

Online Appendix for  
Some Evidence on Secular Drivers of U.S. Safe Real Rates

Kurt G. Lunsford  
Federal Reserve Bank of Cleveland

Kenneth D. West  
University of Wisconsin

December 2017  
Last revised: September 2018

APPENDIX 1

This appendix includes:

- Some derivations omitted from the text to save space: A1: derivation of (2.4); A2: derivation of asymptotic standard errors on the estimate of  $\rho^{MA}$ .
- Tables A3, A4. Expanded versions of Tables 3 and 4, which present 90% as well as 68% confidence intervals, and includes long run correlations for some correlates omitted from the paper to save space. See notes below.
- Table A5. Estimates of the intertemporal IS using low frequency data.
- Tables A6, A7: Summary statistics on the correlates, 1890-2016 (Table A6) and 1950-2016 (Table A7).

Notes on Tables A3 and A4:

1. For correlates that are in Tables 3 and 4, the point estimates and 68 percent confidence intervals are identical to the values in Tables 3 and 4. The second interval is the asymptotic 90% confidence interval. “\*” and “\*\*” indicate that the 68% and 90% intervals do not include zero.
2. In Tables A3, and A4 row numbers in the far left of a row are identical to those in Tables 3 and 4. There is no row number at the far left if the correlate is new to this appendix.
4. Data sources are the same as those stated in the paper (e.g., the source for world demographic variables, including the ones omitted from the paper to save space, was The UN’s *World Population Prospects*). Only one of the variables new to this appendix came from a new data source: World debt/GDP was obtained from Abbas et al. (2014), and runs 1900-2011.
5. To prevent confusion: as noted in the text, the correlate labeled “Federal deficit” in Tables 3 and 4 is the primary deficit. In Tables A3 and A4, for clarity we have relabeled this “Federal primary deficit,” and label the total deficit (including interest payments) “Federal total deficit”.

Additional reference:

Abbas S. M. Ali, Laura Blattner, Mark De Broeck, Asmaa El-Ganainy and Malin Hu , 2014, “Sovereign Debt Composition in Advanced Economies : A Historical Perspective,” working paper, International Monetary Fund.

A1: Derivation of (2.4)

Derivation of (2.4): The intertemporal marginal rate of substitution is (Campbell et al. (1997,p319))

$$\text{IMRS} = \beta^0 (R_{m,t+1})^{\theta-1} (C_{t+1}/C_t)^{-\theta/\psi} \equiv \beta^0 S_{t+1}.$$

Let  $s_{t+1} \equiv \ln(S_{t+1})$ . Then

$$(A1.1) \quad s_{t+1} = (\theta-1)r_{m,t+1} - (\theta/\psi)\Delta c_{t+1}.$$

The FOCs for a nominally safe asset and for the market portfolio are

$$1 = \beta^0 E_t[S_{t+1}(1+i_t)(P_{t+1}/P_t)^{-1}], \quad 1 = \beta^0 E_t[S_{t+1}R_{m,t+1}].$$

Log linearizing the rhs and taking log of lhs of each:

$$(A1.2) \quad 0 = \theta \ln \beta + E_t s_{t+1} + 0.5 \text{var}_t s_{t+1} + \ln(1+i_t) - E_t \pi_{t+1} + 0.5 \text{var}_t \pi_{t+1} - \text{cov}_t(s_{t+1}, \pi_{t+1}),$$

$$(A1.3) \quad 0 = \theta \ln \beta + E_t s_{t+1} + 0.5 \text{var}_t s_{t+1} + E_t r_{m,t+1} + 0.5 \text{var}_t r_{m,t+1} + \text{cov}_t(s_{t+1}, r_{m,t+1}).$$

Subtract (A1.2) from (A1.3) and rearrange

$$(A1.4) \quad E_t r_{m,t+1} = \ln(1+i_t) - E_t \pi_{t+1} - 0.5 \text{var}_t r_{m,t+1} - \text{cov}_t(s_{t+1}, r_{m,t+1}) + 0.5 \text{var}_t \pi_{t+1} - \text{cov}_t(s_{t+1}, \pi_{t+1}).$$

Using the equation (A1.1) definition of  $s_{t+1}$  in the  $\text{cov}_t(s_{t+1}, r_{m,t+1})$  term, (A1.4) becomes

$$(A1.5) \quad E_t r_{m,t+1} = \ln(1+i_t) - E_t \pi_{t+1} - 0.5 \text{var}_t r_{m,t+1} + (1-\theta) \text{var}_t r_{m,t+1} + (\theta/\psi) \text{cov}_t(\Delta c_{t+1}, r_{m,t+1}) \\ + 0.5 \text{var}_t \pi_{t+1} - \text{cov}_t(s_{t+1}, \pi_{t+1}).$$

Now write out (A1.2) in detail

$$(A1.6) \quad 0 = \theta \ln \beta + (\theta-1) E_t r_{m,t+1} - (\theta/\psi) E_t \Delta c_{t+1} \\ + 0.5(\theta-1)^2 \text{var}_t r_{m,t+1} + 0.5(\theta/\psi)^2 \text{var}_t \Delta c_{t+1} - (\theta-1)(\theta/\psi) \text{cov}_t(\Delta c_{t+1}, r_{m,t+1}) \\ + \ln(1+i_t) - E_t \pi_{t+1} + 0.5 \text{var}_t \pi_{t+1} - \text{cov}_t(s_{t+1}, \pi_{t+1}).$$

Here

$$(A1.7) \quad \text{cov}_t(s_{t+1}, \pi_{t+1}) = (\theta-1)\text{cov}_t(r_{mt+1}, \pi_{t+1}) - (\theta/\psi)\text{cov}_t(\Delta c_{t+1}, \pi_{t+1}).$$

Rearrange (A1.6):

$$(A1.8) \quad \begin{aligned} \ln(1+i_t) - E_t \pi_{t+1} &= -\theta \ln \beta + (1-\theta)E_t r_{mt+1} + (\theta/\psi)E_t \Delta c_{t+1} \\ &\quad - 0.5(\theta-1)^2 \text{var}_t r_{mt+1} - 0.5(\theta/\psi)^2 \text{var}_t \Delta c_{t+1} + (\theta-1)(\theta/\psi)\text{cov}_t(\Delta c_{t+1}, r_{mt+1}) \\ &\quad - 0.5 \text{var}_t \pi_{t+1} + \text{cov}_t(s_{t+1}, \pi_{t+1}). \end{aligned}$$

Use (A1.5) to substitute out for  $E_t r_{mt+1}$ . After some rearrangement, the result is

$$(A1.9) \quad \begin{aligned} \theta [\ln(1+i_t) - E_t \pi_{t+1}] &= -\theta \ln \beta + \frac{1}{2} \theta(\theta-1) \text{var}_t r_{mt+1} - 0.5\theta \text{var}_t \pi_{t+1} + (\theta/\psi)E_t \Delta c_{t+1} - 0.5(\theta/\psi)^2 \text{var}_t \Delta c_{t+1} \\ &\quad + \theta \text{cov}_t(s_{t+1}, \pi_{t+1}). \end{aligned}$$

After dividing both sides by  $\theta$  and using (A1.7), the result is (2.4) in the text.

## A2 Derivation of asymptotic standard errors on the estimate of $\rho^{MA}$

Apart from symbols with superscript  $MA$ , notation in this section may not match notation elsewhere in this appendix or in the paper.

For a given correlate  $x_t$ , let  $\mu_r$ ,  $\mu_x$ ,  $\gamma_r$  and  $\gamma_x$  denote the means and variances of  $r_t^{MA}$  and  $x_t^{MA}$ , with  $\gamma_{rx} \equiv \text{cov}(r_t^{MA}, x_t^{MA})$ . Thus

$$(A2.1) \quad \rho^{MA} = \gamma_{rx} / (\gamma_r \gamma_x)^{1/2}.$$

Let

$$(A2.2) \quad \theta \equiv (\mu_r, \mu_x, \gamma_r, \gamma_x, \gamma_{rx})'.$$

Write (A2.1) as

$$(A2.3) \quad \rho^{MA} = f(\theta)$$

where the function  $f: \mathbf{R}^5 \rightarrow \mathbf{R}$  is defined on the r.h.s. of (A2.1).

Let  $S$  be the  $5 \times 5$  asymptotic variance-covariance matrix of estimates of  $\theta$ . ( $S$  is derived below.) Then the asymptotic variance of the estimate of  $\rho^{MA}$  is

$$(A2.4) \quad (\partial f / \partial \theta)' S (\partial f / \partial \theta).$$

Here,  $(\partial f / \partial \theta)'$  is  $1 \times 5$ . The standard error on the estimate of  $\rho^{MA}$  was computed by evaluating  $\partial f / \partial \theta$  at sample moments and using an estimate of  $S$  that is about to be described.

Derivation of  $S$ : The following  $5 \times 1$  orthogonality condition was used to estimate  $\theta$ :

$$(A2.5) \quad E h_t(\theta) = 0, \quad h_t(\theta) = [r_t^{MA} - \mu_r, x_t^{MA} - \mu_x, (r_t^{MA} - \mu_r)^2 - \gamma_r, (x_t^{MA} - \mu_x)^2 - \gamma_x, (r_t^{MA} - \mu_r)(x_t^{MA} - \mu_x) - \gamma_{rx}]'.$$

$\hat{\theta}$  was constructed by setting  $T^{-1} \sum_{t=1}^T h_t(\hat{\theta}) = 0$ —that is,  $\hat{\theta}$  was constructed in the usual way, e.g.,  $\hat{\mu}_r = T^{-1} \sum_{t=1}^T r_t^{MA}$ .

Then under suitable conditions  $\sqrt{T}(\hat{\theta} - \theta)$  is asymptotically normally with variance-covariance matrix

$$(A2.6) \quad (E \partial h_t / \partial \theta)^{-1} \times (\text{long run variance of } h_t) \times (E \partial h_t / \partial \theta)^{-1'} \equiv S.$$

Since  $E \partial h_t / \partial \theta = -I_5$ , this means  $\sqrt{T}(\hat{\theta} - \theta)$  is asymptotically normal with  $S = \text{long run variance of } h_t$ .

This long run variance was estimated using Newey and West (1994). The Bartlett kernel was used. To obtain the bandwidth, Newey and West's weight vector  $w$  was set to  $(0 \ 0 \ 1 \ 1 \ 1)'$  and the parameter that Newey and West (1994) call  $n$  was set to  $4(T/100)^{2/9}$ .

In early work, we did some experimentation with the resulting bandwidth, checking whether results were sensitive to modest increases or decreases in the value produced by the algorithm described in the preceding paragraph. There was little sensitivity.

A3. Expanded Table 3

Long run correlations, 1890-2016

(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter-I(0) $\hat{\rho}^{LP}$	(3b) R <sup>2</sup>	(4a) Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	(4b) R <sup>2</sup>	(4c) median $\hat{\rho}^{LP}$	(5a) Lowpass filter-I(1) $\hat{\rho}^{LP}$	(5b) R <sup>2</sup>
(1a) GDP growth (+)	-0.15 (-0.31,0.01) (-0.42,0.12)	0.01 (-0.19,0.20) (-0.32,0.33)	0.00	-0.02 (-0.19,0.13) (-0.41,0.32)	0.04	-0.01	0.20 (-0.01,0.38) (-0.14,0.49)	0.04
(1b) Consumption growth (+)	0.06 (-0.11,0.23) (-0.22,0.34)	0.04 (-0.16,0.23) (-0.29,0.36)	0.00	0.03 (-0.13,0.14) (-0.26,0.38)	0.03	0.04	-0.01 (-0.21,0.19) (-0.33,0.32)	0.00
(1c) World GDP growth (+)	0.09 (-0.06,0.24) (-0.16,0.34)	0.18 (-0.02,0.36) (-0.15,0.47)	0.03	0.12 (-0.06,0.34) (-0.21,0.46)	0.06	0.10	0.26* (0.06,0.43) (-0.08,0.53)	0.07
(1d) TFP growth (+)	-0.55** (-0.64,-0.45) (-0.70,-0.39)	-0.36** (-0.52,-0.17) (-0.61,-0.03)	0.13	-0.31* (-0.50,-0.08) (-0.75,0.03)	0.14	-0.33*	-0.01 (-0.21,0.19) (-0.33,0.31)	0.00
Labor productivity growth	-0.57** (-0.69,-0.45) (-0.77,-0.37)	-0.39** (-0.54,-0.21) (-0.63,-0.08)	0.16	-0.30* (-0.50,-0.08) (-0.80,0.05)	0.14	-0.32*	-0.11 (-0.30,0.09) (-0.41,0.22)	0.01
(2a) var <sub>t</sub> (GDP growth) (-)	-0.62** (-0.78,-0.46) (-0.88,-0.35)	-0.49** (-0.62,-0.31) (-0.70,-0.19)	0.24	-0.30* (-0.65,-0.10) (-0.70,0.20)	0.12	-0.32*	-0.23* (-0.41,-0.03) (-0.51,0.11)	0.05
var <sub>t</sub> (C growth) (-)	-0.61** (-0.76,-0.46) (-0.86,-0.36)	-0.43** (-0.58,-0.22) (-0.67,-0.08)	0.18	-0.26* (-0.60,-0.03) (-0.70,0.20)	0.11	-0.26*	-0.19 (-0.39,0.03) (-0.50,0.17)	0.04
(2b) var <sub>t</sub> ( $\pi$ _GDP) (-)	-0.17* (-0.29,-0.03) (-0.38,0.05)	-0.04 (-0.24,0.16) (-0.36,0.29)	0.00	0.11 (-0.40,0.39) (-0.60,0.65)	0.08	0.10	0.26* (0.06,0.43) (-0.08,0.53)	0.07
var <sub>t</sub> ( $\pi$ _PCE) (-)	-0.17 (-0.35,0.02) (-0.47,0.14)	-0.04 (-0.25,0.18) (-0.39,0.32)	0.00	0.09 (-0.35,0.34) (-0.50,0.65)	0.07	0.06	0.25* (0.03,0.44) (-0.12,0.55)	0.06

A3. Expanded Table 3, continued

Long run correlations, 1890-2016

(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter-I(0) $\hat{\rho}^{LP}$	(3b) $R^2$	(4a) Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	(4b) $R^2$	(4c) median $\hat{\rho}^{LP}$	(5a) Lowpass filter-I(1) $\hat{\rho}^{LP}$	(5b) $R^2$
(2c) $\text{var}_t(\text{mkt return}) (-)$	-0.59** (-0.73,-0.46) (-0.82,-0.35)	-0.51** (-0.64,-0.34) (-0.71,-0.21)	0.26	-0.35* (-0.75,-0.11) (-0.85,0.01)	0.17	-0.44*	-0.19 (-0.37,0.02) (-0.48,0.15)	0.04
$\text{var}_t(\text{mkt return})$ (PCE defl.) (-)	-0.55** (-0.69,-0.40) (-0.79,-0.30)	-0.46** (-0.61,-0.26) (-0.70,-0.12)	0.21	-0.33* (-0.65,-0.11) (-0.80,0.20)	0.15	-0.39*	-0.25* (-0.44,-0.03) (-0.55,0.11)	0.06
(3a) $\text{cov}_t(\pi\_GDP, \text{GDP growth}) (-)$	-0.42** (-0.57,-0.26) (-0.67,-0.16)	-0.31* (-0.47,-0.11) (-0.57,0.02)	0.10	-0.18 (-0.60,0.08) (-0.70,0.45)	0.11	-0.23	0.10 (-0.10,0.29) (-0.23,0.41)	0.01
$\text{cov}_t(\pi\_PCE, C \text{ growth}) (-)$	-0.66** (-0.78,-0.55) (-0.85,-0.47)	-0.49** (-0.63,-0.29) (-0.71,-0.15)	0.24	-0.33* (-0.70,-0.11) (-0.85,0.03)	0.16	-0.41*	-0.13 (-0.34,0.09) (-0.46,0.23)	0.02
(3b) $\text{cov}_t(\pi\_GDP, \text{mkt return}) (-)$	0.02 (-0.14,0.18) (-0.24,0.29)	-0.08 (-0.28,0.13) (-0.40,0.26)	0.01	-0.18 (-0.42,0.40) (-0.75,0.60)	0.09	-0.20	-0.35** (-0.51,-0.14) (-0.60,-0.01)	0.12
$\text{cov}_t(\pi\_PCE, \text{mkt return}) (-)$	-0.58* (-0.72,-0.44) (-0.81,-0.35)	-0.19 (-0.39,0.03) (-0.50,0.18)	0.04	-0.21 (-0.55,0.20) (-0.75,0.40)	0.10	-0.23	-0.25* (-0.44,-0.03) (-0.55,0.12)	0.06
(4a) Labor hours growth (+)	0.31* (0.09,0.53) (-0.06,0.67)	0.27* (0.07,0.44) (-0.07,0.54)	0.07	0.24* (0.05,0.43) (-0.10,0.50)	0.09	0.23*	0.24* (0.04,0.42) (-0.09,0.52)	0.07
(4b) Growth in capital per hour (-)	0.23* (0.07,0.39) (-0.03,0.49)	0.13 (-0.07,0.32) (-0.20,0.44)	0.02	0.12 (-0.05,0.34) (-0.16,0.43)	0.05	0.10	-0.15 (-0.34,0.05) (-0.45,0.18)	0.02
Growth in capital per capita (-)	0.67** (0.59,0.75) (0.53,0.80)	0.60** (0.44,0.71) (0.33,0.77)	0.36	0.49** (0.30,0.65) (0.16,0.90)	0.27	0.50**	0.25* (0.05,0.42) (-0.08,0.53)	0.06

A3. Expanded Table 3, continued

Long run correlations, 1890-2016

(1)	(2)	(3a)	(3b)	(4a)	(4b)	(4c)	(5a)	(5b)
Correlate (expected sign)	10Y moving avg. $\hat{\rho}^{MA}$	Lowpass filter-I(0) $\hat{\rho}^{LP}$	$R^2$	Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	$R^2$	median $\hat{\rho}^{LP}$	Lowpass filter-I(1) $\hat{\rho}^{LP}$	$R^2$
(5a) Dependency ratio (+)	0.36** (0.17,0.56) (0.04,0.68)	0.38** (0.18,0.53) (0.05,0.62)	0.14	0.25* (0.10,0.65) (-0.15,0.70)	0.09	0.23*	0.17 (-0.03,0.35) (-0.17,0.47)	0.03
(5b) Percent aged 40-64 (-)	-0.43** (-0.61,-0.25) (-0.72,-0.14)	-0.43** (-0.58,-0.25) (-0.66,-0.12)	0.19	-0.26** (-0.55,-0.23) (-0.55,-0.10)	0.08	-0.23**	-0.24* (-0.42,-0.04) (-0.52,0.10)	0.06
MY ratio (-)	-0.34** (-0.51,-0.16) (-0.62,-0.05)	-0.31* (-0.48,-0.12) (-0.57,0.02)	0.10	-0.14 (-0.55,0.10) (-0.70,0.40)	0.08	-0.23	-0.01 (-0.21,0.19) (-0.33,0.32)	0.00
$\Delta$ Dependency ratio (-)	-0.47** (-0.58,-0.37) (-0.64,-0.30)	-0.43** (-0.58,-0.25) (-0.66,-0.12)	0.19	-0.29* (-0.80,-0.04) (-0.85,0.45)	0.13	-0.32*	-0.25* (-0.42,-0.05) (-0.52,0.09)	0.06
(5c) $\Delta$ Percent aged 40-64 (+)	0.14 (0.00,0.27) (-0.08,0.35)	0.18 (-0.02,0.37) (-0.16,0.48)	0.03	0.19 (-0.03,0.70) (-0.50,0.80)	0.10	0.22	0.11 (-0.10,0.30) (-0.23,0.41)	0.01
$\Delta$ MY ratio (+)	0.02 (-0.13,0.18) (-0.24,0.29)	0.05 (-0.15,0.25) (-0.27,0.37)	0.00	0.04 (-0.40,0.60) (-0.70,0.70)	0.08	0.00	-0.06 (-0.26,0.14) (-0.38,0.27)	0.00
(5d) $\Delta$ Life expectancy (-)	-0.14 (-0.27,0.00) (-0.36,0.09)	0.17 (-0.04,0.35) (-0.17,0.46)	0.03	0.08 (-0.13,0.21) (-0.14,0.41)	0.04	0.10	0.51* (0.34,0.64) (0.21,0.71)	0.26
Fed total deficit/GDP (+)	-0.53** (-0.69,-0.38) (-0.79,-0.28)	-0.47** (-0.60,-0.29) (-0.68,-0.16)	0.22	-0.41** (-0.57,-0.21) (-0.71,-0.04)	0.20	-0.42**	-0.23* (-0.41,-0.03) (-0.51,0.10)	0.05
(6a) Fed primary deficit/GDP (+)	-0.56** (-0.68,-0.43) (-0.76,-0.35)	-0.48** (-0.61,-0.30) (-0.69,-0.17)	0.23	-0.43** (-0.59,-0.26) (-0.72,-0.06)	0.22	-0.44*	-0.25* (-0.42,-0.05) (-0.53,0.09)	0.06

A3. Expanded Table 3, continued

Long run correlations, 1890-2016

(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter-I(0) $\hat{\rho}^{LP}$	(3b) $R^2$	(4a) Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	(4b) $R^2$	(4c) median $\hat{\rho}^{LP}$	(5a) Lowpass filter-I(1) $\hat{\rho}^{LP}$	(5b) $R^2$
(6b) Fed debt/GDP (+)	-0.59** (-0.79,-0.39) (-0.91,-0.27)	-0.57** (-0.68,-0.40) (-0.75,-0.29)	0.32	-0.44** (-0.75,-0.27) (-0.85,-0.10)	0.22	-0.46**	-0.29* (-0.45,-0.09) (-0.56,0.04)	0.08
World debt/GDP (+)	-0.68** (-0.80,-0.55) (-0.88,-0.47)	-0.62** (-0.73,-0.46) (-0.79,-0.33)	0.38	-0.50** (-0.71,-0.30) (-0.90,-0.16)	0.29	-0.52**	-0.31* (-0.49,-0.10) (-0.59,0.04)	0.10
(7) Current account/GDP (+)	-0.15 (-0.35,0.05) (-0.48,0.18)	-0.18 (-0.36,0.03) (-0.47,0.16)	0.03	-0.22 (-0.49,0.15) (-0.64,0.45)	0.11	-0.27	-0.47** (-0.61,-0.30) (-0.69,-0.17)	0.22
(8) Relative price inv. goods (+)	-0.01 (-0.17,0.15) (-0.27,0.25)	0.07 (-0.13,0.27) (-0.25,0.38)	0.01	0.12 (-0.11,0.42) (-0.45,0.60)	0.08	0.10	0.17 (-0.03,0.36) (-0.16,0.47)	0.03
(9) Top 10% income share (-)	-0.03 (-0.22,0.15) (-0.34,0.28)	-0.05 (-0.27,0.17) (-0.40,0.32)	0.00	0.11 (-0.40,0.38) (-0.50,0.50)	0.07	0.10	0.27* (0.04,0.46) (-0.11,0.57)	0.07
(10) Baa-10 yr Treasury spread (-)	0.30* (0.11,0.48) (-0.01,0.60)	0.21 (-0.02,0.41) (-0.18,0.53)	0.04	0.17 (-0.20,0.47) (-0.50,0.75)	0.10	0.18	0.22 (-0.01,0.42) (-0.16,0.54)	0.05
(11a) $\pi_{GDP}$ (-)	-0.30** (-0.45,-0.16) (-0.54,-0.06)	-0.43** (-0.57,-0.25) (-0.66,-0.12)	0.19	-0.42* (-0.67,-0.10) (-0.73,0.25)	0.23	-0.46*	-0.69** (-0.78,-0.56) (-0.82,-0.46)	0.48
(11b) $\pi_{PCE}$ (-)	-0.38** (-0.55,-0.22) (-0.65,-0.11)	-0.48** (-0.62,-0.30) (-0.69,-0.18)	0.23	-0.46* (-0.71,-0.21) (-0.79,0.30)	0.27	-0.50*	-0.70** (-0.78,-0.58) (-0.83,-0.48)	0.49
(12a) M1 growth (-)	-0.34* (-0.56,-0.12) (-0.70,0.02)	-0.44** (-0.60,-0.23) (-0.69,-0.09)	0.20	-0.39* (-0.59,-0.16) (-0.85,0.00)	0.21	-0.44*	-0.38** (-0.54,-0.16) (-0.64,-0.01)	0.14
(12b) M2 growth (-)	-0.33** (-0.50,-0.16) (-0.61,-0.06)	-0.33** (-0.49,-0.14) (-0.59,-0.01)	0.11	-0.30* (-0.53,-0.05) (-0.60,0.07)	0.14	-0.33*	-0.28* (-0.45,-0.08) (-0.55,0.05)	0.08



A4. Expanded Table 4

Long run correlations, 1950-2016

(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter-I(0) $\hat{\rho}^{LP}$	(3b) $R^2$	(4a) Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	(4b) $R^2$	(4c) median $\hat{\rho}^{LP}$	(5a) Lowpass filter-I(1) $\hat{\rho}^{LP}$	(5b) $R^2$
(1a) GDP growth (+)	0.61** (0.48,0.74) (0.39,0.83)	0.23 (-0.06,0.46) (-0.25,0.60)	0.05	0.16 (-0.07,0.43) (-0.27,0.65)	0.09	0.15	-0.01 (-0.28,0.27) (-0.45,0.44)	0.00
(1b) Consumption growth (+)	0.56** (0.40,0.72) (0.29,0.83)	0.46** (0.19,0.64) (0.00,0.74)	0.21	0.31* (0.03,0.60) (-0.12,0.80)	0.16	0.34*	0.20 (-0.09,0.44) (-0.27,0.58)	0.04
(1c) World GDP growth (+)	0.05 (-0.23,0.33) (-0.41,0.51)	0.00 (-0.27,0.28) (-0.44,0.44)	0.00	-0.02 (-0.50,0.27) (-0.65,0.70)	0.08	-0.01	-0.15 (-0.40,0.14) (-0.55,0.31)	0.02
(1d) TFP growth (+)	-0.23* (-0.46,-0.01) (-0.61,0.14)	-0.25 (-0.49,0.04) (-0.62,0.22)	0.06	-0.18 (-0.46,0.06) (-0.70,0.45)	0.11	-0.20	-0.38* (-0.58,-0.10) (-0.70,0.09)	0.14
Labor productivity growth (+)	-0.18 (-0.43,0.07) (-0.59,0.23)	-0.08 (-0.34,0.21) (-0.50,0.38)	0.01	-0.05 (-0.34,0.23) (-0.65,0.60)	0.08	-0.03	-0.23 (-0.47,0.06) (-0.60,0.25)	0.05
(2a) $\text{var}_t(\text{C growth}) (-)$	-0.22 (-0.48,0.03) (-0.65,0.20)	-0.40* (-0.60,-0.13) (-0.71,0.07)	0.16	-0.37* (-0.65,0.00) (-0.85,0.35)	0.20	-0.40*	-0.54** (-0.70,-0.29) (-0.78,-0.10)	0.29
$\text{var}_t(\text{GDP growth}) (-)$	-0.35* (-0.59,-0.10) (-0.75,0.06)	-0.38* (-0.58,-0.10) (-0.69,0.09)	0.14	-0.29* (-0.50,-0.10) (-0.70,0.05)	0.13	-0.27*	-0.42* (-0.61,-0.14) (-0.72,0.04)	0.17
(2b) $\text{var}_t(\pi\_NDS) (-)$	-0.16 (-0.43,0.12) (-0.62,0.30)	-0.31* (-0.53,-0.03) (-0.65,0.16)	0.10	-0.23 (-0.60,0.00) (-0.75,0.23)	0.11	-0.27	-0.20 (-0.45,0.08) (-0.59,0.27)	0.04
$\text{var}_t(\pi\_GDP) (-)$	-0.28* (-0.54,-0.02) (-0.71,0.14)	-0.33* (-0.54,-0.05) (-0.66,0.14)	0.11	-0.23* (-0.50,0.00) (-0.70,0.18)	0.11	-0.27*	-0.28 (-0.50,0.01) (-0.63,0.20)	0.08

A4. Expanded Table 4, continued

Long run correlations, 1950-2016

(1)	(2)	(3a)	(3b)	(4a)	(4b)	(4c)	(5a)	(5b)
Correlate (expected sign)	10Y moving avg. $\hat{\rho}^{MA}$	Lowpass filter-I(0) $\hat{\rho}^{LP}$	$R^2$	Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	$R^2$	median $\hat{\rho}^{LP}$	Lowpass filter-I(1) $\hat{\rho}^{LP}$	$R^2$
(2c) $\text{var}_t(\text{mkt return}) (-)$	-0.34* (-0.58,-0.11) (-0.73,0.05)	-0.59** (-0.73,-0.35) (-0.81,-0.17)	0.34	-0.48* (-0.80,-0.27) (-0.91,0.10)	0.28	-0.50*	-0.52** (-0.68,-0.26) (-0.77,-0.07)	0.27
$\text{var}_t(\text{mkt return}) (\text{GDP defl.}) (-)$	-0.43** (-0.64,-0.22) (-0.78,-0.09)	-0.62** (-0.75,-0.39) (-0.82,-0.21)	0.38	-0.48* (-0.80,-0.27) (-0.91,0.10)	0.29	-0.50*	-0.50** (-0.67,-0.24) (-0.76,-0.05)	0.25
(3a) $\text{cov}_t(\pi\_NDS, \text{C growth}) (-)$	-0.76** (-0.83,-0.69) (-0.87,-0.64)	-0.64** (-0.77,-0.42) (-0.84,-0.25)	0.41	-0.45** (-0.80,-0.23) (-0.91,0.00)	0.25	-0.46**	-0.37* (-0.58,-0.09) (-0.69,0.10)	0.14
$\text{cov}_t(\pi\_GDP, \text{GDP growth}) (-)$	-0.24* (-0.42,-0.06) (-0.53,0.06)	-0.36* (-0.57,-0.08) (-0.68,0.11)	0.13	-0.25 (-0.65,0.00) (-0.85,0.18)	0.13	-0.29	-0.06 (-0.33,0.21) (-0.49,0.39)	0.00
(3b) $\text{cov}_t(\pi\_NDS, \text{mkt return}) (-)$	-0.27 (-0.54,0.00) (-0.72,0.17)	0.02 (-0.25,0.30) (-0.42,0.46)	0.00	0.15 (-0.30,0.46) (-0.55,0.70)	0.10	0.16	0.31* (0.02,0.53) (-0.16,0.65)	0.10
$\text{cov}_t(\pi\_GDP, \text{mkt return}) (-)$	0.016 (-0.32,0.35) (-0.54,0.57)	0.10 (-0.19,0.36) (-0.36,0.52)	0.01	0.13 (-0.20,0.45) (-0.55,0.70)	0.09	0.10	0.22 (-0.07,0.46) (-0.25,0.59)	0.05
(4a) Labor hours growth (+)	0.81** (0.76,0.87) (0.72,0.90)	0.38* (0.10,0.58) (-0.09,0.69)	0.15	0.36* (0.06,0.65) (-0.08,0.77)	0.20	0.40*	0.11 (-0.17,0.37) (-0.35,0.52)	0.01
(4b) Growth in capital per hour (-)	0.10 (-0.14,0.33) (-0.29,0.48)	0.34* (0.05,0.55) (-0.14,0.67)	0.11	0.24 (-0.01,0.50) (-0.18,0.70)	0.12	0.27	0.27 (-0.02,0.50) (-0.21,0.63)	0.07
Growth in capital per capita (-)	0.77** (0.68,0.86) (0.62,0.92)	0.72** (0.53,0.82) (0.38,0.87)	0.51	0.57** (0.36,0.80) (0.16,0.93)	0.37	0.60**	0.51** (0.26,0.68) (0.07,0.77)	0.26

