A Bias of Screening

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Online Appendix

Lemma 2. *Every unbounded finite-expectation impact variable has screening biases.*

Proof. Assume that V is unbounded from above. Without loss of generality, fix $\mathbb{E}[V] = 0$ and take some large $n > 0$. Consider a noise variable N where $N \in \{0, n, -n\}$ with equal probabilities. Clearly,

$$\mathbb{E}[V|V + N \geqslant b] = \frac{\mathbb{E}\left[V\mathbb{1}_{\{V \geqslant b+n\}}\right] + \mathbb{E}\left[V\mathbb{1}_{\{V \geqslant b\}}\right] + \mathbb{E}\left[V\mathbb{1}_{\{V \geqslant b-n\}}\right]}{\Pr(V \geqslant b + n) + \Pr(V \geqslant b) + \Pr(V \geqslant b - n)}.$$

If $b = \sqrt{n}$ and n tends to infinity, then $\mathbb{E}[V|V + N \geqslant \sqrt{n}] \to E[V]$. However, if $b = 0$ and n tends to infinity, we get

$$\mathbb{E}[V|V + N \geqslant 0] \to \frac{\mathbb{E}\left[V\mathbb{1}_{\{V \geqslant 0\}}\right] + \mathbb{E}\left[V\right]}{\Pr(V \geqslant 0) + 1} > E[V].$$

Thus, the result holds for a sufficiently large n.

In case V is bounded from above, we can combine the proof of the previous case with the proof of Theorem 1. Consider, w.l.o.g., a tight upper bound of 1 and a noise variable N where $N \in \{0, n, -n\}$ with equal probabilities. Fix $b_1 = 1 - \delta < 1 + \delta = b_2$ where $0 < \delta < 1$. If n is sufficiently large and δ is sufficiently small, we get

$$\mathbb{E}[V|V + N \geqslant b_1] \approxeq \frac{\mathbb{E}[V\mathbb{1}_{\{V \geqslant 1-\delta\}}] + E[V]}{\Pr(V \geqslant 1 - \delta) + 1} > \mathbb{E}[V] \approxeq E[V|V + N \geqslant b_2],$$

which concludes the proof. ∎