The Polarization of the U.S. Labor Market: Theory Appendix

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1 Motivation

Motivating ideas:

1. Workplace tasks may be usually subdivided into three broad groups:

   (a) Abstract problem-solving and managerial tasks. These tasks are not well structured and require non-routine cognitive skills.

   (b) Routine tasks. These may either be cognitive or physical tasks that they follow closely prescribed sets of rules and procedures and are executed in a well-controlled environment.

   (c) Manual tasks. These tasks (which ALM call non-routine manual) do not require abstract problem solving or managerial skills but are nevertheless difficult to automate because they require some flexibility in a less than fully predictable environment. Examples include: truck drivers, security guards, unskilled medical personnel, janitors and house-cleaners, high speed check-keyers, construction workers, many in-person servers.

2. Routine tasks are complementary to Abstract tasks and perhaps also to Manual tasks (though probably less so).

3. Routine tasks are readily substituted with computer capital. The continually falling price of computer capital drives this substitution.

4. The falling price of computer capital generates an incentive for workers engaged in Routine tasks to switch to other tasks.

5. It is more difficult for workers displaced from Routine tasks to shift 'up' to Abstract tasks than it is for them to shift 'down' to Manual tasks.

2 Model

2.1 Production

Aggregate output in this economy (priced at unity) is given by the Cobb-Douglas production function

$$Y = A^{\alpha} R^{\beta} M^{\gamma},$$

where A, R and M are Abstract, Routine and Manual tasks, with exponents $\alpha, \beta, \gamma \in (0, 1)$ respectively, and $\alpha + \beta + \gamma = 1$. Abstract and manual tasks can *only* be performed by workers who supply labor inputs, L_A and L_M. Routine tasks can be performed *either* by workers who supply L_R or by computer capital, K, measured in efficiency units, which is a perfect substitute for L_R.

2.2 Factor supplies

Computer capital is supplied perfectly elastically to Routine tasks at price ρ per efficiency unit. The secularly declining price of computer capital is the exogenous driving force in this model.

There is a large number of income-maximizing workers in this economy, each endowed with a vector of three skills, $S_i = (a_i, r_i, m_i)$, where lower-case letters denote an individual's skill endowment for the three production tasks.

Workers are of two types. A fraction $\theta \in (0,1)$ are High School (H) workers and the remaining $1-\theta$ are College (C) workers. For simplicity, all college workers are assumed identical. Each is endowed with one efficiency unit of Abstract skill: $S^C(a,r,m) = (1,0,0)$.[1] The labor supply of each college worker is $L^C = (1,0,0)$.

All high school workers are equally skilled in Manual tasks but differ in their ability in Routine tasks. We write the skill endowment of high school worker i as $S_i^H(a,r,m_i) = (0,\eta_i,1)$, where η is a continuous variable distributed on the unit interval with positive probability mass at all points $\eta \in (0,1)$. The labor supply of High School worker i is $L_i^H(a,r,m) = (0,\lambda_i\eta_i,(1-\lambda_i))$ where $\lambda_i \in [0,1]$. Each High School worker chooses λ_i to maximize earnings.

2.3 Equilibrium concept

Equilibrium in this model occurs when:

1. Productive efficiency is achieved—that is, the economy operates on the demand curve of the aggregate production function for each factor.

2. All factors are paid their marginal products.

3. The labor market clears; no worker wishes to reallocate labor input among tasks.

2.4 Productive efficiency

The wage of each factor is given by:

$$\begin{aligned} w_a &= \frac{\partial Y}{\partial A} = \alpha A^{\alpha-1} R^{\beta} M^{\gamma}, \\ w_r &= \frac{\partial Y}{\partial R} = \beta A^{\alpha} R^{\beta-1} M^{\gamma}, \\ w_m &= \frac{\partial Y}{\partial M} = \gamma A^{\alpha} R^{\beta} M^{\gamma-1}. \end{aligned}$$

[1] It would be a very minor matter to instead assume that $L_C(a,r,m) = (1,1,1)$ with $w_a > w_r, w_m$ for all relevant cases. This would ensure that college-workers always supply Abstract labor—and of course this tendency would only be reinforced by a falling price of computer capital.

2.5 Self-selection of workers to tasks

The supply of College labor to Abstract tasks is inelastic.

The supply of High School labor to Routine and Manual tasks is determined by self-selection. Each High School worker i chooses to supply one efficiency unit of labor to Manual tasks if $\eta_i < w_m/w_r$, and supplies η_i efficiency units of labor to Routine tasks otherwise. We can write the labor supply functions to Manual and Routine tasks as $L_M(w_m/w_r) = \theta \sum_i 1[\eta_i < w_m/w_r]$ and $L_R(w_m/w_r) = \theta \sum_i \eta_i \cdot 1[\eta_i \geq w_m/w_r]$, where $1[\cdot]$ is the indicator function. Observe that $L'_M(\cdot) \geq 0$ and $L'_R(\cdot) \leq 0$.

2.6 Equilibrium and comparative statics

Since computer capital is a perfect substitute for routine labor input, it is immediate that $w_r = \rho$ and hence a decline in ρ reduces w_r one for one.[2]

We are interested in the effect of a decline in ρ on:

1. The equilibrium quantity of Routine task input
2. The allocation of labor between Routine and Manual tasks
3. The wage paid to each task
4. The observed wage in each job type (which in may differ from the wage per efficiency unit in Routine tasks)

A decline in ρ raises demand for Routine tasks, since own-factor demand curves are downward sloping ($R'(\rho) < 0$). This demand can be supplied by either additional computer capital or Routine labor input. Due to worker self-selection, the additional demand will be supplied by computer capital.

