# Spillovers and the Direction of Innovation: An Application to the Clean Energy Transition

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# This Paper

## Clean Innovation is Critical for Decarbonization

- Policymakers seek to redirect innovation
  - Dirty $\rightarrow$ Clean
- Is it enough? (Acemoglu et al., 2012, 2016)
  - "Big push" generates complete transition, or nothing

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- Overcome the historic advantage of fossil fuels
  - Tesla Prototype Mule 1

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## Cross-Technology Knowledge Spillovers

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## Develop General Endogenous Growth Model

- Clean and dirty innovation in multiple sectors
- Network of cross-technology knowledge spillovers

# Results

## Impact of Policy Reform

- Size and speed of technological transition depends on spillovers vs substitutability
  - Aggregate to increasing returns statistic determining lock-in
  - Size and speed of transition are inversely related

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## Quantitative Application to US Transport & Electricity

- Use model to measure increasing returns to innovation
  - Spillovers mid-sized: Prevent lock-in of dirty technology
  - Impact of realistic policy reform (Biden Administration's SCC, IRA)
  - Slow transition: Half-lives of $\approx$ 90 years
  - Large long-run impact: Clean technology increases by $\approx$ 95%

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## Optimal Clean Innovation Policy

- Recursive innovation subsidy formula with any carbon price
  - No need for "big push"
  - Lower, more stable clean innovation subsidies

# Example Model

## Production

- Technologies: $\{A_{ct}, A_{dt}\}$
- Output: $\mathcal{Y}_t = \Omega_t \big( Y_{ct}^{\frac{\sigma-1}{\sigma}} + Y_{dt}^{\frac{\sigma-1}{\sigma}} \big)^{\frac{\sigma}{\sigma-1}}$
- Dirty Production $\Rightarrow$ Emissions $\Rightarrow$ Future Damages

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## Innovation

- Tech Evolution: $A_{jt} = \gamma^{z_{jt}} A_{jt-1}$
- Innovation: $z_{it} = \chi \underbrace{s_{it}^{\eta}}_{\text{Scientists}} \underbrace{A_{it-1}^{\tilde{\varphi}_{ii}} A_{jt-1}^{\tilde{\varphi}_{ij}} / A_{it-1}}_{\text{Spillovers}}$ ($\Phi \equiv \tilde{\varphi}_{cd} + \tilde{\varphi}_{dc}$)
- Purchase of Innovation $\Rightarrow$ One-Period Monopoly

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## Equilibrium

$$\Big(\frac{s_{ct}}{s_{dt}}\Big)^{1-\eta} = \mathcal{G}_t \Big(\frac{\xi_{ct}}{\xi_{dt}}\Big) \Big(\frac{r_d + \omega_d \tau_t}{r_c}\Big)^{(\sigma-1)\alpha} \Big(\frac{A_{ct-1}}{A_{dt-1}}\Big)^{(\sigma-1)(1-\alpha)-\Phi}$$

# Do we need a Big Push?

## Think in Relative Technology

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- BGP becomes fixed point
- Technological lock-in whenever BGP is unstable

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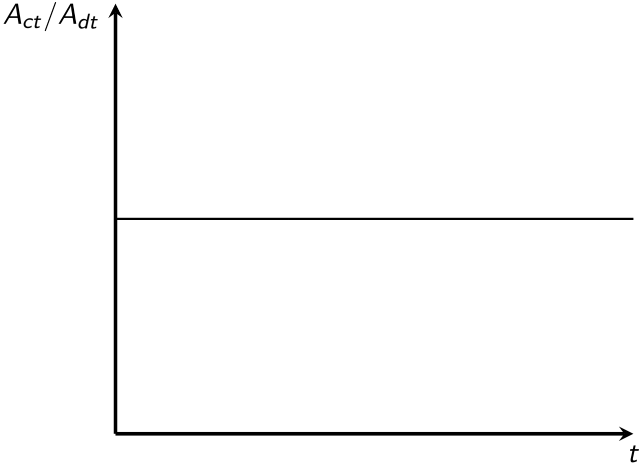
## Lock-In Condition

$$\Phi < \eta(1-\alpha)(\sigma-1)$$

- Spillovers are small relative to substitutability
- Without big push, initial leader takes over the economy
- If condition is not met, what is the impact of policy?

