

Online Appendix for
Missing Growth from Creative Destruction

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A Imputation in the CPI

For compiling a price index, accurately adjusting for quality changes poses a challenge. Let v denote an item produced at date t and which is replaced by a new item $v + 1$ at date $t + 1$. To integrate the corresponding item change in the overall price index, the statistical office needs to infer a value for either price $P(v + 1, t)$ or price $P(v, t + 1)$ when it has information only about $P(v, t)$ and $P(v + 1, t + 1)$. According to the U.S. General Accounting Office (1999) and to the Handbook of Methods from U.S. Bureau of Labor Statistics (2015), the BLS largely chooses among four possible courses of action to handle these *item substitutions*.¹

The first course of action simply involves setting

$$P(v + 1, t) = P(v, t).$$

This no-adjustment strategy is pursued by the BLS when it deems the new and old item as *comparable*, by which the BLS means that the old and new items are essentially the same, so that no quality difference exists between the two items.

The interesting case is when the BLS judges the new and old items to be *noncomparable*. Then, the BLS typically chooses between three remaining strategies. First is *direct quality adjustment*. This is when the BLS can perform hedonic regressions or has information on manufacturers' production costs. Direct quality adjustment involves the BLS setting

$$P(v + 1, t) = P(v, t) \cdot QA(t).$$

Viewed through the lens of our model, BLS quality adjustments are an estimate of the step size of innovations.

For those noncomparable substitutions where the BLS lacks the

¹We use italics to highlight terminology used by the BLS.

information to make direct quality adjustments, it resorts to *class-mean imputation* or *linking*. Class-mean imputation is based on the rate of price changes experienced by other item substitutions — those which the BLS considers comparable or can directly adjust. Linking, meanwhile, uses the average rate of price change among items without substitution, items with comparable substitutions, and items with noncomparable substitutions subject to direct quality adjustments. Both imputations are usually carried out within the item's category or category-region.

Based on Klenow and Kryvtsov (2008), the BLS judged 52% of item substitutions to be comparable from 1988–2004; the prices for these items entered the CPI without adjustment. The remaining 48% (the noncomparable substitutions) broke down as follows:²

- 31.4% direct quality adjustments
- 32.4% class-mean imputations
- 36.2% linking.

To estimate the fraction of creative destruction innovations that were effectively subject to imputation based on all surviving items (those not creatively destroyed), we make the following three assumptions:

1. Comparable item substitutions do not involve any innovation.
2. Direct adjustments are implemented when incumbents improve their own products (OI).
3. Creative destruction (CD) results in imputation by class-mean or linking in the proportions stated above.

²These figures are quite close to those in the publicly available statistics for 1997 in U.S. General Accounting Office (1999).

Under these assumptions, we estimate that creative destruction (CD) innovations were treated with the equivalent of all-surviving-items imputation 90% of the time from 1988–2004. To see why, let D , C , and L denote the numbers of item substitutions subject to direct adjustment, class-mean imputation, and linking, respectively. Let N denote the number of comparable item substitutions.

The number of item substitutions for which some form of imputation is done is $L + C$. The imputation in the two strategies, however, is based on different sets of products. Whereas linking imputes from all surviving products (as in our theoretical model), class-mean imputation is based on other (comparable and noncomparable) substitutions. We are looking for the fraction E of the products $L + C$ for which imputation is effectively based on all surviving products, as opposed to just those surviving products with incumbent own innovations (fraction $1 - E$). These include all cases of linking plus a fraction (call it x) of class-mean imputations:

$$E = \frac{L + x \cdot C}{L + C}. \quad (1)$$

How do we determine x ? Class-mean imputations C use a weighted average for inflation from item substitutions for which there was either no adjustment (fraction $N/(D + N)$) or a direct adjustment (fraction $D/(D + N)$). Since 48% of all substitutions over the period 1988–2004 were noncomparable (31.4% of which were direct adjustments) and 52% of all substitutions were comparable, we get:

$$\frac{D}{D + N} = \frac{0.314 \cdot 0.48}{0.314 \cdot 0.48 + 0.52} \approx 0.225.$$

Using the assumptions above and results from Klenow and Kryvtsov (2008), the fraction of incumbent own-innovations (OI) among surviving products (those not creatively destroyed) is $\lambda_i \approx 0.60\%$ monthly.³ If the fraction of direct

³Together with a monthly rate of product exit of 3.9% this number is obtained as $(0.039 \cdot 0.48 \cdot 0.314) / (0.961 + 0.039[0.52 + 0.48 \cdot 0.314])$.

quality adjustments in class-mean imputations was also 0.60%, we would say class-mean imputation is just like linking (imputation based on all products not creatively destroyed). Because the fraction of direct quality adjustments in class-mean imputations (at 22.5%) was higher than 0.60%, we infer that class-mean imputation puts extra weight on OI:

$$\frac{D}{D + N} = x \cdot \lambda_i + (1 - x) \cdot 1, \quad (2)$$

where x is the weight on all surviving items (only fraction λ_i of which were innovations) and $1 - x$ is the weight on those surviving products which did experience incumbent innovations. Rearranging (2) and using the above percentages we get

$$x = \frac{N/(D + N)}{1 - \lambda_i} \approx \frac{0.775}{1 - 0.0060} \approx 0.780.$$