To see this, let η^* equal the Manual skill level of the marginal worker, such that $\eta^* = w_m/w_r$. Rewriting η^* using the marginal productivity conditions:

$$\eta^* = \frac{w_m}{w_r} = \frac{\gamma R}{\beta L_M(\eta^*)}.$$

Differentiating with respect to $-\rho$ (a decline in the price of K) gives

$$-\frac{\partial \eta^*}{\partial \rho} = \frac{\gamma}{\beta}\left[\frac{\partial \eta^*}{\partial \rho} \cdot \frac{R L'_M(\eta^*)}{L_M(\eta^*)^2} - \frac{\partial R/\partial \rho}{L_M(\eta^*)}\right] = -\frac{\gamma L_M(\eta^*) \cdot \partial R/\partial \rho}{\beta L_M(\eta^*)^2 + \gamma R L'_M(\eta^*)} > 0. \tag{1}$$

A decline in ρ raises the relative Manual/Routine wage.

[2] Technically, ρ only binds w_r from above. In an earlier period, when computer capital was far more expensive, it's plausible that $w_r < \rho$. For the period under study, we assume that this constraint binds.

Summing up these wage implications:

$$
\begin{aligned}
-\frac{\partial w_r}{\partial_\rho} &= -1, \\
-\frac{\partial w_m}{\partial \rho} &= -\gamma A^{\alpha}\left[\underbrace{\beta {L_M}^{\gamma-1} R^{\beta-1}\frac{\partial R}{\partial \rho}}_{(-)} + \underbrace{(\gamma-1) R^{\beta} L_M (\eta^*)^{\gamma-2} L'_M(\eta^*)\frac{\partial \eta^*}{\partial \rho}}_{(+/0)}\right] \lesseqgtr 0, \\
-\frac{\partial w_a}{\partial \rho} &= -\alpha A^{\alpha-1}\left[\underbrace{\beta R^{\beta-1}\frac{\partial R}{\partial \rho}}_{(-)} + \underbrace{\gamma L_M (\eta^*)^{\gamma-1} L'_M(\eta^*)\frac{\partial \eta^*}{\partial \rho}}_{(-)}\right] > 0.
\end{aligned}
$$

A decline in the price of computer capital lowers the wage of Routine labor input, raises the wage of Abstract labor input through two channels of q-complementarity—increased use of Routine task input and increased labor supply to Manual task input—and has ambiguous implications for the wage of Manual task input (due to the countervailing effects of q-complementarity between Routine and Manual tasks and increased labor supply to Manual tasks).

Though, as established above, a decline in ρ yields a larger proportionate fall in the wage of Routine than Manual tasks ($-\partial (w_m/w_r)/\partial\rho > 0$), the *observed* log wage differential between workers in Routine and Manual jobs may *rise* despite the fall in w_r/w_m. The reason is that a decline in w_r leads to marginal workers with lower values of η to exit Routine jobs, inducing a *positive* compositional shift in the pool of workers in Routine occupations ($-\partial E[\eta|\eta > \eta^*]/\partial\rho > 0$).

Summarizing:

1. A decline in the price of computer capital causes an increase in demand for Routine task input.

2. This increase is entirely supplied by computer capital as the price decline causes a corresponding reduction in labor supply to Routine tasks and an increase in labor supply to Manual tasks.

3. The reduction in the price of computer capital has the following implications for wages levels measured in *efficiency units:*

$$-\frac{\partial w_r}{\partial_\rho} < 0, -\frac{\partial w_m}{\partial \rho} \lesseqgtr 0, -\frac{\partial w_r/w_m}{\partial \rho} < 0, -\frac{\partial w_a}{\partial \rho} > 0.$$

4. The reduction in the price of computer capital has the following implications for *observed* wage levels:

$$-\frac{\partial \hat{w}_r}{\partial_\rho} \lesseqgtr 0, -\frac{\partial \hat{w}_m}{\partial \rho} \lesseqgtr 0, -\frac{\partial w_r/w_m}{\partial \rho} \lesseqgtr 0, -\frac{\partial \hat{w}_a}{\partial \rho} > 0,$$

   where 'hats' over wage variables denote observed values that do not adjust for changes in occupational skill composition (e.g., $\partial E[\eta|\eta > \eta^*]/\partial\rho$).[3]

[3] For Manual and Abstract tasks, compositional shifts are nil by assumption.

Remark 1: The model does not pin down the ranking of wages in Abstract, Routine and Manual tasks; these levels depend on labor supplies and ρ. For workers who switch from Routine to Manual tasks as ρ falls, the manual wage must be higher than the Routine wage ($w_m > \eta_i w_r$). But for inframarginal Routine workers, it must be the case that the Routine wage is higher than the Manual wage. This observation has an important empirical implication: if there are *any* workers remaining in the Routine job, their *observed* wage (i.e., not accounting for composition) must be higher than the wage in the Manual job since there is no skill heterogeneity in Manual tasks. Hence, even in cases where $w_r < w_m$, it will be true that $\hat{w}_r > \hat{w}_m$ provided that $L_R\,(w_m/w_r) > 0$.

Remark 2: In an equilibrium in which H workers supply both Routine tasks and Manual tasks, K is a direct substitute for some H workers and a complement to others. In this setting, a decline in ρ causes a 'widening' of wage inequality by lowering w_m relative to w_a (moreover, w_m may fall in absolute terms). In an equilibrium with ρ sufficiently low such that no workers remain in Routine tasks ($w_m/w_r > 1$), further declines in ρ unambiguously benefit both High School and College workers and hence do not augment inequality. (Given the Cobb-Douglas form, both groups benefit equally, so this has no effect on w_a/w_m.)