A1. Proof of Proposition 1

The proof proceeds as follows. First, three lemmas show that no profitable one-shot deviation exists after any history of the game. Then, we show that no one-shot deviation implies that no profitable deviation exists.

Lemma A.1 *There exists no profitable one-shot deviation for the principal.*

PROOF:

First, observe that the principal cannot profitably deviate when allocating the prize. Thus, we only need to consider deviations in instances when the principal chooses whether to stop or continue the contest. If $T < \infty$ there is a final period. In this final period the contest has to end and thus the principal has no action to take. Next, consider any period $t < T$. Suppose an innovation of value $\theta^k \geq \theta^g$ has been submitted to the principal. Stopping yields $\theta^k - p$, whereas continuing yields $-m + \delta(\Delta(\theta^k, n) - p)$. Thus, stopping is optimal whenever

$$m \geq p(1-\delta) + \delta\Delta(\theta^k, n) - \theta^k.$$

Recall that $m = p(1-\delta) + \delta\Delta(\theta^g, n) - \theta^g + \varepsilon$. Simple algebra shows that $\Delta(\theta, n) - \theta$ is strictly decreasing in θ. Thus, the principal will stop the contest if a value $\theta^k \geq \theta^g$ has been submitted.

Suppose now a value $\theta^k < \theta^g$ has been submitted. We will show in three steps that stopping is not optimal. Steps 1 and 2 cover the case when T is finite, while Step 3 deals with the infinite horizon case.

Step 1. Denote with $U_0(\sigma|\theta^k, t)$ the expected utility to the principal of having the highest value θ^k in period t and given a strategy σ. We will prove by induction that $U_0(\sigma|\theta^k, t) > \theta^k - p$, which shows that no profitable one-shot deviation occurs. For the base step, we show that $U_0(\sigma|\theta^k, T-1) > \theta^k - p$. We can write

$$U_0(\sigma|\theta^k, T-1) = -m + \delta\left(F^n(\theta^k)\theta^k + \sum_{j=k+1}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j\right) - \delta p$$

$$= \theta^g - p - \delta(\Delta(\theta^g, n) - \Delta(\theta^k, n)) - \varepsilon. \tag{A1}$$

Thus, $U_0(\sigma|\theta^k, T-1) > \theta^k - p$ if and only if

$$\theta^g - \theta^k - \varepsilon - \delta(\Delta(\theta^g, n) - \Delta(\theta^k, n)) > 0.$$

If $g > k$, then simple algebra gives

$$\Delta(\theta^g, n) - \Delta(\theta^k, n) = \sum_{j=k}^{g-1} F^n(\theta^j)(\theta^{j+1} - \theta^j). \tag{A2}$$

Using this in the inequality above, $U_0(\sigma|\theta^k, T-1) > \theta^k - p$ if and only if

$$\theta^g - \theta^k - \delta \sum_{j=k}^{g-1} F^n(\theta^j)(\theta^{j+1} - \theta^j) - \varepsilon > 0$$
$$\sum_{j=k}^{g-1} (1 - \delta F^n(\theta^j))(\theta^{j+1} - \theta^j) - \varepsilon > 0,$$

which is satisfied whenever

$$(1 - \delta F^n(\theta^{g-1}))(\theta^g - \theta^{g-1}) > \varepsilon.$$

Step 2. To prove the inductive step, we show that if $U_0(\sigma|\theta^k, t+1) > \theta^k - p$ then also $U_0(\sigma|\theta^k, t) > \theta^k - p$. We can write

$$U_0(\sigma|\theta^k, t) =$$
$$-m + \delta \left(F^n(\theta^k) U_0(\sigma|\theta^k, t+1) + \sum_{j=k+1}^{K} (F^n(\theta^j) - F^n(\theta^{j-1})) U_0(\sigma|\theta^j, t+1) \right).$$

Since $U_0(\sigma|\theta^j, t+1) = \theta^j - p$ for all $j \geq g$, and by the previous step $U_0(\sigma|\theta^j, t) > \theta^j - p$ for all $k \leq j < g$, then we can write

$$\begin{aligned} U_0(\sigma|\theta^k, t) &\geq -m + \delta \left(F^n(\theta^k)(\theta^k - p) + \sum_{j=k+1}^{K} (F^n(\theta^j) - F^n(\theta^{j-1}))(\theta^j - p) \right) \\ &= -m - \delta p + \delta \Delta(\theta^k, n) \\ &= \theta^g - p - \delta(\Delta(\theta^g, n) - \Delta(\theta^k, n)) - \varepsilon. \end{aligned}$$

Observe that the last expression is identical as equation (A1) and the proof that $U_0(\sigma|\theta^k, t) > \theta^k - p$ proceeds analogously.

