Debt Indexation, Determinacy, and Inflation

Tobias Kawalec

University of Oxford

November 28, 2025

Presentation at the University of Oxford

Motivation: the prevalence of inflation-indexed debt

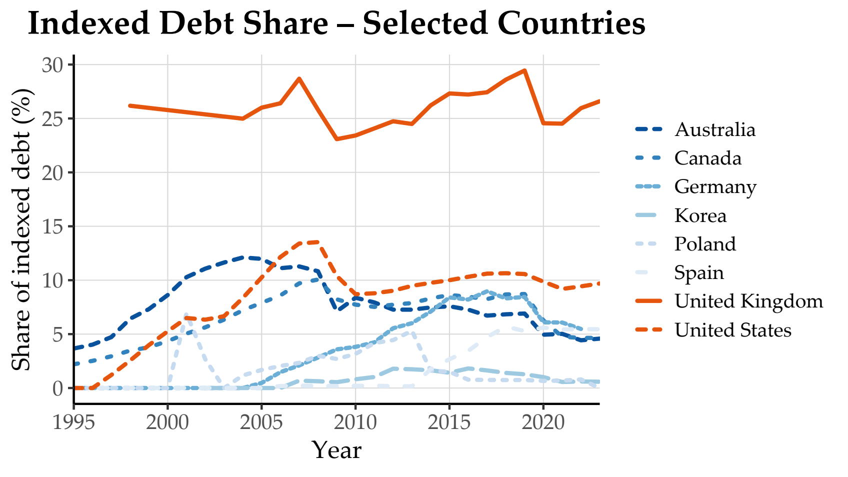


Figure: The prevalence of indexed debt in selected countries over time. Data source: BIS Aggregate Debt Statistics.

Motivation: is indexed debt correlated with lower inflation rates?

Campbell and Shiller (1996): ”[T]he use of indexed debt removes the incentive for the government to erode the real value of its obligations by creating inflation.”

Motivation: is indexed debt correlated with lower inflation rates?

Campbell and Shiller (1996): "[T]he use of indexed debt removes the incentive for the government to erode the real value of its obligations by creating inflation."

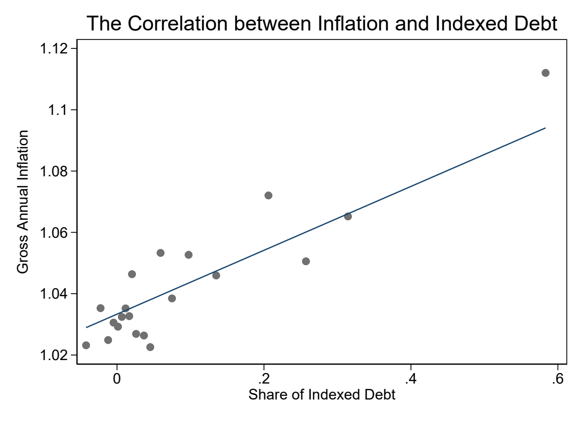


Figure: The correlation between inflation and the share of inflation-indexed debt in BIS member countries since 1999

This paper: inflation-indexed debt and price level dynamics

How does *indexed debt* (TIPS) affect the response of inflation to fiscal deficit shocks?

- Result #1: inflation-indexed debt is a significant driver of inflationary dynamics in response to fiscal deficit shocks both empirically and theoretically
- Result #2: the inflationary effect of indexed debt magnifies especially under a *fiscally-led policy mix* (Bianchi et al., 2023)
- Cause of interesting dynamics: inflation-indexed debt cannot be devalued in nominal terms through inflation (*positive wealth effect for HHs*)

Why did we not care about indexed debt yet?

Contribution

1. We identify indexed debt as a magnifier of fiscal-monetary interactions
   - Deficit-inflation multipliers - 0.13 in base case, 0.53 with high levels of indexed debt
   - Volatility of inflation $\uparrow$, volatility of output and consumption non-linearly affected
   - Sargent and Wallace (1981), Leeper (1991), Sims (1994), Leeper and Leith (2016), Bassetto and Cui (2018), Ascari et al. (2023), Bianchi et al. (2023), Campos et al. (2024), Smets and Wouters (2024), Angeletos et al. (2025)
2. We characterize analytically the value of (non-)indexed debt in a dynamic equilibrium model with imperfect risk-sharing among households
   - Indexed debt influences policy- and heterogeneity-driven channels of inflation differently
   - Fischer (1975), Barr and Campbell (1997), Angeletos (2007), Bassetto and Cui (2018), Brunnermeier et al. (2020), Miao and Su (2021), Kaplan et al. (2023), Angeletos et al. (2025), Jiang et al. (2024), Rachel and Ravn (2025)
3. We provide empirical evidence on the deficit-inflation multiplier with indexed debt
   - Multiplier range of 0.22-0.47
   - Banerjee et al. (2022), Ascari et al. (2024), Hazell and Hobler (2024), Barro and Bianchi (2025)

Table of Contents

A Blanchard-Yaari type model with indexed debt

Households: maximize lifetime utility s.t. mortality risk ω, being able to save in a fixed-share portfolio of government assets R_t^P, with share θ of inflation-indexed bonds.

$$\max_{\{C_{t+k},\, L_{t+k},\, A_{t+k+1}\}_{k=0}^{\infty}} \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k \left[\frac{C_{t+k}^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} - \nu \frac{L_{t+k}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right] \right] \qquad \text{s.t.}$$

$$P_{t+1}A_{t+1} = \frac{R_t^P}{\omega} P_t \left(A_t + \underbrace{Y_t}_{\equiv W_t L_t + \text{Transfers}_t} - C_t - T_t \right), \quad \text{where}$$

$$R_t^P = \theta I_t \frac{P_{t+1}}{P_t} + (1-\theta) I_t = I_t \left(1 + \theta \left(\frac{P_{t+1}}{P_t} - 1 \right) \right), \qquad A_t \equiv \theta b_t + (1-\theta) B_t.$$

Indexed debt magnifies the output-inflation demand-side link

In equilibrium (HH optimality + market clearing), we obtain an **aggregate demand equation**:

- Without indexed debt (Angeletos et al., 2025):

$$y_t = (1-\beta\omega)\underbrace{\left(d_t + \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s (y_{t+s} - t_{t+s})\right)}_{\text{Overall wealth / "PIH"}}$$

$$\underbrace{-\beta\left(\sigma\omega - (1-\beta\omega)\frac{D^{SS}}{Y^{SS}}\right)\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\omega)^s r_{t+s}\right]}_{\text{Income/substitution effect of real interest rates...}}$$

Indexed debt magnifies the output-inflation demand-side link

In equilibrium (HH optimality + market clearing), we obtain an **aggregate demand equation**:

- *With* indexed debt:

$$y_t = (1-\beta\omega)\underbrace{\left(d_t + \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s (y_{t+s} - t_{t+s})\right)}_{\text{Overall wealth / "PIH"}}$$

$$- \underbrace{\beta\left(\sigma\omega - (1-\beta\omega)\frac{D^{SS}}{Y^{SS}}\right)\mathbb{E}_t\left[\sum_{s=0}^{\infty} (\beta\omega)^s (r_{t+s} + \theta\pi_{t+1+s})\right]}_{\text{Income/substitution effect of real rates adjusted for windfall gains from indexed debt}}$$

Indexed debt magnifies the output-inflation demand-side link

$$y_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t \sum_{s=0}^{\infty}(\beta\omega)^s(y_{t+s} - t_{t+s})\right)$$
$$-\beta\left(\sigma\omega - (1-\beta\omega)\frac{D^{SS}}{Y^{SS}}\right)\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\omega)^s(r_{t+s} + \theta\pi_{t+1+s})\right]$$

$\Rightarrow$ Consistent with idea that with *lower* fiscal surpluses, budget balance requires:

1. A concurrent change in equilibrium real interest rates, or
2. A devaluation of the outstanding stock of *non-indexed* debt.

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \sum_{j=0}^{\infty}\prod_{l=1}^{j}\frac{1}{1+r_{t+l}}s_{t+j}$$

Generalization of the debt valuation equation to long-term debt

Formal proof: indexed debt and Ricardian Equivalence (RE)

- Assume monetary+fiscal rules that *partially* absorb the effect of indexed debt (CB sets policy rates in proportion to the *portfolio* return earned by HHs)

$$r_t = \phi y_t - (1 - \omega + \theta)\mathbb{E}_t \pi_{t+1}; \qquad t_t = -\varepsilon_t + \tau_d(d_t + \varepsilon_t) + \tau_y y_t + \beta \frac{D^{SS}}{Y^{SS}} \theta \mathbb{E}_t \pi_{t+1}$$

- We can then prove that **the fiscal-monetary policy mix conditions the link between inflation-indexed debt and deficit-driven inflation**

Formal proof: indexed debt and Ricardian Equivalence (RE)

- Assume monetary+fiscal rules that *partially* absorb the effect of indexed debt (CB sets policy rates in proportion to the *portfolio* return earned by HHs)

$$r_t = \phi y_t - (1 - \omega + \theta)\mathbb{E}_t \pi_{t+1}; \qquad t_t = -\varepsilon_t + \tau_d(d_t + \varepsilon_t) + \tau_y y_t + \beta \frac{D^{SS}}{Y^{SS}} \theta \mathbb{E}_t \pi_{t+1}$$

- We can then prove that **the fiscal-monetary policy mix conditions the link between inflation-indexed debt and deficit-driven inflation**

Proposition (Inflation-indexed debt and non-Ricardian fiscal policy)

Once $\theta > 0$ and fiscal policy is non-Ricardian, there exists a discrete jump in the inflationary impact of deficit shocks between fiscally-led ($\phi < \phi*$) and monetary-led ($\phi \geq \phi*$) policy mixes.

$\Rightarrow$ Proof relies on wealth effects of inflation-indexed debt increasing in household mortality risk.

$\Rightarrow$ Important result in the light of recent debates about the *necessity* of policy rules implying fiscally-driven equilibrium determination (Angeletos et al., 2025; Rachel and Ravn, 2025)

Visualizing the inflationary impact

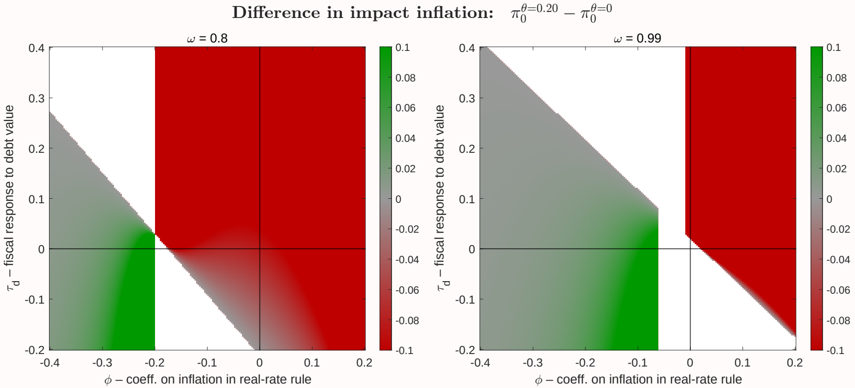


Figure: The space of policy coefficients under which inflation-indexed debt boosts inflation, in line with the precious proposition.

Table of Contents

Heterogeneous households in the spirit of Auclert et al. (2021)

Households maximize

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_{it})-v(N_{it})\right)\right]$$

subject to two budget constraints - one for the aggregate household budget, and one for the semantically separate evolution of indexed debt:

$$P_t c_{it} + Q_t B_{it} = \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta}di}(1-\tau_t)W_t N_{it} + B_{i,t-1} - d_{it}\mathbb{1}_{\{adj_{it}=1\}},$$

$$q_t b_{it} = \Pi_t b_{i,t-1} + d_{it}\mathbb{1}_{\{adj_{it}=1\}},$$

and borrowing constraints + no-Ponzi conditions on the two types of debt.

⇒ Transfers d_{it} from non-indexed to indexed bond holdings can only happen when $adj_{it}=1$ (with probability ν) (Graham and Wright, 2007; Auclert et al., 2024b).