Thus, class-mean imputation effectively puts 78% weight on all surviving items and 22% weight on innovating survivors. Given that class-mean imputation was used 32% of time and linking was used 36% of time, we estimate that the BLS used imputation based on all surviving items the equivalent of 90% of the time from 1988–2004. More exactly, we substitute the numerical values for x , L and C into (1) to get

$$E = \frac{L + x \cdot C}{L + C} \approx \frac{0.362 \cdot 1 + 0.324 \cdot 0.780}{0.362 + 0.324} \approx 0.896.$$

B Derivations and proofs

Here we lay out the growth accounting model in whole to show the relationship between missing growth and underlying innovations. Time is discrete and in each period there is a representative household that supplies L units of labor. The household's utility is CES:

$$C_t = \left(\int_0^{N_t} [q_t(j) c_t(j)]^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \quad (3)$$

where $c(j)$ denotes quantity and $q(j)$ the quality of variety j . N is the number of varieties available, which can grow over time. Here $\sigma > 1$ denotes the constant elasticity of substitution between varieties.⁴

The aggregate price index In the following we derive the exact welfare-based aggregate price index. The results follow from the consumer choosing $\{c(j)\}_{j=0}^N$ to minimize the cost of acquiring one unit of CES composite consumption given prices $\{p(j)\}_{j=0}^N$.

Proposition 1 (i) *the demand for consumption good $c(j)$ of quality $q(j)$ sold at price $p(j)$ is given by*

$$c_t(j) = q_t(j)^{\sigma-1} \left[\frac{P_t}{p_t(j)} \right]^\sigma C_t, \forall j. \quad (4)$$

(ii) *the aggregate price index is given by*

$$P_t = \left(\int_0^{N_t} \left[\frac{p_t(j)}{q_t(j)} \right]^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (5)$$

Proof. Let $M = \Pi + WL$ denote the household's nominal spending/income. The first-order conditions when maximizing (3) subject to the (budget)

⁴In Section F, we use the Cobb-Douglas case with $\sigma \rightarrow 1$ to highlight channels of missing growth.

constraint $M = \int_0^N c(j)p(j)dj$, can be written as

$$\xi p(j) = q(j)^{\frac{\sigma-1}{\sigma}} c(j)^{-\frac{1}{\sigma}} \left(\int_0^N [q(j')c(j')]^{\frac{\sigma-1}{\sigma}} d(j') \right)^{\frac{1}{\sigma-1}}, \forall j \in [0, N],$$

where ξ is the Lagrange multiplier attached to the budget constraint. Integrating both sides of this equation over all j 's and combining it with (3) yields

$$\xi = \frac{C}{M} = \frac{1}{P}.$$

Together with the above first-order conditions, this yields (4). Next, to derive expression (5) for P , note that (4) implies that

$$p(j)c(j) = \frac{M}{P} q(j)^{\sigma-1} P^\sigma p(j)^{1-\sigma}.$$

Integrating both side of this equation over all j 's then immediately yields (5). This establishes the proposition. ■

We assume monopolistic competition across varieties so that the price of each consumer variety is⁵

$$p(j) = \frac{\sigma}{\sigma-1} \cdot W, \quad (6)$$

where W is the nominal wage in the competitive labor market. Substituting (6) into (5) links the aggregate price index to the quality of each variety

$$P_t = \mu W_t \left(\int_0^{N_t} q_t(j)^{\sigma-1} dj \right)^{\frac{1}{1-\sigma}}. \quad (7)$$

⁵We assume Bertrand competition within varieties. But we also assume an infinitesimal overhead cost of production that must be expended before choosing prices and output. The overhead cost allows the highest quality producer to charge the monopoly markup $\frac{\sigma}{\sigma-1}$, as the next lowest quality competitor will be deterred by zero ex post profits under Bertrand competition. Without this assumption, firms would engage in limit pricing and markups would be heterogeneous across varieties.

Innovation process At each point in time, and for each variety j there is a common exogenous probability of creative destruction $\lambda_d \in [0, 1)$. I.e., with probability λ_d the incumbent firm of input j is replaced by a new producer. We assume that the new producer (who may be an entrant or an incumbent firm) improves upon the previous producer's quality by a factor $\gamma_d > 1$. The previous producer cannot profitably produce due to limit pricing by the new producer. If j is an existing variety where quality is improved upon by a new producer, we have

$$q_{t+1}(j) = \gamma_d q_t(j).$$

We refer to this innovation process as *creative destruction*.

In addition, for products j where the incumbent producer is not eclipsed by creative destruction, there is each period an exogenous arrival rate $\lambda_i \in [0, 1)$ of an innovation that improves their by factor $\gamma_i > 1$. Hence, if j is a variety where quality is improved upon by the incumbent producer, we have

$$q_t(j) = \gamma_i q_{t-1}(j).$$

We call this *incumbent own innovation*. The main difference from creative destruction is that the producer of j changes with creative destruction, whereas it stays the same with incumbent own innovation. The arrival rates and step sizes of creative destruction and incumbent own innovation are constant over time and across varieties.

