The Currency Composition of Sovereign Debt

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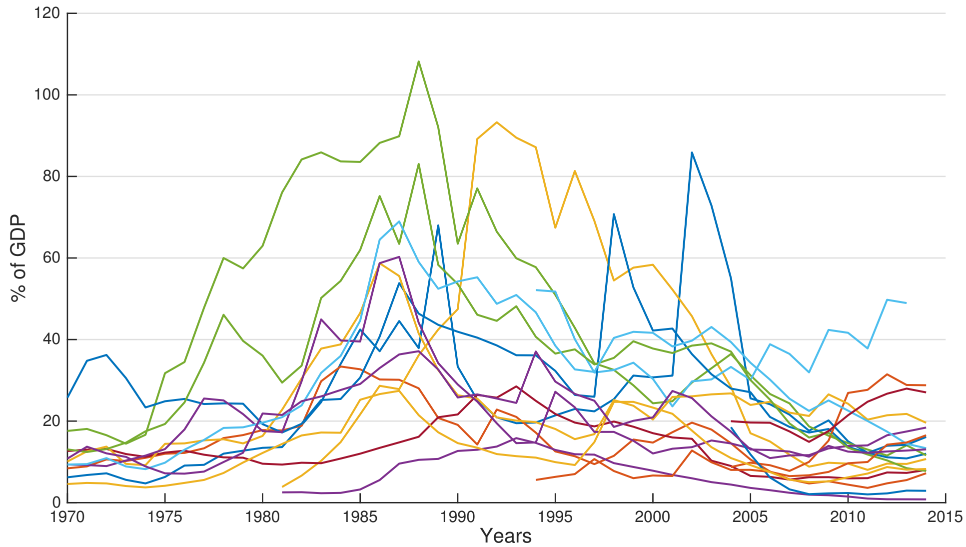
Online Appendix

Appendix A. Additional Figures and Tables

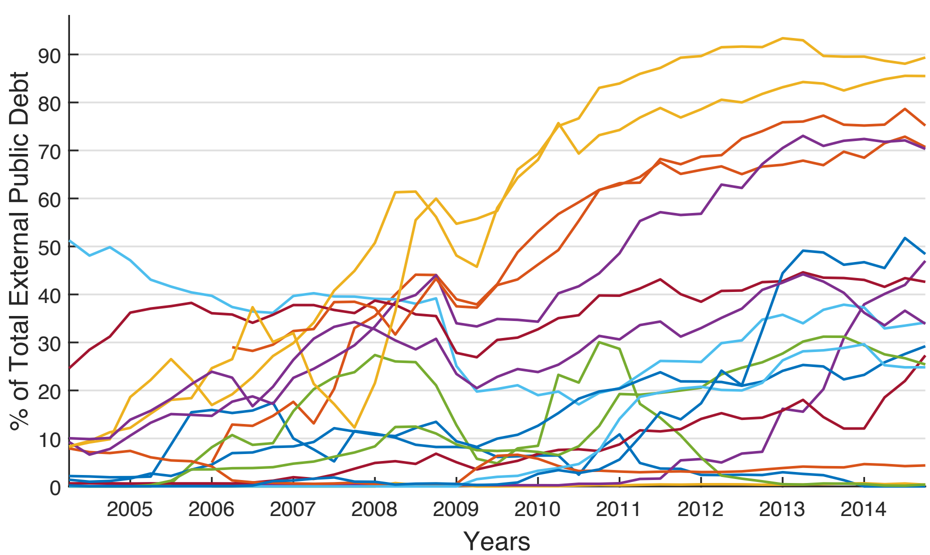
A.1. *Additional Figures*

Figure A.1. Evolution of Total External Public Debt



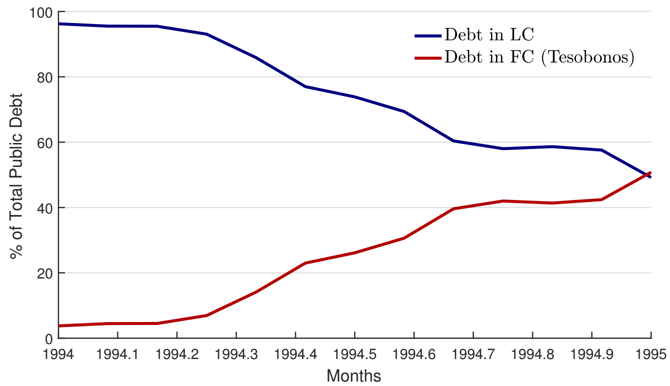
Notes: Based on WDI data. Stock of external public debt as a percentage of annual GDP for the countries in the sample.

Figure A.2. Evolution of the Currency Composition of External Public Debt



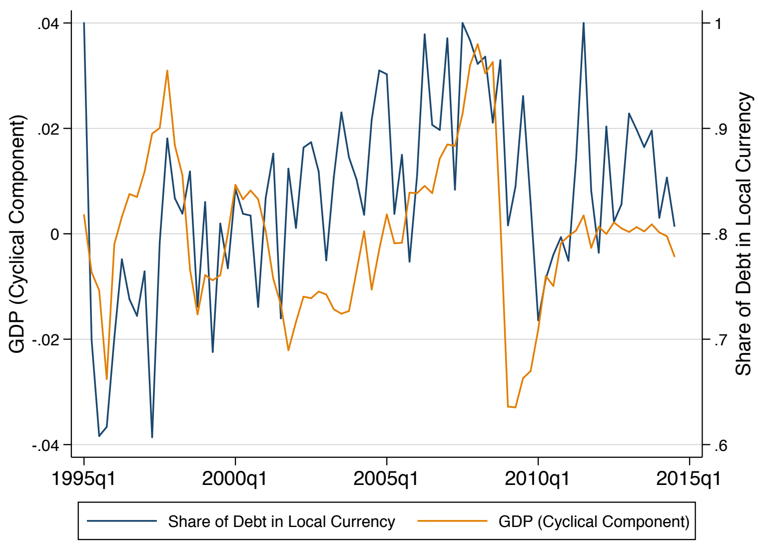
Notes: Based on Arslanalp & Tsuda (2014). Share of external public debt in local currency as a fraction of total external public debt for the countries in the sample.

Figure A.3. Currency Composition of Sovereign Debt: The Case of Mexico During the Tequila Crisis



Notes: This figures shows the share of total public debt in local and foreign currency. It is based on data from Banxico. Local currency debt includes Ajustabonos, CETES and Bondes. Foreign-currency-denominated debt includes Tesobonos.

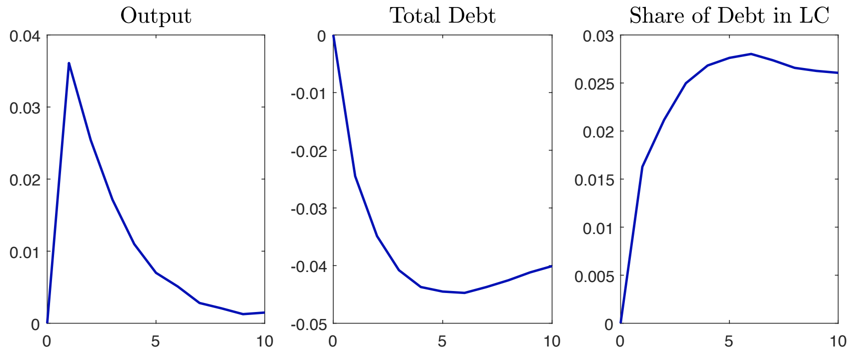
Figure A.4. Issuance Data: Cyclicality of Currency Denomination



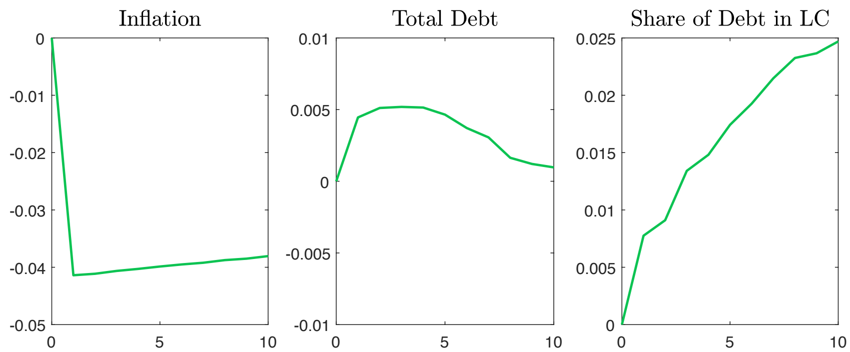
Notes: The share of issuance of public debt in local currency is computed as the sum of the face value of sovereign bonds in local currency divided by the sum of the face value of total sovereign bond issued in a given quarter by a given country. The blue line plots the average share of issuance of public debt in local currency across all countries in our sample. The orange line plots the average cyclical component of real GDP. Trend GDP is computed with HP filter (smoothing parameters 1600).

