Mixed Frequency Functional VARs for Nowcasting the Income Distribution in the UK

ASSA 2026

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Introduction

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- $\hookrightarrow$ Evaluate policy effect on targeted income quantiles

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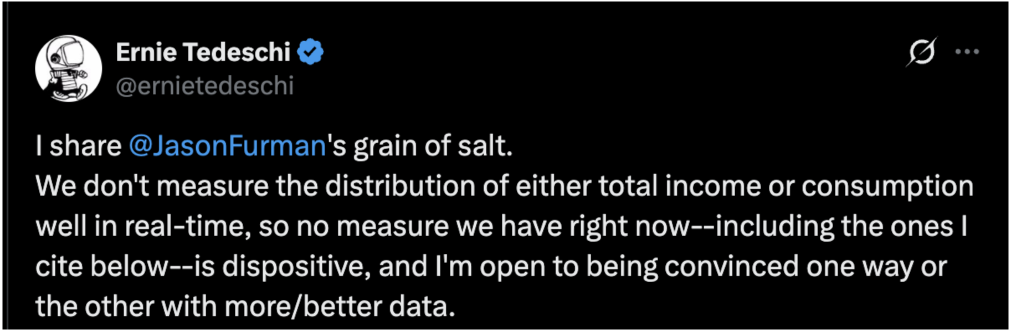
$\hookrightarrow$ Evaluate policy effect on targeted income quantiles

More recently, models with firms or household heterogeneity have been used to study the distributional impact of macroeconomic policies.

$\hookrightarrow$ Heterogeneous features evolve with aggregate quantities

Motivation



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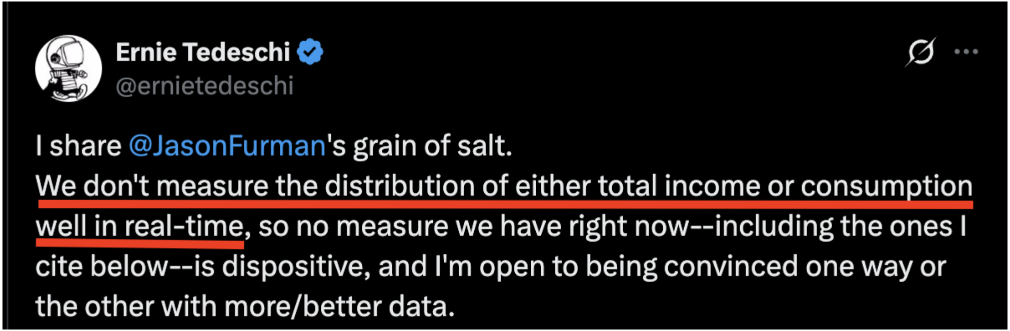

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From the policymaker perspective, it seems relevant to have a timely and accurate measurement of distributional features of the population to improve policy planning.

In this paper

We develop a state-space model to capture the interaction between aggregate fluctuations and distributional dynamics at business cycle frequencies.

$\hookrightarrow$ Parsimoniously captures salient features of the distributions and models the joint dynamics with macro data

$\hookrightarrow$ Timely *nowcasts* of the underlying micro-distribution as implied by developments in macroeconomic aggregates

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- ↪ Parsimoniously captures salient features of the distributions and models the joint dynamics with macro data
- ↪ Timely *nowcasts* of the underlying micro-distribution as implied by developments in macroeconomic aggregates

We propose a method to evaluate the predictive accuracy of functional forecasts

- ↪ Generalization of the Continuously Ranked Probability Score (CRPS)
- ↪ Can highlight predictive accuracy across subsets of the prediction support

Structural analysis

- ↪ Response of income distribution to MP shock

A quick look at the data

Income distribution

ONS survey data about the income distribution in the UK are available from 1970s.