# New Steady-State & Transition



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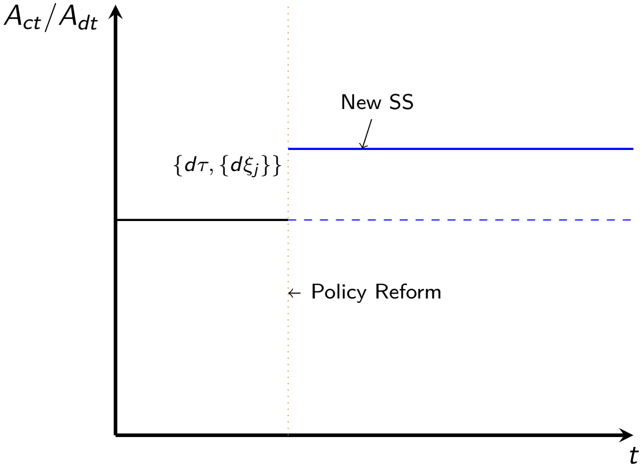

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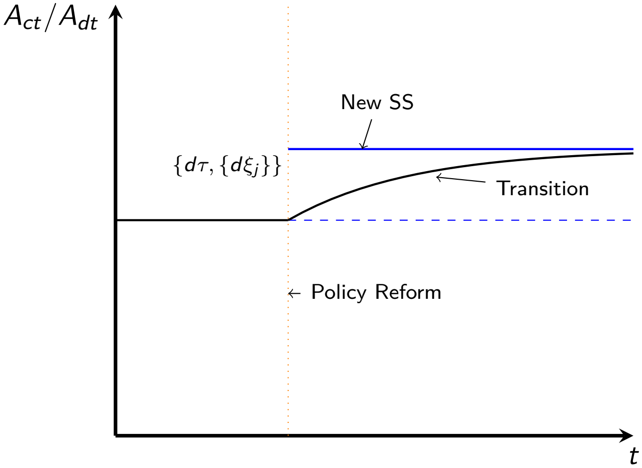

# Long-Run Impact of Policy Reform

### Proposition

*Policy reforms induce first-order steady-state change*

$$d \ln \left(\bar{A}_{ss}\right) = \eta \mathcal{M} \left[ d \ln \left( \xi_c / \xi_d \right) + \alpha(\sigma - 1) d \ln \left( r_d + \omega_d \tau \right) \right]$$

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- $\uparrow \Phi \Rightarrow$ Decrease Long-Run Impact
- $\uparrow \sigma \Rightarrow$ Increase Long-Run Impact

# Transition Path Following Policy Reform

## Proposition

*Given initial condition $\bar{A}_0$, technology follows transition path*

$$\ln(\bar{A}_t) - \ln(\bar{A}_{ss}) \approx \mathcal{J}^t \left( \ln(\bar{A}_0) - \ln(\bar{A}_{ss}) \right)$$

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Long-Run vs. Transition

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- $\uparrow \Phi \Rightarrow$ Fast Convergence

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- Path dependence if $\mathcal{J} > 1 \iff \Phi < \eta(1-\alpha)(\sigma-1)$

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## Interpretation

- $\mathcal{J}$ quantifies increasing returns to innovation

Long-Run vs. Transition

# General Case

## General Setup

- $J$ Technologies, CRTS Production & Spillovers
- Sufficient Statistic Matrices
  - $\Phi \rightarrow \mathbf{\Phi}_t$
  - $\sigma \rightarrow \mathbf{\Sigma}_t$

Detail

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## Same Formulas

$$\boldsymbol{\mathcal{M}} = \left[\mathbf{\Phi} - \eta(1-\alpha)\left(\mathbf{\Sigma} - \mathbf{I}\right)\right]^{-1}$$

$$\boldsymbol{\mathcal{J}} = \left[(1-\eta)\mathbf{I} - g\eta(1-\alpha)\left(\mathbf{\Sigma} - \mathbf{I}\right)\right]^{-1}\left[(1-\eta)\mathbf{I} - g\mathbf{\Phi}\right]$$

- Same formulas, now in terms of sufficient statistics
- Spectral radius of $\mathcal{J}$ quantifies increasing returns to innovation

Detail

# Quantitative Application to US Transport & Electricity

## Granted US Patents from PatentsView

- Assignment $\rightarrow$ IPC/CPC Codes (Lanzi et al., 2011; Aghion et al., 2016)
- General Patents $\rightarrow$ Not Transport or Electricity

Heat Map | Parametric Assumptions | Parameter Choices | Centrality Stability | Reduced Form Evidence

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## Calibrate $\tilde{\varphi}$ with Patent Citation Network

$$\tilde{\varphi}_{ij} = \frac{cites_{ij}}{\sum_k cites_{ik}}$$

- Citation Shares (Liu and Ma, 2021)
- Patents widely accepted spillover proxy (Jaffe et al., 1993; Hall et al., 2005)