FIGURE A.5. Impulse Response Functions

(A) Response to a shock to y_T



(B) Response to a shock to the inflation regime



Notes: The top panel plots the response of variables to a one-standard-deviation shock to tradable endowment (y_T) in the calibrated model with monetary shocks. For details on the model calibration, see section 5.4. Variables are expressed in deviations from their value before the shock hits ($t = 0$). The response of output is expressed in percentage points. The response of debt is expressed in percent of GDP. The share of debt is measured as the difference between debt in local and foreign currency expressed as percent of GDP. The bottom panel plots the response of variables to a change from the 'high inflation costs' regime to the 'low inflation costs' regime. The response of inflation is expressed in percentage points. In both panels, the graphs display the average response of variables to exogenous shocks starting from different states in the ergodic distribution. The impulse response functions were computed in the model with monetary shocks.

A.2. *Additional Tables*

Table A1. Cyclicality of the Currency Composition of Sovereign Debt

	Correlation with Output				
	Debt in LC	Debt in FC	Share of Debt in LC		
Country	HP-trend Filtering	HP-trend Filtering	HP-trend Filtering	Linear-trend Filtering	Share of LC debt at constant XR
Argentina	19%	-18%	24%	26%	17%
Brazil	38%	-64%	72%	88%	47%
Bulgaria	55%	-47%	53%	35%	54%
China	-21%	2%	-23%	-83%	-22%
Egypt	27%	-5%	31%	66%	30%
Hungary	31%	-58%	50%	10%	44%
India	6%	-69%	15%	-1%	7%
Indonesia	17%	-39%	27%	15%	9%
Lithuania	-55%	-43%	-70%	-63%	-71%
Malaysia	57%	-11%	44%	3%	37%
Mexico	53%	-51%	59%	49%	44%
Peru	63%	-63%	64%	8%	64%
Philippines	-16%	-53%	-7%	9%	-7%
Poland	-20%	-34%	5%	-21%	-27%
Russia	22%	-48%	28%	-25%	30%
South Africa	41%	13%	16%	-15%	22%
Thailand	30%	-9%	-11%	-16%	-13%
Turkey	58%	-40%	62%	68%	58%
Average	22%	-35%	24%	9%	18%
Median	28%	-42%	27%	9%	26%
Std. Dev.	33%	25%	36%	44%	35%

Notes: Debt in LC and debt in FC refer, respectively, to public debt denominated in local currency and in foreign currency over GDP. The share of debt in LC refers to the share of external public debt denominated in local currency. The correlations between output and debt in LC, debt in FC, and the share of debt in LC refer to the correlations between the cyclical component of real GDP and the cyclical component debt in LC, debt in FC, and the share of debt in LC. In the first three columns variables are detrended using the HP filter. In the fourth column they are detrended with a linear trend. The last column computes the same moment as the third column but with the share of debt in local currency measured at constant exchange rates of 2006.Q1. The share of debt in local currency and the correlation with GDP is computed for the period 2004-2014, when data by currency becomes available.

The average external public debt is computed over the period 1990-2014. The share of debt in LC refers to the share of external public debt denominated in local currency. The correlation between output and the share of debt in LC refers to the correlation between the cyclical component of real GDP and the cyclical component of the share of external public debt denominated in local currency. Both variables are detrended using the HP filter. The share of debt in local currency and the correlation with GDP is computed for the period 2004-2014, when data by currency becomes available. Δ 2014-04 refers to the difference between the share of debt in local currency in 2014 and in 2004. For countries in which the 2004 data was not available, the difference is taken from the earliest data available.

Table A2. Facts on Public and Private External Debt

	Average Debt		Corr. Debt & GDP		Corr. Public
Country	Public	Private	Public	Private	& Private Debt
Argentina	28%	9%	-84%	-76%	92%
Brazil	12%	10%	-77%	-44%	55%
Bulgaria	43%	29%	-44%	66%	-34%
China	7%	2%	50%	-33%	-33%
Egypt	34%	0%	-65%	19%	-0%
Hungary	38%	44%	1%	-14%	11%
India	14%	5%	-49%	-28%	-42%
Indonesia	30%	16%	-71%	-36%	88%
Lithuania	19%	n.a.	-62%	n.a.	n.a.
Malaysia	23%	12%	-89%	-11%	14%
Mexico	17%	5%	-69%	21%	12%
Peru	31%	7%	-73%	-20%	30%
Philippines	35%	11%	-53%	-3%	43%
Poland	22%	n.a.	-66%	n.a.	n.a.
Russia	5%	n.a.	-56%	n.a.	n.a.
South Africa	9%	8%	-35%	-25%	15%
Thailand	12%	17%	-85%	-17%	49%
Turkey	20%	12%	-70%	-6%	5%
Average	22%	12%	-55%	-14%	20%
Median	21%	10%	-66%	-17%	14%
Std. Dev.	11%	11%	34%	33%	41%

Notes: The averages of external public and private debt are computed over the period 1990-2014. The correlation between output and the public (private) external debt refers to the correlation between the cyclical component of real GDP and the cyclical component the public (private) external debt to GDP ratio. Both variables are detrended using HP filter (smoothing parameter 100). Data source: WDI.

Table A3. GDP and Exchange Rates in Emerging Economies

Country	$\rho_{y,e}$	$\rho_{y,rxr}$	$\rho_{e,rxr}$
Argentina	-61%	-79%	90%
Brazil	-81%	-83%	95%
Bulgaria	-63%	-62%	94%
China	39%	-21%	47%
Egypt	-66%	-66%	93%
Hungary	-57%	-43%	90%
India	-65%	-78%	83%
Indonesia	-85%	-72%	97%
Lithuania	-64%	-17%	41%
Malaysia	-77%	-78%	99%
Mexico	-48%	-69%	82%
Peru	2%	-58%	49%
Philippines	-30%	-16%	97%
Poland	-18%	-25%	74%
Russia	-58%	-64%	96%
South Africa	-21%	-6%	98%
Thailand	-87%	-84%	97%
Turkey	-53%	-53%	100%
Average	-50%	-54%	85%
Median	-59%	-63%	94%
Std. Dev.	33%	26%	19%

Notes: $\rho_{y,e}$ refers to the correlation coefficient between the cyclical component of real GDP and the cyclical component of the nominal exchange rate, measured as units of local currency per unit of foreign currency. $\rho_{y,rxr}$ refers to the correlation coefficient between the cyclical component of real GDP and the cyclical component of the real exchange rate, measured as the ratio of the US CPI index expressed in local currency to the domestic CPI index. $\rho_{e,rxr}$ refers to the correlation coefficient between the cyclical component of the nominal exchange rate and the real exchange rate. Correlations are computed for the period 1990-2014. Data are at the annual frequency, trends are computed with HP filter (smoothing parameters 100). Data source: WDI.

Appendix B. Data Sources and Estimation (For Online Publication)

B.1. *Data sources*

The data on sovereign debt comes from two sources. Annual data on sovereign external debt by country for the period 1990-2014 comes from WDI. Quarterly data on sovereign external debt in foreign and local currency by country for the period 2004-2014 comes from Arslanalp and Tsuda (2014). The authors collect data on central government debt by currency denomination. They estimate foreign and domestic holdings of this debt issued in global and local markets. The definition of foreign and domestic investors they use follows the residency principle of external debt statistics. For this they use data from the IMF, BIS and national sources. In order to reach the final estimates the authors make certain assumptions in cases in which the necessary data was unavailable.[19] These assumptions are stated in the paper.