- Income survey have changed over time
  - FES (1961-2001)
  - Expenditure and Food Survey (2001-2007)
  - Living Costs and Food survey (2007-)
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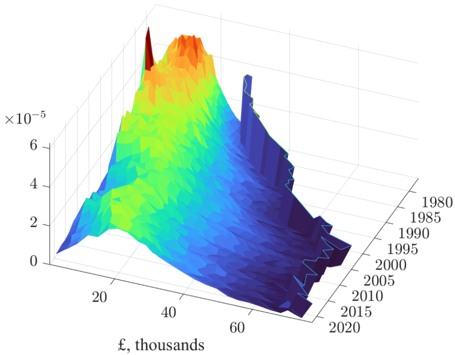

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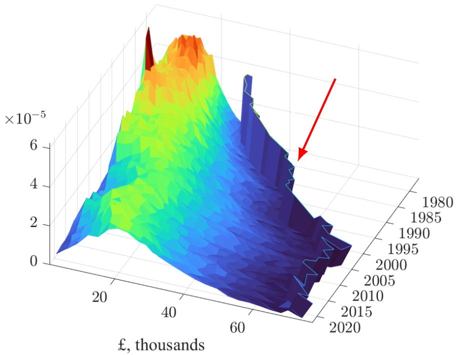
- Data top-coded from 2006.



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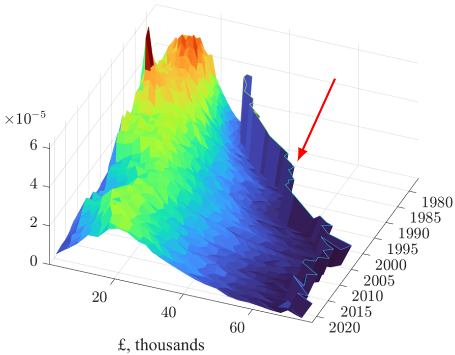
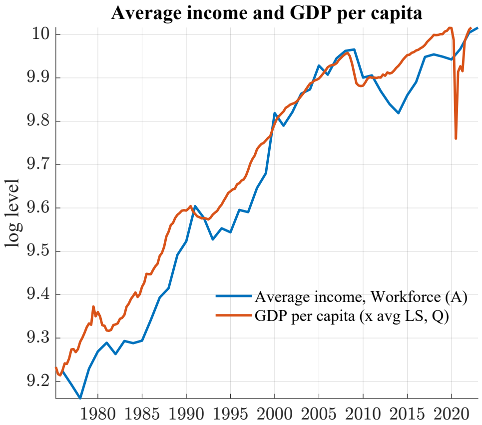


We use income data for the workforce (employed & unemployed people)

Micro and Macro data

Increase in average income appears proportional to GDP per capita



Average income and output per capita - ONS survey data

Model & Methods

Model - the problem

Let $\boldsymbol{y}_t = [\boldsymbol{y}_{1,t}, \ldots, \boldsymbol{y}_{n_{macro},t}]'$ contain n_{macro} variables. We can model their joint dynamics as a Vector Autoregression (VAR)

$$Y_t = \Phi_1 Y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}\left(0, \Sigma\right)$$

Model - the problem

Let $\boldsymbol{y}_t = [\boldsymbol{y}_{1,t}, \ldots, \boldsymbol{y}_{n_{macro},t}]'$ contain n_{macro} variables. We can model their joint dynamics along with any distribution as a functional-VAR

$$\boldsymbol{y}_t = \Phi_{yy}\boldsymbol{y}_{t-1} + \int_{\mathbb{R}+} \Phi_{y\delta}(\tilde{x})\delta_{t-1}(\tilde{x})d\tilde{x} + \varepsilon_{y,t}$$
$$\delta_t(x) = \Phi_{\delta y}(x)Y_{t-1} + \int_{\mathbb{R}+} \Phi_{\delta\delta}(x,\tilde{x})\delta_{t-1}(\tilde{x})d\tilde{x} + \varepsilon_{\delta,t}(x)$$

Good news!

$\hookrightarrow$ We can add the entire *distribution of micro-outcomes* at time t as $\delta_t(x) = \ln p_t(x)$ and expand the model.

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Good news!

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However

$\hookrightarrow$ This is an infinite-dimensional model, that poses several challenges.

Model - the (approximate) solution

We follow Chang et al. (2024) and approximate $\delta_t(x)$ with a K-dimensional cubic spline, which allows us to keep the dimension of the model tractable. That is, *we only need K $\alpha_{k,t}$ factors* to recover the full density as

$$\delta_t(x) = \sum_{k=1}^{K} \alpha_{k,t}\zeta_k(x)$$

where $\zeta(x)$ is spline functions.

