Lemma 2. Every unbounded finite-expectation impact variable has screening biases.

Proof. Assume that $V$ is unbounded from above. Without loss of generality, fix $E[V] = 0$ and take some large $n > 0$. Consider a noise variable $N$ where $N \in \{0, n, -n\}$ with equal probabilities. Clearly,

$$E[V|V + N \geq b] = \frac{E[V1_{V \geq b+n}] + E[V1_{V \geq b}]}{Pr(V \geq b + n) + Pr(V \geq b) + Pr(V \geq b - n)}.$$  

If $b = \sqrt{n}$ and $n$ tends to infinity, then $E[V|V + N \geq \sqrt{n}] \rightarrow E[V]$. However, if $b = 0$ and $n$ tends to infinity, we get

$$E[V|V + N \geq 0] \rightarrow \frac{E[V1_{V \geq 0}] + E[V]}{Pr(V \geq 0) + 1} > E[V].$$  

Thus, the result holds for a sufficiently large $n$.

In case $V$ is bounded from above, we can combine the proof of the previous case with the proof of Theorem 1. Consider, w.l.o.g., a tight upper bound of $1$ and a noise variable $N$ where $N \in \{0, n, -n\}$ with equal probabilities. Fix $b_1 = 1 - \delta < 1 + \delta = b_2$ where $0 < \delta < 1$. If $n$ is sufficiently large and $\delta$ is sufficiently small, we get

$$E[V|V + N \geq b_1] \approx \frac{E[V1_{V \geq 1-\delta}] + E[V]}{Pr(V \geq 1 - \delta) + 1} > E[V] \approx E[V|V + N \geq b_2],$$

which concludes the proof.