A4. Expanded Table 4, continued

Long run correlations, 1950-2016

(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter-I(0) $\hat{\rho}^{LP}$	(3b) $R^2$	(4a) Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	(4b) $R^2$	(4c) median $\hat{\rho}^{LP}$	(5a) Lowpass filter-I(1) $\hat{\rho}^{LP}$	(5b) $R^2$
(5a) Dependency ratio (+)	-0.04 (-0.30,0.23) (-0.47,0.40)	0.10 (-0.18,0.36) (-0.36,0.52)	0.01	0.13 (-0.11,0.50) (-0.50,0.70)	0.09	0.11	0.13 (-0.16,0.39) (-0.34,0.54)	0.02
(5b) Dependency ratio, world (+)	0.00 (-0.27,0.26) (-0.44,0.43)	0.08 (-0.21,0.34) (-0.38,0.50)	0.01	0.08 (-0.23,0.32) (-0.33,0.55)	0.06	0.10	0.08 (-0.21,0.34) (-0.38,0.50)	0.01
(5c) Percent aged 40-64 (-)	-0.62** (-0.79,-0.45) (-0.90,-0.34)	-0.54** (-0.70,-0.29) (-0.78,-0.11)	0.29	-0.28* (-0.65,-0.10) (-0.80,0.10)	0.12	-0.27*	-0.25 (-0.48,0.04) (-0.61,0.23)	0.06
(5d) Percent aged 40-64,world (-)	-0.41** (-0.64,-0.18) (-0.79,-0.03)	-0.40* (-0.60,-0.12) (-0.71,0.07)	0.16	-0.21 (-0.55,0.00) (-0.75,0.23)	0.09	-0.23	-0.18 (-0.43,0.10) (-0.57,0.29)	0.03
MY ratio (-)	-0.52** (-0.70,-0.35) (-0.81,-0.24)	-0.35* (-0.56,-0.07) (-0.68,0.12)	0.12	-0.07 (-0.60,0.23) (-0.75,0.55)	0.08	-0.03	0.05 (-0.23,0.32) (-0.40,0.48)	0.00
MY ratio, world (-)	-0.52** (-0.71,-0.32) (-0.84,-0.19)	-0.45* (-0.64,-0.18) (-0.74,0.01)	0.20	-0.17 (-0.55,0.10) (-0.75,0.27)	0.09	-0.23	-0.03 (-0.30,0.25) (-0.46,0.42)	0.00
$\Delta$ Dependency ratio (-)	-0.45** (-0.62,-0.29) (-0.73,-0.18)	-0.49** (-0.67,-0.23) (-0.76,-0.04)	0.24	-0.28* (-0.75,0.00) (-0.85,0.30)	0.14	-0.30*	-0.22 (-0.46,0.07) (-0.60,0.26)	0.05
$\Delta$ Dependency ratio, world (-)	-0.65** (-0.78,-0.52) (-0.87,-0.44)	-0.68** (-0.79,-0.47) (-0.86,-0.31)	0.46	-0.48* (-0.80,-0.27) (-0.90,0.05)	0.28	-0.49*	-0.55** (-0.70,-0.30) (-0.79,-0.11)	0.30
(5e) $\Delta$ Percent aged 40-64 (+)	0.18 (-0.01,0.36) (-0.12,0.48)	0.33* (0.04,0.55) (-0.14,0.67)	0.11	0.28* (0.01,0.70) (-0.45,0.80)	0.14	0.30*	0.32* (0.04,0.54) (-0.15,0.66)	0.11

A4. Expanded Table 4, continued

Long run correlations, 1950-2016

(1)	(2)	(3a)	(3b)	(4a)	(4b)	(4c)	(5a)	(5b)
Correlate (expected sign)	10Y moving avg. $\hat{\rho}^{MA}$	Lowpass filter-I(0) $\hat{\rho}^{LP}$	R <sup>2</sup>	Lowpass filter-I(d) mean $\hat{\rho}^{LP}$	R <sup>2</sup>	median $\hat{\rho}^{LP}$	Lowpass filter-I(1) $\hat{\rho}^{LP}$	R <sup>2</sup>
(5f) $\Delta$ Percent aged 40-64, world (+)	0.20 (-0.01,0.42) (-0.15,0.56)	0.24 (-0.04,0.48) (-0.23,0.61)	0.06	0.18 (-0.07,0.55) (-0.50,0.80)	0.11	0.19	0.14 (-0.15,0.39) (-0.33,0.54)	0.02
(5g) $\Delta$ Life expectancy (-)	-0.23* (-0.37,-0.08) (-0.47,0.01)	-0.33* (-0.54,-0.04) (-0.66,0.15)	0.11	-0.32 (-0.60,0.15) (-0.75,0.45)	0.17	-0.36	-0.59** (-0.73,-0.35) (-0.81,-0.18)	0.35
$\Delta$ MY ratio (+)	0.36** (0.17,0.55) (0.05,0.67)	0.22 (-0.07,0.46) (-0.25,0.60)	0.05	0.09 (-0.21,0.60) (-0.60,0.75)	0.09	0.10	0.04 (-0.24,0.31) (-0.41,0.47)	0.00
$\Delta$ MY ratio, world (+)	0.35** (0.20,0.49) (0.11,0.58)	0.29* (0.00,0.51) (-0.19,0.64)	0.08	0.20 (-0.04,0.50) (-0.55,0.80)	0.11	0.21	0.21 (-0.08,0.45) (-0.26,0.59)	0.04
Fed total deficit/GDP (+)	0.05 (-0.32,0.42) (-0.55,0.66)	-0.04 (-0.30,0.24) (-0.47,0.41)	0.00	-0.05 (-0.36,0.27) (-0.70,0.65)	0.08	-0.03	-0.09 (-0.36,0.19) (-0.51,0.37)	0.01
(6a) Fed primary deficit/GDP (+)	-0.26 (-0.58,0.05) (-0.79,0.26)	-0.26 (-0.49,0.03) (-0.62,0.22)	0.07	-0.17 (-0.48,0.08) (-0.75,0.55)	0.11	-0.19	-0.16 (-0.42,0.13) (-0.56,0.30)	0.03
(6b) Fed debt/GDP (+)	-0.58** (-0.76,-0.40) (-0.87,-0.28)	-0.62** (-0.76,-0.40) (-0.83,-0.22)	0.39	-0.37* (-0.80,-0.13) (-0.85,0.01)	0.18	-0.42*	-0.28 (-0.51,0.01) (-0.64,0.19)	0.08
World debt/GDP (+)	-0.25* (-0.45,-0.05) (-0.58,0.08)	-0.45* (-0.64,-0.17) (-0.74,0.03)	0.20	-0.29* (-0.70,-0.01) (-0.80,0.11)	0.14	-0.32*	-0.22 (-0.47,0.08) (-0.62,0.27)	0.05
(7) Current account/GDP (+)	0.18 (-0.09,0.44) (-0.26,0.61)	0.12 (-0.17,0.38) (-0.35,0.53)	0.01	0.06 (-0.23,0.39) (-0.60,0.70)	0.09	0.03	0.07 (-0.22,0.33) (-0.39,0.49)	0.00

A4. Expanded Table 4, continued

Long run correlations, 1950-2016

(1)	(2)	(3a)	(3b)	(4a)	(4b)	(4c)	(5a)	(5b)
Correlate (expected sign)	10Y moving avg. $\hat{\rho}^{MA}$	Lowpass filter-I(0) $\hat{\rho}^{LP}$	$R^2$	Lowpass filter-I(d)			Lowpass filter-I(1)	
				mean $\hat{\rho}^{LP}$	$R^2$	median $\hat{\rho}^{LP}$	$\hat{\rho}^{LP}$	$R^2$
(8) Relative price inv. goods (+)	0.22 (-0.06,0.50) (-0.24,0.68)	0.20 (-0.09,0.44) (-0.27,0.58)	0.04	0.10 (-0.13,0.32) (-0.32,0.55)	0.07	0.10	0.10 (-0.19,0.36) (-0.36,0.51)	0.01
(9) Top 10% income share (-)	-0.25 (-0.50,0.00) (-0.66,0.16)	-0.29* (-0.51,-0.01) (-0.64,0.18)	0.08	-0.18 (-0.40,0.01) (-0.60,0.27)	0.08	-0.23	-0.18 (-0.43,-0.10) (-0.57,0.29)	0.03
(10) Baa-10 yr Treasury spread (-)	-0.04 (-0.37,0.29) (-0.58,0.50)	-0.07 (-0.33,0.22) (-0.49,0.39)	0.00	-0.09 (-0.43,0.35) (-0.70,0.60)	0.09	-0.07	-0.16 (-0.41,0.13) (-0.56,0.31)	0.03
(11a) $\pi_{GDP}$ (-)	0.33** (0.15,0.51) (0.03,0.63)	0.26 (-0.02,0.50) (-0.22,0.63)	0.07	0.06 (-0.25,0.60) (-0.50,0.75)	0.09	0.03	-0.10 (-0.37,0.18) (-0.52,0.36)	0.01
(11b) $\pi_{NDS}$ (-)	0.39** (0.21,0.56) (0.10,0.67)	0.31* (0.02,0.53) (-0.17,0.65)	0.09	0.08 (-0.21,0.60) (-0.45,0.75)	0.09	0.05	-0.16 (-0.41,0.13) (-0.56,0.31)	0.03
(12a) M1 growth (-)	0.22 (-0.08,0.51) (-0.27,0.71)	0.00 (-0.28,0.27) (-0.44,0.44)	0.00	-0.04 (-0.36,0.27) (-0.70,0.60)	0.08	-0.03	-0.03 (-0.30,0.25) (-0.46,0.42)	0.00
(12b) M2 growth (-)	0.11 (-0.12,0.34) (-0.27,0.49)	0.15 (-0.14,0.40) (-0.32,0.55)	0.02	0.08 (-0.21,0.38) (-0.55,0.75)	0.09	0.05	-0.02 (-0.30,0.26) (-0.45,0.42)	0.00

## A5 Low frequency estimation of the intertemporal IS

The intertemporal IS equation (2.4) provides a simple framework to investigate how a combined set of variables relates to the safe rate. Imposing values of  $\psi$  and  $\gamma$ , we run a band spectral regression of  $r_t$  the right hand side of (2.4) and on the right hand side of (2.5).

Specifically, for given values of  $\psi$ ,  $\gamma$  and  $\theta \equiv (1-\gamma) / (1-\psi^{-1})$  let

$$(A5.1) \quad z_t \equiv (1/\psi)E_t\Delta c_{t+1} - \frac{1}{2} [(\theta/\psi^2)\text{var}_t\Delta c_{t+1} + \text{var}_t\pi_{t+1} + 2(\theta/\psi)\text{cov}_t(\Delta c_{t+1},\pi_{t+1})] \\ + \frac{1}{2} (\theta-1)[\text{var}_t r_{mt+1} + 2 \text{cov}_t(r_{mt+1},\pi_{t+1})].$$

In constructing  $z_t$ , we use the measures of conditional second moments described in the text, rescaled so that the units are commensurate with the units for  $E_t\Delta c_{t+1}$ . We then execute a band spectral regression of low pass  $r_t$  on low pass  $z_t$ , yielding a regression slope of  $\hat{\beta}$ :

$$(A5.2) \quad r_t^{LP} = \hat{\beta} z_t^{LP} + \text{residual}.$$

There is no constant term in (A5.2) because Müller and Watson's filter sweeps out sample means. If our imposed parameter values are reasonable, then, under equation (2.4), the regression (A5.2) should deliver  $\hat{\beta} \approx 1$  and  $\bar{R}^2$  near 1.

We execute the regression both under the assumption of I(0) data, and using Müller and Watson's (2018) (*A, B, c, d*) model. For the latter, we report posterior means, labeled I(*d*) in Table A5 below. The frequency cutoff is again 10 years. For confidence intervals, we followed Müller and Watson's (2015, "Low Frequency Econometrics," NBER Working Paper 21564) recommendation to use a *t* distribution with degrees of freedom that are dependent on sample size and number of regressors. For our application, that means 20 degrees of freedom for the 1910-2016 sample and 12 degrees of freedom for the 1951-2016 sample.

The parameter values we used are listed in column (1) of Table A5 below. Consistent with Tables 3 and 4, the regression performs better for the 1951-2016 sample (columns (4) and (5)) than for the 1910-2016 sample (columns (2) and (3)): adjusted  $\bar{R}^2$ 's are higher, and  $\hat{\beta}$  is closer to 1. However, with the exception of row (1) in the 1951-2016 sample, none of the confidence intervals include 1, and in that sense one can reject the null hypothesis that  $\beta=1$ .

In both panels the  $\bar{R}^2$  is notably higher for the Epstein-Zin-Weil specification (2.4) than for the constant elasticity specification (2.5). This is consistent with the literature cited in the text that finds that the additional structure provided by Epstein-Zin-Weil substantially improves our understanding of the safe real rate. However, the fact that the coefficient estimate is far from 1 indicates that the model still is hardly the final word on trends in the safe real rate.

Table A5

Band Spectral Regression of Low Pass  $r_t$  on Low Pass  $z_t$ 

(1)	1910-2016				1951-2016			
	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)	(5a)	(5b)
	I(0)		I(d)		I(0)		I(d)	
	$\hat{\beta}$	R <sup>2</sup>	$\hat{\beta}$	R <sup>2</sup>	$\hat{\beta}$	R <sup>2</sup>	$\hat{\beta}$	R <sup>2</sup>
(1) $\gamma = \psi^{-1} = 1 \equiv \sigma$	-0.01 (-0.34,0.33)	0.00	0.04 (-0.34,0.39)	0.04	1.02 (0.45,1.59)	0.23	0.94 (0.28,1.57)	0.18
(2) $\gamma = \psi^{-1} = 2 \equiv \sigma$	0.00 (-0.16,0.17)	0.00	0.03 (-0.16,0.20)	0.04	0.51 (0.23,0.80)	0.23	0.47 (0.14,0.78)	0.18
(3) $\gamma = 10, \psi = 1.5$	0.07 (0.04,0.09)	0.28	0.07 (0.03,0.10)	0.18	0.13 (0.08,0.17)	0.41	0.12 (0.07,0.18)	0.31
(4) $\gamma = 3, \psi = 1.5$	0.25 (0.15,0.35)	0.26	0.25 (0.12,0.39)	0.18	0.47 (0.31,0.63)	0.43	0.45 (0.26,0.64)	0.33

Notes to Table A5: See discussion on previous page.

Table A6

## Summary Statistics on the Correlates, 1890-2016

Correlate	(1) Mean	(2) St. Dev.	(3) Median	(4) 1 <sup>st</sup> Order Autocorr	(5) Max	(6) Min
(1a) GDP growth	1.97	4.38	2.16	0.40	16.21	-14.43
(1b) Consumption growth	1.98	3.05	2.11	0.06	10.69	-10.02
(1c) World GDP growth	1.77	3.15	2.15	0.38	7.95	-13.76
(1d) TFP growth	1.15	3.04	1.03	0.08	9.45	-7.92
Labor productivity growth	2.10	2.50	2.11	-0.10	10.16	-6.18
(2a) var <sub>t</sub> (GDP growth)	14.27	14.42	6.35	0.99	52.17	0.99
var <sub>t</sub> (C growth)	7.59	6.71	6.60	0.98	22.85	0.57
(2b) var <sub>t</sub> ( $\pi$ _GDP)	12.70	16.66	6.88	0.98	58.95	0.13
var <sub>t</sub> ( $\pi$ _PCE)	10.64	14.15	2.39	0.98	46.01	0.23
(2c) var <sub>t</sub> (mkt return)	305.99	152.63	257.50	0.98	670.14	101.29
var <sub>t</sub> (mkt return) (PCE defl.)	326.32	139.72	290.02	0.97	636.22	96.26
(3a) cov <sub>t</sub> ( $\pi$ _GDP,GDP growth)	3.21	6.02	0.49	0.96	20.29	-6.09
cov <sub>t</sub> ( $\pi$ _PCE,C growth)	1.18	3.27	-0.01	0.95	11.85	-3.63
(3b) cov <sub>t</sub> ( $\pi$ _GDP,mkt return)	-18.84	26.08	-10.62	0.97	8.52	-94.58
cov <sub>t</sub> ( $\pi$ _PCE,mkt return)	-17.98	19.08	-15.41	0.96	8.50	-69.84
(4a) Labor hours growth	1.14	3.95	1.74	0.30	12.23	-13.19
(4b) Growth in capital per hour	2.24	3.92	1.93	0.32	15.42	-10.58
Per capita capital growth	2.08	1.35	1.81	0.83	5.73	-1.07



Table A6, continued

		Summary Statistics on the Correlates, 1890-2016					
Correlate	(1) Mean	(2) St. Dev.	(3) Median	(4) 1 <sup>st</sup> Order Autocorr	(5) Max	(6) Min	
(5a) Dependency ratio	44.20	3.06	44.32	1.00	49.98	39.85	
(5b) Percent aged 40-64	25.29	3.91	25.85	1.00	33.33	18.21	
MY ratio	0.77	0.18	0.75	0.99	1.13	0.52	
$\Delta$ Dependency ratio	-0.07	0.28	-0.14	0.86	0.65	-0.52	
(5c) $\Delta$ Percent aged 40-64	0.11	0.19	0.11	0.90	0.57	-0.26	
$\Delta$ MY ratio	0.00	0.02	0.00	0.86	0.06	-0.05	
$\Delta$ Life expectancy	0.27	2.18	0.20	-0.40	15.60	-11.80	
Fed total deficit/GDP	2.28	4.40	1.00	0.75	26.86	-4.29	
(6a) Fed primary deficit/GDP	1.06	4.39	-0.01	0.74	26.11	-5.87	
(6b) Fed debt/GDP	42.50	28.83	39.50	0.98	118.96	2.43	
World debt/GDP	58.17	26.38	58.45	0.96	172.81	26.31	
(7) Current account/GDP	-0.29	1.98	0.13	0.91	5.83	-5.79	
(8) Relative price inv. goods	1.25	0.13	1.28	0.97	1.57	0.96	
(9) Top 10% income share	0.39	0.06	0.38	0.96	0.50	0.31	
(10) Baa-10 yr Treasury spread	2.03	0.93	1.98	0.84	5.52	0.59	
(11a) $\pi$ _GDP	2.44	4.39	2.25	0.51	21.01	-15.99	
(11b) $\pi$ _PCE	2.39	4.32	1.99	0.65	16.42	-12.56	
(12a) M1 growth	5.51	6.09	4.87	0.70	26.61	-13.41	
(12b) M2 growth	6.44	5.16	6.52	0.58	23.39	-16.91	

Table A7

## Summary Statistics on the Correlates, 1950-2016

Correlate	(1) Mean	(2) St. Dev.	(3) Median	(4) 1 <sup>st</sup> Order Autocorr	(5) Max	(6) Min
(1a) GDP growth	2.00	2.20	2.11	0.19	6.31	-3.69
(1b) Consumption growth	1.95	1.22	2.02	0.48	4.00	-2.00
(1c) World GDP growth	2.27	1.71	2.35	0.39	5.67	-4.25
(1d) TFP growth	1.12	1.77	0.88	0.05	6.06	-3.74
Labor productivity growth	2.09	1.49	2.05	0.11	6.40	-1.67
(2a) var <sub>t</sub> (C growth)	1.40	1.28	0.97	0.96	8.59	0.43
var <sub>t</sub> (GDP growth)	10.01	12.20	4.44	0.99	44.73	0.99
(2b) var <sub>t</sub> ( $\pi$ _NDS)	2.22	2.50	1.48	0.98	12.80	0.29
var <sub>t</sub> ( $\pi$ _GDP)	3.08	4.09	1.21	0.99	17.71	0.13
(2c) var <sub>t</sub> (mkt return)	246.04	75.86	250.85	0.90	393.55	96.57
var <sub>t</sub> (mkt return) (GDP defl.)	246.80	81.75	252.57	0.91	529.54	101.29
(3a) cov <sub>t</sub> ( $\pi$ _NDS,C growth)	0.03	0.75	0.05	0.75	4.70	-0.71
cov <sub>t</sub> ( $\pi$ _GDP,GDP growth)	-0.32	1.95	-0.04	0.79	7.69	-6.09
(3b) cov <sub>t</sub> ( $\pi$ _NDS,mkt return)	-11.08	7.54	-11.91	0.94	0.62	-24.64
cov <sub>t</sub> ( $\pi$ _GDP,mkt return)	-10.42	9.69	-10.97	0.95	1.77	-32.69
(4a) Labor hours growth	1.05	2.39	1.51	0.23	5.68	-7.35
(4b) Capital deepening	2.52	2.30	2.13	0.25	7.92	-1.73
Per capita capital growth	2.42	1.32	2.28	0.80	5.15	-0.79

Table A7, continued

## Summary Statistics on the Correlates, 1950-2016

Correlate	(1) Mean	(2) St. Dev.	(3) Median	(4) 1 <sup>st</sup> Order Autocorr	(5) Max	(6) Min
(5a) Dependency ratio	43.31	2.98	41.93	0.99	48.83	39.85
(5b) Dependency ratio, world	42.50	2.61	42.59	1.00	46.13	38.98
(5c) Percent aged 40-64	27.75	2.86	26.55	1.00	33.33	24.53
(5d) Percent aged 40-64, world	28.34	2.60	26.98	1.00	33.58	25.93
MY ratio	0.87	0.18	0.91	0.99	1.13	0.55
MY ratio, world	0.87	0.13	0.82	1.00	1.09	0.70
$\Delta$ Dependency ratio	-0.02	0.33	-0.03	0.95	0.65	-0.52
$\Delta$ Dependency ratio, world	-0.07	0.17	-0.10	0.97	0.24	-0.44
(5e) $\Delta$ Percent aged 40-64	0.08	0.24	-0.01	0.94	0.57	-0.26
(5f) $\Delta$ Percent aged 40-64, world	0.11	0.15	0.08	0.96	0.36	-0.20
$\Delta$ Life expectancy	0.18	0.23	0.15	-0.06	0.80	-0.61
$\Delta$ MY ratio	0.00	0.03	0.00	0.94	0.05	-0.05
$\Delta$ MY ratio, world	0.00	0.01	0.00	0.97	0.03	-0.02

Table A7, continued

## Summary Statistics on the Correlates, 1950-2016

Correlate	(1) Mean	(2) St. Dev.	(3) Median	(4) 1 <sup>st</sup> Order Autocorr	(5) Max	(6) Min
Fed total deficit/GDP	2.20	2.33	2.14	0.79	9.80	-2.30
(6a) Fed primary deficit/GDP	0.43	2.30	0.16	0.77	8.50	-4.46
(6b) Fed debt/GDP	56.30	19.60	55.02	0.98	105.25	30.98
World debt/GDP	51.50	17.81	49.82	0.99	96.91	26.31
(7) Current account/GDP	-1.27	1.85	-0.60	0.95	1.17	-5.79
(8) Relative price inv. goods	1.22	0.16	1.31	0.99	1.43	0.96
(9) Top 10% income share	0.37	0.06	0.35	0.98	0.50	0.31
(10) Baa-10 yr Treasury spread	1.85	0.80	1.81	0.81	4.04	0.59
(11a) $\pi_{\text{GDP}}$	3.14	2.13	2.30	0.81	8.98	0.79
(11b) $\pi_{\text{NDS}}$	3.33	2.20	2.81	0.79	10.34	0.13
(12a) M1 growth	5.08	3.83	4.83	0.69	14.29	-3.34
(12b) M2 growth	6.43	2.38	6.52	0.63	12.00	1.02

## Some Evidence on Secular Drivers of U.S. Safe Real Rates

Kurt G. Lunsford  
Federal Reserve Bank of Cleveland

Kenneth D. West  
University of Wisconsin

October 2017  
Revised August 2018

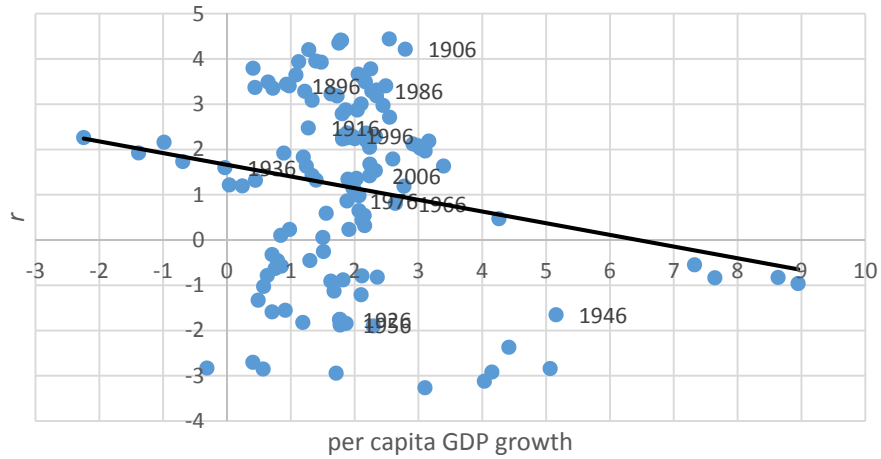
### APPENDIX 2

This appendix presents the following graphs, for each correlate:

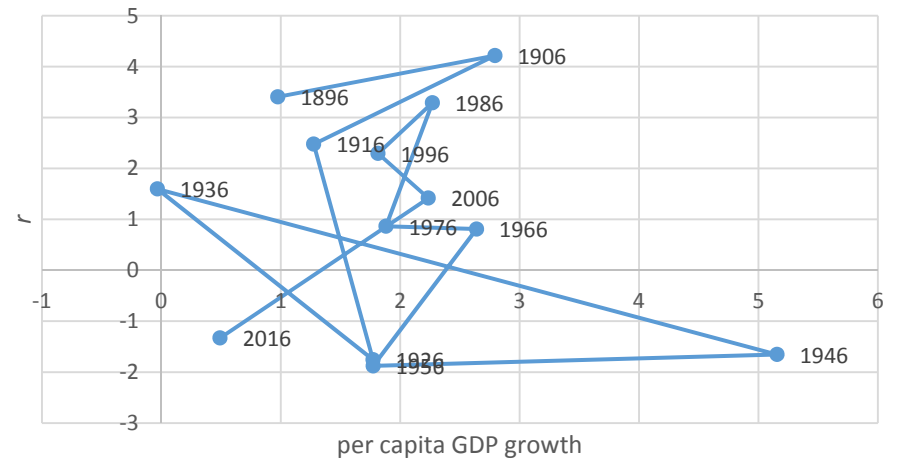
- Scatterplot of 10 year averages, correlate vs.  $r$
- Every 10<sup>th</sup> observation of the preceding scatterplot (to make it easier to see movement over time)
- Low frequency component of a correlate and  $r$ , computed per Müller and Watson (2018), 1890-2016
- Low frequency component of a correlate and  $r$ , computed per Müller and Watson (2018), 1950-2016

If data on a correlate is not available over the entire indicated period, the sample period is adjusted accordingly. If a correlate is used in only one sample period but not the other, it appears in only one of the two low frequency plots. The values of  $r$ , and of Müller and Watson's (2018) the low frequency component of  $r$ , are the same across graphs.

