Online Appendix for
"Sovereign Debt and Structural Reforms"

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Appendix B

B.1 Analysis of Alternative Decentralizations (Section III.E)

Proposition B1 *Assume the economy has an AJ market structure, where the sovereign can issue nonrenegotiable debt contingent on the realization of w and ϕ, subject to borrowing constraints. Then, the COA can be decentralized by an AJ equilibrium with borrowing constraints $b'_{AJ,\phi} \leq \hat{b}(\phi)$. The borrowing constraint is binding for $\phi \leq \Phi(B(b))$ and slack otherwise, where b denotes the current debt repayment and B and Φ are defined in Definition 1.*

Proof of Proposition B1. Let ϕ-specific recession-contingent debt be denoted $b'_{AJ,\phi} = b'_\phi$ and let the recession- and normal-time value functions be given by W_{AJ} and $\bar{W}_{AJ}$, respectively. Since debt is nonrenegotiable, the debt prices in recession and normal time are $(1-p_{AJ})f(\phi)R^{-1}$ and $p_{AJ}R^{-1}$, respectively, where p_{AJ} is the expected effort. The problem for a sovereign who owes b in recession is then

$$\begin{aligned} W_{AJ}(b) &= \max_{\{b'_\phi\}_{\phi\in\mathcal{N}},\bar{b}'_{AJ}} \left\{ u\left(\underline{w} - b + p_{AJ}R^{-1}\times \bar{b}'_{AJ} + (1-p_{AJ})R^{-1}\times \int_{\mathcal{N}} b'_\phi f(\phi)\,d\phi\right) \right. \\ &\quad \left. -X(p_{AJ}) + \beta\left[p_{AJ}\bar{W}_{AJ}\left(\bar{b}'_{AJ}\right) + (1-p_{AJ})\int_{\mathcal{N}} W_{AJ}\left(b'_\phi\right)\,dF(\phi)\right]\right\}, \end{aligned}$$

where $p_{AJ} = \arg\max_{p\in[\underline{p},\bar{p}]} -X(p) + \beta\left[p\bar{W}_{AJ}\left(\bar{b}'_{AJ}\right) + (1-p)\int_{\mathcal{N}} W_{AJ}\left(b'_\phi\right)dF(\phi)\right]$, subject to a set of no-default borrowing constraints $b'_\phi \leq J(\phi)\ \forall\phi\in\mathcal{N}$, and $b'_\phi \geq \underline{b}$, $\bar{b}'_{AJ} \in [\underline{b},\tilde{b}]$.

The proof strategy is to show that the market allocation of Definition 1 is feasible, rules out default, is consistent with the FOCs, and yields the same expected utility in the AJ economy. Let the policy functions $\hat{b}(\phi)$, $\Phi(b)$, $\Pi(b)$, $\bar{B}(\bar{b})$, $B(b)$, $\mathcal{B}(b,\phi)$, $C(b)$, $\bar{C}(\bar{b})$, $W(b)$, $\Psi(b,\bar{b})$, and $\bar{\mu}(\bar{b})$ be given by the market allocation.

Guess that (i) the borrowing constraints are $J(\phi) = \hat{b}(\phi)$, (ii) optimal debt issuance is given by:

$$b'_\phi = \begin{cases} B(b) & \text{for } \phi \geq \Phi(B(b)) \\ \hat{b}(\phi) & \text{for } \phi < \Phi(B(b)) \end{cases},$$

and $\bar{b}'_{AJ} = \bar{B}(b)$, and (iii) optimal consumption is $C_{AJ}(b) = C(b)$ and $\bar{C}_{AJ}(\bar{b}) = \bar{C}(\bar{b})$. Since $b'_\phi = \mathcal{B}(B(b),\phi)$, the realized debt payments are equivalent to the market allocation. Equilibrium effort p_{AJ} must therefore be as in the market economy. Recession debt revenue is identical in the two economies, since

$$\begin{aligned} (1-p_{AJ})R^{-1}\times\int_{\mathcal{N}} b'_\phi f(\phi)\,d\phi &= \left(1-\Psi(B(b),\bar{B}(b))\right)R^{-1}\times\left(\begin{array}{c}(1-F(\Phi(B(b))))\cdot B(b) \\ +\int_{\phi_{\min}}^{\Phi(B(b))}\hat{b}(\phi)\,f(\phi)\,d\phi\end{array}\right) \\ &= \left(1-\Psi(B(b),\bar{B}(b))\right)R^{-1}\times\Pi(B(b)). \end{aligned}$$

Hence, the proposed allocation $\left\{b'_\phi, \bar{b}'_{AJ}, C_{AJ}, \bar{C}_{AJ}, p_{AJ}\right\}$ satisfies the budget constraint and is therefore feasible. It follows that discounted value must be identical; $\bar{W}_{AJ}(b) = \bar{\mu}(b)$ and $W_{AJ}(b) = W(b)$. Since $W_{AJ}(J(\phi)) = W(\hat{b}(\phi)) = \alpha - \phi \; \forall \phi \in \mathcal{N}$, it follows that $J(\phi)$ is not too tight, in the sense that the PC holds with equality at the borrowing constraint. Hence, J rules out default.

Finally, it is straightforward to verify that the proposed allocation satisfies the FOCs in the AJ economy. Since the proposed allocation is feasible, satisfies the AJ optimality conditions, and at the same time attains the same utility as in the COA, it must represent an equilibrium allocation in the AJ economy. ■

B.2 Analysis of the One-Asset Economy (Section IV)

In this section, we provide the technical analysis of the results summarized in Section IV and one related figure. We consider the (Markov) market equilibrium for an economy where the sovereign can issue only a one-period renegotiable noncontingent bond.[32]

We provide (i) a definition of the market equilibrium for the one-asset economy; (ii) a proof of existence and uniqueness of W^R and the associated equilibrium functions when the effort is exogenously given; (iii) a derivation of the CEE in equation (38). All proofs are in a separate section of this appendix below.

Definition B1 *A market equilibrium with noncontingent renegotiable debt is a set of value functions $\left\{V^R, W^R\right\}$, a threshold renegotiation function Φ^R, an equilibrium debt price function Q^R, and a set of optimal decision rules $\{\mathcal{B}^R, B^R, C^R, \Psi^R\}$ such that, conditional on the state vector $(b, \phi) \in \left([\underline{b}, \tilde{b}] \times [\phi_{\min}, \phi_{\max}]\right)$, the sovereign maximizes utility, the creditors maximize profits, and markets clear. More formally:*

- *The value function V^R satisfies*

$$V^R(b, \phi) = \max\left\{W^R(b), \alpha - \phi\right\},$$

where $W^R(b)$ is the value function conditional on the debt level b being honored,

$$W^R(b) = \max_{b' \in [\underline{b}, \tilde{b}]} u\left(Q^R(b') \times b' + \underline{w} - b\right) + Z^R(b'), \tag{B1}$$

continuation utility Z^R is defined as

$$Z^R(b') = \max_{p \in [\underline{p}, \bar{p}]} -X(p) + \beta\left(p \times \bar{\mu}^R(b') + (1-p) \times \mu^R(b')\right),$$

the value of starting in recession with debt b and in normal time with debt $\bar{b}$ are $\mu^R(b) = \int_{\mathcal{N}} V^R(b, \phi)\, dF(\phi)$ and $\bar{\mu}^R(\bar{b}) = u\left(\bar{w} - \left(1 - R^{-1}\right)\bar{b}\right)/(1-\beta)$, respectively.

[32] The extension in which we rule out renegotiation is qualitatively similar. The only difference is that the bond price when renegotiation is allowed, given by equation (B2) in the below Definition B1, becomes

$$Q^{NR}(b') = R^{-1}\left(\Psi^{NR}(b') + \left[1 - \Psi^{NR}(b')\right] \times \left[1 - F\left(\Phi^{NR}(b')\right)\right]\right)$$

when renegotiation is ruled out.

- *The threshold renegotiation function Φ^R satisfies*

$$\Phi^R(b) = \alpha - W^R(b).$$

- *The debt price function satisfies the following arbitrage condition:*

$$Q^R(b') \times b' = R^{-1}\left[\Psi^R(b') \times b' + \left[1 - \Psi^R(b')\right] \times \Pi^R(b')\right] \tag{B2}$$

where $\Pi^R(b')$ is the expected repayment of the noncontingent bond conditional on next period being a recession,

$$\Pi^R(b) = \left(1 - F\left(\Phi^R(b)\right)\right) b + \int_{\phi_{\min}}^{\Phi^R(b)} \hat{b}^R(\phi) \times dF(\phi),$$

and where $\hat{b}^R(\phi) = \left(\Phi^R\right)^{-1}(\phi)$ is the new post-renegotiation debt after a realization ϕ.

- *The set of optimal decision rules comprises:*

  1. *A take-it-or-leave-it debt renegotiation offer:*

$$\mathcal{B}^R(b,\phi) = \begin{cases} \hat{b}^R(\phi) & \text{if} \quad \phi \leq \Phi^R(b), \\ b & \text{if} \quad \phi > \Phi^R(b). \end{cases}$$

  2. *An optimal debt accumulation and an associated consumption decision rule:*

$$\begin{aligned} B^R\left(\mathcal{B}^R(b,\phi)\right) &= \arg\max_{b' \in [\underline{b}, \tilde{b}]} u\left(Q^R(b') \times b' + \underline{w} - \mathcal{B}^R(b,\phi)\right) + Z^R(b') \\ C^R\left(\mathcal{B}^R(b,\phi)\right) &= Q^R\left(B^R\left(\mathcal{B}^R(b,\phi)\right)\right) \times B^R\left(\mathcal{B}^R(b,\phi)\right) + \underline{w} - \mathcal{B}^R(b,\phi). \end{aligned}$$

  3. *An optimal effort decision rule:*

$$\Psi^R(b') = \arg\max_{p \in [\underline{p}, \bar{p}]} -X(p) + \beta\left(p \times \bar{\mu}^R(b') + (1-p) \times \mu^R(b')\right).$$

- *The equilibrium law of motion of debt is $b' = B^R\left(\mathcal{B}^R(b,\phi)\right)$.*
- *The probability that the recession ends is $p = \Psi^R(b')$.*

Since the haircut $\hat{b}^R(\phi)$ keeps the sovereign indifferent between accepting the creditors' offer and defaulting, this implies the following indifference condition,

$$W^R(\hat{b}^R(\phi)) = \alpha - \phi. \tag{B3}$$

With some abuse of notation, let $W^R(b;\alpha)$ denote the value function conditional on honoring debt b in an economy with exogenous outside option α as defined above. In the analysis of Section IV, we assume that in each economy (with and without renegotiation) the outside option is given by the market equilibrium with a zero debt position. Namely, $\alpha_R = W^R(0;\alpha_R)$ in the case with renegotiation, and $\alpha_{NR} = W^{NR}(0;\alpha_{NR})$, when we rule out renegotiation.

We prove, next, that for an exogenously given effort $\Psi^R = p$ a market equilibrium satisfying Definition B1 exists and that the set of equilibrium functions $\{V^R, W^R, \Phi^R, Q^R, \mathcal{B}^R\}$ is unique.

Proposition B2 *Assume $\Psi^R = p \in [\underline{p}, \bar{p}] \subset \mathbb{R}^+$. Then the Markov equilibrium with noncontingent debt exists and is unique: (A) The equilibrium functions $\{V^R, W^R, \Phi^R, Q^R, \mathcal{B}^R\}$ satisfying Definition B1 exist and are unique. The value functions $\left\{V^R, W^R\right\}$ are continuous, W^R is strictly decreasing in b, and V^R is nonincreasing in b; (B) There exists a unique α_R satisfying the fixed point $\alpha_R = W^R(0; \alpha_R)$.*

Finally, we derive formally the CEE of equation (38). Since the CEE is derived from first-order conditions, the proposition must first establish appropriate differentiability properties to ensure that the first-order conditions are necessary conditions for an equilibrium. It turns out that in the one-asset economy with renegotiation the equilibrium functions are not continuous and differentiable everywhere. However, we can prove that they are differentiable at all interior level of debt that can be the result of an optimal choice. Moreover, the discontinuities in the policy functions do not invalidate the fact that the FOCs are necessary conditions for an equilibrium. It is then useful to define $\hat{B}^R$ as the set of debt levels b' that can be the result of an optimal interior choice given debt b.

Definition B2 $\hat{B}^R = \{b' \in (\underline{b}, \tilde{b}) | B^R\left(\mathcal{B}^R(b, \phi)\right) = b'$, *for* $b \in [\underline{b}, \tilde{b}]\}$.