⇒ Supply-side block characterized by standard NKPC ($\pi_t = \kappa y_t + \beta\mathbb{E}_t[\pi_{t+1}]$)

Fiscal and monetary policy rules

- Fiscal policy faces budget constraint $B_{t-1} + \Pi_t b_{t-1} = P_t s_t + Q_t B_t + q_t b_t$.
- Fiscal *rule* manages distortionary tax rates τ_t, with policy coefficients γ_B and γ_b:

$$\frac{\tau_t}{\tau} = \left(\frac{v_{B,t-1}}{v_B}\right)^{\gamma_B} \left(\frac{v_{b,t-1}}{v_b}\right)^{\gamma_b} e^{\varepsilon_t^g}, \qquad v_{B,t} \equiv \frac{Q_t B_t}{P_t y_t}, \; v_{b,t} \equiv \frac{q_t b_t}{P_t y_t}$$

  - Bond prices follow from household SDFs: $Q_t = \beta \mathbb{E}_t \left[\frac{u'(c_{i,t+1})}{u'(c_{it})} \frac{P_t}{P_{t+1}}\right] := \mathbb{E}_t \left[\mathscr{M}_{i,t,t+1}\right]$ and $q_t = \beta \mathbb{E}_t \left[\frac{u'(c_{i,t+1})}{u'(c_{it})}\right] = \mathbb{E}_t \left[\mathscr{M}_{i,t,t+1} \Pi_{t+1}\right]$
- Monetary rule follows common Taylor-type specification:

$$\left(\frac{1+i_t}{1+i}\right) = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} e^{\nu_t}.$$

The debt valuation equation with heterogeneous households

- Because of HH heterogeneity and incomplete markets, simple transversality condition on government debt may fail (Brunnermeier et al., 2024)

Proposition (The government debt valuation equation with indexed debt)

The government debt valuation equation is given by:

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \tilde{\mathscr{M}}_{t,t+k} \bar{A}_{t+k} \right], \tag{1}$$

where $\tilde{\mathscr{M}}_{t,t+k}$ is the weighted average SDF across all households i, and

$$A_{it} \equiv \underbrace{c_{it} - \varepsilon_{it}(1-\tau_{it})\, w_t N_t}_{\text{surpluses}} + \underbrace{\left[\mathrm{Cov}_t\big(\mathscr{M}_{i,t,t+1}, \Pi_{t+1}\big) + \mathscr{M}_{i,t,t+1}\big(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1}\big) \right] \frac{b_{it}}{P_t}}_{\text{wealth effect from surprise inflation through indexed debt}}$$

The debt valuation equation with heterogeneous households

- Because of HH heterogeneity and incomplete markets, simple transversality condition on government debt may fail (Brunnermeier et al., 2024)

Proposition (The government debt valuation equation with indexed debt)

The government debt valuation equation is given by:

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t\left[\sum_{k=0}^{\infty} \tilde{\mathscr{M}}_{t,t+k}\bar{A}_{t+k}\right], \tag{1}$$

where $\tilde{\mathscr{M}}_{t,t+k}$ is the weighted average SDF across all households i, and

$$A_{it} \equiv \underbrace{c_{it} - \varepsilon_{it}(1-\tau_{it})\,w_t N_t}_{\textit{surpluses}} + \underbrace{\left[\mathrm{Cov}_t\big(\mathscr{M}_{i,t,t+1}, \Pi_{t+1}\big) + \mathscr{M}_{i,t,t+1}\big(\mathbb{E}_t\Pi_{t+1} - \Pi_{t+1}\big)\right]\frac{b_{it}}{P_t}}_{\textit{wealth effect from surprise inflation through indexed debt}}$$

⇒ Standard definitions of competitive equilibrium and stationary competitive equilibrium

Full derivation of (1) | Formal equilibrium definition | Proposition: stationary equilibrium uniqueness

Table of Contents

Scenario-dependent calibration

Debt/GDP shares	$\mathbb{P}$(adjustment)
Main calibration: UK debt portfolio	
$B = 0.8176,\ b = 0.3024$	$\nu = 0.2293$
Counterfactual: US debt shares	
$B = 1.0171,\ b = 0.1029$	$\nu = 0.1385$
Counterfactual: no indexed debt	
$B = 1.12,\ b = 0$	$\nu = 0.0052$

(a) Debt shares

	AF/PM	AM/PF	Possible range
ϕ	0.5	1.5	$[0, \infty)$
γ_B	0.5	1.5	$[0, \infty)$
γ_b	0.5	1.5	$[0, \infty)$

(b) Policy combinations

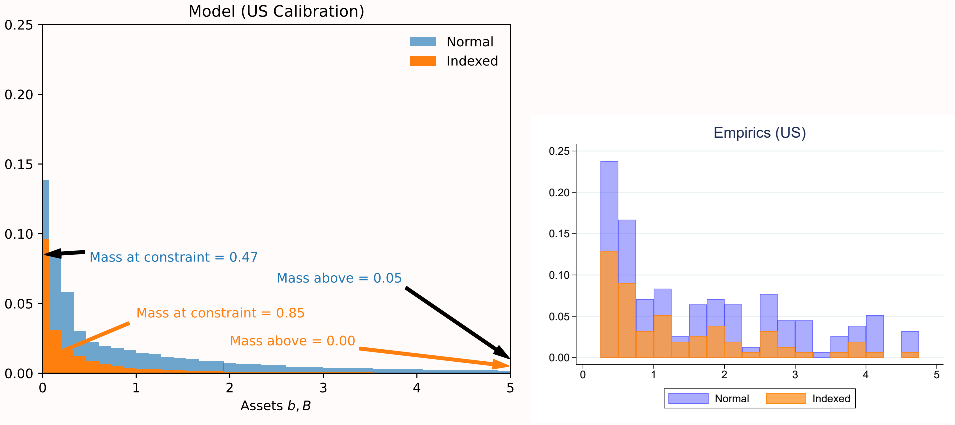
Table: Crucial parameters across different calibration scenarios

$\Rightarrow$ Variation of parameters across scenarios is reasonable and untargeted moments (e.g. $G/Y,\ G/T$) line up with empirical data.

$\Rightarrow$ Solution algorithm of the model based on Auclert et al. (2021) (Sequence-Space Jacobians)

Details on computational approach | Parametrization of remaining model | A primer on the policy parametrizations and determinacy

Non-targeted distribution matches empirics well



(a) Model-based distribution

(b) Empirical distribution (suppressing constrained HHs)

Figure: Comparison of model-based and empirical distributions of debt holdings

Table of Contents

IRFs to persistent government spending shocks I

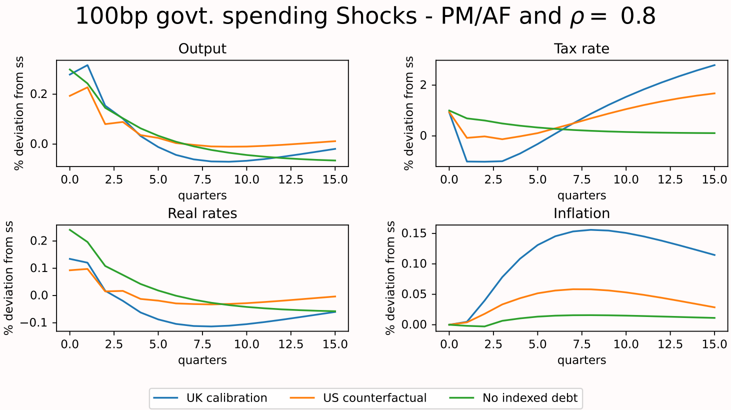


Figure: IRFs to a 100bp government spending shock with a conventional fiscally-led policy mix.

Result I: The (annual) deficit-inflation multiplier peaks at 0.13 without indexed debt, and at 0.53 in the calibration to UK debt shares. Shocks without persistence

IRFs to persistent government spending shocks II

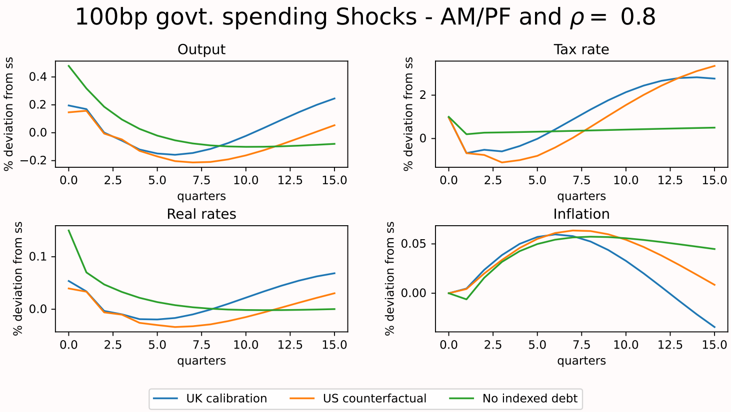


Figure: IRFs to a 100bp government spending shock with a conventional monetary-led policy mix.

Result II: Under monetary-led policy mixes, indexed debt does not amplify inflationary pressure. Shocks without persistence

Decomposing the response of the price level

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \tilde{\mathscr{M}}_{t+k} \left[(\textit{Surpluses} + \textit{Issuance premia}) \frac{b_{i,t+k}}{P_{t+k}} \right] \right]$$

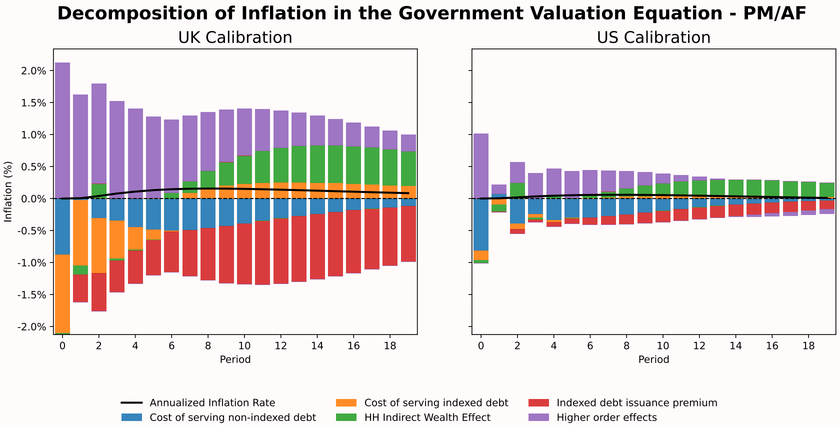


Figure: Decomposition of inflation in response to a 1% government spending shock under a fiscally-led policy mix.

Estimated dynamic moments

	Normalized standard deviations across policy scenarios					
	PM/AF-UK	PM/AF-US debt	PM/AF-NoIndex	AM/PF-UK	AM/PF-US debt	AM/PF-NoIndex
π	0.211	0.183	0.106	0.121	0.113	0.086
Y	0.855	0.761	0.883	1.066	1.055	0.863
C	0.400	0.395	0.331	0.951	0.961	0.346

Table: Normalized standard deviations of aggregate variables in response to fiscal shocks

Result III: indexed debt *barely matters* for inflation volatility under a monetary-led policy mix.

Estimated dynamic moments

	Normalized standard deviations across policy scenarios					
	PM/AF-UK	PM/AF-US debt	PM/AF-NoIndex	AM/PF-UK	AM/PF-US debt	AM/PF-NoIndex
π	0.211	0.183	0.106	0.121	0.113	0.086
Y	0.855	0.761	0.883	1.066	1.055	0.863
C	0.400	0.395	0.331	0.951	0.961	0.346

Table: Normalized standard deviations of aggregate variables in response to fiscal shocks

Result IV: under a fiscally-led policy mix, moving from 0% to 25% indexed debt *doubles* relative inflation volatility.

Estimated dynamic moments

	Normalized standard deviations across policy scenarios					
	PM/AF-UK	PM/AF-US debt	PM/AF-NoIndex	AM/PF-UK	AM/PF-US debt	AM/PF-NoIndex
π	0.211	0.183	0.106	0.121	0.113	0.086
Y	0.855	0.761	0.883	1.066	1.055	0.863
C	0.400	0.395	0.331	0.951	0.961	0.346

Table: Normalized standard deviations of aggregate variables in response to fiscal shocks

Result V: the effect of inflation-indexed debt on inflation volatility is *non-linear*.

