1. Introduction

In this online appendix, I provide some proofs that were omitted from Lagerlöf (2020). In particular, I here show the calculations that were used for Figures 2, 4, and 5 of that paper.

2. Proofs of Results Not Proven in the Paper

2.1. Calculations Used for Figure 2

Assume a CES production function, a CSF of the generalized Tullock form (as in eq. (9) in Lagerlöf, 2020), and that \( t = 1 \) and \( r \leq 1 \). Under these assumptions, condition (i) in Assumption 1 is satisfied for all \( \sigma \leq 1 \). Thus suppose that \( \sigma > 1 \). Table 1 in Lagerlöf (2020) tells us that, under the stated assumptions, \( \eta \left( \frac{1}{p_i} \right) = (\frac{\alpha}{1-\alpha})^\sigma p_i^{\sigma-1} / \left[ (\frac{\alpha}{1-\alpha})^\sigma p_i^{\sigma-1} + 1 \right] \). For \( \sigma > 1 \), this expression is strictly increasing in \( p_i \). Therefore, since \( p_i \leq 1 \), an upper bound on \( \eta \left( \frac{1}{p_i} \right) \) is given by \( (\frac{\alpha}{1-\alpha})^\sigma / \left[ (\frac{\alpha}{1-\alpha})^\sigma + 1 \right] \). It follows that condition (i) in Assumption 1 (i.e., \( r \eta \left( \frac{1}{p_i} \right) \sigma \leq 2 \)) is satisfied for all \( p_i \in [0,1] \) if

\[
\frac{r}{\left( \frac{\alpha}{1-\alpha} \right)^\sigma} \sigma \leq 2 \iff (r\sigma - 2) \left( \frac{\alpha}{1-\alpha} \right)^\sigma \leq 2.
\]
This inequality is satisfied for all $\sigma \leq 2/r$. Suppose $\sigma > 2/r$. Then the inequality can be rewritten as

$$
\alpha \leq \frac{\left(\frac{1}{\alpha^2} - 2\right)^\frac{1}{2}}{1 + \left(\frac{1}{\alpha^2} - 2\right)^\frac{1}{2}} \equiv \Theta(\sigma, r).
$$

This is the function that is graphed in Figure 2 in Lagerlöf (2020). Note that the derivative of $\Theta(\sigma, r)$ has the same sign as the derivative of $\frac{1}{\sigma} \ln(2 - \ln(r\sigma - 2))$. Differentiating the latter expression with respect to $\sigma$ yields

$$
\frac{\ln(r\sigma - 2) - \ln(2 - \frac{r\sigma}{\sigma - 2})}{\sigma^2},
$$

(S1)

which clearly is negative for all $r\sigma \leq 4$. Moreover, the numerator in (S1) is increasing in $\sigma$ and for sufficiently large values of $\sigma$ the numerator is positive. Thus, for all $\sigma \leq 4/r$, $\Theta(\sigma, r)$ is downward-sloping and there is a unique $\sigma$, such that $\sigma > 4/r$, for which $\Theta(\sigma, r)$ is minimized. This value of $\sigma$, which I denote by $\sigma = \sigma^*$, is characterized by $\ln(2\sigma^* - 2) - \ln(2 - \frac{r\sigma^*}{\sigma^* - 2}) = 0$. The values of $\sigma^*$ shown in the table in Figure 2 in Lagerlöf (2020) are obtained by, using Maple, solving this equation for different $r$ values. The table also shows the associated minimized values of $\Theta(\sigma, r)$, denoted by $\alpha^* = \Theta(\sigma^*, r)$. □

2.2. Calculations Used for Figure 4

In Figure 4 in Lagerlöf (2020) there are two graphs that indicate the part of the parameter space where $R^H$ is decreasing in $n$ (at $n = 10$). I here describe how these graphs were obtained. By assuming a CES production function (which implies $h(n) = \left(\frac{\alpha}{1 - \alpha}n\right)^\sigma$) and by setting $t = r = 1$, we can write

$$
h(n) > 2^\frac{1}{2}n \iff \left(\frac{\alpha}{1 - \alpha}\right)^\sigma > \frac{(n - 1)^2(\sigma - 1) - 2n}{2n^2} - \frac{1}{2n} \sqrt{(n - 1)^2(\sigma - 1) - 2n}^2 - 4 \iff
$$

$$
\left(\frac{\alpha}{1 - \alpha}\right)^\sigma > \frac{(n - 1)^2(\sigma - 1) - 2n - \sqrt{(n - 1)^2(\sigma - 1) - 2n}^2 - 4n^2}{2n^2 - \sigma} \iff
$$

$$
\frac{(n - 1)^2(\sigma - 1) - 2n + \sqrt{(n - 1)^2(\sigma - 1) - 2n}^2 - 4n^2}{2n^2 - \sigma} \iff
$$

$$
\frac{2n^\sigma}{(n - 1)^2(\sigma - 1) - 2n + \sqrt{(n - 1)^2(\sigma - 1) - 2n}^2 - 4n^2} \iff
$$

$$
\frac{\alpha}{1 - \alpha} > \sqrt{\frac{2^\frac{1}{2}n}{(n - 1)^2(\sigma - 1) - 2n + \sqrt{(n - 1)^2(\sigma - 1) - 2n}^2 - 4n^2}} \iff
$$

$$
\alpha > \frac{2^\frac{1}{2}n}{2^\frac{1}{2}n + (n - 1)^2(\sigma - 1) - 2n + \sqrt{(n - 1)^2(\sigma - 1) - 2n}^2 - 4n^2} \iff
$$