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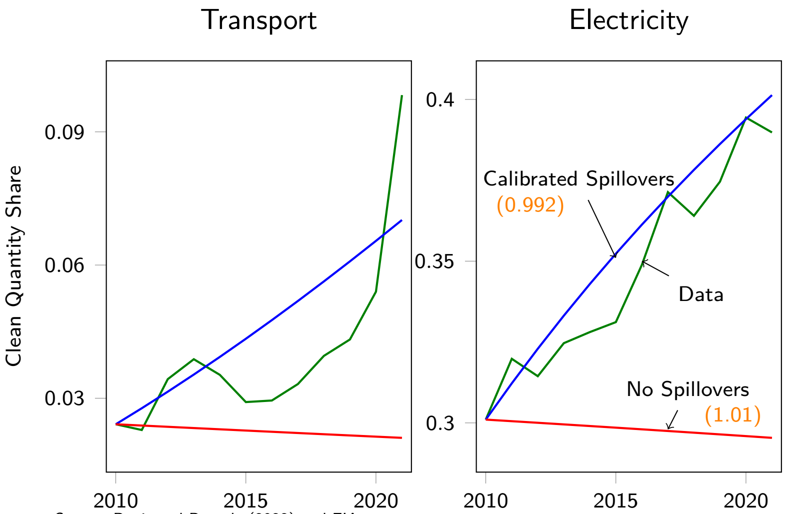
# Model with Spillovers Matches 2010s Clean Advance




*Source*: Davis and Boundy (2022) and EIA.

- Evidence rejects dirty technology lock-in for both sectors

Solar Prices

# Policy Reform

## Realistic Carbon Price & Clean Innovation Subsidy

- $\tau = \$51$ (Biden Administration's lower SCC)
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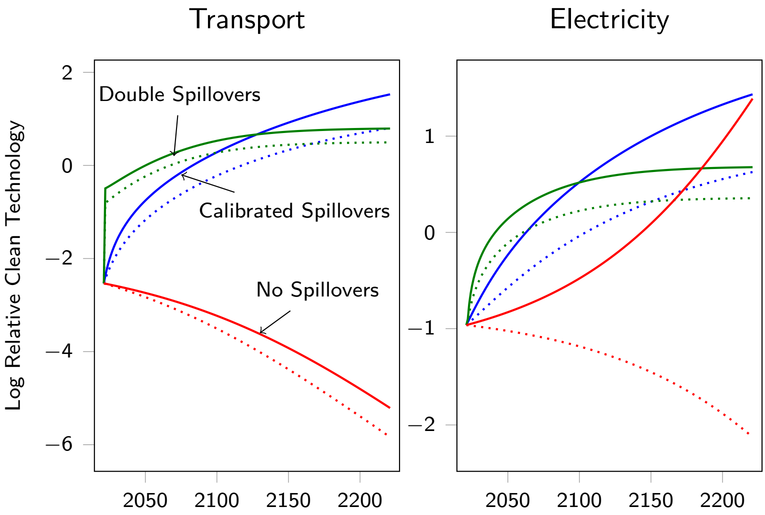
## Three Cases for Spillover Network

- Zero, Calibrated, Double
- Varies increasing returns to innovation

# Technology Path

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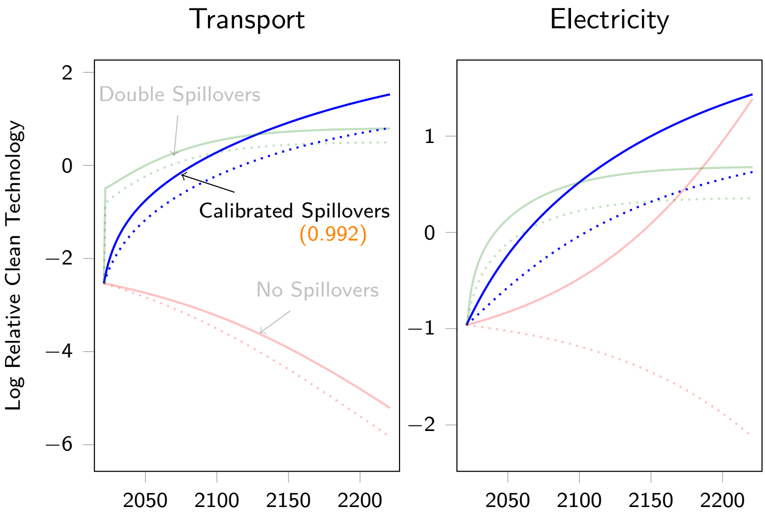