Step 3. In the infinite horizon case, the expected utility when the principal follows σ, given any $\theta^k < \theta^g$ is equal to the value of search. Thus we can write

$$U_0(\sigma|\theta^k) = -m + \delta \left(F^n(\theta^{g-1})U_0(\sigma|\theta^k) + \sum_{j=g}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))(\theta^j - p) \right)$$

$$(1-\delta F^n(\theta^{g-1}))U_0(\sigma|\theta^k) = -m - \delta(1 - F^n(\theta^{g-1}))p + \delta \sum_{j=g}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j$$

$$(1-\delta F^n(\theta^{g-1}))U_0(\sigma|\theta^k) = \theta^g(1-\delta F^n(\theta^{g-1})) - p(1-\delta F^n(\theta^{g-1})) - \varepsilon$$

$$U_0(\sigma|\theta^k) = \theta^g - p - \frac{\varepsilon}{1-\delta F^n(\theta^{g-1})}.$$

Thus, $U_0(\sigma|\theta^k) > \theta^k - p$ for all $k < g$.

Lemma A.2 *There is no profitable one-shot deviation at the submission stage for the agent.*

PROOF:

Observe that submitting an innovation that has value below θ^g is never profitable. Thus we only need to consider the decision of an agent who has an innovation of value $\theta^k \geq \theta^g$. Suppose the state of the world is such that another agent has a value $\theta \geq \theta^k$. Then, submitting yields a weakly higher payoff, as it could mean the agent wins the prize, whereas not submitting yields zero as the contest ends for sure. Finally, suppose the state of the world is such that no other agent has a value $\theta \geq \theta^k$.

We need to consider two cases: when $\theta^k = \theta^K$ and when $\theta^k < \theta^K$. Suppose first that $\theta^k = \theta^K$. The payoff of following the equilibrium strategy and submitting is p. One-shot deviation is to not submit, then not do research, and then submit. The payoff of this deviation is

$$\frac{m}{n} + \delta \mathcal{P}_{t+1}(\sigma'|\theta^K, t)p$$

where $\mathcal{P}_{t+1}(\sigma'|\theta^K, t)$ is the probability that the agent wins the contest in period $t+1$ given that he has the quality θ^K in period t and follows the deviation strategy σ'. The deviation will not be profitable if $p \geq m/n + \delta \mathcal{P}_{t+1}(\sigma'|\theta^K, t)p$. It is sufficient to show that $p \geq m + \delta \mathcal{P}_{t+1}(\sigma'|\theta^K, t)p$. Substituting $m = (1-\delta)p + \delta\Delta(\theta^g, n) - \theta^g + \varepsilon$ and rearranging, this is equivalent to

$$p \geq \frac{\delta\Delta(\theta^g, n) - \theta^g + \varepsilon}{\delta(1 - \mathcal{P}_{t+1}(\sigma'|\theta^K, t))}.$$

The probability of the event that all the opponents obtain θ^K and one of them wins is given by $(1 - F(\theta^{K-1}))^{n-1}(n-1)/n$. Since $\mathcal{P}_{t+1}(\sigma'|\theta^K, t) < 1 - (1 -$

$F(\theta^{K-1}))^{n-1}(n-1)/n$, it is sufficient to show that

$$p \geq \frac{n(\delta\Delta(\theta^g, n) - \theta^g + \varepsilon)}{\delta(n-1)(1 - F(\theta^{K-1}))^{n-1}}.$$

which always holds since $p \geq \bar{p}$.

In the case $\theta^k < \theta^K$ the one-shot deviation is to not submit, invest, and then submit. However, observe that the quality in the next period cannot be greater than θ^K, which implies that the deviation payoff is less than $m/n + \delta(\mathcal{P}_{t+1}(\sigma|\theta^K, t+1)p - C)$. As this is less than in the previous case, this deviation is also not profitable.