Why is the effect nonlinear?

1. Relative shift towards indexed debt in debt valuation equation *relatively* larger when its base level is low
2. Overcoming market incompleteness only for richest HH at first $\Rightarrow$ Strong wealth effect for these HHs $\Rightarrow$ $Cov_t\left(\mathscr{M}_{i,t,t+1}, \Pi_{t+1}\right) \downarrow\downarrow \Rightarrow$ Initial units of b_t contribute more to volatility

Estimated dynamic moments

	Normalized standard deviations across policy scenarios					
	PM/AF-UK	PM/AF-US debt	PM/AF-NoIndex	AM/PF-UK	AM/PF-US debt	AM/PF-NoIndex
π	0.211	0.183	0.106	0.121	0.113	0.086
Y	0.855	0.761	0.883	1.066	1.055	0.863
C	0.400	0.395	0.331	0.951	0.961	0.346

Table: Normalized standard deviations of aggregate variables in response to fiscal shocks

Result VI: the link between output volatility and the share of indexed debt is non-linear under the fiscally-led policy mix.

Is the effect of indexed debt linear in the reaction of fiscal policy?

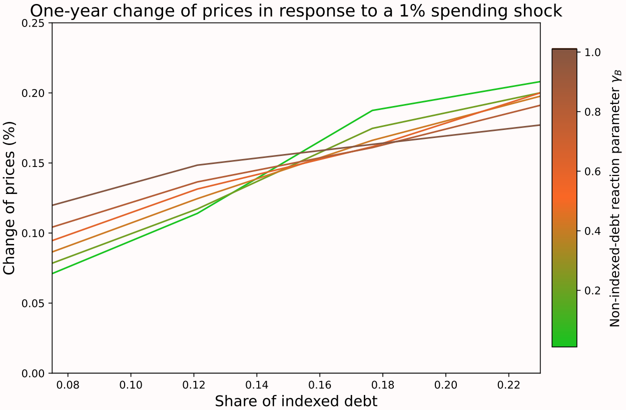
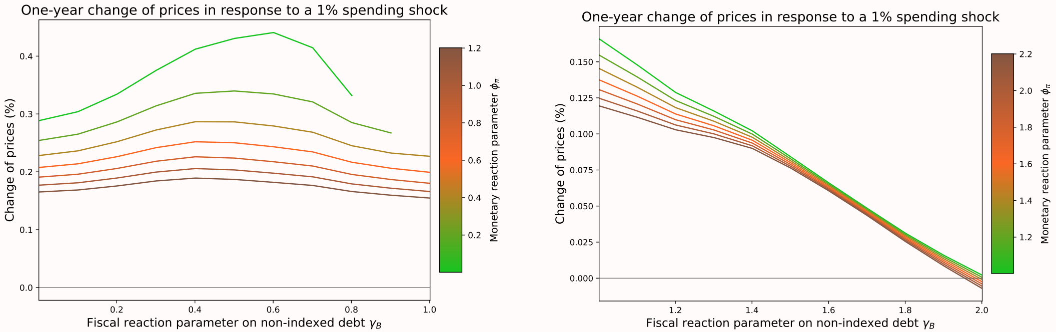


Figure: Cumulative one-year reaction of prices in response to fiscal spending shocks under a fiscally-led policy mix.

Result VII: The more active fiscal policy is, the more sensitive inflationary pressure is to the presence of indexed debt.

Does it matter whether equilibrium is fiscally-led or monetary-led?



(a) Fiscally-led policy mix

(b) Monetary-led policy mix

Figure: The one-year deficit-inflation multiplier in dependence on the monetary and fiscal policy mix in place.

Result VIII: The deficit-inflation multiplier is higher under a fiscally-led policy mix, but the *passive* policy authority has a larger effect on inflation *conditional on being passive*.

Determinacy properties of the quantitative model

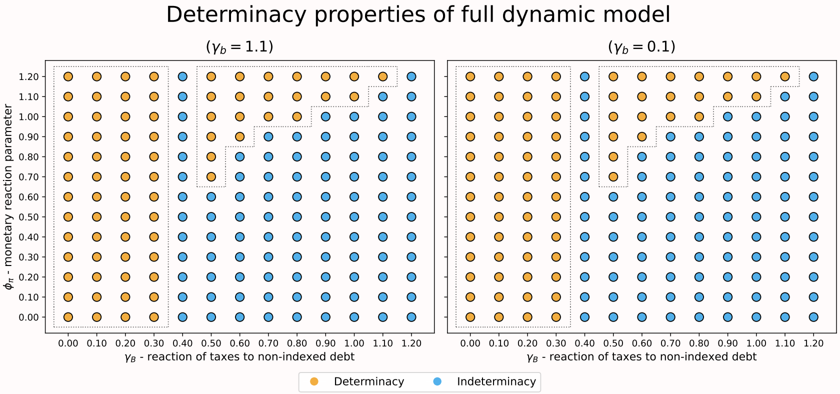


Figure: Determinacy of the generalized Jacobian in relation to choices for the fiscal and monetary policy reaction coefficients.

Result IX: With inflation-indexed debt, so-called "active/active" policy mixes can still yield determinate equilibria.

Analytical derivation of wider determinacy bounds in a conventional pure monetary model

Table of Contents

Expoiting the high-powered 2022 'mini-budget shock'

- Following Hazell and Hobler (2024), we identify an *unexpected* fiscal shock in the UK: the '2022 mini-budget' (GBP 60 billion shortfall)
- Narrative shock component: GBP 47.4 billion (1.27% of annual GDP)
- We then use inflation swaps to back out the expected effects on future inflation:

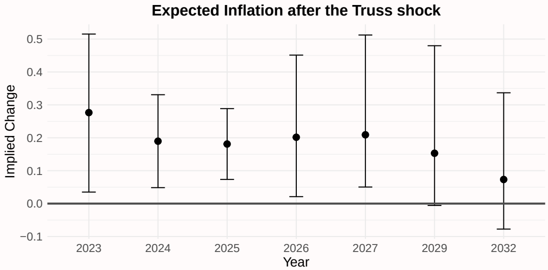


Figure: Implied change in expected inflation within one trading-day after the 2022 'mini-budget' announcement.

$\Rightarrow$ **Estimated fiscal inflation multiplier:** 0.22%-0.47%.

Timing of $\mathbb{E}$(inflation) movements around the mini-budget shock

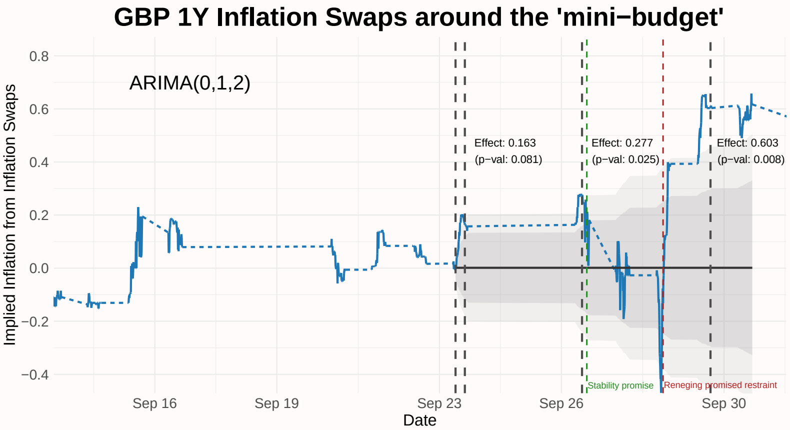


Figure: Implied inflation expectations from one-year GBP Inflation swaps in the period around the 'mini-budget' shock, with data normalized to 0 for September 23, 2022, 09:30am. The gray fan-chart depicts 68% and 95% confidence intervals for implied inflation based on a forecast of the swap price from the moment of the shock onward.

Evidence on the inflationary effect of inflation-indexed debt

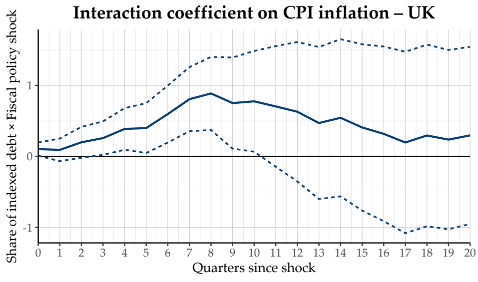
But does the presence of inflation-indexed debt matter for *realized* inflation rates?

$\Rightarrow$ To answer this question, we estimate a country-specific local projection (1981-2019):

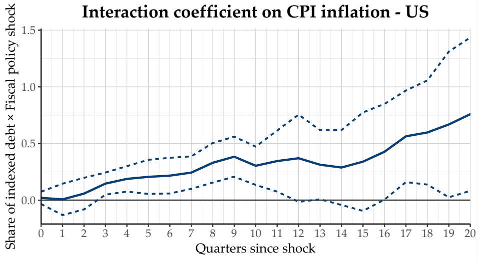
$$\log P_{t+h} - \log P_{t-1} = \alpha_h + \beta_h \Delta\omega_t \varepsilon_t^F + \delta_{1h}\Delta\omega_t + \delta_{2h}\varepsilon_t^F + \Gamma_h Z_{t-1} + e_{t+h}, \quad (2)$$

- Of particular interest to us: coefficient β_h, which captures the cross-effect of the identified fiscal shock ε_t^F and the change in the share of inflation-indexed debt $\Delta\omega_t$
  - Z_{t-1} is a vector of control variables (ΔGDP, Δ unemployment rate, $\mathbb{E}(\pi)$, MPR)
  - Source of shock series: Mierzwa (2024)-narrative shocks (based on Romer and Romer (2010)-method)

Local projection results in the two largest indexed debt markets



(a) United Kingdom



(b) United States

Figure: IRFs implied by the local projection (2). Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1981 Q1 - 2019 Q4.

Additional evidence from high-frequency data on UK bond revaluations

LPs for US in periods of fiscally-led policy mixes

Table of Contents

Conclusion

This paper:

- Provided empirical evidence that inflation-indexed debt boosts inflation in response to sovereign deficit shocks
- Introduced inflation-indexed debt in a state-of-the-art heterogeneous-agent model with rich fiscal-monetary policy interactions
    - Price level becomes a state variable without further ado
    - Price level uniqueness under incomplete markets established
    - Changing the share of inflation-indexed government debt from zero to UK levels increases the fiscal inflation multiplier by 0.40 by under a fiscally-led policy mix
    - A one percentage point increase in the share of inflation-indexed debt in overall government debt increases the volatility of the response of inflation by $\sim 2.6\%$.

Thank you!

References I

Acemoglu, D. and M. K. Jensen (2015). Robust Comparative Statics in Large Dynamic Economies. Journal of Political Economy 123(3), 587–640.

Angeletos, G.-M. (2007). Uninsured Idiosyncratic Investment Risk and Aggregate Saving. Review of Economic Dynamics 10(1), 1–30.

Angeletos, G.-M., C. Lian, and C. Wolf (2025). Deficits and Inflation: HANK meets FTPL. NBER Working Paper (w33102).

Ascari, G., P. Beck-Friis, A. Florio, and A. Gobbi (2023). Fiscal Foresight and the Effects of Government Spending: It's All in the Monetary-Fiscal Mix. Journal of Monetary Economics 134, 1–15.

Ascari, G., D. Bonam, L. Mori, and A. Smadu (2024). Fiscal Policy and Inflation in the Euro Area. DNB Working Paper No. 820.

References II

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub (2021). Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models. Econometrica 89(5), 2375–2408.

Auclert, A., R. Rigato, M. Rognlie, and L. Straub (2024). New Pricing Models, Same Old Phillips Curves? The Quarterly Journal of Economics 139(1), 121–186.

Auclert, A., M. Rognlie, and L. Straub (2023). Determinacy and Existence in the Sequence Space. Manuscript.

