Aggregating Distortions in Networks with Multi-Product Firms

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Motivation

- Aggregate productivity determined by technological and allocative efficiency
- Large literature decomposes aggregate TFP into resource misallocation and a technological residual
- Standard assumption is that each firm produces a single product
- In the data, most transactions involve multiproduct firms (e.g., 99% in Chile)

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- In the data, most transactions involve multiproduct firms (e.g., 99% in Chile)
- With multiproduct firms, the allocation of resources across products within firms matters
- Assigning inputs to outputs within firms is hard or even impossible (e.g., oil refinery)

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- Assigning inputs to outputs within firms is hard or even impossible (e.g., oil refinery)

How do multiproduct firms with joint (non-separable) production technology change the measurement of misallocation?

This Paper

Framework for measuring allocative efficiency with multi-product firms and joint production.

- Joint production captures how easy it is for firms to adjust their product mix (e.g., gas/diesel)
- Finding: technological barriers in adjusting product mix attenuate reallocation effects on TFP
- Develop sufficient stats using product level prices to capture curvature of firm-level PPF

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Framework for measuring allocative efficiency with multi-product firms and joint production.

- Joint production captures how easy it is for firms to adjust their product mix (e.g., gas/diesel)
- Finding: technological barriers in adjusting product mix attenuate reallocation effects on TFP
- Develop sufficient stats using product level prices to capture curvature of firm-level PPF

Quantify framework using firm-product level transactions and prices in Chile

- Conduct aggregate growth accounting to decompose aggregate TFP growth into allocative and technical efficiency given past observed data
- Estimate curvature of firms' PPF and use it to conduct counterfactuals
- Ignoring joint production results in overestimation of reallocation effects

Related Papers

- Multi-product firms & Joint production
  - Multi-product firms: Klette and Kortum (2004); Bernard et al. (2010, 2011); Mayer et al. (2014); De Loecker et al. (2016); Hottman et al. (2016); Mayer et al. (2021); Wang and Yang (2023)
  - Joint production: Diewert (1971); Lau (1972); Hall (1973); Dhyne et al. (2017); Almunia et al. (2021); Dhyne et al. (2022); Ding (2023); Carrillo et al. (2023)
  - Study misallocation with multi-product firms that engage in joint production
  - Provide empirical evidence consistent with joint production
- Aggregation & (In)efficiency in production networks
  - Hulten (1978); Basu and Fernald (2002); Restuccia and Rogerson (2008); Petrin and Levinsohn (2012); Jones (2013); Liu (2019); Baqaee and Farhi (2020); Bigio and La'O (2020); Kikkawa (2022); Baqaee et al. (2023)
  - Generalize theory to allow for joint production and quantify using firm-to-firm product level data

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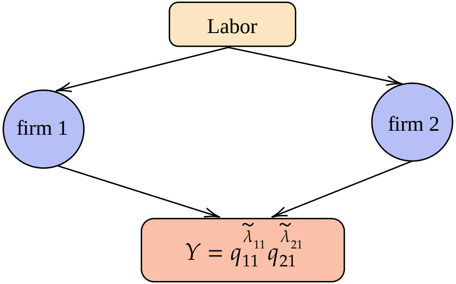
Illustrative Example

- Demonstrate that aggregate TFP can change without technological progress
- Illustrate how the change in TFP varies with and without joint production
- Show how sufficient statistics work
- Much of the intuition from the simple example survives in general setup

Illustrative Example with Single Product Firms

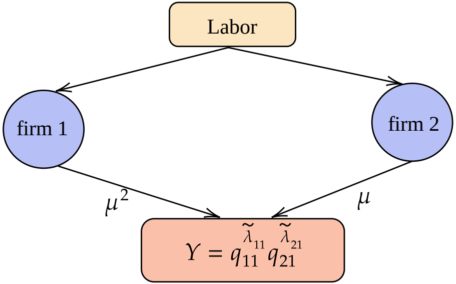


- First index: firm. Second index: each firm's product
- Production functions: $q_{11} = L_{11}, \; q_{21} = L_{21}$
- Each firm sets its price as $p_{i1} = \mu_{i1} mc_{i1}$, where μ_{i1} is exogenous wedge (markup)
- Household maximizes $Y = q_{11}^{\tilde{\lambda}_{11}} q_{21}^{\tilde{\lambda}_{21}}$ subject to

$$\sum_{i=1}^{2} p_{i1} q_{i1} = wL + \sum_{i=1}^{2} (1 - 1/\mu_{i1}) p_{i1} q_{i1},$$

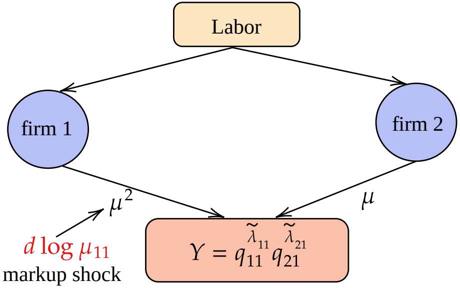
- Market clearing condition: $L = L_{11} + L_{21}$
- $TFP = Y/L$

Misallocation with Single Product Firms



- Firm 1 is underproduced due to higher wedge

Misallocation with Single Product Firms



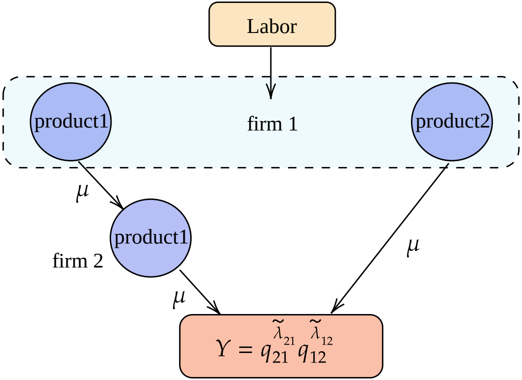
- First order TFP response to markup change:

$$\Delta \log TFP = \tilde{\lambda}_{11} \underbrace{\left(\frac{\bar{\mu}}{\mu_{11}} - 1 \right)}_{<0} d \log \mu_{11}$$

where $\bar{\mu} = \left(\tilde{\lambda}_{11} \mu_{11}^{-1} + \tilde{\lambda}_{21} \mu_{21}^{-1} \right)^{-1}$

- An increase in wedges for products with higher wedges decreases TFP

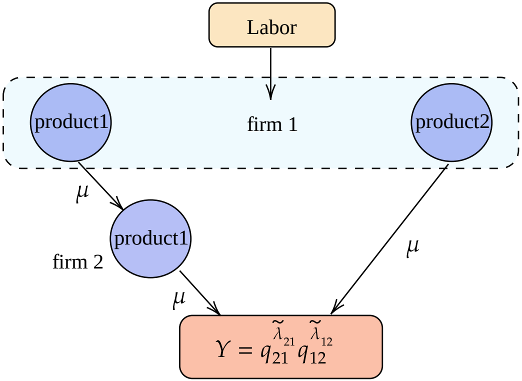
Now with Multi-Product Firms but not Joint Production



- Introduce multi-product firms and downstream firms:
- Production functions:
  $q_{11} = L_{11}, \;\; q_{12} = L_{12}, \;\; q_{21} = q_{11}$
- Household: $Y = q_{21}^{\tilde{\lambda}_{21}} q_{12}^{\tilde{\lambda}_{12}}$
- Market clearing:

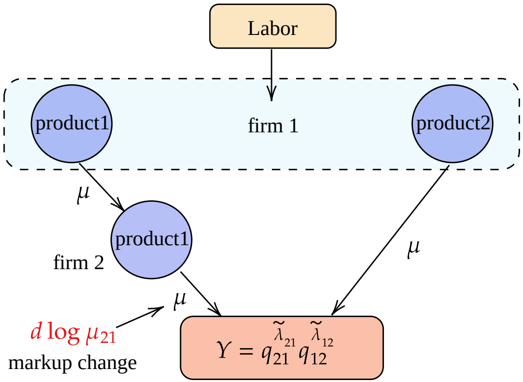
$$L = L_{11} + L_{12}$$

Misallocation with Multi-Product Firm



- Product 1 is underproduced due to downstream wedge
- Define $\Gamma_{11} = \mu^2$ and $\Gamma_{12} = \mu$ to be cumulative wedges
- Define $\tilde{\lambda}_{11} = \tilde{\lambda}_{21}$ to capture the (indirect) cost effect of firm 1's product 1 on final demand

Misallocation with Multi-Product Firm



- Cumulative wedges: $\Gamma_{11} = \mu^2$ and $\Gamma_{12} = \mu$
- First order TFP response to markup change:

$$\Delta \log TFP = \tilde{\lambda}_{21} \underbrace{\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1 \right)}_{<0} d \log \mu_{21}$$

where $\bar{\Gamma}_1 = \left(\tilde{\lambda}_{11} \Gamma_{11}^{-1} + \tilde{\lambda}_{12} \Gamma_{12}^{-1} \right)^{-1}$

- An increase in cumulative wedges for products with high cumulative wedges decreases TFP

Production Possibility Frontier with Joint Production

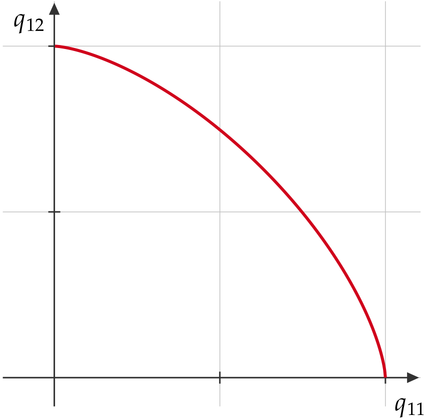
Multiproduct firms use shared input to make multiple outputs

(Diewert (1971); Lau (1972); Hall (1973)) details

- Constant-Elasticity of Transformation (CET)

$$\underbrace{\left(q_{11}^{\frac{\sigma+1}{\sigma}}+q_{12}^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}}}_{\text{output bundle}}=L$$

- σ is elasticity of transformation



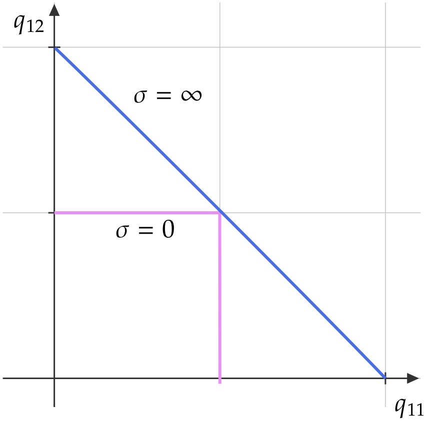
Production Possibility Frontier with Joint Production

Multiproduct firms use shared input to make multiple outputs
(Diewert (1971); Lau (1972); Hall (1973))

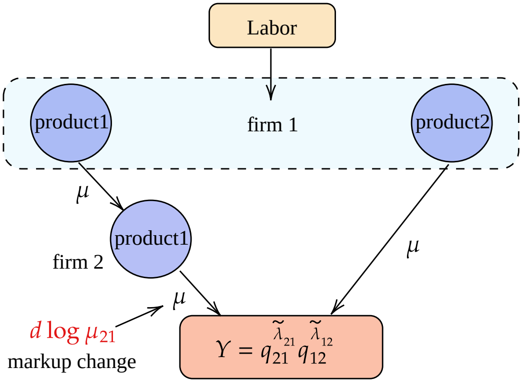
- Constant-Elasticity of Transformation (CET)

$$\underbrace{\left(q_{11}^{\frac{\sigma+1}{\sigma}} + q_{12}^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}}}_{\text{output bundle}} = L$$

- $\sigma \to \infty$: separable production
- $\sigma \to 0$: outputs can only be produced in a constant ratio



Misallocation with Joint Production



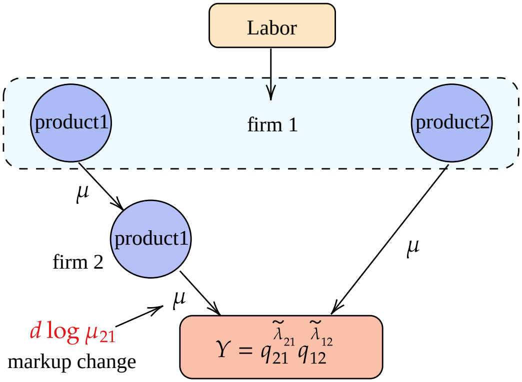
$$\left(q_{11}^{\frac{\sigma+1}{\sigma}} + q_{12}^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}} = L$$

- First order TFP response to the same markup change:

$$\Delta \log TFP = \left(1 - \frac{1}{\sigma+1}\right) \tilde{\lambda}_{21} \left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d \log \mu_{21}$$

depends on ease of adjusting product mix, σ

Misallocation with Joint Production



$$\left(q_{11}^{\frac{\sigma+1}{\sigma}} + q_{12}^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}} = L$$

- First order TFP response to the same markup change:

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depends on ease of adjusting product mix, σ

$$\Delta \log TFP = \begin{cases} \tilde{\lambda}_{21} \left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d \log \mu_{21} & \textit{if } \sigma \to \infty \\ 0 & \textit{if } \sigma \to 0 \end{cases}$$

- Joint production attenuates TFP response

Using Prices Instead of PPF Curvature

- To derive sufficient statistics, rewrite the TFP response in a way that is not dependent on σ
- With joint production, relative price changes are associated with changes in production ratio:

$$d\log\left(p_{11}/p_{12}\right) = \frac{1}{\sigma} d\log\left(q_{11}/q_{12}\right)$$

Using Prices Instead of PPF Curvature

- With joint production, relative price changes are associated with changes in production ratio:

$$d\log(p_{11}/p_{12}) = \frac{1}{\sigma} d\log(q_{11}/q_{12})$$

- Solving for prices: $d\log(p_{11}/p_{12}) = -\frac{1}{\sigma+1} d\log\mu_{21}$, the TFP response is expressed by: deri

$$\Delta\log TFP = \left(1 - \frac{1}{\sigma+1}\right)\tilde{\lambda}_{21}\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d\log\mu_{21}$$

$$= \underbrace{\tilde{\lambda}_{21}\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d\log\mu_{21}}_{\text{Single-Product Term}} + \underbrace{d\log(p_{11}/p_{12})\,\tilde{\lambda}_{21}\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right)}_{\text{Multi-Product Term}}$$

Using Prices Instead of PPF Curvature

- With joint production, relative price changes are associated with changes in production ratio:

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$$\Delta\log TFP = \left(1 - \frac{1}{\sigma+1}\right)\tilde{\lambda}_{21}\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d\log\mu_{21}$$

$$= \underbrace{-\underbrace{d\log\Lambda}_{\text{Labor Share}} - \tilde{\lambda}_{21} d\log\mu_{21}}_{\text{Single-Product Term}} + \underbrace{Cov_{s_1}\left(d\log p_{(1,\cdot)}, \frac{\bar{\Gamma}_1}{\Gamma_{(1,\cdot)}}\right)}_{\text{Multi-Product Term}}$$