Finally, each period t , a flow of $\lambda_n N_t$ new product varieties $\iota \in (N_t, N_{t+1}]$ are created and available to final goods producers from $t+1$ onward. Consequently, the law of motion for the number of varieties is

$$N_t = (1 + \lambda_n) N_{t-1}.$$

We allow the (relative) quality of new product varieties to be higher than the “average” quality of pre-existing varieties by a factor γ_n . More formally, we

assume that a firm that introduces in period t a new variety ι starts with a quality that equals $\gamma_n > 0$ times the “average” quality of pre-existing varieties $j \in [0, N_{t-1}]$ in period $t - 1$, that is:

$$q_t(\iota) = \gamma_n \left(\frac{1}{N_{t-1}} \int_0^{N_{t-1}} q_{t-1}(j)^{\sigma-1} dj \right)^{\frac{1}{\sigma-1}}, \quad \forall \iota \in (N_{t-1}, N_t]. \quad (8)$$

The true inflation rate Using (7), we can compute the true inflation rate as a function of the arrival rates and the step sizes of the various types of innovations. We obtain the following proposition:

Proposition 2 *The true gross inflation rate in the economy is given by*

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} [1 + \lambda_d (\gamma_d^{\sigma-1} - 1) + (1 - \lambda_d) \lambda_i (\gamma_i^{\sigma-1} - 1) + \lambda_n \gamma_n^{\sigma-1}]^{\frac{1}{1-\sigma}}. \quad (9)$$

Proof. Taking gross growth factors of both sides of (7) gives

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} \left(\int_0^{N_t} q_t(j')^{\sigma-1} dj' \right)^{\frac{1}{\sigma-1}} \left(\int_0^{N_{t+1}} q_{t+1}(j)^{\sigma-1} dj \right)^{\frac{1}{1-\sigma}}. \quad (10)$$

Next, note that the term, $\int_0^{N_{t+1}} q_{t+1}(j)^{\sigma-1} dj$, can be written as

$$\int_0^{N_{t+1}} q_{t+1}(j)^{\sigma-1} dj = \int_0^{N_t} q_{t+1}(j)^{\sigma-1} dj + \int_{N_t}^{N_{t+1}} q_{t+1}(\iota)^{\sigma-1} d\iota. \quad (11)$$

Furthermore, with (8) and $\frac{N_{t+1} - N_t}{N_t} = \lambda_n$, we obtain

$$\int_{N_t}^{N_{t+1}} q_{t+1}(\iota)^{\sigma-1} d\iota = \lambda_n \gamma_n^{\sigma-1} \int_0^{N_t} q_t(j)^{\sigma-1} dj. \quad (12)$$

The first term on the right-hand side of (11), $\int_0^{N_t} q_{t+1}(j)^{\sigma-1} dj$, can be rewritten as

$$\int_0^{N_t} q_{t+1}(j)^{\sigma-1} dj = \gamma_d^{\sigma-1} \int_{\iota \in \mathcal{D}_t} q_t(\iota)^{\sigma-1} d\iota + \gamma_i^{\sigma-1} \int_{j' \in \mathcal{O}_t} q_t(j')^{\sigma-1} dj' + \int_{\iota' \in \tilde{\mathcal{N}}_t} q_t(\iota')^{\sigma-1} d\iota'. \quad (13)$$

where \mathcal{D}_t and \mathcal{O}_t is the set of products with a successful creative destruction or incumbent own innovation and $\tilde{\mathcal{N}}_t = [0, N_t] \setminus \{\mathcal{D}_t \cup \mathcal{O}_t\}$ is the set of surviving incumbents that do not improve the quality of their product between t and $t+1$. We also know that $|\mathcal{D}_t| = \lambda_d N_t$ and $|\mathcal{O}_t| = (1 - \lambda_d)\lambda_i N_t$. Then, because the arrival rate of an innovation is independent of $q_t(j)$ (and there is a continuum of varieties) the distribution of productivity of the varieties with and without innovation coincide and then by the law of large numbers we have

$$\begin{aligned} \int_{l \in \mathcal{D}_t} q_t(l)^{\sigma-1} dl &= \lambda_d \int_0^{N_t} q_t(j)^{\sigma-1} dj, \\ \int_{j' \in \mathcal{O}_t} q_t(j')^{\sigma-1} dj' &= (1 - \lambda_d)\lambda_i \int_0^{N_t} q_t(j)^{\sigma-1} dj, \\ \int_{l' \in \tilde{\mathcal{N}}_t} q_t(l')^{\sigma-1} dl' &= [1 - \lambda_d - (1 - \lambda_d)\lambda_i] \int_0^{N_t} q_t(j)^{\sigma-1} dj. \end{aligned}$$

This in turn implies that (13) can be expressed as

$$\frac{\int_0^{N_t} q_{t+1}(j)^{\sigma-1} dj}{\int_0^{N_t} q_t(j)^{\sigma-1} dj} = 1 + \lambda_d (\gamma_d^{\sigma-1} - 1) + (1 - \lambda_d)\lambda_i (\gamma_i^{\sigma-1} - 1). \quad (14)$$

Putting equations (10), (12), and (14) together establishes the proposition. ■

Proposition 2 shows how the arrival rates and step sizes of the different type of innovation affect the inflation rate (for a given change in wages). The term $\lambda_n \gamma_n^{\sigma-1}$ captures the effect of variety expansion on inflation, and the inflation rate is indeed falling in λ_n and γ_n . The term $(1 - \lambda_d)\lambda_i (\gamma_i^{\sigma-1} - 1)$ summarizes the effect of incumbent own innovation on price growth. The term $\lambda_d (\gamma_d^{\sigma-1} - 1)$ captures the effect from creative destruction on the inflation rate.

Imputation and measured inflation

Assumption 1 *In the presence of product entry and exit the statistical office resorts to imputation, i.e., the set of products with a surviving incumbent*

producer is assumed to be representative and the economy-wide inflation rate is imputed from this subset of products.

Products of continuing producers can be either subject to incumbent own innovation or no innovation at all. We denote the statistical office's estimates for the frequency and step size of quality-improving innovations on surviving products as $\widehat{\lambda}_i$ and $\widehat{\gamma}_i$.