Proposition B3 *Let $\bar{C}^R(b)$ denote the consumption function in normal time. The equilibrium functions $W^R(b')$, $\Phi^R(b')$, $Q^R(b')$, and $\Psi^R(b')$ are differentiable for all $b' \in \hat{B}^R$. Moreover, for any $b' \in \hat{B}^R$, the FOC $(\partial/\partial b')u(Q^R(b')b' + \underline{w} - b) + (\partial/\partial b')Z^R(b') = 0$ and the envelope condition $\partial W^R(b')/\partial b' = -u'(C^R(b'))$ holds true, such that the conditional Euler equation (CEE)*

$$
\begin{aligned}
& \frac{\Psi^R(b')}{(1-\Psi^R(b'))\left[1-F(\Phi^R(b'))\right]+\Psi^R(b')} \frac{u'(\bar{C}^R(b'))}{u'(C^R(b))} + \frac{(1-\Psi^R(b'))\left[1-F(\Phi^R(b'))\right]}{(1-\Psi^R(b'))\left[1-F(\Phi^R(b'))\right]+\Psi^R(b')} \frac{u'(C^R(b'))}{u'(C^R(b))} \\
= \quad & 1 + \frac{\left(\Psi^R\right)'(b')\left[b' - \Pi^R(b')\right]}{(1-\Psi^R(b'))\left[1-F(\Phi^R(b'))\right]+\Psi^R(b')}, \qquad \text{(B4)}
\end{aligned}
$$

is a necessary condition for an interior optimum.

Note that the left-hand side of (B4) is the expected ratio between next-period and current-period marginal utility conditional on debt being honored in the next period. More precisely, the term $u'(C^R(b'))/u'(C^R(b))$ is the ratio of marginal utilities if the recession continues, whereas the term $u'(\bar{C}^R(b'))/u'(C^R(b))$ is the ratio of marginal utilities if the recession ends. Therefore, equation (B4) is identical to equation (38).

B.2.1 Figure B1

Figure B1 illustrates the properties of the one-asset economy with and without renegotiation. This figure is discussed in Section IV.A in the text.

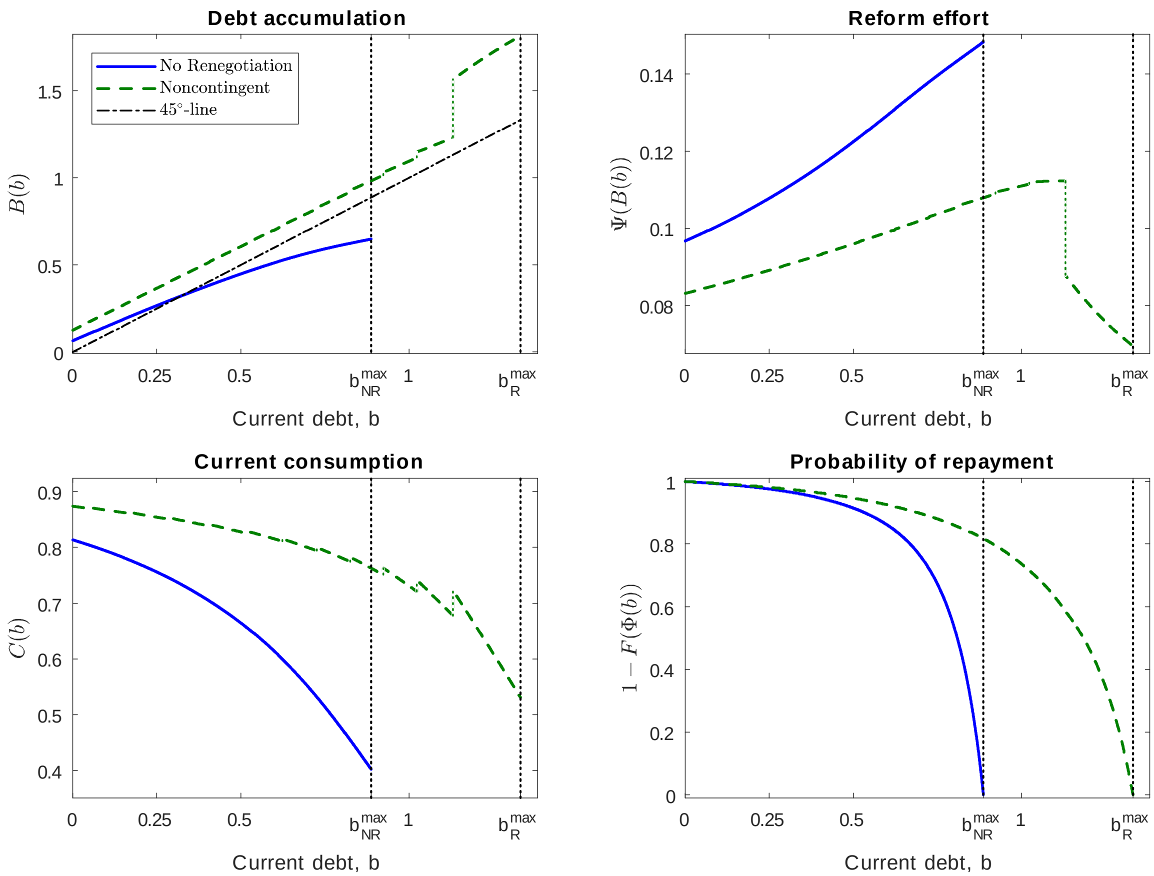

Figure B1: The top panels show the equilibrium debt issuance (top-left) and effort (top-right) conditional on repayment. The bottom panels show the equilibrium policy function for consumption (bottom-left) and the equilibrium probability of repayment (bottom-right). The dashed lines illustrate the Markov equilibrium with renegotiation, while the solid lines the equilibrium where renegotiation is ruled out. The parameterization of these economies is described in Section B.4.

B.3 Proofs

B.3.1 First-Best Allocation (Section II.A)

In this section, we provide the proof of Proposition 1.

Proof of Proposition 1. In the first part of the proof we take as given that the profit function $\bar{P}$ has the solution (9) and verify this below. Moreover, we also take as given that P is strictly decreasing, strictly concave, and differentiable in ν. We defer the formal proof of these properties to Lemma B1 further below in this appendix.

The Lagrangian of the planner's problem in recession reads as

$$\begin{aligned}\mathcal{L} &= \int_{\mathcal{N}} \left[\underline{w} - c_\phi + \beta \left(p_\phi \bar{P}\left(\bar{\omega}_\phi\right) + \left(1-p_\phi\right) P\left(\omega_\phi\right)\right)\right] f(\phi) d\phi \\ &\quad + \vartheta \left(\int_{\mathcal{N}} \left[u\left(c_\phi\right) - X\left(p_\phi\right) + \beta\left(p_\phi \bar{\omega}_\phi + \left(1-p_\phi\right)\omega_\phi\right)\right] f(\phi) d\phi - \nu \right)\end{aligned}$$

where the Lagrange multiplier on the PK is given by ϑ. The FOCs with respect to the controls c_ϕ, ω_ϕ, $\bar{\omega}_\phi$, and p_ϕ yield:

$$f\left(\phi\right) = u'\left(c_\phi\right) \vartheta f\left(\phi\right), \tag{B5}$$

$$\vartheta f\left(\phi\right) = -P'\left(\omega_\phi\right) f\left(\phi\right), \tag{B6}$$

$$\vartheta f\left(\phi\right) = -\bar{P}'\left(\bar{\omega}_\phi\right) f\left(\phi\right), \tag{B7}$$

$$\beta\left(\bar{P}\left(\bar{\omega}_\phi\right) - P\left(\omega_\phi\right)\right) f\left(\phi\right) = \vartheta f\left(\phi\right)\left(X'\left(p_\phi\right) - \beta\left(\bar{\omega}_\phi - \omega_\phi\right)\right), \tag{B8}$$

while the envelope condition is given by

$$-P'\left(\nu\right) = \vartheta. \tag{B9}$$

First, since $f(\phi) > 0$ over the relevant support of ϕ the optimal allocation is independent of the default cost realization. Thus, the planner fully insures the agent against the risk in ϕ. The optimality condition in (B7) implies that $\vartheta > 0$, since $-\bar{P}'\left(\bar{\omega}_\phi\right) > 0$. The optimality conditions (B6) and (B9) imply $\omega^{FB}(\nu) = \nu$ such that promised utility, consumption, and reform effort stay constant during recessions. Equations (B5)-(B7) together with (9) imply that the planner provides the agent with full consumption insurance across the income states, $u'\left(c^{FB}(\nu)\right) = u'\left(\bar{c}(\bar{\omega}^{FB}(\nu))\right) \Leftrightarrow c^{FB}(\nu) = \bar{c}(\bar{\omega}^{FB}(\nu)) = u^{-1}\left[(1-\beta)\bar{\omega}^{FB}(\nu)\right]$. Given the constant allocation, equation (8) in Proposition 1 follows immediately from the PK (2). Moreover, since in normal time the agent gets the same consumption as in recession (but reform effort is absent), equation (8) implies that promised utility in normal time can be expressed as $\bar{\omega}^{FB}(\nu) = \nu + X\left(p^{FB}\left(\nu\right)\right) / \left(1-\beta\left(1-p^{FB}\left(\nu\right)\right)\right) = u(c^{FB}(\nu))/(1-\beta)$. The FOC with respect to effort (B8) can then be expressed as

$$\beta\left(\bar{P}\left(\bar{\omega}^{FB}(\nu)\right) - P(\nu)\right) = u'(c^{FB}(\nu))^{-1}\left(X'\left(p^{FB}(\nu)\right) - \beta\left(\bar{\omega}^{FB}(\nu) - \nu\right)\right), \tag{B10}$$

where

$$\begin{aligned}\bar{P}\left(\bar{\omega}^{FB}(\nu)\right) - P(\nu) &= \bar{w} - \underline{w} + \beta\left(1 - p^{FB}\left(\nu\right)\right)\left(\bar{P}\left(\bar{\omega}^{FB}(\nu)\right) - P(\nu)\right) \\ &= \frac{1}{1-\beta\left(1-p^{FB}\left(\nu\right)\right)}\left(\bar{w} - \underline{w}\right) \\ \bar{\omega}^{FB}(\nu) - \nu &= \frac{X\left(p^{FB}\left(\nu\right)\right)}{1-\beta\left(1-p^{FB}\left(\nu\right)\right)}.\end{aligned}$$

Substituting for $\bar{P}\left(\bar{\omega}^{FB}(\nu)\right)-P(\nu)$ and $\bar{\omega}^{FB}(\nu)-\nu$ in (B10) yields equation (7) in Proposition 1. Note that the profit function in recession

$$\begin{aligned} P(\nu) &= \frac{\underline{w}-c^{FB}(\nu)}{1-\beta\left(1-p^{FB}(\nu)\right)}+\frac{\beta p^{FB}(\nu)}{1-\beta}\frac{\bar{w}-c^{FB}(\nu)}{1-\beta\left(1-p^{FB}(\nu)\right)} \\ &= \frac{\bar{w}-c^{FB}(\nu)}{1-\beta}-\frac{\bar{w}-\underline{w}}{1-\beta\left(1-p^{FB}(\nu)\right)}, \end{aligned} \tag{B11}$$

defines a positively sloped locus in the plane (p,c), while equation (7) defines a negatively sloped locus in the same plane. The two equations pin down a unique interior solution for $p^{FB}(\nu)$ and $c^{FB}(\nu)$. Now, consider the comparative statics with respect to ν. An increase in ν yields a strict increase in consumption $c^{FB}(\nu)$ according to (B5) and (B9) since P is strictly concave, while (7) is independent of ν such that the increase in $c^{FB}(\nu)$ must come with a strict decrease in $p^{FB}(\nu)$. Finally, set $\underline{w}=\bar{w}$ in (B11) to see that the profit function in normal time is indeed given by (9). This concludes the proof of the proposition. ■

B.3.2 Constrained Optimum without Moral Hazard (Section II.B)

In this section, we provide the proof of Proposition 2. As a preliminary step, we state Lemma B1 that is used in the proof of Proposition 2.

Lemma B1 The profit functions P and $\bar{P}$ that solve the Bellman equation (1) subject to (2)-(3), or, subject to (2)-(5), are strictly decreasing, strictly concave, and differentiable at the interior of their support. The FOCs of the planning problem are necessary and sufficient to characterize the COA.

Since the proof of Lemma B1 is long and uses standard methods, we defer it to Section B.3.6 below.

Proof of Proposition 2. In this proof we take as given Lemma B1, that is proved separately. We limit the proof to the arguments that do not overlap with those in the proof of Proposition 3 in Appendix A.

The Lagrangian of the planner's problem is the same as in (A1), except that we can drop all terms that involve the incentive constraint ($\chi_\phi=0$). The FOCs yield, then:

$$f(\phi) = u'(c_\phi)\left(\vartheta f(\phi)+\lambda_\phi\right), \tag{B12}$$

$$\vartheta f(\phi)+\lambda_\phi = -P'(\omega_\phi)\, f(\phi), \quad \forall \omega_\phi>\underline{\nu}, \tag{B13}$$

$$\vartheta f(\phi)+\lambda_\phi = -\bar{P}'(\bar{\omega}_\phi)\, f(\phi), \tag{B14}$$

$$\beta\left(\bar{P}(\bar{\omega}_\phi)-P(\omega_\phi)\right) f(\phi) = \left(\vartheta f(\phi)+\lambda_\phi\right)\left(X'(p_\phi)-\beta\left(\bar{\omega}_\phi-\omega_\phi\right)\right). \tag{B15}$$

The envelope condition yields $-P'(\nu)=\vartheta>0$, $\forall \nu>\underline{\nu}$, and the slackness condition for θ_ϕ reads $0=\theta_\phi\left(\omega_\phi-\underline{\nu}\right)$.

