

Online Appendix to ‘Optimal Taxation with Risky Human Capital’

Marek Kapička
CERGE-EI

Julian Neira
University of Exeter

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This online appendix explores a number of additional robustness exercises and extensions.

1 Verification of the First Order Approach

Because second period utility is not monotonically increasing in human capital realizations, the sufficient conditions in Proposition 1 are not satisfied. Therefore, we directly verify that the expected lifetime utility is concave in the effort. Figure 1 shows that this is indeed the case, and that the first order approach is valid.

Figure 1 also serves to illustrate the point that, as harsh as the welfare losses seem for high ability agents, they still enjoy the highest welfare. This, of course, is the direct outcome of the private information constraint.

Figure 2 illustrates that the savings wedge for the main calibration with effort share $\eta = 0.13$ and for the HVY economy with $\eta = 1$. The savings wedge in both cases is strictly increasing in ability. Since high-ability agents face the riskiest second period consumption, they have the highest incentive to self-insure through savings. In order to discourage this, the social planner imposes a higher savings tax on them. This result stands in contrast to the previous literature that features exogenous i.i.d. shocks (e.g. [Albanesi and Sleet 2006](#)), where high-ability agents typically face less consumption risk, and a lower savings wedge than low-ability agents.

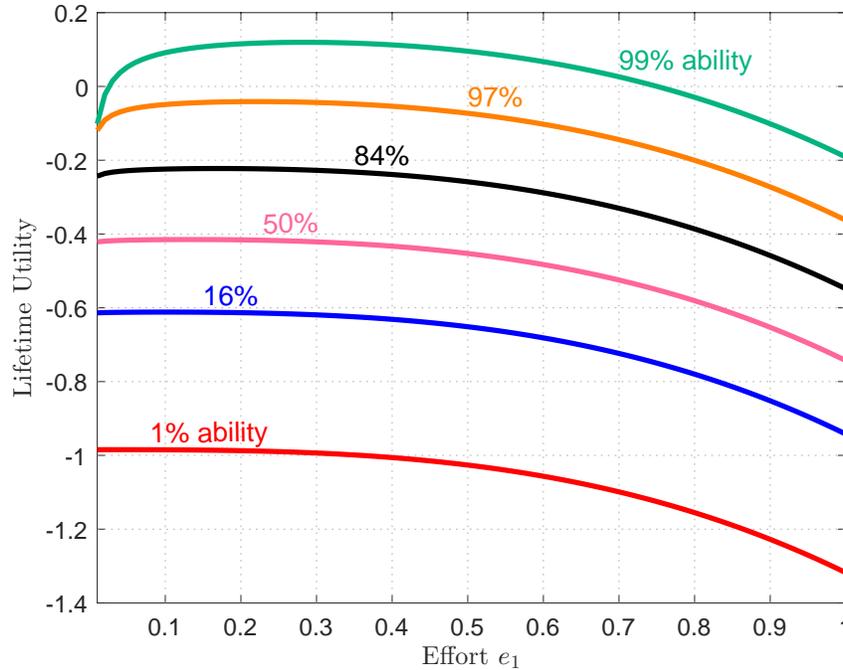


Figure 1: Verifying the concavity of effort decision

The main difference between the two specifications is in the magnitude. For the case with $\eta = 0.13$, the highest ability agents face a savings wedge of 23 percent. This translates into an efficient annual tax on capital income of 33.8 percent.¹ Thus, even for the highest ability agents, the efficient tax on capital income is about 4 percent lower than the status quo U.S. tax rate. An individual with median abilities would, on the other hand, face a capital income tax rate of only 6.7 percent, substantially less than under the current U.S. tax code. For the case with $\eta = 1$, the savings wedge is uniformly higher for all ability levels. For the highest ability, the savings wedge is 28. In this case, the planner is more reliant on higher consumption risk, so higher savings wedges are necessary to discourage precautionary savings from higher ability agents.

2 Savings Wedge

It is useful to compare the magnitudes of the capital tax to [Farhi and Werning \(2013\)](#) and [Goloso, Troshkin, and Tsyvinski \(2016\)](#). While [Farhi and Werning \(2013\)](#) does not

¹We use the relationship $1 - \delta = \beta(1 + r(1 - \tau^k))$ to recover the tax on capital earnings over 20 years τ^k , and then convert it to an annual value.