Lemma A.3 *There exists no profitable one-shot deviation at the research stage for the agent.*

PROOF:

Suppose that the highest quality agent i has in period t is θ^k. In what follows, we show that for $p \geq \bar{p}$ investing is optimal for all $\theta^k < \theta^K$ and it is not optimal for $\theta^k = \theta^K$. Let σ' be a strategy profile that coincides with the equilibrium candidate σ with the exception of the agent i's action in the investment stage in period t. Thus, it is a one-shot deviation. First note that a deviation in the case $\theta^k = \theta^K$ would imply investing when the agent has the highest feasible quality. This is trivially never optimal, as the agent incurs research costs without an increase in quality. Thus, focus on the case $\theta^k < \theta^K$ where a deviation is to not invest.

Denote the expected utility of agent i following the strategy σ from period t in which his highest quality is θ^k with $U_i(\sigma|\theta^k, t)$. A one-shot deviation is not profitable if

$$U_i(\sigma|\theta^k, t) - U_i(\sigma'|\theta^k, t) \geq 0. \tag{A3}$$

As before, let $\mathcal{P}_s(\sigma|\theta^k, t)$ be the probability that the agent i wins the contest in period $s \geq t$, following the strategy σ from period t in which the highest quality was θ^k.

First consider the case $\theta^g \leq \theta^k < \theta^K$. In this case, the game will end with certainty in period t and the LHS of inequality (A3) reads

$$U_i(\sigma|\theta^k, t) - U_i(\sigma'|\theta^k, t) = -C + p(\mathcal{P}_t(\sigma|\theta^k, t) - \mathcal{P}_t(\sigma'|\theta^k, t)).$$

Due to perfect recall, in any state of the world in which the agent i wins following the strategy σ', he also wins following the strategy σ. Following strategy σ', agent i has a zero probability of winning if all the opponents have θ^K, while that probability is positive if he follows σ. The event that all the opponents draw θ^K and agent i following the strategy σ wins, happens with probability of at least

$(1-F(\theta^{K-1}))^n/n$. Thus, to show that the inequality (A3) holds, it is sufficient to show that

$$\text{(A4)} \qquad -C + p\frac{(1-F(\theta^{K-1}))^n}{n} \geq 0$$
$$p \geq \frac{nC}{(1-F(\theta^{K-1}))^n}$$

which always holds since $p \geq \bar{p}$.

The only remaining case is $\theta^k < \theta^g$, which we now consider. In this case, the agent i could not have observed a deviation by the principal, so he believes all of his opponents have values below θ^g. First suppose that $t = T$ so that the contest ends with certainty in period t. Then the analysis above applies. Next, suppose that $t < T$. Since σ and σ' coincide after t, that we can write the expected utilities in the following way:

$$U_i(\sigma'|\theta^k, t) = F^{n-1}(\theta^{g-1})\left[\frac{m}{n} + \delta U_i(\sigma|\theta^k, t+1)\right]$$
$$U_i(\sigma|\theta^k, t) = p\mathcal{P}_t(\sigma|\theta^k, t) + F(\theta^k)F^{n-1}(\theta^{g-1})\left[\frac{m}{n} + \delta U_i(\sigma|\theta^k, t+1)\right]$$
$$+ \sum_{j=k+1}^{g-1}\left(F(\theta^j) - F(\theta^{j-1})\right)F^{n-1}(\theta^{g-1})\left[\frac{m}{n} + \delta U_i(\sigma|\theta^j, t+1)\right] - C.$$

Since $U_i(\sigma|\theta^j, t+1) \geq U_i(\sigma|\theta^k, t+1)$ for all $\theta^j > \theta^k$, we can write

$$U_i(\sigma|\theta^k, t) \geq p\mathcal{P}_t(\sigma|\theta^k, t) + F(\theta^{g-1})F^{n-1}(\theta^{g-1})\left[\frac{m}{n} + \delta U_i(\sigma|\theta^k, t+1)\right] - C.$$

Furthermore, since $\mathcal{P}_t(\sigma|\theta^k, t) \geq (1-F(\theta^{g-1}))F^{n-1}(\theta^{g-1}) + (1-F(\theta^{K-1}))^n/n$ and $p \geq m/n + \delta U_i(\sigma|\theta^k, t+1)$ by Lemma A.2, we can write

$$U_i(\sigma|\theta^k, t) \geq \frac{(1-F(\theta^{K-1}))^n}{n}p + F^{n-1}(\theta^{g-1})\left[\frac{m}{n} + \delta U_i(\sigma|\theta^k, t+1)\right] - C.$$

Then,

$$U_i(\sigma|\theta^k, t) - U_i(\sigma'|\theta^k, t) \geq \frac{(1-F(\theta^{K-1}))^n}{n}p - C \geq 0$$

which holds by the same argument as for Inequality (A4).