Auclert, A., M. Rognlie, and L. Straub (2024a). Fiscal and Monetary Policy with Heterogeneous Agents. NBER Working Paper (w32991).

Auclert, A., M. Rognlie, and L. Straub (2024b). The Intertemporal Keynesian Cross. Journal of Political Economy 132(12), 000–000.

References III

Banerjee, R., V. Boctor, A. N. Mehrotra, and F. Zampolli (2022). Fiscal Deficits and Inflation Risks: The Role of Fiscal and Monetary Regimes. Bank for International Settlements, Monetary and Economic Department.

Barr, D. G. and J. Y. Campbell (1997). Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices. Journal of Monetary Economics 39(3), 361–383.

Barro, R. J. and F. Bianchi (2025). Fiscal Influences on Inflation in OECD Countries, 2020-2023. The Economic Journal, ueaf066.

Bassetto, M. and W. Cui (2018). The Fiscal Theory of the Price Level in a World of Low Interest Rates. Journal of Economic Dynamics and Control 89, 5–22.

Benigno, G., B. Hofmann, G. N. Barrau, and D. Sandri (2024). Quo vadis, r*? The Natural Rate of Interest After the Pandemic. BIS Quarterly Review 4.

References IV

Benigno, P. and G. B. Eggertsson (2023). It's Baaack: The Srge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve. National Bureau of Economic Research WP No. w31197.

Bianchi, F., R. Faccini, and L. Melosi (2023). A Fiscal Theory of Persistent Inflation. The Quarterly Journal of Economics 138(4), 2127–2179.

Brunnermeier, M. K., S. Merkel, and Y. Sannikov (2024). Safe Assets. Journal of Political Economy 132(11), 3603–3657.

Brunnermeier, M. K., S. A. Merkel, and Y. Sannikov (2020). The Fiscal Theory of Price Level with a Bubble. Technical report, National Bureau of Economic Research.

Campbell, J. Y. and R. J. Shiller (1996). A Scorecard for Indexed Government Debt. NBER Macroeconomics Annual 11, 155–197.

References V

Campos, R. G., J. Fernández-Villaverde, G. Nuño, and P. Paz (2024). Navigating by Falling Stars: Monetary Policy with Fiscally Driven Natural Rates. Technical report, National Bureau of Economic Research.

Chen, X., E. M. Leeper, and C. Leith (2022). Strategic Interactions in US Monetary and Fiscal Policies. Quantitative Economics 13(2), 593–628.

Cloyne, J. (2013). Discretionary Tax Changes and the Macroeconomy: New Narrative Evidence from the United Kingdom. American Economic Review 103(4), 1507–1528.

Cochrane, J. H. (2011). Determinacy and Identification with Taylor Rules. Journal of Political Economy 119(3), 565–615.

Fischer, S. (1975). The Demand for Index Bonds. Journal of Political Economy 83(3), 509–534.

References VI

Gagliardone, L., M. Gertler, S. Lenzu, and J. Tielens (2023). Anatomy of the Phillips Curve: Micro Evidence and Macro Implications. National Bureau of Economic Research WP No. w31382.

Graham, L. and S. Wright (2007). Nominal Debt Dynamics, Credit Constraints and Monetary Policy. The BE Journal of Macroeconomics 7(1).

Hagedorn, M. (2021). A Demand Theory of the Price Level. CEPR discussion paper no. DP11364.

Hazell, J., J. Herreno, E. Nakamura, and J. Steinsson (2022). The Slope of the Phillips Curve: Evidence from US States. The Quarterly Journal of Economics 137(3), 1299–1344.

Hazell, J. and S. Hobler (2024). Do Deficits Cause Inflation? A High Frequency Narrative Approach. mimeo.

Jiang, Z., H. Lustig, S. Van Nieuwerburgh, and M. Z. Xiaolan (2024). The US Public Debt Valuation Puzzle. Econometrica 92(4), 1309–1347.

References VII

Kaplan, G., G. Nikolakoudis, and G. L. Violante (2023). Price Level and Inflation Dynamics in Heterogeneous Agent Economies. National Bureau of Economic Research WP no. w31433.

Leeper, E. M. (1991). Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies. Journal of Monetary Economics 27(1), 129–147.

Leeper, E. M. and C. Leith (2016). Understanding Inflation as a Joint Monetary–Fiscal Phenomenon. In Handbook of Macroeconomics, Volume 2, pp. 2305–2415. Elsevier.

Miao, J. and D. Su (2021). Fiscal and Monetary Policy Interactions in a Model with Low Interest Rates. Unpublished Working Paper. Retrieved from: https://people.bu.edu/miaoj/MiaoSuN08.pdf.

Mierzwa, S. (2024). Spillovers from Tax Shocks to the Euro Area. Oxford Economic Papers, gpae024.

References VIII

Nakamura, E., V. Riblier, and J. Steinsson (2025). Beyond the Taylor Rule. Presented at the Jackson Hole Economic Policy Symposium.

Rachel, Ł. and M. Ravn (2025). Brothers in Arms: Monetary-Fiscal Interactions Without Ricardian Equivalence.

Reiter, M. (2009). Solving Heterogeneous-Agent Models by Projection and Perturbation. Journal of Economic Dynamics and Control 33(3), 649–665.

Romer, C. D. and D. H. Romer (2010). The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks. American Economic Review 100(3), 763–801.

Sargent, T. J. and N. Wallace (1981). Some Unpleasant Monetarist Arithmetic. Federal Reserve Bank of Minneapolis Quarterly Review 5(3), 1–17.

Sims, C. A. (1994). A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy. Economic Theory 4, 381–399.

References IX

Smets, F. and R. Wouters (2024). Fiscal Backing, Inflation and US Business Cycles.

Walsh, C. E. (2017). Monetary Theory and Policy. MIT press.

Appendix

A Fisherian model with inflation-indexed debt I

A simple exposition of the importance of inflation-indexed debt for price level determination can be done in a Fisherian model with representative households receiving a constant stream of goods

$$\max_{\{c_t, B_t, b_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where B_t and b_t denote nominal quantities of non-indexed and inflation-indexed debt, respectively, subject to the flow budget constraint

$$P_t c_t + Q_t B_t + q_t b_t = P_t(Y - T_t) + B_{t-1} + \Pi_t b_{t-1}.$$

Optimality conditions yield standard bond pricing kernels:

A Fisherian model with inflation-indexed debt II

$$Q_t = \beta \mathbb{E}_t \left(\frac{P_t}{P_{t+1}} \right); \qquad q_t = \beta.$$

Government: The simple flow budget constraint of the government is given by

$$B_{t-1} + \Pi_t b_{t-1} = P_t T_t + Q_t B_t + q_t b_t.$$

For simplicity, we assume here that $s_t = T_t$.

Standard fiscal policy rule in a Fisherian model: government reacts to deviations of both types of debt in *real* terms from their respective steady-state levels:

$$\frac{\tau_t}{\tau} = \left(\frac{s_{B,t-1}}{s_B} \right)^{\gamma_B} \left(\frac{s_{b,t-1}}{s_b} \right)^{\gamma_b} e^{\zeta_t},$$

A Fisherian model with inflation-indexed debt III

where $\tau_t \equiv \frac{T_t}{Y}$ are surpluses raised by the government as a fraction of output, and $s_{B,t} \equiv \frac{Q_t B_t}{P_t Y}$, $s_{b,t} \equiv \frac{q_t b_t}{P_t Y}$ are the real market values of the two existing types of debt. ζ_t is a standard AR(1) shock to the quantity of lump-sum taxes raised, and the policy reaction coefficients are given by γ_B and γ_b.

The central bank follows a simplified monetary rule:

$$\frac{R_{n,t}}{R_n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi},$$

where $R_{n,t} = 1 + i_t$ is the gross nominal interest rate. Note also that under the present setting $Q_t = \frac{1}{R_{n,t}}$, i.e., the price of the nominal bond must be the inverse of the gross nominal interest rate.

A Fisherian model with inflation-indexed debt IV

Linearizing the simple Fisherian model: we denote variables in their log-deviations from steady-state with hats. A simple (log)-linearization around the zero-inflation steady-state gives us the following system of difference equations:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi \hat{\pi}_t, \tag{A.1}$$

$$\begin{aligned} \hat{s}_{B,t} + \frac{b}{B}\hat{s}_{b,t} = \frac{1}{\beta}\left[\hat{s}_{B,t-1} + \frac{b}{B}\hat{s}_{b,t-1}\right] + \frac{1}{\beta}\left[\hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\frac{B+b}{B}\zeta_t\right] \\ - \frac{1-\beta}{\beta}\frac{B+b}{B}\left[\gamma_B \hat{s}_{B,t-1} + \gamma_b \hat{s}_{b,t-1}\right]. \end{aligned} \tag{A.2}$$

The presence of indexed debt introduces a third first-differenced variable into this policy-side system of equations, $\hat{s}_b$. We therefore need to close this system with some further condition.

A Fisherian model with inflation-indexed debt V

The approach taken here to close the economy is the 'shadow economy' trick used by Bianchi et al. (2023). Following Bianchi et al. (2023), we construct a 'shadow economy' that has the same monetary block, but a simplified fiscal block with *only* non-indexed debt: we set $b_t = 0 \; \forall t$. The underlying assumption behind this 'shadow economy' is tantamount to postulating that the fiscal authority *only* reacts with non-indexed debt in response to fiscal disturbances: when a spending shortage or surplus occurs, the fiscal authority only reacts by adjusting the stock of non-indexed debt. This simplified fiscal block is summarized by the log-linearized equation

$$\hat{s}_{B,t} = \frac{1}{\beta}\left[1 - (1-\beta)\gamma_B\right]\hat{s}_{B,t-1} + \frac{1}{\beta}\left[\hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\zeta_t\right], \tag{A.3}$$

which is the standard Fisherian model with non-indexed debt only. Combining, we obtain the following system in state-space form:

A Fisherian model with inflation-indexed debt VI

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{\beta} & \frac{b}{B} & 1 \\ \frac{1}{\beta} & 0 & 1 \end{bmatrix}}_{=A_0} \mathbb{E}_t \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{s}_{b,t+1} \\ \hat{s}_{B,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi & 0 & 0 \\ 0 & \frac{1}{\beta}\left[\frac{b}{B} - (1-\beta)\frac{B+b}{B}\gamma_b\right] & \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right] \\ 0 & 0 & \frac{1}{\beta}\left[1 - (1-\beta)\gamma_B\right] \end{bmatrix}}_{=A_1} \begin{bmatrix} \hat{\pi}_t \\ \hat{s}_{b,t} \\ \hat{s}_{B,t} \end{bmatrix} + C \begin{bmatrix} \hat{r}_{n,t} \\ \zeta_t. \end{bmatrix}$$

The determinacy properties of this system depend on $Z \equiv A_0^{-1} A_1$. This matrix Z is given by:

$$Z = \begin{bmatrix} \phi & 0 & 0 \\ 0 & \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{b}\gamma_b\right] & -\frac{1-\beta}{\beta}\gamma_B \\ 0 & 0 & \frac{1}{\beta}\left[1 - (1-\beta)\gamma_B\right], \end{bmatrix}$$

with corresponding eigenvalues

$$\left\{ \phi, \quad \frac{1}{\beta}\left[1 - (1-\beta)\gamma_B\right], \quad \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{b}\gamma_b\right] \right\},$$

A Fisherian model with inflation-indexed debt VII

and since the system consists of one forward-looking and two backward-looking variables, we now need one eigenvalue outside the unit circle in modulus and two inside to ensure determinacy. Relevant policy mixes inducing saddle-path stability are thus given by:[1]

- *AM/PF*: $\phi > 1, \gamma_B > 1, \gamma_b > \frac{b}{B+b}$;
- *PM/AF$_B$/PF$_b$*: $\phi < 1, \gamma_B < 1, \gamma_b > \frac{b}{B+b}$;
- *PM/AF$_B$/PF$_b$*: $\phi < 1, \gamma_B > 1, \gamma_b < \frac{b}{B+b}$,

such that active fiscal policy can choose what type of debt to actively take on in response to fiscal shocks. For better intuition, it is instructive to consider two specific types of debt policies:

- $\gamma_b = \gamma_B \frac{b}{B+b}$: under such a debt issuance rule, there are only two distinct eigenvalues, similar to the case above due to the induced co-movement of both types of debt in response to deviations from steady-state.