Proposition 3 *Under Assumption 1, the measured inflation rate is given by*

$$\left(\frac{\widehat{P}_{t+1}}{P_t}\right) = \frac{W_{t+1}}{W_t} \left[1 + \widehat{\lambda}_i (\widehat{\gamma}_i^{\sigma-1} - 1)\right]^{\frac{1}{1-\sigma}}. \quad (15)$$

Proof. Under Assumption 1 we have

$$\left(\frac{\widehat{P}_{t+1}}{P_t}\right) = \frac{W_{t+1}}{W_t} \left(\int_{\mathcal{I}_t} q_t(j')^{\sigma-1} dj'\right)^{\frac{1}{\sigma-1}} \left(\int_{\mathcal{I}_t} q_{t+1}(j)^{\sigma-1} dj\right)^{\frac{1}{1-\sigma}}, \quad (16)$$

where $\mathcal{I}_t = [0, N_t] \setminus \mathcal{D}_t$ is the set of surviving products with the same producer in period t and $t+1$. Note that a fraction λ_i of these surviving products experiences incumbent own innovation (and the quality improves by a factor of γ_i) whereas for the remaining fraction, $1 - \lambda_i$, quality remains unchanged. Hence, we have $\int_{\mathcal{I}_t} q_{t+1}(j)^{\sigma-1} dj = \left(\int_{\mathcal{I}_t} q_t(j')^{\sigma-1} dj'\right) [1 - \lambda_i + \lambda_i \gamma_i^{\sigma-1}]$. Using this equation in (16) and replacing γ_i and λ_i by their estimates yields (15). ■

Henceforth we assume that the statistical office perfectly observes the frequency and step size of incumbent own innovations, i.e., we have $\widehat{\lambda}_i = \lambda_i$ and $\widehat{\gamma}_i = \gamma_i$. We make this assumption to isolate missing growth due to imputation. In Section D, we show how missing growth would change if the quality improvement of incumbents is not perfectly measured.

Missing growth from imputation We close the economy with market clearing condition $y(j) = c(j)$ for all j and $\int_j l(j) dj = L$. This implies that aggregate nominal output is equal to consumption expenditure $PY = PC$.

Using the welfare based index, aggregate real output growth in logs is then

$$\log \frac{Y_t}{Y_{t-1}} = \log \frac{P_t Y_t}{P_{t-1} Y_{t-1}} - \pi_t.$$

Assuming the statistical agencies measure nominal output accurately and use the measured inflation (15) to deflate, measured real output growth is then

$$\log \frac{\widehat{Y}_t}{Y_{t-1}} = \log \frac{P_t Y_t}{P_{t-1} Y_{t-1}} - \widehat{\pi}_t.$$

Combining the above two equation yields missing growth as the difference between measured and true inflation rate: $\widehat{\pi}_t - \pi_t$. Substituting in (9) and (15) allows us to express missing growth as

$$MG_t = \frac{\log [1 + \lambda_d (\gamma_d^{\sigma-1} - 1) + (1 - \lambda_d) \lambda_i (\gamma_i^{\sigma-1} - 1) + \lambda_n \gamma_n^{\sigma-1}] - \log (1 + \lambda_i (\gamma_i^{\sigma-1} - 1))}{\sigma - 1}.$$

C Missing growth with capital

The purpose of this section of the Online Appendix is to extend our “missing growth” framework to a production technology with capital as an input, and to see how this affects estimated missing growth as a fraction of “true” growth.

C.1 A simple Cobb-Douglas technology with capital

Instead of the linear technology in the main text, we assume the following Cobb-Douglas production technology for each variety

$$y(j) = (k(j)/\alpha)^\alpha (l(j)/(1 - \alpha))^{1-\alpha}.$$

It is straightforward to see how this generalization affects the main equations in the paper. If R denotes the rental rate of capital, then the true aggregate price index becomes

$$P = p \left(\int_0^N q(j)^{\sigma-1} dj \right)^{\frac{1}{1-\sigma}},$$

with just $p = p(j) = \mu R^\alpha W^{1-\alpha}$.

Again we assume that the statistical office perfectly observes the nominal price growth $\frac{p_{t+1}(j)}{p_t(j)}$ of the surviving incumbent products. Since the Cobb-Douglas production technologies are identical across all varieties the capital-labor ratio equalizes across all firms and we have in equilibrium

$$y(j) = (\alpha)^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \left(\frac{K}{L} \right)^\alpha l(j),$$

where K and L denote the aggregate capital and labor stocks in the economy.

We assume that labor supply is constant over time and we assume a closed economy where profits, Π , labor earnings and capital income are spent on the

final output good such that

$$P \cdot Y = W \cdot L + R \cdot K + \Pi.$$

Then we can derive the equilibrium output of variety j which yields

$$y_t(j) = (\alpha)^{-\alpha} (1 - \alpha)^{-(1-\alpha)} K_t^\alpha L^{1-\alpha} q_t(j)^{\sigma-1} \left(\int_0^{N_t} q_t(j')^{\sigma-1} dj' \right)^{-1}. \quad (17)$$

The aggregate production function can now be written in reduced form as

$$Y_t = (\alpha)^{-\alpha} (1 - \alpha)^{-(1-\alpha)} Q_t K_t^\alpha L^{1-\alpha},$$

where $Q_t \equiv \left(\int_0^{N_t} q_t(j)^{\sigma-1} dj \right)^{\frac{1}{\sigma-1}}$. The term Q_t summarizes how quality/variety gains affect total productivity for given capital stock K_t .