- If $\sigma \to \infty$, multi-product term is zero
- If $\sigma \to 0$, $\Delta\log TFP$ is zero

Sufficient Statistics for Simple Example

Proposition

In the simple example, TFP response to the markup shock can be expressed as

$$\Delta \log TFP = \underbrace{Cov_{\mathbf{s}_1}\left(d \log p_{(1,\cdot)}, \frac{\bar{\Gamma}_1}{\Gamma_{(1,\cdot)}}\right)}_{\text{Multi-Product Term}} \underbrace{- \underbrace{d \log \Lambda}_{\text{Labor Share}} - \tilde{\lambda}_{21} d \log \mu_{21}}_{\text{Single-Product Term}}$$

where $d \log p_{(1,\cdot)} = (d \log p_{11}, d \log p_{12})$, $\Gamma_{(1,\cdot)} = (\Gamma_{11}, \Gamma_{12})$, $\bar{\Gamma}_1 = \left(\sum \tilde{\lambda}_{11} \Gamma_{11}^{-1} + \tilde{\lambda}_{12} \Gamma_{12}^{-1}\right)^{-1}$ *and* $\mathbf{s}_1 = \left(\tilde{\lambda}_{11}, \tilde{\lambda}_{12}\right)$

taste shock

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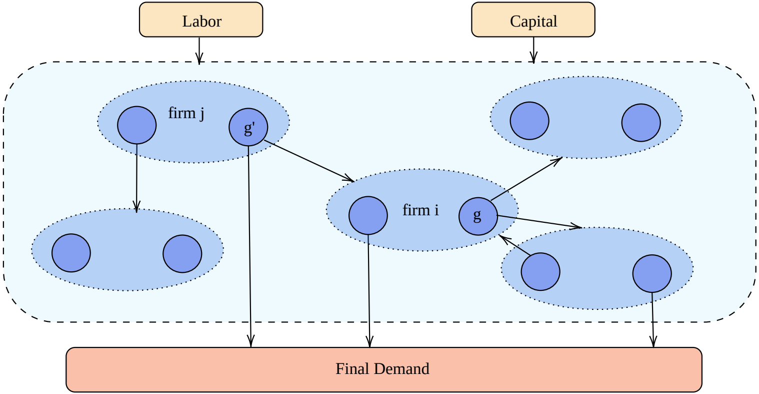
General Theory for Measurement

- Introduce a general framework to derive sufficient statistics that can accommodate any number of products and arbitrary input-output linkages



General Setup

- Firm $i \in \mathcal{N}$ makes product $g \in \mathcal{G}$ using g' from firm j and factors as inputs
- CRS joint production with firm-level technology shock (technology shock can be product specific with additional assumption): details

$$F_i^Q\left(\underbrace{\{q_{ig}\}_{g\in\mathcal{G}}}_{\text{outputs}}\right) = A_i F_i^X\left(\underbrace{\{x_{i,jg'}\}_{j\in\mathcal{N},g'\in\mathcal{G}}}_{\text{product } g' \text{ from j}}, L_i, K_i\right)$$

- Firm i sets its products' price as $p_{ig} = \mu_{ig} mc_{ig}$, where μ_{ig} is the wedge (markup)
- Agnostic about microfoundations of wedges (e.g., market power, sticky prices, etc.)

General Setup

- Final demand as maximizer of homothetic aggregator, $Y = D(c_{ig}, ..., c_{NG})$: subject to:

$$\underbrace{\sum_{i\in\mathcal{N}}\sum_{g\in\mathcal{G}} p_{ig}c_{ig}}_{\text{Final goods expenditure}} = \underbrace{\sum_{f\in\{L,K\}} w_f L_f}_{\text{Factor income}} + \underbrace{\sum_{i\in\mathcal{N}}\sum_{g\in\mathcal{G}} (1 - 1/\mu_{ig}) p_{ig}q_{ig}}_{\text{Profits}}$$

- Resource constraints: $q_{ig} = c_{ig} + \sum_{j\in\mathcal{N}} x_{jig}, \quad \sum_{i\in\mathcal{N}} L_i = L, \quad \sum_{i\in\mathcal{N}} K_i = K$

Definition (General Equilibrium)

Given productivities $\boldsymbol{A}$ *and product-level markups* $\boldsymbol{\mu}$*, an equilibrium consists of prices* p_{ig}, w_f*, intermediate input choices* $x_{ijg'}$*, factor inputs* (L_i, K_i)*, outputs* q_{ig}*, and consumption* c_{ig}*, such that (i) each firm minimizes its costs and sets prices as markups over marginal costs, (ii) final demand maximizes the aggregator subject to budget constraints, (iii) and all goods and factor markets clear.*

Growth Accounting with Multi-Product Firms

Proposition

To the first order, aggregate TFP is decomposed into allocative efficiency and technology :

$$\Delta TFP = \underbrace{\underbrace{\sum_i \tilde{\lambda}_i Cov_{\mathbf{s}_i}\left(d\log p_{(i,\cdot)}, \frac{\bar{\Gamma}_i}{\Gamma_{(i,\cdot)}}\right)}_{\text{Multi-Product Term}} \underbrace{- \sum_f \tilde{\Lambda}_f d\log \Lambda_f - \sum_i \tilde{\lambda}_i d\log \mu_i}_{\text{Single-Product Term}}}_{\Delta \text{Allocative Efficiency}} + \underbrace{\sum_i \tilde{\lambda}_i d\log A_i}_{\Delta \text{Technology(residual)}}$$

where $d\log p_{(i,\cdot)} = (d\log p_{i1}, \ldots, d\log p_{iG}), \Gamma_{(i,\cdot)} = (\Gamma_{i1}, \ldots, \Gamma_{iG})$ *and* $\bar{\Gamma}_i = \mathbb{E}_{\mathbf{s}_i}\left(\Gamma_{(i,g)}^{-1}\right)^{-1}$

- Cumulative wedge, Γ and Domar weights, $\tilde{\lambda}$ ($\tilde{\Lambda}$ for factors) IO matrix cummulative wedge
  - Functions of products' markups, sales, and expenditure shares

formal definition efficient benchmark interpretation

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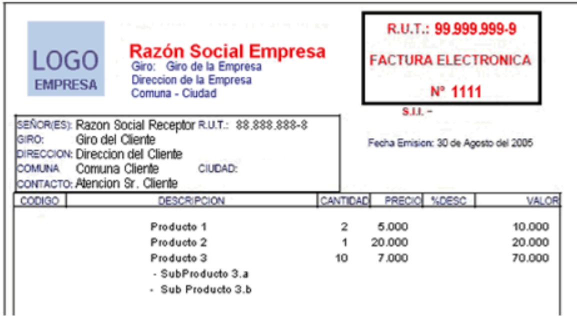
Quantifying Misallocation

- Perform growth accounting to decompose Chile's cumulative TFP changes into allocative efficiency and technology components
- Construct sufficient statistics using firm-to-firm transaction data from Chile
- Compare results with and without accounting for joint production

Data: Administrative Chilean Invoice between Firms

- Electronic invoices between all formal firms, including service firms
- Information on buyers, sellers, product, quantity, and price distribution
- Combined with balance sheets information

Figure: Example of Invoice



LOGO EMPRESA

Razón Social Empresa
Giro: Giro de la Empresa
Direccion de la Empresa
Comuna - Ciudad

R.U.T.: 99.999.999-9
FACTURA ELECTRONICA
N° 1111
S.I.I. -

SEÑOR(ES): Razon Social Receptor R.U.T.: 88.888.888-8
GIRO: Giro del Cliente
DIRECCION: Direccion del Cliente
COMUNA: Comuna Cliente CIUDAD:
CONTACTO: Atencion Sr. Cliente