Since P is strictly concave, Lemma 1 implies that the solution is characterized by the unique threshold $\tilde{\phi}(\nu)$ in (13). The PC binds only for $\phi<\tilde{\phi}(\nu)$.

The FOCs (B12)–(B15) imply equations (10)-(12) in the text. Combine equation (B13) and the envelope condition to yield $P'(\nu)=P'(\omega_\phi)+\lambda_\phi/f(\phi)\geq P'(\omega_\phi)$ for $\nu,\omega_\phi>\underline{\nu}$. Since P is strictly concave this implies that promised utility is weakly increasing conditional on staying in recession, $\omega_\phi\geq\nu>\underline{\nu}$. Note that this property extends to the lower bound where $\nu=\underline{\nu}=\omega(\nu)$ if the PC is slack. This can be proved by a contradiction argument: Suppose that $\exists\nu>\underline{\nu}$ such that $\omega(\underline{\nu})\geq\omega(\nu)$. The optimal allocation associated with $\omega(\nu)$ yields utility $\nu > \underline{\nu}$. Thus, $\omega(\underline{\nu}) \geq \omega(\nu)$ would violate the PK, which is not feasible. In summary, promised utility is weakly increasing $\omega_\phi \geq \nu$, such that its lower bound is never relevant and we can drop the multiplier θ_ϕ from the subsequent analysis.

Since λ_ϕ enters the optimality conditions, the solution will depend on whether the PC is slack or binding:

1. When the PC is binding and the recession continues, $\phi < \tilde{\phi}(\nu)$, $\lambda_\phi > 0$, $\omega_\phi > \nu$, and
$$u\left(c_\phi\right) - X\left(p_\phi\right) + \beta\left[p_\phi \bar{\omega}_\phi + \left(1 - p_\phi\right)\omega_\phi\right] = \alpha - \phi. \tag{B16}$$
Then, (10), (11), (12), and (B16) determine jointly the solution for $(c_\phi, p_\phi, \omega_\phi, \bar{\omega}_\phi)$. In this case, there is no history dependence, i.e., ν does not matter.

2. When the PC is not binding, $\phi \geq \tilde{\phi}(\nu)$ and $\lambda_\phi = 0$. Then, $\omega_\phi = \nu$, and $c_\phi = c\left(\nu\right)$, $p_\phi = p(\nu)$, and $\bar{\omega}_\phi = \bar{\omega}_\phi(\nu)$ are determined by (14), (10), and (12), respectively. The solution is history dependent.

By the same argument made in the proof of Proposition 3, $c\left(\nu\right)$ must be strictly increasing in ν. In turn $1/u'(c\left(\nu\right)) = 1/u'(\bar{c}\left(\bar{\omega}(\nu)\right))$ implies that also $\bar{\omega}(\nu)$ is strictly increasing in ν. Finally, equation (12) implies that
$$u'\left(c\left(\nu\right)\right)\left[\bar{P}\left(\bar{\omega}\left(\nu\right)\right) - P\left(\nu\right)\right] + \left[\bar{\omega}\left(\nu\right) - \nu\right] = \beta^{-1}X'\left(p\left(\nu\right)\right).$$
For $\nu > \underline{\nu}$, differentiating the left-hand side yields
$$\begin{aligned} & \underbrace{u''\left(c\left(\nu\right)\right)c'\left(\nu\right) \times \left[\bar{P}\left(\bar{\omega}\left(\nu\right)\right) - P\left(\nu\right)\right]}_{<0} + \left[u'\left(c\left(\nu\right)\right)P'\left(\nu\right) + 1\right]\left(\bar{\omega}'\left(\nu\right) - 1\right) \\ = \; & u''\left(c\left(\nu\right)\right)c'\left(\nu\right) \times \left(\bar{P}\left(\bar{\omega}\left(\nu\right)\right) - P\left(\nu\right)\right) < 0 \end{aligned}$$
since (11) implies that $P'\left(\nu\right) = -1/u'\left(c\left(\nu\right)\right)$, and we establish below that $\bar{P}\left(\bar{\omega}\left(\nu\right)\right) - P(\omega(\nu)) = \bar{P}\left(\bar{\omega}\left(\nu\right)\right) - P\left(\nu\right) > 0$. This implies that the right-hand side must also be strictly decreasing in ν. Since X is convex and increasing, this implies in turn that $p\left(\nu\right)$ must be strictly decreasing in $\nu > \underline{\nu}$. Note that this property extends to the lower bound, $p(\underline{\nu}) > p(\nu)$ $\forall \nu > \underline{\nu}$. Suppose not, $p(\underline{\nu}) \leq p(\nu)$, then $\nu = \omega(\nu) > \omega(\underline{\nu}) = \underline{\nu}$ which contradicts the fact that $\omega(\nu)$ and $p(\nu)$ are optimal given ν and the same ϕ. Thus, effort $p(\nu)$ must be strictly decreasing.

Finally, we must establish that $\bar{P}(\bar{\omega}_\phi(\nu)) - P(\omega_\phi(\nu)) > 0$, $\forall \omega_\phi(\nu) > \underline{\nu}$. Suppose, to derive a contradiction, that $\bar{P}(\bar{\omega}_\phi(\nu)) - P(\omega_\phi(\nu)) \leq 0$. For simplicity, we write ω_ϕ for $\omega_\phi(\nu)$. The FOCs of the planner problem in equations (10) and (11) imply
$$\bar{c}(\bar{\omega}_\phi) = c_\phi = c(\omega_\phi).$$
Recall that $\omega_\phi \geq \nu$. Then, once the economy recovers, promised-utility and profits remains constant such that consumption can be written as
$$\begin{aligned} \bar{c}(\bar{\omega}_\phi) &= \bar{w} - \bar{P}(\omega_\phi) + \beta\bar{P}(\omega_\phi) = \bar{w} - \bar{P}(\bar{\omega}_\phi) + \frac{p_\phi}{R}\bar{P}(\bar{\omega}_\phi) + \frac{1 - p_\phi}{R}\bar{P}(\bar{\omega}_\phi) \\ &> \underline{w} - \bar{P}(\bar{\omega}_\phi) + \frac{p_\phi}{R}\bar{P}(\bar{\omega}_\phi) + \frac{1 - p_\phi}{R}\bar{P}(\bar{\omega}_\phi) \\ &\geq \underline{w} - P(\omega_\phi) + \frac{p_\phi}{R}\bar{P}(\bar{\omega}_\phi) + \frac{1 - p_\phi}{R}P(\omega_\phi) \\ &\geq \underline{w} - P(\nu) + \frac{p_\phi}{R}\bar{P}(\bar{\omega}_\phi) + \frac{1 - p_\phi}{R}P(\omega_\phi) = c_\phi. \end{aligned}$$

To see why, note that $[(1-p_\phi)/R-1]\,\bar{P}(\bar{\omega}_\phi) \geq [(1-p_\phi)/R-1]\,P(\omega_\phi)$ since $[(1-p_\phi)/R-1] < 0$ and (by assumption) $\bar{P}(\bar{\omega}_\phi) \leq P(\omega_\phi)$. The last inequality follows from $\omega_\phi \geq \nu$ and $P'(\nu) < 0$. The conclusion that $\bar{c}(\bar{\omega}_\phi) > c_\phi$ contradicts $\bar{c}(\bar{\omega}_\phi) = c_\phi$ which was derived above. Thus, we have proven that $\bar{P}(\bar{\omega}_\phi) - P(\omega_\phi) > 0$.

This concludes the proof of Proposition 2. ■

B.3.3 Constrained Optimum with Moral Hazard (Section II.B)

This section contains three lemmas that are used in the analysis of the planner problem with moral hazard in Section II.B. Lemma B2 provides a sufficient condition for the effort function to be falling in promised utility when ν is sufficiently large. Lemmas B3 and B4 are instrumental to prove Proposition 3 in Appendix A.

Lemma B2 Suppose $\lim_{c\to\infty} u'(c) = 0$ and that $\lim_{p\to\underline{p}} X''(p) > 0$. Then $\lim_{\nu\to\infty} p(\nu) = \underline{p}$.

Proof of Lemma B2. We conjecture that in the limit the COA is given by $\lim_{\nu\to\infty}\{\bar{\omega}(\nu)-\omega(\nu)\} = 0$, $\lim_{\nu\to\infty} c(\nu) = \lim_{\nu\to\infty} c(\omega(\nu)) = \lim_{\nu\to\infty} \bar{c}(\bar{\omega}(\nu)) = \infty$, and $\lim_{\nu\to\infty} p(\nu) = \underline{p}$, where $p(\nu) \equiv \Upsilon(\bar{\omega}-\omega)$. We verify that this allocation satisfies the necessary FOCs of the COA. First, eq. (2) implies $\lim_{\nu\to\infty}\omega(\nu) = \lim_{\nu\to\infty}\bar{\omega}(\nu) = \infty$. Second, $\lim_{\nu\to\infty} X'(p(\nu)) = 0$ satisfies eq. (15). Third, the lower bound on ω and the PC (4) become irrelevant when ν is sufficiently large. Fourth, note that $\Upsilon'(\bar{\omega}-\omega) = \beta/X''(\Upsilon(\bar{\omega}-\omega))$. Equations (16)-(17) can then be rewritten as

$$1-p(\nu) = (1-p(\nu))\frac{u'(c(\nu))}{u'(c(\omega(\nu)))} + u'(c(\nu))\frac{\beta}{X''(p(\nu))}\left[\bar{P}(\bar{\omega}(\nu)) - P(\omega(\nu))\right] \tag{B17}$$

$$p(\nu) = p(\nu)\frac{u'(c(\nu))}{u'(\bar{c}(\bar{\omega}(\nu)))} - u'(c(\nu))\frac{\beta}{X''(p(\nu))}\left[\bar{P}(\bar{\omega}(\nu)) - P(\omega(\nu))\right]. \tag{B18}$$

Consider the limit when $\nu\to\infty$. Equations (B17)-(B18) hold as $\nu\to\infty$ since $\lim_{\nu\to\infty} p(\nu) = \underline{p}$, $\lim_{\nu\to\infty}\left\{\bar{P}(\bar{\omega}(\nu)) - P(\omega(\nu))\right\} = (\bar{w}-\underline{w})\,/\left[1-\beta(1-\underline{p})\right]$, $\lim_{\nu\to\infty} u'(c(\nu)) = 0$, $\lim_{\nu\to\infty}\{u'[c(\nu)]/u'[c(\omega(\nu))]\}$ $= \lim_{\nu\to\infty}\{u'[c(\nu)]/u'[\bar{c}(\bar{\omega}(\nu))]\} = 1$, and $X''(p)$ is bounded away from zero by assumption. Finally, note that the conjectured COA coincides with the FB in the limit, since $\lim_{\nu\to\infty} u'(c^{FB}(\nu))\,(\bar{w}-\underline{w})$ $= 0$ in (7). Thus, it must yield the maximal profits given ν. This implies that the conjectured limiting allocation is indeed the COA. ■

Lemma B3 Assume P is strictly concave. Then, P is differentiable at the interior of its support with $P'(\nu) = -1/u'(c(\nu)) < 0$.