We conclude the proof by showing that since no one-shot deviation exists, then no profitable deviation exists at all. First, observe that if T is finite, then the result follows by Theorem 1 of Hendon, Jacobsen and Sloth (1996). If T is infinite, then the game is continuous at infinity and the result follows by Corollary 2 of

Hendon, Jacobsen and Sloth (1996).

A2. *Proof of Proposition 2*

The claim follows from the lemmas below. Throughout the proof, we will again use $\mathcal{P}(\sigma|\theta_i^{max}, t)$ to denote the probability that the agent i wins the contest in period t, given strategy σ and his current highest value θ_i^{max}. Similarly, $\mathcal{P}^c(\sigma|\theta_i^{max}, t)$ will denote the probability that the contest continues to the next period. We will use $U_i(\sigma|\theta_i^{max}, t)$ to denote the expected payoff of player i (agent or the principal). We will denote the threshold of the principal's strategy with θ^s. The principal's strategy is denoted with $\sigma_0(\cdot;\theta^s)$. We will omit θ^s and write just $\sigma_0(\cdot)$ when there is no risk of confusion.

Lemma A.4 *In any PBE,* $\sigma_0(\theta^K) = NCont$.

PROOF:

Suppose not. We show that $NCont$ is a profitable one-shot deviation. We can write

$$U_0(NCont, \sigma|\theta^K) = \theta^K - p.$$

The principal's expected utility after $Cont$, given a strategy that terminates the contest in $\bar{t} \leq \infty$ periods, is

$$U_0(Cont, \sigma|\theta^K) = \delta^{\bar{t}}(\theta^K - p) - \sum_{t=0}^{\bar{t}-1} \delta^t m.$$

Then,

$$\begin{aligned} U_0(NCont, \sigma|\theta^K) - U_0(Cont, \sigma|\theta^K) &= \left(1 - \delta^{\bar{t}}\right)\left(\theta^K - p\right) + \sum_{t=0}^{\bar{t}-1} \delta^t m \\ &= \frac{1-\delta^{\bar{t}}}{1-\delta}\left((1-\delta)\theta^K + \delta\Delta(\theta^g, n) - \theta^g + \varepsilon\right) \\ &\geq \frac{1-\delta^{\bar{t}}}{1-\delta}\left(\theta^K - \theta^g + \varepsilon\right) > 0. \end{aligned}$$

Lemma A.5 *At any submission stage and for any* $\theta_i^{max} \in \Theta$

$$\mathcal{P}(S, \sigma|\theta_i^{max}, t) + \mathcal{P}^c(S, \sigma|\theta_i^{max}, t) \geq \mathcal{P}(NS, \sigma|\theta_i^{max}, t) + \mathcal{P}^c(NS, \sigma|\theta_i^{max}, t).$$

PROOF:

If $t = T$ or $\theta^s = \theta^0$, then the contest ends for sure in the current period. Since the probability of winning in case the contest ends is minimized after

NS, submitting can only weakly increase the probability of winning. Next, consider $t < T$ or $\theta^s > \theta^0$. Then $\mathcal{P}(NS, \sigma|\theta_i^{max}, t) = 0$ and following NS by agent i, the contest continues only if all agents either do not submit or submit a value below θ^s. If $\theta_i^{max} < \theta^s$ then the contest continues in all the same states of the world as with NS. Hence $\mathcal{P}^c(S, \sigma|\theta_i^{max}, t) = \mathcal{P}^c(NS, \sigma|\theta_i^{max}, t)$ and $\mathcal{P}(S, \sigma|\theta_i^{max}, t) = \mathcal{P}(NS, \sigma|\theta_i^{max}, t) = 0$. If $\theta_i^{max} \geq \theta^s$ then the agent wins the contest in all the states of the world in which the contest would continue after NS. Hence, $\mathcal{P}(S, \sigma|\theta_i^{max}, t) \geq \mathcal{P}^c(NS, \sigma|\theta_i^{max}, t)$ and since $\mathcal{P}(NS, \sigma|\theta_i^{max}, t) = 0$ the conclusion follows.