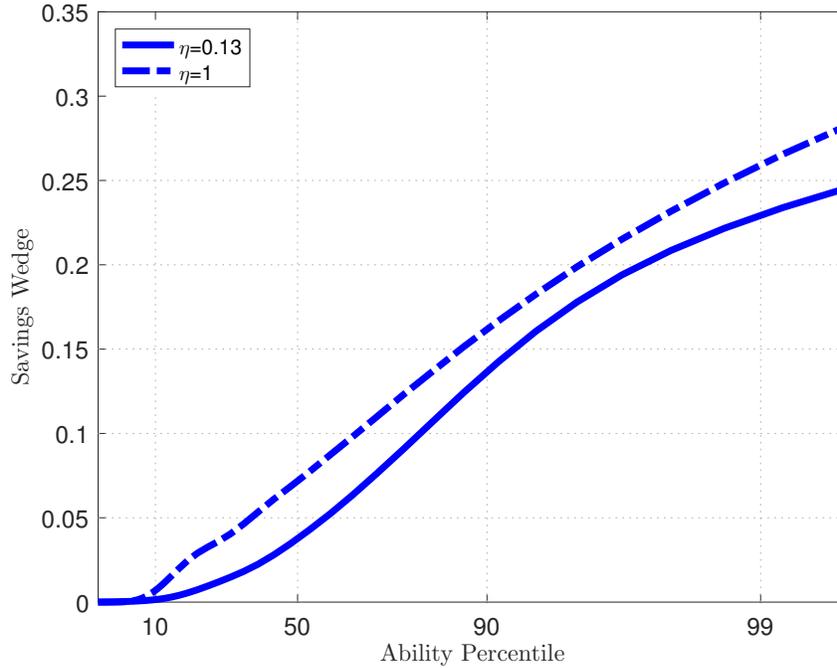


Figure 2: Savings Wedge

report how the implied tax on capital varies with current ability or current income, but report that it decreases with age. It is on average about 12 percent at the beginning of one’s lifecycle, and then decreases to zero. In our model, the average capital tax is 7.3 percent annually, which is about 40 percent lower. In [Goloso, Troshkin, and Tsyvinski \(2016\)](#), agents with the highest income level typically face about 100 -125 percent implied tax on capital², which is substantially more than our tax in either of the two scenarios. In both models, the tax on savings is higher than in our model. The cross-sectional pattern of the savings wedge in [Goloso, Troshkin, and Tsyvinski \(2016\)](#) is, however, similar to ours in that it increases in income.

3 No moral hazard

We set the Lagrange multiplier on effort $\phi(a)$ equal to 0 for all abilities to study the impact of moral hazard on human capital investments. Without the moral hazard con-

²The savings wedge is about 0.02-0.025 for the highest income agents, $\beta = 0.98$ and $1 + r = \beta^{-1}$.

straint there is no need to incentivize effort indirectly via consumption risk or schooling subsidies less than one (see Figure 3a). Instead, effort can be elicited directly by distorting the first order condition in effort. Learning effort is significantly higher for all agents, as shown in Figure 3b. Schooling increases for high ability agents as well, as it no longer imposes a cost on keeping the Euler in effort undistorted. As this cost was the highest for high ability agents, it is these agents that experience the largest increase in schooling, albeit small. The increase in schooling is shown in Figure 3c.