Fecha Emision: 30 de Agosto del 2005

CODIGO	DESCRIPCION	CANTIDAD	PRECIO	%DESC	VALOR
	Producto 1	2	5.000		10.000
	Producto 2	1	20.000		20.000
	Producto 3	10	7.000		70.000
	- SubProducto 3.a				
	- Sub Producto 3.b				

Construction of Sufficient Statistics

- Domar weights: from firm-to-firm transactions with balance sheets details
- Cumulative wedge Variance decomposition

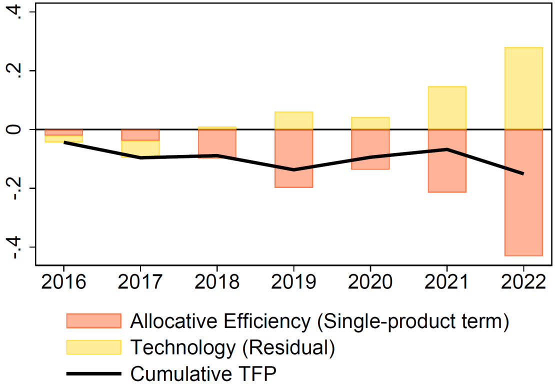
$$\Gamma_{ig} \equiv \underbrace{\frac{\tilde{\lambda}_{ig}}{\text{sales share}_{ig}}}_{\text{downstream wedge}} \times \underbrace{\mu_{ig}}_{\text{own markup}}$$

  - Own markup: accounting approach (common $\mu_{ig} = \mu_i$ from $sales_i / cost_i$)
  - Sensitivity: obtain μ_{ig} using Dhyne, Petrin, Smeets & Warzynski (2022) with estimated σ markup estimation
- Factor shares & aggregate TFP are computed from aggregated microdata

Cumulative TFP Decomposition: Ignoring Multi-Product Term

$$\Delta\text{TFP} = \underbrace{\Delta\text{Technology}}_{\text{Residual}} + \underbrace{\Delta\text{Single-Product Term}}_{\text{Allocative Efficiency}} + \Delta\text{Multi-Product Term}$$

- Large negative allocative efficiency: firms with high-cumulative wedges shrink and firms with low-cumulative wedges expand in terms of inputs use
- Since TFP does not fall by as much, technology residual must be large and positive

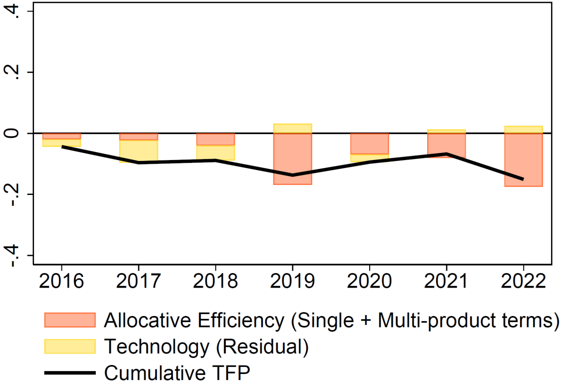


Cumulative TFP Decomposition with Multi-Product Term

$$\Delta\text{TFP} = \underbrace{\Delta\text{Technology}}_{\text{Residual}} + \underbrace{\Delta\text{Single-Product Term} + \Delta\text{Multi-Product Term}}_{\text{Allocative Efficiency}}$$

- Adding the multi-product term attenuates the strong reallocation effects
- Technology residual is now small

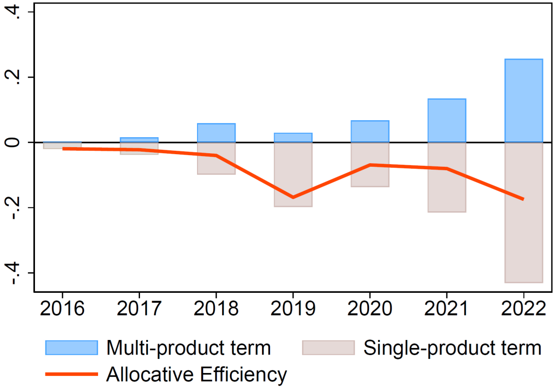
Robustness



Cumulative TFP Decomposition with Multi-Product Term

$$\Delta\text{TFP} = \underbrace{\Delta\text{Technology}}_{\text{Residual}} + \underbrace{\Delta\text{Single-Product Term} + \Delta\text{Multi-Product Term}}_{\text{Allocative Efficiency}}$$

- The multi-product term attenuates the single-product term as the theory suggests
- Rising prices limit the expansion of low cumulative wedge products, mitigating negative reallocation



Robustness

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Quantifying Distance to Frontier

- Use parametric model to measure TFP gains (distance to frontier) when removing all wedges:

$$\text{Distance to Frontier} \equiv \log Y(\mu)/Y(1)$$

- Parameterizing the general framework's production technologies using CET and CES specifications:

$$F_i^Q\left(\underbrace{\{q_{ig}\}_{g\in\mathcal{G}}}_{\text{outputs}}\right) = A_i F_i^X\left(\underbrace{\{x_{i,jg'}\}_{j\in\mathcal{N},g'\in\mathcal{G}}}_{\text{product } g' \text{ from j}}, L_i, K_i\right)$$

where

$$F_i^Q(\cdot) = \left(\sum \delta_{ig} q_{ig}^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}}, \quad F_i^X(\cdot) = \left(\sum \omega_{i,jg'} q_{i,jg'}^{\frac{\theta-1}{\theta}} + \omega_{i,L} L_i^{\frac{\theta-1}{\theta}} + \omega_{i,K} K_i^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$

distance to frontier

Calibration and Results

- Use Chilean invoice data to account for firm-to-firm linkages
- Comparison between estimated σ and $\sigma = \infty$ (independent product lines benchmark)
- For simplicity, we set $\theta = 2.5$ across inputs following Arkolakis et al. (2023)
- Joint production attenuates TFP losses

Specification	Distance to Frontier
Baseline Estimate ($\sigma = 1.2$)	12.3%
Independent Products ($\sigma \to \infty$)	18.7%

Conclusion

- Sufficient statistics to measure misallocation with joint production
- Provide empirical evidence consistent with joint production
- Develop and implement distance to frontier formula with joint production
- Ignoring joint production overestimates misallocation

Decreasing Return to Scale in Firm Level

- Production function from Almunia et al. (2021)

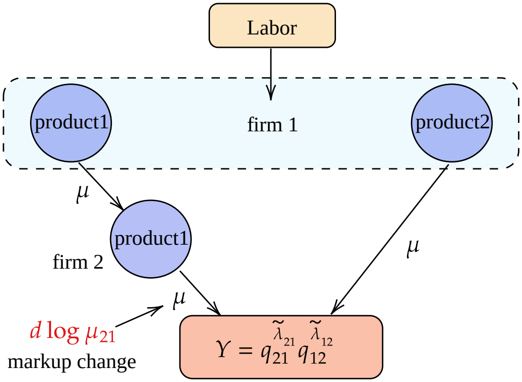
$$q_{11} + q_{12} = L_1^{\alpha}$$

- This is a special case of parametric version of our specification as $\sigma \to \infty$

$$\left(q_{11}^{\frac{\sigma+1}{\sigma}} + q_{12}^{\frac{\sigma+1}{\sigma}} \right)^{\frac{\sigma}{\sigma+1}} = L_1^{\alpha} K^{1-\alpha}$$

where K is firm specific factor

Derivation



$$\left(q_{11}^{\frac{\sigma+1}{\sigma}} + q_{12}^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}} = L$$