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Therefore,

$$\boldsymbol{z}_t = \sum_{p=1}^{P} \Phi_p \boldsymbol{z}_{t-p} + u_t, \quad u_t \sim \mathcal{N}(0, \Xi)$$

where $\boldsymbol{z}_t = [\boldsymbol{y}_t \;\; \boldsymbol{\alpha}_t]'$ is a $n = n_{macro} + K$ vector of observables, and Ξ is unrestricted.

Quick digression - Few alternatives

Functional VARs with spline basis are quite flexible and relatively easy to work with, but this is not the only way! Alternative measure that can be used are:

- measures of inequality, Gini coefficient (Mumtaz and Theophilopoulou, 2017)
  $\hookrightarrow$ this coefficient is defined over [0,1], and they miss info about the distribution
- quantiles of the distribution
  $\hookrightarrow$ similar to our approach, but difficult to impose non-crossing restriction
- pseudo-individuals (Koop et al., 2024)
  $\hookrightarrow$ this approach amounts to slicing the income distribution by individuals' characteristics.

Methods - Sieve Estimation

Chang et al. (2024) show that the filtering problem for this type of functional-VAR (fVAR) can easily be broken into a two-step procedure.

1. Estimate the sieve coefficients, α_t for each year by fitting the earning distribution to a K-knots cubic spline via ML.
2. Add α_t to the VAR and estimate the joint dynamics.

Additional details:

$\hookrightarrow$ Top-coding issue addressed via penalized ML.

$\hookrightarrow$ Earnings data are detrended by a factor of $\approx (.6\,GDP_t)^{-1}$

$\hookrightarrow$ We apply an inverse hyperbolic sine transformation to the earnings data to ensure non-negativity of the density functions.

Methods - Data

We use quarterly data for the period 1975Q1 to 2021Q4 for the macro variables:

- Inflation rate
- Real GDP pc
- Real consumption pc
- Unemployment
- Labour share
- Bank rate

Data are appropriately transformed to guarantee *stationarity* and demeaned.

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We have a mixed frequency problem!

Methods - Intertemporal restrictions

We cast the mixed-frequency problem in state-space

$$\boldsymbol{z}_t = \sum_{p=1}^{P} \Phi_p \boldsymbol{z}_{t-p} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \Sigma) \tag{1}$$

$$\boldsymbol{Y}_t = \boldsymbol{H} \boldsymbol{z}_t \tag{2}$$

$\boldsymbol{z}_t$ contains *high-frequency estimates* of the micro-density, and $\boldsymbol{H}$ imposes intertemporal restriction to recover annual sieve factors, $\boldsymbol{\alpha}_t$.

$\hookrightarrow$ Average accumulator: $\boldsymbol{\alpha}_t = \frac{1}{4} \sum_{q=1}^{4} \boldsymbol{z}_{t,q} \rightarrow$ Annual frequency.

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- $\hookrightarrow$ Average accumulator: $\boldsymbol{\alpha}_t = \frac{1}{4}\sum_{q=1}^{4} \boldsymbol{z}_{t,q} \rightarrow$ Annual frequency.
- $\hookrightarrow$ We account for the estimation uncertainty via measurement error in the annual sieve coefficients

Application

Nowcasting exercise

We employ the model within a nowcasting exercise to monitor developments in the income distribution in real-time.

- ↪ Pseudo-vintages since 1990, and real-time vintages for the macro data from mid 2015 (ONS data)
- ↪ Micro data are released at the date of the first version of the survey results

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We compare our benchmark MF-fVAR with K=6 against:

- a simple “micro-VAR” with annual sieve coeffitients only
- a VAR on K quantiles of the data
- a VAR on K quantiles of the data and macro variables

How to evaluate functional forecasts?

Consider the Continuously Ranked Probability Score (CRPS)

$$CRPS(\hat{y}, y) = \int_{\mathbb{R}} \left(F(z) - \mathbb{I}_{\{y \leq z\}}\right)^2 dz \tag{3}$$

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Our target is the *whole distribution* of UK earnings, F_Y, not a single realization, y!

How to evaluate functional forecasts?

Consider the functional Continuously Ranked Probability Score (CRPS)

$$fCRPS(\hat{Y}, Y) = \int_{\mathbb{R}} \omega(q) \left(F(z) - F_Y\right)^2 dz \tag{3}$$

Our target is the *whole distribution* of UK earnings, F_Y, not a single realization, y!

$\hookrightarrow$ This metric is akin to the *Cramér-von Mises* criterion.