(S2)
values of the right-hand sides of (S2) and (S3), evaluated at principle, be plotted with the help of some appropriate software. However, I have instead computed associated pairs of 

Recall from the proof of Proposition 10 in Lagerlöf (2020) that

Similarly we can write

\[
\begin{align*}
\hat{p}_1 &= 0.50 .517 .528 .542 .550 .555 .559 .562 .564 .565 .567 .573 .577 .578 .580 \\
\hat{w}_1 &= 1 .743 .599 .436 .344 .285 .243 .212 .188 .169 .153 .080 .033 .026 .013 \\
\hat{v}_1 &= 1 .704 .547 .381 .294 .239 .202 .174 .153 .137 .064 .036 .026 .013 .011 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \hat{p}_1 )</th>
<th>( \hat{w}_1 )</th>
<th>( \hat{v}_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.500</td>
<td>.517</td>
<td>.528</td>
</tr>
<tr>
<td>.5</td>
<td>.500</td>
<td>.510</td>
<td>.517</td>
</tr>
<tr>
<td>.9</td>
<td>.500</td>
<td>.502</td>
<td>.504</td>
</tr>
</tbody>
</table>

Table 1: Computed values of \( \hat{p}_1 \) and \( \hat{w}_1 \) used in Figure 5 of Lagerlöf (2020).

Similarly we can write

\[
\begin{align*}
\alpha < \frac{(n-1)^2(\sigma-1) - 2n + \sqrt{[(n-1)^2(\sigma-1) - 2n]^2 - 4n^2}}{2n^2 - \sigma} \\
\alpha < \frac{[n-1]^2(\sigma-1) - 2n - \sqrt{[(n-1)^2(\sigma-1) - 2n]^2 - 4n^2}}{2n^2 - \sigma} \\
\end{align*}
\]

The expressions in (S2) and (S3) are then evaluated at \( n = 10 \). The resulting expressions can then, in principle, be plotted with the help of some appropriate software. However, I have instead computed values of the right-hand sides of (S2) and (S3), evaluated at \( n = 10 \) and different \( \sigma \)'s. Then I plotted the associated pairs of \((\sigma, \alpha)\) using the \LaTeX\ package \texttt{TikZ}.

**2.3. Calculations Used for Figure 5**

Recall from the proof of Proposition 10 in Lagerlöf (2020) that \( \hat{p} \) is characterized by \( F(\hat{p}_1) = 0 \), where

\[
F(p_1) = \frac{(1 - 2p_1)(r\beta + 1)}{p_1(1 - p_1)} + \frac{r\beta(v_1 - v_2)}{p_1v_1 + (1 - p_1)v_2 + v_1 + v_2}.
\]
Also recall that \( \hat{w}_1 \) is given by
\[
\hat{w}_1 = w_2 \left( \frac{\hat{p}_1}{1 - \hat{p}_1} \right)^{1+r\beta} \left( \frac{r\beta (1 - \hat{p}_1) + 1}{r\beta \hat{p}_1 + 1} \right)^{rt}.
\]

Now set \( r = t = v_2 = 1 \). Moreover, to start with, assume \( \alpha = \beta = \frac{1}{2} \). We then get
\[
F(p_1) = \frac{(1 - 2p_1) \left( \frac{1}{2} + 1 \right)}{p_1 (1 - p_1) \left( \frac{1}{2} p_1 (1 - p_1) + \frac{1}{2} + 1 \right)} + \frac{\frac{1}{2} (v_1 - 1)}{2 (p_1 v_1 + 1 - p_1) + v_1 + 1}
\]  
(S4)

and
\[
\hat{w}_1 = w_2 \left( \frac{\hat{p}_1}{1 - \hat{p}_1} \right)^{\frac{3}{2}} \left[ \frac{1}{2} (1 - \hat{p}_1) + 1 \right] \frac{1}{v_1} = w_2 \left( \frac{\hat{p}_1}{1 - \hat{p}_1} \right)^{\frac{3}{2}} \left[ \frac{3 - \hat{p}_1}{2 + \hat{p}_1} \right].
\]  
(S5)

By using Maple and the expression in (S4), the equality \( F(\hat{p}_1) = 0 \) can be solved for \( \hat{p}_1 \), given various values of \( v_1 \). Thereafter, by plugging \( \hat{p}_1 \) into (S5), we can compute \( \hat{w}_1 \). Doing this yields the numbers in rows 3 and 4 (i.e., the ones for \( \alpha = 0.5 \)) of Table 1 in the present document. The numbers for \( \alpha = 0.1 \) and \( \alpha = 0.9 \) are obtained similarly.

References