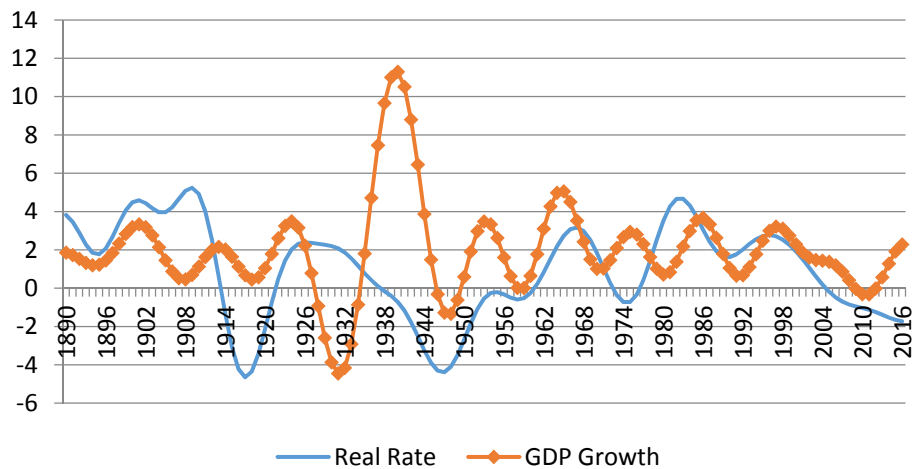
***r* vs. per capita GDP growth (10 yr averages)**



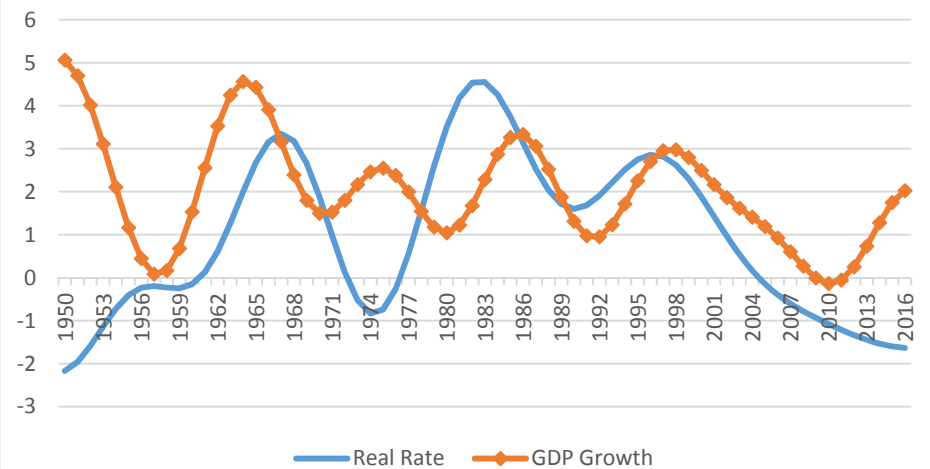
***r* vs. per capita GDP growth (10 yr averages)**



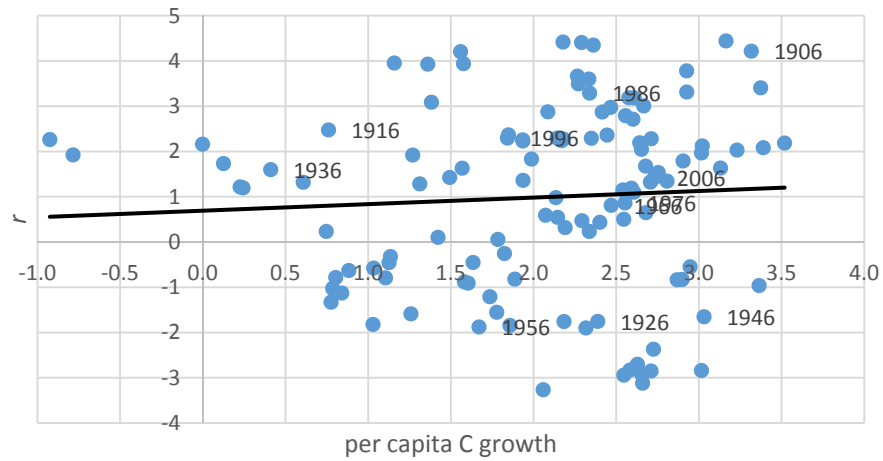
***r* and per capita GDP growth (lowpass filter)**



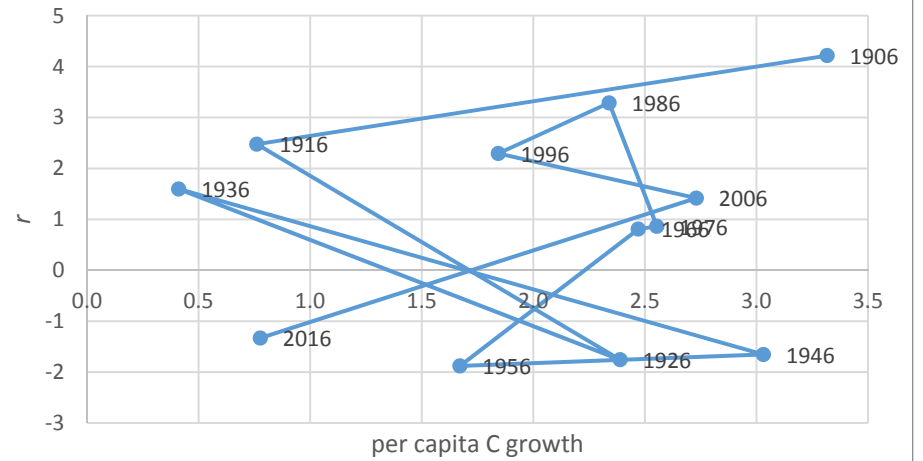
***r* and per capita GDP growth (lowpass filter)**



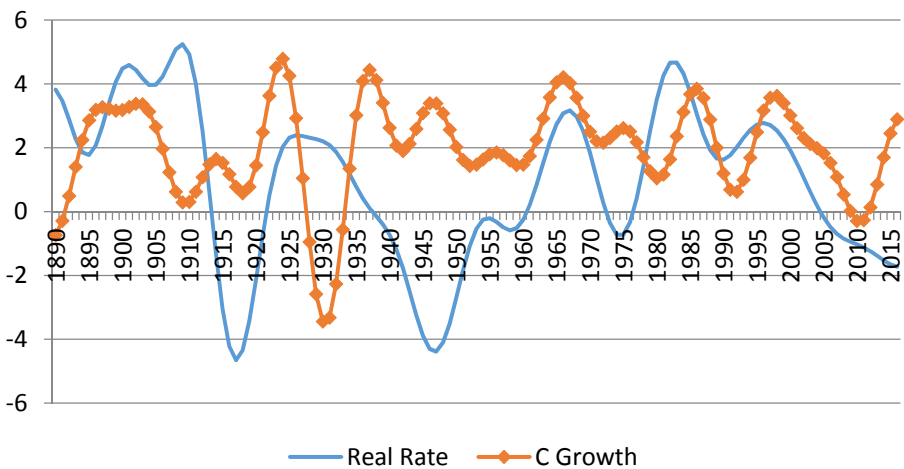
***r* vs. per capita C growth (10 yr averages)**

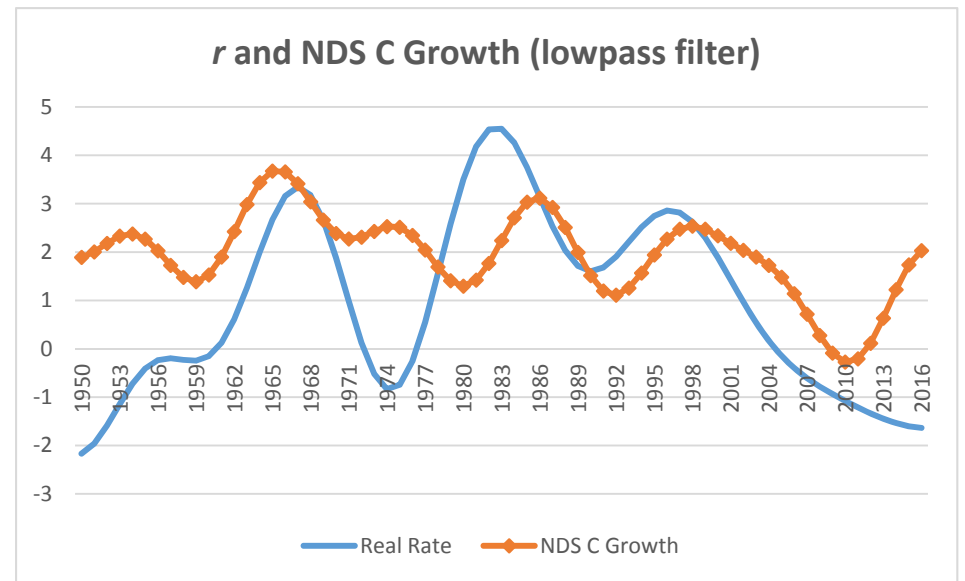
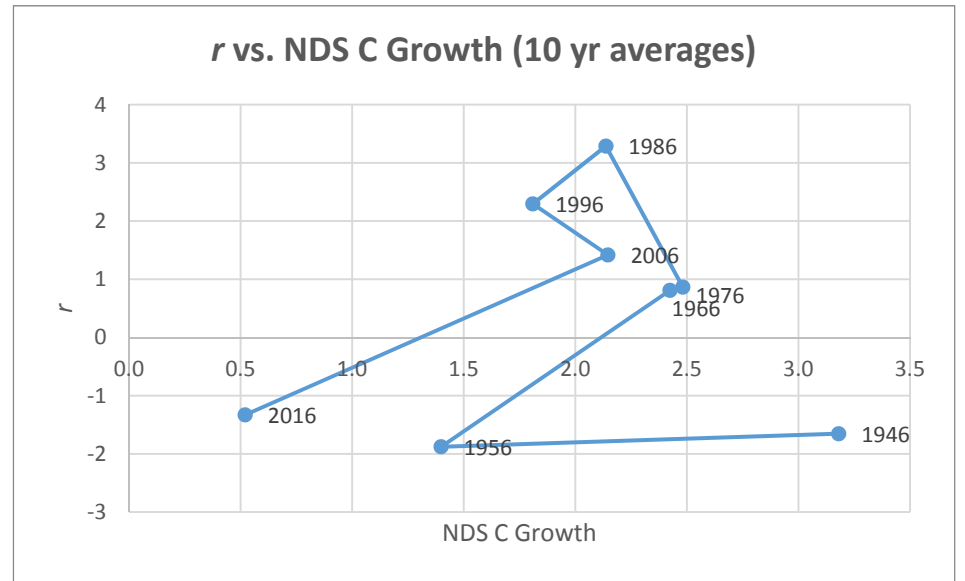
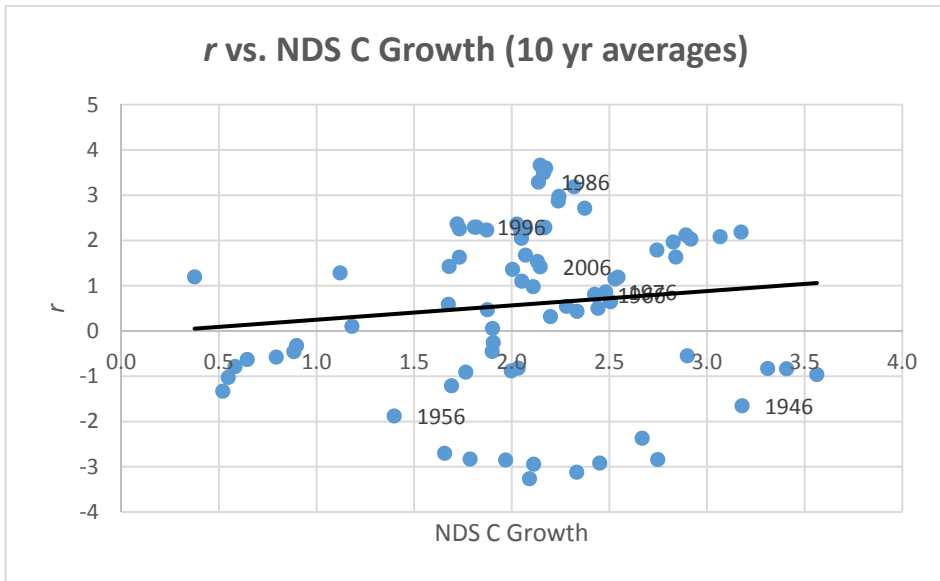


***r* vs. per capita C growth (10 yr averages)**



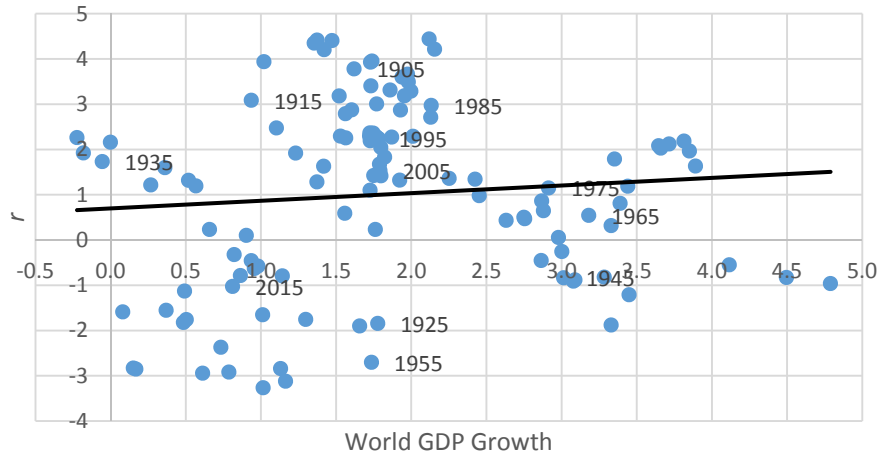
***r* and per capita C growth (lowpass filter)**



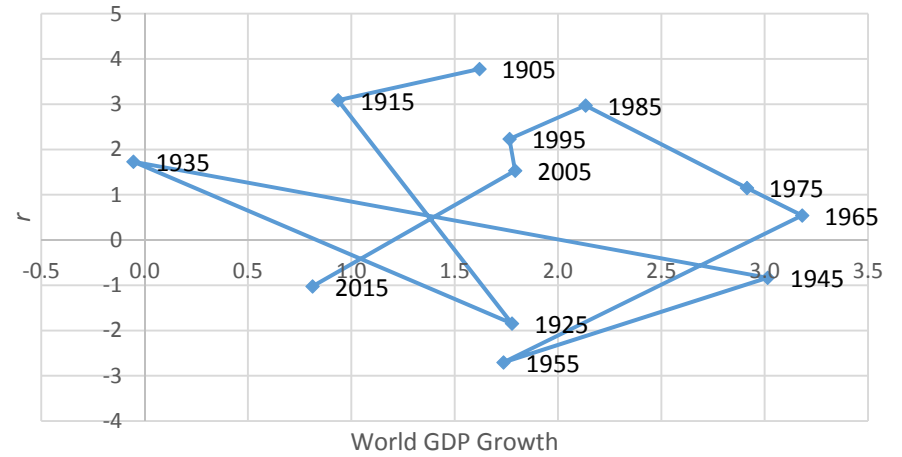




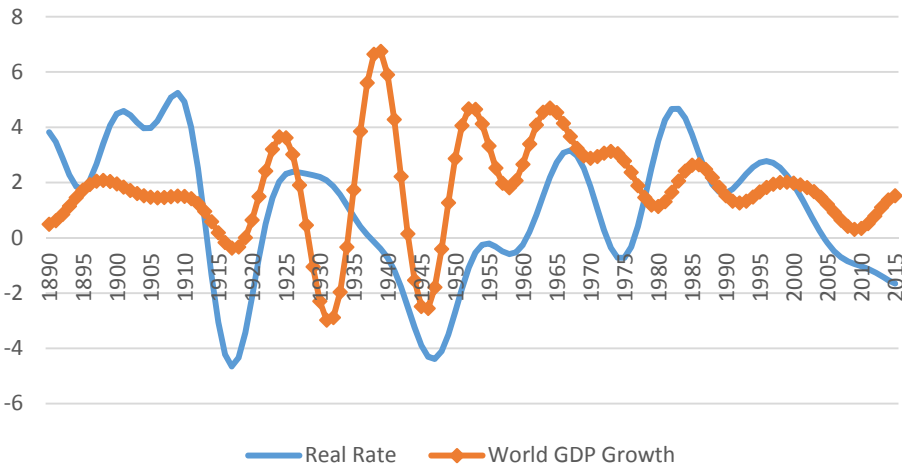
***r* vs. World GDP Growth (10 yr averages)**



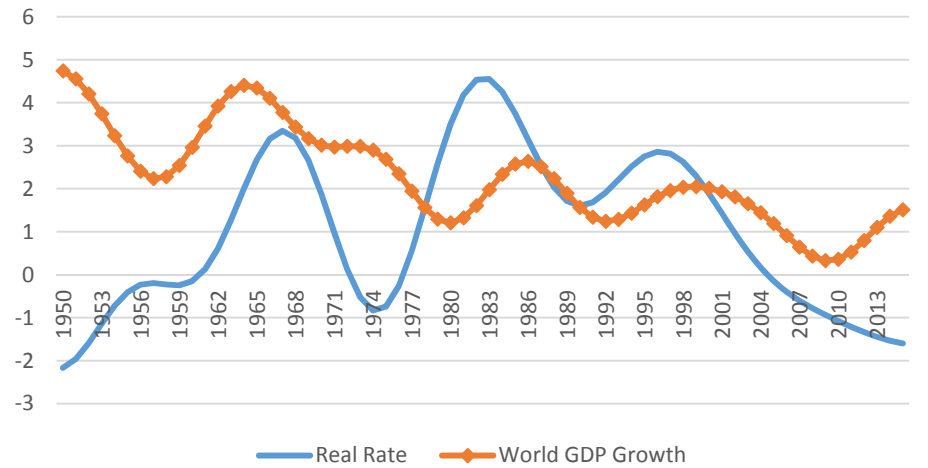
***r* vs. World GDP Growth (10 yr averages)**

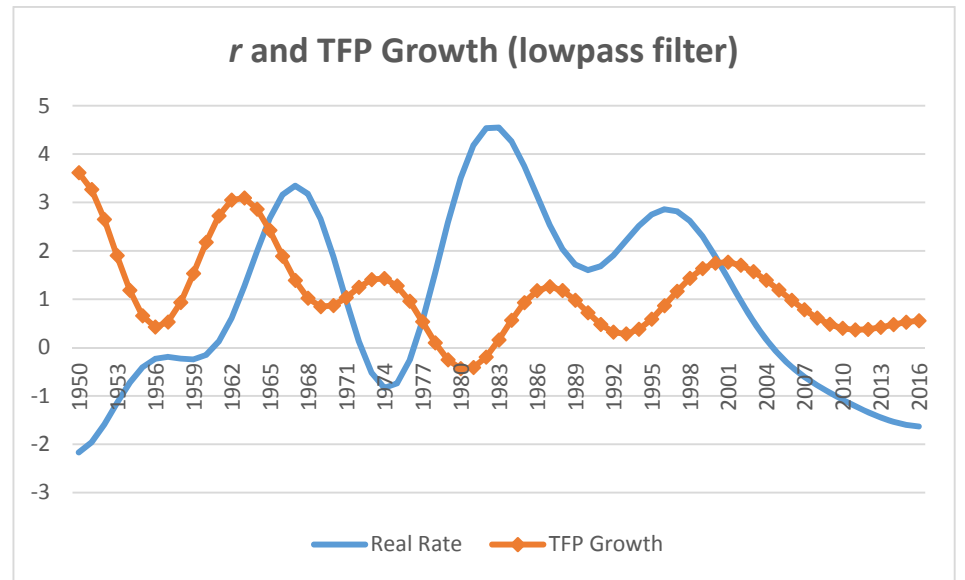
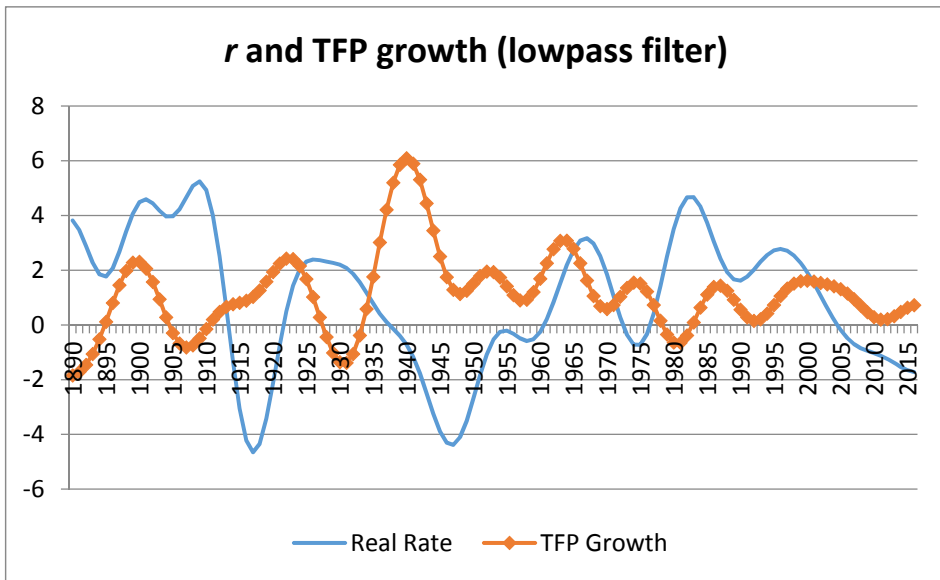
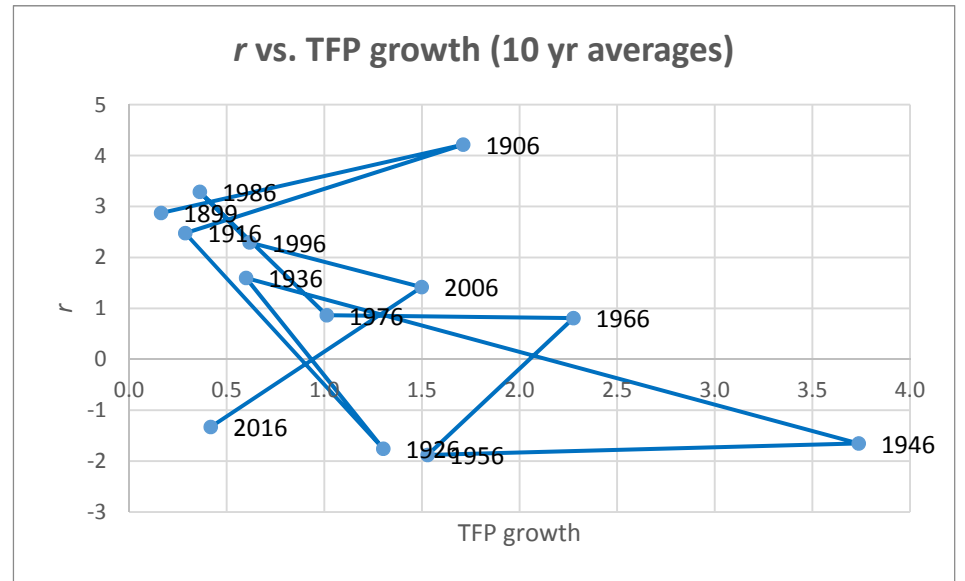
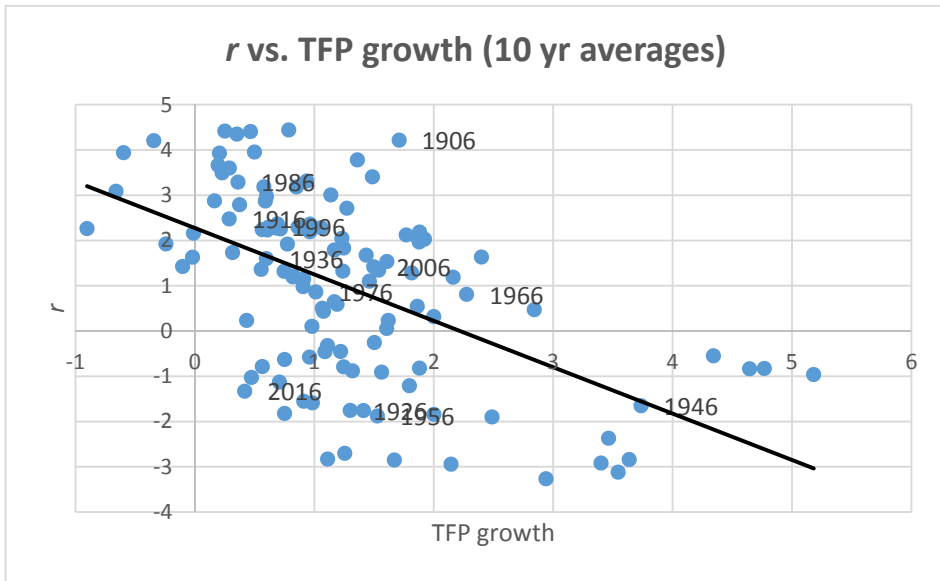


***r* and World GDP Growth (lowpass filter)**

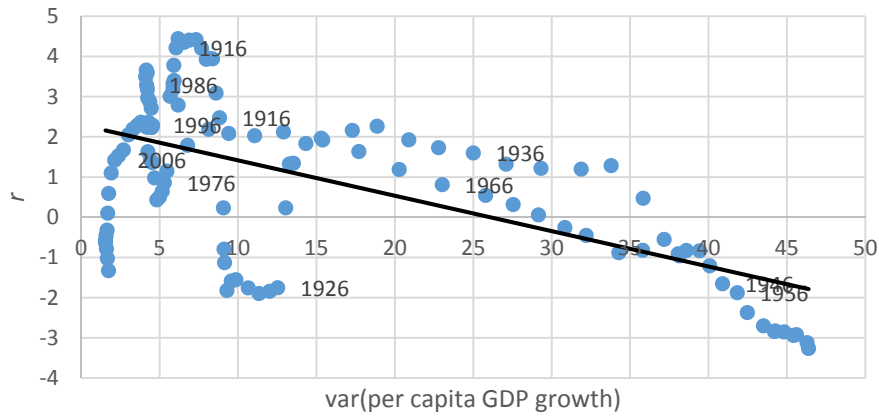


***r* and World GDP Growth (lowpass filter)**

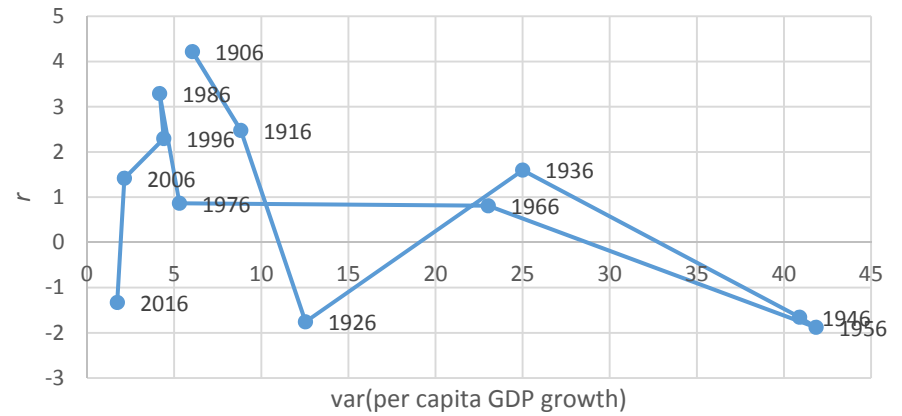




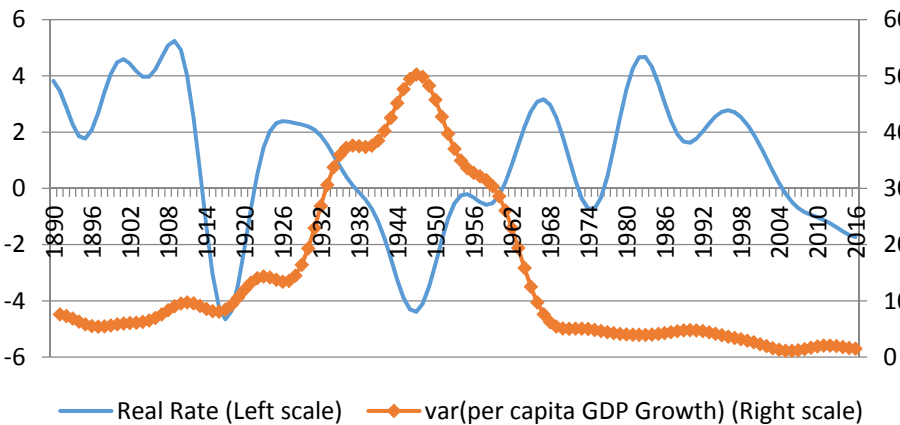
**$r$  vs.  $\text{var}_t(\text{per capita GDP growth})$  (10 yr averages)**



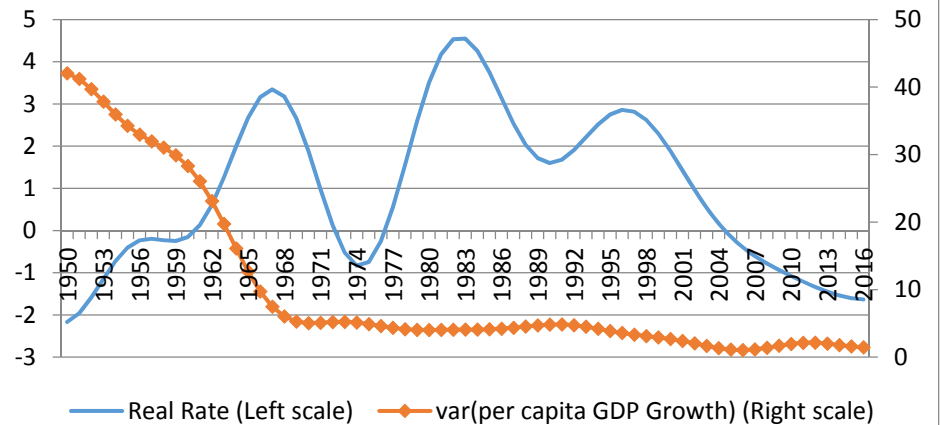
**$r$  vs.  $\text{var}_t(\text{per capita GDP growth})$  (10 yr averages)**

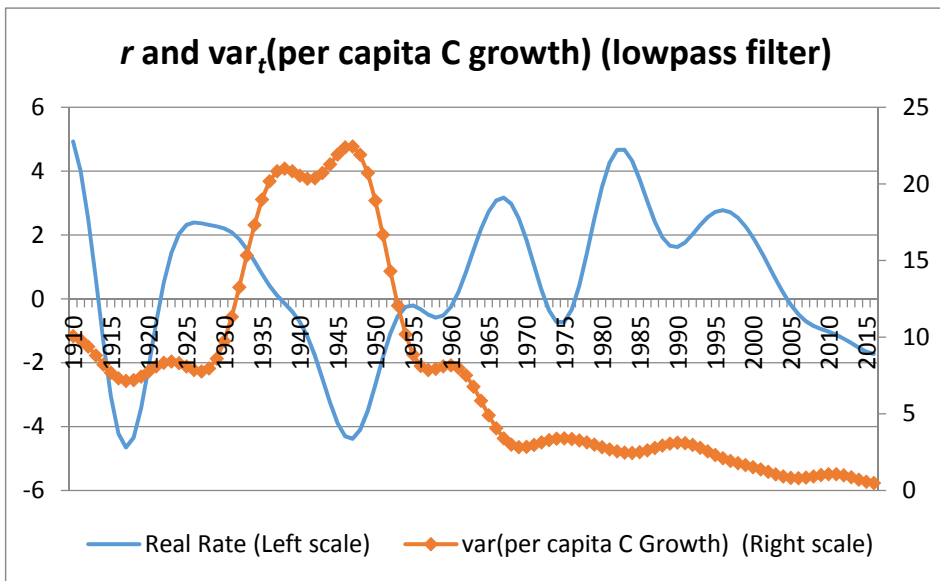
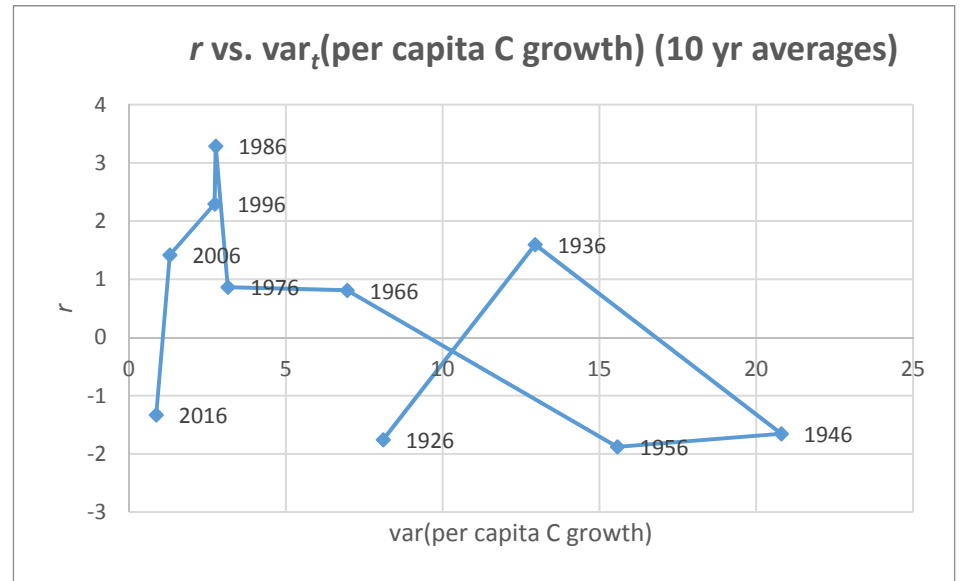
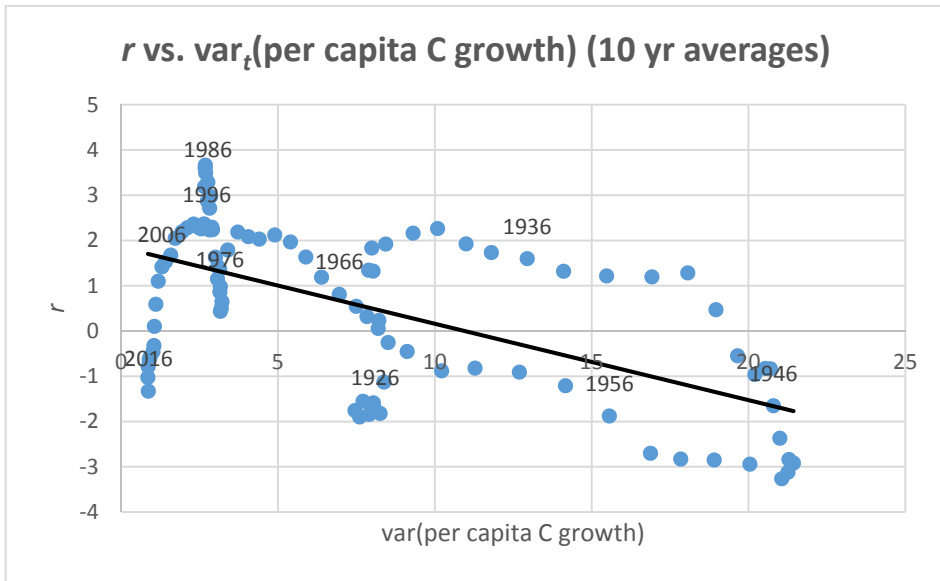