Data on bond issuance comes from Bloomberg. We collected data on all bonds issuances recorded in the Bloomberg terminal for all countries in our sample during the period 1990-2014. For each bond we have data on the institutional name of the debtor, its face value, maturity, date of issuance and currency of denomination. We do not know the residence of who purchased the bonds, nor the market of issuance. Therefore, our data can include bonds that were purchased by domestic creditors. To address this issue we exclude all bonds with very short maturities, with very small face values and those issued by central banks. These bonds are likely to be acquired by domestic investors (IMF and World Bank (2016)). Specifically, we exclude all bonds that satisfy any of the following three conditions: 1. the word "bank" or "banco" appears in the name of the issuing institution, 2. the face value is less than 0.1 percent of GDP and 3. the maturity is less than 6 months. As we argue next, our main results are invariant to this filtering procedure. We also excluded bonds associated to debt restructuring and bank recapitalizations (Argentina, Russia and Indonesia). Once we filter out these observations we are left with a total of 14,745 bonds. We then compute the total issuance as the sum of the face value of all the bonds in a given country and year, expressed as a percentage of GDP. The share of issuance in local currency is computed as the sum of the face value of all issued bonds denominated in local currency divided by total bond issuance.

We check whether the micro-data on bond issuances aggregated at the country level is consistent with the aggregate data on debt levels. We do so by comparing at the country level the average levels of debt with the average levels of debt issuances, and the average share of

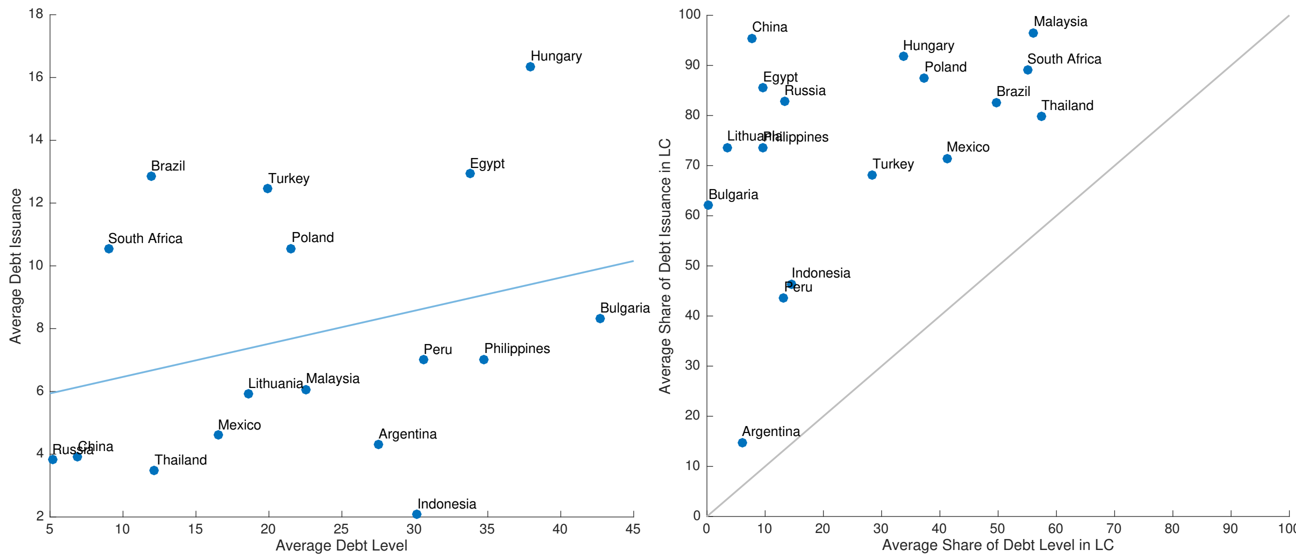
[19]For example, to estimate foreign bank holdings of public debt the authors use data on BIS about foreign bank holdings of non-bank debt and multiply it by the share of non-bank debt that corresponds to public debt.

debt levels in local currency with the average share debt issuance in local currency. The cross-country correlation between the average level of debt and the average level of debt issuance for the same period of time is 30 percent (see Figure B.1a). The cross-country correlation between the share of debt levels in local currency and the share of debt issuance in local currency is 49 percent (see Figure B.1b). Additionally, the average share of debt issuance in local currency is larger than the average share of debt levels in local currency for all countries. This is consistent with the fact that the share of debt levels in local currency has grown over time (given that debt issuance is one component of the change in debt levels). Overall, this evidence is suggestive of rough consistency between the two datasets. However, there are some discrepancies in specific countries. For example, in Bulgaria, while the share of debt levels in local currency is near zero, the average share of debt issuance in local currency is around 60 percent. This inconsistency may be due to imprecisions in our method for filtering out bond issuances that are purchased by domestic investors and/or imprecisions in the estimates in debt levels, which in fact are acknowledged in Arslanalp and Tsuda (2014) for the case of Bulgaria.[20] Since both datasets are constructed based on assumptions and proxies we view them as complements.

[20] For certain years in their sample, data on the sum of external debt holdings by investors collected from several sources exceeded the total level of external debt reported in IMF data. In those years, the authors 'calculated alternative measures of foreign bank holdings using other emerging markets as a benchmark'.

FIGURE B.1. Data on Debt Levels and Issuance: A Comparison

(A) Debt and Issuance: Levels (B) Debt and Issuance: Share of Debt in LC



Notes: Panel A compares data on debt levels and on debt issuance. The horizontal axis shows the average level of sovereign external debt measured as a percent of GDP by country for the period 1990-2014. The source of this data is WDI. The vertical axis shows the average annual sovereign debt issuance as a percent of GDP by country for the period 1990-2014. The data was computed from micro-data on bond issuance. The blue line is the best linear fit. Panel B compares data on the share of debt in local currency for debt levels and for debt issuance. The horizontal axis shows the average share of sovereign external debt levels in local currency by country for the period 1990-2014. This data is based on Arslanalp & Tsuda (2014). The vertical axis shows the average share of sovereign debt issuance in local currency by country for the period 1990-2104. The data was computed from micro-data on bond issuance. The green line is the 45 degree line.

Finally, we also assess the robustness of the analysis of the issuance data by computing the same analysis with the data without filtering out bonds with small maturities and small face values. Results, shown in Table B1, indicate that the main stylized facts are robust to the analysis of unfiltered data.

Data on GDP at an annual frequency comes from WDI. Data on real GDP at a quarterly frequency was obtained from national sources and IMF. Two measures of tradable output were constructed and considered. The first one is the sum of agriculture and industry value added. The second is industrial production. We used the one that had data availability for the longest time period for each country.

Table B1. Facts on Sovereign Debt Issuance by Currency: Unfiltered Data

Country	Annual Issuance Average (% of GDP)	Share of Issuance in LC Avg. 90-03 (% of Issuance)	Avg. 04-14 (% of Issuance)	Correlation with Output
Argentina	5.2%	7%	38%	-17%
Brazil	13.6%	66%	99%	43%
Bulgaria	62.8%	54%	82%	26%
China	6.4%	85%	100%	-5%
Egypt	24.4%	98%	97%	5%
Hungary	22.5%	96%	93%	6%
Indonesia	1.6%	15%	69%	-18%
Lithuania	8.4%	88%	73%	53%
Malaysia	7.0%	95%	100%	39%
Mexico	9.2%	66%	97%	19%
Peru	97.1%	72%	77%	1%
Philippines	13.3%	76%	94%	-3%
Poland	11.6%	79%	88%	24%
Russia	6.4%	64%	94%	45%
South Africa	14.3%	95%	98%	16%
Thailand	7.7%	76%	99%	-71%
Turkey	14.9%	54%	93%	38%
Average	19.2%	70%	88%	12%
Median	11.6%	76%	94%	16%
Std. Dev.	24.4%	26%	16%	31%

Notes: Average annual issuance of public debt is computed over the period 1990-2014. It is computed as the sum of the face value of all sovereign bonds issued in a given year as a percentage of GDP and then averaged across years. The high values of Bulgaria and Peru in average issuance reflect valuation effects during years of hyperinflation. The share of issuance in LC refers to the share of bond issuance denominated in local currency. The correlation between output and the share of debt in LC refers to the correlation between the cyclical component of real GDP and the cyclical component of the share of bond issuance denominated in local currency. Both variables are detrended using the HP filter. These data contains all bond issuances without filtering out bonds with low face values and low maturities. See Appendix B.1 for details on the issuance data.

Data on inflation refers to $\pi_t = \frac{P_t}{P_{t-1}}$ where P_t is the country's CPI and was obtained from WDI. A number of countries of our sample experienced hyperinflation during the 1980s and 1990s (for a detail of hyperinflation episodes see, for example, Hanke and Krus (2013)). We excluded data on inflation above 100 percent so that moments on inflation are not influenced by these extreme episodes. Data on nominal exchange rates refers to units of local currency per U.S. dollar and was obtained from Bloomberg. The real exchange rate was also computed vis-a-vis the US dollar, i.e. $RER_t = \frac{e_t P_t^*}{P_t}$, where P_t^* is the US CPI and P_t is the country's CPI.

