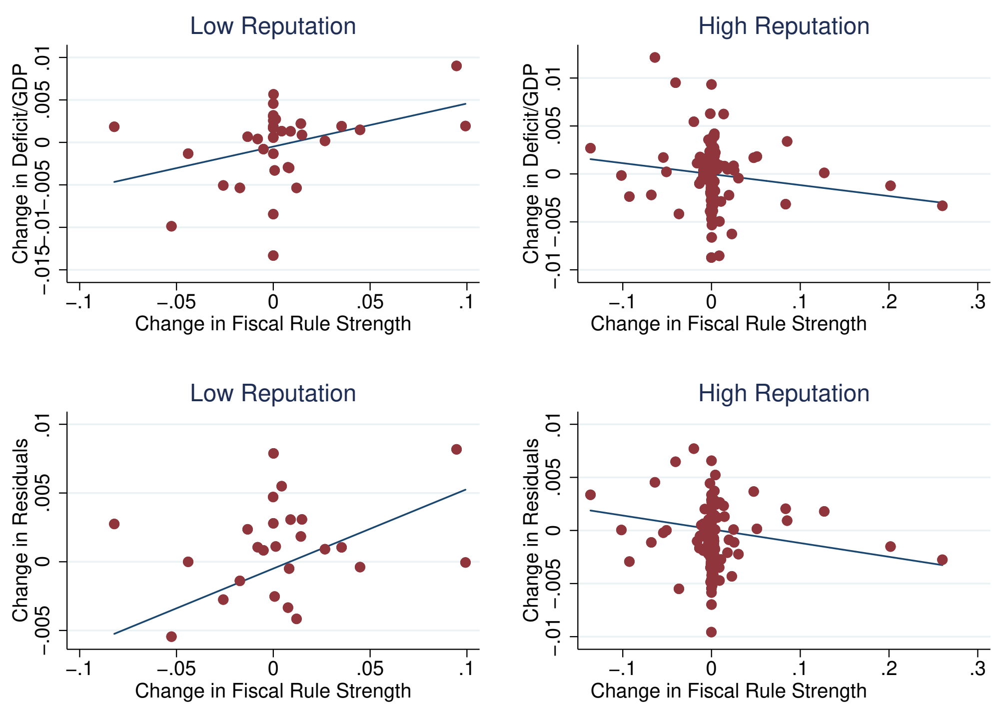
   (d) unemployment from the World Economic Outlook;

(e) legislative election dummy taking the value of 1 if a national legislative election was held in that year, and zero otherwise, from the Database for Political Institutions (DPI).

2. World Bank's Worldwide Governance Indicators (WGI) data. This dataset consists of data on the quality of governance provided by a large number of enterprise, citizen, and expert survey respondents in industrial and developing countries. The WGI consists of aggregate indicators of six broad dimensions of governance: (i) Voice and Accountability, (ii) Political Stability and Absence of Violence/Terrorism, (iii) Government Effectiveness, (iv) Regulatory Quality, (v) Rule of Law, and (vi) Control of Corruption. The governance indicator ranges from around -2.5 to 2.5, with higher values implying better outcomes. The data on government efficiency are biannual from 1996 until 2002 and then annual. We use linear interpolation to add observations in 1997, 1999, and 2001. Our preferred measure of reputation, π, is Government Effectiveness.

In figure 1 we plot the raw data and look at the changes in deficits for contemporaneous changes in fiscal rules.

Figure 1: Scatter plot of changes in primary deficits to changes in fiscal rule strength



In the bottom panels of Figure 1 in the paper and Figure 1, we report the change in residuals after controlling for an estimated fiscal reaction function. In particular, we run the following regression

$$\mathtt{deficit}_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 \mathtt{deficit}_{it-1} + f_i + \varepsilon_{it},$$

where $\mathtt{deficit}_{it}$ is the primary deficit; X_{it} is a vector of control variables (including lags) consisting of output gap, population above 65 years of age, unemployment, legislative election dummy, and inflation; f_i is a country fixed effect; and ε_{it} is the residual from the regression. The figures plot the change in the average residual across two consecutive fiscal rule regimes.

References

KOTIA, A. AND V. D. LLEDÓ (2016): *Do Subnational Fiscal Rules Foster Fiscal Discipline? New Empirical Evidence from Europe*, International Monetary Fund. 18