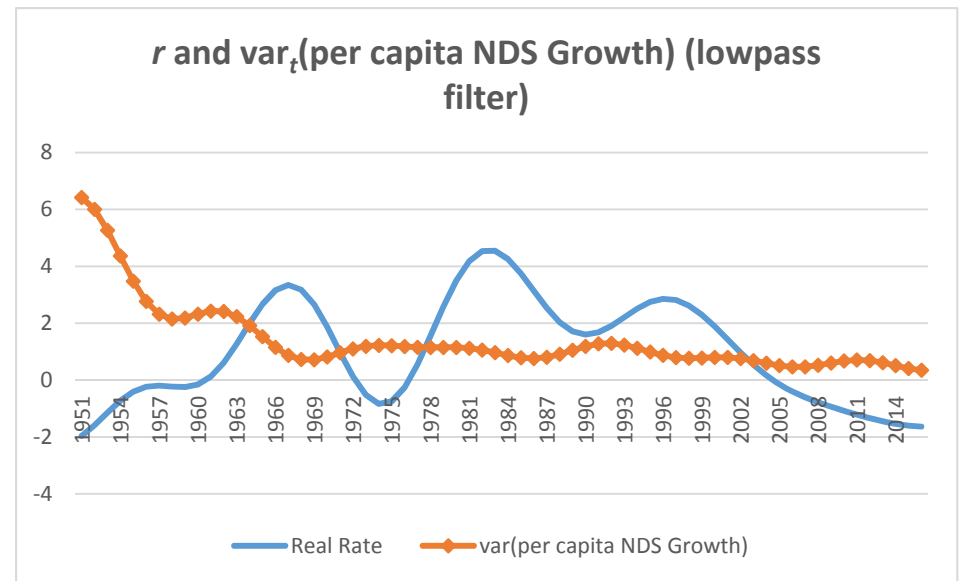
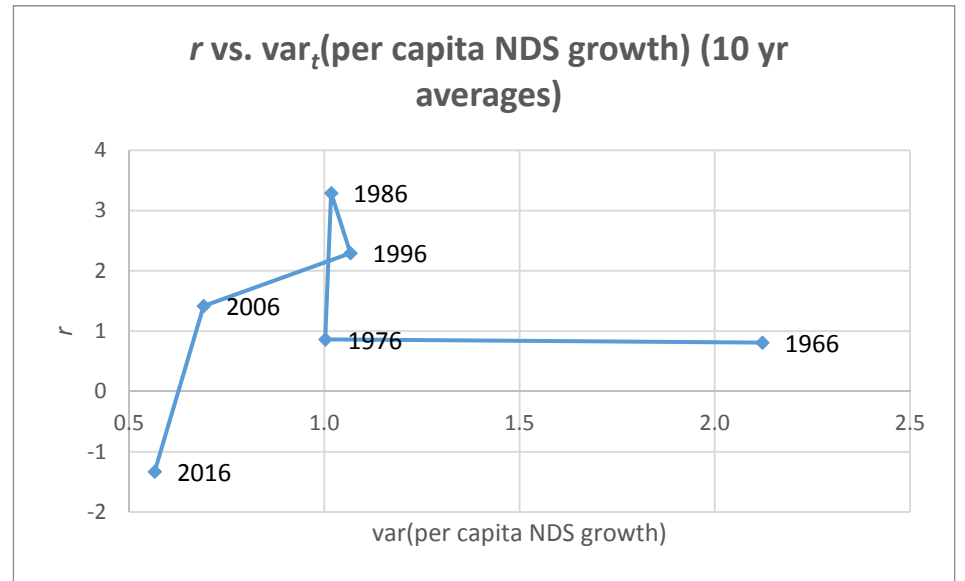
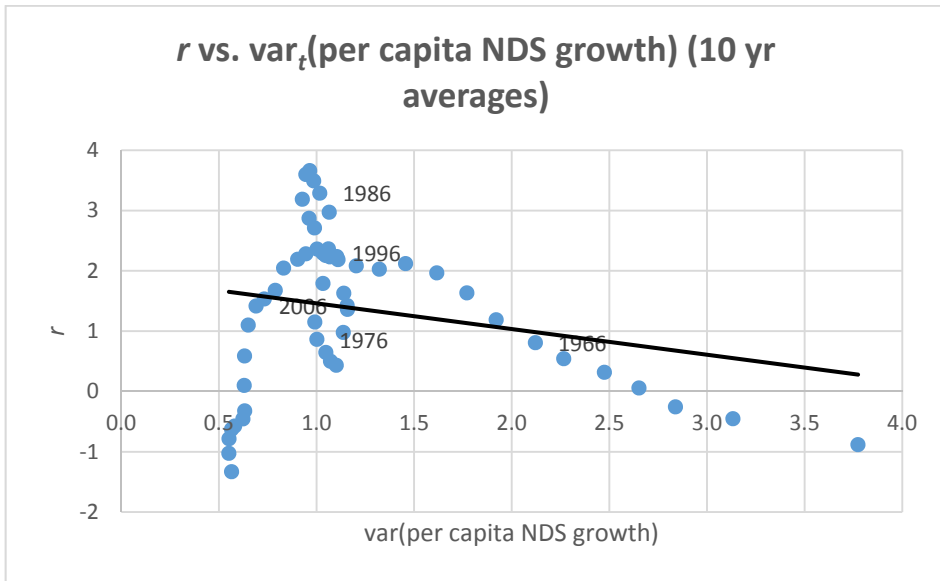
**$r$  and  $\text{var}_t(\text{per capita GDP growth})$  (lowpass filter)**

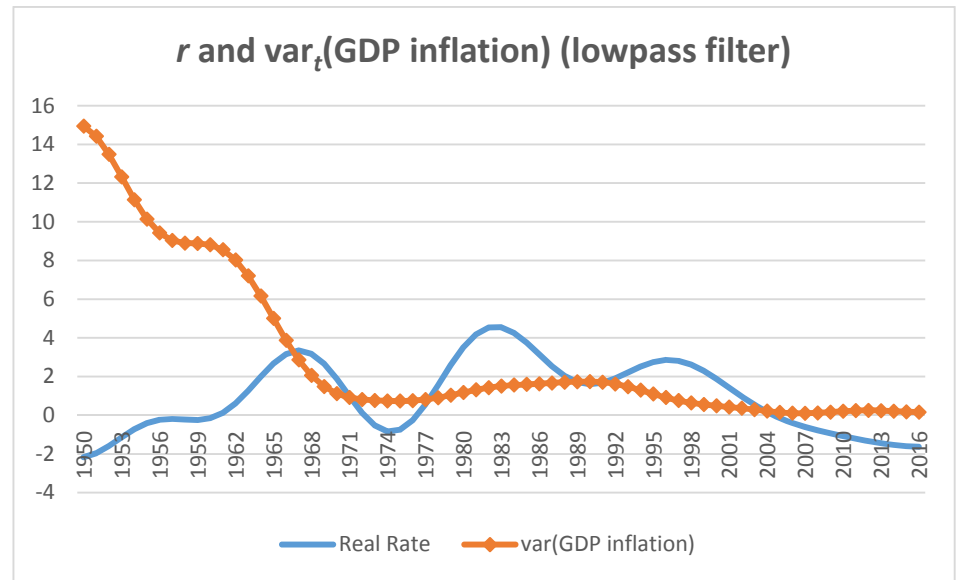
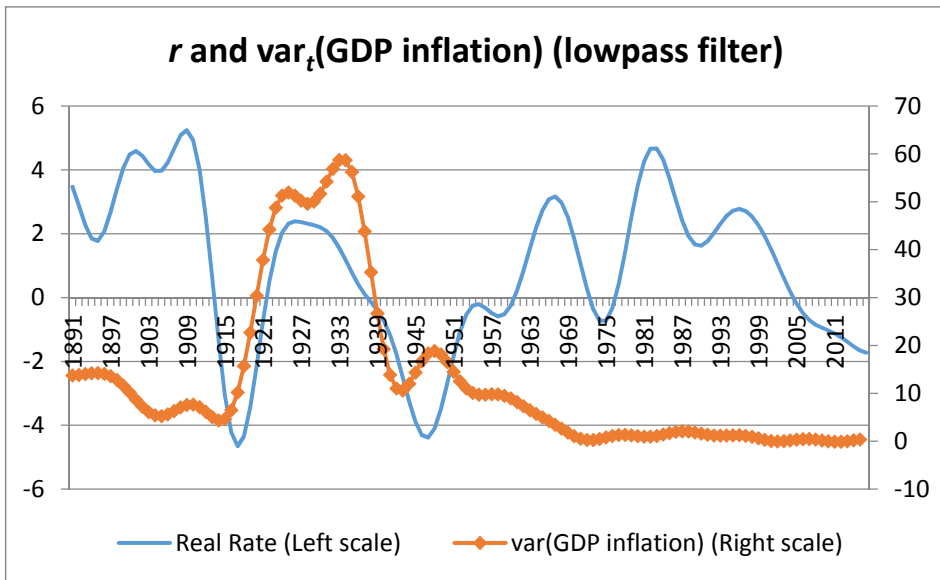
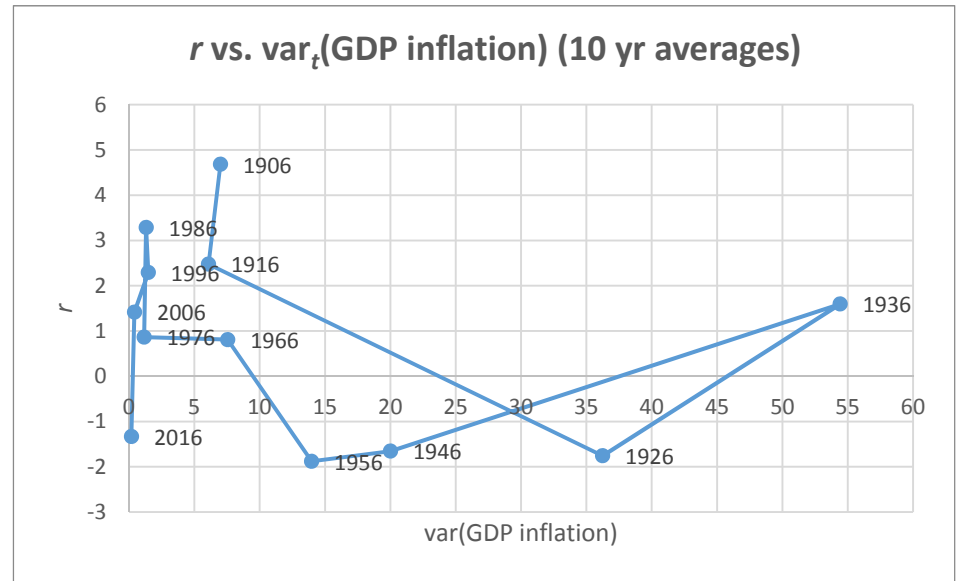
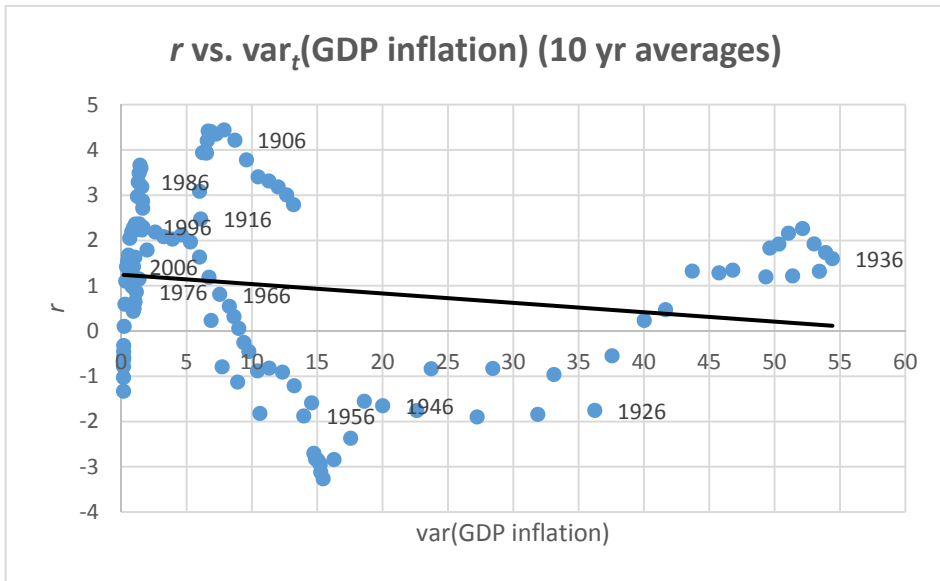


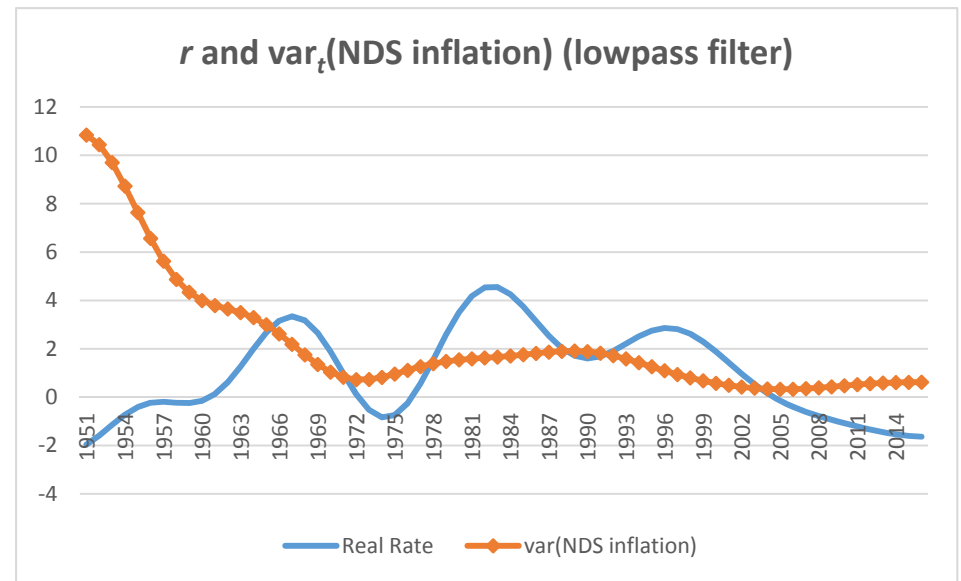
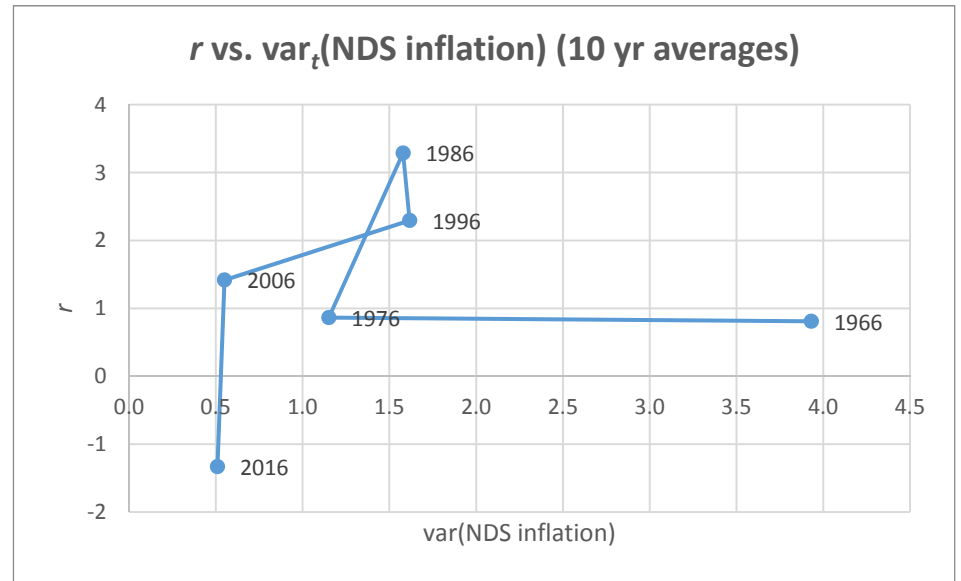
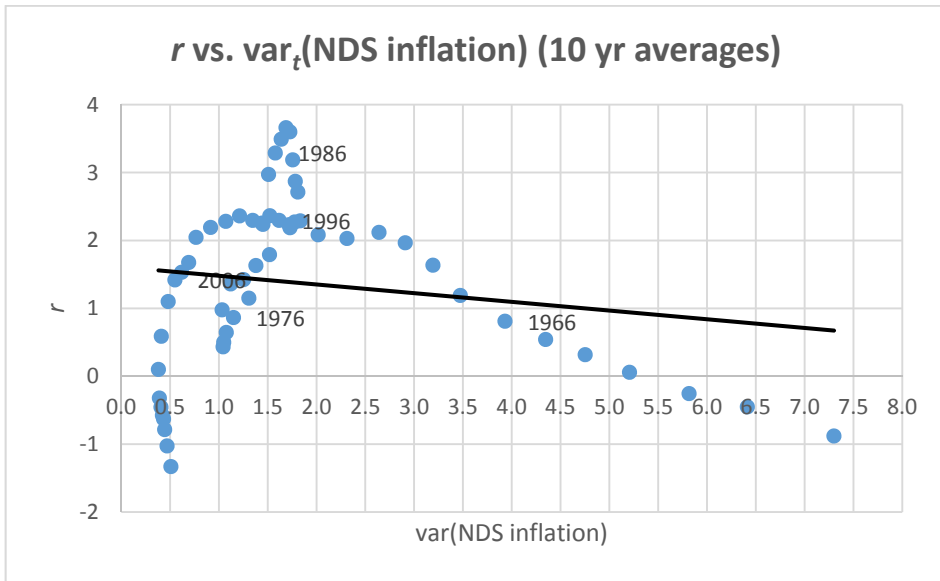
**$r$  and  $\text{var}_t(\text{per capita GDP growth})$  (lowpass filter)**



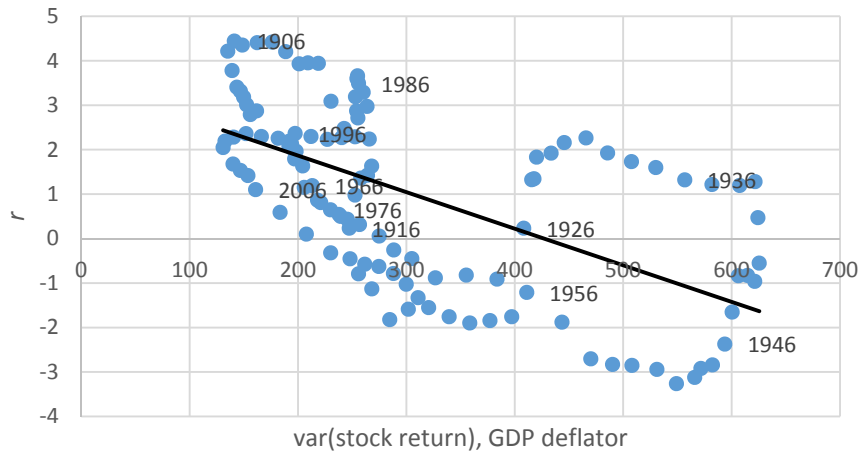




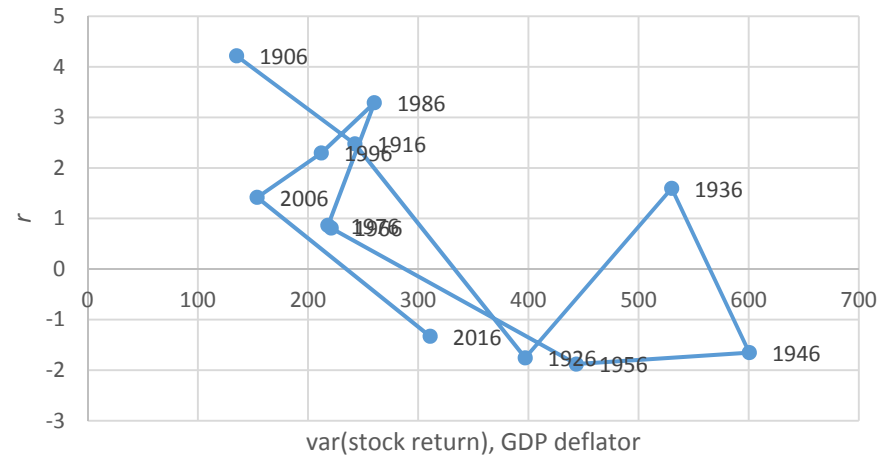




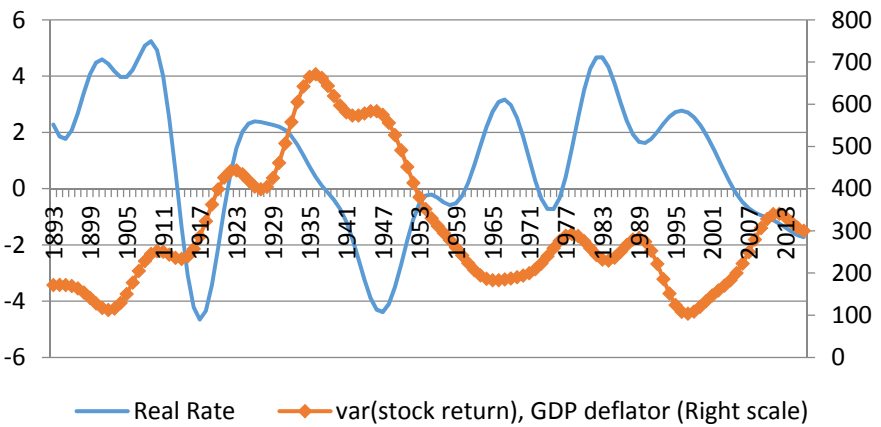
**$r$  vs.  $\text{var}_t(\text{stock return})$ , GDP deflator (10 yr averages)**



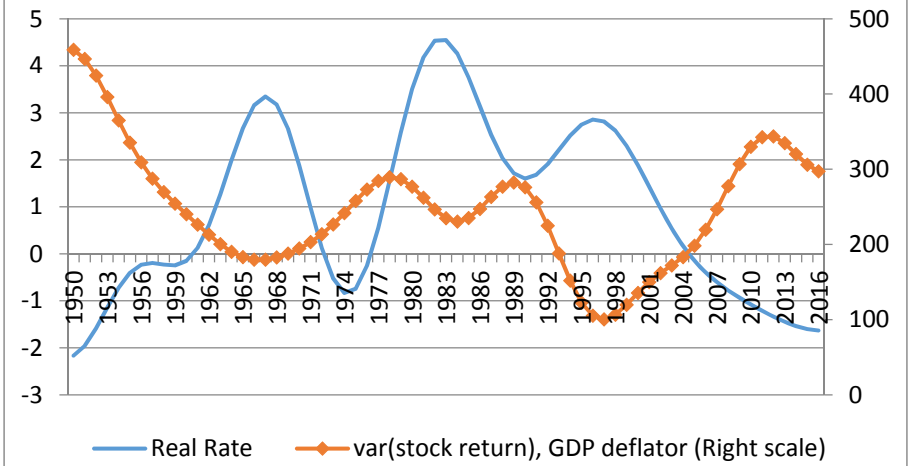
**$r$  vs.  $\text{var}_t(\text{stock return})$ , GDP deflator (10 yr averages)**



**$r$  and  $\text{var}_t(\text{stock return})$ , GDP deflator (lowpass filter)**

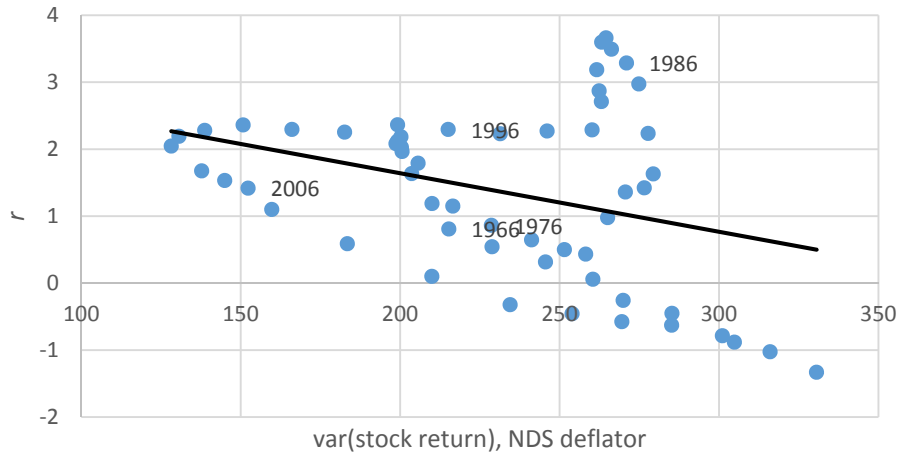


**$r$  and  $\text{var}_t(\text{stock return})$ , GDP deflator (lowpass filter)**

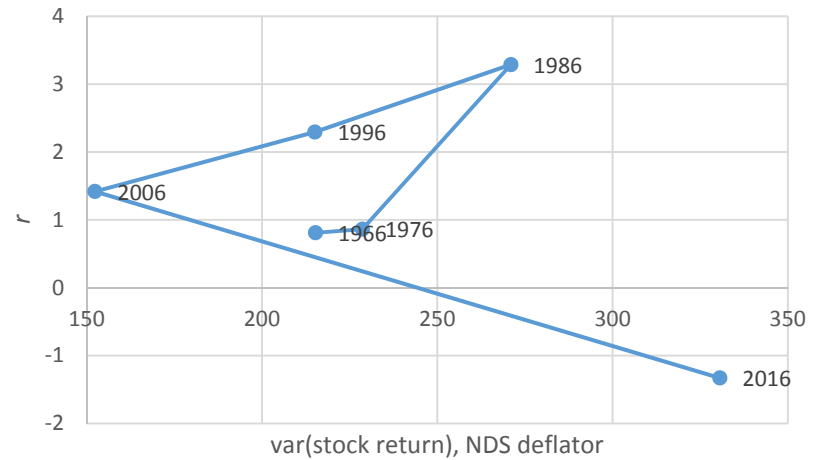




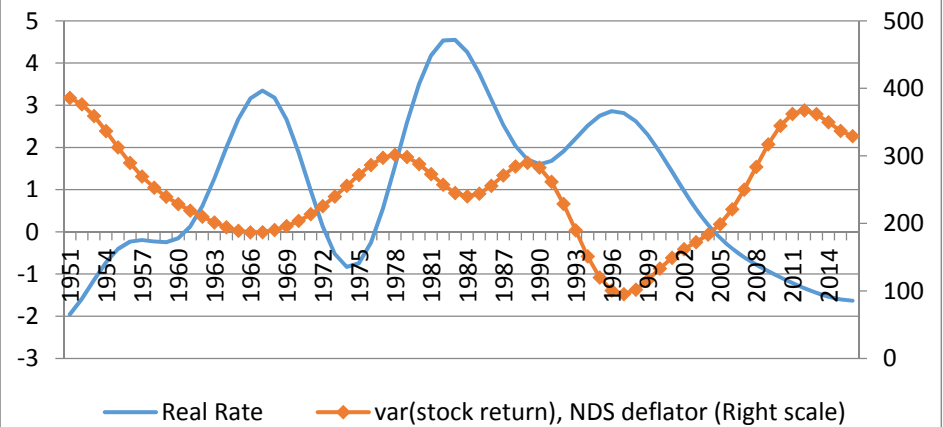
**$r$  vs.  $\text{var}_t(\text{stock return})$ , NDS deflator (10 yr averages)**



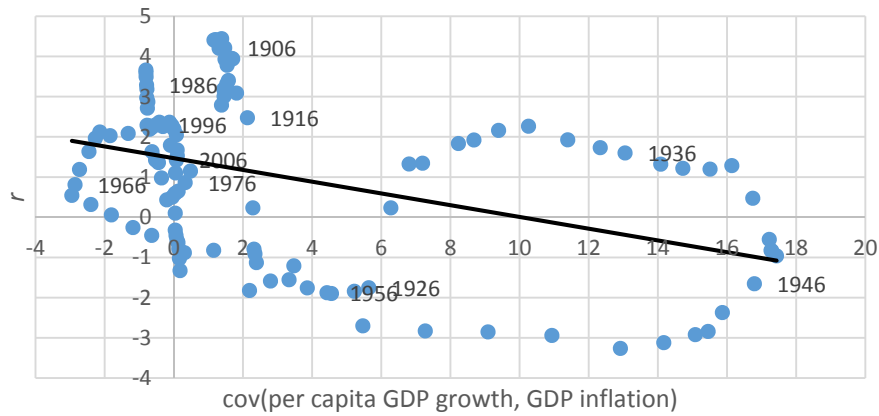
**$r$  vs.  $\text{var}_t(\text{stock return})$ , NDS deflator (10 yr averages)**



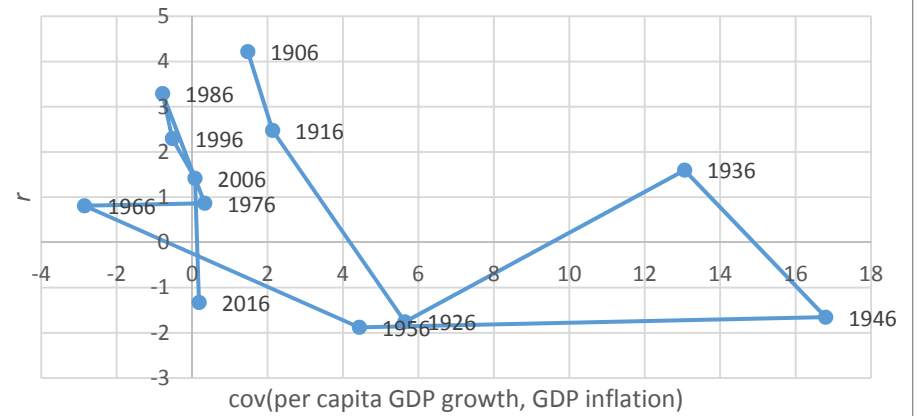
**$r$  and  $\text{var}_t(\text{stock return})$ , NDS deflator (lowpass filter)**



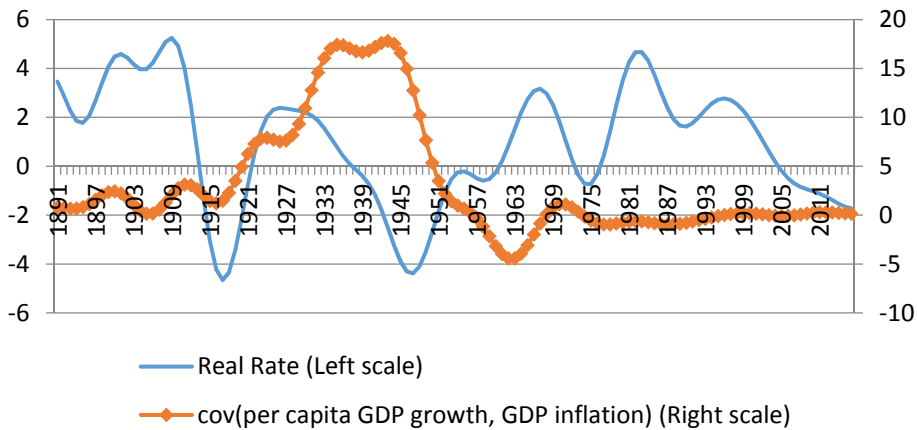
**$r$  vs.  $cov_t(\text{per capita GDP growth, GDP inflation})$   
(10 yr averages)**