Proof of Lemma B3. The proof is an application of Benveniste and Scheinkman (1979, Lemma 1). Consider the profit of a pseudo planner that is committed to deliver the initial promise $\tilde{\nu}$, but suboptimally chooses effort and future promised-utility like in the optimal contract given an initial promise ν

$$\begin{aligned}\widetilde{P}(\tilde{\nu},\nu) \;\equiv\; & \int_{\phi_{\min}}^{\tilde{\phi}(\tilde{\nu})}\left[\underline{w} - x(\phi, p_\phi(\nu), \bar{\omega}_\phi(\nu), \omega_\phi(\nu)) + \beta\left[\begin{array}{c} p_\phi(\nu)\bar{P}(\bar{\omega}_\phi(\nu)) \\ +(1-p_\phi(\nu))P(\omega_\phi(\nu))\end{array}\right]\right]dF(\phi) \\ & + \int_{\tilde{\phi}(\tilde{\nu})}^{\phi_{\max}}\left[\underline{w} - x(\phi(\tilde{\nu}), p_\phi(\nu), \bar{\omega}_\phi(\nu), \omega_\phi(\nu)) + \beta\left[\begin{array}{c} p_\phi(\nu)\bar{P}(\bar{\omega}_\phi(\nu)) \\ +(1-p_\phi(\nu))P(\omega_\phi(\nu))\end{array}\right]\right]dF(\phi),\end{aligned}$$

where consumption provided by the pseudo planner is determined by

$$x(\phi, p, \bar{\omega}, \omega) = u^{-1} \left(\alpha - \phi + X(p) - \beta \left[p\bar{\omega} + (1-p)\omega\right]\right)$$

and $p_\phi(\nu) = \Upsilon(\bar{\omega}_\phi(\nu) - \omega_\phi(\nu))$. Note that for $\tilde{\nu} = \nu$, the pseudo planner achieves the same profit as in the optimal contract, $\widetilde{P}(\tilde{\nu}, \tilde{\nu}) = P(\tilde{\nu})$, but profits must be weakly lower otherwise, $\widetilde{P}(\tilde{\nu}, \nu) \leq P(\tilde{\nu})$. Furthermore, $\widetilde{P}(\tilde{\nu}, \nu)$ is twice differentiable in $\tilde{\nu}$ and strictly concave. Then, Lemma 1 in Benveniste and Scheinkman (1979) applies and the profit function $P(\nu)$ is differentiable at the interior of its support ν with derivative

$$P'(\nu) = \widetilde{P}_1(\nu, \nu) = -1/u'\left(c(\nu)\right) < 0.$$

This concludes the proof of the lemma. ■

Lemma B4 The FOCs of the planning problem are necessary for optimality.

Proof of Lemma B4. That the optimal effort is interior follows from the assumed properties of the X function ($X'(\underline{p}) = 0$, $X'(p) > 0$ for $p > \underline{p}$, and $\lim_{p \to \bar{p}} X'(\bar{p}) = +\infty$). The optimality condition for effort $X'(p_\phi(\nu)) = \beta(\bar{\omega}_\phi(\nu) - \omega_\phi(\nu))$ implies then $\bar{\omega}_\phi(\nu) > \omega_\phi(\nu)$, and that there exists an interior maximum effort level $p^+ = \max\{p_\phi(\nu)\} < \bar{p}$. Since $\omega_\phi(\nu) \geq \underline{\nu}$, then $\bar{\omega}_\phi(\nu) > \underline{\nu}$. In conclusion, the optimal choice of p_ϕ is interior and $\bar{\omega}_\phi(\nu)$ will never be at the lower bound $\underline{\omega}$. Note that the possibility $\omega_\phi(\nu) = \underline{\nu}$ is taken into account by the stated FOCs.

Next, consider the upper bound $\tilde{\omega}$ for $\omega_\phi(\nu)$ which is sufficiently high that none of the PCs will bind if the economy starts at $\tilde{\omega}$, i.e., $\omega_\phi(\tilde{\omega}) = \omega(\tilde{\omega}) > \alpha - \phi_{\min}$. Then, the FOC with respect to ω_ϕ (17) implies that - if the planner was not constrained by $\omega_\phi \leq \tilde{\omega}$ - profits are maximized when $\omega_\phi(\tilde{\omega}) < \tilde{\omega}$. This allocation is feasible in the constrained problem thus it must also be the optimal choice when $\omega_\phi(\nu)$ is bounded by $\tilde{\omega}$. The same applies to any level of promised-utility below $\tilde{\omega}$ when the PC is slack. Finally, in states where the PC binds, $\omega_\phi(\nu)$ always remains below $\alpha - \phi_{\min} < \tilde{\omega}$. Thus, $\omega_\phi(\nu)$ always remains strictly below $\tilde{\omega}$. In turn, the optimality condition for effort then implies that the $\bar{\omega}_\phi(\nu)$ can never be higher than $X'(p^+)/\beta + \tilde{\omega} < \tilde{\bar{\omega}}$, where $X'(p^+) < +\infty$. In summary, the optimal choices of $\bar{\omega}_\phi$ and ω_ϕ are also interior (apart from the corner solution, $\omega_\phi = \underline{\nu}$). Finally, consumption must always be positive since $\lim_{c \to 0} u(c) = -\infty$ and it is interior because promised-utility and effort is interior. Thus, the solution to the planner problem must be interior and the stated FOCs are necessary. ■

B.3.4 Decentralization (Section III.B)

This section contains the complete proofs of two lemmas that are stated in the proof of Proposition 4 in Appendix A.

Proof of Lemma A1. We prove that the mapping T_δ defined in eq. (A6), satisfies Blackwell's sufficient conditions on the complete metric space (Γ, d_∞), thereby being a contraction mapping. Therefore, $W_\delta = \lim_{n \to \infty} T_\delta^n(\gamma)$ exists and is unique (see Stokey, Lucas, and Prescott 1989, Theorem 3.3).

Claim (i): T_δ is strictly decreasing in b for all b such that $T_\delta(\gamma)(b) > \alpha - \phi_{\max}$, otherwise $T_\delta = \alpha - \phi_{\max}$, in particular, $T_\delta(\gamma)(\tilde{b}) = \alpha - \phi_{\max}$. Proof of the claim: For any $\varepsilon > 0$, $\widetilde{T}_\delta(\gamma)(b + \varepsilon) < \widetilde{T}_\delta(\gamma)(b)$ since

$$\begin{aligned} \widetilde{T}_\delta(\gamma)(b+\varepsilon) &= O(b+\varepsilon, B^*_\gamma(b+\varepsilon), \bar{B}^*_\gamma(b+\varepsilon), \gamma(B^*_\gamma(b+\varepsilon)); \delta) \\ &< O(b, B^*_\gamma(b+\varepsilon), \bar{B}^*_\gamma(b+\varepsilon), \gamma(B^*_\gamma(b+\varepsilon)); \delta) \\ &\leq O(b, B^*_\gamma(b), \bar{B}^*_\gamma(b), \gamma(B^*_\gamma(b)); \delta) = \widetilde{T}_\delta(\gamma)(b), \end{aligned}$$

where $\langle \bar{B}^*_\gamma(b), B^*_\gamma(b)\rangle = \arg\max_{\bar{b}',b'\in[\underline{b},\tilde{b}],\Pi_\delta(b')\leq \bar{b}'} O(b,b',\bar{b}',\gamma(b');\delta)$. The strict inequality follows from $b+\varepsilon > b$ and $u'(\cdot)>0$. The weak inequality follows from the fact that $\langle \bar{B}^*_\gamma(b), B^*_\gamma(b)\rangle$ is the optimal policy for the debt level b. Since $\widetilde{T}_\delta$ is strictly decreasing in $b\in(\underline{b}, b_0(\delta))$, and $\lim_{b\to b_0(\delta)}\widetilde{T}_\delta(\gamma)(b) = -\infty < \alpha-\phi_{\max}$, then, $T_\delta(\gamma)(b)$ is strictly decreasing in b for all b such that $T_\delta(\gamma)(b) > \alpha - \phi_{\max}$, being constant at $\alpha-\phi_{\max}$ otherwise. Finally, since $b_0(\delta) < \tilde{b}$ (A5) implies that $T_\delta(\gamma)(\tilde{b}) = \alpha - \phi_{\max}$.

Claim (ii): T_δ maps Γ into Γ. Proof of the claim: Recall that T_δ is bounded from below by $\alpha-\phi_{\max}$. T_δ is also bounded from above because consumption, reform effort, the support of the default cost, and the elements of Γ and Λ are bounded. Continuity of $\widetilde{T}_\delta$ in $b\in[\underline{b}, b_0(\delta))$ follows by the Theorem of the Maximum. Since $\lim_{b\to b_0(\delta)}\widetilde{T}_\delta(\gamma)(b) = -\infty < \alpha-\phi_{\max}$, then $T_\delta(\gamma)(b) = \max\left\{\widetilde{T}_\delta(\gamma)(b), \alpha-\phi_{\max}\right\}$ is also continuous in b. Finally, we have already established above that T_δ is nonincreasing in b and that $T_\delta(\gamma)(\tilde{b}) = \alpha-\phi_{\max}$. Thus, $T_\delta(\gamma)\in\Gamma$.

Claim (iii): T_δ discounts: for any scalar $a\geq 0$ and $\gamma\in\Gamma$, $T_\delta(\gamma+a)(b)\leq T_\delta(\gamma)(b)+\beta a$. Proof of the claim: Let $a\geq 0$ be a real constant. Then

$$\widetilde{T}_\delta(\gamma+a)(b) \leq \max_{\bar{b}',b'\in[\underline{b},\tilde{b}],\Pi_\delta(b')\leq\bar{b}'} u\begin{pmatrix} \underline{w}-b+\beta\Psi^*\left(\gamma(b')+a,\bar{b}'\right)\bar{b}' \\ +\beta\left(1-\Psi^*\left(\gamma(b')+a,\bar{b}'\right)\right)\Pi_\delta(b')\end{pmatrix} + Z^*\left(\gamma(b'),\bar{b}'\right)+\beta a \quad \text{(B19)}$$

$$\leq \max_{\bar{b}',b'\in[\underline{b},\tilde{b}],\Pi_\delta(b')\leq\bar{b}'} u\begin{pmatrix} \underline{w}-b+\beta\Psi^*\left(\gamma(b'),\bar{b}'\right)\bar{b}' \\ +\beta\left(1-\Psi^*\left(\gamma(b'),\bar{b}'\right)\right)\Pi_\delta(b')\end{pmatrix} + Z^*\left(\gamma(b'),\bar{b}'\right)+\beta a \quad \text{(B20)}$$

$$= \widetilde{T}_\delta(\gamma)(b)+\beta a.$$

The first inequality follows from an envelope argument which implies that

$$dZ^*(\gamma(b')+a,\bar{b}')/da = \beta\left(1-\Psi^*\left(\gamma\left(b'\right)+a,\bar{b}'\right)\right)\left[1-F\left(\Phi^*(\gamma(b')+a)\right)\right]\in[0,\beta].$$

Therefore, a linear expansion yields $Z^*\left(\gamma(b')+a,\bar{b}'\right)\leq Z^*\left(\gamma(b'),\bar{b}'\right)+\beta a$. The second inequality follows from observing that $\Psi^*\left(\gamma(b')+a,\bar{b}'\right)\leq\Psi^*\left(\gamma(b'),\bar{b}'\right)$ and $\bar{b}'\geq\Pi_\delta(b')$, implying that debt revenue and utility are (weakly) higher in (B20) than in (B19). So, $\tilde{T}_\delta$ discounts. The definition of T_δ implies that if $\tilde{T}_\delta$ discounts, so does T_δ.

Claim (iv): T_δ is a monotone mapping, i.e., $\forall\gamma,\gamma^+\in\Gamma$ such that $\gamma^+(b)\geq\gamma(b)$, $T_\delta(\gamma^+)(b)\geq T_\delta(\gamma)(b)\ \forall b\in[\underline{b},\tilde{b}]$. Proof of the claim: Let $\gamma,\gamma^+\in\Gamma$ with $\gamma^+(b)\geq\gamma(b)$, $\forall b\in[\underline{b},\tilde{b}]$. We first establish that $\gamma^+(b)\geq\gamma(b)\Rightarrow\widetilde{T}_\delta(\gamma^+)(b)\geq\widetilde{T}_\delta(\gamma)(b)$. To this aim, let $(b',\bar{b}') = \left(B^*_{\gamma^+}(b), \bar{B}^*_{\gamma^+}(b)\right)$ denote the optimal debt issuance under the function γ^+. Let $\xi(b)\geq 0$ be such that $\gamma^+\left(B^*_\gamma(b)+\xi(b)\right) = \gamma(B^*_\gamma(b))$. (That such a positive function exists follows immediately from the properties of γ^+ and γ.) Suppose, first, that b is such that $\Pi_\delta\left(B^*_\gamma(b)+\xi(b)\right)\leq\bar{B}^*_\gamma(b)$, so that the LSS constraint is satisfied. Then, the following sequence of inequalities holds true:

$$\begin{aligned}
\widetilde{T}_\delta\left(\gamma^+\right)(b) &= O\left(b, B^*_{\gamma^+}(b), \bar{B}^*_{\gamma^+}(b), \gamma^+\left(B^*_{\gamma^+}(b)\right);\delta\right) \\
&\geq O\left(b, B^*_\gamma(b)+\xi(b), \bar{B}^*_\gamma(b), \gamma^+\left(B^*_\gamma(b)+\xi(b)\right);\delta\right) \\
&= O\left(b, B^*_\gamma(b)+\xi(b), \bar{B}^*_\gamma(b), \gamma\left(B^*_\gamma(b)\right);\delta\right) \\
&\geq O\left(b, B^*_\gamma(b), \bar{B}^*_\gamma(b), \gamma\left(B^*_\gamma(b)\right);\delta\right) = \widetilde{T}_\delta(\gamma)(b).
\end{aligned}$$

The first inequality follows from the fact that, under γ^+, the choice $(b',\bar{b}') = \left(B^*_\gamma(b)+\xi(b), \bar{B}^*_{\gamma^+}(b)\right)$ is feasible and suboptimal. The second inequality follows from the fact that the expression in the third line has a larger b' than that in the fourth line, while effort is held constant across the two expressions.