A Fisherian model with inflation-indexed debt VIII

- $\gamma_b = \gamma_B$: such a fiscal rule would indicate a fiscal authority reacting with similarly-sized deviations of both types of debt from their respective steady-state values in response to shocks to the revenue generated by taxation. In that case, we recover the eigenvalues

$$\left\{\phi, \frac{1}{\beta}\left[1-(1-\beta)\gamma_B\right], \frac{1}{\beta}\left[1-(1-\beta)\frac{B+b}{b}\gamma_B\right]\right\}, \tag{A.4}$$

depending on one fiscal policy reaction parameter and one monetary policy reaction parameter only. Policy combinations supporting saddle-path stability are then given by:

  - *AM/PF*: $\phi > 1$, $\gamma_B > 1$, $\gamma_B > \frac{b}{B+b}$;
  - *PM/AF-1*: $\phi < 1$, $\gamma_B > 1$, $\gamma_B < \frac{b}{B+b}$;
  - *PM/AF-2*: $\phi < 1$, $\gamma_B < 1$, $\gamma_B > \frac{b}{B+b}$.

A Fisherian model with inflation-indexed debt IX

The only viable active fiscal policy combination here is PM/AF-2. Clearly, relative to a standard Leeper (1991)-model, it implies tighter bounds, ruling out 'fully active' fiscal policies of the type $\gamma_B = 0$ so long as $b > 0$: fiscal policy therefore cannot be 'fully active' in the traditional sense, as such behavior would mean an unbounded devaluation of the debt stock as not enough surpluses are raised to service the spiralling costs of indexed debt.

Back (to GE analysis)

More in-depth evidence from the UK for a specific fiscal shock

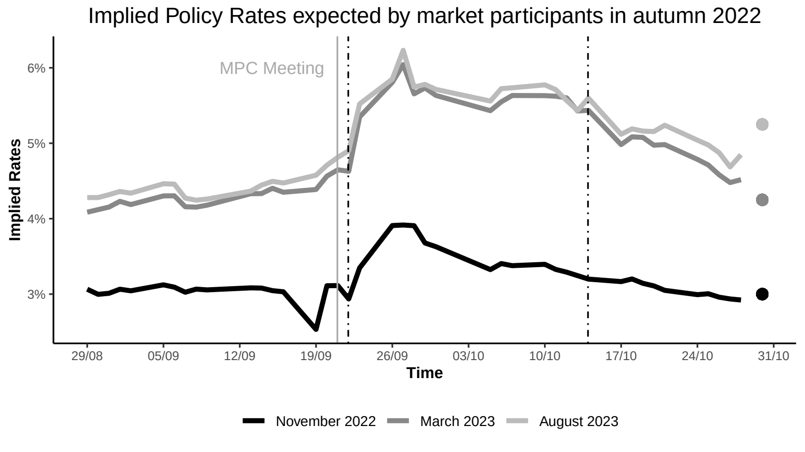


Figure: Expectations of nominal interest rates in the UK for the three MPC meetings after the 'mini-budget' announced in September 2022. The dots at the end reflect the factual values of nominal policy rates after each meeting has taken place.

Back

Evidence from high-frequency bond data I

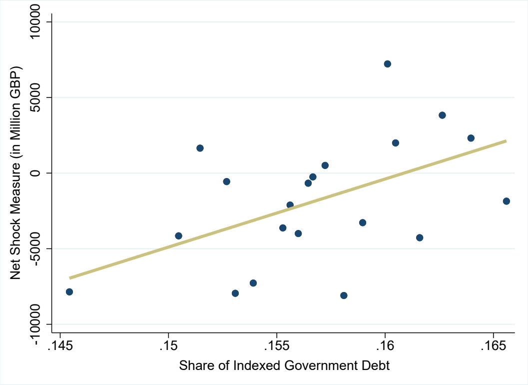
From equation (H.1), we can derive an identity capturing *unexpected* sovereign bond revaluations:

$$NetShock_{t+1} = \Delta \mathbb{E}_{t+1} \left[\sum_{j=0}^{\infty} \beta^j s_{t+1+j} - \sum_{t=0}^{\infty} b_t^{(t+j)} \left(q_{t+1}^{(t+j)} \right) - \frac{\sum_{j=0}^{\infty} B_t^{(t+1+j)} \left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}} \right)}{P_t} \right]. \tag{B.5}$$

$\Rightarrow \Delta \mathbb{E}_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}$ is coming from Cloyne (2013). Using this specification, we estimate:

$$NetShock_t^s = \alpha + \beta \frac{\sum_{j=0}^{\infty} B_t^{*(t+j)}}{\sum_{j=0}^{\infty} b_t^{(t+j)} + B_t^{(t+j)}} + \Gamma_s X_s + \varepsilon_t^s. \tag{B.6}$$

Evidence from high-frequency bond data II



	Dependent variable: Shock Measure	
Share of indexed debt	450.134***	445.734***
	(154.966)	(153.894)
Recession indicator	.	1.588
	.	(3.729)
Constant	−61.745***	−61.88***
	(20.662)	(20.796)
Year-FE	Yes	Yes
Observations	88	88
R^2	0.2907	0.2928
Note:		$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table and Figure: OLS results for the relationship between the share of indexed debt and the new net shock measure in the UK, 2000-2010. The figure shows results for our preferred specification (2). Standard errors are robust to heteroskedasticity. Back

Evidence from Local projections in the US - additional binary indicator on 'only active fiscal policy'

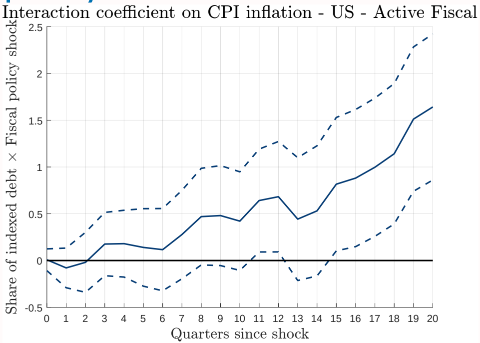


Figure: IRFs implied by the local projection (2) - in the US in periods of active fiscal policy following Chen et al. (2022). The control vector Z consists of the first four lags of the real GDP growth rate, the short-run nominal interest rate, the change in the weighted real exchange rate, and a same-period recession indicator. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1981 Q1 - 2019 Q4.

Back

Motivation: left-skewed inflation-indexed sovereign bond holdings

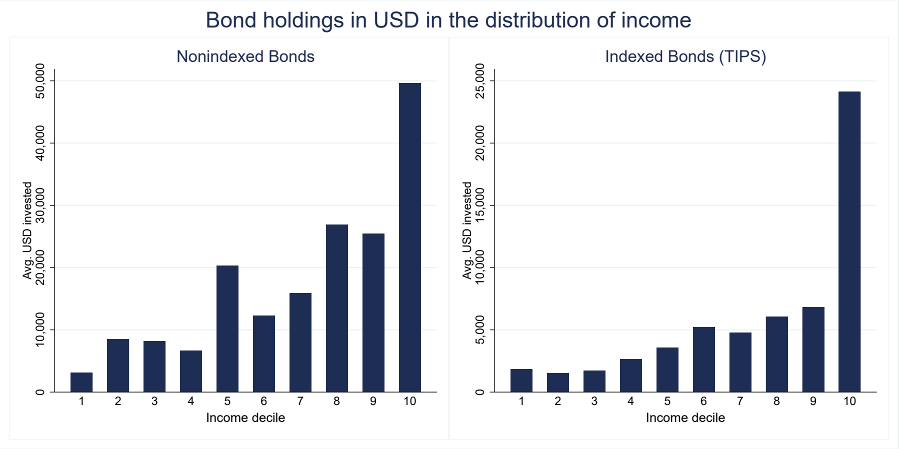


Figure: Distribution of non-indexed and indexed sovereign debt holdings in the US SCF in 2019 by income deciles. Data source: US Survey of Consumer Finances.

Back

Sample IRFs in partial equilibrium - ”news shock”

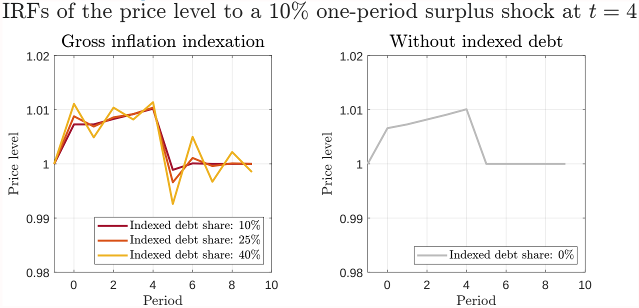


Figure: Normalized IRFs to a 10% decrease in one-period surpluses in $t = 4$ for lower levels of indexed debt.

- Future surplus cut $\Rightarrow$ price level increase today
- With *inflation* indexation: oscillations from over- and under-shooting of adjustment of real value of indexed debt before the factual surplus change
- Initial upwards trend in price level from continuing devaluation of PDV of surpluses

IRFs to one-period government spending shocks

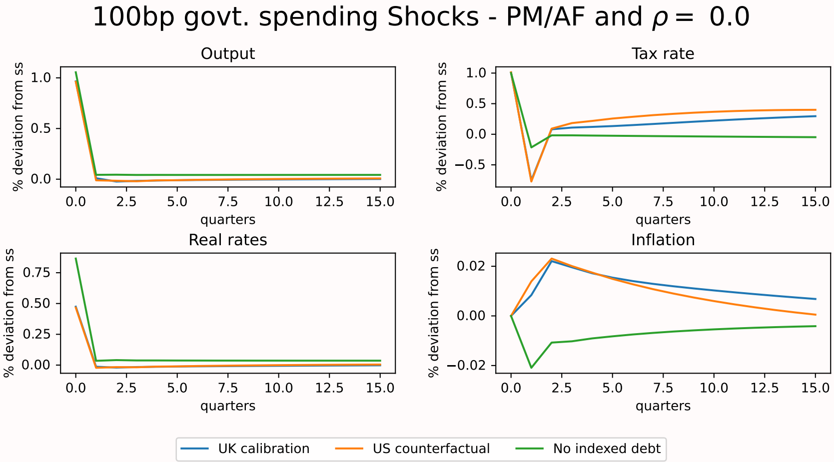


Figure: IRFs to the government spending shock with active fiscal policy - shock without persistence.

Back

IRFs to one-period government spending shocks

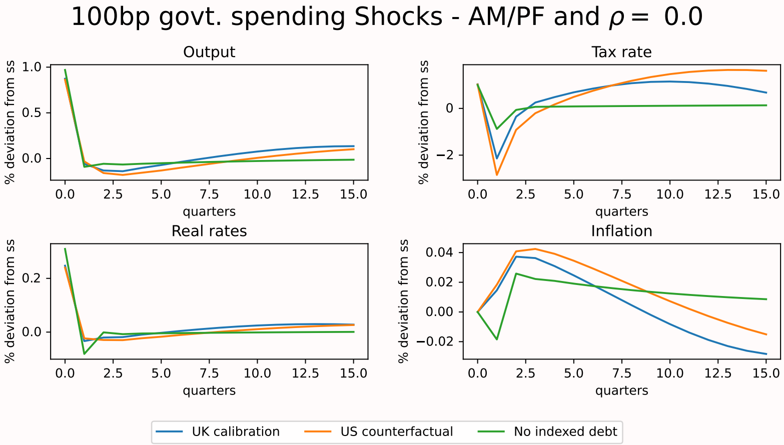


Figure: IRFs to the government spending shock with active monetary policy - shock without persistence.