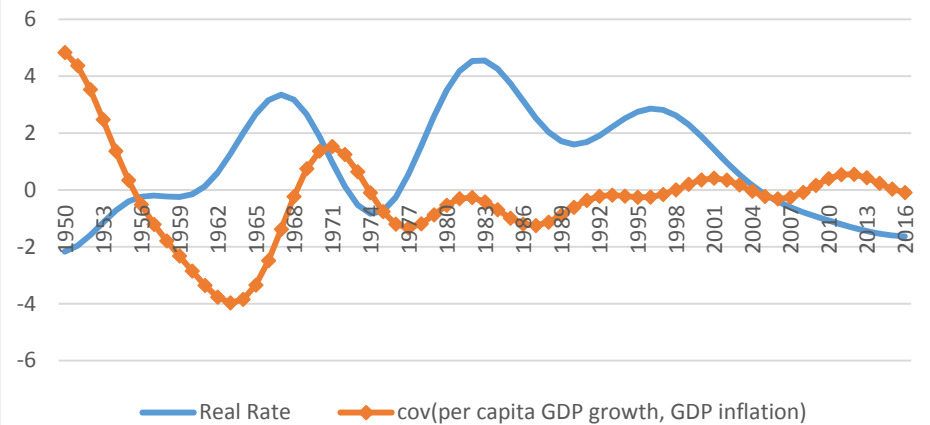
**$r$  vs.  $cov_t(\text{per capita GDP growth, GDP inflation})$   
(10 yr averages)**



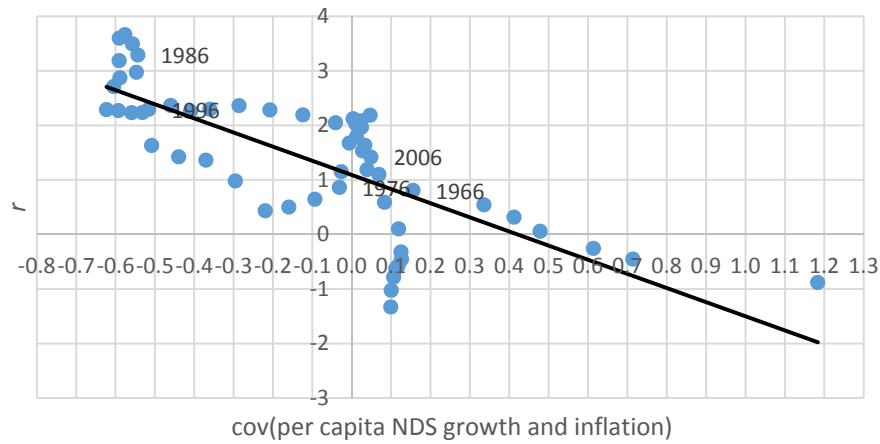
**$r$  and  $cov_t(\text{per capita GDP growth, GDP inflation})$  (lowpass filter)**



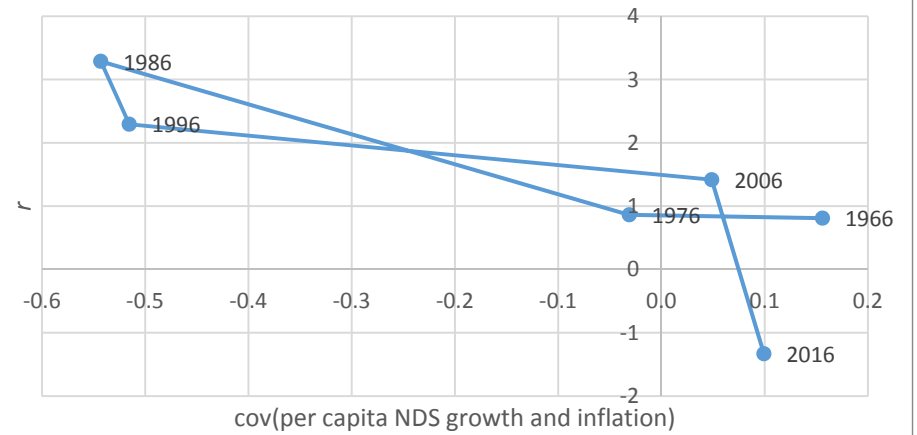
**$r$  and  $cov_t(\text{per capita GDP growth, GDP inflation})$  (lowpass filter)**



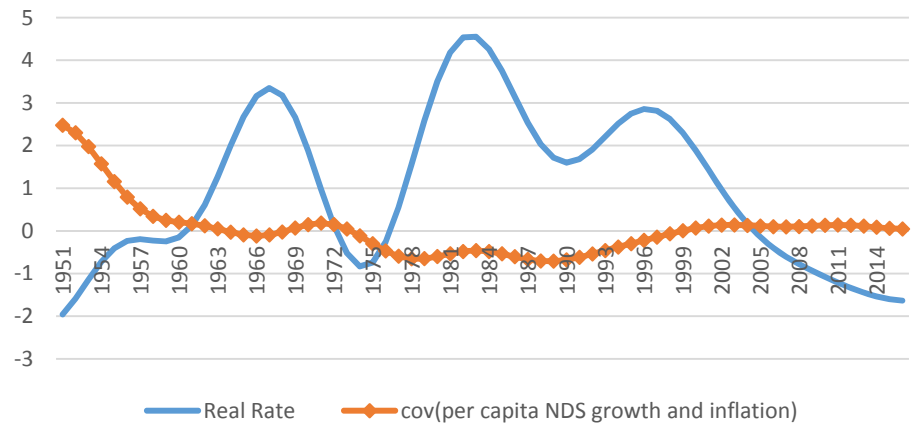
**$r$  vs.  $cov_t(\text{per capita NDS growth, NDS inflation})$   
(10 yr averages)**

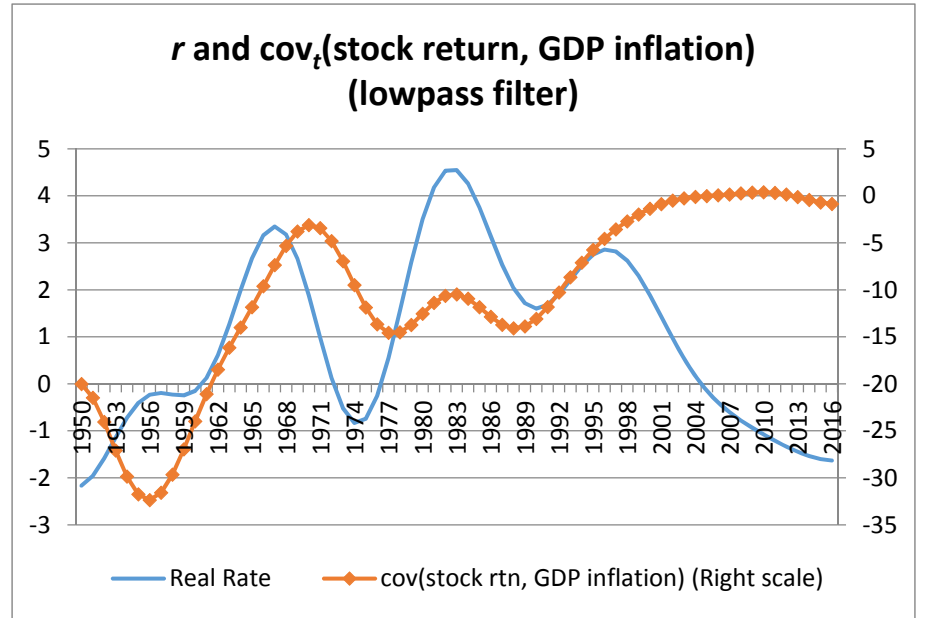
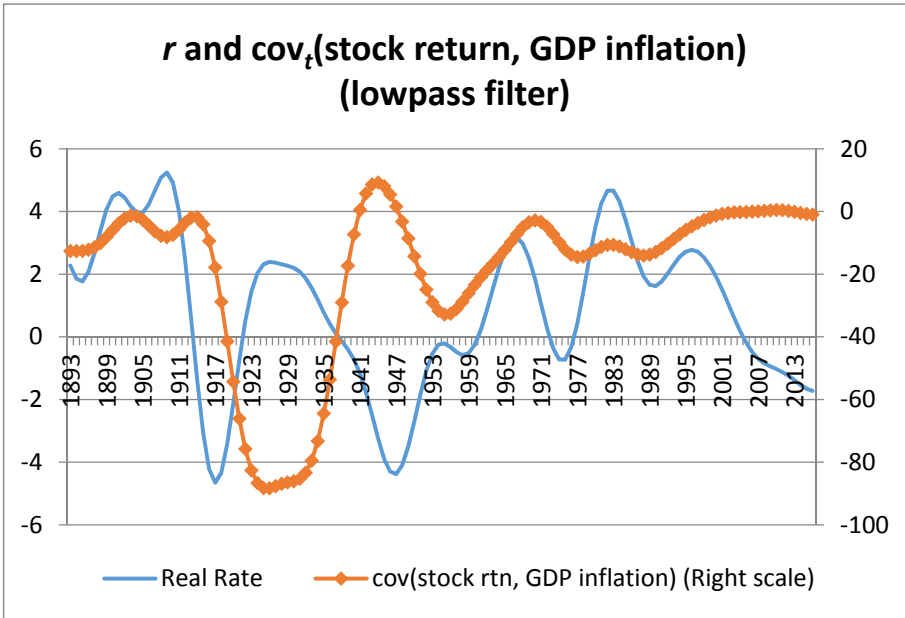
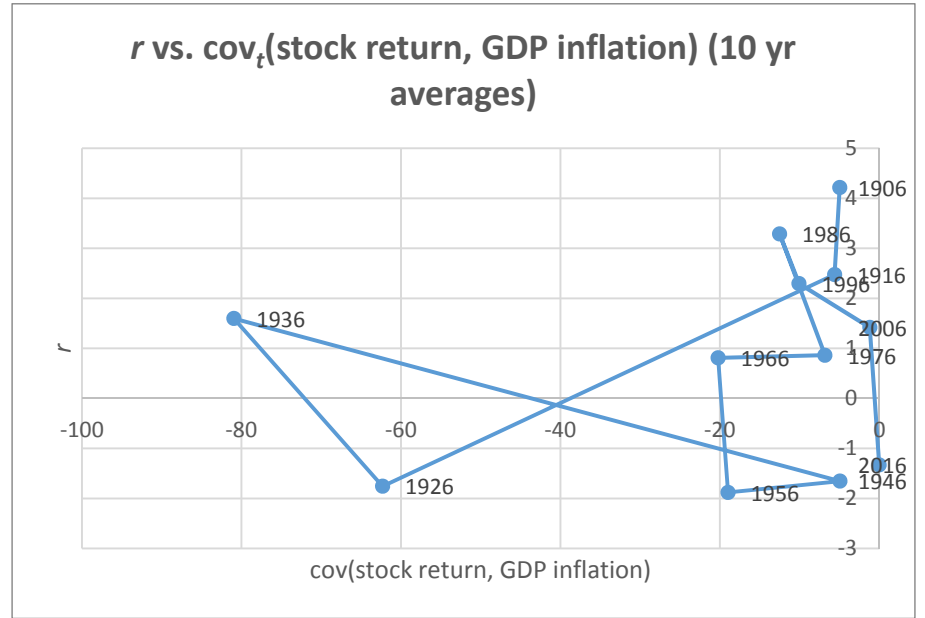
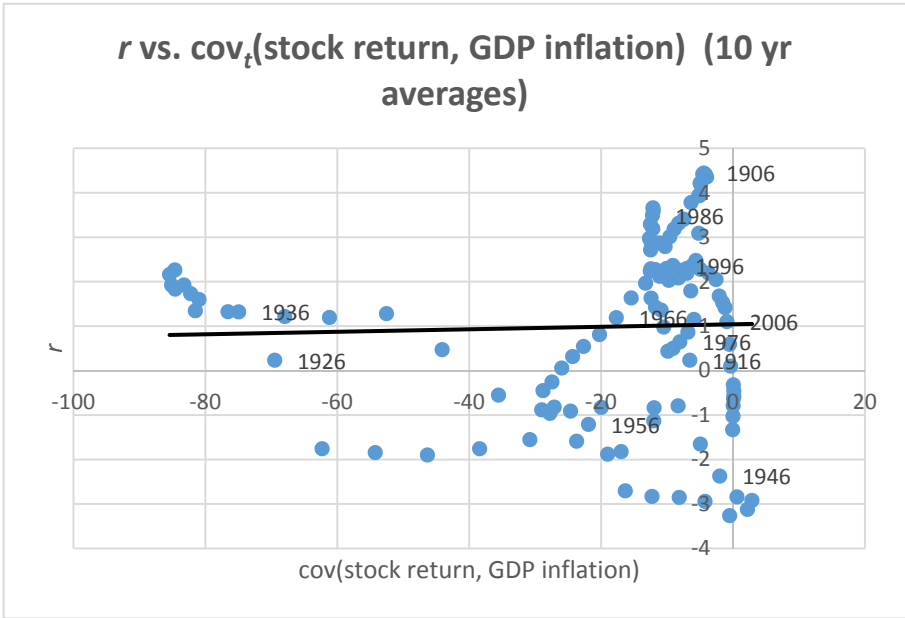


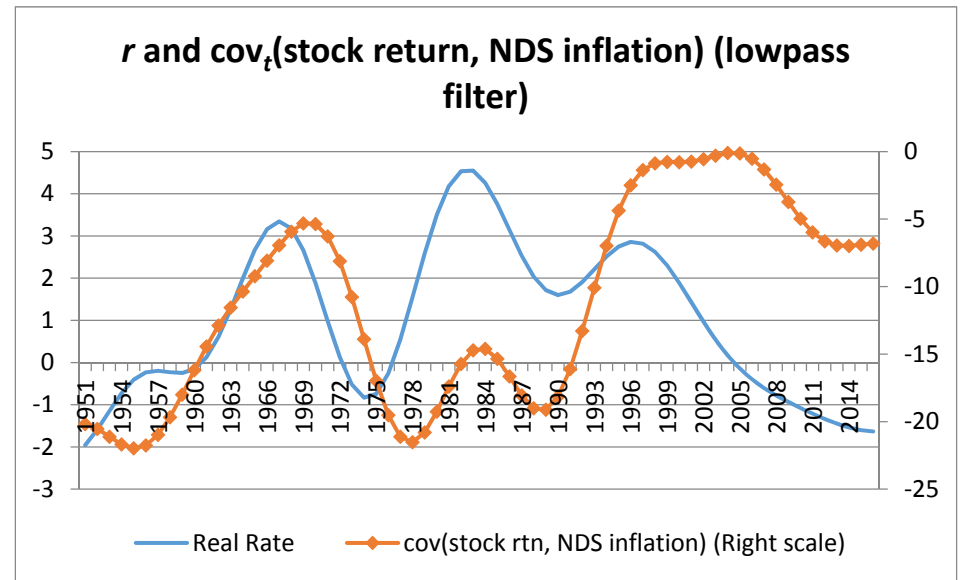
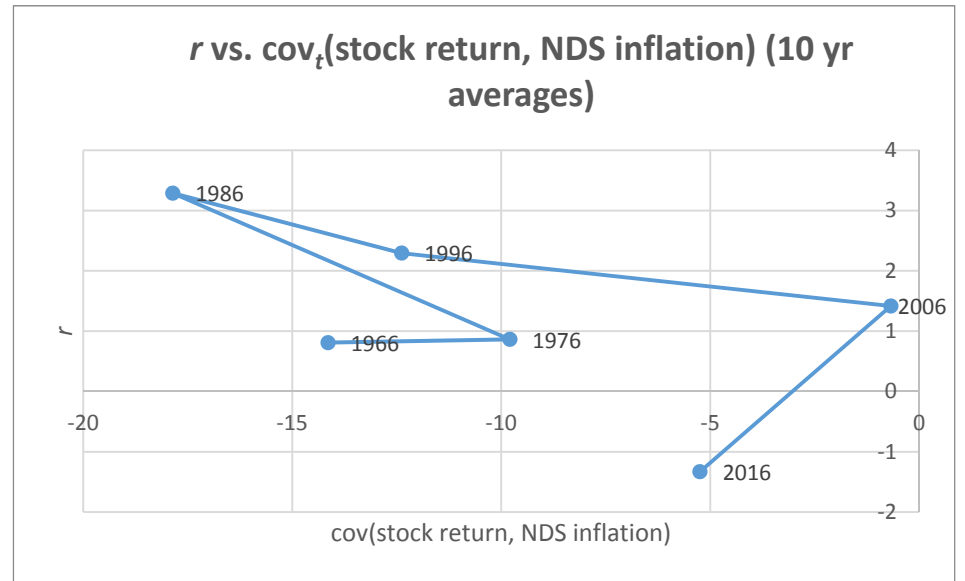
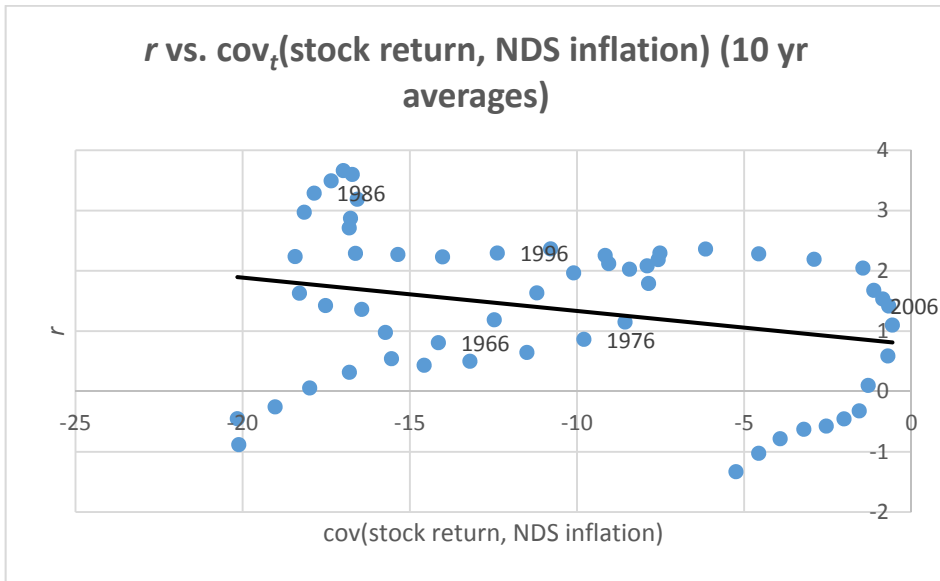
**$r$  vs.  $cov_t(\text{per capita NDS growth, NDS inflation})$   
(10 yr averages)**



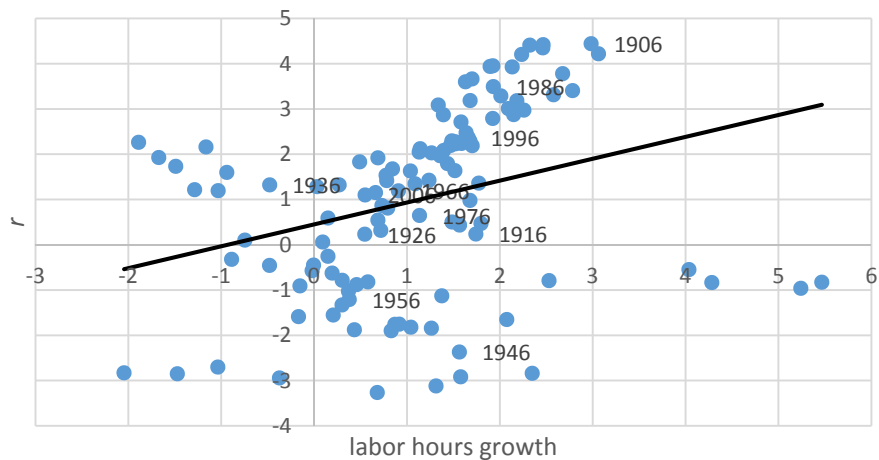
**$r$  and  $cov_t(\text{per capita NDS growth, NDS inflation})$  (lowpass filter)**



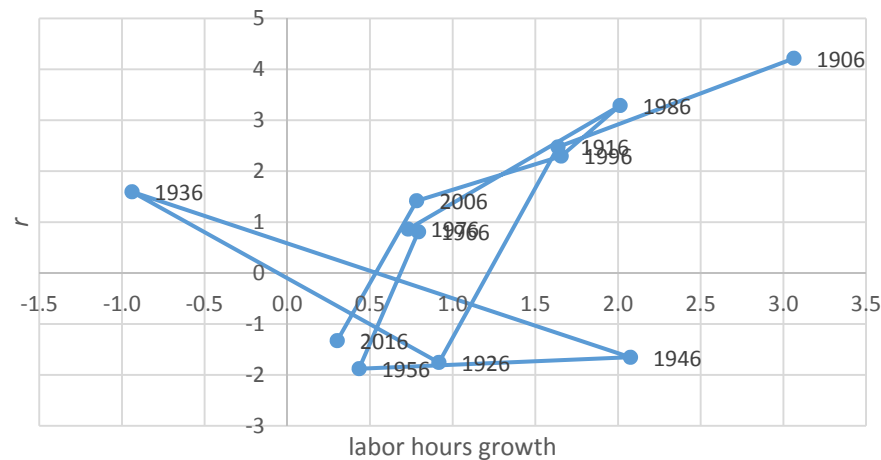




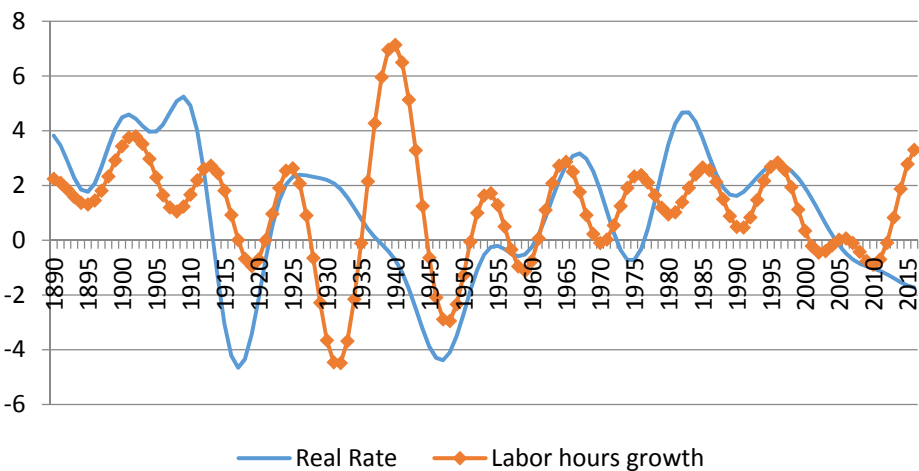
***r* vs. labor hours growth (10 yr averages)**



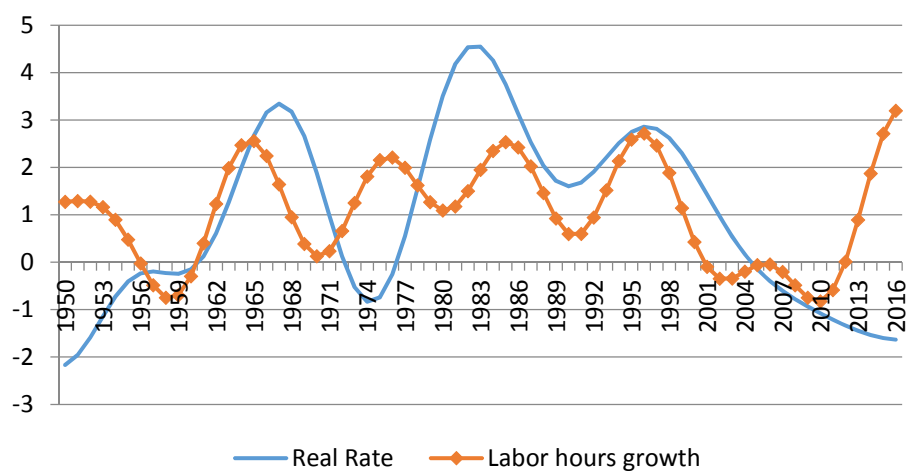
***r* vs. labor hours growth (10 yr averages)**



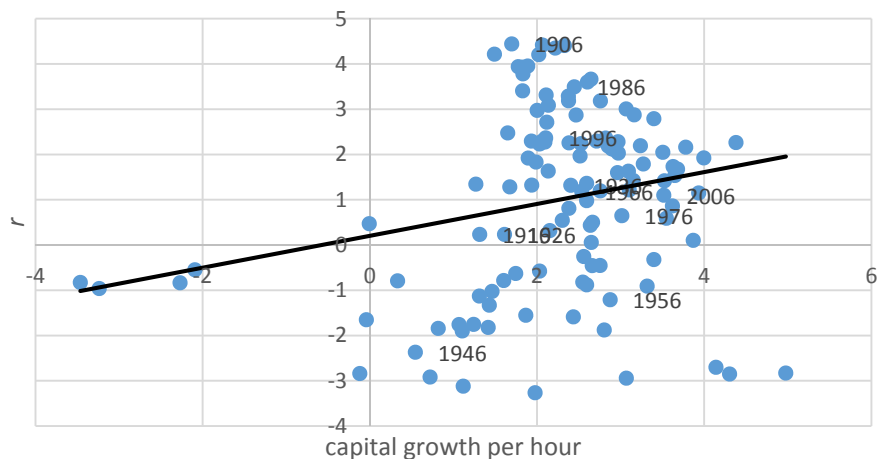
***r* and labor hours growth (lowpass filter)**



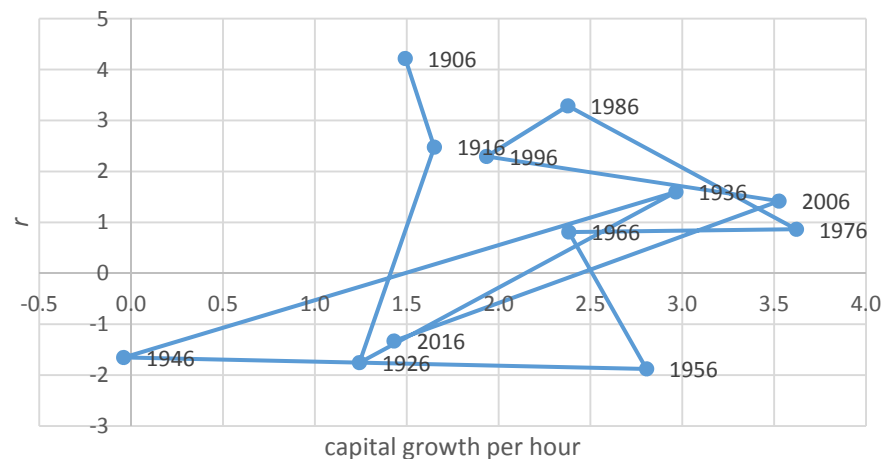
***r* and labor hours growth (lowpass filter)**



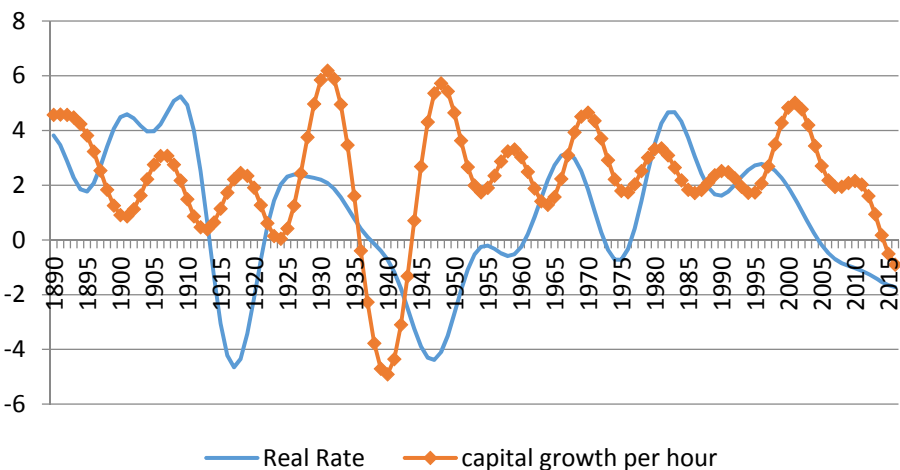
***r* vs. capital growth per hour (10 yr averages)**



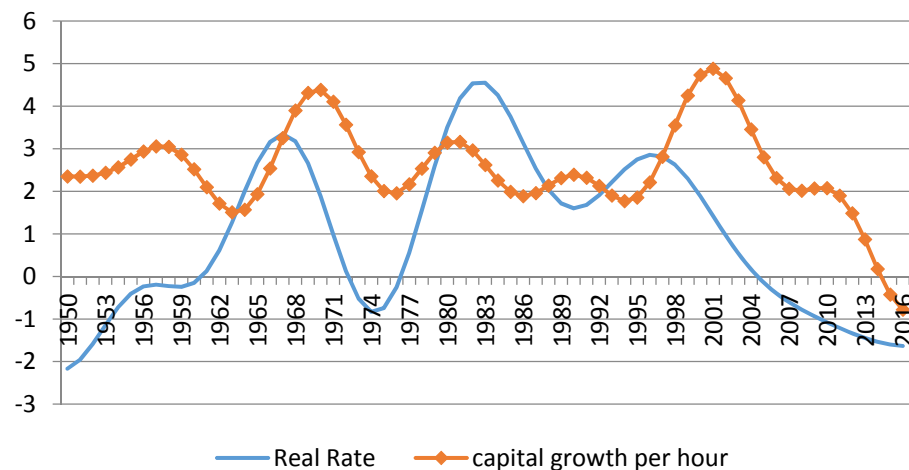
***r* vs. capital growth per hour (10 yr averages)**



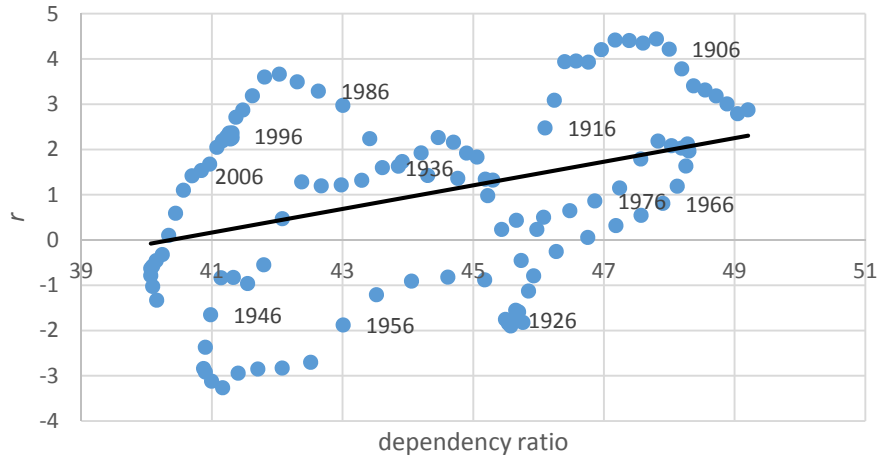
***r* and capital growth per hour (lowpass filter)**