$\hookrightarrow$ $\omega(q)$ is a weighting function specified over the quantiles of the predictive distributions that allows to highlight different regions of interest.
We consider four specifications for $\omega(q)$

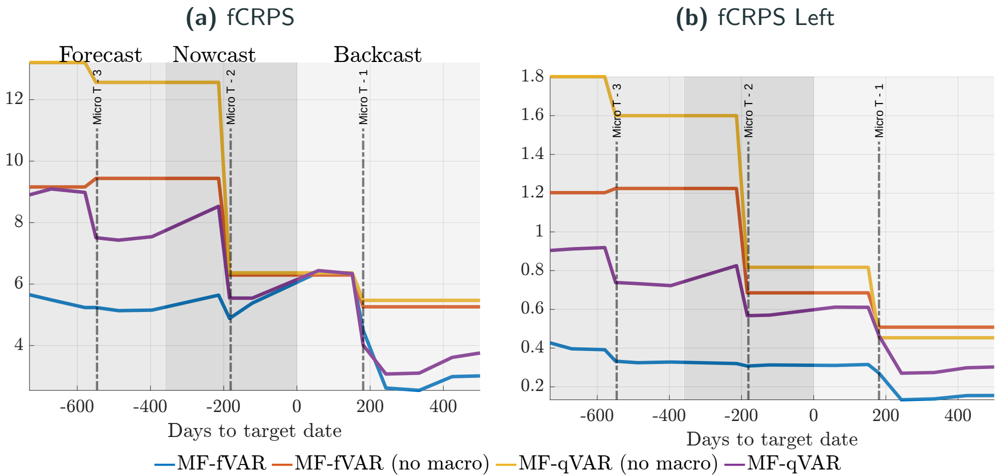
- fCRPS: $\omega\left(q \in [0\ 1]\right) = 1$
- Left tail: $\omega(q < 0.2) = 1$
- Center: $\omega\left(0.2 \leq q \leq 0.8\right) = 1$
- Right tail: $\omega(q > 0.8) = 1$

Real-time forecasting

	MF-fVAR	MF-fVAR (no macro) / MF-fVAR	MF-qVAR / MF-fVAR	MF-qVAR (no macro) / MF-fVAR	MF-fVAR	MF-fVAR (no macro) / MF-fVAR	MF-qVAR / MF-fVAR	MF-qVAR (no macro) / MF-fVAR
	fCRPS				**fCRPS Center**			
Mean	5.272	**1.458** (0.010)	**2.187** (0.000)	**1.614** (0.000)	3.578	**1.596** (0.007)	**2.253** (0.000)	**1.639** (0.000)
Forecast	6.003	**1.495** (0.029)	**2.472** (0.000)	**1.814** (0.000)	4.050	**1.669** (0.023)	**2.620** (0.000)	**1.876** (0.000)
Nowcast	5.371	1.172 (0.524)	1.186 (0.345)	**1.264** (0.066)	3.791	1.207 (0.521)	1.071 (0.481)	**1.256** (0.094)
Backcast	3.414	1.304 (0.467)	**1.386** (0.076)	1.135 (0.145)	2.272	1.350 (0.539)	1.252 (0.411)	1.052 (0.371)
	fCRPS Left				**fCRPS Right**			
Mean	0.347	**2.812** (0.000)	**4.343** (0.000)	**2.368** (0.000)	1.348	0.744 (0.998)	**1.457** (0.002)	**1.352** (0.002)
Forecast	0.411	**2.912** (0.000)	**4.835** (0.002)	**2.513** (0.000)	1.542	0.660 (0.998)	**1.454** (0.010)	**1.465** (0.011)
Nowcast	0.319	**2.151** (0.005)	**2.577** (0.007)	**2.098** (0.005)	1.261	0.820 (0.930)	1.180 (0.234)	1.076 (0.313)
Backcast	0.209	**2.221** (0.016)	**2.166** (0.010)	**1.956** (0.011)	0.934	0.985 (0.592)	**1.538** (0.009)	1.155 (0.120)

Note: the table reports fCRPS measures for the fVAR, and relative scores for the mVAR, qVAR and qmVAR relative to the fVAR. The fCRPS and fCRPS Right measures have been censored at the 96th quantile to reflect the top-coding in the data. The out-of-sample exercise runs from 1990 to 2022.