B.2. *Estimation of exogenous processes*

In the full model of section 5 we assume that $y_{T,t}$ follows an AR(1) process in logs as specified in (14). This process is estimated with annual data on the cyclical component of tradable output for the period 1990-2014 for all countries in our sample. Since we have country level data, we need to estimate a dynamic panel version of (14) with country-fixed effects. We estimate this panel with OLS. These estimates may be subject to the bias identified in Nickell (1981). To assess whether this bias is important in our estimates, we also estimate individual processes for each country (which are not subject to any bias) and compare the estimated parameters from the dynamic panel with the histogram of estimates from country-specific regressions (see Figure B.2). As can be seen in the first two panels, the estimates of ρ_{y_T} and $\sigma^2_{y_T}$ are very close to the average estimates from country-specific regressions.

In the debt problem we assume that $(y_{T,t}, \hat{e}_t^{-1})$ follow a first-order VAR process in logs as specified in (15). Data for $\hat{e}_t^{-1}$ is obtained by computing the cyclical component of the inverse of the ratio between the nominal exchange rate and the lagged CPI, also for the period 1990-2014. Cyclical components were computed by estimating a log-linear trend.

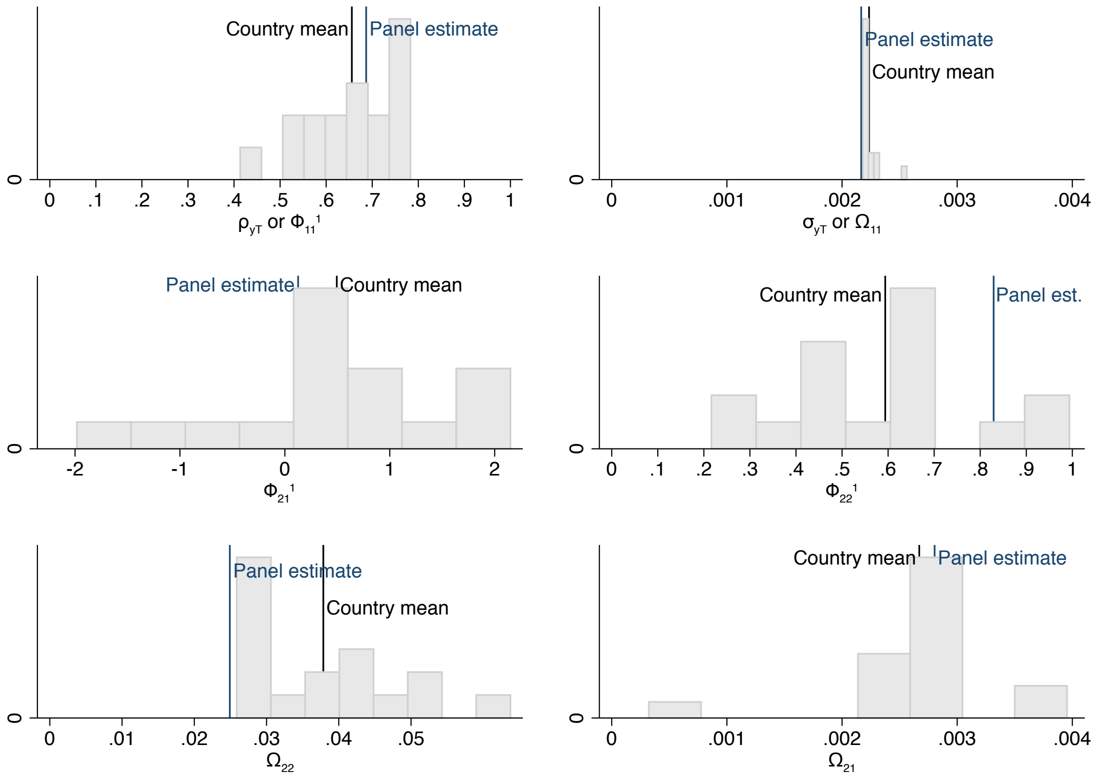
The following pooled-OLS estimates were obtained (including in the estimation country fixed effects):

$$\widehat{\Phi}^1 = \begin{bmatrix} .687 & .000 \\ .126 & .829 \end{bmatrix}, \ \widehat{\Omega} = \begin{bmatrix} .0021 & .0028 \\ .0028 & .0249 \end{bmatrix}.$$

Since the estimation procedure is pooled OLS, the same process for y_T is estimated in both the debt problem and full model. Consistent with the results in Table A3, exhibits an implicit negative relationship between the nominal exchange rate and output, as reflected by the positive entries on the off-diagonal elements of the covariance matrix $\widehat{\Omega}$.

We also perform country-specific VAR estimations and compare the estimated coefficients with those estimated in the panel VAR. The auto-regressive component of the inverse of the

Figure B.2. Country-Specific Exogenous Processes: Histogram of Estimated Parameters



Notes: These plots show the histograms of the each estimated coefficient in country-specific versions of the VAR (15). The average coefficient and the panel estimate are included in each graph. All regressions are estimated with OLS. The panel regression is estimated with pooled OLS. The first two panels also correspond to the country-specific estimation for the AR process for y_T (14).

exchange rate and the covariance of innovations are close the average of their country-specific estimates.

Appendix C. Additional Derivations and Numerical Solution (For Online Publication)

C.1. *Stationary problem*

Define $\hat{b} = \frac{b}{P_{-1}}$. Using (9), the resource constraint can be re-expressed as

$$\begin{aligned} c_T &= y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) P_{-1}\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[r\left(P', \frac{c_T'}{y_N}\right)(1+\delta q')\right]\left(\hat{b}' - \delta\frac{\hat{b}}{\pi}\right)P, \\ &= y_T - b^* - r\left(\pi, \frac{c_T}{y_N}\right)\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[r\left(\pi', \frac{c_T'}{y_N}\right)(1+\delta q')\right]\left(\hat{b}' - \delta\frac{\hat{b}}{\pi}\right). \end{aligned} \tag{23}$$

Similarly, we can de-trend the recursive expression for the price of debt in local currency

$$\begin{aligned} q &= \frac{1}{r\left(P, \frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[r\left(P', \frac{c_T'}{y_N}\right)(1+\delta q')\right], \\ &= \frac{1}{r\left(1, \frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[r\left(\pi', \frac{c_T'}{y_N}\right)(1+\delta q')\right]. \end{aligned}$$

Define $\hat{\mathbf{s}} = (b^*, \hat{b}, y)$ as the detrended state. We can then write the detrended version of problem (P1) as

$$V(\hat{\mathbf{s}}) = \max_{b^{*\prime}, \hat{b}', \pi, c_T} u(C(c_T, y_N)) - l(\pi) + \beta\mathbb{E}\left[V(\hat{\mathbf{s}}')\right] \tag{P1'}$$

subject to

$$c_T = y_T - b^* - r\left(\pi, \frac{c_T}{y_N}\right)\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}'\right)\right]\left(\hat{b}' - \delta\frac{\hat{b}}{\pi}\right).$$

The government takes as given $\mathcal{X}(\hat{\mathbf{s}})$ when solving the problem. In equilibrium this object solves the following fixed point

$$\mathcal{X}(\hat{\mathbf{s}}) = r\left(\pi, \frac{c_T}{y_N}\right)(1 + \delta q(\hat{\mathbf{s}})),$$

where the price of local currency is determined by the following recursive equation

$$q(\hat{\mathbf{s}}) = \frac{1}{r\left(1, \frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}'\right)\right]. \tag{24}$$

Similarly, we can then write the detrended version of problem (P2) as

$$V(\hat{\mathbf{s}},\epsilon^{\pi}) = \max_{b^{*\prime},\hat{b}^{\prime},\pi,c_T} u(C(c_T,y_N)) - l(\Pi(\hat{\mathbf{s}},\epsilon^{\pi},b^{*'},b^{'})) + \beta\mathbb{E}\left[V(\hat{\mathbf{s}}^{\prime})\right] \tag{P2'}$$

subject to

$$c_T = y_T - b^* - r\left(\Pi(\hat{\mathbf{s}},\epsilon^{\pi},b^{*'},b^{'},\frac{c_T}{y_N}\right)\hat{b} + \frac{1}{R-\delta}\left(b^{*\prime}-\delta b^*\right) + \frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right]\left(\hat{b}^{\prime}-\delta\frac{\hat{b}}{\pi}\right).$$

Governments take $\mathcal{X}(\hat{\mathbf{s}})$ as given. In equilibrium, it solves the following recursive structure

$$\mathcal{X}\left(\hat{\mathbf{s}}\right) = r\left(\Pi(\hat{\mathbf{s}},\epsilon^{\pi},b^{*'},b^{'}),\frac{c_T}{y_N}\right)(1+\delta q(\hat{\mathbf{s}})),$$

where the price of local currency is determined by the following recursive equation

$$q(\hat{\mathbf{s}}) = \frac{1}{r\left(1,\frac{c_T}{y_N}\right)}\frac{1}{R}\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right].$$

C.2. *Derivation of Euler Equations*

This subsection derives the Euler equations shown in section 4. We assume there is no uncertainty and $\delta = 0$, and derive one generalized Euler equation for each type of debt, that embeds the Euler equations for the case of dilution through inflation and the case of dilution through real exchange rate.