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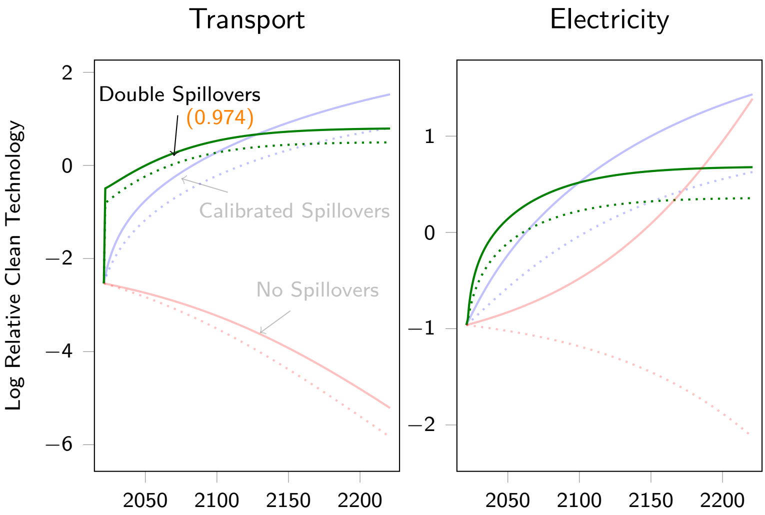


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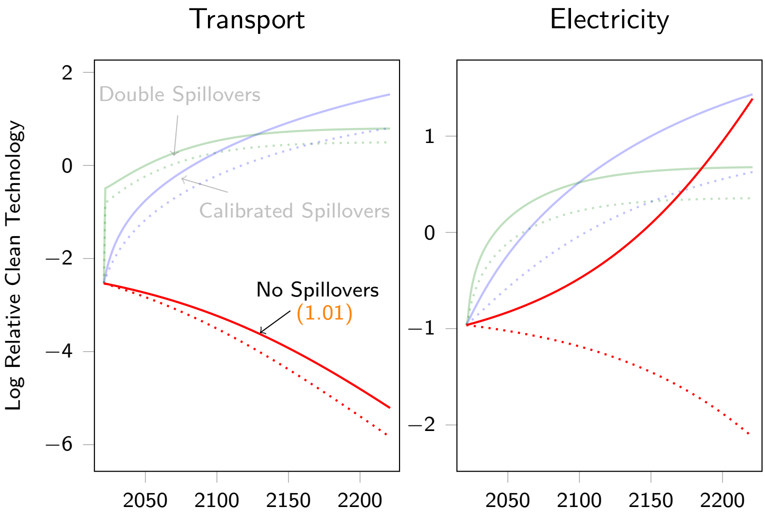


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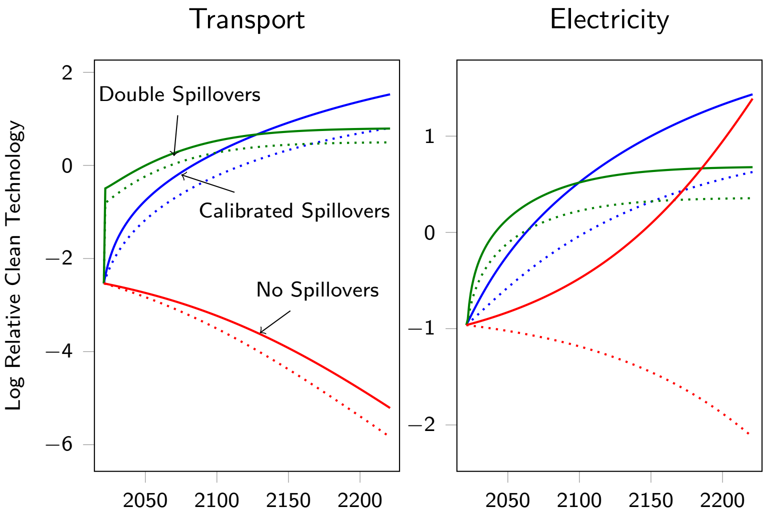


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# Conclusion

### Cross-Technology Spillovers Critical to Understand Transition

- Exclusion leads to false conclusion of dirty lock-in
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Optimal Policy

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## Going Forward

- Importance of substitutability long-recognized (Hicks, 1932)
- Cross-technology spillovers deserve a similar status
- As with substitutability, insights could be applied elsewhere
  - Skill-biased technological change, automation, etc