Allowing for capital does not change anything in the model-based market share approach since we still have

$$\frac{S_{I_t,t+1}}{S_{I_t,t}} = \left(\frac{P_{t+1}}{P_t} \right)^{\sigma-1} \left(\frac{\widehat{P}_{t+1}}{P_t} \right)^{-(\sigma-1)}.$$

This equation can (still) be used to estimate missing growth as in the main text.⁶ Hence the missing growth figures we obtained in Section 3.3 of the main text are unaffected when we introduce capital as specified above. The only important thing to note here is that this missing growth is “missing growth in the Q term” since under the assumption that nominal price growth is perfectly well observed by the statistical office we have:

$$MG_{t+1} = \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{\widehat{P}_{t+1}}{P_t} \right) = \left(\frac{Q_{t+1}}{Q_t} \right) \left(\frac{\widehat{Q}_t}{Q_{t+1}} \right).$$

What may (potentially) change when introducing capital is how this

⁶This also easily generalizes to any constant return to scale production function.

missing growth should be compared to measured productivity growth. This issue is discussed in the remaining sections of this Online Appendix.

C.2 Finding “true” growth

So far we saw that our market share analysis in the main text remains valid when introducing capital, in the sense that it allows us to compute the bias in $\frac{Q_{t+1}}{Q_t}$. We now want to combine this missing growth estimate with information on measured growth to calculate “true” growth. The main question then is: what is the “right” estimate for measured growth $\left(\widehat{\frac{Q_{t+1}}{Q_t}}\right)$? Once we have found this “right” estimate of measured growth we can simply calculate true growth as

$$\left(\frac{Q_{t+1}}{Q_t}\right) = MG \cdot \left(\widehat{\frac{Q_{t+1}}{Q_t}}\right), \quad (18)$$

where MG is 1.0056 for the whole period in the baseline specification.

A potentially difficulty here is that the capital stock, K_t , may itself grow over time.⁷ Suppose K_t is growing at a constant rate over time, then part of the aggregate output growth $\frac{Y_{t+1}}{Y_t}$ is generated by capital deepening. Relatedly, if the capital stock grows over time the question arises as to whether this capital growth is perfectly measured or not. Finally, the long-run growth path of the capital stock will also matter and consequently we need to specify the saving and investment behaviors which underlie this growth of capital stock, and also need to take a stand as to whether there is investment specific technical change etc. The answer to all these questions have implication for the interpretation of the measured TFP growth and how it relates to $\widehat{\frac{Q_{t+1}}{Q_t}}$.

We first assume that the long-run growth rate of K_t results from a constant (exogenous) saving rate and abstract from investment specific technical change (see Section C.2.1). Furthermore we assume that all growth due to

⁷If instead K_t was like “land”, i.e., constant over time then the measured $\left(\widehat{\frac{Q_{t+1}}{Q_t}}\right)$ would be equal to the measured Hicks-neutral TFP growth.

capital deepening is perfectly well observed and measured by the statistical office (see Section C.2.2) . Then, in Section C.2.3, we consider two alternative assumptions as to which part of physical capital growth is measured and analyze how these affect true growth estimates.

C.2.1 Capital accumulation

We assume that the final output good can be either consumed or invested. Furthermore we assume a constant exogenous saving/investment rate in the economy (we thus abstract from intertemporal optimization), i.e.,

$$K_{t+1} = K_t(1 - \delta) + sY_t, \quad (19)$$

where s is the constant savings rate and δ is the depreciation rate of capital.

Suppose that $Q_{t+1}/Q_t = g$ is constant over time. This in turn implies that in the long run the capital-output ratio will stabilize at

$$\frac{K}{Y} = \frac{s}{g^{\frac{1}{1-\alpha}} - 1 + \delta}. \quad (20)$$

Along this balanced growth path investment, capital, and wages all grow at the same constant gross rate $g^{\frac{1}{1-\alpha}}$.

C.2.2 Measured output growth

Under the above assumption for capital accumulation, in the long run, true output growth is given by

$$\frac{Y_{t+1}}{Y_t} = \frac{Q_{t+1}}{Q_t} \left(\frac{Q_{t+1}}{Q_t} \right)^{\frac{\alpha}{1-\alpha}}. \quad (21)$$

Note that the first term on the right-hand side captures direct quality/variety gains, whereas the second term captures output growth due to capital deepening. In the following we assume that the second term is perfectly well

measured whereas the first term is mismeasured as specified in our theory.⁸ Under this assumption, measured output growth is equal to

$$\frac{\widehat{Y}_{t+1}}{Y_t} = \frac{\widehat{Q}_{t+1}}{Q_t} \left(\frac{Q_{t+1}}{Q_t} \right)^{\frac{\alpha}{1-\alpha}}. \quad (22)$$

C.2.3 Two alternative approaches on measured growth in capital stock

Next, we need to take a stand on how to measure the growth rate of capital stock. For given measured capital growth, the statistical office can compute the rate of Hicks-neutral TFP growth implicitly through the following equation:

$$\frac{\widehat{Q}_{t+1}}{Q_t} \left(\frac{Q_{t+1}}{Q_t} \right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\widehat{K}_{t+1}}{K_t} \right)^{\alpha} \frac{\widehat{TFP}_{t+1}}{TFP_t}. \quad (23)$$