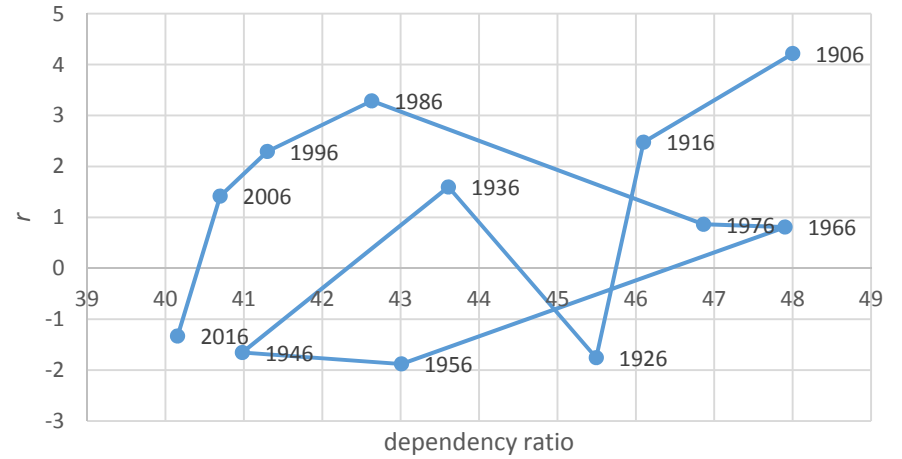
***r* and capital growth per hour (lowpass filter)**



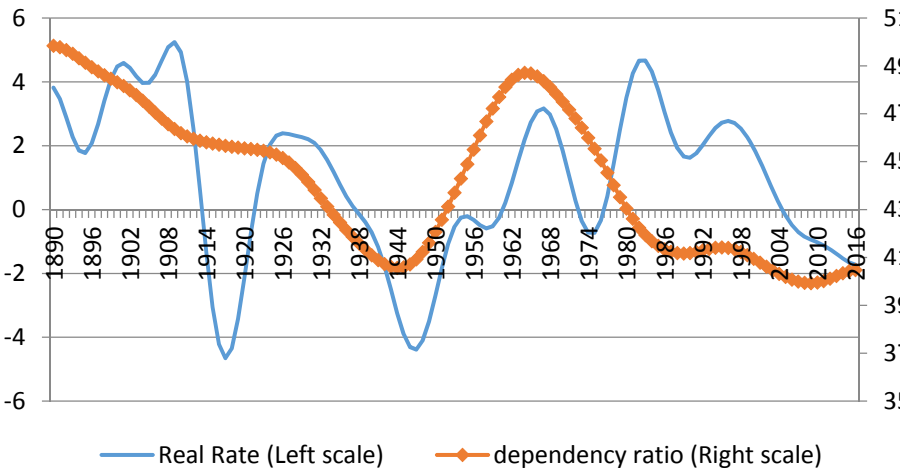
***r* vs. dependency ratio (10 yr averages)**



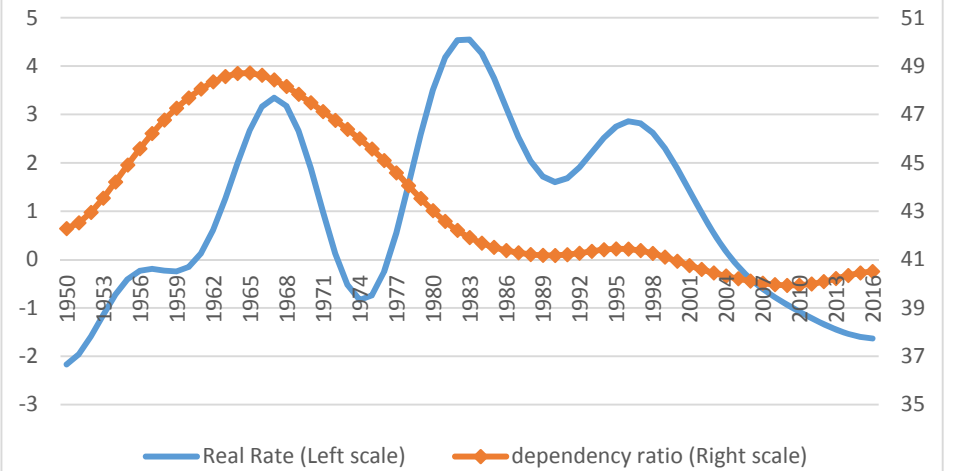
***r* vs. dependency ratio (10 yr averages)**



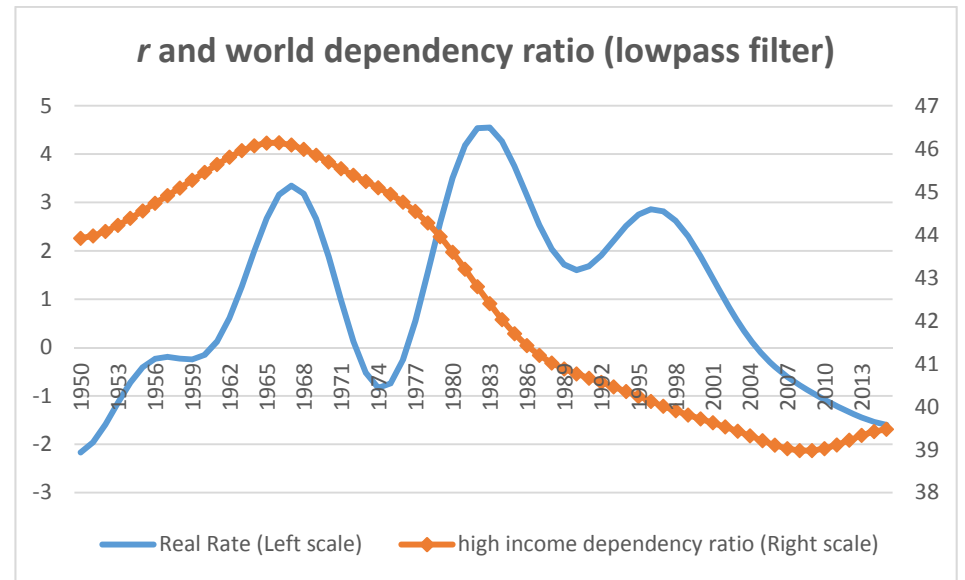
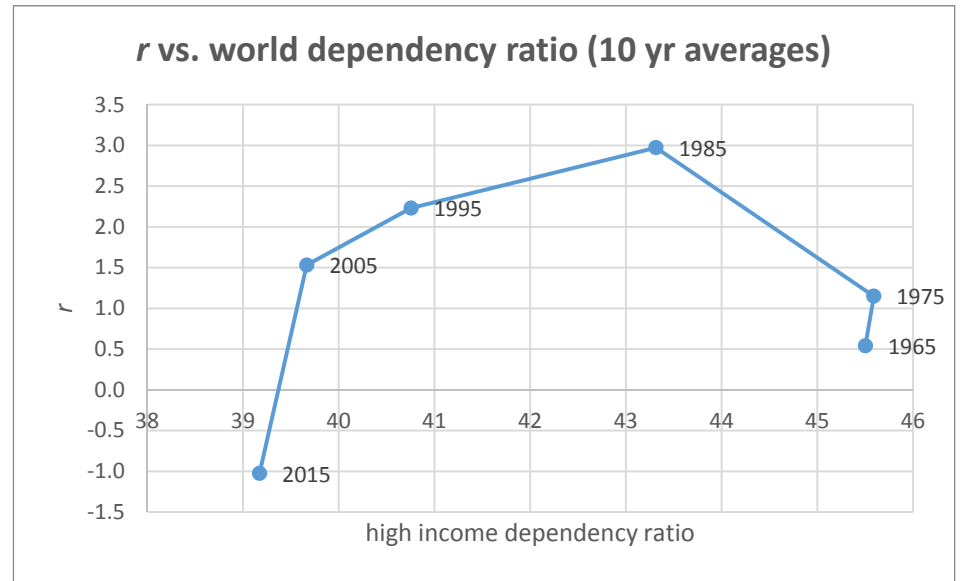
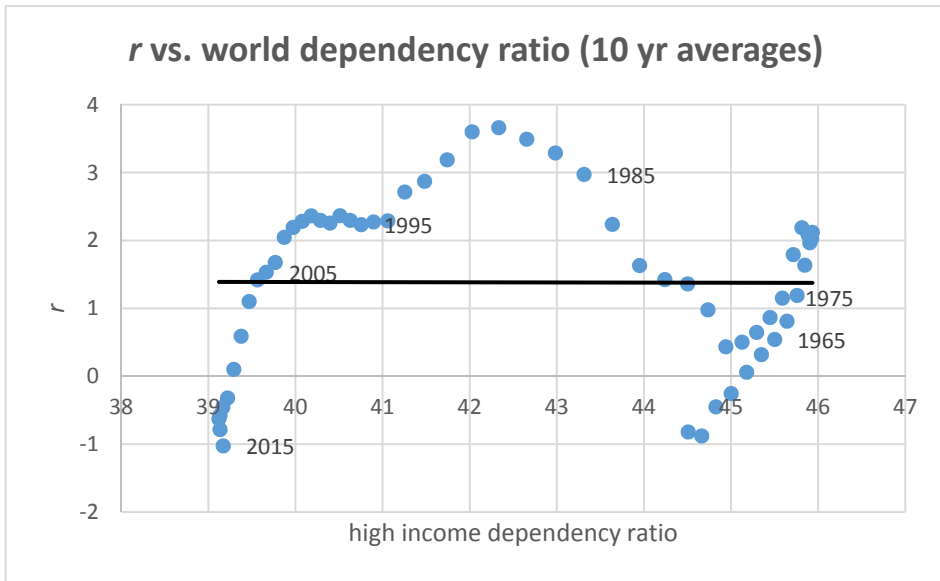
***r* and dependency ratio (lowpass filter)**



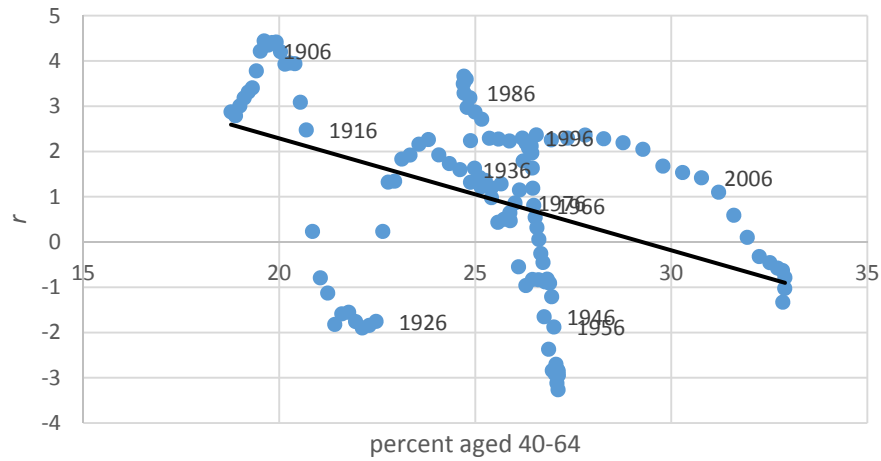
***r* and dependency ratio (lowpass filter)**



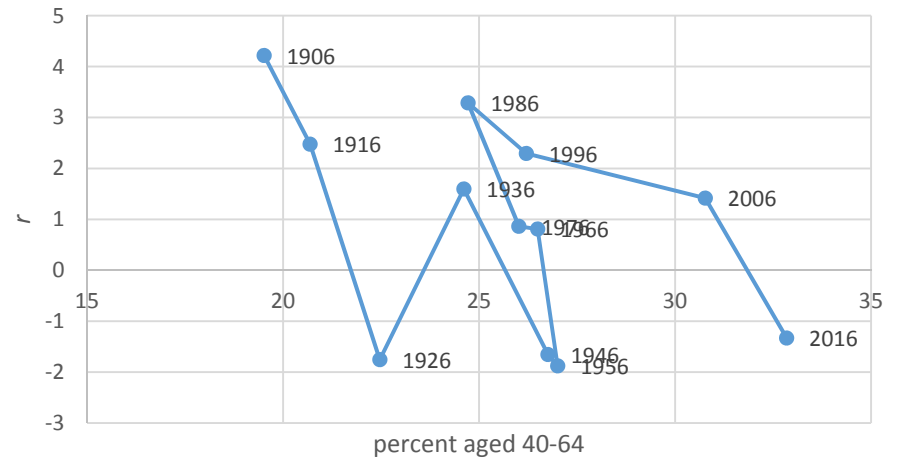




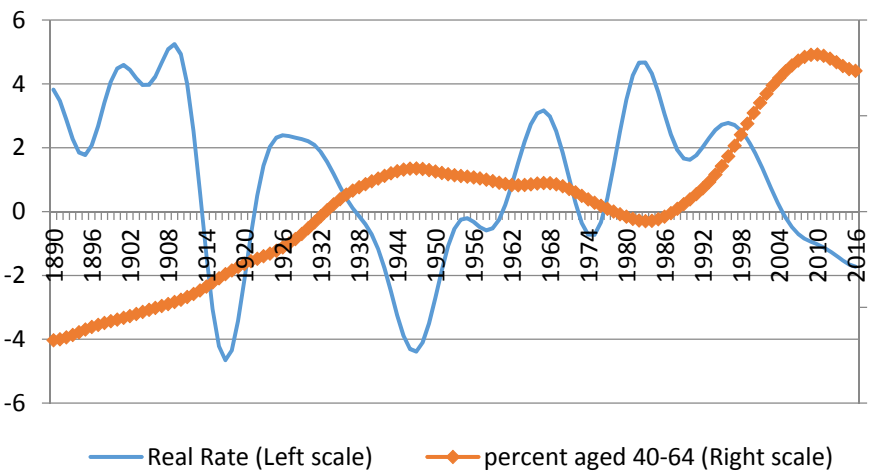
***r* vs. percent aged 40-64 (10 yr averages)**



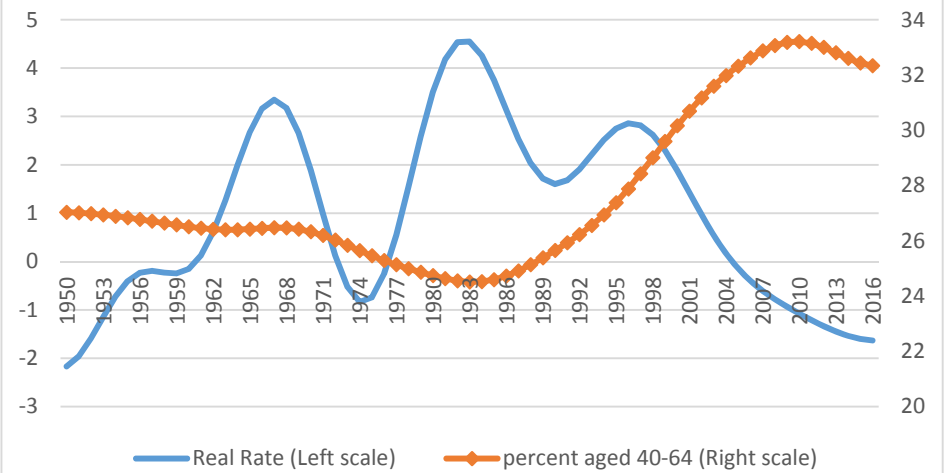
***r* vs. percent aged 40-64 (10 yr averages)**



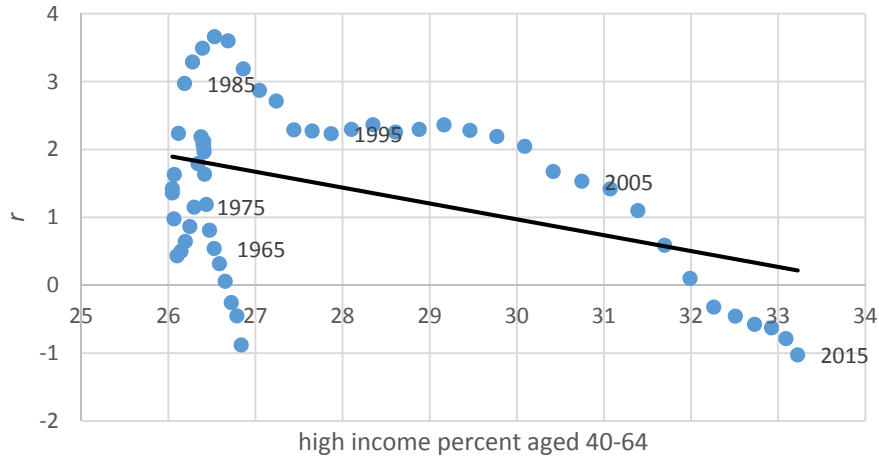
***r* and percent aged 40-64 (lowpass filter)**



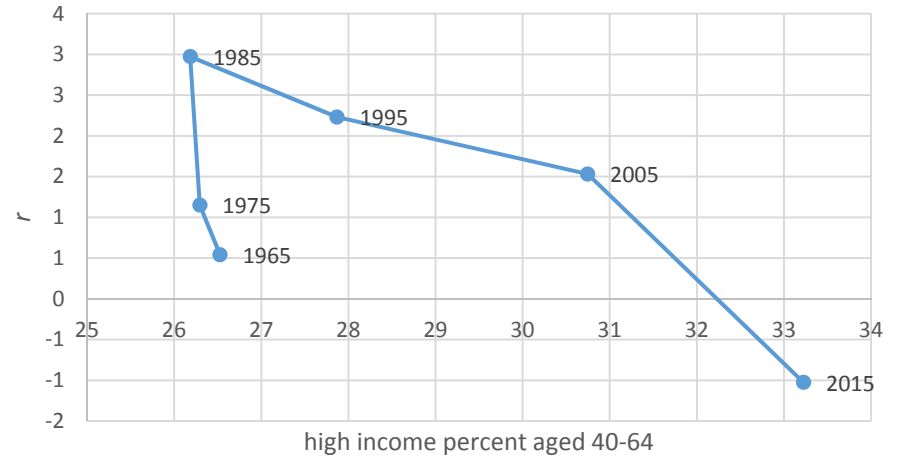
***r* and percent aged 40-64 (lowpass filter)**



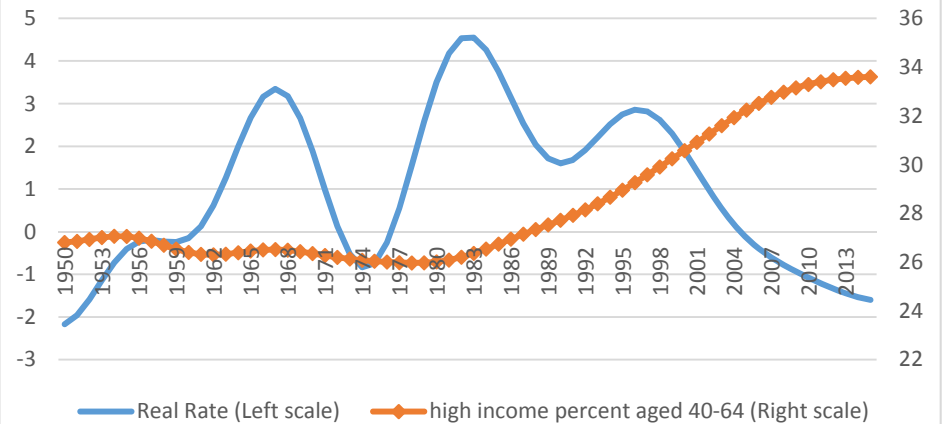
***r* vs. world percent aged 40-64 (10 yr averages)**



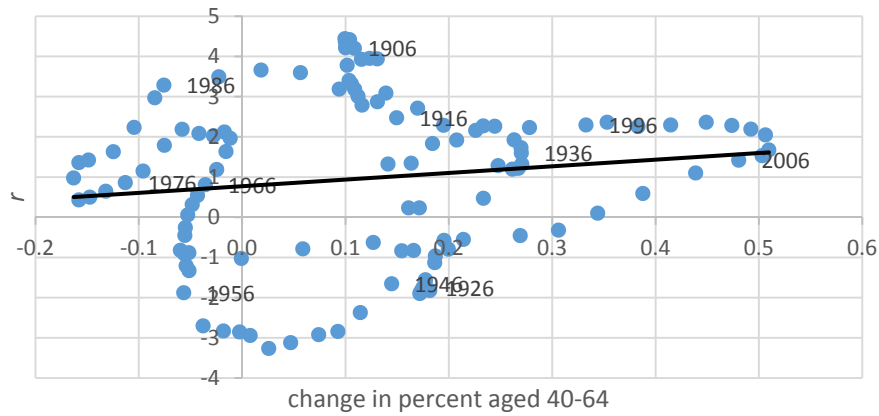
***r* vs. world percent aged 40-64 (10 yr averages)**



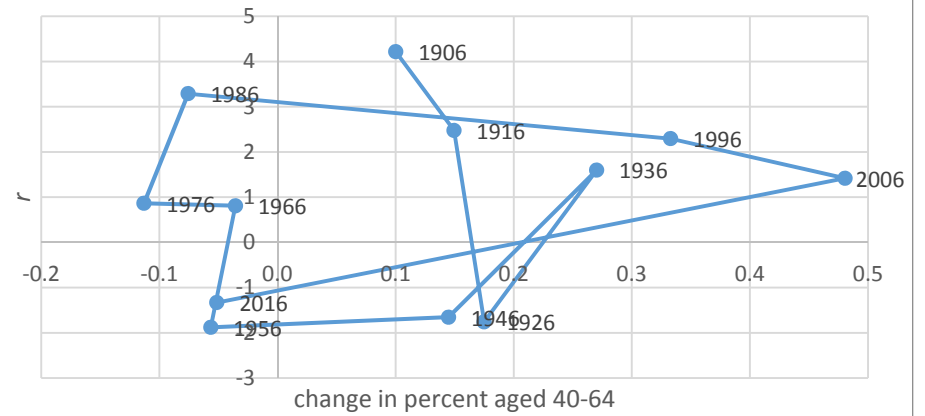
***r* and world percent aged 40-64 (high income) (lowpass filter)**



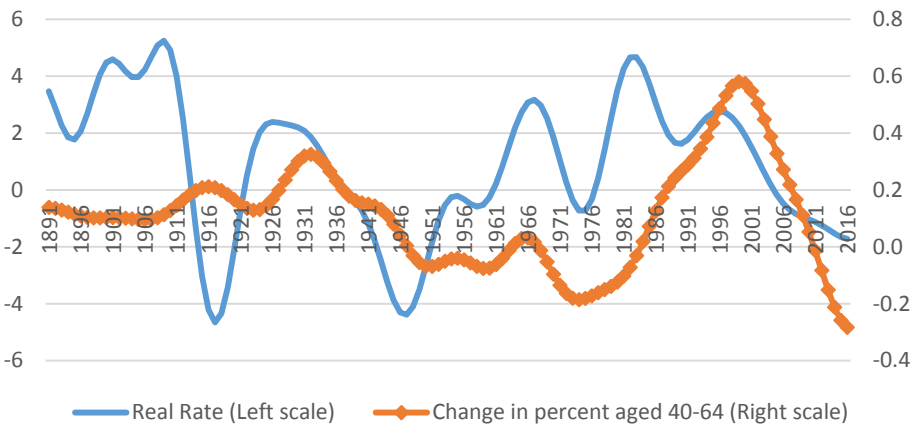
***r* vs. change in percent aged 40-64 (10 yr averages)**



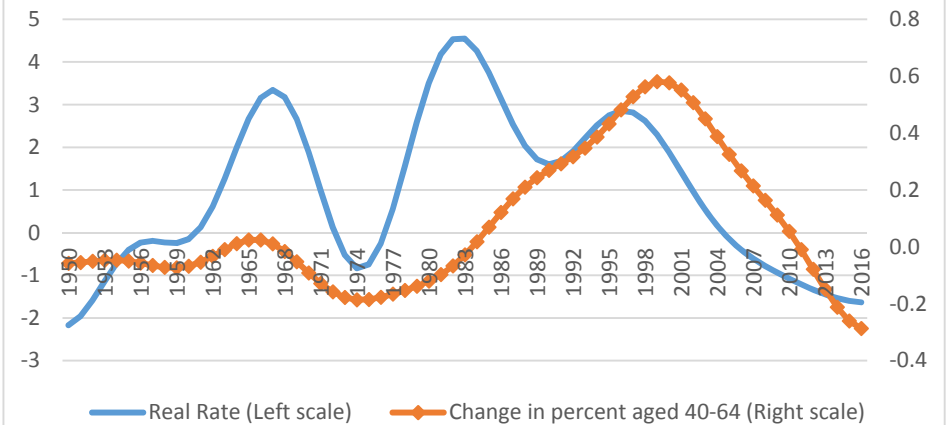
***r* vs. change in percent aged 40-64 (10 yr averages)**

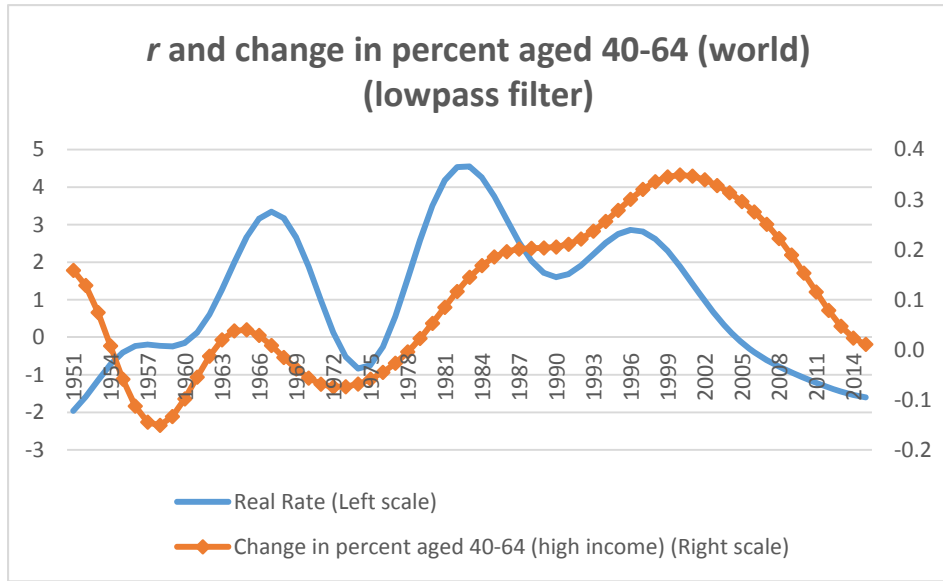
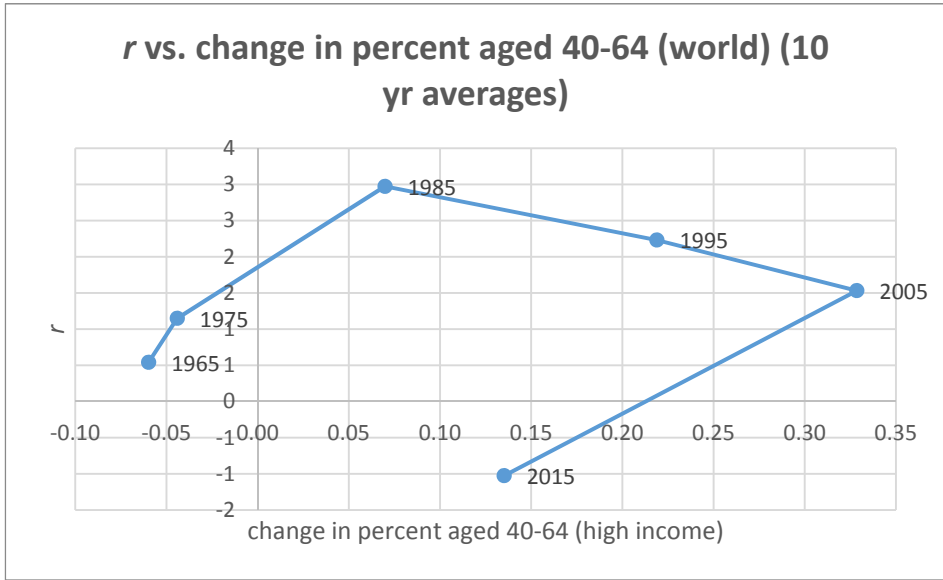
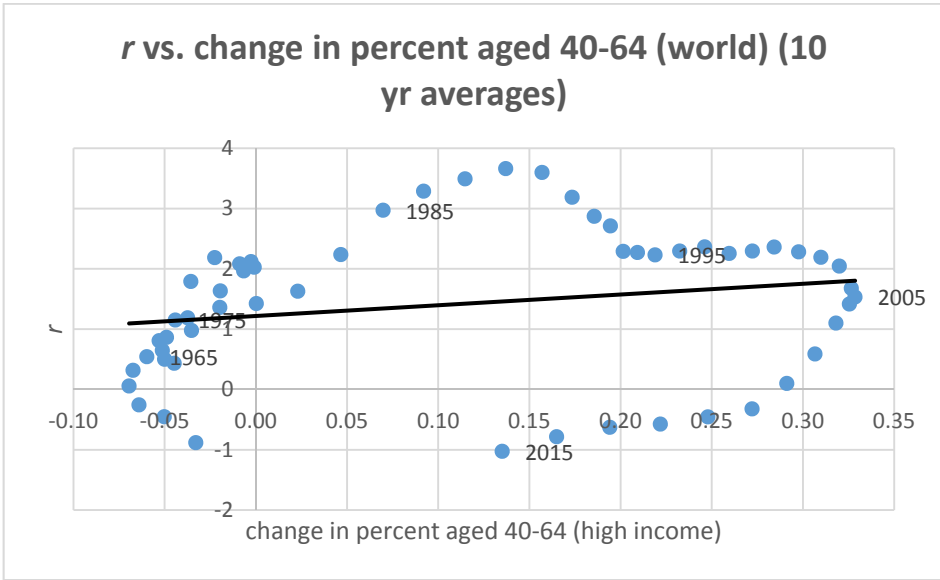