Real-time forecasting



Structural Analysis

The effect of monetary policy shocks

- Augment a macro-financial VAR with K sieve coefficients

  ▸ Data

- Identification via recursive ordering: Slow moving $\rightarrow$ Income $\rightarrow$ Financial

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Impulse responses to the identified shock follow from the MA representation

$$\begin{aligned} \boldsymbol{Y}^*_{t+h} &= \boldsymbol{H}\boldsymbol{z}_{t+h} + \boldsymbol{H}\boldsymbol{A}_+^h \boldsymbol{b}_{mp}, \quad h = 0, 1, \ldots, H \\ &= \boldsymbol{H}(\boldsymbol{Z}_{t+h} + IRF_h^{\boldsymbol{Y}(mp)}) \end{aligned}$$

where $IRF_h^{\boldsymbol{Y}(mp)} = \left(\boldsymbol{\Phi}^h + \boldsymbol{\Phi}^{h-1} I_{\{h-1\geq 0\}} + \boldsymbol{\Phi}^{h-2} I_{\{h-2\geq 0\}} + \boldsymbol{\Phi}^{h-3} I_{\{h-3\geq 0\}}\right) \boldsymbol{b}_{mp}$.

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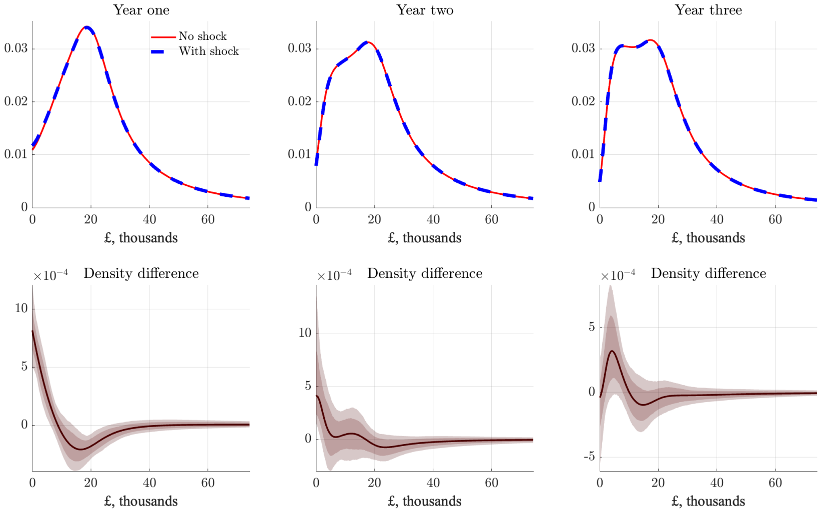
The distributional IRFs are defined as

$$dIRF^{(j)}_{t+h} = p_{\pounds}(\boldsymbol{\alpha}^{\boldsymbol{A}^*}_{t+h}) - p_{\pounds}(\boldsymbol{\alpha}^{\boldsymbol{A}}_{t+h}),$$

where $\boldsymbol{\alpha}^{\boldsymbol{A}^*}_{t+h} = E_t\left[\boldsymbol{\alpha}^{\boldsymbol{A}}_{t+h} | u_t^{(1)} \neq 0\right]$ and $\boldsymbol{\alpha}^{\boldsymbol{A}}_{t+h} = E_t\left[\boldsymbol{\alpha}^{\boldsymbol{A}}_{t+h} | u_t^{(1)} = 0\right]$.

dIRFs

Response to 100bps monetary policy shock



£-value effect of monetary policy shock

Table 1: The effect of monetary policy shocks on income quantiles in 2019

	Q5	Q10	Q20	Q80	Q90	Q95
No Shock	4676.441	7528.756	11388.639	31166.801	40775.869	50736.890
1 year (% change)	-4.265	-2.510	-0.755	0	0	0
2 years (% change)	-1.197	-0.919	-0.785	0	-0.708	0
3 years (% change)	-1.148	-0.917	0	0	0	0

The table reports the percentage changes from the initial £-value for selected quantiles of the income distribution after a shock of 1000 bps hits the economy.