- The downstream markup change implies that sales of product 1 of firm 1 changes by $-d\log\mu_{21}$ back
- Sales of product 2 do not change (due to Cobb-Douglas)
- Combining these with $d\log(p_{11}/p_{12}) = \frac{1}{\sigma} d\log(q_{11}/q_{12})$ implies

$$d\log(p_{11}/p_{12}) = -\frac{1}{\sigma+1} d\log\mu_{21}$$

Example with Taste Shock

- Instead of markup shock, consider taste shock: $d\tilde{\lambda}_2 = -d\tilde{\lambda}_1$ back
- Using $d\log(p_{11}/p_{12}) = \frac{1}{\sigma+1}\frac{1}{\tilde{\lambda}_2} d\log\tilde{\lambda}_1$, the TFP response can be expressed by:

$$\Delta \log TFP = \left(1 - \frac{1}{\sigma+1}\right)\frac{\tilde{\lambda}_1}{\tilde{\lambda}_2}\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d\log\tilde{\lambda}_1,$$

$$= \underbrace{\frac{\tilde{\lambda}_1}{\tilde{\lambda}_2}\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d\log\tilde{\lambda}_1}_{\text{Single-Product Term}} + \underbrace{\tilde{\lambda}_1\left(\frac{\bar{\Gamma}_1}{\Gamma_{11}} - 1\right) d\log p_{11}/p_{12}}_{\text{Multi-Product Term}}$$

- If $\sigma \to \infty$, multi-product term is zero
- If $\sigma \to 0$, $\Delta \log TFP$ is zero

Sufficient Statistics with Taste Shock

Proposition

In the simple example, TFP response to the taste shock is expressed by

$$\Delta \log TFP = \underbrace{Cov_{s_1}\left(d\log p_{(1,\cdot)}, \frac{\bar{\Gamma}_1}{\Gamma_{(1,\cdot)}}\right)}_{\text{Multi-Product Term}} - \underbrace{\underbrace{d\log\Lambda}_{\text{Labor Share}}}_{\text{Single-Product Term}}$$

where $d\log p_{(1,\cdot)} = (d\log p_{11}, d\log p_{12})$, $\Gamma_{(1,\cdot)} = (\Gamma_{11}, \Gamma_{12})$, $\bar{\Gamma}_1 = \left(\tilde{\lambda}_{11}\Gamma_{11}^{-1} + \tilde{\lambda}_{12}\Gamma_{12}^{-1}\right)^{-1}$ *and* $\mathbf{s}_1 = \left(\tilde{\lambda}_{11}, \tilde{\lambda}_{12}\right)$

Joint Production

Let $J(\boldsymbol{q}, \boldsymbol{x})$ be the joint production function (Hall (1973))

$$J(\boldsymbol{q}, \boldsymbol{x}) = 0,$$

where $\boldsymbol{q}$; output vector and $\boldsymbol{x}$: input vector

Assumptions:

(1) CRS: $J(\boldsymbol{q}, \boldsymbol{x}) = 0$ implies $J(\lambda \boldsymbol{q}, \lambda \boldsymbol{x}) = 0$

(2) Separability between input and output bundles: $J(\boldsymbol{q}, \boldsymbol{x}) = -F^Q(\boldsymbol{q}) + F^X(\boldsymbol{x})$

Example Constant-Elasticity of Transformation and CES Input (CET-CES): cost function

$$\underbrace{\left(\sum_g q_g^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}}}_{\text{Output bundle}} = A \underbrace{\left(\omega_L L^{\frac{\theta-1}{\theta}} + \omega_K K^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}}_{\text{Input Bundle}}$$

back to simple example | back to network settings

Cost Function

Example

CET-CES Cost Function

$$C(\mathbf{q}, w, r; A) = \frac{1}{A}\left(\sum_g \delta^g \left[q_g\right]^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}} \left(\alpha_L w^{1-\theta} + \alpha_K r^{1-\theta}\right)^{\frac{1}{1-\theta}},$$

- w and r are (factor) prices
- marginal cost

$$mc_g = \frac{1}{A}\frac{\partial C(\mathbf{q}, w, r; A)}{\partial q_g}$$

back to joint production

Input-Output Objects: Revenue-Based

- To derive sufficient statistics, define Ω to be $(\mathcal{N}\mathcal{G}+\mathcal{F})\times(\mathcal{N}\mathcal{G}+\mathcal{F})$ input-output matrix: back

$$\Omega_{ig,jg'} = \underbrace{\frac{p_{jg'}x_{i,jg'}}{\mu_{ig}\left(\sum_{j,g'} p_{jg'}x_{i,jg'} + \sum_f w_f L_{if}\right)}}_{\text{Revenue share of } jg'},$$

- Leontief inverse matrix, Ψ:

$$\Psi \equiv (I-\Omega)^{-1},$$

- Domar weights, λ and:

$$\begin{aligned}\lambda' \equiv& b'\Psi,\\ =& b' + b'\Omega + b'\Omega^2 + \dots\end{aligned}$$

λ = sales share (over GDP)

Input-Output Objects: Revenue vs Cost-Based

- Ω and $\tilde{\Omega}$ are input output matrices: back

$$\Omega_{ig,jg'} = \underbrace{\frac{p_{jg'} x_{i,jg'}}{\mu_{ig}\left(\sum_{j,g'} p_{jg'} x_{i,jg'} + \sum_f w_f L_{if}\right)}}_{\text{Revenue share of } jg'}, \qquad \tilde{\Omega}_{ig,jg'} = \underbrace{\frac{p_{jg'} x_{i,jg'}}{\sum_{j,g'} p_{jg'} x_{i,jg'} + \sum_f w_f L_{if}}}_{\text{Cost share of } jg'}$$

- Leontief inverse matrices:

$$\Psi \equiv (I - \Omega)^{-1}, \qquad \tilde{\Psi} \equiv (I - \Omega)^{-1}$$

- Domar weights ($\tilde{\Lambda}$ for factors) :

$$\lambda' \equiv b'\Psi, \qquad \tilde{\lambda}' \equiv b'\tilde{\Psi}$$

If there is no markup, $\lambda = \tilde{\lambda}$ = sales share (over GDP)

Cumulative Wedge

- **Cumulative wedge** (for product g of firm i): back

$$\Gamma_{ig} \equiv \underbrace{\tilde{\lambda}_{ig}/\lambda_{ig}}_{\text{downstream wedge}} \times \underbrace{\mu_{ig}}_{\text{own wedge}},$$

- Captures indirect wedges through the downstream supply chain
- In the previous simple example: $\Gamma_{11} = \mu^2 > \Gamma_{12} = \mu$
- Firm-level (cost-based) Domar weights and their shares: $\tilde{\lambda}_i = \sum_g \tilde{\lambda}_{ig}, \quad s_{ig} = \frac{\tilde{\lambda}_{ig}}{\tilde{\lambda}_i}$

Decomposition of TFP into Allocative Efficiency and Technology

- $\mathcal{Y}(X, A)$: output Y given firm level productivities A and shares $X_{ijg} = x_{ijg}/q_{jg}$
- Change in response to shocks:

$$d \log Y = \underbrace{\frac{\partial \log \mathcal{Y}}{\partial \log X} d \log X}_{\Delta \text{Allocative Efficiency}} + \underbrace{\frac{\partial \log \mathcal{Y}}{\partial \log A} d \log A}_{\Delta \text{Technology}}$$