Thus, the former grants (weakly) higher consumption than the latter.
Consider, next, the case when the LSS binds, i.e., $\Pi_\delta\left(B^*_\gamma(b)+\xi(b)\right) > \bar{B}^*_\gamma(b)$. Define $\tilde{\xi}(b)$ such that $\Pi_\delta(B^*_\gamma(b)+\tilde{\xi}(b)) = \bar{B}^*_\gamma(b)$ and note that $0 \le \tilde{\xi}(b) < \xi(b)$. Then,

$$\begin{aligned}
\widetilde{T}_\delta\left(\gamma^+\right)(b) &= O\left(b, B^*_{\gamma^+}(b), \bar{B}^*_{\gamma^+}(b), \gamma^+\left(B^*_{\gamma^+}(b)\right); \delta\right) \\
&\ge O\left(b, B^*_\gamma(b)+\tilde{\xi}(b), \bar{B}^*_\gamma(b), \gamma^+\left(B^*_\gamma(b)+\tilde{\xi}(b)\right); \delta\right) \\
&= u\left(\underline{w}-b+\beta\bar{B}^*_\gamma(b)\right) + Z^*\left(\gamma^+\left(B^*_\gamma(b)+\tilde{\xi}(b)\right), \bar{B}^*_\gamma(b)\right) \\
&\ge O\left(b, B^*_\gamma(b), \bar{B}^*_\gamma(b), \gamma\left(B^*_\gamma(b)\right); \delta\right) = \widetilde{T}_\delta(\gamma)(b).
\end{aligned}$$

The first inequality follows from the fact that, under γ^+, the choice $\left(b', \bar{b}'\right) = \left(B^*_\gamma(b)+\tilde{\xi}(b), \bar{B}^*_{\gamma^+}(b)\right)$ is suboptimal. The second equality follows from the fact that $p\Pi_\delta(B^*_\gamma(b)+\tilde{\xi}(b))+(1-p)\bar{B}^*_\gamma(b) = \bar{B}^*_\gamma(b)$. The second inequality follows from the observation that both consumption and continuation utility are higher in the expression in the third line than in the one in the fourth line. Consumption is higher because (by the LSS) $\bar{B}^*_\gamma(b) \ge \Pi_\delta(B^*_\gamma(b))$. Continuation utility is higher because (i) $\tilde{\xi}(b) \le \xi(b) \Rightarrow \gamma^+\left(B^*_\gamma(b)+\tilde{\xi}(b)\right) \ge \gamma^+\left(B^*_\gamma(b)+\xi(b)\right) = \gamma\left(B^*_\gamma(b)\right)$ and (ii) $Z^*\left(y^+, x\right) \ge Z^*\left(y, x\right)$.
This establishes that $\tilde{T}_\delta$ is a monotone mapping. Next, observe that $T_\delta\left(\gamma^+\right) = \max\{\widetilde{T}_\delta\left(\gamma^+\right), \alpha - \phi_{\max}\} \ge \max\{\widetilde{T}_\delta(\gamma), \alpha-\phi_{\max}\} = T_\delta(\gamma)$. Thus, T_δ is also a monotone mapping.

By Blackwell's theorem, Claims (i)–(iv) jointly imply that $T_\delta(\gamma)$ is a contraction operator on the complete metric space (Γ, d_∞), thus its fixed point $W_\delta = \lim_{n\to\infty} T^n_\delta(\gamma)$ exists in Γ and is unique. ■

Proof of Lemma A2. First, note that $S(\delta)$ is bounded by $[\underline{b}, \tilde{b}]$, continuous and nondecreasing in ϕ since $W_\delta(b)$ is continuous and nonincreasing in b. Thus, S is an operator on the complete metric space (d_∞, Λ). We now verify Blackwell's sufficient conditions for S being a contraction mapping.

Monotonicity: Let $\delta^+, \delta \in \Lambda$ and assume $\delta^+ \ge \delta$. We claim that $S(\delta^+) \ge S(\delta)$. Corollary B1 below establishes that $\delta^+ \ge \delta \Rightarrow W_{\delta^+}(b) \ge W_\delta(b)$, $\forall b \in [\underline{b}, \tilde{b}]$. This property must also hold true for $b = S(\delta)(\phi)$, so $W_{\delta^+}(S(\delta)(\phi)) \ge W_\delta(S(\delta)(\phi))$. To prove the claim, we distinguish two cases:
(i) $W_\delta(\underline{b}) \ge \alpha - \phi$: The definition of S implies that $W_{\delta^+}(S(\delta^+)(\phi)) = \alpha - \phi$ and $W_\delta(S(\delta)(\phi)) = \alpha - \phi$, implying that $W_{\delta^+}(S(\delta^+)(\phi)) = W_\delta(S(\delta)(\phi))$. Joint with the above inequality this yields $W_{\delta^+}(S(\delta)(\phi)) \ge W_{\delta^+}(S(\delta^+)(\phi))$. Since $W_{\delta^+}(b)$ is monotone decreasing in b, then, $S(\delta)(\phi) \le S(\delta^+)(\phi)$.
(ii) $W_\delta(\underline{b}) < \alpha - \phi$: The definition of S yields $S(\delta)(\phi) = \underline{b}$, implying $S(\delta^+)(\phi) \ge S(\delta)(\phi) = \underline{b}$.

Discounting: Let $a \ge 0$. We claim that $S(\delta+a) \le S(\delta) + \beta a$. Corollary B1 below establishes that for any $b \in [\underline{b}, \tilde{b}]$ $\exists\tilde{\beta}(b) \in [0, \beta]$ such that $W_\delta(b) = W_{\delta+a}(b+\tilde{\beta}(b)a)$. This property must also hold for $b = S(\delta)(\phi)$. Thus, $\exists\tilde{\beta}_\phi \in [0,\beta]$ such that $W_\delta\left(S(\delta)(\phi)\right) = W_{\delta+a}\left(S(\delta)(\phi)+\tilde{\beta}_\phi a\right)$. We now distinguish three cases to prove the claim.
(i) $W_\delta(\underline{b}) \ge \alpha - \phi$: The definition of S implies $W_\delta\left(S(\delta)(\phi)\right) = \alpha - \phi$ and $W_{\delta+a}\left(S(\delta+a)(\phi)\right) = \alpha - \phi$. Thus, $W_{\delta+a}(\delta+a)\left(S(\delta+a)(\phi)\right) = W_\delta\left(S(\delta)(\phi)\right) = W_{\delta+a}\left(S(\delta)(\phi)+\tilde{\beta}_\phi a\right)$, implying that $S(\delta+a)(\phi) = S(\delta)(\phi) + \tilde{\beta}_\phi a \le S(\delta)(\phi) + \beta a$.
(ii) $W_\delta(\underline{b}) < \alpha-\phi$ and $W_{\delta+a}(\underline{b}) \ge \alpha-\phi$: The definition of S implies $S(\delta)(\phi) = \underline{b}$ and $W_{\delta+a}\left(S(\delta+a)(\phi)\right) = \alpha-\phi$. The above equality implies that $W_\delta(\underline{b}) = W_{\delta+a}(\underline{b}+\tilde{\beta}(\underline{b})a) = \alpha-\phi$. It follows that $W_{\delta+a}\left(S(\delta+a)(\phi)\right) = \alpha - \phi = W_{\delta+a}(\underline{b}+\tilde{\beta}(\underline{b})a)$ and therefore $S(\delta+a)(\phi) = \underline{b}+\tilde{\beta}(\underline{b})a \le S(\delta)+\beta a$.
(iii) $W_{\delta+a}(\underline{b}) < \alpha - \phi$: The definition of S implies that $S(\delta)(\phi) = \underline{b}$ and $S(\delta+a)(\phi) = \underline{b}$, implying that $S(\delta+a)(\phi) = S(\delta)(\phi) \le S(\delta)(\phi) + \beta a$ is necessarily satisfied.

Thus, S is a contracting operator with a unique fixed point $\hat{\delta}(\phi) = \lim_{n\to\infty} S^n(\delta)$ in Λ. This concludes the proof of the lemma. ■

The following corollary of Lemma A1 was used in the proof of Lemma A2.

Corollary B1 W_δ has the following properties: (a) W_δ is monotone in δ: if $\delta^+ \geq \delta$, then $W_{\delta^+}(b) \geq W_\delta(b)$ $\forall b \in [\underline{b}, \tilde{b}]$; (b) for any $b \in [\underline{b}, \tilde{b}]$ and $a \geq 0$, $\exists \tilde{\beta}(b) \in [0, \beta]$: $W_{\delta+a}(b + \tilde{\beta}(b)a) = W_\delta(b)$.

Proof of Corollary B1. **Part (a)**: First, note that $T_{\delta^+}(\gamma)(b) \geq T_\delta(\gamma)(b)$. The reason is that $\delta^+ \geq \delta \Rightarrow \Pi_{\delta^+}(b') \geq \Pi_\delta(b')$. Moreover, any feasible debt revenue under δ $(\beta\left[p\bar{b}' + (1-p)\Pi_\delta(b')\right])$ is also feasible under δ^+ while yielding a weakly higher continuation value than $Z^*\left(\gamma(b'), \bar{b}'\right)$. Since T_δ is a monotone contraction mapping, it follows that $W_{\delta^+}(b) \geq W_\delta(b)$. **Part (b)**: Part (a) implies that $\widetilde{T}_\delta(\gamma)(b)$ is bounded from above by $\widetilde{T}_{\delta+a}(\gamma)(b)$. $\widetilde{T}_\delta(\gamma)(b)$ is also bounded from below by

$$
\begin{aligned}
\widetilde{T}_\delta(\gamma)(b) &= \max_{\bar{b}', b' \in [\underline{b}, \tilde{b}], \Pi_\delta(b') \leq \bar{b}'} u\left(\begin{array}{c} \underline{w} - b + \beta\Psi^*\left(\gamma(b'), \bar{b}'\right)\bar{b}' \\ +\beta\left(1 - \Psi^*\left(\gamma(b'), \bar{b}'\right)\right)\Pi_\delta(b') \end{array}\right) + Z^*\left(\gamma(b'), \bar{b}'\right) \\
&\geq \max_{\bar{b}', b' \in [\underline{b}, \tilde{b}], \Pi_\delta(b') \leq \bar{b}'} u\left(\begin{array}{c} \underline{w} - b + \beta\Psi^*\left(\gamma(b'), \bar{b}'\right)\bar{b}' \\ +\beta\left(1 - \Psi^*\left(\gamma(b'), \bar{b}'\right)\right)\left[\Pi_{\delta+a}(b') - a\right] \end{array}\right) + Z^*\left(\gamma(b'), \bar{b}'\right) \\
&\geq \max_{\bar{b}', b' \in [\underline{b}, \tilde{b}], \Pi_{\delta+a}(b') \leq \bar{b}'} u\left(\begin{array}{c} \underline{w} - b - \beta a + \beta\Psi^*\left(\gamma(b'), \bar{b}'\right)\bar{b}' \\ +\beta\left(1 - \Psi^*\left(\gamma(b'), \bar{b}'\right)\right)\Pi_{\delta+a}(b') \end{array}\right) + Z^*\left(\gamma(b'), \bar{b}'\right) \\
&= \widetilde{T}_{\delta+a}(\gamma)(b + \beta a).
\end{aligned}
$$

The first inequality follows from the fact that $\Pi_{\delta+a}(b) - a \leq \Pi_\delta(b)$ for all $b \in [\underline{b}, \tilde{b}]$. The second inequality follows from $1 - \Psi^*\left(\gamma(b'), \bar{b}'\right) \leq 1$ and the fact that $\Pi_{\delta+a}(b') \leq \bar{b}'$ is a tighter constraint than $\Pi_\delta(b') \leq \bar{b}'$. Since the function $\widetilde{T}_{\delta+a}(\gamma)(x)$ is continuous in x, there must exist a $\tilde{\beta}(b) \in [0, \beta]$ such that $\widetilde{T}_{\delta+a}(\gamma)(b + \tilde{\beta}(b)a) = \widetilde{T}_\delta(\gamma)(b)$. This implies that $T_{\delta+a}(\gamma)(b + \tilde{\beta}(b)a) = T_\delta(\gamma)(b)$. Since T_δ is a contraction mapping, the same holds true at the fixed point. ■

B.3.5 Less Complete Markets (Section III.B)

Proof of Proposition B2. **Part (A):** We follow the same strategy used to prove Proposition 4. First (**Step 1**), we define an inner operator T_δ that maps value functions into value functions conditional on an *arbitrary* debt threshold function $\delta(\phi)$. We show that this operator has a unique fixed point $W_\delta = \lim_{n\to\infty} T_\delta^n$ that satisfies (B1). Second (**Step 2**), we define an outer operator S that maps debt threshold functions into debt threshold functions. We show that this operator has a unique fixed point $\hat{b}^R(\phi) = \lim_{n\to\infty} S^n(\delta)$ that satisfies (B3) when W_δ is evaluated at $\delta(\phi) = \hat{b}^R(\phi)$, i.e., $W_{\hat{b}^R}(\hat{b}^R(\phi)) = \alpha - \phi$. Let $W^R = W_{\hat{b}^R}$. Then, the fixed point $\left\langle W^R, \hat{b}^R \right\rangle$ must be unique. The uniqueness of the remaining equilibrium functions follows then from Definition (B1).