Alternative identification scheme

- Exploit mixed-frequency specification to deploy monthly model
- Proxy-MF-fSVAR using Braun et al. (2025) UKMPD factors

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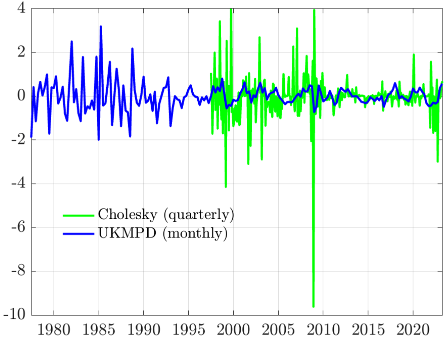

- Distributional impact qualitatively similar to quarterly model: significant impact on lower quantiles.

Conclusions

What we have so far

There is scope to inform the low-frequency dynamics of the UK income distribution with business cycle developments.

$\hookrightarrow$ Macro data add relevant information to predict micro outcomes

$\hookrightarrow$ Our nowcasting model seems to fare well compared to competing models, and can easily address the mixed-frequency nature of the problem

$\hookrightarrow$ MP shocks impact mainly lower quantiles of the distribution of income and benefits from nowcasting

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Appendix

Recovering cross-sectional densities

At every period t, we observe N_t^{survey} draws, $x_{i,t}$, from $p_t^{(K)}(x)$. Defining $\delta_t^{(K)}(x) = \log p_t^{(K)}(x)$, we can then compute

$$p_t^{(K)}(x) = \frac{exp\left(\delta_t^{(K)}(x)\right)}{\int_{\mathbb{R}+} exp\left(\delta_t^{(K)}(\tilde{x})\right) d\tilde{x}}.$$

In the model, we only observe α_t, and so

$$p_t^{(K)}(x) = exp\left(N_t^{survey} \mathcal{F}^{(K)}\left(\alpha_t | X_t\right)\right)$$

$$\mathcal{F}^{(K)}\left(\alpha_t | X_t\right) = \zeta_k(X_t)\alpha_t - \varphi(\alpha_t)$$

$$\varphi(\alpha_t) = \int_{\mathbb{R}+} exp\left(\zeta_k(\tilde{x})\alpha_t\right) d\tilde{x}.$$

Back

Proxy SVARs: Identification via External Instruments (I)

Reduced form:

$$Y_t = B(L)Y_{t-1} + u_t, \quad u_t \sim (0, \Sigma_u)$$

Y_t: $n \times 1$ vector of macroeconomic variables.

$B(L)$: lag polynomial.

u_t: reduced-form residuals.

Structural form:

$$u_t = A\varepsilon_t, \quad \mathbb{E}[\varepsilon_t \varepsilon_t'] = I$$

A: impact matrix.

ε_t: structural shocks, not directly observed.

Mertens and Ravn (2013) propose a method to identify structural shocks in VARs using *external instruments* (also known as *proxy variables*).

Back

Proxy SVARs: Identification via External Instruments (II)

Identify one structural shock (e.g., a monetary policy shock) using a *proxy variable* z_t:

- correlated with the target structural shock ε_{1t},
- uncorrelated with all other shocks ε_{jt}, $j \neq 1$.

Instrument relevance and exogeneity:

$$\text{Cov}(z_t, \varepsilon_{1t}) \neq 0, \quad \text{Cov}(z_t, \varepsilon_{jt}) = 0 \quad \forall j \neq 1$$

Operationally: Use z_t as an instrument for u_{1t} in a 2SLS regression:

$$\hat{A}_1 = \text{Cov}(u_t, z_t)\text{Var}(z_t)^{-1}$$

is a consistent estimate of the first column of A corresponding to the structural shock.

Back

Monetary policy shocks in the UK

Monetary policy shocks can be identified using changes in asset prices around the Bank of England,Äôs Monetary Policy Committee,Äôs announcements.

Surprises are computed over a narrow window around Bank of England Monetary Policy Committee (MPC) announcements (10 min before, 20 min after)

$$z_t = P_t^{\text{post}} - P_t^{\text{pre}}$$

Braun et al. (2025) provide *Target*, *Path* and *QE* factors extracted from revision in market expectations of interest rate futures, treasury (gilt) yields and overnight index swaps, the stock market and exchange rates.

These factors can be used in a proxySVAR.

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Regional response to a UK monetary policy shock

We estimate a Bayesian VAR on quarterly UK macroeconomic and financial data:

- BoE rate
- BAA credit spread
- CPI
- 1-year Gilt yield
- FTSE-100 index
- Nominal FX

Data are available from 1997Q1 to 2019 Q4, and we consider 8 lags.

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