First “macro” approach Here we assume that the bias in the measure of capital stock is the same as that for measuring real output.⁹ Then the measured growth rate of capital stock in the long run is equal to

$$\frac{\widehat{K}_{t+1}}{K_t} = \frac{\widehat{Y}_{t+1}}{Y_t} = \frac{\widehat{Q}_{t+1}}{Q_t} \left(\frac{Q_{t+1}}{Q_t} \right)^{\frac{\alpha}{1-\alpha}}. \quad (24)$$

Substituting this expression for measured capital growth in (23) in turn yields

$$\frac{\widehat{TFP}_{t+1}}{TFP_t} = \left(\frac{\widehat{Q}_{t+1}}{Q_t} \right)^{1-\alpha} \left(\frac{Q_{t+1}}{Q_t} \right)^{\alpha}. \quad (25)$$

⁸This assumption rests on the view that the part of growth driven by capital deepening materializes—for given quality and variety—in increasing $y(j)$ (see (17)) which the statistical office should be able to capture (otherwise we would have still another source of missing growth).

⁹This is a reasonable assumption to the extent that: (i) the same final good serves both as consumption good and as investment good; (ii) if the long-run growth rate of Q_t is constant, i.e., $Q_{t+1}/Q_t = g$, then the bias in measuring capital stock growth (when using a perpetual inventory method) is in the long run identical to the bias in measuring real output growth.

Substituting this into (18) then leads to:

$$\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{1}{1-\alpha}} = MG_{t+1} \cdot \left(\frac{\widehat{TFP}_{t+1}}{TFP_t}\right)^{\frac{1}{1-\alpha}}. \quad (26)$$

In other words, one should add MG to measured growth in TFP (in labor augmenting units) to get total “true” quality/variety growth in labor augmenting units. This is exactly what we are doing in our core analysis in the main text. Thus under the assumptions underlying this first approach the whole analysis and quantification of missing growth in our core analysis carries over to the extended model with capital. Let us repeat what underlies this approach: first, the focus is on the long-run when the capital-output ratio stabilizes at its balanced growth level; second, investment specific technical change is ruled out, so that the bias in measuring the growth in capital stock is the same as that in measuring the growth in real output.¹⁰

Second “micro” approach Here we assume that the growth in capital stock is perfectly measured by the statistical office, i.e.,

$$\frac{\widehat{K}_{t+1}}{K_t} = \left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{1}{1-\alpha}}. \quad (28)$$

Plugging this expression in (23) gives

$$\frac{\widehat{TFP}_{t+1}}{TFP_t} = \frac{\widehat{Q}_{t+1}}{Q_t}, \quad (29)$$

¹⁰To get some intuition, note that we can also write the production function as

$$Y_t = (\alpha)^{-\alpha} (1 - \alpha)^{-(1-\alpha)} Q_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} L. \quad (27)$$

Since under the assumptions above the growth rate in the capital-output ratio, $\frac{K_t}{Y_t}$, (which is zero in the long run) is properly measured, we see that missing growth automatically obtains a labor-augmenting interpretation and should consequently be compared to TFP growth estimates expressed in labor augmenting terms.

so that

$$\frac{Q_{t+1}}{Q_t} = MG_{t+1} \cdot \frac{\widehat{TFP}_{t+1}}{TFP_t}. \quad (30)$$

This in turn implies that our missing growth estimate should be added to measured TFP growth in Hick-neutral terms to obtain Hicks-neutral “true” TFP growth. Assuming $\alpha = 1/3$, this approach would increase missing growth as a fraction of true growth from 22% ($= 0.54/(1.87 + 0.54)$) see Table 3 in the main text) to 30% ($= 0.54/(1.87 \cdot 2/3 + 0.54)$)¹¹.

C.3 Wrapping-up

In this Appendix we argued that our core analysis can easily be extended to production technologies involving physical capital. Under our first (macro) approach the missing growth estimates remain exactly the same as in our core analysis based on the model without capital. And moving to our second (micro) approach only increases our missing growth estimates. In that sense, the macro approach can be viewed as being more conservative.

¹¹We see this approach as being more “micro” for the following reason. Suppose we only have data about the only one industry. Then we could use our market share approach together with data about the revenue shares of different products to estimate missing output growth in this particular industry. It would then be reasonable to compare this number to the Hicks-neutral TFP growth in this industry, within the implicit assumption that the statistical office perfectly measures the growth in capital stock in the industry when calculating TFP growth. Next, one could sum-up “missing growth” and measured Hicks-neutral TFP growth to compute “true” TFP growth. This true TFP growth would of course itself be mismeasured if there is mismeasurement in the growth of capital stock: this would add yet another source of missing growth.

D Other robustness checks

The gains from variety Our theory does not impose much discipline in terms of how the gains from specialization/variety are calibrated. Our baseline specification makes the standard assumption connecting the gains from specialization to the elasticity of substitution. It assumes the increasing the available product variety by one percent increases final output by $1/(\sigma - 1)$ percent. This only affects missing growth from variety expansion. In our second quantification approach, in the next section, we show that missing growth mainly originates from creative destruction as opposed to variety expansion. Consequently, we expect this assumption not to be as critical as it first seems.