Decomposition of TFP into Allocative Efficiency and Technology

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- Efficient economy (set all markups equal 1) implies Hulten (1978):

$$\underbrace{d\log Y - \sum_f \tilde{\Lambda}_f d\log F_f}_{\Delta TFP} = \underbrace{0}_{\Delta \text{Allocative Efficiency}} + \underbrace{\sum \tilde{\lambda}_i d\log A_i}_{\Delta \text{Technology}}$$

back

Allocative Efficiency: Multi-Product Term

$$\text{Multi-Product Term} = \sum_i \tilde{\lambda}_i Cov_{\mathbf{s}_i}\left(\underbrace{d\log p_{(i,\cdot)}}_{\text{Price changes of firm } i\text{'s products}}, \underbrace{\frac{\bar{\Gamma}_i}{\Gamma_{(i,\cdot)}}}_{\text{Cumulative wedges of firm } i\text{'s products}} \right)$$

- Inside firm:
  - Joint production $\Rightarrow$ the opportunity cost to modify firm i's product mix is *not* constant
  - $\downarrow$ price of more distorted products, high Γ_{ig} $\Rightarrow$ beneficial reallocation
- Aggregation:
  - Sum up the covariance using firm's Domar weights, $\tilde{\lambda}_i$ back

Allocative Efficiency: Single-Product Term

$$\text{Single-Product Term} = -\sum_f \underbrace{\tilde{\Lambda}_f d\log\Lambda_f}_{\text{(1) Aggregate Factor share}} - \sum_i \underbrace{\tilde{\lambda}_i d\log\mu_i}_{\text{(2) Firm-level Markup}}$$

(1) A decline in factor shares $\Rightarrow$ resources shifting to high markup firms that are underproducing

(2) The direct change in factor shares due to markup changes: To isolate changes in allocative efficiency, need to subtract this back

Efficient Benchmark

- Efficient economy (set all markups equal 1) implies Hulten (1978):

$$\Delta \log TFP = \underbrace{0}_{\Delta \text{Allocative Efficiency}} + \underbrace{\sum \tilde{\lambda}_i d \log A_i}_{\Delta \text{Technology}}$$

back

Table of Contents

Empirical Evidence of Joint Production and Parametric Counterfactual

- Sufficient statistics approach is useful for growth accounting given past observed data
- Develop a complementary parametric approach for counterfactuals, which requires the knowledge of the curvature of firm-level PPF
- Assume joint production function with constant elasticity of transformation, σ
- Present evidence consistent with joint production and estimate σ

Motivating Empirical Evidence of Joint Production

Assume:

- The cost function is $C(q_1,\ldots,q_G) = P_I\left(\sum_{i=1}^{G}\omega_g q_g^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}}$
- Demand for each product is : $q_g = D_g p_g^{-\theta_g}$
- Price of each product is set to maximize profits given $\{D_g\}$ and input prices variable markup

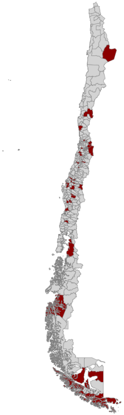
Proposition

A negative demand shock to product m ($d\log D_m < 0$) lowers quantity and raises the price of every other product $g \neq m$, if, and only if, $\sigma < \infty$.

- Intuitively, due to complementarities in production, a decrease in production of product *m* raises the cost of production of other products and decreases their production

Data Sources

- Provide evidence of joint production using product-level transaction data and product-specific demand shocks
- Data: Universe of electronic invoices between formal firms, aggregated at the firm-product level
- Demand shock: COVID lockdowns affecting buyers as product-specific demand shocks buyer regression



Locked down municipalities in March 2020

Simple Event Study Specification

- Define main product (m) to be the product with the highest revenue for firm i
- Estimate the impact of demand shocks to main product (m) on other products $g \neq m$
- Main product receives a demand shock if at least one buyer is in a locked down municipality in March 2020
- For all non-main products $g \neq m$,

$$\log X_{igt} = \sum_{j=-11}^{j=10} \beta_j \underbrace{Lockdown_{i,t-j}}_{\text{shock to main product}} + FE_{ig} + FE_t + \varepsilon_{igt}$$

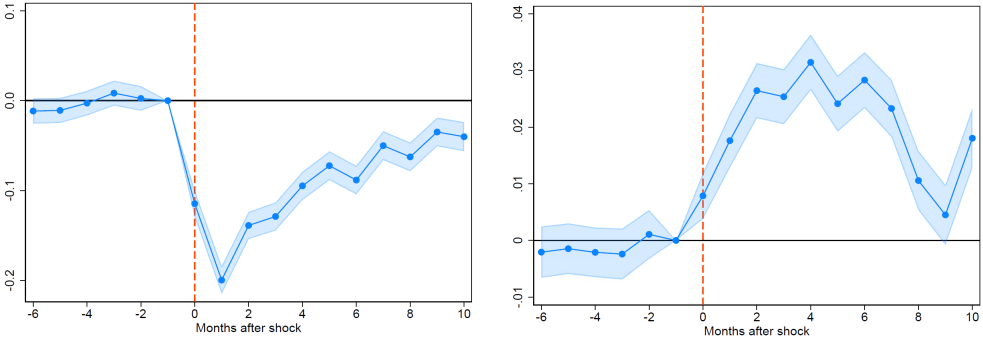
where $Lockdown_{i,t-j} = 1$ if firm i 's main product experienced the demand shock j months ago. X represents either quantity or price

Sample Restriction

- If your buyers are in a locked down area, you, buyers of your other products, or your suppliers may also be in locked down areas
- To deal with this, limit to a subsample of firms satisfying two conditions:
  (1) No direct supply shocks: Firms and their suppliers are not in locked down areas
  (2) No other demand shocks: Non-main product g has no buyers in locked down areas sample

Results

- Negative demand shock to main product reduces quantity and increases price of other products robustness



(a) log quantity (b) log price

Comparison with Alternative Within-Firm Spillover Mechanisms

- Venting Out (Almunia, Antràs, Lopez-Rodriguez, and Morales (2021))
  - Domestic demand $\downarrow \rightarrow$ Firm specific resources shift to exports $\rightarrow$ Exports $\uparrow$
  - Our results predict opposite sign
- Knowledge Linkages (Ding (2023))
  - US census: Product demand $\downarrow \rightarrow$ knowledge-linked products $\downarrow$
  - 5-year vs. monthly data and R&D intensity difference

Estimating Elasticity of Transformation σ

$$\Delta \log\left(\frac{p_{ig}}{p_{im}}\right) = \alpha + \beta \Delta \log\left(\frac{q_{ig}}{q_{im}}\right) + \gamma FE_{pt} + \xi_{igt}$$

	(1)	(2)	(3)
β	0.949***	1.207***	0.865***
	(0.0015)	(0.0586)	(0.0209)
Implied $\sigma(=1/\beta)$	1.053	0.828	1.155
Time FE	Y	Y	N
Product $\times$ Time FE	N	N	Y
First Stage F stats	-	276.7	274.6

Notes: The standard errors are clustered at the firm level. Columns (1) report a result by ordinary least squares (OLS), while columns (2) and (3) report results by 2SLS. Product FE refer to harmonized product code. Three stars indicate statistical significance at the 1% level.

Buyer Level Regression

$$\log \text{Total Intermediate Input Purchases}_{it} = \alpha Lockdown_{it} + FE_t + FE_i + \varepsilon_{it},$$

	(1)	(2)	(3)
Lockdown Dummy	-0.222***	-0.230***	-0.191***
	(0.0524)	(0.00521)	(0.0589)
Firm FE	Y	Y	Y
Time FE	N	N	Y
Sector $\times$ Time FE	N	Y	N
Restricted sample	N	N	Y
Observations	4,345,534	4,345,534	378,646

Notes: Standard errors are clustered at the country level. Columns (1) and (2) report results for the full sample, while column (3) reports results restricted to firms where none of the suppliers are located in the lockdown area. Three stars indicate statistical significance at the 1% level.