Lemma A.6 *In any ITC, for any σ, θ_i^{max} and t,*

$$p - (m/n + \delta U_i(\sigma|\theta_i^{max}, t+1)) \geq \frac{(n-1)(1-\delta)p - \delta\Delta(\theta^g, n) + \theta^g - \varepsilon}{n} > 0.$$

PROOF:

If the contest terminates in $\bar{t}$ periods, where $1 \leq \bar{t} \leq \infty$, then

$$\sum_{t=0}^{\bar{t}-1} \delta^t \frac{m}{n} + \delta^{\bar{t}} p \geq \frac{m}{n} + \delta U_i(\sigma|\theta_i^{max}, t+1)$$

which implies

$$\begin{aligned} p - (m/n + \delta U_i(\sigma|\theta_i^{max}, t+1)) \geq & \; p - \left(\sum_{t=0}^{\bar{t}-1} \delta^t \frac{m}{n} + \delta^{\bar{t}} p\right) \\ = & (1-\delta^{\bar{t}}) \left(\frac{(1-\delta)(n-1)p - \delta\Delta(\theta^g, n) + \theta^g - \varepsilon}{(1-\delta)n}\right) \\ \geq & \frac{(1-\delta)(n-1)p - \delta\Delta(\theta^g, n) + \theta^g - \varepsilon}{n}. \end{aligned}$$

Finally, note that

$$p > \bar{\bar{p}} \geq \frac{\delta\Delta(\theta^g, n) - \theta^g + \varepsilon}{(1-\delta)(n-1)},$$

which implies

$$\frac{(1-\delta)(n-1)p - \delta\Delta(\theta^g, n) + \theta^g - \varepsilon}{n} > 0.$$

Lemma A.7 *If σ is a PBE and $\sigma_0(\cdot; \theta^s)$ is a threshold strategy, then for each agent i who has not observed a deviation by the principal $\sigma_i(\theta_i^{max}) = S$ for all $\theta_i^{max} \geq \theta^s > \theta^0$.*

PROOF:

Suppose not. Then there exists some history, some agent i and some $\theta_i^{max} \geq \theta^s > \theta^0$, such that $\sigma_i(\theta_i^{max}) = NS$ in a PBE. We will show that S is a profitable deviation. If $\theta_i^{max} \geq \theta^s$ then

$$U_i(S, \sigma|\theta_i^{max}, t) = \mathcal{P}(S, \sigma|\theta_i^{max}, t)p$$

and

$$\begin{aligned} U_i(NS, \sigma|\theta_i^{max}, t) = \mathcal{P}(NS, \sigma|\theta_i^{max}, t)p \\ + \mathcal{P}^c(NS, \sigma|\theta_i^{max}, t)\left(\frac{m}{n} + \delta U_i(\sigma|\theta_i^{max}, t+1)\right). \end{aligned}$$

Since the agent has not observed a deviation by the principal, then his beliefs are formed by Bayesian updating. Given that $\theta_i^{max} > \theta^0$, then $\mathcal{P}(S, \sigma|\theta_i^{max}, t) > \mathcal{P}(NS, \sigma|\theta_i^{max}, t)$. Next, by Lemma A.5, $\mathcal{P}(S, \sigma|\theta_i^{max}, t) \geq \mathcal{P}(NS, \sigma|\theta_i^{max}, t) + \mathcal{P}^c(NS, \sigma|\theta_i^{max}, t)$ and by Lemma A.6, $p > (m/n + \delta U_i(\sigma|\theta_i^{max}, t+1))$. Thus $U_i(S, \sigma|\theta_i^{max}, t) > U_i(NS, \sigma|\theta_i^{max}, t)$.

Lemma A.8 *If σ is a PBE and $\sigma_0(\cdot; \theta^s)$ is a threshold strategy, then $\sigma_i(\theta_i^{max}) = I$ for all $\theta_i^{max} < \theta^s$ and for each agent i.*

PROOF:

Suppose not. Then there exists some history, some agent i and some $\theta_i^{max} < \theta^s$, such that $\sigma_i(\theta_i^{max}) = NI$ in a PBE. We will show that I is a profitable one-shot deviation. If $t < T$, we can write

$$\begin{aligned} U_i(I, \sigma|\theta_i^{max}, t) = -C + \mathcal{P}(I, \sigma|\theta_i^{max}, t)p \\ + \mathcal{P}^c(I, \sigma|\theta_i^{max}, t)\left(\frac{m}{n} + \delta \mathbb{E}_{\theta_i^{max}} U_i(\sigma|\theta_i^{max}, t+1)\right) \end{aligned}$$

and

$$U_i(NI, \sigma|\theta_i^{max}, t) = \mathcal{P}^c(NI, \sigma|\theta_i^{max}, t)\left(\frac{m}{n} + \delta U_i(\sigma|\theta_i^{max}, t+1)\right).$$