Define $\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}),\hat{\mathcal{B}}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{B}^*(\hat{b}_{t+1},b^*_{t+1})$ the expected consumption, inflation and debt policies in local and foreign currency, respectively. In an equilibrium, these expectations are consistent with optimal policies. Without loss of generality we set $y_N = 1$, and assume y_T is constant. The recursive government problem (P1') can be expressed as:

$$V(\hat{b}_t,b^*_t) = \max_{\hat{b}_{t+1},b^*_{t+1},c_t,\pi_t} u(C(c_{Tt},1)) - l(\pi_t) + \beta V(\hat{b}_{t+1},b^*_{t+1})$$

subject to

$$y_T - c_{Tt} + \frac{1}{R}r(\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1} - r(\pi_t,c_{Tt})\hat{b}_t + \frac{1}{R}b^*_{t+1} - b^*_t = 0,$$

The first-order conditions of this problem are given by:

$$[c_t]: \qquad u'(c_t)C_{c_T,t} = \lambda_t(1 + r_c(\pi_t, c_{Tt})\hat{b}_t),$$

$$[\pi_t]: \qquad -l'(\pi_t) = \lambda_t r_P(\pi_t, c_{Tt})\hat{b}_t,$$

$$[\hat{b}_{t+1}]: \qquad \beta V_{\hat{b}}(\hat{b}_{t+1}, b^*_{t+1}) = -\lambda_t \frac{1}{R}\frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}},$$

$$[b^*_{t+1}]: \qquad \beta V_{b^*}(\hat{b}_{t+1}, b^*_{t+1}) = -\lambda_t \frac{1}{R}\left[1 + \frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))}{\partial b^*_{t+1}}\hat{b}_{t+1}\right].$$

Envelope conditions:

$$[\hat{b}_t]: \qquad V_{\hat{b}}(\hat{b}_t, b^*_t) = -\lambda_t r(\pi_t, c_{Tt}),$$

$$[b^*_t]: \qquad V_{b^*}(\hat{b}_t, b^*_t) = -\lambda_t.$$

Combining these equations we get two Euler equations:

$$\frac{u'(c_t)C_{c_T,t}}{1 + r_c(\pi_t, c_{Tt})\hat{b}_t}\frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} = \beta R \frac{u'(c_{t+1})C_{c_T,t+1}}{1 + r_c(\pi_{t+1}, c_{Tt+1})\hat{b}_{t+1}} r(\pi_{t+1}, c_{Tt+1}), \quad (25)$$

$$\frac{u'(c_t)C_{c_T,t}}{1 + r_c(\pi_t, c_{Tt})\hat{b}_t}\left[1 + \frac{\partial r(\mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}))}{\partial b^*_{t+1}}\hat{b}_{t+1}\right] = \beta R \frac{u'(c_{t+1})C_{c_T,t+1}}{1 + r_c(\pi_{t+1}, c_{Tt+1})\hat{b}_{t+1}}. \quad (26)$$

The price sensitivity of debt can be calculated. Differentiating the resource constraint at $t+1$ with respect to $\hat{b}_{t+1}$ and b^*_{t+1}:

$$\begin{aligned}\frac{\partial \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1})}{\partial \hat{b}_{t+1}} =& \frac{1}{R}\frac{\partial r(\mathcal{C}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})), \mathcal{P}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})))\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}) + \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})}{\partial \hat{b}_{t+1}} \\ &- \frac{\partial r(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}}, \end{aligned} \quad (27)$$

$$\begin{aligned}\frac{\partial \mathcal{C}(\hat{b}_{t+1}, b^*_{t+1})}{\partial b^*_{t+1}} =& \frac{1}{R}\frac{\partial r(\mathcal{C}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})), \mathcal{P}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})))\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}) + \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})}{\partial b^*_{t+1}} \\ &- \frac{\partial r(\mathcal{C}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{P}(\hat{b}_{t+1}, b^*_{t+1}))\hat{b}_{t+1}}{\partial b^*_{t+1}} - 1. \end{aligned} \quad (28)$$

To simplify notation denote

$$\frac{\partial r(c_{Tt+2}, \pi_{Tt+2})\hat{b}_{t+2} + b^*_{t+2}}{\partial \hat{b}_{t+1}} \equiv$$

$$\frac{\partial r(\mathcal{C}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})), \mathcal{P}(\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}), \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})))\mathcal{B}(\hat{b}_{t+1}, b^*_{t+1}) + \mathcal{B}^*(\hat{b}_{t+1}, b^*_{t+1})}{\partial \hat{b}_{t+1}}.$$

and an analogous expression for the derivative with respect to b^*_{t+1}. Applying the chain rule to the second term of the right hand side of equations (27) and (28) yields

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} =& r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{C}_b(\hat{b}_{t+1},b^*_{t+1})\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1},b^*_{t+1}), \qquad (29)\\ \frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial b^*_{t+1}} =& \hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{C}_{b^*}(\hat{b}_{t+1},b^*_{t+1})\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_{b^*}(\hat{b}_{t+1},b^*_{t+1}). \qquad (30)\end{aligned}$$

With these equations we can derive the Euler equations. First we derive the Euler equation for debt in local currency. Combining (27) and (29) we get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} =& r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{Tt+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial \hat{b}_{t+1}}\\ &-\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}}\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1},b^*_{t+1}).\end{aligned}$$

We re-arrange the same equation to get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}} =& \frac{1}{1+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))}\Big[r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{Tt+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial \hat{b}_{t+1}}\\ &+\hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1},b^*_{t+1})\Big].\end{aligned}$$

Substituting this equation in (25) we get the modified Euler equation for debt in local currency

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1} \underbrace{\left(1+r_{c,t}\hat{b}_t\right)}_{\text{Dilution through RXR}} \frac{1}{\underbrace{1+\frac{\hat{b}_{t+1}\left(r_{c,t+1}\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{t+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial \hat{b}_{t+1}}+r_{P,t+1}\pi_b(\hat{b}_{t+1},b^*_{t+1})\right)}{r(\pi_{t+1},c_{Tt+1})}}_{\text{Discipline Effect}}}. \qquad (31)$$

Now we derive the Euler equation for debt in foreign currency. We follow a similar approach as before. Combining (28) and (30) we get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1}),b^*_{t+1})\hat{b}_{t+1}}{\partial b^*_{t+1}} =& + \hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{Tt+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial b^*_{t+1}}\\ &- \hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1}),b^*_{t+1})\hat{b}_{t+1}}{\partial b^*_{t+1}}\\ &- \hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+ \hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_b(\hat{b}_{t+1},b^*_{t+1}).\end{aligned}$$

We re-arrange the same equation to get

$$\begin{aligned}\frac{\partial r(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1}),b^*_{t+1})\hat{b}_{t+1}}{\partial b^*_{t+1}} =& \frac{1}{1+\hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))}\Big[- \hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\\ &+ \hat{b}_{t+1}r_c(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{Tt+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial b^*_{t+1}}\\ &+ \hat{b}_{t+1}r_P(\mathcal{C}(\hat{b}_{t+1},b^*_{t+1}),\mathcal{P}(\hat{b}_{t+1},b^*_{t+1}))\mathcal{P}_{b^*}(\hat{b}_{t+1},b^*_{t+1})\Big].\end{aligned}$$

Substituting this equation in (26) we get the modified Euler equation for debt in foreign currency

$$u'(c_t)C_{c_T,t} = \beta R u'(c_{t+1})C_{c_T,t+1}\underbrace{\left(1+r_{c,t}\hat{b}_t\right)}_{\text{Dilution thr. RXR}}\underbrace{\frac{1}{1+\hat{b}_{t+1}\left(r_{c,t+1}\frac{1}{R}\frac{\partial r(c_{Tt+2},\pi_{t+2})\hat{b}_{t+2}+b^*_{t+2}}{\partial b^*_{t+1}}+r_{P,t+1}\pi_{b^*,t+1}\right)}}_{\text{Discipline Effect}}. \tag{32}$$