Step 1: Let Γ be the space of bounded, continuous, and nonincreasing functions $\gamma : [\underline{b}, \tilde{b}] \to [\alpha - \phi_{\max}, u(\bar{w} + (1-\beta)\underline{b})/(1-\beta)]$. Moreover, let Λ be the space of bounded and continuous functions $\delta : \mathcal{N} \to [\underline{b}, \tilde{b}]$. Define $d_\infty(y, z) \equiv \sup_{x\in\mathcal{X}} |y(x) - z(x)|$ such that (Γ, d_∞) and (Λ, d_∞) are complete metric spaces. Let $\gamma \in \Gamma$ and $\delta \in \Lambda$ and define the mapping:

$$
\begin{aligned}
\widetilde{T}_\delta(\gamma)(b) &= \max_{b' \in [\underline{b}, \tilde{b}]} u\left(\begin{array}{c} \underline{w} - b + \beta p b' \\ +\beta(1-p)\Pi_\delta(b') \end{array}\right) + Z^*\left(\gamma(b'), b'\right), \\
&\equiv \max_{b' \in [\underline{b}, \tilde{b}]} O\left(b, b', \gamma(b'); \delta\right), \qquad \text{(B21)}
\end{aligned}
$$

where $\Pi_\delta(b') = \int_{\mathcal{N}} \min\{b', \delta(\phi)\} dF(\phi)$ and $Z^*(\gamma(b'), b') = -X(p) + \beta\left[p\bar{\mu}(b') + (1-p)\mu^*(b')\right]$. In turn, $\mu^*(\gamma(b'))$ is defined in analogy with $\mu(b')$ in Definition B1, i.e., $\mu^*(\gamma(b')) = (1 - F(\Phi^*(\gamma(b')))) \times \gamma(b') + \int_{\phi_{\min}}^{\Phi^*(\gamma(b'))} [\alpha - \phi] \, dF(\phi)$, where $\Phi^*(\gamma(b')) = \alpha - \gamma(b')$. Replacing $1/R$ with β in (B21) will be convenient throughout the proof. Note that $\widetilde{T}_\delta(\gamma)$ maps γ for a given debt threshold function $\delta(\phi)$ and effort is exogenously given by p. Define, next,

$$T_\delta(\gamma)(b) = \begin{cases} \max\left\{\widetilde{T}_\delta(\gamma)(b), \alpha - \phi_{\max}\right\} & \text{if} \quad b \in [\underline{b}, b_0(\delta)] \\ \alpha - \phi_{\max} & \text{if} \quad b \in (b_0(\delta), \tilde{b}] \end{cases}, \tag{B22}$$

where

$$b_0(\delta) \equiv \underline{w} - b + \beta p\tilde{b} + \beta(1-p)\Pi_\delta(\tilde{b}) < \tilde{b} = \bar{w}/(1-\beta),$$

denotes the largest b for which nonnegative consumption is feasible. The definition of T_δ extends the mapping to a possible range in which $\widetilde{T}_\delta$ is not well-defined.

Lemma B5 *For any $\delta \in \Lambda$ the mapping $T_\delta(\gamma)$ has a unique fixed point $W_\delta = \lim_{n\to\infty} T_\delta^n(\gamma) \in \Gamma$.*

Proof of Lemma B5. The same argument used in the proof of Lemma A1 establishes that T_δ in (B22) maps Γ into Γ and is strictly decreasing in b for all b such that $T_\delta(\gamma)(b) > \alpha - \phi_{\max}$, otherwise $T_\delta = \alpha - \phi_{\max}$. Moreover T_δ discounts by the same argument used in the proof of Lemma A1 – noting that effort is exogenous. Finally, T_δ is also monotone since $\gamma^+(b) \geq \gamma(b) \Rightarrow Z^*(\gamma^+(b'), b') \geq Z^*(\gamma(b'), b')$ such that

$$\begin{aligned} \widetilde{T}_\delta(\gamma^+)(b) &\geq O(b, B_\gamma^*(b), \gamma^+(B_\gamma^*(b)); \delta) \\ &\geq O(b, B_\gamma^*(b), \gamma(B_\gamma^*(b)); \delta) = \widetilde{T}_\delta(\gamma)(b), \end{aligned}$$

where $B_\gamma^*(b) = \arg\max_{b' \in [\underline{b}, \tilde{b}]} O(b, b', \gamma(b'); \delta)$. Then, $\widetilde{T}_\delta$ satisfies Blackwell's sufficient conditions (monotonicity and discounting) on the complete metric space (Γ, d_∞), thereby being a contraction mapping. Therefore, $W_\delta = \lim_{n\to\infty} T_\delta^n(\gamma)$ exists and is unique (see Stokey, Lucas, and Prescott 1989, Theorem 3.3). ■

Step 2: We establish that there exists a unique threshold function $\hat{b}^R \in \Lambda$ such that $\left\langle W_{\hat{b}^R}, \hat{b}^R \right\rangle$ satisfies equation (B3). Let $\delta \in \Lambda$. Define the mapping:

$$S(\delta)(\phi) = \begin{cases} \min\left\{b \in [\underline{b}, \tilde{b}] : W_\delta(b) = \alpha - \phi\right\} & \text{if} \quad W_\delta(\underline{b}) \geq \alpha - \phi \\ \underline{b} & \text{if} \quad W_\delta(\underline{b}) < \alpha - \phi \end{cases}, \quad \forall \phi \in \mathcal{N}.$$

Lemma A2 then establishes that S has a unique fixed point $\hat{\delta} \equiv \lim_{n\to\infty} S^n(\delta) \in \Lambda$ and $W_{\hat{\delta}}(\hat{\delta}(\phi)) = \alpha - \phi$, $\forall \phi \in \mathcal{N}$.

Since the fixed point $\hat{\delta}$ meets the indifference condition (B3) and $W_{\hat{\delta}}$ satisfies the Bellman equation of the market equilibrium, Lemma B5 and Lemma A2 imply that $\left\langle W_{\hat{b}^R}, \hat{b}^R \right\rangle$ is the unique pair of value and threshold functions satisfying the market equilibrium conditions.

Then, the uniqueness of V^R, Φ^R, Q^R, and $\mathcal{B}^R$ follows from Definition B1. Note that we do not claim uniqueness of B^R and C^R. The continuity of the value function $W^R(b)$ in b follows from the Theorem of the Maximum, and implies that also the equilibrium functions V^R, Φ^R, and Q^R are continuous in b. Since $\widetilde{T}_{\hat{b}^R}(\gamma)$ maps decreasing functions into strictly decreasing functions, it follows that the fixed-point W^R is strictly decreasing in b and, hence, $V^R(b, \phi) = \max\left\{W^R(b), \alpha - \phi\right\}$ is nonincreasing in b.

Part (B): Having proved the existence and uniqueness of a fixed point W^R conditional on α, we now show that there exists a unique $\alpha_R \in [W_{MIN}, W_{MAX}] \equiv [\alpha - \phi_{\max}, u(\bar{w} + (1-\beta)\underline{b})/(1-\beta)]$ such that $W^R(0;\alpha_R) = \alpha_R \in (W_{MIN}, W_{MAX})$ (with slight abuse of notation we add α as a function argument in $W^R(b;\alpha)$ and $\Phi^*(\gamma(b);\alpha))$. To see why, note that, by the Theorem of the Maximum, $W^R(b;\alpha)$ is continuous in α. Moreover, since $W^R \in [W_{MIN}, W_{MAX}]$, then, Brouwer's fixed-point theorem ensures that there exists an $\alpha \in [W_{MIN}, W_{MAX}]$ such that $W^R(0;\alpha) = \alpha$. Since $W^R(0;\alpha) = \alpha - \Phi^*\left(W^R(0;\alpha);\alpha\right)$, then $\Phi^*\left(W^R(0;\alpha);\alpha\right) = 0$. To prove that such an α is unique, we note that Φ^* is monotone increasing in α (the set of states of nature in which the outside option is preferred expands as α increases). Therefore, there exists a unique fixed point α_R such that $\Phi^*\left(W^R(0;\alpha_R), \alpha_R\right) = 0$. In particular, $\alpha_R = W^R(0;\alpha_R) \in (W_{MIN}, W_{MAX})$.

This concludes the proof of the Proposition B2. ■

Proof of Proposition B3. The proof is an application of the generalized envelope theorem in Clausen and Strub (2016) which allows for discrete choices (i.e., repayment or renegotiation) and nonconcave value functions. Consider the program $W^R(b) = \max_{b' \in [\underline{b}, \tilde{b}]} O(b')$, $O(b') \equiv u(Q^R(b')b' - b + \underline{w}) + \beta Z^R(b')$. Theorem 1 in Clausen and Strub (2016) ensures that if we can find a *differentiable lower support function* (DLSF) for O, then O is differentiable at all interior optimal debt choices $b' \in \hat{B}^R$ where $\hat{B}^R$ was defined in Definition B2 above.

To construct a DLSF for O, we follow the strategy of Benveniste and Scheinkman (1979), and consider the value function of a pseudo borrower with post-renegotiation debt b that chooses debt issuance $b' = B^R(x)$ instead of the optimal $b' = B^R(b)$,

$$\widetilde{W}(b,x) \equiv u\left(Q^R\left(B^R(x)\right)B^R(x) - b + \underline{w}\right) + Z^R\left(B^R(x)\right).$$

Note that $\widetilde{W}$ is differentiable and strictly decreasing in b. Since debt issuance is chosen suboptimally, it must be that $\widetilde{W}(b,x) \le W^R(b)$ with equality holding at $x = b$. Furthermore, let the pseudo borrower set the default threshold at the level $\widetilde{\Phi}(b,x) = W^R(0) - \widetilde{W}(b,x)$, where $\widetilde{\Phi}(b,x) \ge \Phi^R(b)$. Thus, the pseudo borrower renegotiates even for some ϕ larger than $\Phi^R(b)$. Note that $\widetilde{\Phi}(b,x)$ is differentiable and strictly increasing in b. Thus, the inverse function exists and is such that $\widetilde{\Phi}_x^{-1}(\phi) \le \hat{b}^R(\phi)$ (where we define $\widetilde{\Phi}_x(b) \equiv \widetilde{\Phi}(b,x)$).

Let

$$\widetilde{O}\left(b',x\right) = u\left(\widetilde{Q}(b',x)b' - b + \underline{w}\right) + \widetilde{Z}(b',x),$$

where $\widetilde{Q}(b',x)b'$ and $\widetilde{Z}(b',x)$ are given by

$$\begin{aligned}
\widetilde{Q}\left(b',x\right)b' &= R^{-1}\left[\left(1 - \widetilde{\Psi}(b',x)\right)\left(\left[1 - F(\widetilde{\Phi}\left(b',x\right))\right]b' + \int_{\phi_{\min}}^{\widetilde{\Phi}(b',x)} \widetilde{\Phi}_x^{-1}(\phi)dF(\phi)\right) + \widetilde{\Psi}(b',x)b'\right], \\
\widetilde{Z}(b',x) &= -X(\widetilde{\Psi}(b',x)) + \beta\left[\begin{array}{c} \widetilde{\Psi}(b',x)\bar{\mu}^R(b') + (1 - \widetilde{\Psi}(b',x)) \\ \times\left(\left[1 - F(\widetilde{\Phi}\left(b',x\right))\right]\widetilde{W}\left(b',x\right) + \int_{\phi_{\min}}^{\widetilde{\Phi}(b',x)}\left[W^R(0) - \phi\right]dF(\phi)\right) \end{array}\right],
\end{aligned}$$

having defined $\widetilde{\Psi}(b',x)$ as

$$\widetilde{\Psi}(b',x) = \left(X'\right)^{-1}\left(\beta\left[\bar{\mu}^R(b') - \left(\left[1 - F(\widetilde{\Phi}\left(b',x\right))\right]\widetilde{W}\left(b',x\right) + \int_{\phi_{\min}}^{\widetilde{\Phi}(b',x)}\left[W^R(0) - \phi\right]dF(\phi)\right)\right]\right).$$

Note that $\widetilde{Q}$, $\widetilde{Z}$ and $\widetilde{\Psi}$ are differentiable in b' since we established above that $\widetilde{W}$ and $\widetilde{\Phi}$ are differentiable.