In every state of the world in which the agent i loses the contest in period t following the action I, he also loses after the action NI. Thus

$$\mathcal{P}(I, \sigma|\theta_i^{max}, t) + \mathcal{P}^c(I, \sigma|\theta_i^{max}, t) \geq \mathcal{P}^c(NI, \sigma|\theta_i^{max}, t).$$

By Lemma A.4, the principal stops the contest if $\theta_0^{max} = \theta^K$. Thus,

$$\mathcal{P}(I, \sigma|\theta_i^{max}, t) \geq \frac{1 - F(\theta^{K-1})}{n}.$$

Furthermore, since U_i is non-decreasing in θ_i^{max} then

$$\begin{aligned} U_i(I,\sigma|\theta_i^{max},t) - U_i(NI,\sigma|\theta_i^{max},t) \\ &\geq -C + \frac{1-F(\theta^{K-1})}{n}\left(p - \frac{m}{n} - \delta U_i(\sigma|\theta_i^{max},t+1)\right) \\ &\geq \frac{1-F(\theta^{K-1})}{n}\left(\frac{(n-1)(1-\delta)p - \delta\Delta(\theta^g,n) + \theta^g - \varepsilon}{n}\right) - C > 0 \end{aligned}$$

where the second inequality follows from Lemma A.6 and the third inequality follows from

$$p > \bar{\bar{p}} \geq \frac{n^2C + (\delta\Delta(\theta^g,n) - \theta^g + \varepsilon)\left(1-F(\theta^{K-1})\right)}{(1-\delta)(n-1)\left(1-F(\theta^{K-1})\right)}.$$

If $t = T$, then the contest ends for sure. The proof is analogous to the proof of Lemma A.9 below, and is therefore omitted.

Lemma A.9 *If σ is a PBE and $\sigma_0(\cdot;\theta^s)$ is a threshold strategy with $\theta^s < \theta^K$, then $\sigma_i(\theta^s) = I$ for each agent i.*

PROOF:

Suppose not. Then there exists some history and some agent i, such that $\sigma_i(\theta^s) = NI$ in a PBE. We will show that I is a profitable one-shot deviation. Since $\theta_i^{max} \geq \theta^s$ and the principal follows a threshold strategy, the contest ends in period t for sure. Then, we can write

$$\begin{aligned} U_i(I,\sigma|\theta_i^{max},t) &= -C + \mathcal{P}(I,\sigma|\theta_i^{max},t)p \\ U_i(NI,\sigma|\theta_i^{max},t) &= \mathcal{P}(NI,\sigma|\theta_i^{max},t)p. \end{aligned}$$

Observe that in every state of the world in which agent i wins following NI, he also wins following I. We now show that, whatever the beliefs of agent i, and for any strategy profile of his opponents and the principal which are compatible with PBE, his probability of winning increases by at least $(1-F(\theta^{K-1}))^n/(2n)$ after action I.

At any state of the world, one of the three following cases holds.

- Case 1: $\max_{j\neq i}\theta_j^{max} > \theta_i^{max}$. In this case, agent i loses the contest for sure after NI, while his winning probability is at least $(1-F(\theta^{K-1}))/n$ after I.
- Case 2: $\max_{j\neq i}\theta_j^{max} < \theta_i^{max}$. In this case, all opponents have a value strictly below θ^s, so by Lemma A.8 they all invest in the current period. Thus, agent i's increase in the probability of winning is at least $(1-F(\theta^{K-1}))^n/n$.
- Case 3: $\max_{j\neq i}\theta_j^{max} = \theta_i^{max}$. In this case, agent i loses with probability at least 1/2. If he invested, his probability of winning would increase by at least $(1-F(\theta^{K-1}))/(2n)$.