***r* and change in percent aged 40-64 (lowpass filter)**

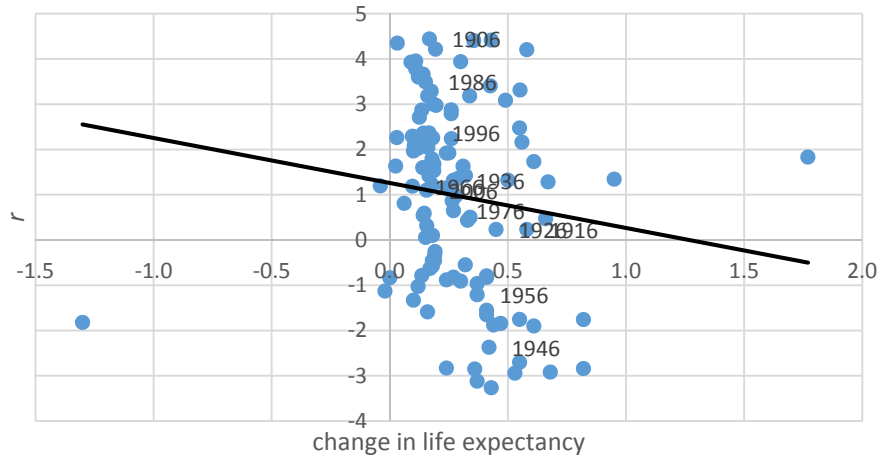


***r* and change in percent aged 40-64 (lowpass filter)**

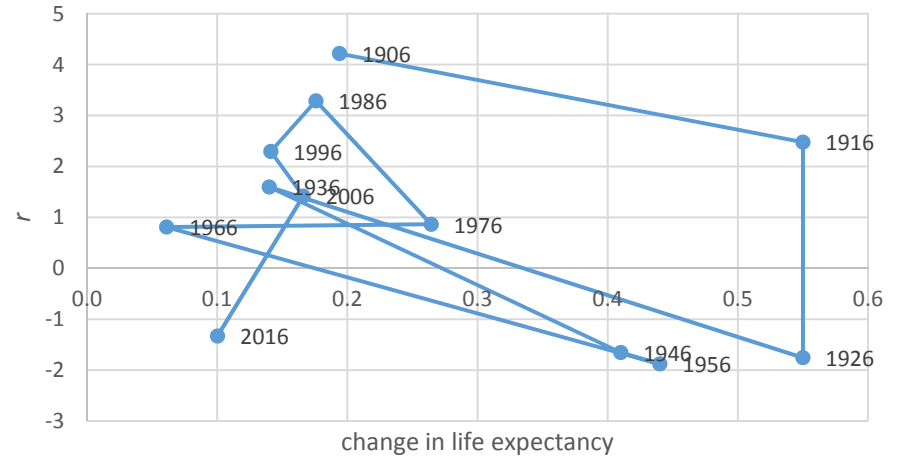




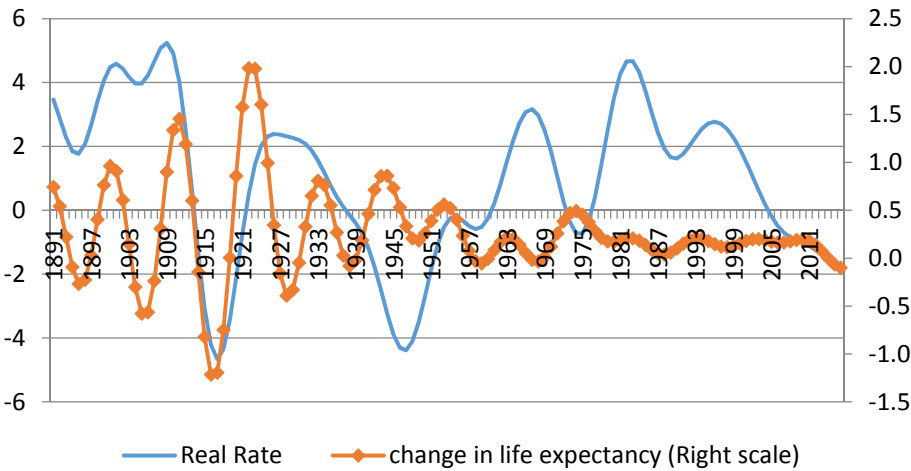
***r* vs. change in life expectancy (10 yr averages)**



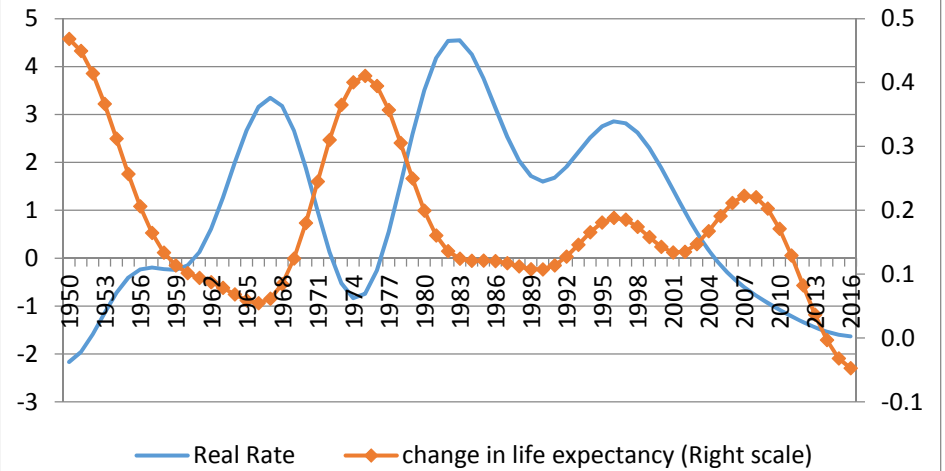
***r* vs. change in life expectancy (10 yr averages)**



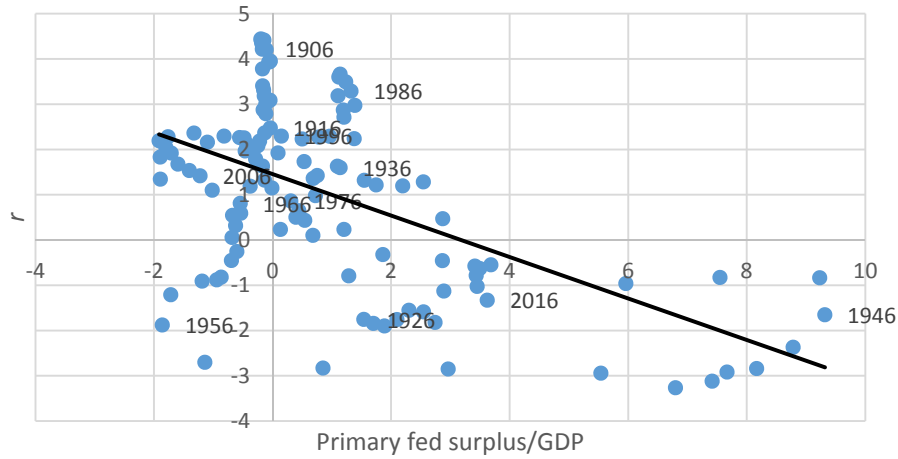
***r* and change in life expectancy (lowpass filter)**



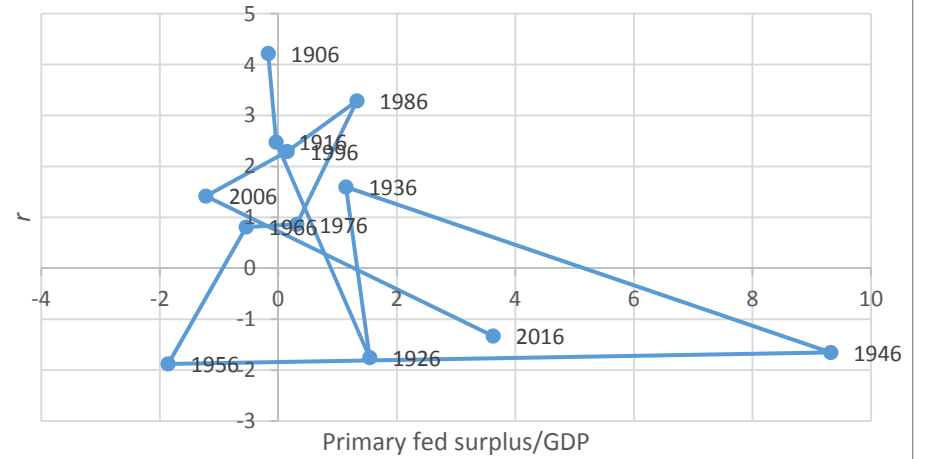
***r* and change in life expectancy (lowpass filter)**



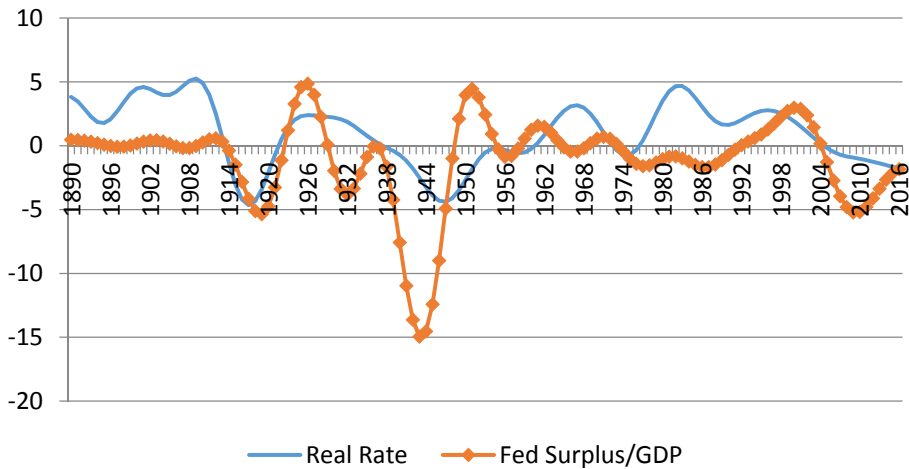
***r* vs. Primary fed surplus/GDP (10 yr averages)**



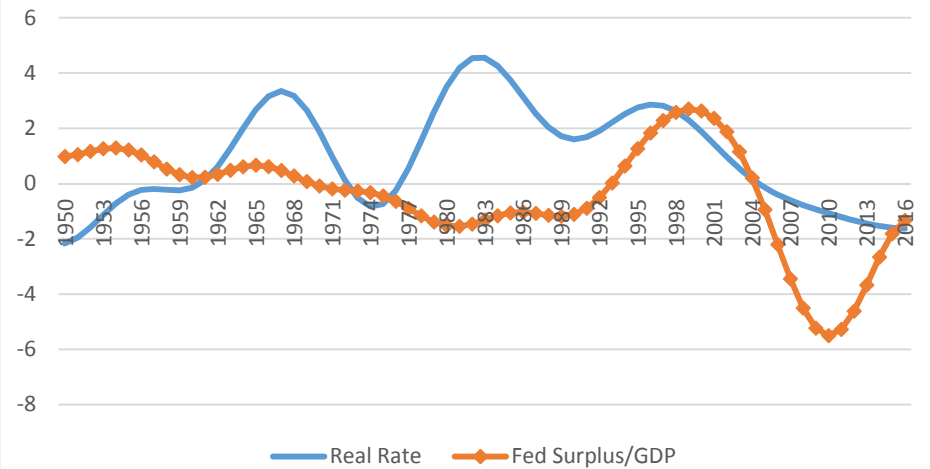
***r* vs. Primary fed surplus/GDP (10 yr averages)**



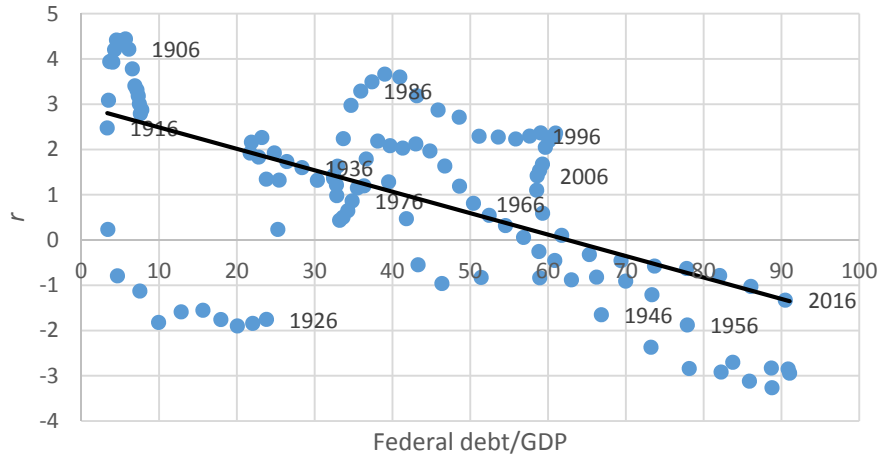
***r* and Fed surplus/GDP (lowpass filter)**



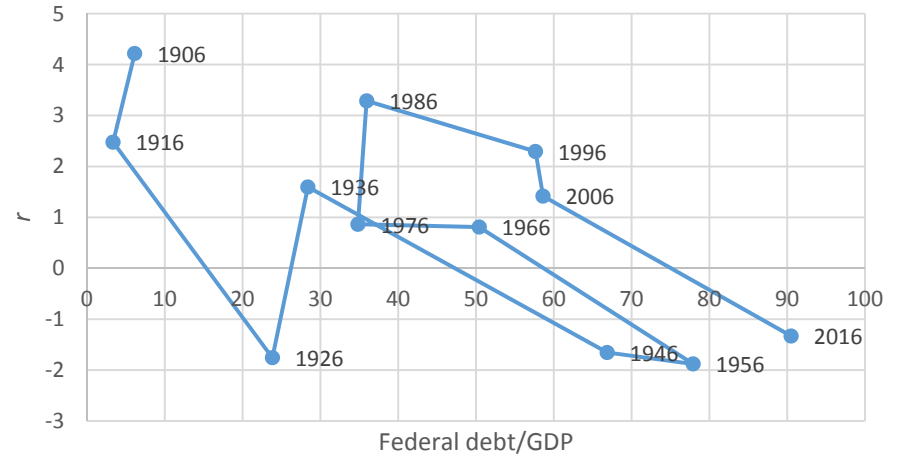
***r* and Fed surplus/GDP (lowpass filter)**



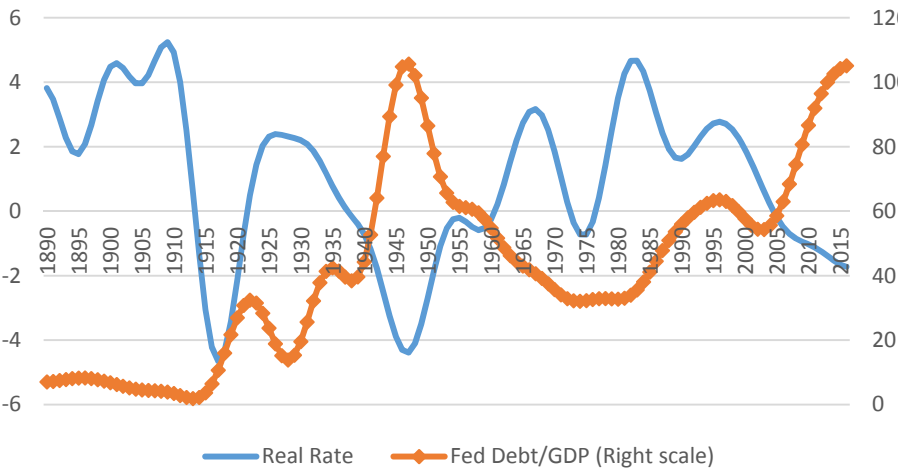
***r* vs. Federal debt/GDP (10 yr averages)**



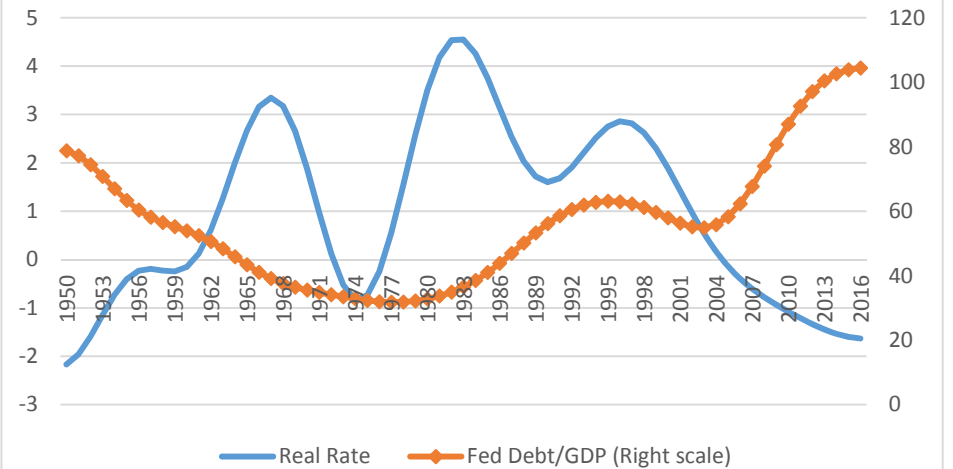
***r* vs. Federal debt/GDP (10 yr averages)**



***r* and Fed debt/GDP (lowpass filter)**

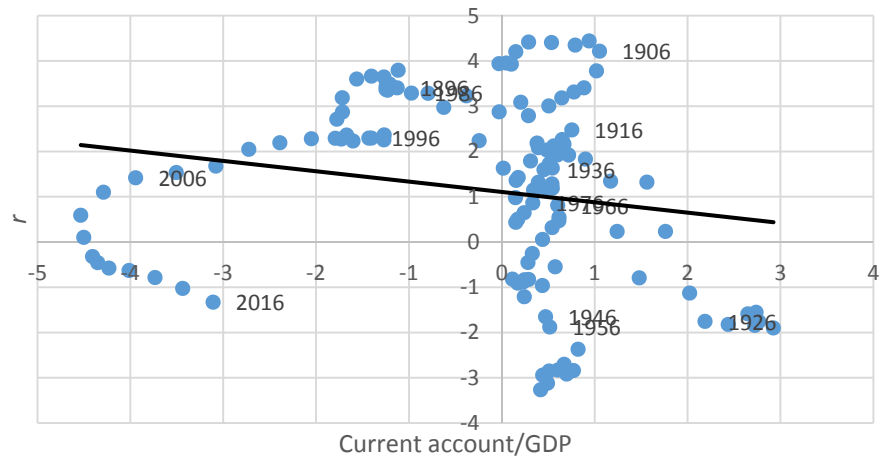


***r* and Fed debt/GDP (lowpass filter)**

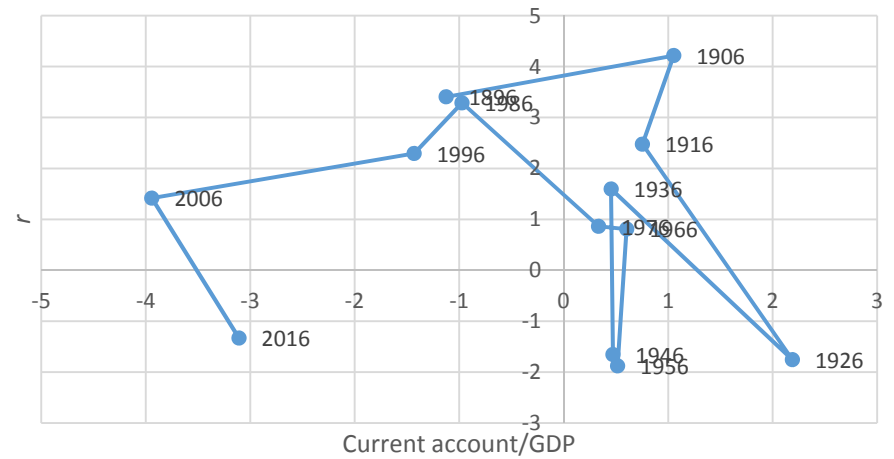




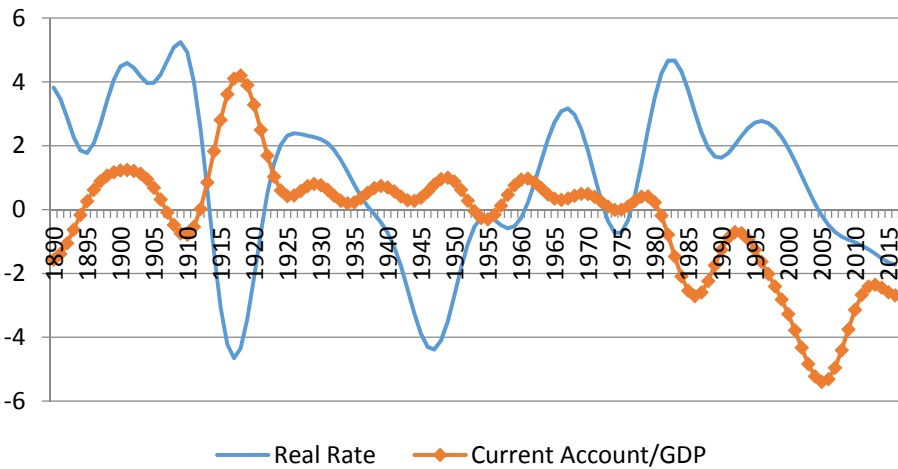
***r* vs. Current account/GDP (10 yr averages)**



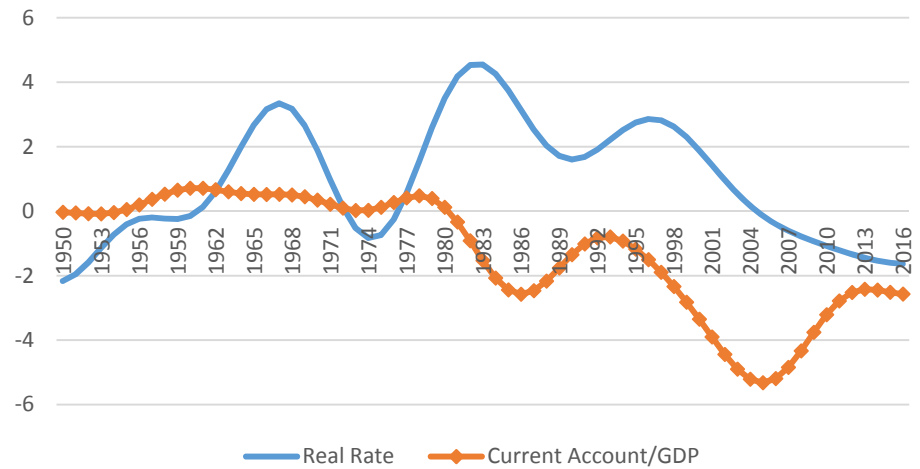
***r* vs. Current account/GDP (10 yr averages)**



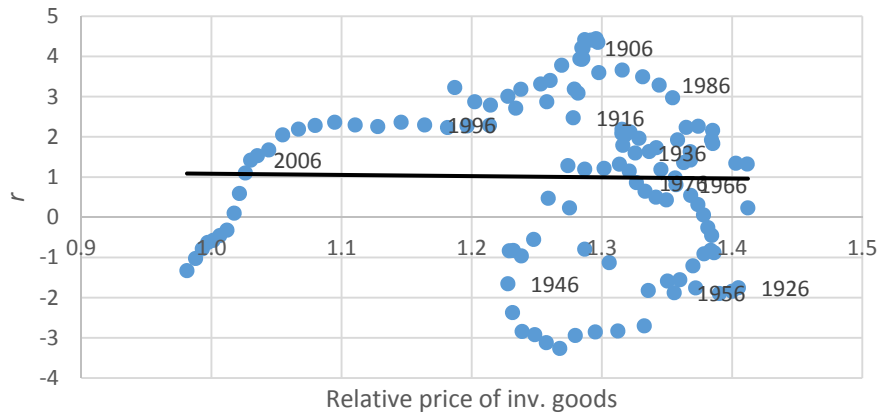
***r* and Current account/GDP (lowpass filter)**



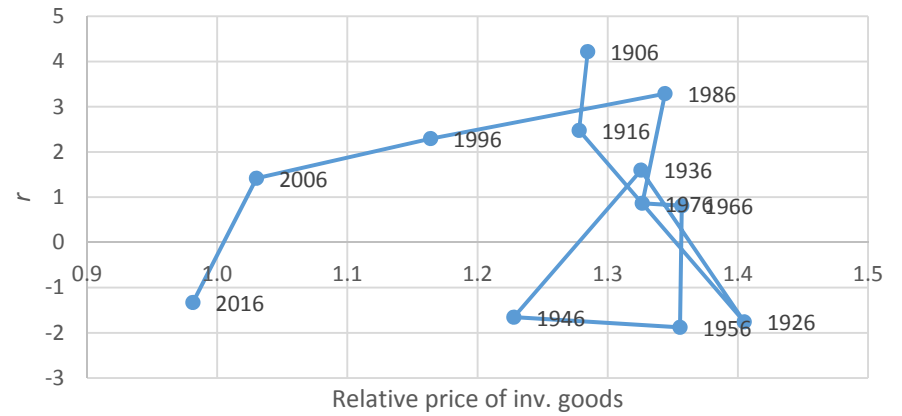
***r* and Current account/GDP (lowpass filter)**



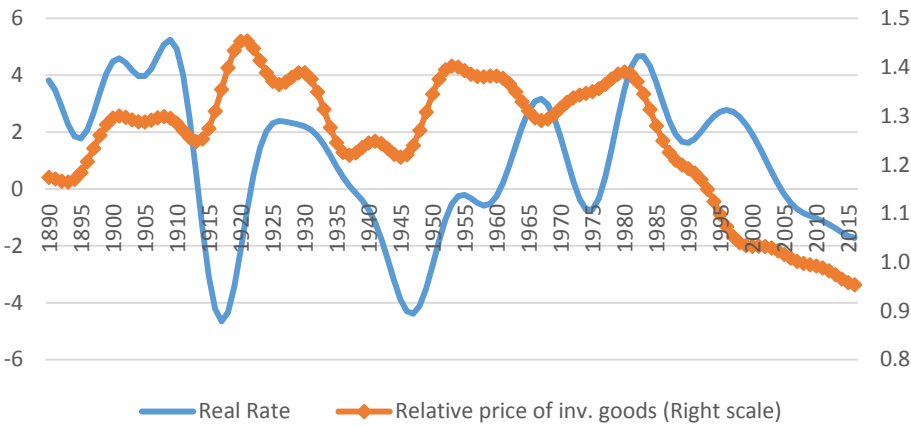
***r* vs. Relative price of inv. goods (10 yr averages)**



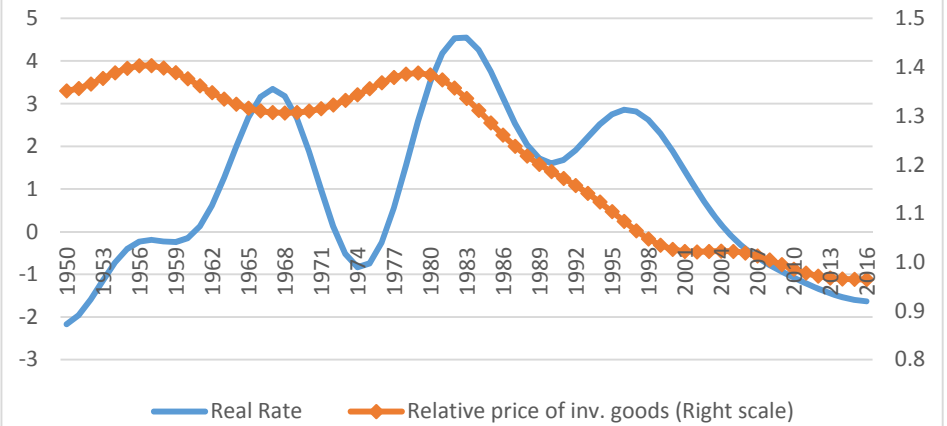
***r* vs. Relative price of inv. goods (10 yr averages)**



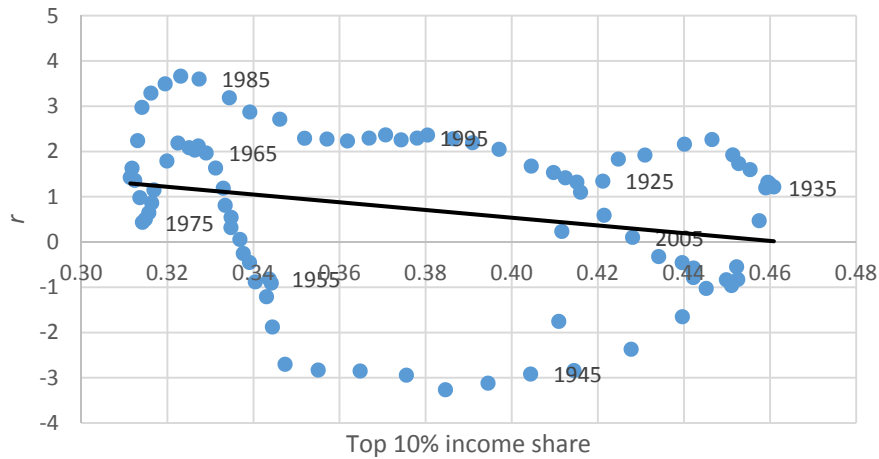
***r* and Relative price of inv. goods (lowpass filter)**



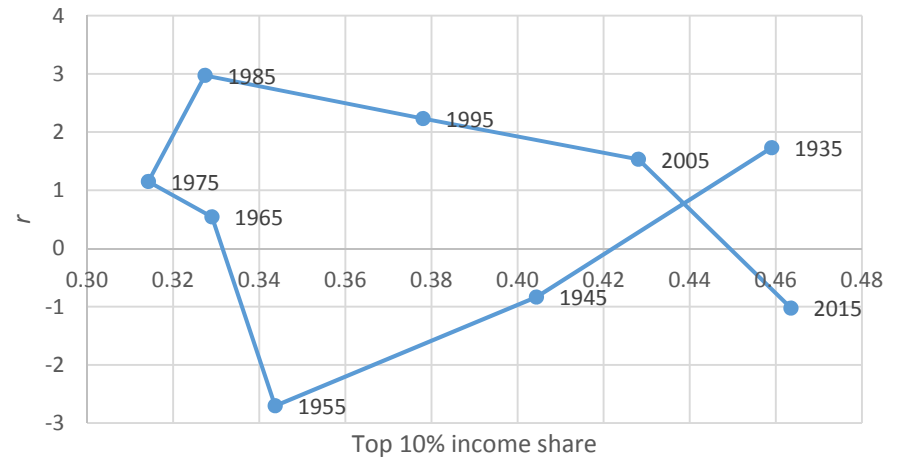
***r* and Relative price of inv. goods (lowpass filter)**



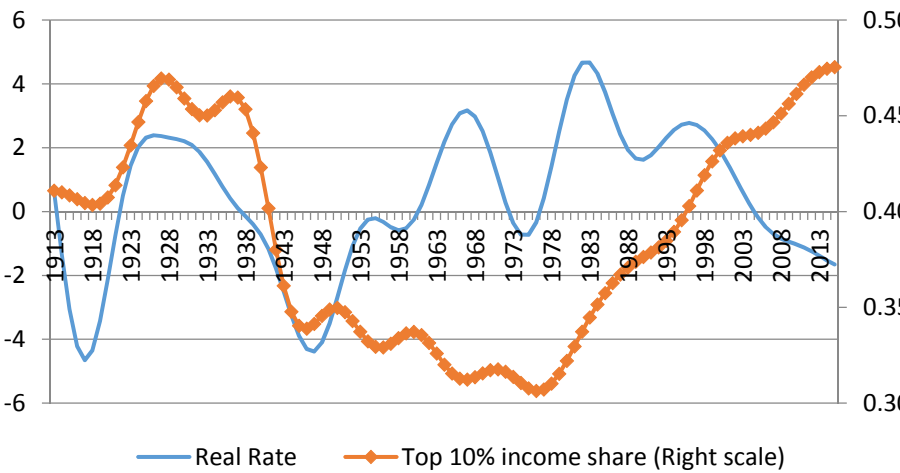
***r* vs. Top 10% income share (10 yr averages)**



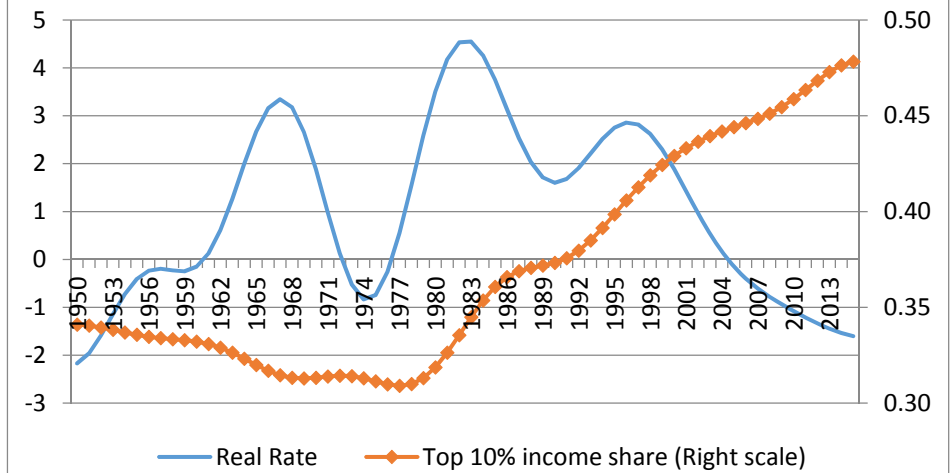
***r* vs. Top 10% income share (10 yr averages)**