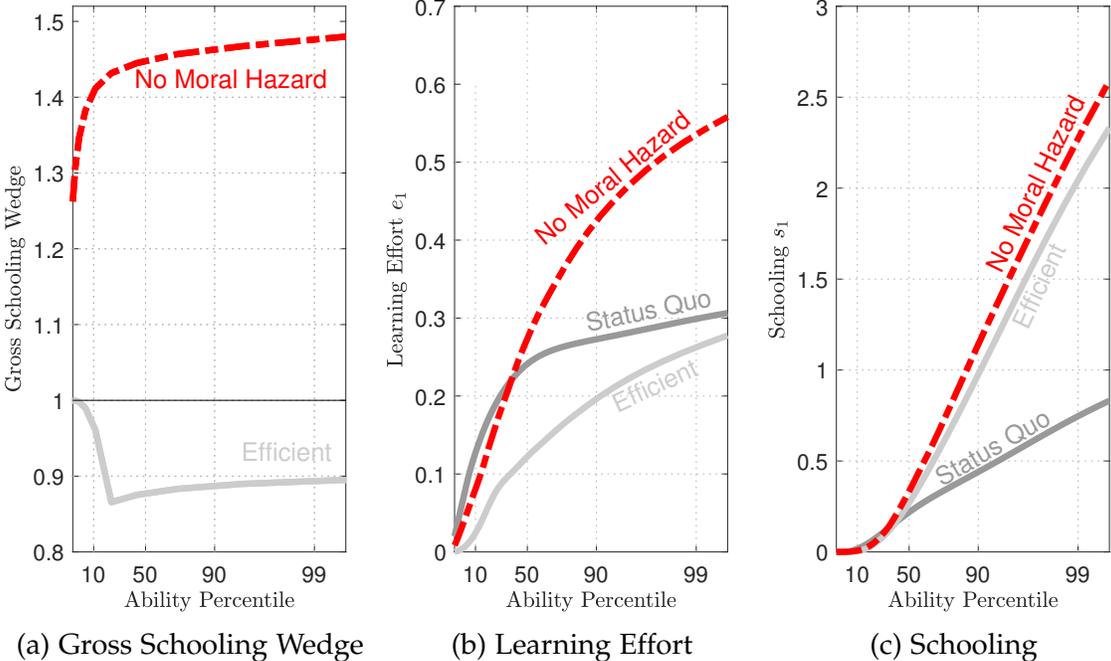
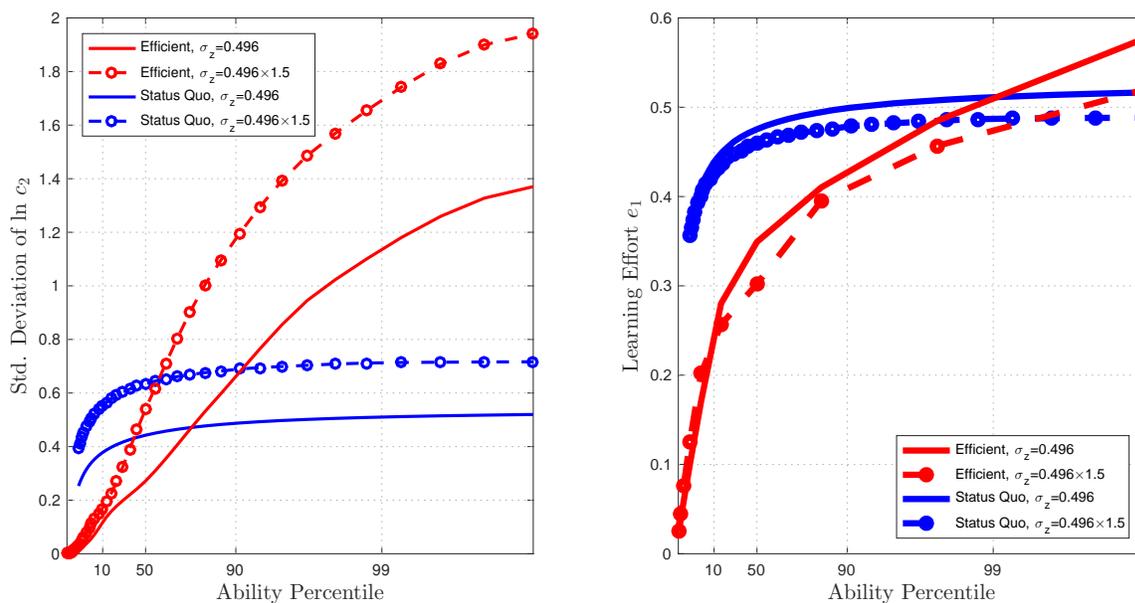


Figure 3: Schooling wedge, effort and schooling without moral hazard.

4 Higher Dispersion of Shocks

If we increase the variance of shocks, we find that the optimal allocation prescribes higher consumption risk for all abilities than the optimal allocation with lower σ_z (see Figures 6a and 6b). Interestingly, the effect on learning effort is uneven across the ability distribution. Low ability agents exert higher effort, whereas higher ability agents exert lower effort. In other words, a riskier shock process makes it harder to provide insurance for low ability agents while at the same time making it harder to provide incentives for

human capital accumulation for high types. This is because a riskier shock process, by itself, discourages learning effort, as evident by the lower effort for all abilities in the Status Quo economy with higher risk in Figure 6b. Figures 7a and 7b show that none of the other allocations change much with a riskier shock process.



(a) Standard deviation of log-consumption in second period

(b) Effort

Figure 4: Consumption Risk and Effort, higher dispersion of shocks.