Purchases of intermediate inputs from lockdown counties decreased by about 20%. back

Summary Statistics

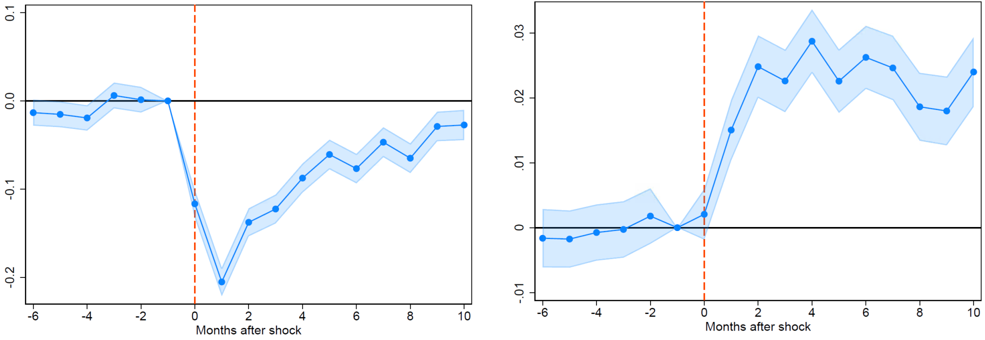
	Treatment Firms	Control Firms
Number of firms	26,411	96,321
Number of products sold	16	10
Number of producers	107	119
Number of buyers	59	26
Annual revenue (million pesos)	186	101

Notes: This Table presents the characteristics of treated firms (those whose major product buyers experienced lockdowns in March 2020) and control firms, showing values from February 2020, the month before the shock. The rows display the median of each statistic.

back

Results

- Use the samples with only above the 80th percentile of sales distribution robustness



(c) log quantity (d) log price

DiD Specification

$$\log X_{igt} = \beta \underbrace{Lockdown_{i,t}}_{\text{shock to main product}} + FE_{ig} + FE_t + \varepsilon_{igt}$$

where $Lockdown_{i,t} = 1$ if firm i 's main product experienced the demand shock and after March 2020. back

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\log q$	-0.117***		-0.117***		-0.102***		-0.106***		-0.125***		-0.159***	
	(0.0046)		(0.0046)		(0.0053)		(0.0188)		(0.0049)		(0.0062)	
$\log p$		0.0175***		0.0184***		0.0191***		0.0193***		0.0168***		0.0177***
		(0.0021)		(0.0022)		(0.0022)		(0.0096)		(0.0023)		(0.0024)
Input price control	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Large firms	N	N	N	N	Y	Y	N	N	N	N	N	N
Only manufacturing firms	N	N	N	N	N	N	Y	Y	N	N	N	N
Time × product FE	N	N	N	N	N	N	N	N	Y	Y	N	N
Continuous treatment	N	N	N	N	N	N	N	N	N	N	Y	Y
Observations	7,693,066	7,693,066	7,669,848	7,669,848	4,394,166	4,394,166	1,399,617	1,399,617	7,594,918	7,594,918	7,693,066	7,693,066

Notes: Input price control indicates inclusion of the Tornqvist input price index. Large firms restricts to firms above the 80th percentile in total sales. Only manufacturing firms restricts to the manufacturing sector. Time × Product FE are time-varying fixed effects at the harmonized product level. Continuous treatment uses the share of transaction values to lockdown destinations in firm's main product instead of the binary lockdown variable. *** denote significance at 1%.

Variable Markup

- If Marshall's second law of demand hold (the higher q, the higher the markup, the smaller the pass-through), the same proposition hold
- Intuitively, markups offset some of the price changes back

Proposition

Under the marshall's second law of demand hold, A negative demand shock to product k ($d \log D_k < 0$) affects product $g \neq k$:

(i) If $\sigma < \infty$, quantity: $d \log q_g < 0$, price: $d \log p_g > 0$,

(ii) If $\sigma = \infty$, quantity: $d \log q_g = 0$, price: $d \log p_g = 0$

Data and Growth Accounting

(1) Sales, materials, investment: F29 (2014-2022)

(2) Wage bill, employment: DJ1887 (2014-2022)

(3) Capital: F22 (2014-2022)
- Capital stock using perpetual inventory methods combining capital stock with investment
- UCC: interest rate - inflation expectation + depreciation rate from LA-Klems database + external financing premium (5 percent)

(4) Product, I-O matrices and output and input prices: F2F electronic receipts (2014-2022)

(5) Official deflectors for aggregate real variables back

Data Cleaning

- The final sample does not include firms with a missing variable of sales, capital, wage bill, or materials
- Winzorized labor, capital and materials shares over sales at 1% of both tails of the distribution
- Firms with negative value added (sales minus materials), less than two workers, or capital less than 10.000 CLP (USD 15) are excluded

Multi-Product Firms Dominate Intermediate Transactions

- Multi-product firms account for 99 percent of intermediate transactions back

Percentile	Number of products by firm
1%	1
5%	2
10%	4
25%	36
50%	**475**
75%	2,459
90%	32,195
95%	37,422
99%	62,372

Table: Share of firm-to-firm transactions made by firms with no more than X product

Unweighted Distribution of the Number of Products

Percentile	Number of products by firm
1%	1
5%	1
10%	1
25%	2
50%	**7**
75%	26
90%	119
95%	290
99%	1,253

Table: Distribution of the Number of Products by Firm (Unweighted)

back

Markup Estimation

- Following Dhyne, Petrin, Smeets & Warzynski (2022) with CET output function
- Cobb-Douglas production function for input bundle using three inputs (K, L, M) (lower case variables denote logs)

$$Q_{it} = \beta_0 + \beta_K K_{it} + \beta_L L_{it} + \beta_M M_{it} + \omega_{it}$$

where $Q_i = \left(\sum_{i=1}^{N} \left(q_{ig} \right)^{\frac{\sigma+1}{\sigma}} \right)^{\frac{\sigma}{\sigma+1}}$ using estimated σ

- GMM Estimation was performed separately by firm's sector
- Time invariant output elasticities recover product level markup back

$$\mu_{ig} = \frac{\beta_M^g}{\left(q_{ig} \right)^{\frac{\sigma+1}{\sigma}} / \sum_{g'=1}^{N} \left(q_{ig'} \right)^{\frac{\sigma+1}{\sigma}}} \frac{R_{ig}}{P_i^M M_i}$$

where R_{ig} is the revenue from product g and $P_i^M M_i$ is the expenditure on materials.

Variance Decomposition

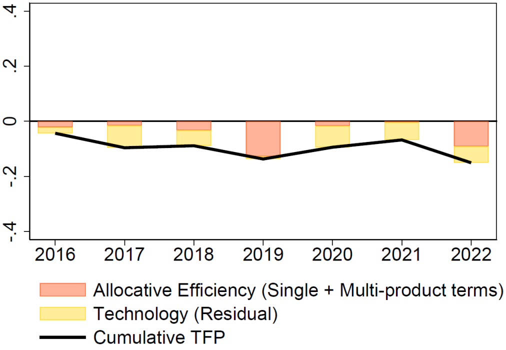
$$\Gamma_{ig} \equiv \frac{\tilde{\lambda}_{ig}}{salesshare_{ig}} \times \mu_{ig},$$