Optimal Policy

# Tesla's First Prototype

◂ Return

# Different Impacts at Different Time Horizons

## Corollary

*Long-run impact and transition speed are inversely related*

$$\mathcal{M} = \frac{g(1-\mathcal{J})^{-1}}{(1-\eta) - g\eta(1-\alpha)(\sigma-1)}$$

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## Cross-Technology Spillovers

- Prevent lock-in of dirty technology
- Speed up transitions
- Reduce long-run impact

◂ Return

# General Setup

## Setup with $J$ Technologies

- Production Structure: CES $\rightarrow \underbrace{F(\{Y_{jt}\})}_{\text{CRTS}}$

Return

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## Clean and Dirty Technologies

- Dirty inputs emit $\omega_j > 0$ units of carbon pollution
- Emissions in each period $\mathcal{E}_t = \sum_j \omega_j \Lambda_{jt}$

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## Innovation

- Spillover Structure: Cobb-Douglas $\rightarrow \underbrace{\tilde{\phi}_i(\{A_{jt-1}\})}_{\text{CRTS}}$
- Normalize $\phi_{it} \equiv \tilde{\phi}_{it}/A_{it-1}$
- Spillover Network: $\varphi_{ijt} \equiv \frac{\partial \ln(\phi_{it})}{\partial \ln(A_{jt-1})}$ $(\varphi_{ijt}|_{i \neq j} \geq 0)$

Return

# Sufficient Statistic Matrices

## Spillover Matrix

$$\Phi \to \Phi_t$$

- $\mathbf{\Phi}_{ijt} \equiv \frac{\partial \ln(\phi_{Jt}/\phi_{it})}{\partial \ln(A_{jt-1}/A_{Jt-1})}$
- Measures size of cross-technology spillovers Example
- Summarizes spillover structure $\phi(.)$

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## Substitution Matrix

$$\sigma \to \boldsymbol{\Sigma}_t$$

- $\boldsymbol{\Sigma}_{ijt} \equiv \frac{\partial \ln(Y_{it}/Y_{Jt})}{\partial \ln(p_{Jt}/p_{jt})}$
- Measures degree of substitutability in production
- Summarizes production structure $F(.)$

Return

# Impact of Policy Reform

## Proposition

*Policy reform* $d \ln(\Xi_j) \equiv d \ln\left(\frac{\xi_j}{\xi_J}\right)$ *and* $d \ln(\mathcal{R}_j) \equiv d \ln\left(\frac{r_j + \omega_j \tau}{r_J + \omega_J \tau}\right)$ *induces first-order steady-state change*

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*with transition dynamics*

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# Eigendecomposition

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*The transition and amplification matrices can be decomposed in terms of $\mathcal{J}$'s eigenvectors $\boldsymbol{Q}$ and eigenvalues $\kappa$*

$$\mathcal{J}^t = \boldsymbol{Q}\boldsymbol{D}(\kappa)^t\boldsymbol{Q}^{-1}$$

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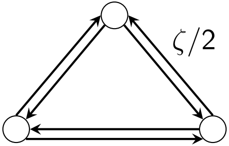
## Implications

- Spectral radius quantifies increasing returns to innovation
  - $(\kappa_j \to 0) \Rightarrow$ Fast Convergence/Small Long-Run Impact
  - $(\kappa_j \to 1) \Rightarrow$ Slow Convergence/Large Long-Run Impact
  - $(\kappa_j > 1) \Rightarrow$ Technological Lock-In

Return Spectral Analysis

# Example Spillover Network




### Spillover Network

$$\boldsymbol{\varphi} = \begin{pmatrix} -\zeta & \zeta/2 & \zeta/2 \\ \zeta/2 & -\zeta & \zeta/2 \\ \zeta/2 & \zeta/2 & -\zeta \end{pmatrix}$$

- Uniform cross-technology spillover $\zeta$

### Spillover Matrix

$$\mathbf{\Phi} \to \zeta \mathbf{I}$$

Return

# Spectral Analysis

## Corollary

*Let* $\bar{\mathcal{A}}_t \equiv \ln(\bar{A}_t) - \ln(\bar{A}_{ss})$*. The transition path of technology follows*

$$\bar{\mathcal{A}}_t \approx \sum_j \kappa_j^t \beta_j Q_j$$

## Components

1. Eigenvalues $\kappa$
2. "Eigenstates" $\boldsymbol{Q}$ (Kleinman et al., 2023)
3. Initial State's Loading on Eigenstates $\beta = \boldsymbol{Q}^{-1}\bar{\mathcal{A}}_0$