Then, $\widetilde{O}$ is a DLSF for O such that $\widetilde{O}(b', x) \leq O(b')$ with equality (only) at $b' = x$. Thus, Theorem 1 in Clausen and Strub (2016) ensures that $O(b')$ is differentiable at all optimal interior choices $b' \in \hat{B}^R$ and that $\partial O(B^R(b))/\partial B^R(b) = \partial \widetilde{O}(B^R(b), B^R(b))/\partial B^R(b) = 0$. In this case, a standard FOC holds

$$\frac{\partial u\left(Q^R(B^R(b))B^R(b) - b + \underline{w}\right)}{\partial B^R(b)} + \frac{\partial Z^R(B^R(b))}{\partial B^R(b)} = 0.$$

Moreover, Lemma 3 in Clausen and Strub (2016) ensures that also the functions $W^R(b')$, $Z^R(b')$, $\Phi^R(b')$, $Q^R(b')$, and $\Psi^R(b')$ are differentiable in $b' \in \hat{B}^R$ and that a standard envelope condition applies, namely,

$$\begin{aligned}
\frac{\partial Z^R(B^R(b))}{\partial B^R(b)} &= \beta\left[(1 - \Psi^R(B^R(b)))\left[1 - F(\Phi^R(B^R(b)))\right]\frac{\partial W^R(B^R(b))}{\partial B^R(b)} + \Psi^R(B^R(b))\frac{\partial \bar{\mu}^R(B(b))}{\partial B^R(b)}\right], \\
\frac{\partial W^R(B^R(b))}{\partial B^R(b)} &= -u'\left(C^R(B^R(b))\right) < 0.
\end{aligned}$$

This proves that the FOC stated in Proposition B3 is necessary for an interior optimum. ■

B.3.6 Proof of Lemma B1

In this section, we prove Lemma B1. The proof strategy follows Thomas and Worrall (1990, Proof of Proposition 1). We show first that the planner's problem is a contraction mapping with a strictly concave fixed-point P. The differentiability of P follows from Benveniste and Scheinkman (1979, Lemma 1). Note that $\bar{P}$ is given by (9) and has the same properties. Finally, we prove that P and $\bar{P}$ pin down uniquely interior promised utilities, effort and consumption.

We prove the results in the form of five claims. Each of them has a separate proof below. We demonstrate the proof for the COA. The properties of the first-best planning problem follow immediately by dropping the PC and adjusting the boundary conditions appropriately.

Define, first, the mapping $T(\gamma)(\nu)$ as the right-hand side of the planner's functional equation

$$T(\gamma)(\nu) = \max_{\left(\{c_\phi, p_\phi, \bar{\omega}_\phi, \omega_\phi\}_{\phi\in\mathcal{N}}\right)\in\Lambda(\nu)} \int_{\mathcal{N}} \left[\underline{w} - c_\phi + \beta\left[\begin{array}{c} p_\phi \bar{P}(\bar{\omega}_\phi) \\ +(1 - p_\phi)\gamma(\omega_\phi)\end{array}\right]\right] dF(\phi)$$

where maximization is constrained by the set $\Lambda(\nu)$ defined by

$$\begin{aligned}
\int_{\mathcal{N}} \left[u(c_\phi) - X(p_\phi) + \beta\left[p_\phi\bar{\omega}_\phi + (1 - p_\phi)\omega_\phi\right]\right] dF(\phi) &= \nu \\
u(c_\phi) - X(p_\phi) + \beta\left[p_\phi\bar{\omega}_\phi + (1 - p_\phi)\omega_\phi\right] &\geq \alpha - \phi, \quad \forall\phi \in \mathcal{N}, \\
c_\phi \in [0, \tilde{c}],\ p_\phi \in [\underline{p}, \bar{p}],\ \nu, \omega_\phi \in [\underline{\nu}, \tilde{\omega}],\ \bar{\omega}_\phi &\in [\underline{\omega}, \tilde{\bar{\omega}}].
\end{aligned}$$

We take as given that $\bar{P}$ is strictly concave and bounded between $\bar{P}_{MIN}$ and $\bar{P}_{MAX}$.

Claim 1 *$T(\gamma)$ maps concave functions into strictly concave functions.*

Proof of Claim 1. Let $\nu' \neq \nu'' \in [\underline{\nu}, \tilde{\omega}]$, $\delta \in (0, 1)$, $\nu = \delta\nu' + (1 - \delta)\nu''$, $P_k(\nu) = T(P_{k-1})(\nu)$, and P_{k-1} be concave. Then,

$$P_{k-1}(\delta\nu' + (1 - \delta)\nu'') \geq \delta P_{k-1}(\nu') + (1 - \delta)P_{k-1}(\nu'').$$

We follow the strategy of Thomas and Worrall (1990, Proof of Proposition 1), i.e., we construct a feasible but (weakly) suboptimal contract, $\left\{c^o_\phi(\nu), p^o_\phi(\nu), \bar{\omega}^o_\phi(\nu), \omega^o_\phi(\nu)\right\}_{\phi\in\mathcal{N}}$, such that even the profit generated by the suboptimal contract $P^o_k(\delta\nu' + (1-\delta)\nu'') \leq P_k(\delta\nu' + (1-\delta)\nu'')$ is higher than the linear combination of maximal profits $\delta P_k(\nu') + (1-\delta)P_k(\nu'')$. Define the weights $\underline{\delta}, \bar{\delta} \in (0,1)$ and the 4-tuple $(c^o_\phi(\nu), p^o_\phi(\nu), \omega^o_\phi(\nu), \bar{\omega}^o_\phi(\nu))$ such that

$$\begin{aligned}
\underline{\delta} &\equiv \frac{\delta\left[1-p_\phi(\nu')\right]}{\delta(1-p_\phi(\nu')) + (1-\delta)(1-p_\phi(\nu''))} \equiv \delta\frac{1-p_\phi(\nu')}{1-p^o_\phi(\nu)} \\
\bar{\delta} &\equiv \frac{\delta p_\phi(\nu')}{\delta p_\phi(\nu') + (1-\delta)p_\phi(\nu'')} \equiv \delta\frac{p_\phi(\nu')}{p^o_\phi(\nu)} \\
\omega^o_\phi(\nu) &= \underline{\delta}\omega_\phi(\nu') + (1-\underline{\delta})\omega_\phi(\nu'') \\
\bar{\omega}^o_\phi(\nu) &= \bar{\delta}\bar{\omega}_\phi(\nu') + (1-\bar{\delta})\bar{\omega}_\phi(\nu'') \\
c^o_\phi(\nu) &= u^{-1}\left[\begin{array}{c} \delta u(c_\phi(\nu')) + (1-\delta)u(c_\phi(\nu'')) \\ -\left[\delta X(p_\phi(\nu')) + (1-\delta)X(p_\phi(\nu''))\right] + X\left(\delta p_\phi(\nu') + (1-\delta)p_\phi(\nu'')\right) \end{array}\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
(1-p^o_\phi(\nu))\omega^o_\phi(\nu) &= \delta\left(1-p_\phi(\nu')\right)\omega_\phi(\nu') + (1-\delta)(1-p_\phi(\nu''))\omega_\phi(\nu'') \\
p^o_\phi(\nu)\bar{\omega}^o_\phi(\nu) &= \delta p_\phi(\nu')\bar{\omega}_\phi(\nu') + (1-\delta)p_\phi(\nu'')\bar{\omega}_\phi(\nu'') \\
u\left(c^o_\phi(\nu)\right) - X(p^o_\phi(\nu)) &= \delta u(c_\phi(\nu')) + (1-\delta)u(c_\phi(\nu'')) - \left[\delta X(p_\phi(\nu')) + (1-\delta)X(p_\phi(\nu''))\right]
\end{aligned}$$

By construction the suboptimal allocation satisfies

$$c^o_\phi(\nu) \in [0,\tilde{c}],\ p^o_\phi(\nu) \in [\underline{p},\bar{p}],\ \omega^o_\phi(\nu) \in [\underline{\nu},\tilde{\omega}],\ \bar{\omega}^o_\phi(\nu) \in [\underline{\omega},\tilde{\bar{\omega}}],$$

and, given the promised-utility ν, is also consistent with the PK

$$\begin{aligned}
&\int_{\mathcal{N}} \left[u\left(c^o_\phi(\nu)\right) - X(p^o_\phi(\nu)) + \beta\left[p^o_\phi(\nu)\bar{\omega}^o_\phi(\nu) + (1-p^o_\phi(\nu))\omega^o_\phi(\nu)\right]\right] dF(\phi) \\
= &\int_{\mathcal{N}} \left[\begin{array}{c} \delta u\left(c_\phi(\nu')\right) + (1-\delta)u\left(c_\phi(\nu'')\right) - \left[\delta X(p_\phi(\nu') + (1-\delta)X(p_\phi(\nu''))\right] \\ +\beta\left[\delta(1-p_\phi(\nu'))\omega_\phi(\nu') + (1-\delta)(1-p_\phi(\nu''))\omega_\phi(\nu'')\right] \\ +\beta\left[\delta p_\phi(\nu')\bar{\omega}_\phi(\nu') + (1-\delta)p_\phi(\nu''))\bar{\omega}_\phi(\nu'')\right] \end{array}\right] dF(\phi) \\
= &\ \delta\nu' + (1-\delta)\nu'' = \nu.
\end{aligned}$$

Moreover, the PC for any ϕ yields

$$\begin{aligned}
&u\left(c^o_\phi(\nu)\right) - X(p^o_\phi(\nu)) + \beta\left[p^o_\phi(\nu)\bar{\omega}^o_\phi(\nu) + (1-p^o_\phi(\nu))\omega^o_\phi(\nu)\right] \\
= &\left[\begin{array}{c} \delta u\left(c_\phi(\nu')\right) + (1-\delta)u\left(c_\phi(\nu'')\right) - \left[\delta X(p_\phi(\nu') + (1-\delta)X(p_\phi(\nu''))\right] \\ +\beta\left[\delta p_\phi(\nu')\bar{\omega}_\phi(\nu') + (1-\delta)p_\phi(\nu''))\bar{\omega}_\phi(\nu'')\right] \\ +\beta\left[\delta(1-p_\phi(\nu'))\omega_\phi(\nu') + (1-\delta)(1-p_\phi(\nu''))\omega_\phi(\nu'')\right] \end{array}\right] \\
= &\ \delta\left[u\left(c_\phi(\nu')\right) - X(p_\phi(\nu') + \beta p_\phi(\nu')\bar{\omega}_\phi(\nu') + \beta(1-p_\phi(\nu'))\omega_\phi(\nu')\right] \\
&+(1-\delta)\left[u(c_\phi(\nu'')) - X(p_\phi(\nu'')) + \beta p_\phi(\nu''))\bar{\omega}_\phi(\nu'') + \beta(1-p_\phi(\nu''))\omega_\phi(\nu'')\right] \\
\geq &\ \delta\left(\alpha-\phi\right) + (1-\delta)\left(\alpha-\phi\right) = \alpha-\phi,
\end{aligned}$$

Thus, we have proven that the suboptimal allocation $\left\{c^o_\phi(\nu), p^o_\phi(\nu), \omega^o_\phi(\nu), \bar{\omega}^o_\phi(\nu)\right\}_{\phi\in\mathcal{N}}$ is feasible. Namely, it satisfies the PCs and delivers promised utility ν. The profit function evaluated at the optimal contract $\{c_\phi(\nu), p_\phi(\nu), \omega_\phi(\nu), \bar{\omega}_\phi(\nu)\}_{\phi\in\mathcal{N}}$ then implies the following inequality,