Back

IRFs to Monetary Policy Shocks

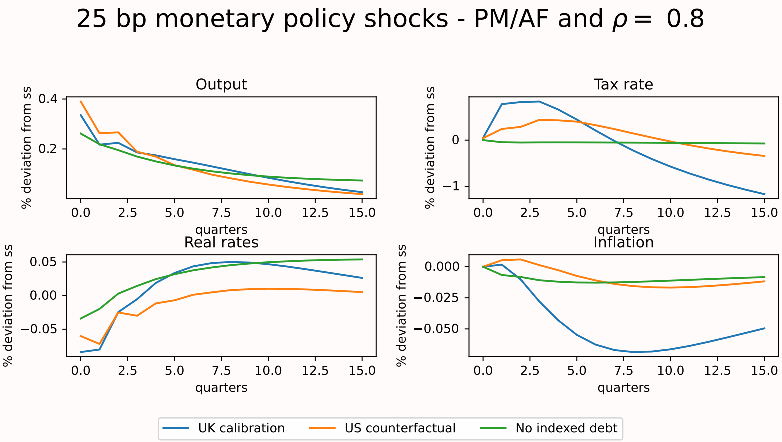


Figure: IRFs to a 25bps expansionary monetary shock - with active fiscal policy and $\rho_G = 0.8$.

Conditions for uniqueness of equilibrium

Proposition (Conditions for stationary equilibrium uniqueness)

Under incomplete markets, with positive steady-state inflation, and abstracting from aggregate uncertainty, the FTPL can determine a unique initial price level in stationary equilibrium even in the presence of inflation-indexed debt and a positive inflation rate if $\frac{b}{b+B} < 1$, $r_{ss} > 0$, *and a steady-state asset demand function* $\mathscr{S}(r_{SS})$ *exists and is invertible.*
The jump variable is P_0 *itself, which is pinned down through the government valuation equation as*

$$P_0 = \frac{\frac{B_0}{1+\pi_{ss}} + \frac{r_{ss}}{1+r_{ss}} b_0}{\frac{\tau_{ss}}{1+i_{ss}} + B_0}. \tag{C.7}$$

Outline of proof Back

Mathematical intuition for stationary equilibrium uniqueness

Eq. conditions (FTPL, asset market, Fisher) can be condensed to one mapping of $\hat{P}$ on itself:

$$\hat{P} = \frac{B_{ss} + b_{ss}(1 + \pi_{ss})}{\tau_{ss} \frac{(1+i_{ss})}{(1+\pi_{ss})}} f(\hat{P}). \tag{C.8}$$

What is π_{ss} here? Two possibilities:

- Independent of taxation, pinned down by Fisher equation (real value of debt may vary)
- Linked to tax schedule $\rightarrow$ constant real value of debt ('true BGP') Definition of tax schedule

$\Rightarrow$ For the FTPL to work with incomplete markets, it is necessary to take away the 'double burden' of determining *both* the real interest rate and the price level from it.

$\Rightarrow$ If the FTPL is meant to do both, we will have either collinearity or no solution

Back

Proof outline of proposition 3 I

We first show that determinacy can indeed be achieved with the FTPL when indexed debt is present, provided that we include a suitable theory of the real interest rate, before showing how indexed debt translates into a model where taxation is assumed to cover all interest expenses over time, following Hagedorn (2021). We therefore maintain a 'true BGP' with a constant real value of the debt portfolio thanks to an appropriate taxation schedule.

To apply the framework of Hagedorn (2021), we have to rewrite the steady-state taxation function to account for possible non-zero steady-state inflation and some positive level of indexed debt, since the presence of both changes the nominal value of taxation over time. We still aim to find an asset demand function depending only on model primitives.[2] To do so, we must pin down steady-state asset demand under incomplete markets in a closed-form solution, for which we will leverage the results of Acemoglu and Jensen (2015).

To find the steady-state level of taxation consistent with the bond issuance schedule that keeps the real value of bonds constant (provided that inflation devalues the non-indexed bonds), we begin with an arbitrary per-period government budget constraint (setting $G_t = 0$, such that real surpluses are $s_t = \tau_t$, or, in nominal terms, $P_t s_t = P_t \tau_t =: T_t$):

$$B_{t-1} + \frac{P_t}{P_{t-1}} b_{t-1} = T_t + Q_t B_t + q_t b_t .$$

Q_t and q_t must be equal to some constant values in steady-state. Without aggregate uncertainty, the bond prices arising through asset demand must solely depend on the offered interest rates, since cross-sectional risks average out. Thus, *in steady-state*, we have that:

Proof outline of proposition 3 II

$$B_{ss} + \Pi_{ss} b_{ss} = T_{ss} + Q_{ss} B_{ss} + q_{ss} b_{ss}$$
$$\Leftrightarrow B_{ss} + \Pi_{ss} b_{ss} = T_{ss} + \frac{1}{1+i_{ss}} B_{ss} + \frac{1}{1+r_{ss}} b_{ss}$$
$$\Leftrightarrow T_{ss} = \left(1 - \frac{1}{1+i_{ss}}\right) B_{ss} + \left(\Pi_{ss} - \frac{1}{1+r_{ss}}\right) b_{ss}.$$

Using the Fisher equation, we can see that $\Pi_{ss} - \frac{1}{1+r_{ss}} = \frac{1+i_{ss}}{1+r_{ss}} - \frac{1}{1+r_{ss}} = \frac{i_{ss}}{1+r_{ss}}$, and therefore:

$$T_{ss} = \frac{i_{ss}}{1+i_{ss}} B_{ss} + \frac{i_{ss}}{1+r_{ss}} b_{ss},$$

which can be expressed in real terms (as the household cares about real taxation) as

$$\tau_{ss} = \frac{r_{ss}}{1+i_{ss}} B_{ss} + \frac{r_{ss}}{1+r_{ss}} b_{ss}.$$

Define by $S_t\left(\Omega_t, \{1+r_l, \tau_l\}_t^\infty\right)$ the cumulative asset demand function under incomplete markets, which depends on the household distribution of wealth Ω_t, real interest rates $1+r_t$, and tax rates τ_t, and is well-defined under standard regularity conditions (Acemoglu and Jensen, 2015). To relate steady-state taxation more clearly to gross asset demand, we fix the shares of

Proof outline of proposition 3 III

B_{ss} and b_{ss} of gross asset demand S_{ss} in steady-state. Denoting by ω the share of indexed debt b_{ss} in the steady-state asset portfolio, the taxation term in steady-state finally becomes

$$\tau_{ss} = \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}.$$

Under such steady-state taxes, the gross asset demand function arising from heterogeneous household demand ($S_{t+1} = \mathscr{S}(\Omega_t; 1+r_t, 1+r_{t+1}, 1+r_{t+2}, \ldots; \tau_t, \tau_{t+1}, \ldots)$) simplifies to the following mapping in steady-state:

$$S_{ss} = \mathscr{S}\left(\Omega_{ss}; 1+r_{ss}, 1+r_{ss}, 1+r_{ss}, \ldots; \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}, \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}, \ldots\right).$$

With i_{ss} being equal to some constant set by the monetary policymaker in steady-state and the taxation function that we just derived, asset demand can be derived by finding the fixed point of the above equation, which would yield asset demand as a function of the real interest rate r_{ss}, following Acemoglu and Jensen (2015):

$$\underline{\text{Asset demand:}} \qquad S(r).$$

Proof outline of proposition 3 IV

From our previous derivations, we directly leverage asset supply in real terms as the left-hand side of our derivations of the fiscal theory equation evaluated in steady-state, such that the stationary asset market equilibrium must be pinned down by

$$S(r) = \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}(1+\pi_{ss})},$$

or, making use of the Fisher equation,

$$S(r) = \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}} \frac{(1+r_{ss})}{(1+i_{ss})}.$$

An important question relates to the source of π_{ss}, the posited non-zero steady-state inflation rate in this economy. Following the contribution of Hagedorn (2021), we posit that the only possible non-zero steady-state inflation rate is the one consistent with a corresponding increase in taxation over time alongside this inflationary path:

$$1 + \pi_{ss} = \frac{T' - T}{T},$$

where variables with a prime denote next period values. Since T represents nominal taxes, the above statement is equivalent to the claim that *real* taxes remain constant.

Proof outline of proposition 3 V

Given the bond portfolio on offer, we can express the above condition as follows:

$$
\begin{aligned}
1+\pi_{ss} &= (1-\omega)\frac{B'-B}{B} + \omega\frac{b'-b}{b}\cdot(1+\pi_{ss}) \\
\Leftrightarrow 1+\pi_{ss} &= \frac{(1-\omega)\frac{B'-B}{B}}{1-\omega\frac{b'-b}{b}},
\end{aligned}
$$

where the inflation-adjustment on the right-hand side in the first line follows from the adjustment of the face value of inflation-indexed debt. This bond issuance schedule therefore can be considered to pin down steady-state inflation.

*Using the FTPL to determine the price **level***: We can now invoke the above derivations within the FTPL to pin down the price level uniquely, provided that we can recover the real interest rate from the asset market.

Following our above reasoning, that steady-state real interest rate can indeed be recovered from the asset market through household demand, provided that this demand function is invertible, as

$$
r_{ss} = S^{-1}\left(\frac{B}{\tilde{P}} + \frac{b}{\tilde{P}(1+\pi_{ss})}\right),
$$

Proof outline of proposition 3 VI

which we can insert in the stationary intertemporal FTPL equilibrium ($\frac{B}{\tilde{P}} + \frac{B}{\tilde{P}(1+\pi_{ss})} = \sum_{j=0}^{\infty} \left(\frac{1}{1+r_{ss}}\right)^j \bar{s}$) with $r_{ss} > 0$ (such that the right-hand side can be rewritten as a geometric sum, $\sum_{j=0}^{\infty} \left(\frac{1}{1+r_{ss}}\right)^j = \frac{1+r_{ss}}{r_{ss}}$) to get the following condition:

$$\frac{B_{ss} + b_{ss}(1+\pi_{ss})}{\tilde{P}} = \bar{s}\frac{1+r_{ss}}{r_{ss}},$$

and the fixed point of this equation pins down the price level uniquely, given asset market optimality. To be precise, given our earlier definition of the surplus process, i.e., $\bar{s} = \tau_{ss} = \frac{r_{ss}}{1+i_{ss}} B_{ss} + \frac{r_{ss}}{1+r_{ss}} b_{ss}$, we have

$$\frac{B_{ss} + b_{ss}(1+\pi_{ss})}{\tilde{P}} = \left[\frac{r_{ss}}{1+i_{ss}} B_{ss} + \frac{r_{ss}}{1+r_{ss}} b_{ss}\right] \frac{1+r_{ss}}{r_{ss}}.$$

Using the Fisher equation ($(1+i_{ss}) = (1+r_{ss})(1+\pi_{ss})$), we can simplify this equilibrium relation to:

$$\frac{B_{ss}}{\tilde{P}} + \frac{b}{\tilde{P}(1+\pi_{ss})} = (1+\pi_{ss})B + b,$$

which eventually pins down the price level as

Proof outline of proposition 3 VII

$$\tilde{P} = \frac{B_{ss} + b_{ss}(1+\pi_{ss})}{(1+\pi_{ss})B_{ss} + b_{ss}}.$$

From the taxation schedule (which is a fiscal variable itself, actively managed by fiscal policy), we can recover the steady-state inflation rate. We simplify this by utilizing the steady-state growth rates $\frac{B'-B}{B} =: g_B$ and $\frac{b'-b}{b} =: g_b$, such that steady-state inflation becomes $1+\pi_{ss} = \frac{(1-\omega)\frac{B'-B}{B}}{1-\omega\frac{b'-b}{b}} = \frac{(1-\omega)g_B}{1-\omega g_b}$. Thus, the initial price level in this steady-state is given by:

$$\tilde{P} = \frac{B_{ss} + b_{ss}\frac{(1-\omega)g_B}{1-\omega g_b}}{B_{ss}\frac{(1-\omega)g_B}{1-\omega g_b} + b_{ss}},$$

with the bond growth rates themselves being fiscal choice variables in the stationary equilibrium.

Back

Full derivation of FTPL equation with heterogeneous agents I

We here present the derivations underlying a *dynamic trading perspective* for asset valuation laid out in Brunnermeier et al. (2024), which avoids fallacies related to a possibly nonexistent aggregate transversality condition by clearly defining the valuation differences of government debt between households and the government based off the insurance properties that government bonds bear for households. This allows us to leverage household-level transversality conditions to derive an aggregate FTPL-type condition that only holds for one initial candidate price level.