C.3. *Solution Method*

We solve for equilibrium using a global numerical method that combines value function iteration and policy function iteration. Solving the model implies finding policy functions $\{b^{*\prime}(\hat{\mathbf{s}}), \hat{b}'(\hat{\mathbf{s}}), \pi(\hat{\mathbf{s}})\}$ that solve (P1'). The algorithm to solve for the policies numerically follows these steps:

(1) Generate a discrete grid for variable x state space $G_x = x_1, x_2, ..., x_{N_x}$, for $x = y_T, b^*, \hat{b}$. The total aggregate state space is given by $S = G_{y_T} \times G_{b^*} \times G_{\hat{b}}$.
(2) Conjecture a multi-dimensional object $EQ(b^{*\prime}, \hat{b}', y_T)$ as a guess for $\mathbb{E}\left[\mathcal{X}(\hat{\mathbf{s}}')\right]$, which is the expectation term associated to the price of debt in local currency.
(3) Solve for tradable consumption $c_T(\hat{\mathbf{s}}, b^{*\prime}, \hat{b}', \pi)$ using the resource constraint (23). This is a non-linear equation in consumption since it also appears in the exchange rate expression.
(4) Solve problem (P1') using value function iteration method. To achieve numerical accuracy we solve in finer grids and use numerical optimizers. Once the maximum was over

the finer grids was identified, we use a numerical optimizer routine to find the maximum in a continuous neighborhood around the initially identified point. We use quadrature methods to compute all expectations and piecewise linear interpolation to interpolate policies outside the grids.

(5) Compute $q(\hat{\mathbf{s}}, b^{*\prime}, \hat{b}^{\prime})$ using (24). Then compute $\mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right]$.

(6) If $\sup_{\hat{\mathbf{s}}} \left\| EQ(b^{*\prime}, \hat{b}^{\prime}, y_T) - \mathbb{E}\left[\mathcal{X}\left(\hat{\mathbf{s}}^{\prime}\right)\right] \right\| < \epsilon$, for small ϵ stop. Otherwise, update the guess EQ and start from the first point again.

Appendix D. Robustness and Extensions (For Online Publication)

D.1. *The Role of Debt Maturity and Risk Aversion*

In this section we study how our results are affected by the degree of risk aversion of households and the maturity of debt. The degree of risk aversion can affect the trade-off associated to the currency composition of debt since it determines how valuable are the hedging properties of debt in local currency. We solve the model with a lower degree of risk aversion and find that, consistent with the above-mentioned argument, the share of debt in local currency is smaller than in the baseline model (see Table D1).

Table D1. Sensitivity Analysis: Risk Aversion and Bond Maturity

Moment	Data	Baseline ($\sigma = 5$, $\delta = 0.76$)	Low Risk Av. ($\sigma = 2$, $\delta = 0.76$)	Short Term Debt ($\sigma = 5$, $\delta = 0$)
Average Levels				
Debt	22.0%	22.0%	22.7%	19.8%
Share of Debt in LC	24.7%	8.8%	5.4%	6.0%
Inflation	8.7%	8.5%	8.5%	8.5%
Standard deviation				
Debt	4.2%	3.1%	3.0%	2.9%
Debt in LC - Debt in FC	2.3%	2.9%	2.8%	2.8%
Inflation	3.9%	0.1%	0.1%	0.1%
Exchange Rate	13.2%	0.5%	0.5%	0.7%
GDP	3.3%	2.2%	2.2%	2.2%
Correlations with GDP				
Debt	-55.5%	-50.3%	-62.4%	-56.3%
Debt in LC - Debt in FC	50.0%	35.0%	47.7%	43.2%
Inflation	7.4%	3.0%	5.5%	-2.5%
Exchange Rate	-50.6%	-73.0%	-74.7%	-75.9%

Notes: The column *Data* refers to average moments for the sample countries detailed in Table 1, using annual data, for the period 1990-2014. For countries in which the 1990 data for a given variable was not available, we consider the earliest data available for that variable. Debt refers to external public debt over GDP; the average share of debt in LC refers to the ratio between external public debt in local currency to total external public debt; to measure the standard deviation and correlation with GDP of the debt currency composition we use an alternative measure (Debt in LC - Debt in FC) computed as the difference between external public debt in local currency over GDP and external public debt in foreign currency over GDP. Average inflation was computed excluding observations with inflation rates above 100 percent. For details, see Data Appendix B. Standard deviations and correlations with GDP were computed using the cyclical component of each variable, using HP filter (smoothing parameter 100). The last three columns refer to the moments of simulations of models with alternative parameterizations. The column *Baseline* refers to the simulations of the baseline model. The columns *Low Risk Av.* and *Short Term Debt* refer to the simulations of the of models with $\sigma = 2$ and $\delta = 0$, respectively. In these two alternative calibrations, we leave all remaining parameters as in the baseline model with the exception of the discount factor that is recalibrated to match the level of total debt and the inflation cost parameter that is recalibrated to deliver the targeted welfare costs of inflation.

The maturity of debt can affect the quantitative relevance of incentive problems since it determines the extent to which the government can spread the inflation costs of dilution over time (Cochrane (2001)), and can also affect the level of debt in local currency necessary to attain certain degree of hedging.[21] We solve the model with short-term debt and find that the share of debt in local currency is lower than in the baseline model (see Table D1), pointing to the fact that the reduction in the scale of debt in local currency to attain certain hedging predominates over the inability of the government to smooth its inflationary indiscipline over time.

D.2. *Models with Detailed Inflation Costs*

In this section we show that models that feature cash-in-advance constraints or money in the utility function can give rise inflation losses featured as a decreasing function in the household's preferences. We also show that under certain functional form assumptions these inflation losses show up as a separable function, similar to our baseline specification. We embed the cash-in-advance and money in the utility function specifications in the context of our model of currency composition of sovereign debt to maintain the parallelism with our baseline model as close as possible.

D.2.1. *Cash-in-advance model*

Consider a variant of our baseline economy in which there are three goods: tradable and non-tradable goods, and the cash good. The cash good is produced with labor with production function $c_{Xt} = n_t$, where c_{Xt} denotes household's consumption of the cash good and n_t the labor supplied by households. We adopt the Svensson (1985) timing convention by which money brought from the previous period can only be used to purchase the cash good. This specification gives rise to inefficiencies due to realized inflation. Consuming the cash good yields utility $v(c_{Xt})$ which is increasing and concave, and labor entails disutility $g(n_t) = \chi n_t$. The household problem now involves choices of money holdings and labor in addition to intra-temporal consumption:

$$\max_{c_{Tt},c_{Nt},c_{Xt},n_t,M_t} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt},c_{Nt})) + v\left(c_{Xt}\right) - \chi n_t\right]$$

[21]The flow repayment of debt (as opposed to the stock) is what determines the scale of the hedging. Hence, the necessary stock of domestic debt to attain a given degree of hedging is increasing in the maturity of debt.

subject to

$$p_{Xt}c_{Xt} + e_t c_{Tt} + p_{Nt}c_{Nt} + M_t = p_{Xt}n_t + e_t y_{Tt} + p_{Nt}y_{Nt} + T_t + M_{t-1} \tag{33}$$

$$p_{Xt}c_{Xt} \leq M_{t-1} \tag{34}$$

given M_0. We implicitly imposed that wages are equal to prices using zero-profits for firms in the cash good sector. The first order conditions of this problem are given by:

$$\begin{aligned} u'(c_t) &= \lambda_t P_t, \\ v'(c_{Xt}) &= (\lambda_t + \mu_t)p_{Xt}, \\ \chi &= \lambda_t p_{Xt}, \\ \lambda_t &= \mathbb{E}[\beta(\lambda_{t+1} + \mu_{t+1})], \end{aligned}$$

where $c_t = C(c_{Tt}, c_{Nt})$ is the consumption aggregator, P_t is the ideal price index, and λ_t and μ_t are the Lagrange multipliers of the budget constraint (33) and the cash-in-advance constraint (34). We focus on the case in which the cash-in-advance constraint is binding. Define the surplus utility for the cash good as $\tilde{v}(c_{Xt}) \equiv v(c_{Xt}) - \chi c_{Xt}$. Imposing market clearing in the cash good and the non-tradable good we can express the household utility as

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, y_N)) + \tilde{v}\left(\frac{m_{t-1}}{\pi_t}\frac{u'(c_t)}{\chi}\right)\right],$$

where $m_t \equiv \frac{M_t}{P_t}$ are the real balances and their law of motion satisfies

$$\mathbb{E}\left[\beta\tilde{v}'\left(\frac{m_t}{\pi_{t+1}}\frac{u'(c_{t+1})}{\chi}\right)\frac{1}{p_{Xt}}\right] = \frac{u'(c_t)}{P_t} - \mathbb{E}\left[\beta\frac{u'(c_{t+1})}{P_{t+1}}\right].$$

This expression is decreasing in π_t. Hence, inflation shows up as a loss in the utility function.