Bias in measuring incumbent own innovation In the main text we assume that quality improvements from incumbent own innovation are correctly measured, i.e., that $\hat{\gamma}_i = \gamma_i$ and $\hat{\lambda}_i = \lambda_i$. Without this assumption, missing growth in our model is given by

$$MG_{t+1} = \frac{1}{\sigma - 1} \left[\log \left(\frac{1 + \lambda_i(\gamma_i^{\sigma-1} - 1)}{1 + \hat{\lambda}_i(\hat{\gamma}_i^{\sigma-1} - 1)} \right) + \log \left(\frac{S_{I_t,t}}{S_{I_t,t+1}} \right) \right]. \quad (31)$$

Understating incumbent own innovation adds log-linearly to missing growth, contributing directly and making the bias from imputation larger.

Imports and outsourcing Our model did not taken into account the possibility that plants may outsource the production of some items to other plants. Nor did it consider the role of imports as an additional source of new products. On outsourcing, our answer is twofold: (i) if the outsourcing is to another incumbent plant or leads an incumbent plant to shut down, then then outsourcing will not affect our analysis and results; (ii) if outsourcing is to a new plant then it can be viewed as an instance of creative destruction since the

reason for such outsourcing is presumably that the new plant produces at lower (quality-adjusted) price; it will be treated as such in our market share approach.

Outsourcing may indeed create a bias in our missing growth estimates if incumbent plants survive but outsource overseas. Our LBD dataset only covers domestic employment.¹²

Finally, imports are known to affect manufacturing the most, as manufacturing goods are the most tradable. Very little of our missing growth, however, comes from manufacturing (see Table 1 in the main text). This suggests that overall missing growth is not affected much by what happens in import-competing sectors.

¹²Domestic M&A should not affect missing growth in the same way because we are looking at plants, not firms. If firm A acquires firm B and all firm B plants remain in operation, then these plants will be counted as surviving plants. If some of firm B's plants close as a result of the M&A, then we rightly count them as exiting. One might want to compute the fraction of aggregate missing growth associated with M&A, but we leave that for future research.

E Implementation of GHK

We made the following changes to the GHK algorithm (Table E displays the a mapping between the notation used in GHK and our paper).

1. The original GHK methodology assumes that the statistical office measures growth perfectly. Hence, the algorithm chooses parameters such that true growth, given by equation (A1), matches measured growth in the data. We modify the algorithm to allow measured and true growth to differ. Instead of matching to true growth, we choose parameters so that measured growth in (A2) matches the observed growth rates: 1.66% for 1983–1993, 2.29% for 1993–2003 and 1.32% for 2003–2013.
2. We impose an additional restriction that comes from the CPI micro data. We restrict the sum of the (unconditional) arrival rates of OI and CD to equal the cumulative rate of non-comparable substitutions from the CPI over 5 years. This substitution rate averages 3.75% per 2 months in the CPI.¹³ Using the notation in our market share model, we impose that $\lambda_i(1 - \lambda_{e,d} - \lambda_{i,d}) + \lambda_{i,d} + \lambda_{e,d} = 0.68$.¹⁴
3. Since the original GHK code estimates 5-year arrival rates and step sizes, whereas BLS substitutions and imputations happen at a monthly or bimonthly frequency (depending on the item), we convert 5-year arrival rates into bimonthly arrival rates by imposing $(1 - X^{(b)})^{30} = 1 - X^{(5)}$, where $X^{(b)}$ and $X^{(5)}$ denote the bimonthly and five-year arrival rates, respectively. We then scale the bimonthly OI and CD arrival rates in equal proportion so that their sum equals the bimonthly CPI non-comparable substitution rate of 3.75%. Finally, we adjust the step sizes of NV and CD so that: (i) annualized bimonthly measured growth equals the observed

¹³Klenow and Kryvtsov (2008).

¹⁴ $0.68 = 1 - (1 - 0.0375)^{30}$. 30 compounds the bi-monthly arrival rate to 60 months (5 years).

annual measured growth; and (ii) the relative contributions of CD and NV to growth stay the same as those estimated using 5-year parameters.

Table 1: GHK NOTATIONS VS. OUR NOTATION

Parameter	Our model	GHK equivalent
Share of non-obsolete products with OI innovation	$\lambda_i(1 - \lambda_d)$	$\frac{\lambda_i}{(1-\delta_o)}$
Share of non-obsolete products having incumbent CD	0	$\frac{\delta_i(1-\lambda_i)}{(1-\delta_o)}$
Share of non-obsolete products having entrant CD	λ_d	$\frac{\delta_e(1-\lambda_i)}{(1-\delta_o)}$
Measure of incumbent or entrant NV in $t + 1$ relative to the number of products in t	λ_n	$\kappa_i + \kappa_e + \delta_o$
Share of obsolescence	0	δ_o
Net expected step size of CD innovation	$\gamma_d^{\sigma-1} - 1$	$\frac{1-\delta_o}{1-\delta_o\psi}(E[s_q^{\sigma-1}] - 1)$
Net expected step size of OI innovation	$\gamma_i^{\sigma-1} - 1$	$\frac{1-\delta_o}{1-\delta_o\psi}(E[s_q^{\sigma-1}] - 1)$
Quality of NV innovation relative to average productivity last period	γ_n	$s\kappa^{\frac{1}{\sigma-1}}$
Average quality of product becoming obsolete in $t + 1$ relative to average quality in t	n/a	ψ
Elasticity of substitution	σ	σ

F An illustrative example: the Cobb-Douglas case

Even though this may not be the most realistic case, we use the special case where the consumption function is Cobb-Douglas to illustrate how creative destruction can lead to missing growth. So let us assume

$$C = N \exp \left[\frac{1}{N} \int_0^N \log [q(j)c(j)] dj \right]. \quad (32)$$

We assume the number of varieties N is fixed here because there is no love-of-variety under Cobb-Douglas aggregation.