The starting point for this valuation equation of government debt is the household budget constraint, which we recall was given by

$$P_t c_{it} + Q_t B_{it} + q_t b_{it} = \varepsilon_{it}(1 - \tau_{it}) P_t w_t N_t + B_{i,t-1} + \Pi_t b_{i,t-1}$$

for each household i. Following our results derived in the household block, we let households price bonds in accordance with their *SDF*:

$$B_{i,t-1} + \Pi_t b_{i,t-1} = \mathbb{E}_t \left(\mathscr{M}_{i,t,t+1} \right) B_{it} + \mathbb{E}_t \left(\Pi_{t+1} \mathscr{M}_{i,t,t+1} \right) b_{it} + P_t (c_{it} - \varepsilon_{it} w_t N_t (1 - \tau_{it})).$$

Full derivation of FTPL equation with heterogeneous agents II

Splitting up the second expectation term, we get

$$B_{i,t-1} + \Pi_t b_{i,t-1} = \mathbb{E}_t\left(\mathscr{M}_{i,t,t+1}\right) B_{it} + \mathbb{E}_t\left(\mathscr{M}_{i,t,t+1}\right) \mathbb{E}_t\left(\Pi_{t+1}\right) b_{it}$$
$$+ b_{it} Cov_t\left(\mathscr{M}_{i,t,t+1}, \Pi_{t+1}\right) + P_t(c_{it} - \varepsilon_{it} w_t N_t(1-\tau_{it})).$$

We divide all elements by P_t and add/subtract relevant terms on the right-hand side to ensure that we can iterate on the resulting expression:

$$\frac{B_{i,t-1} + \Pi_t b_{i,t-1}}{P_t} = \mathbb{E}_t\left(\mathscr{M}_{i,t,t+1}\right) \Pi_{t+1} \left[\frac{B_{it} + \Pi_{t+1} b_t}{P_{t+1}}\right] + \left(c_{it} - \varepsilon_{it}(1-\tau_{it}) w_t N_t\right)$$
$$+ Cov_t\left(\mathscr{M}_{i,t,t+1}, \Pi_{t+1}\right) \frac{b_{it}}{P_t} + \mathbb{E}_t\left(\mathscr{M}_{i,t,t+1}\right) \frac{b_{it}}{P_t}\left(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1}\right).$$

We can now start iterating on this expression. The first iteration yields:

Full derivation of FTPL equation with heterogeneous agents III

$$\frac{B_{i,t-1}+\Pi_t b_{i,t-1}}{P_t} = \mathbb{E}_t\left(\mathscr{M}_{i,t,t+1}\right)\Pi_{t+1}\left[\mathbb{E}_{t+1}\left(\mathscr{M}_{i,t+1,t+2}\right)\Pi_{t+2}\left[\frac{B_{i,t+1}+\Pi_{t+2}b_{i,t+1}}{P_{t+2}}\right]\right.$$
$$(c_{i,t+1}-\varepsilon_{i,t+1}(1-\tau_{i,t+1})w_{t+1}N_{t+1}) + Cov_{t+1}\left(\mathscr{M}_{i,t+1,t+2},\Pi_{t+2}\right)\frac{b_{i,t+1}}{P_{t+1}}$$
$$\left. + \mathbb{E}_{t+1}\left(\mathscr{M}_{i,t+1,t+2}\right)\frac{b_{i,t+1}}{P_{t+1}}\left(\mathbb{E}_{t+1}\Pi_{t+2}-\Pi_{t+2}\right)\right]$$
$$+(c_{it}-\varepsilon_{it}(1-\tau_{it})w_t N_t) + Cov_t\left(\mathscr{M}_{i,t,t+1},\Pi_{t+1}\right)\frac{b_{it}}{P_t} + \mathbb{E}_t\left(\mathscr{M}_{i,t,t+1}\right)\frac{b_{it}}{P_t}\left(\mathbb{E}_t\Pi_{t+1}-\Pi_{t+1}\right).$$

Continuing rolling over, applying the LIE, and simplifying SDFs by making use of the identity $\mathscr{M}_{i,t,t+k}\mathscr{M}_{i,t+k,t+l} = \mathscr{M}_{i,t,t+l}\ \forall t,k,l$, we eventually end up with:

$$\frac{B_{i,t-1}+\Pi_t b_{i,t-1}}{P_t} = \mathbb{E}_t\left[\sum_{k=0}^{\infty}\mathscr{M}_{i,t,t+k}\Pi_{t+1,t+k+1}\left\{(c_{i,t+k}-\varepsilon_{i,t+k}(1-\tau_{i,t+k})w_{t+k}N_{t+k})\right.\right.$$
$$\left.\left. + \left[Cov_{t+k}\left(\mathscr{M}_{i,t+k,t+k+1},\Pi_{t+k+1}\right)+\mathscr{M}_{t+k,t+k+1}\left(\mathbb{E}_{t+k}\Pi_{t+k+1}-\Pi_{t+k+1}\right)\right]\frac{b_{i,t+k}}{P_{t+k}}\right\}\right] \tag{D.9}$$
$$+\lim_{T\to\infty}\left\{\mathbb{E}_t\left[\mathscr{M}_{i,t,t+T}\left(\frac{B_{i,t+T}+\Pi_{t+T+1}b_{i,t+T}}{P_{t+T}}\right)\right]\right\},$$

Full derivation of FTPL equation with heterogeneous agents IV

where we use the notation $\Pi_{t+1,t+k+1}$ to define gross inflation from period $t+1$ to period $t+k+1$. This is the integrated household budget constraint at optimality, from which we hope to derive the government valuation equation.

Crucially, we note that household optimality implies $\lim_{T\to\infty} \frac{B_{i,T}+\Pi_{T+1}b_{i,T}}{P_T} \leq 0$, while a no-Ponzi condition on household debt holdings ensures that $\lim_{T\to\infty} \frac{B_{i,T}+\Pi_{T+1}b_{i,T}}{P_T} \geq 0$. Furthermore, by the definition of the SDF and the properties of a standard CRRA utility function, $\lim_{T\to\infty} \mathscr{M}_{i,t,T} \neq \pm\infty$. Therefore, the final limit converges to 0 and must not be considered.

The formulation of equation (D.9) is intuitive: the real value of household bond holdings is equal to its expected discounted consumption benefits from today to infinity (as future net consumption earnings are suitably discounted with the SDF, which is a mirror image of the price of the two bonds), adjusted suitably for additional surprise earnings enjoyed from holdings of *indexed* sovereign debt: these are decreased by surprise inflation through its (negative) covariance with the SDF (as higher *future* inflation pushes the SDF down), and increased by surprise inflation through a level effect (since such inflation yields a windfall gain relative to what was paid for the indexed bond in the previous period).

Full derivation of FTPL equation with heterogeneous agents V

We now aggregate these individual household bond constraints up to an integrated government valuation equation. We make use of the asset market clearing conditions $B_t = \sum_i B_{it}$ and $b_t = \sum_i b_{it}$ and of the idea that the household TVCs hold individually to get the following expression:

$$\frac{B_{t-1} + \Pi_t b_{t-1}}{P_t} = \sum_i \left\{ \mathbb{E}_t \left[\sum_{k=0}^{\infty} \mathscr{M}_{i,t,t+k} \Pi_{t+1,t+k+1} \left\{ \left(c_{i,t+k} - \varepsilon_{i,t+k}(1 - \tau_{i,t+k}) w_{t+k} N_{t+k} \right) \right. \right. \right.$$
$$\left. \left. \left. + \left[Cov_{t+k} \left(\mathscr{M}_{i,t+k,t+k+1}, \Pi_{t+k+1} \right) + \mathscr{M}_{t+k,t+k+1} \left(\mathbb{E}_{t+k} \Pi_{t+k+1} - \Pi_{t+k+1} \right) \right] \frac{b_{i,t+k}}{P_{t+k}} \right\} \right] \right\}. \quad \text{(D.10)}$$

We simplify this equation by noting that we can take the summation into the expectation and switch around the order of summation. To further simplify the integrated government budget valuation equation, we create the variable A_{it} which captures the surpluses raised by the government from each household i:

$$A_{it} \equiv c_{it} - \varepsilon_{it}(1 - \tau_{it}) w_t N_t + \left[Cov_t \left(\mathscr{M}_{i,t,t+1}, \Pi_{t+1} \right) + \mathscr{M}_{i,t,t+1} \left(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right) \right] \frac{b_{it}}{P_t},$$

which is the full portfolio return of household i of holding an additional unit of net worth. Alternatively, one can view this as what the government factually can raise as surplus from each household i.

Full derivation of FTPL equation with heterogeneous agents VI

We additionally define $\bar{A}_t = \sum_i A_{it}$ as the sum of all individual-level surpluses. We can then rewrite the implied intertemporal government valuation equation (D.10) to:

$$\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\sum_i \mathscr{M}_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}} \right) \bar{A}_{t+k} \right],$$

or, defining the *household value-weighted SDF* $\tilde{\mathscr{M}}_{t,t+k} = \sum_i \mathscr{M}_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}}$, we finally arrive at:

$$\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \tilde{\mathscr{M}}_{t,t+k} \bar{A}_{t+k} \right], \tag{D.11}$$

where $\tilde{\mathscr{M}}_{t,t+k}$ is now the weighted average SDF across all households i, adjusted for inflation, with weights being proportionate to A_{it}, consisting of the net utility gain from saving, the insurance premium on indexed debt (captured through the covariance term), and the possible windfall gain/loss from surprise inflation (captured through the last term in the definition of A_{it}). Equation (D.11) is 'the FTPL equation' that is used to pin down the price level at time t, given some previous price level P_{t-1}. Back

Taxation schedule for constant real debt value on BGP I

If the real value of the debt stock is to be constant on the BGP, we need to first determine the real interest cost of debt and subsequently define the appropriate taxation schedule, which in turn will feed back to the inflation rate.

Thus, *in steady-state*, we have that:

$$
\begin{aligned}
B_{ss} + \Pi_{ss} b_{ss} &= T_{ss} + Q_{ss} B_{ss} + q_{ss} b_{ss} \\
\Leftrightarrow B_{ss} + \Pi_{ss} b_{ss} &= T_{ss} + \frac{1}{1 + i_{ss}} B_{ss} + \frac{1}{1 + r_{ss}} b_{ss} \\
\Leftrightarrow T_{ss} &= \left(1 - \frac{1}{1 + i_{ss}}\right) B_{ss} + \left(\Pi_{ss} - \frac{1}{1 + r_{ss}}\right) b_{ss}.
\end{aligned}
$$

Using the Fisher equation, we can see that $\Pi_{ss} - \frac{1}{1+r_{ss}} = \frac{1+i_{ss}}{1+r_{ss}} - \frac{1}{1+r_{ss}} = \frac{i_{ss}}{1+r_{ss}}$, and therefore:

$$
T_{ss} = \frac{i_{ss}}{1 + i_{ss}} B_{ss} + \frac{i_{ss}}{1 + r_{ss}} b_{ss},
$$

which can be expressed in real terms (as the household cares about real taxation) as

Taxation schedule for constant real debt value on BGP II

$$\tau_{ss} = \frac{r_{ss}}{1+i_{ss}} B_{ss} + \frac{r_{ss}}{1+r_{ss}} b_{ss}.$$

To relate steady-state taxation more clearly to gross asset demand, we fix the shares of B_{ss} and b_{ss} of gross asset demand S_{ss} in steady-state. Denoting by ω the share of indexed debt b_{ss} in the steady-state asset portfolio, the taxation term in steady-state finally becomes

$$\tau_{ss} = \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}.$$

Following the Hagedorn (2021)-DTPL contribution, I posit that the only possible non-zero steady-state inflation rate is the one consistent with a corresponding increase in taxation over time alongside this inflationary path:

$$1+\pi_{ss} = \frac{T'-T}{T},$$

where variables with a prime denote next period values. Since T represents nominal taxes, the above statement is equivalent to the claim that *real* taxes remain constant.