Table: Variance decomposition of $\log \Gamma$

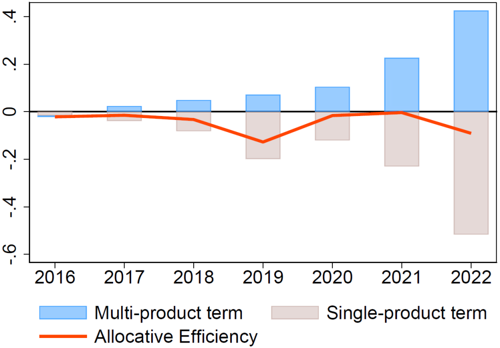
Year	Downstream distortions	Own markup	Covariance
2016	106.4%	1.8%	-8.2%
2017	107.4%	2.1%	-9.5%
2018	107.3%	2.2%	-9.5%
2019	107.6%	2.4%	-10.0%
2020	107.6%	2.6%	-10.2%
2021	107.3%	2.7%	-10.0%
2022	107.2%	2.8%	-10.0%

- Most of the variation stem from downstream distortions back

Growth Accounting with Production Function Approach



(e) Growth accounting with mutiproduct term



(f) Decomposition of allocative efficiency

back

Growth Accounting with Production Function Approach

$$Cov_{\mathbf{s}_i}\left(d\log p_{(i,\cdot)}, \frac{\bar{\Gamma}_i}{\Gamma_{(i,\cdot)}}\right) = Cov_{\mathbf{s}_i}\left(d\log \mu_{(i,\cdot)}, \frac{\bar{\Gamma}_i}{\Gamma_{(i,\cdot)}}\right) + Cov_{\mathbf{s}_i}\left(d\log mc_{(i,\cdot)}, \frac{\bar{\Gamma}_i}{\Gamma_{(i,\cdot)}}\right)$$

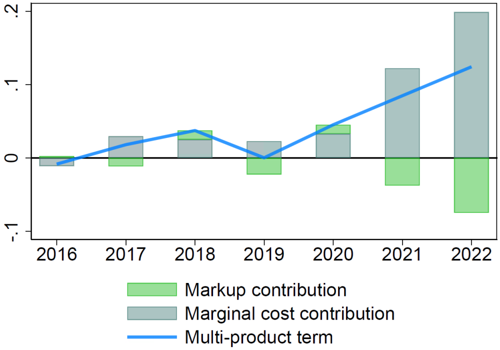
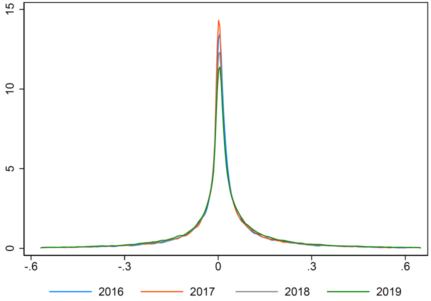


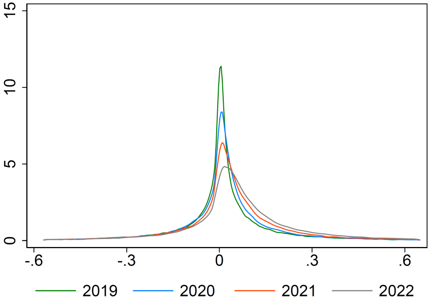
Figure: Decomposition of multiproduct term

Distributions of Multi-Product Term, $Cov_{\mathbf{s}_i}\left(d\log p_{(i,\cdot)}, \frac{\bar{\Gamma}_i}{\Gamma_{(i,\cdot)}}\right)$

- Stable distribution in normal times, but the distribution is skewed to the right after periods of turmoil
- Firms may have adjusted their product mixes significantly in response to the disturbance suggestive evidence



(a) 2016~2019



(b) 2019~2022

Large Product Mix Adjustments and Efficiency: Suggestive Evidence

- Firms may have adjusted their product mix significantly to respond to the recent supply chain disruption

back

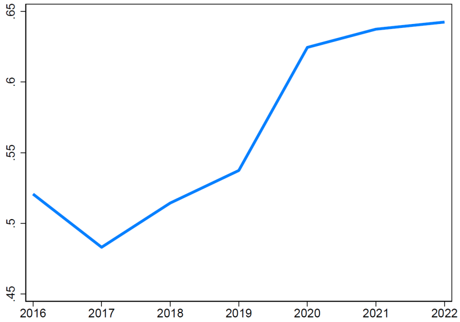


Figure: Variance of Product Quantity Changes: $Var_{\lambda_i}(d \log q_{ig})$

Distance to the Pareto-Efficient Frontier with Joint Production

Proposition

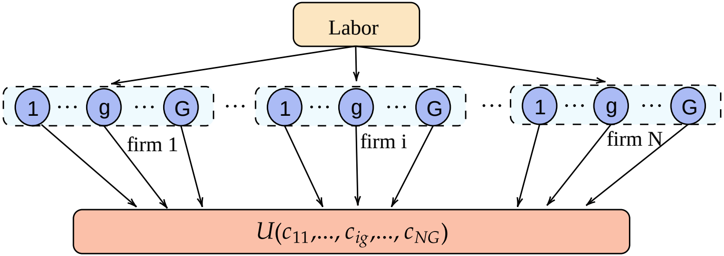
Starting at an efficient equilibrium, up to second order, and in response to the introduction of wedges, changes in TFP are given by Domar-weighted Harberger triangles:

$$\mathcal{L} = \frac{1}{2} \sum_{ig} \lambda_{ig} \underbrace{d \log q_{ig}}_{quantity} \underbrace{d \log \mu_{ig}}_{markup}$$

where λ_{ig} is the Domar weight of product g from firm i back

Horizontal Economy

- A representative consumer (CES with elasticity θ)
- N multi-product firms use labor to make G products using CET technology (elasticity σ)
- Markups μ_{ig} are heterogeneous across products and firms



Analytical Structural Result: Horizontal Economy

Proposition

Starting at an efficient equilibrium, up to second order, the distance to the frontier in the horizontal economy is given by:

$$\mathcal{L} = -\frac{1}{2}\theta\left(Var_{\boldsymbol{\lambda}}(d\log\mu_{ig}) - \frac{1}{\sigma+1}\mathbb{E}_{\bar{\boldsymbol{\lambda}}}\left\{Var_{\boldsymbol{s}_i}(d\log\mu_{ig})\right\}\right)$$

where $\boldsymbol{\lambda} = (\lambda_{11}, \lambda_{12}, \ldots, \lambda_{NG})$, $\bar{\boldsymbol{\lambda}} = (\lambda_1, \lambda_2, \ldots, \lambda_N)$ *with* $\lambda_i = \sum_g \lambda_{ig}$, *and* $\boldsymbol{s}_i = (\lambda_{i1}/\lambda_i, \lambda_{i2}/\lambda_i, \ldots, \lambda_{iG}/\lambda_i)$

- Joint production attenuates the distance to the frontier given wedges

Analytical Structural Result: Horizontal Economy

Proposition

Starting at an efficient equilibrium, up to second order, the distance to the frontier in the horizontal economy is given by:

$$\mathcal{L} = -\frac{1}{2}\theta \left(Var_{\boldsymbol{\lambda}}(d\log\mu_{ig}) - \frac{1}{\sigma+1}\mathbb{E}_{\bar{\boldsymbol{\lambda}}}\left\{Var_{\mathbf{s}_i}(d\log\mu_{ig})\right\}\right)$$

where $\boldsymbol{\lambda} = (\lambda_{11}, \lambda_{12}, \ldots, \lambda_{NG})$, $\bar{\boldsymbol{\lambda}} = (\lambda_1, \lambda_2, \ldots, \lambda_N)$ *with* $\lambda_i = \sum_g \lambda_{ig}$, *and* $\mathbf{s}_i = (\lambda_{i1}/\lambda_i, \lambda_{i2}/\lambda_i, \ldots, \lambda_{iG}/\lambda_i)$

- Joint production attenuates the distance to the frontier given wedges

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