If we further assume that $v(c_{xt}) = \nu \log(c_{xt}) + \chi c_{xt}$ then we get the following expression for utility in which inflation enters as a separable negative cost (like in our baseline model)

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, y_N)) - \nu\log(\pi_t) + \nu\log\left(\frac{m_{t-1}u'(c_t)}{\chi}\right)\right]$$

where the law of motion of m_t satisfies

$$m_t = \frac{\beta\nu}{u'(c_t) - \mathbb{E}\left[\beta\frac{u'(c_{t+1})}{\pi_{t+1}}\right]}.$$

D.2.2. *Model with Preferences for Real Money Balances*

Consider another variant of our baseline economy in which households have preferences for real money balances, with the equivalent timing as the cash-in-advance model. The household problem now involves choices of money holdings and consumption:

$$\max_{c_{Tt}, c_{Nt}, M_t} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, c_{Nt})) + v\left(\frac{M_{t-1}}{P_t}\right)\right]$$

subject to

$$p_{Xt}c_{Xt} + e_t c_{Tt} + M_t = e_t y_{Tt} + p_{Nt} y_{Nt} + T_t + M_{t-1} \tag{35}$$

given M_0, where $v(\cdot)$ is an increasing and concave function. It follows directly from the definition of real balances that utility is decreasing in realized inflation π_t. Additionally, for the particular case of $v\left(\frac{M_{t-1}}{P_t}\right) = \nu \log\left(\frac{M_{t-1}}{P_t}\right)$ we can show that realized inflation enters utility in the following separable decreasing function

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C(c_{Tt}, y_N)) - \nu \log(\pi_t) + \nu \log(m_{t-1})\right]$$

where the law of motion of m_t satisfies

$$m_t = \frac{\beta \nu}{u'(c_t) - \mathbb{E}\left[\beta \frac{u'(c_{t+1})}{\pi_{t+1}}\right]}.$$

Note that the model with cash in advance and money in the utility function are symmetric with the only difference that in the cash in advance model real balances are defined over the price of the cash good instead of the ideal price index.

D.2.3. *Quantitative Analysis of the Model with Preferences for Real Balances*

In this section we calibrate and analyze the quantitative predictions of the model with money in the utility function. We find that this model delivers quantitative results that are in line with those of the baseline model.

We adopt the log specification for utility associated to real balances. The recursive problem of the government is given by

$$V(b^*, b, m_{-1}, y_T) = \max_{b^{*\prime}, b', \pi, c_T} u(C(c_T, y_N) + \nu \log(m_{-1}) - \nu \log(\pi) + \beta \mathbb{E}\left[V(\mathbf{s}')\right]$$

subject to

$$
\begin{aligned}
c_T &= y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) b + \frac{1}{R-\delta}\left(b^{*\prime} - \delta b^*\right) + \tilde{q}\left(b' - \delta b\right), \\
\tilde{q} &= \frac{1}{R}\mathbb{E}\left[\mathcal{R}(\mathbf{s}') + \delta \tilde{q}(\mathbf{s}')\right], \\
m &= \frac{\beta\nu}{u'(c) - \mathbb{E}\left[\beta \frac{u'(c')}{\pi'}\right]} \\
P &= \pi P_{-1}.
\end{aligned}
$$

There are two main differences with the baseline specification. First, the losses of inflation are concave here and convex in the baseline specification. Second, there is an extra term $\nu \log(m_{-1})$ that enters utility which the government takes into account when choosing consumption. Next we argue that the presence of this new term is not important quantitatively for the optimal choices of the government.

To calibrate the model we set the same parameter values as in the baseline specification with the exception of β, which we calibrate to match the total level of external public debt. The calibrated value of the discount factor is $\beta = 0.96$. The new parameter is ν which governs the preferences for real balances. We follow a symmetric approach as in our baseline model and calibrate ν to obtain the same welfare loss of inflation as in the baseline model. That is, we calibrate ν so that a increase in inflation of 1 percent has associated a loss of 0.1 percent in consumption equivalent terms. The calibrated value is $\nu = 0.002$.

We then simulate data from this model and compare the moments from the simulated data with those of the data and the baseline model. Results are shown in the third column of Table D3. The main quantitative results of the model with money in the utility function are in line with those of the baseline model. In particular, the share of debt in local currency is 9 percent compared to 10 percent in the baseline model. The remaining moments are also similar, including the rate of inflation. This finding suggest that we do not loose generality by focusing on the baseline model in which inflation costs enter in a reduced-form way.

D.3. *Model with Costs of Currency Depreciation*

Consider a variation of the baseline model in which the losses come from fluctuations in the nominal exchange rate, $l(\Delta e_t)$, where $\Delta e_t \equiv \frac{e_t}{e_{t-1}}$. As in section 4.2, we focus on the case of short-term debt ($\delta = 0$). Without loss of generality we set $y_N = 1$, and assume y_T is constant.

The recursive government problem can be expressed as:

$$V(\hat{b}_t, b_t^*) = \max_{\hat{b}_{t+1}, b_{t+1}^*, c_t, \Delta e_t} u(C(c_{Tt}, 1)) - l(\Delta e_t) + \beta V(\hat{b}_{t+1}, b_{t+1}^*)$$

subject to

$$y_T - c_{Tt} + \frac{1}{R} r(\mathcal{D}(\hat{b}_{t+1}, b_{t+1}^*))\hat{b}_{t+1} - r(\Delta e_t)\hat{b}_t + \frac{1}{R} b_{t+1}^* - b_t^* = 0,$$

where $r(\Delta e_t) = \frac{1}{\Delta e_t}$ is the repayment function (which now only depends on the depreciation rate), and $\mathcal{D}(\hat{b}_{t+1}, b_{t+1}^*)$ is the expected nominal currency depreciation. The first-order conditions of this problem are given by:

$$[c_t]: \quad u'(c_t) C_{c_T,t} = \lambda_t,$$

$$[\pi_t]: \quad -l'(\Delta e_t) = \lambda_t r'(\Delta e_t)\hat{b}_t,$$

$$[\hat{b}_{t+1}]: \quad \beta V_{\hat{b}}(\hat{b}_{t+1}, b_{t+1}^*) = -\lambda_t \frac{1}{R} \frac{\partial r(\mathcal{D}(\hat{b}_{t+1}, b_{t+1}^*))\hat{b}_{t+1}}{\partial \hat{b}_{t+1}},$$

$$[b_{t+1}^*]: \quad \beta V_{b^*}(\hat{b}_{t+1}, b_{t+1}^*) = -\lambda_t \frac{1}{R} \left[1 + \frac{\partial r(\mathcal{D}(\hat{b}_{t+1}, b_{t+1}^*))}{\partial b_{t+1}^*} \hat{b}_{t+1}\right].$$

Envelope conditions:

$$[\hat{b}_t]: \quad V_{\hat{b}}(\hat{b}_t, b_t^*) = -\lambda_t r(\Delta e_t),$$

$$[b_t^*]: \quad V_{b^*}(\hat{b}_t, b_t^*) = -\lambda_t.$$

Comparing these optimality conditions with those of the model with inflation costs (baseline model) we can see that the trade-off are similar. In particular the optimal depreciation is countercyclical and the optimal choice of debt take into account disciplining effects. In addition, in the particular case of an infinite cost of depreciation the nominal exchange rate is fixed and the both assets are payoff-equivalent, which leads to portfolio indeterminancy.