Thus, in any of the three possible cases, agent i's probability of winning increases by at least $(1 - F(\theta^{K-1}))^n/(2n)$ after action I. Then we can write

$$U_i(I, \sigma|\theta_i^{max}, t) - U_i(NI, \sigma|\theta_i^{max}, t) \geq -C + p\frac{(1 - F(\theta^{K-1}))^n}{2n} > 0,$$

where the last inequality holds from

$$p > \bar{\bar{p}} \geq \bar{p} \geq \frac{2nc}{(1 - F(\theta^{K-1}))^n}.$$

Lemma A.10 *If σ is a PBE and $\sigma_0(\cdot;\theta^s)$ is a threshold strategy, then $\theta^s = \theta^g$.*

PROOF:

From Proposition 1, we know that $\theta^s = \theta^g$ is an equilibrium. Here we show that $\theta^s \neq \theta^g$ is never an equilibrium. Suppose not. Then there exists a PBE, such that $\sigma_0(\cdot;\theta^s)$ is a threshold strategy and either $\theta^s > \theta^g$ or $\theta^s < \theta^g$. The proof follows in three steps. Steps 1 and 2 show that there exists a profitable one-shot deviation when $\theta^s > \theta^g$. Step 3 shows the same when $\theta^s < \theta^g$.

Step 1. Suppose that T is infinite and $\theta^s > \theta^g$. We will show that there exists a profitable one-shot deviation.

Suppose that $\theta_0^{max} = \theta^{s-1}$. The payoff from deviating to $NCont$ is $\theta^{s-1} - p \geq \theta^g - p$. The principal's expected utility from following $\sigma_0(\cdot;\theta^s)$ is

$$U_0(Cont, \sigma|\theta^{s-1}) = -m + \delta\left(F^n(\theta^{s-1})U_0(Cont, \sigma|\theta^{s-1}) + \sum_{j=s}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))(\theta^j - p)\right)$$

$$U_0(Cont, \sigma|\theta^{s-1}) = \frac{-m - \delta(1 - F^n(\theta^{s-1}))p + \delta\left(\sum_{j=s}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j\right)}{1 - \delta F^n(\theta^{s-1})}$$

The one-shot deviation to $NCont$ is profitable if

$$\theta^g - p > \frac{-m - \delta(1 - F^n(\theta^{s-1}))p + \delta\left(\sum_{j=s}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j\right)}{1 - \delta F^n(\theta^{s-1})}$$

which is equivalent to

$$\begin{aligned}
0 &> -\delta\Delta(\theta^g, n) + \delta F^n(\theta^{s-1})\theta^g - \varepsilon + \delta\left(\sum_{j=s}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j\right) \\
&\geq (F^n(\theta^{s-1}) - F^n(\theta^g))\theta^g - \sum_{j=g+1}^{s-1}\left(F^n(\theta^j) - F^n(\theta^{j-1})\right)\theta^j \\
&\geq (F^n(\theta^{s-1}) - F^n(\theta^g))\theta^g - \left(F^n(\theta^{s-1}) - F^n(\theta^g)\right)\theta^g
\end{aligned}$$

which is always satisfied.

Step 2. Suppose that T is finite and $\theta^s > \theta^g$. We will show that there exists a profitable one-shot deviation when $t = T-1$ and $\theta_0^{max} = \theta^g$.

In this case, the payoff from deviating to $NCont$ is $\theta^g - p$. Now, the contest ends in the next period, so that the principal's expected utility from following $\sigma_0(\cdot;\theta^s)$ is

$$U_0(Cont, \sigma|\theta^g, t) = -m - \delta p + \delta\left(F^n(\theta^g)\theta^g + \sum_{j=g+1}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j\right).$$

The one-shot deviation to $NCont$ is profitable if

$$\theta^g - p > -m - \delta p + \delta\left(F^n(\theta^g)\theta^g + \sum_{j=g+1}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j\right)$$

which is equivalent to

$$\begin{aligned}
-\delta F^n(\theta^g)\theta^g - \delta\sum_{j=g+1}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j + \delta\Delta(\theta^g, n) + \varepsilon > 0 \\
-\delta\Delta(\theta^g, n) + \delta\Delta(\theta^g, n) + \varepsilon > 0.
\end{aligned}$$

Step 3. Suppose that $\theta^s < \theta^g$. We will show that there exists a profitable one-shot deviation when $\theta_0^{max} = \theta^s$.