## Convergence Speed

- Half-Lives $t_{ES_j}^{(1/2)} = \left\lceil \frac{\ln(1/2)}{\ln(\kappa_j)} \right\rceil$

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## Components

1. Eigenvalues $\kappa$
2. "Eigenstates" $\boldsymbol{Q}$ (Kleinman et al., 2023)
3. Initial State's Loading on Eigenstates $\beta = \boldsymbol{Q}^{-1}\bar{\mathcal{A}}_0$

## Convergence Speed

- Half-Lives $t_{ES_j}^{(1/2)} = \left\lceil \frac{\ln(1/2)}{\ln(\kappa_j)} \right\rceil$

Return

# Parametric Assumptions

## Production Structure

$$\mathcal{Y}_t = \Omega_t \Big( \sum_\theta \nu_\theta^{\frac{1}{\lambda}} E_{\theta t}^{\frac{\lambda-1}{\lambda}} \Big)^{\frac{\lambda}{\lambda-1}}$$

$$E_{\theta t} = \Big( Y_{\theta ct}^{\frac{\sigma-1}{\sigma}} + Y_{\theta dt}^{\frac{\sigma-1}{\sigma}} \Big)^{\frac{\sigma}{\sigma-1}}$$

- $\lambda < 1 < \sigma$
- $\theta \in \{\text{Transport, Electricity, General}\}$

## Spillover Structure

$$\phi_{it} = \frac{\prod_j A_{jt-1}^{\tilde{\varphi}_{ij}}}{A_{it-1}}$$

- Gross Spillover Network $\tilde{\varphi}$

Return | Additional Assumptions | Climate Module

# Further Parametric Assumptions

## Household Utility

$$u(c_t) = \frac{c_t^{1-\vartheta} - 1}{1-\vartheta}$$

- Inverse Intertemporal Elasticity of Substitution $\vartheta$

## Idea Production Function

$$z_{jt} = \chi \left( \frac{s_{jt}}{\nu_{\theta(j)}} \right)^{\eta} \phi_{jt}$$

- Scientists per Economic Task (Acemoglu and Restrepo, 2022)

◂ Return

# Climate Module

## Atmospheric Carbon Concentrations

$$\mathcal{C}_t = \sum_{\hat{t}=1800}^{t} \Big( \underbrace{\psi_p}_{\text{Permanent}} + \underbrace{(1-\psi_p)\psi_0\psi^{t-\hat{t}}}_{\text{Transitory}} \Big)\mathcal{E}_{\hat{t}} + \bar{\mathcal{C}}$$

- Pre-Industrial Carbon $\bar{\mathcal{C}}$ (596.4) (280 PPM)
- Permanent Share $\psi_p$ (0.2) (2007 IPCC)
- Unabsorbed Carbon $\psi_0$ (0.308) (Joos et al., 2013)
- Decay Rate $\psi$ (0.998) (Joos et al., 2013)

Historic Match

## Damages

$$\Omega_t = \exp\big(-\varrho(\mathcal{C}_t - \bar{\mathcal{C}})\big)$$

- Damages Semi-Elasticity $\varrho$ ($5.3 \times 10^{-5}$) (Golosov et al., 2014)

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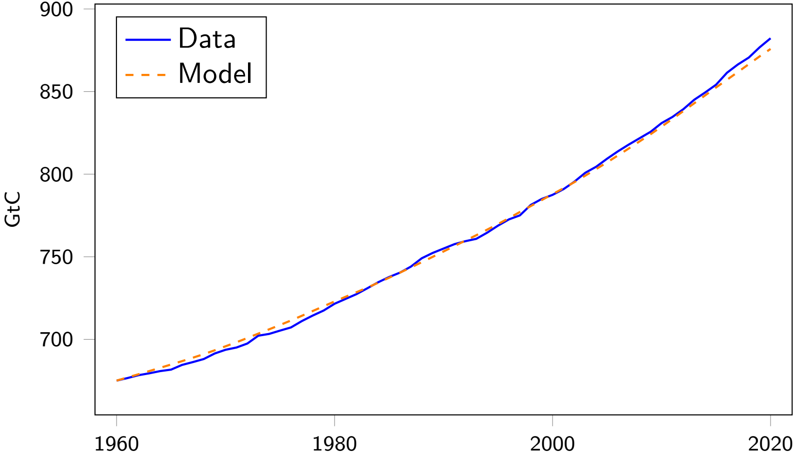
Return