5 Concavity of Human Capital Production Function

We don't have a clear way of mapping a concavity of a human capital production function in a one period model to concavity in a multiperiod model. However, we have performed the following simple exercise: In our model, a period is 20 years, and the human capital production function is

$$h_1 = h_0 + (h_0 e)^\alpha.$$

Consider now 20 periods in a [Huggett, Ventura, and Yaron \(2011\)](#)-like model where a period is a year. Assume that the effort undertaken is $e_{t+1} = e_t / (1 + d)$ each period, i.e.

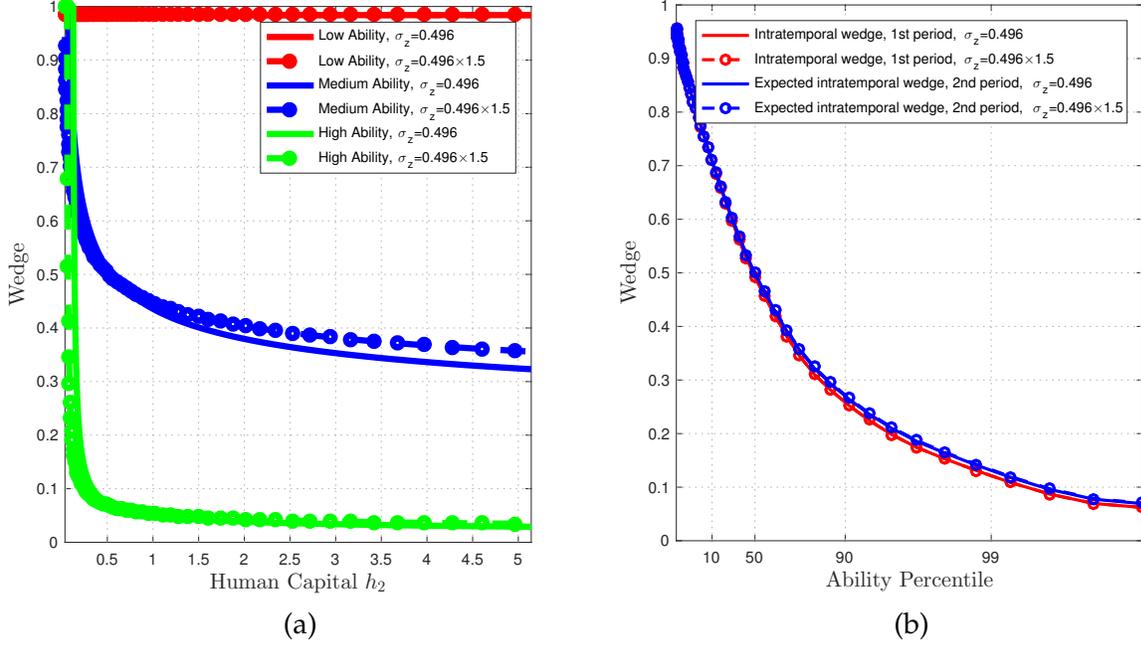


Figure 5: Labor Wedge, higher dispersion of shocks.

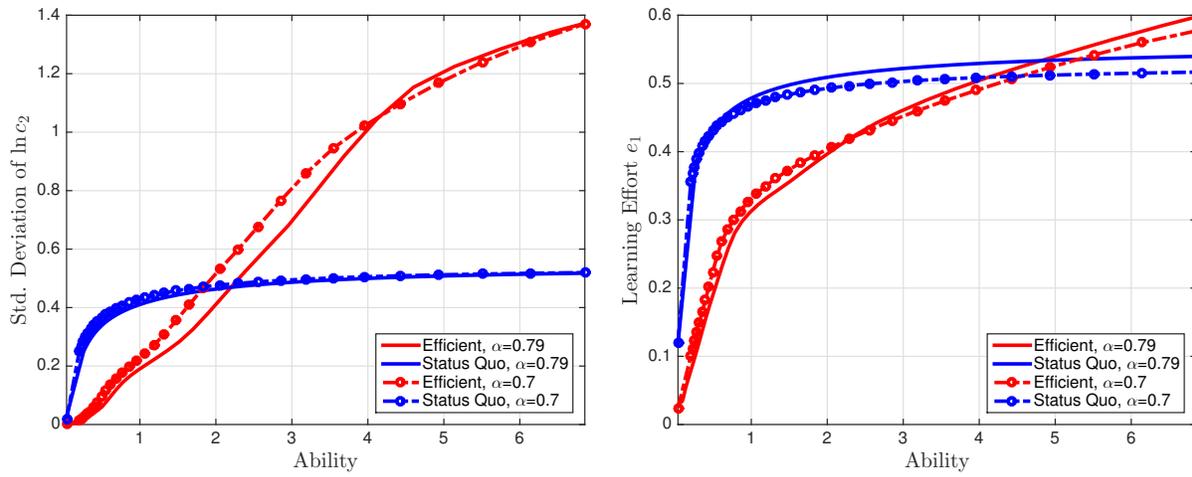
decreases at d percent per year. The human capital evolves according to