Taxation schedule for constant real debt value on BGP III

Given the bond portfolio on offer, we can express the above condition as follows:

$$
\begin{aligned}
1+\pi_{ss} &= (1-\omega)\frac{B'-B}{B} + \omega\frac{b'-b}{b}\cdot(1+\pi_{ss}) \\
\Leftrightarrow 1+\pi_{ss} &= \frac{(1-\omega)\frac{B'-B}{B}}{1-\omega\frac{b'-b}{b}},
\end{aligned}
$$

where the inflation-adjustment on the right-hand side in the first line follows from the adjustment of the face value of inflation-indexed debt. This bond issuance schedule therefore can be considered to pin down steady-state inflation.

Back

Definition: Competitive Equilibrium

A competitive equilibrium of the heterogeneous-agent economy is an allocation $\{C_t, N_t, Y_t, B_t, b_t, Y_{it}, N_{it}, d_t, \tau_t\}_{t=0}^{\infty}$, together with prices $\{P_t, P_{it}, w_t, \pi_t, Q_t, q_t, 1+i_t\}_{t=0}^{\infty}$ and exogenous variables $\{R_t^*, Z_t, G_t, \}_{t=0}^{\infty}$, such that:

- all agents maximize their utility with suitable policy functions on $c(\cdot)$, $N(\cdot)$, $B(\cdot)$, and $b(\dot{})$, solving the type-dependent value functions,
- all firms maximize their PDV of profits,
- the government does not violate its per-period budget constraint, levies taxes in accordance with its fiscal rule, and the price level is determined through equation (1),
- the central bank follows its policy rule (10),
- all markets clear, and
- the distribution of household wealth and productivity $\Gamma_t(B, b, z)$ evolves by its law of motion and is determined in the long-run by the fixed point of its evolution:

$$\Gamma_{t+1}(\mathscr{B},, z') = \int_{\{(B,b,z):g_t(B,b,z)\in(\mathscr{B},)\}} Pr(z'|z)d\Gamma_t(\mathscr{B},, z).$$

Back

Calibration I

Parameter	Description	Value	Source/Target
Firms			
Y	Steady-state output	1	Normalization
ε	Elasticity of substitution between product varieties	9	Firm mark-up of 11% (Auclert et al., 2024a)
κ	Slope of price Phillips curve	0.055	Hazell et al. (2022), Gagliardone et al. (2023), Benigno and Eggertsson (2023)
Households			
σ	Inverse of intertemporal elasticity of substitution	1	Simplification for simulation
φ	Inverse of Frisch elasticity of labor supply	1	Simplification for simulation
$\underline{B}$	Lower bound of non-indexed debt holdings	0	
$\underline{b}$	Lower bound of indexed debt holdings	0	
ρ_z	Persistence of AR(1) shocks to household productivity	0.966	Auclert et al. (2021)
σ_z	Standard deviation of AR(1) shocks to household productivity	0.92	Auclert et al. (2021)

Calibration II

Government			
T/G	Steady-state surplus, measured by the tax-to-government spending ratio	1.03	See explanation below
r^*	Natural rate of interest	0.0125	Benigno et al. (2024)
ρ_M	Inertia in Taylor-type interest rate rule	0	Simplification
ϕ_π	Monetary policy reaction to inflation deviations from steady-state	$\{0.5, 1.5\}$	For fiscally-led/monetary-led policy mix (Bianchi et al., 2023)
ϕ_y	Monetary policy reaction to output deviations from steady-state	0.3	
γ_B	Fiscal policy reaction to deviations of market value of non-indexed debt from steady-state	$\{0.3, 1.5\}$	For fiscally-led/monetary-led policy mix (Bianchi et al., 2023)
γ_b	Fiscal policy reaction to deviations of market value of indexed debt from steady-state	0.6	

Calibration III

Computational parameters			
n_z	Number of points in asset grid for household productivity shock	11	
n_b	Number of points in asset grid for indexed debt	50	
n_B	Number of points in asset grid for non-indexed debt	50	
$\bar{B}$	Maximum holdings of non-indexed debt in asset grid	5000	
$\bar{b}$	Maximum holdings of indexed debt in asset grid	5000	Approximation to Auclert et al. (2024)
T	Number of periods used in simulations of Jacobians	300	Auclert et al. (2021)

Table: Baseline parametrization for the quantitative estimation

Calibration IV

Back

Overview: Why did we not care about indexed debt yet?

- "Standard" NK paradigm: focus on real & inflation determinacy through restrictions on monetary policy rule (Cochrane, 2011; Walsh, 2017)
  - Characterized by "Taylor Rule" ($\hat{i}_t = \varphi \hat{\pi}_t + \varepsilon_t$), where $\varphi > 1$.
  - This is *usually* supported by fiscal policy in the background: $\hat{\tau}_t = \gamma_B \hat{v}_t$, where $\gamma_B > 1$.
  - Here: debt composition not first-order relevant for price level dynamics
  - Idea that $\varphi > 1$ challenged by Nakamura et al. (2025)
- Resurgent trend: joint consideration of monetary and fiscal policy, co-characterized through a 'government debt valuation equation'

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left(\frac{1}{1+\bar{r}} \right)^j (T_{t+j} - G_{t+j}) \quad \text{(H.1)}$$

  - Equilibrium-determining under a 'fiscally-led policy mix' (Leeper, 1991; Bianchi et al., 2023)
  - Present in most macro models (even if in the background) Back

Computational approach

- Main algorithm utilizes sequence-space Jacobians used by Auclert et al. (2021) (refinement of Reiter (2009)-method)
- Solutions are perfect-foresight in aggregates in response to time-zero unexpected disturbances, but are fully nonlinear in idiosyncracies
  1. Solve the heterogeneous household block, taking aggregate prices as given, for both the steady-state policy functions (through backwards iteration) and the steady-state distribution of asset holdings (through forwards iteration)
  2. Use heterogeneous agent block to inform other blocks of the model (such as firm optimality, government policies, and market clearing) and to generate updates of aggregates where necessary
  3. Iterate the two until convergence
- parametrization of computational parameters in line with Auclert et al. (2023)

Back

Inflation in RANK-FD and (Quasi-)HANK-MD

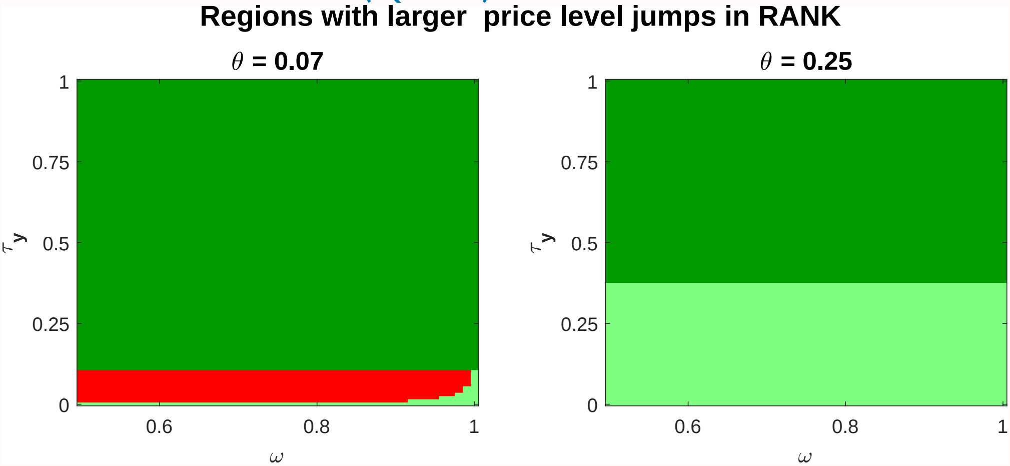


Figure: Comparison of inflation under FD-RANK and MD-HANK in the framework under various levels of inflation-indexed debt. Both green areas depict regions with larger price level movements in FD-RANK. τ_y is the parameter capturing the tax base channel, and ω is the inverse household mortality risk capturing market incompleteness.

Back

Visualizing the effect of indexed debt

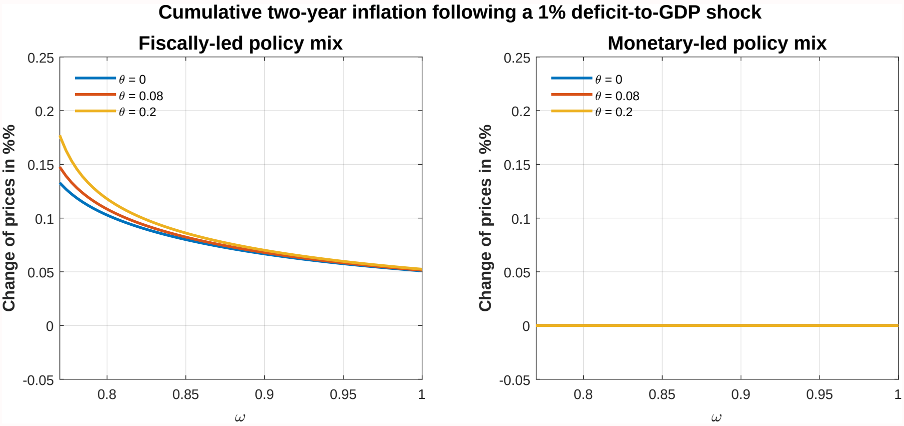


Figure: The role of household (quasi-)heterogeneity and indexed debt across policy regimes. The fiscally-led policy mix is defined by the parameters $\tau_d = 0$ and $\phi = -0.2$, while under the monetary-led policy mix $\tau_d = 0.4$ and $\phi = 0.2$. The remaining calibration is: $D^{SS} = 1, \; Y^{SS} = 1, \; \kappa = 0.025, \; \beta = 0.97, \; \sigma = 1, \; \tau_d = 0, \; \phi = \frac{1-\omega+\epsilon}{\sigma}$.

A brief explainer on active/passive fiscal/monetary policy

- Used interchangeably: active fiscal/monetary policy (Leeper, 1991) $\Leftrightarrow$ fiscally/monetary-led policy mix (Bianchi et al., 2023), and vice versa
- Consider the system from the Blanchard-Yaari type framework:

$$\mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\sigma\phi}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{bmatrix}.$$

- The properties of the model depend on the eigenvalues of the matrix.
- In this special case (lower triangular matrix), its eigenvalues are the elements of its diagonal:

$$\lambda_1 = \frac{1+\sigma\phi}{\omega}; \qquad \lambda_2 = \frac{1}{\beta}: \qquad \lambda_3 = \frac{1}{\beta}(1-\tau_d).$$

- To satisfy the necessary conditions for a *unique saddle-path stable equilibrium* (coherence and completeness), *exactly* two eigenvalues must lie outside the unit circle, as the system has one state variable.
- *Monetary-led policy mix:* $\phi > -\frac{1-\omega}{\sigma}$, $\tau_d > 1-\beta$ - not supported empirically (Nakamura et al., 2025)

Back

The Debt Valuation Equation with Long-Term Indexed Debt

- Under complete markets, $Q_t^{(t+j)} = \mathbb{E}_t \left(\beta^j \frac{P_t}{P_{t+j}} \right), \quad q_t^{(t+j)} = \beta^j$
- $\Rightarrow$ Inflation-indexed debt always has the same price, as its face value accounts for changes to the price level between issuance and redemption.
- NB: indexed debt is *not* equivalent to a real claim - its payout value is not scaled by the prevailing price level.
- Government flow budget:
$B_{t-1}^{(t)} + \Pi_t b_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left(B_t^{(t+j)} - B_{t-1}^{(t+j)} \right) + \sum_{j=1}^{\infty} q_t^{(t+j)} \left(b_t^{(t+j)} - \Pi_t b_{t-1}^{(t+j)} \right)$

Together, they yield the debt valuation equation with inflation-indexed debt:

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} \frac{B_{t-1}^{(t+j)}}{P_t} + \sum_{j=0}^{\infty} q_t^{(t+j)} \frac{b_{t-1}^{(t+j)}}{P_{t-1}} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j},$$

Back