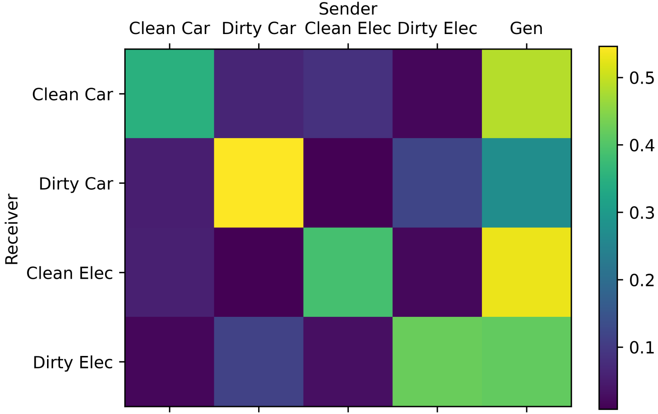
# Climate Model Matches Historical Carbon Concentrations




*Source*: Our World in Data & NOAA's Mauna Loa Observatory

Return

# Heat Map of Gross Spillover Network $\tilde{\varphi}$




Return

# Parameter Choice Summary

| Parameter | Description | Value | Source |
|---|---|---|---|
| $\sigma$ | Clean/Dirty ES | 1.86 | Papageorgiou et al. (2017) |
| $\lambda$ | Cross-Sector ES | 0.1 | Hassler et al. (2021) |
| $\nu_{car}$<br>$\nu_{elec}$ | CES Shares | 0.028<br>0.022 | Income Shares |
| $\gamma$ | Innovation Step Size | 1.07 | Acemoglu et al. (2023) |
| $\alpha$ | Input Share | 0.4 | Standard<br>Barrage (2020) |
| $r_{\theta d}/r_{\theta c}$ | Relative Dirty Input Price | 2.25 | BP (2022) |
| $\eta$ | Research Elasticity | 0.5 | Akcigit and Kerr (2018) |
| $\vartheta$ | Inverse Intertemporal ES | 1 | Standard |
| $\rho$ | Rate of Pure Time Preference | 0.001<br>0.015 | Stern (2007)<br>Nordhaus (2017) |

Return  Internal Calibration  Initial Conditions  2010s Policy  Additional Climate

# Internal Calibration

## CES Shares $\nu$

- Match 2000-2020 income shares
- BEA, EIA: **2.8%, 2.2%**

## Research Productivity $\chi$

- Set laissez-faire SS growth
- Standard: **2%**

## Carbon Intensities $\omega$

- Match 2021 $CO_2$e emissions in MMT
- EPA: **1804.3, 1584.1**

◂ Return

# Technology's Initial Conditions

## Within-Sector $A_{\theta c t_0} / A_{\theta d t_0}$

- Match clean quantity shares
- Transportation: Share of new light vehicles hybrids or EV
  - Department of Energy (Davis and Boundy, 2022)
- Electricity Generation: Share of non-fossil electricity
  - EIA

## Across-Sector $A_{\theta d t_0} / A_{\Theta c t_0}$

- Match income shares
- BEA, EIA

Return

# 2010s US Climate Policy

## Clean Input Subsidies

$$(1-\bar{\xi}_{\theta c})r_{\theta c}$$

$$\frac{\bar{\xi}_{\theta c}}{1-\bar{\xi}_{\theta c}}\sum_{t=2011}^{T_{IF}}\alpha\hat{S}_{ct}^{\theta}=\sum_{t=2011}^{T_{IF}}\hat{cred}_{\theta ct}$$

## Transportation

- Electric Vehicle Tax Credit (US Congressional Research Service Report IF11017)
- $\bar{\xi}_{car,c}=0.011$

## Electricity Generation

- Energy Investment Tax Credit (US Congressional Research Service Report IF10479)
- $\bar{\xi}_{elec,c}=0.03$

Return

# 2010s US Climate Policy

## Innovation Subsidies

$$\frac{\xi_j - 1}{\xi_j} \sum_{t=2010}^{2015} \frac{s_{jt}}{\eta \mathcal{S}} = \sum_{t=2010}^{2015} \hat{pubrd}_{jt}$$

- Public R&D Spending/Total: IEA, BEA

| Transportation | | Electricity Generation | |
|---|---|---|---|
| Description | Codes | Description | Codes |
| Clean | | Clean | |
| Vehicle Batteries/Storage Technologies | 1311 | Renewable Energy Sources | 3 |
| Advanced EV/HEV/FCV Systems | 1312 | –Excluding Biofuels | 34 |
| Electric Vehicle Infrastructure | 1314 | Nuclear | 4 |
| Dirty | | Dirty | |
| Advanced Combustion Engines | 1313 | Coal | 22 |
| Oil & Gas (1/2) | 21 | Oil & Gas (1/2) | 21 |