D.4. *Model with Outright Default*

In this section we extend our baseline model to allow for outright default. We assume that every period the government can choose to repay or default on its debt. We assume that the default decision applies to all debt regardless of its currency of denomination. We follow most quantitative models of default and assume that following a default the government faces a stochastic number of periods during which it is excluded from credit markets. It regains access to credit markets with probability $\theta \in (0,1)$ every period. We follow Arellano (2008) and assume that in those periods in which the country is in autarky the level of tradable output is given by[22]

$$y_{Tt}^{def} = \begin{cases} \hat{y} & \text{if } y_{Tt} > \xi\mathbb{E}[y_T] \\ y & \text{if } y_{Tt} \leq \xi\mathbb{E}[y_T]. \end{cases}$$

Denote $\mathbf{s} = \{b^*, b, y_T, P_{-1}\}$ the aggregate state, and $q(\mathbf{s}, b^{*\prime}, b)$ and $q(\mathbf{s}, b^{*\prime}, b)$ the price schedules of debt in foreign and local currency, respectively, both expressed in foreign currency. The government's problem written in recursive form is given by:

$$V(\mathbf{s}) = \max_{\iota \in \{0,1\}} \iota V^r(\mathbf{s}) + (1-\iota)V^a(y_T, P_{-1}).$$

The value of repaying is given by:

$$V^r(\mathbf{s}) = \max_{b^{*\prime}, b^\prime, \pi, c_T} \{u\left(C(c_T, y_N)\right) - l\left(\pi\right)\} + \beta\mathbb{E}\left[V(\mathbf{s}^\prime)\right]$$

subject to

$$c_T = y_T - b^* - r\left(P, \frac{c_T}{y_N}\right) b + q^*(\mathbf{s}, b^{*\prime}, b)\left(b^{*\prime} - \delta b^*\right) + \tilde{q}(\mathbf{s}, b^{*\prime}, b)\left(b^\prime - \delta b\right),$$

$$P = \pi P_{-1}.$$

The value of default (or being in autarky) is given by:

$$V^d(y_T, P_{-1}) = u\left(C(y_T^{def}, y_N)\right) - l\left(\pi^*\right) + \beta\mathbb{E}\left[\theta V(0, 0, y_T^\prime, P) + (1-\theta)V^d(y_T^\prime, P)\right]$$

Note that we are assuming that the government re-enters with zero debt to markets. We are already imposing that the optimal inflation rate while in autarky is π^*. This is because there are no incentives to incur in costly inflation since there is no debt to dilute. Finally, in equilibrium risk-neutral investors obtain an expected return of R and debt prices satisfy the following recursive expressions

$$q^*(\mathbf{s}, b^{*\prime}, b) = \frac{1}{R}\mathbb{E}\left[\iota(1 + \delta\mathcal{Q}^*(\mathbf{s}^\prime))\right],$$

$$\tilde{q}(\mathbf{s}, b^{*\prime}, b) = \frac{1}{R}\mathbb{E}\left[\iota(\mathcal{R}(\mathbf{s}^\prime) + \delta\tilde{\mathcal{Q}}(\mathbf{s}^\prime))\right],$$

where $\mathcal{R}(\mathbf{s}), \mathcal{Q}^*(\mathbf{s}), \tilde{\mathcal{Q}}(\mathbf{s})$ are the inverse of the nominal exchange rate and prices of debt in foreign and local currency evaluated in the optimal policies.

We calibrate the model using the same functional forms and parameter values as in the baseline model. This specification introduces two new parameters: the re-entry probability

[22]The assumption that a default triggers a drop in y_T results in a real exchange rate depreciation after a default, a common feature of the data.

θ and the parameter that governs the output cost of default ξ. We follow Chatterjee and Eyigungor (2012) and set $\theta = 0.15$ so that the average length of the exclusion period is 6.5 years. We calibrate β and ξ so that we match the average level of total debt and obtain a frequency of default of 3.5 percent, which is in the range of frequencies targeted in quantitative models of default. The calibrated values are $\beta = 0.94$ and $\xi = 0.82$.

Table D3 shows the moments associated to the model with default, compared to the moments from the data and from the baseline model. The main quantitative results remain in the model with default. The average share of debt in local currency is 14.6 percent, which is close to the 8.8 percent share in the baseline model and the 24.7 percent in the data. The remaining models are also very close to those of the baseline model.

Table D2. Models with Real Money Balances and Default: Results

Moment	Data	Baseline Model	Model with Money	Model with Outright Default
Average Levels				
Debt	22.0%	22.0%	13.6%	13.3%
Share of Debt in LC	24.7%	8.8%	8.9%	14.6%
Inflation	8.7%	8.5%	7.3%	8.5%
Standard deviation				
Debt	4.2%	3.1%	1.8%	2.4%
Debt in LC - Debt in FC	2.3%	2.9%	1.7%	6.3%
Inflation	3.9%	0.1%	0.5%	0.4%
Exchange Rate	13.2%	0.5%	0.7%	1.7%
Real Exchange Rate	12.6%	0.4%	0.5%	1.5%
GDP	3.3%	2.2%	2.2%	2.3%
Correlations with GDP				
Debt	-55.5%	-50.3%	-47.2%	-57.7%
Debt in LC - Debt in FC	50.0%	35.0%	38.2%	11.2%
Inflation	7.4%	3.0%	4.0%	-11.0%
Exchange Rate	-50.6%	-73.0%	-53.0%	-68.3%
Real Exchange Rate	-54.1%	-76.9%	-76.2%	-76.0%

Notes: The column *Data* refers to average moments for the sample countries detailed in Table 1, using annual data, for the period 1990-2014. For countries in which the 1990 data for a given variable was not available, we consider the earliest data available for that variable. Debt refers to external debt over GDP; the average share of debt in LC refers to the ratio between debt in local currency to total debt; to compute the standard deviation and correlation with GDP of the share of debt in local currency, we measure this variable as the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP. Average inflation was computed excluding observations with inflation rates above 100 percent. For details, see Data Appendix B. Standard deviations and correlations with GDP were computed using the cyclical component of each variable, using HP filter (smoothing parameter 100).

Table D3. Models with Real Money Balances and Default: Results

Moment	Data	Baseline Model	Model with Money	Model with Outright Default
Average Levels				
Debt	22.0%	22.6%	22.6%	22.1%
Share of Debt in LC	24.7%	9.4%	8.9%	14.6%
Inflation	8.7%	8.7%	7.3%	8.7%
Standard deviation				
Debt	4.2%	3.1%	3.1%	4.0%
Debt in LC - Debt in FC	2.3%	3.0%	2.9%	10.5%
Inflation	3.9%	0.1%	0.5%	0.4%
Exchange Rate	13.2%	0.5%	0.7%	1.7%
Real Exchange Rate	12.6%	0.4%	0.5%	1.5%
GDP	3.3%	2.2%	2.2%	2.3%
Correlations with GDP				
Debt	-55.5%	-50.7%	-51.2%	-59.8%
Debt in LC - Debt in FC	50.0%	34.6%	42.1%	11.8%
Inflation	7.4%	1.8%	4.0%	-11.0%
Exchange Rate	-50.6%	-73.4%	-53.0%	-68.3%
Real Exchange Rate	-54.1%	-77.6%	-76.2%	-76.0%

Notes: The column *Data* refers to average moments for the sample countries detailed in Table 1, using annual data, for the period 1990-2014. For countries in which the 1990 data for a given variable was not available, we consider the earliest data available for that variable. Debt refers to external debt over GDP; the average share of debt in LC refers to the ratio between debt in local currency to total debt; to compute the standard deviation and correlation with GDP of the share of debt in local currency, we measure this variable as the difference between debt denominated in local currency over GDP and debt denominated in foreign currency over GDP. Average inflation was computed excluding observations with inflation rates above 100 percent. For details, see Data Appendix B. Standard deviations and correlations with GDP were computed using the cyclical component of each variable, using HP filter (smoothing parameter 100).

The column *Data* refers to average moments for the sample countries detailed in Table 1, using annual data, for the period 1990-2014. For countries in which the 1990 data for a given variable was not available, we consider the earliest data available for that variable. Debt refers to external public debt over GDP; the average share of debt in LC refers to the ratio between external public debt in local currency to total external public debt; to measure the standard deviation and correlation with GDP of the debt currency composition we use an alternative measure (Debt in LC - Debt in FC) computed as the difference between external public debt in local currency over GDP and external public debt in foreign currency over GDP. Average inflation was computed excluding observations with inflation rates above 100 percent. For details, see Data Appendix B. Standard deviations and correlations with GDP were computed using the cyclical component of each variable, using HP filter (smoothing parameter 100). The column *Baseline Model* reports moments of the simulations of the baseline model. The column *Model with Money* reports moments from the model with money in the utility function. The last column reports the moments from the model with outright default.