$$\hat{h}_{t+1} = \hat{h}_t + D(\hat{h}_t e_t)^\alpha.$$

We set $\alpha = 0.7$ and $d = 0.03$ (the simulations are not very sensitive to reasonable variations in d). We plot h_1 and \hat{h}_{20} as a function of e in the first case, and of $\hat{e} = \sum_{t=1}^{20} e_t$ in the second case, and choose D to match the production function for $e = \hat{e} = 0.5$ (which is approximately the upper bound on effort in our simulations). Figure 3 shows the production function:

There is a considerable uncertainty about the parameters of the human capital production function. Also, while the concavity parameter is typically estimated on annual values, our model period is 20 years. If the agents can adjust within period, the resulting production function will typically be less concave.

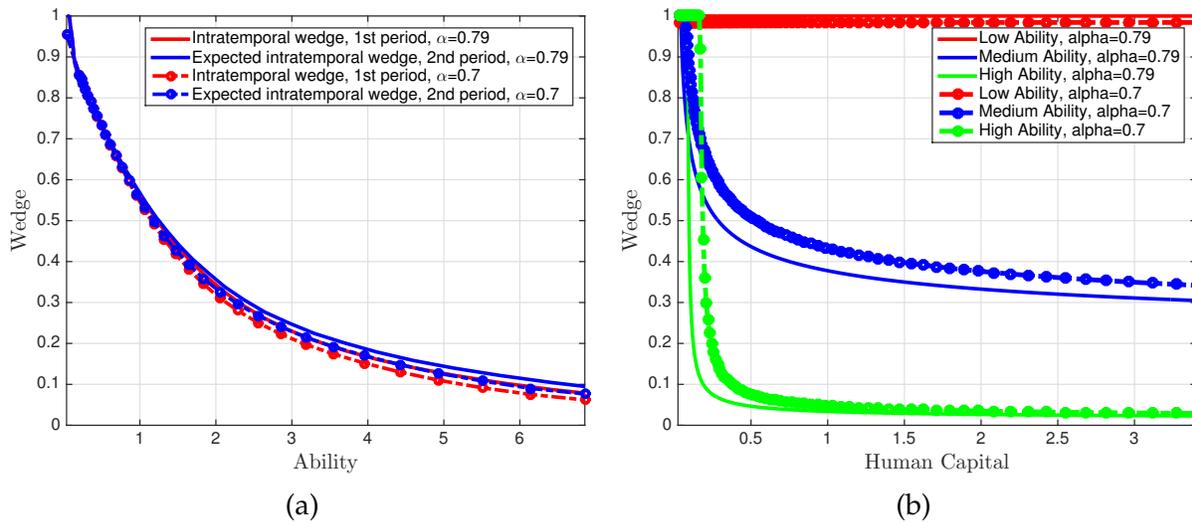
We have therefore re-calibrated the model for $\alpha = 0.79$, instead of $\alpha = 0.7$. Figures 5 and 6 show how the optimum changes.



(a) Standard deviation of log-consumption in second period

(b) Effort

Figure 6: Consumption Risk and Effort, less concave production function



(a)

(b)

Figure 7: Labor Wedge, less concave production function

References

- Albanesi, S. and C. Sleet (2006). Dynamic optimal taxation with private information. *The Review of Economic Studies* 73(1), 1–30. [1](#)
- Farhi, E. and I. Werning (2013). Insurance and taxation over the life cycle. *The Review of Economic Studies* 80(2), 596–635. [2](#)
- Golosov, M., M. Troshkin, and A. Tsyvinski (2016). Redistribution and social insurance. *American Economic Review* 106(2), 359–86. [2](#), [3](#)

Huggett, M., G. Ventura, and A. Yaron (2011). Sources of lifetime inequality. *American Economic Review* 101(7), 2923–54. 5