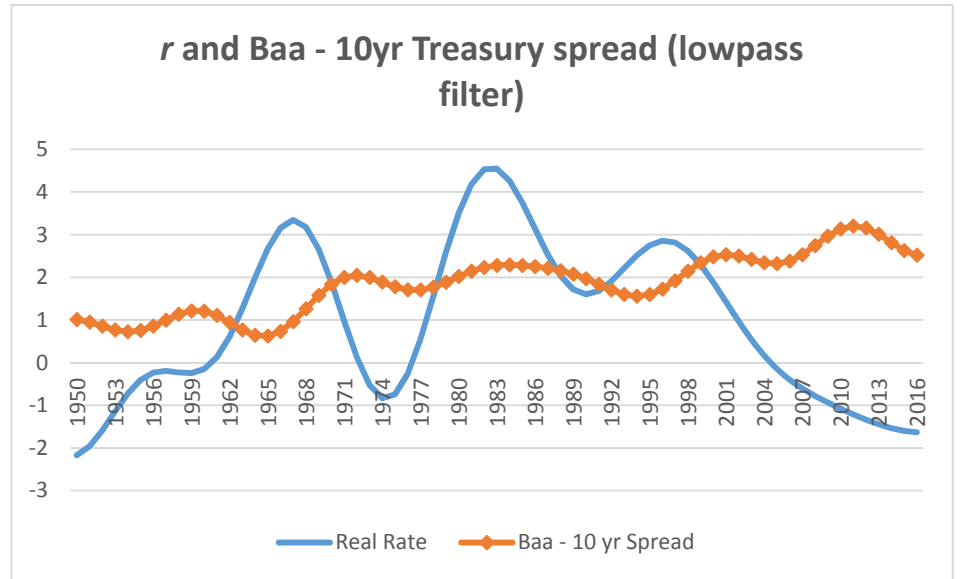
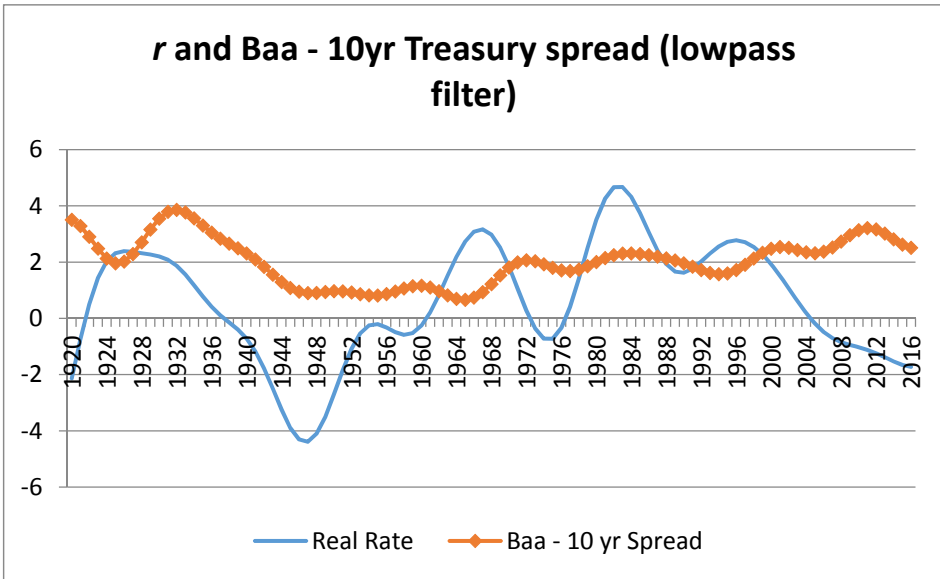
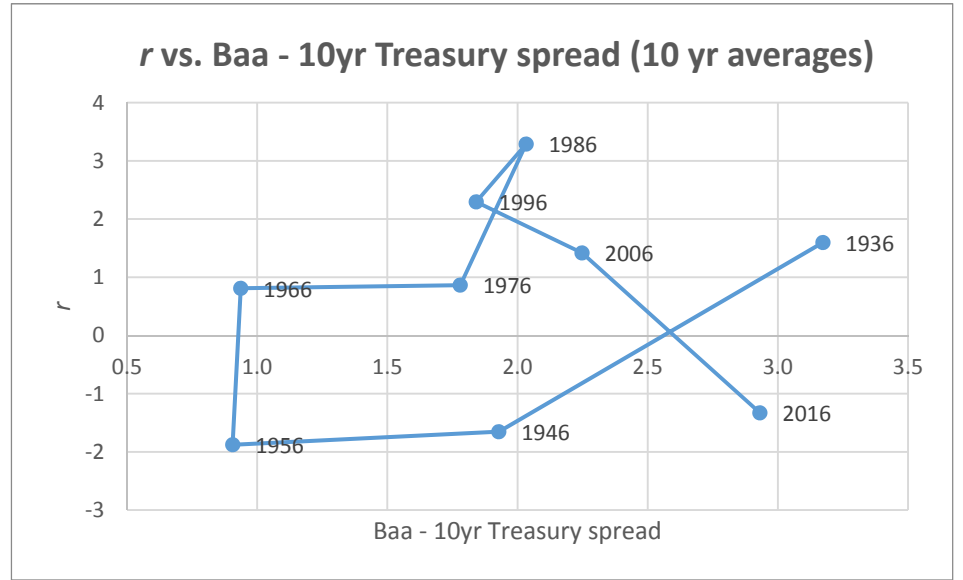
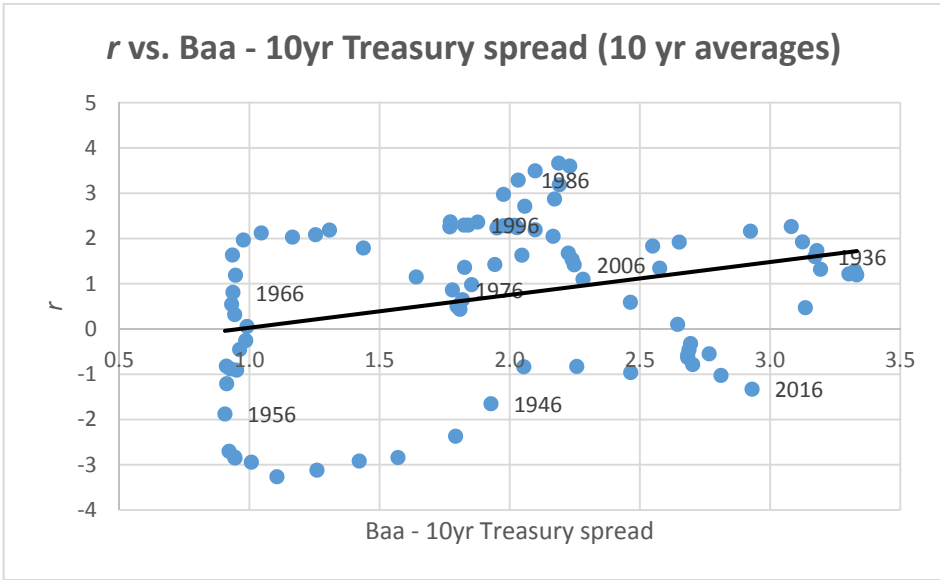


***r* and Top 10% income share (lowpass filter)**

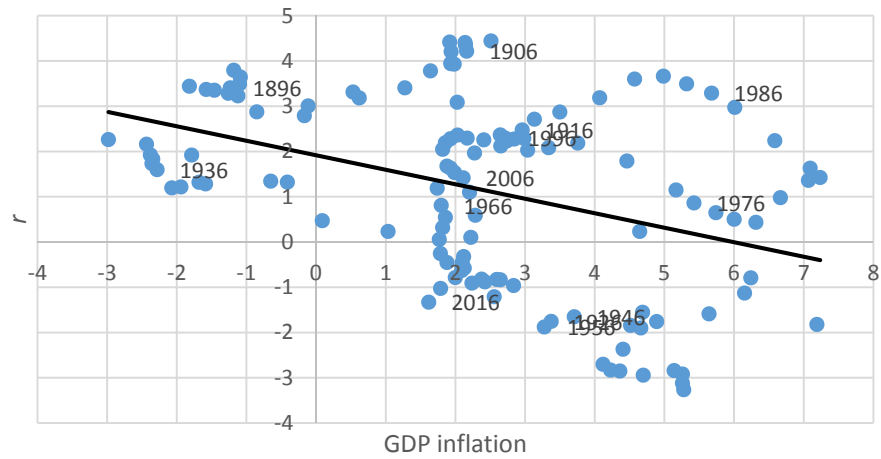


***r* and Top 10% income share (lowpass filter)**

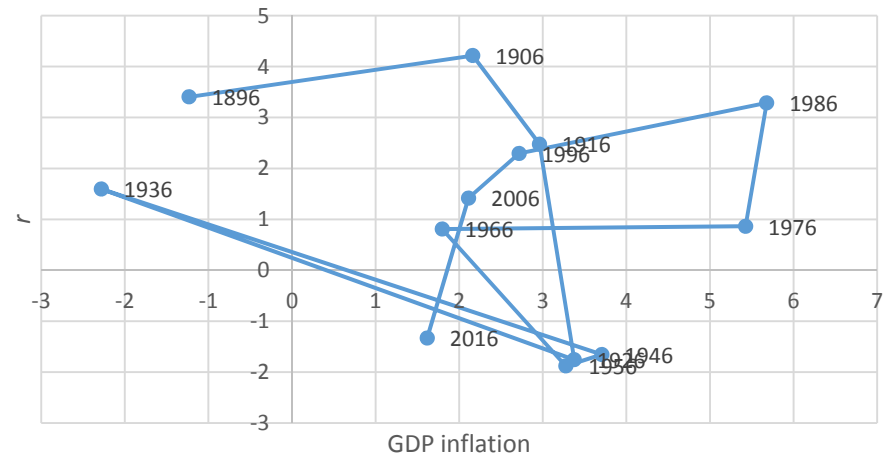




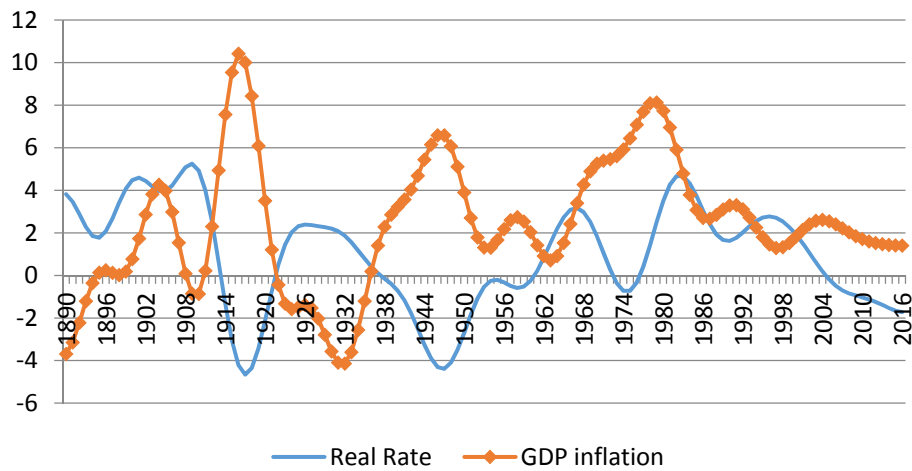
***r* vs. GDP inflation (10 yr averages)**



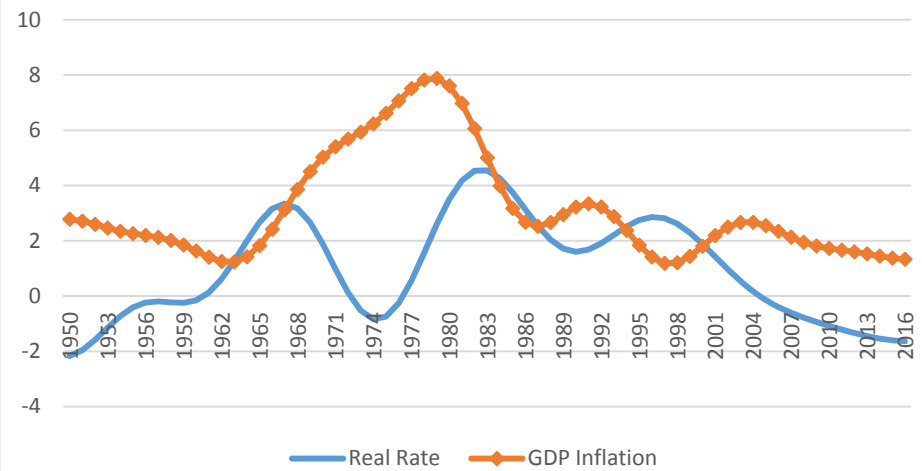
***r* vs. GDP inflation (10 yr averages)**



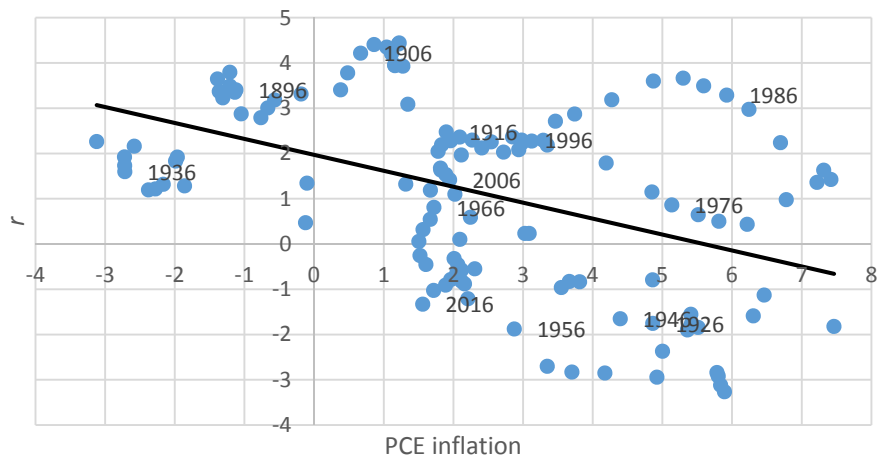
***r* and GDP inflation (lowpass filter)**



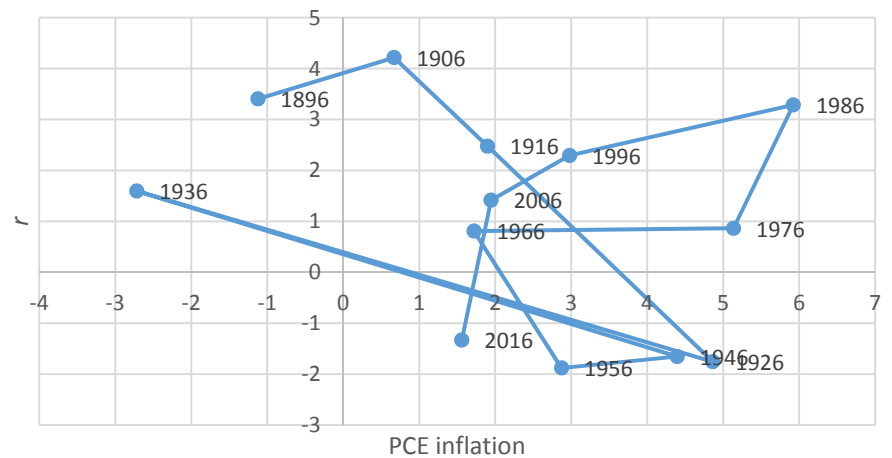
***r* and GDP inflation (lowpass filter)**



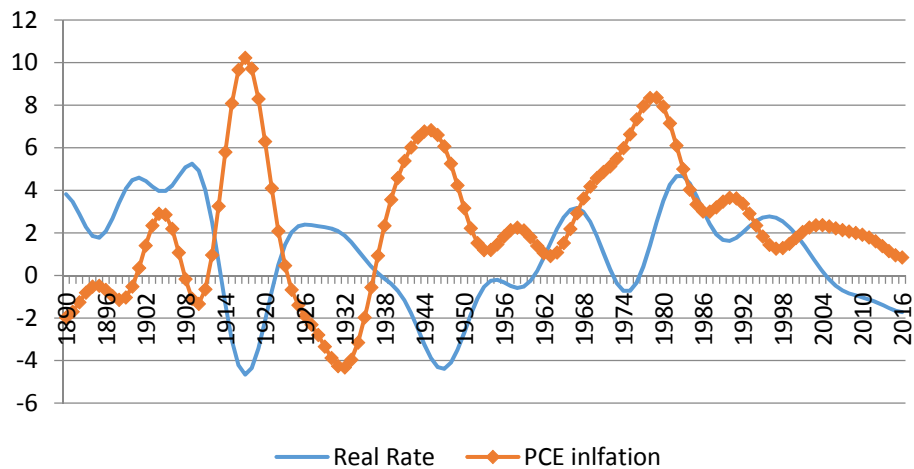
***r* vs. PCE inflation (10 yr averages)**



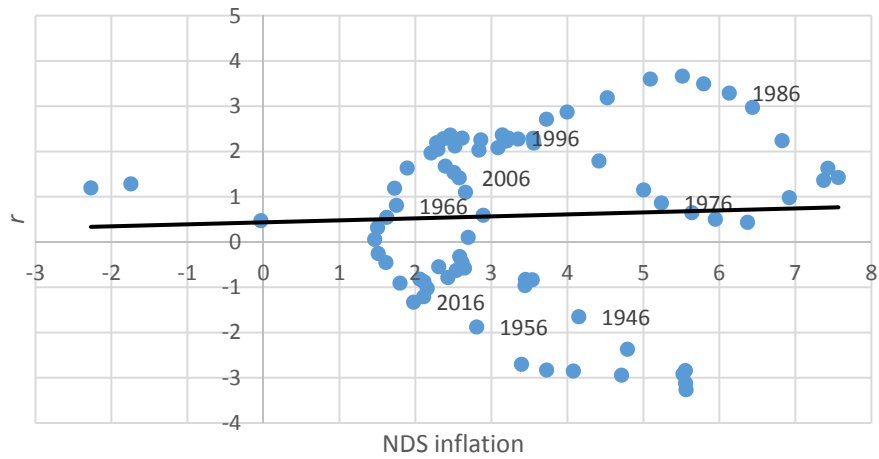
***r* vs. PCE inflation (10 yr averages)**



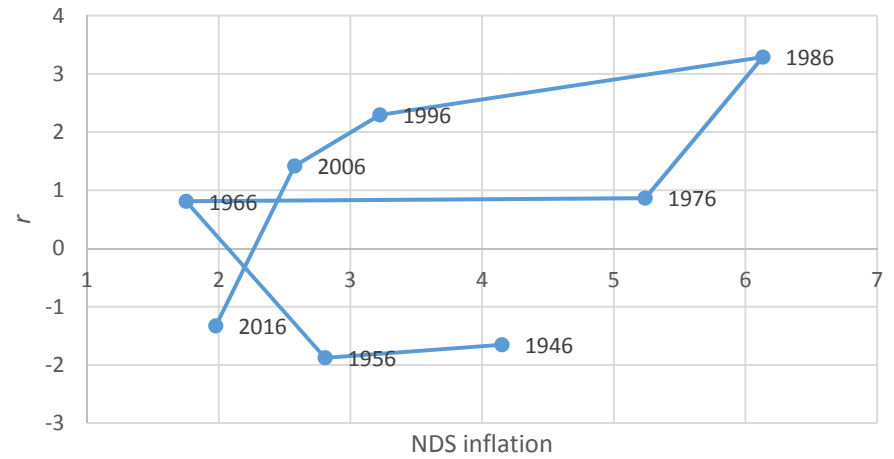
***r* and PCE inflation (lowpass filter)**



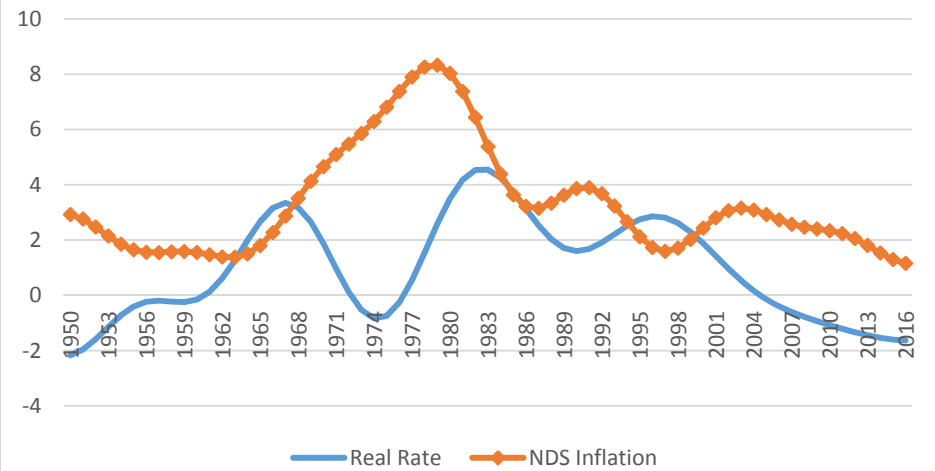
**$r$  vs. NDS inflation (10 yr averages)**



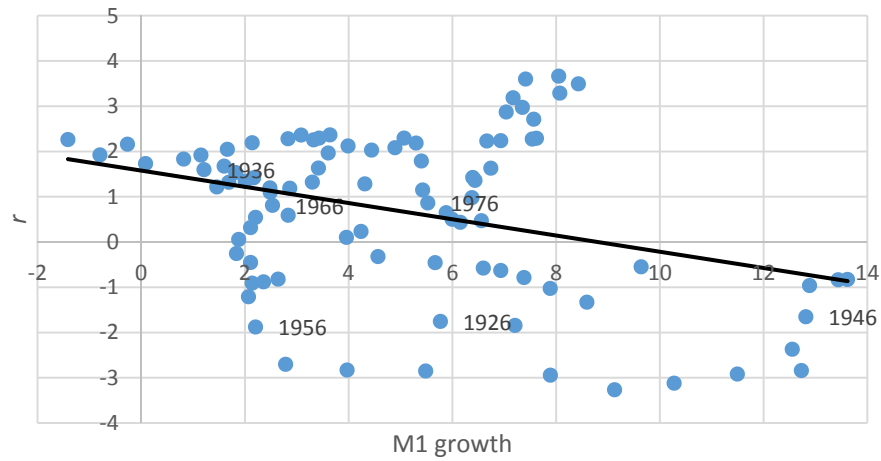
**$r$  vs. NDS inflation (10 yr averages)**



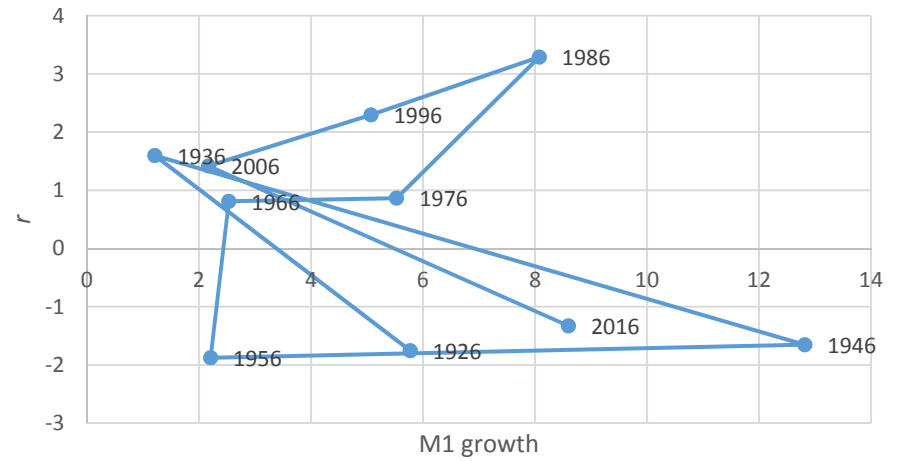
**$r$  and NDS inflation (lowpass filter)**



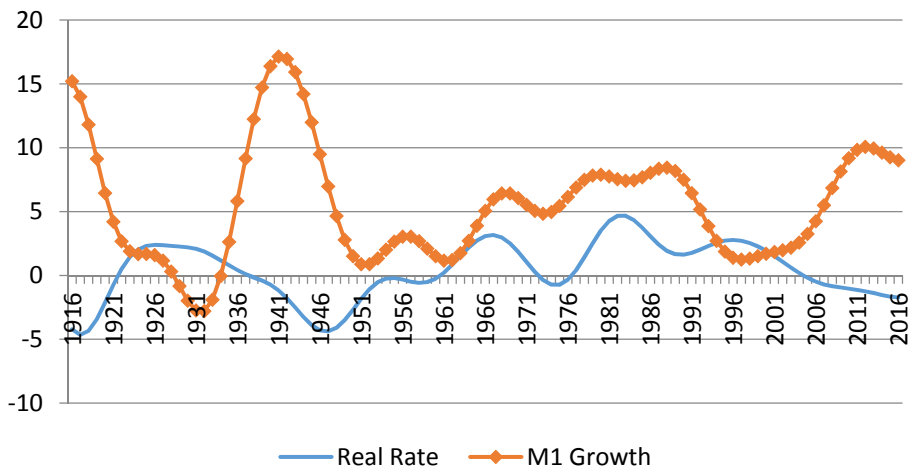
***r* vs. M1 growth (10 yr averages)**



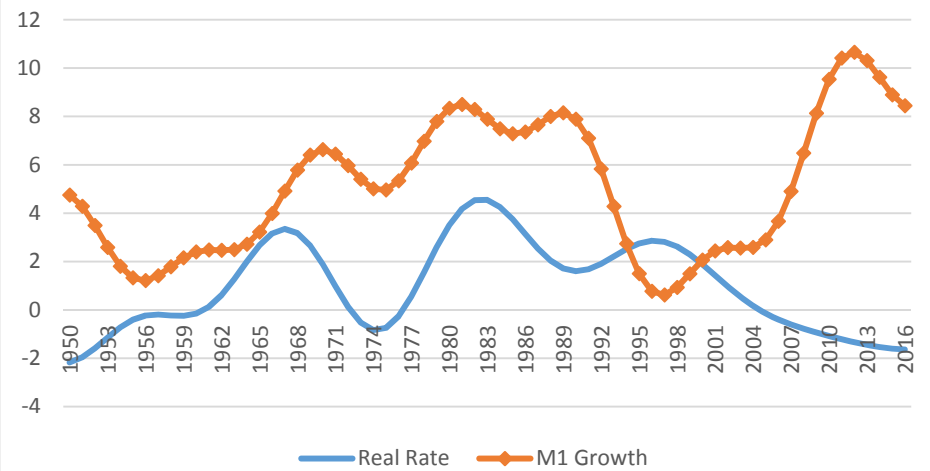
***r* vs. M1 growth (10 yr averages)**



***r* and M1 Growth (lowpass filter)**

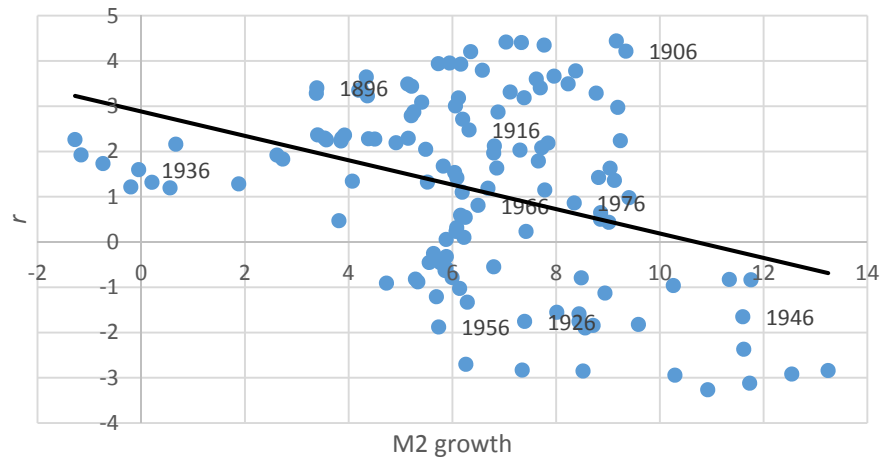


***r* and M1 Growth (lowpass filter)**

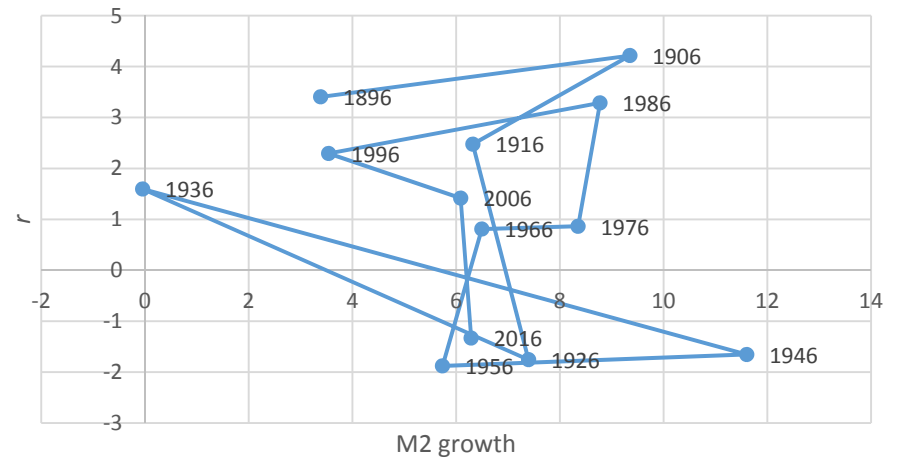




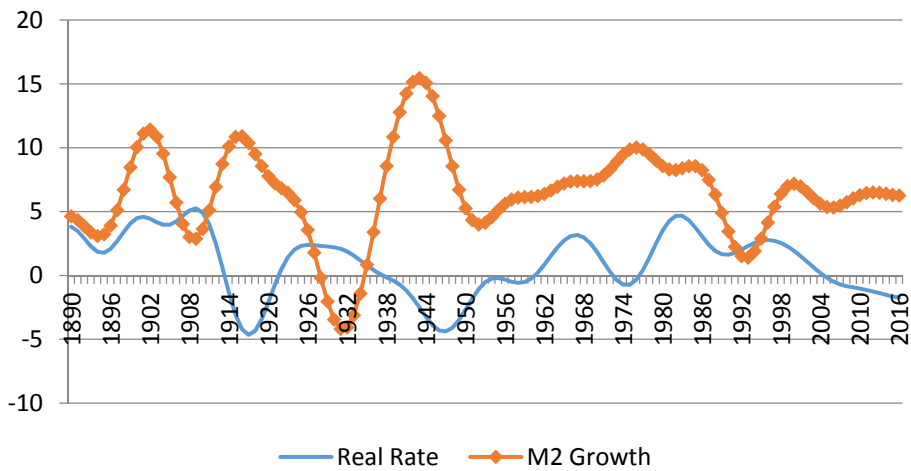
***r* vs. M2 growth (10 yr averages)**



***r* vs. M2 growth (10 yr averages)**



***r* and M2 Growth (lowpass filter)**



***r* and M2 Growth (lowpass filter)**