$$
\begin{aligned}
& \delta P_k(\nu') + (1-\delta)P_k(\nu'') \\
= \ & \delta T(P_{k-1})(\nu') + (1-\delta)T(P_{k-1})(\nu'') \\
= \ & \int_{\mathcal{N}} \left[\begin{array}{c} \underline{w} - [\delta c_\phi(\nu') + (1-\delta)c_\phi(\nu'')] \\ +\beta\left[\delta p_\phi(\nu')\bar{P}(\bar{\omega}_\phi(\nu')) + (1-\delta)p_\phi(\nu'')\bar{P}(\bar{\omega}_\phi(\nu''))\right] \\ +\beta\left[\delta(1-p_\phi(\nu'))P_{k-1}(\omega_\phi(\nu')) + (1-\delta)(1-p_\phi(\nu''))P_{k-1}(\omega_\phi(\nu''))\right] \end{array} \right] dF(\phi) \\
= \ & \int_{\mathcal{N}} \left[\begin{array}{c} \underline{w} - [\delta c_\phi(\nu') + (1-\delta)c_\phi(\nu'')] \\ +\beta p^o_\phi(\nu)\left[\bar{\delta}\bar{P}(\bar{\omega}_\phi(\nu')) + (1-\bar{\delta})\bar{P}(\bar{\omega}_\phi(\nu''))\right] \\ +\beta(1-p^o_\phi(\nu))\left[\underline{\delta}P_{k-1}(\omega_\phi(\nu')) + (1-\underline{\delta})P_{k-1}(\omega_\phi(\nu''))\right] \end{array} \right] dF(\phi) \\
< \ & \int_{\mathcal{N}} \left[\begin{array}{c} \underline{w} - u^{-1}\left(\delta u(c_\phi(\nu')) + (1-\delta)u(c_\phi(\nu''))\right) \\ +\beta p^o_\phi(\nu)\bar{P}(\bar{\delta}\bar{\omega}_\phi(\nu') + (1-\bar{\delta})\bar{\omega}_\phi(\nu'')) \\ +\beta(1-p^o_\phi(\nu))P_{k-1}(\underline{\delta}\omega_\phi(\nu') + (1-\underline{\delta})\omega_\phi(\nu'')) \end{array} \right] dF(\phi) \\
< \ & \int_{\mathcal{N}} \left[\begin{array}{c} \underline{w} - u^{-1}\left(\begin{array}{c} \delta u(c_\phi(\nu')) + (1-\delta)u(c_\phi(\nu'')) \\ -[\delta X(p_\phi(\nu')) + (1-\delta)X(p_\phi(\nu''))] + X\delta p_\phi(\nu') + (1-\delta)p_\phi(\nu'') \end{array} \right) \\ +\beta p^o_\phi(\nu)\bar{P}(\bar{\delta}\bar{\omega}_\phi(\nu') + (1-\bar{\delta})\bar{\omega}_\phi(\nu'')) \\ +\beta(1-p^o_\phi(\nu))P_{k-1}(\underline{\delta}\omega_\phi(\nu') + (1-\underline{\delta})\omega_\phi(\nu'')) \end{array} \right] dF(\phi) \\
= \ & \int_{\mathcal{N}} \left[\underline{w} - c^o_\phi(\nu) + \beta\left[p^o_\phi(\nu)\bar{P}(\bar{\omega}^o_\phi(\nu)) + (1-p^o_\phi(\nu))P_{k-1}(\omega^o_\phi(\nu))\right]\right] dF(\phi) \\
\equiv \ & P^o_k(\nu) \le P_k(\nu) = P_k(\delta\nu' + (1-\delta)\nu'').
\end{aligned}
$$

The first inequality follows from the strict concavity of u and $\bar{P}$, along with the concavity of P_{k-1}. The second inequality follows from $0 \le X\left(\delta p_\phi(\nu') + (1-\delta)p_\phi(\nu'')\right) < \delta X(p_\phi(\nu')) + (1-\delta)X(p_\phi(\nu''))$ since X is strictly convex. The third inequality, $P^o_k(\nu) \le P_k(\nu)$ follows from the fact that the optimal allocation delivers (weakly) larger profits than the suboptimal one. We conclude that $P_k(\delta\nu' + (1-\delta)\nu'') > \delta P_k(\nu') + (1-\delta)P_k(\nu'')$, i.e., P_k is strictly concave. This concludes the proof of the lemma. ■

Let Γ denote the space of continuous functions defined over the interval $[\underline{\nu}, \tilde{\omega}]$ and bounded between $P_{MIN} = \left(\underline{w} - \tilde{c} + \beta\underline{p}\bar{P}_{MIN}\right)/(1-\beta(1-\underline{p}))$ and $P_{MAX} = \bar{w}/(1-\beta)$. Moreover, let d_∞ denote the supremum norm, such that (Γ, d_∞) is a complete metric space.

Claim 2 *The mapping $T(\gamma)$ is an operator on the complete metric space (Γ, d_∞), $T(\gamma)$ is a contraction mapping with a unique fixed-point $P \in \Gamma$.*

Proof of Claim 2. By the Theorem of the Maximum $T(\gamma)(\nu)$ is continuous in ν. Moreover, $T(\gamma)(\nu)$ is bounded between P_{MIN} and P_{MAX} since even choosing zero consumption for any realization of ϕ would induce profits not exceeding P_{MAX}

$$
\begin{aligned}
\underline{w} + \beta\int_{\mathcal{N}} \left[p_\phi\bar{P}(\bar{\omega}_\phi) + (1-p_\phi)\gamma(\omega_\phi)\right] dF(\phi) &< \bar{w} + \beta/(1-\beta)\bar{w} \\
&= \bar{w}/(1-\beta) = P_{MAX},
\end{aligned}
$$

and choosing the maximal consumption $\tilde{c}$ and promised utility $\tilde{\omega}$ and $\tilde{\bar{\omega}}$ for any ϕ would induce profits no lower than P_{MIN}. Thus, $T(\gamma)(\nu)$ is indeed an operator on (Γ, d_∞).

According to Blackwell's sufficient conditions $T(\gamma)$ is a contraction mapping (see Stokey, Lucas, and Prescott 1989, Theorem 3.3) if: (i) T is monotone, (ii) T discounts.

1. Monotonicity: Let $\gamma^+, \gamma \in \Gamma$ with $\gamma^+(\nu) \geq \gamma(\nu)$, $\forall \nu \in [\underline{\nu}, \tilde{\omega}]$. Then

$$\begin{aligned}
T(\gamma^+)(\nu) &= \max_{(\{c_\phi, p_\phi, \bar{\omega}_\phi, \omega_\phi\}_{\phi \in \mathcal{N}}) \in \Lambda(\nu)} \int_{\mathcal{N}} \left[\underline{w} - c_\phi + \beta \left[\begin{array}{c} p_\phi \bar{P}(\bar{\omega}_\phi) \\ +(1-p_\phi)\gamma^+(\omega_\phi) \end{array} \right] \right] dF(\phi) \\
&\geq \max_{(\{c_\phi, p_\phi, \bar{\omega}_\phi, \omega_\phi\}_{\phi \in \mathcal{N}}) \in \Lambda(\nu)} \int_{\mathcal{N}} \left[\underline{w} - c_\phi + \beta \left[\begin{array}{c} p_\phi \bar{P}(\bar{\omega}_\phi) \\ +(1-p_\phi)\gamma(\omega_\phi) \end{array} \right] \right] dF(\phi) \\
&= T(\gamma)(\nu).
\end{aligned}$$

2. Discounting: Let $\gamma \in \Gamma$ and $a \geq 0$ be a real constant. Then

$$\begin{aligned}
T(\gamma + a)(\nu) &= \max_{(\{c_\phi, p_\phi, \bar{\omega}_\phi, \omega_\phi\}_{\phi \in \mathcal{N}}) \in \Lambda(\nu)} \int_{\mathcal{N}} \left[\underline{w} - c_\phi + \beta \left[\begin{array}{c} p_\phi \bar{P}(\bar{\omega}_\phi) \\ +(1-p_\phi)\left(\gamma(\omega_\phi) + a\right) \end{array} \right] \right] dF(\phi) \\
&= T(\gamma)(\nu) + \beta a \int_{\mathcal{N}} (1 - p_\phi) dF(\phi) \\
&\leq T(\gamma)(\nu) + \beta a
\end{aligned}$$

   and $\beta \in (0, 1)$.

Thus, $T(\gamma)$ is indeed a contraction mapping and according to Banach's fixed-point theorem (see Stokey, Lucas, and Prescott 1989, Theorem 3.2) there exists a unique fixed-point $P \in \Gamma$ satisfying the stationary functional equation,

$$P(\nu) = T(P)(\nu).$$

■

Claim 3 *The profit function P is strictly concave.*

Proof of Claim 3. This claim follows immediately from Stokey, Lucas, and Prescott 1989, Corollary 1). Since the unique fixed-point of $T(\gamma)$ is the limit of applying the operator n times starting from any element γ in Γ (and, in particular the concave elements), and the operator $T(\gamma)$ maps concave into strictly concave functions the fixed-point P must be strictly concave. ■

Claim 4 *The profit function P is differentiable at its interior support with $P'(\nu) = 1/u'(c(\nu)) < 0$.*

Proof of Claim 4. Given the strict concavity of the profit function, the proof is the same as for Lemma B3. The only difference is that $p_\phi(\nu)$ denotes the optimal effort stated in Proposition 2 instead of Proposition 3. ■

We can now establish that the FOCs of the COA are necessary and sufficient.

Claim 5 *The FOCs of the planner problem without moral hazard are necessary and sufficient for optimality.*

Proof of Claim 5. Lemma 1 implies that there cannot be two optimal contracts with distinct ω_ϕ and $\bar{\omega}_\phi$. Suppose not, so that there exists a 4-tuple of promised utilities $\left\{\omega'_\phi, \omega''_\phi, \bar{\omega}'_\phi, \bar{\omega}''_\phi\right\}$ such that either $\omega'_\phi(\nu) \neq \omega''_\phi(\nu)$ or $\bar{\omega}'_\phi(\nu) \neq \bar{\omega}''_\phi(\nu)$ (or both). Then, from the strict concavity of P and $\bar{P}$, it would be possible to construct a feasible allocation that dominates the continuation profit implied by the proposed optimal allocations, i.e., either $P(\underline{\delta}\omega'_\phi + (1-\underline{\delta})\omega''_\phi) > \underline{\delta}P(\omega'_\phi) + (1-\underline{\delta})P(\omega''_\phi)$, or $\bar{P}(\bar{\delta}\bar{\omega}'_\phi + (1-\bar{\delta})\bar{\omega}''_\phi) > \bar{\delta}\bar{P}(\bar{\omega}'_\phi) + (1-\bar{\delta})\bar{P}(\bar{\omega}''_\phi)$ (or both). This contradicts the assumption that the proposed allocations are optimal, establishing that the optimal contract pins down a unique pair of promised utilities, $\left\{\omega_\phi, \bar{\omega}'_\phi\right\}$.

Finally, we show that a unique pair of promised utilities pins down uniquely effort and consumption. The assumptions on X rule out corner solutions for effort, the assumptions on u and the fact that promised utility is interior (ω_ϕ and $\bar{\omega}_\phi$ remain constant for $\nu \geq \alpha - \phi_{\min}$) implies that also consumption is interior. Then the FOCs in (10) and (12) imply that

$$\begin{aligned} -\bar{P}'\left(\bar{\omega}_\phi(\nu)\right)^{-1} &= u'(c_\phi(\nu)), \\ X'\left(p_\phi(\nu)\right) &= \beta\left(-\bar{P}'\left(\bar{\omega}_\phi(\nu)\right)^{-1}\left(\bar{P}\left(\bar{\omega}_\phi(\nu)\right) - P\left(\omega_\phi(\nu)\right)\right) + \left(\bar{\omega}_\phi(\nu) - \omega_\phi(\nu)\right)\right), \end{aligned}$$

which shows that, given ν and ϕ, effort and consumption are uniquely determined as well. ■

This concludes the proof of Lemma B1.

B.4 Parameterization

In this section, we provide details of the parameterization underlying the numerical examples shown in the figures of the paper. We focus on the quantitative properties of the one-asset economy (with renegotiation) of Section IV since this is a more realistic positive representation of the world. We choose parameters so as to match salient moments observed for Greece, Ireland, Italy, Portugal, and Spain (GIIPS) during the Great Recession.

A model period corresponds to one year. We normalize the GDP during normal time to $\bar{w} = 1$ and assume that the recession causes a drop in income of 25 percent, i.e., $\underline{w} = 0.75 \times \bar{w}$. This corresponds to the fall of real GDP per capita for Greece between 2007 and 2016.[33] The annual real gross interest rate is set to $R = 1.02$. The utility function is assumed to be CRRA with a relative risk aversion of 2. We assume an isoelastic effort cost function, $X(p) = \xi p^{1+1/10} / (1 + 1/10)$, and calibrate $\xi = 14.371$ so that a country starting with a 100 percent debt-output ratio in recession recovers in expectation after one decade (we have Greece in mind). Finally, we parameterize $f(\phi)$ and its support. The maximum default cost realization $\phi_{\max} = 2.275$ is calibrated to target a debt limit during recession, $b^{\max}/\underline{w}$, of 178 percent in line with Collard, Habib, and Rochet (2015, Table 3, Column 1).[34] Finally, we assume that $\phi_{\max} - \phi$ is distributed exponential with rate parameter $\eta = 1.625$ and truncation point $\phi_{\max}$.[35] The model then predicts an average default premium of 4.04 percent for a country with a debt-output ratio of 100 percent in recession. This overlaps with the average debt and average default premium for the GIIPS during 2008-2012 (Eurostat).

[33] Greece's real GDP per capita fell from 22'700 to 17'100 Euro between 2007 and 2016 (Eurostat, nama_10_pc series). Where the years 2007 and 2016 correspond to the peak and the trough, respectively, of real GDP per capita relative to a 2 percent growth trend with base year 1995.

[34] We ignore the value of 282 percent for Korea which is a clear outlier.

[35] More formally, ϕ has the p.d.f.

$$f(\phi) = \frac{\eta e^{-\eta(\phi_{\max} - \phi)}}{1 - e^{-\eta \phi_{\max}}}, \; \phi \in [0, \phi_{\max}].$$

This also implies that $\phi_{\min} = 0$.

B.5 References for Appendix B

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