Aggregate price index Demand for product $c(j)$ is

$$c(j) = \frac{PC}{Np(j)}.$$

P is the price index:

$$P = \exp \left(\frac{1}{N} \int_0^N \log [p(j)/q(j)] dj \right).$$

Under the optimal price setting rule we get

$$P = \mu W \exp \left(-\frac{1}{N} \int_0^N \log (q(j)) dj \right).$$

The true inflation rate can then be expressed as

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} \gamma_i^{-(1-\lambda_d)\lambda_i} \gamma_d^{-\lambda_d}.$$

Measured inflation and missing growth Under Assumption 1 measured inflation becomes

$$\left(\frac{\widehat{P}_{t+1}}{P_t} \right) = \frac{W_{t+1}}{W_t} \gamma_i^{-\lambda_i}.$$

Consequently, we obtain for missing growth

$$MG = \lambda_d \cdot (\log \gamma_d - \lambda_i \log \gamma_i). \quad (33)$$

This missing growth from creative destruction can be decomposed as

$$\lambda_d (\log \gamma_d - \lambda_i \log \gamma_i) = \lambda_d (1 - \lambda_i) \log \gamma_i + \lambda_d (\log \gamma_d - \log \gamma_i).$$

The first term in this decomposition captures the fact that not all incumbents innovate, whereas the second term captures the step size differential between creative destruction and incumbent own innovation.

Numerical example The Cobb-Douglas case with the following calibration replicates the motivating example of the introduction. Let us assume: (i) no variety expansion; (ii) the same step size for incumbent own innovation (OI) and for creative destruction (CD), i.e., $\gamma_i = \gamma_d = \gamma$, and (iii) annualized arrival rates λ_i and λ_d of OI and CD by new entrants that are both equal to 10%. Finally, assume that the common step size is $\gamma_i = 1.1$, or 10%. Then *measured* annual real output growth is equal to 1.1% ($\lambda_i \log \gamma_i = .011$). From (33), the annual rate of missing growth from creative destruction is equal to

$$MG = 10\% \cdot (1 - 10\%) \cdot 10\% = 0.9\%.$$

True growth is 2% in this example. Hence, roughly half of the growth is missed due to imputation. Although this is just an illustrative exercise, we will see in the next sections that this simple example is not far off from what we obtain using firm-level data on employment dynamics to infer the step sizes and frequencies of each type of innovations.

G Varying markups

Our baseline analysis carries over to the case where markups are heterogeneous but uncorrelated with the age of the firm or with whether or not there was a successful innovation (own incumbent or new entrant innovation).

Now, suppose that: (i) the markups of unchanged products grow at gross rate g ; (ii) the markups of new varieties are equal to g_n times the “average markup” in the economy in the last period; (iii) markups grow at gross rate g_i if there is an incumbent own innovation; (iv) markups after a successful creative destruction innovation is g_d times the markup of the eclipsed product. This amounts to replacing (8) by:¹⁵

$$\frac{q_{t+1}(j)}{\mu_{t+1}(j)} = \frac{\gamma_n}{g_n} \left(\frac{1}{N_t} \int_0^{N_t} \left(\frac{q_t(i)}{\mu_t(i)} \right)^{\sigma-1} di \right)^{\frac{1}{\sigma-1}}, \forall j \in (N_t, N_{t+1}].$$

Under the above assumptions the market share approach can still provide a precise estimate of missing growth, as long as: (a) we still make the assumption that the statistical office is measuring changes in markups of surviving product properly since changes in nominal prices are observed; (b) the market share relates to the quality-adjusted price in the same way for young and old firms, but recall that we are focusing our market share analysis on plants that have appeared in the data set for at least five years.

However, allowing for changing markups affects the expression for missing growth, which now becomes:

$$MG = \frac{1}{\sigma-1} \log \left(1 + \frac{\lambda_d \left[\left(\frac{\gamma_d}{g_d} \right)^{\sigma-1} - g^{1-\sigma} - \lambda_i \left(\left(\frac{\gamma_i}{g_i} \right)^{\sigma-1} - g^{1-\sigma} \right) \right] + \lambda_n \left(\frac{\gamma_n}{g_n} \right)^{\sigma-1}}{g^{1-\sigma} + \lambda_i \left(\left(\frac{\gamma_i}{g_i} \right)^{\sigma-1} - g^{1-\sigma} \right)} \right).$$

¹⁵Note that this covers several possible theories governing the dynamics of markups. In particular it covers the case where firms face a competitive fringe from the producer at the next lower quality rung, in which $g_i > 1$ and $g < 1$. It also covers the case where newly born plants start with a low markup and markups just grow over the live-cycle of a product, in which $g_d < 1$, $g_n < 1$ and $g > 1$.

In particular, allowing for changing markups introduces an additional source of missing growth having to do with the fact that the subsample of (surviving) products are not representative of all firms in their markup dynamics. For example, even if $\lambda_i = 1$ and $\gamma_i = \gamma_d$, there can be missing growth from creative destruction if the markup of creatively destroyed goods grows slower than the markup of products with incumbent own innovation, i.e., if $g_d < g_i$.

In the market share section of our paper. The second and third columns report missing growth in manufacturing and non-manufacturing, respectively. Missing growth in non-manufacturing is about 0.11 percentage points larger than our baseline results but also appears to be constant over time. Missing growth in manufacturing, however, is only 0.04 percentage points on average between 1983–2013.

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