Following the strategy $\sigma_0(\cdot;\theta^s)$ yields $\theta^s - p$. Consider a one-shot deviation to $Cont$. By Lemma A.9, all agents will do research, so the principal's expected utility is

$$U_0(Cont, \sigma|\theta^s, t) = -m - \delta p + \delta\left(F^n(\theta^s)\theta^s + \sum_{j=s+1}^{K}(F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j\right)$$

The deviation is profitable if

$$-m - \delta p + \delta \left(F^n(\theta^s)\theta^s + \sum_{j=s+1}^{K} (F^n(\theta^j) - F^n(\theta^{j-1}))\theta^j \right) > \theta^s - p$$
$$\theta^g - \theta^s - \delta(\Delta(\theta^g, n) - \Delta(\theta^s, n)) - \varepsilon > 0$$

Using equation (A2), a sufficient condition for the above inequality to hold is

$$\sum_{j=s}^{g-1}(\theta^{j+1} - \theta^j) - \delta \sum_{j=s}^{g-1} F^n(\theta^j) \left(\theta^{j+1} - \theta^j\right) > \varepsilon$$
$$\sum_{j=s}^{g-1}(1 - \delta F^n(\theta^j)) \left(\theta^{j+1} - \theta^j\right) > \varepsilon$$

which is always satisfied.

A3. Proof of Proposition 3

The first-best problem corresponds to the optimal search problem in Benkert, Letina and Nöldeke (2018). By Proposition 1 of Benkert, Letina and Nöldeke (2018) the first-best is to draw n^{FB} observations in each period until the value of at least θ^{FB} has been discovered. By Proposition 1, we know that there exists an ITC which can implement the global stopping threshold θ^g_N with n^{FB}_N and $T = \infty$, thus generating the first-best surplus. Then, by setting E appropriately, the principal extracts the entire expected surplus and achieves the first-best outcome.

A4. Proof of Proposition 4

The game induced by a gITC and the equilibrium candidate are analogous to those described above. The proof proceed in similar fashion. We first characterize the optimal sequence of agents $\mathbf{n}^{FB}$. Next, we show that no profitable one-shot deviation exists. Finally, since the game is finite and no profitable one-shot deviation exists, Theorem 1 of Hendon, Jacobsen and Sloth (1996) implies that no profitable deviation exists at all. We assume that agents who have not entered the contest cannot exert effort.

Lemma A.11 *Given Assumption 1, $n^{FB}(0, t) \leq n^{FB}(0, t+1)$ and $n^{FB}(\theta, t) = 0$ for all $\theta \geq \theta^b$ and $t \leq T$.*

PROOF:

It is straightforward that for any quality level $\theta^j \geq \theta^b$ the principal will stop doing research because $\theta^K - \theta^b < C$ and thus the cost of doing more research strictly outweighs the potential benefit. Thus, whenever the principal continues searching, she has a current highest quality of innovation of 0. Hence, the problem is as if the principal had no recall. Proposition 3 in Gal, Landsberger and Levykson (1981), which can be adapted to the current setting with discounting and a discrete set of innovation levels, then implies that the principal will want to employ an increasing number of agents as the deadline draws nearer.

Lemma A.12 *There exists no profitable one-shot deviation for the principal.*

PROOF:
In the final period the principal has no decision to make. Thus we only need to consider deviations in periods $t < T$. Suppose an innovation of value $\theta^k \geq \theta^b$ has been submitted to the principal. Stopping yields $\theta^k - p$, whereas continuing yields $-m_t + \delta(\Delta(\theta^k, n_{t+1}) - p + \mathcal{E}_{t+1})$, where $\mathcal{E}_{t+1} = (n_{t+1} - n_t)E_{t+1}$ is the sum of entry fees received by the principal in period $t+1$. Thus, stopping is optimal whenever

$$m_t \geq p(1-\delta) + \delta(\Delta(\theta^k, n_{t+1}) + \mathcal{E}_{t+1}) - \theta^k.$$

Recall that $m_t = p(1-\delta) + \delta(\Delta(\theta^b, n_{t+1}) + \mathcal{E}_{t+1}) - \theta^g$. Since $\Delta(\theta, n_{t+1}) - \theta$ is strictly decreasing in θ, the principal will stop the contest whenever a value $\theta^k \geq \theta^b$ has been submitted.

Suppose now a value $\theta^k < \theta^b$ has been submitted. Assumption 1 then implies $\theta^k = 0$. The payoff of stopping is $-p$, and continuing (since it constitutes the first best) always has a positive payoff. Hence, stopping is never optimal.

Lemma A.13 *There exists no profitable one-shot deviation at either the submission or the research stage for the agent.*

PROOF:
The proofs are analogous to the proofs of Lemma A.2 and Lemma A.3.