Return

# Additional Climate Information

## Recursion

$$\mathcal{C}_{1t} = \psi_p \mathcal{E}_t + \mathcal{C}_{1t-1}$$
$$\mathcal{C}_{2t} = (1 - \psi_p)\psi_0 \mathcal{E}_t + \psi \mathcal{C}_{2t-1}$$

## Initial Conditions

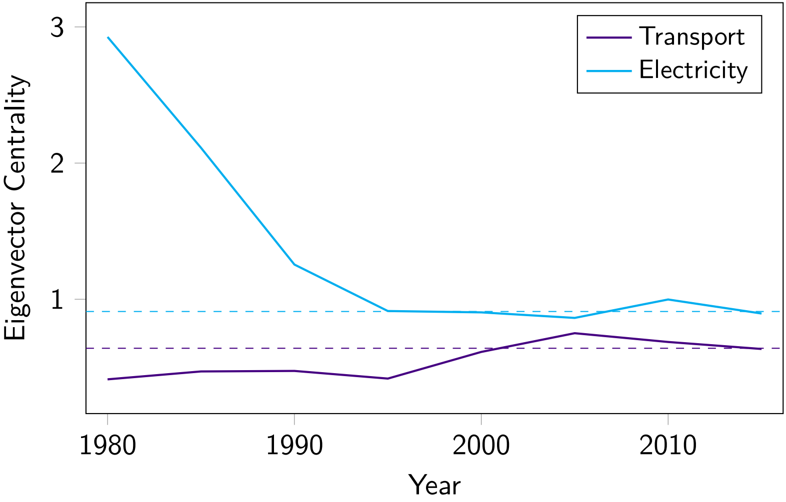
$$\mathcal{C}_{1t_0} = \sum_{\hat{t}=1800}^{t_0} \psi_p \mathcal{E}_{\hat{t}} + \bar{\mathcal{C}}$$
$$\mathcal{C}_{2t_0} = \hat{\mathcal{C}}_{t_0} - \mathcal{C}_{1t_0}$$

## External Emissions

- Optimal path from 2010 RICE (Nordhaus, 2010)
- Subtract US transport & electricity (56.8%)

Return

# Centrality Stability




Return AR(1)

# Stability Regressions

| | Dependent Variable: Spillover Elasticity | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| Lagged Spillover Elasticity | 0.982*** | 0.983*** | 0.983*** |
| | (0.01) | (0.01) | (0.01) |
| Clean Technology Trend | | -0.0001 | |
| | | (0.0002) | |
| Clean Transport Trend | | | 0.00002 |
| | | | (0.0002) |
| Clean Electricity Trend | | | -0.0002 |
| | | | (0.0003) |
| $R^2$ | 0.992 | 0.992 | 0.992 |
| Obs | 200 | 200 | 200 |

*Notes*: Spillover network computed in five year bins from 1970 to 2015.

$$\tilde{\varphi}_{ijt} = \mu \tilde{\varphi}_{ijt-1} + \delta_{ijt}$$

Return

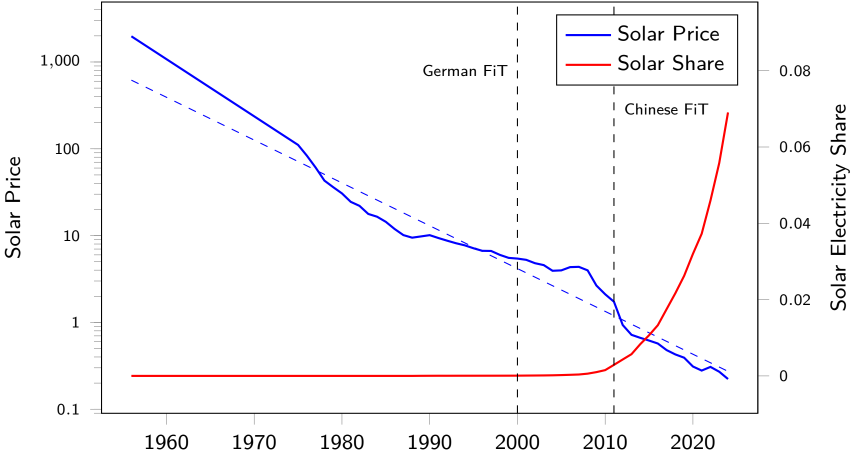
# Reduced Form Evidence

| | Dependent Variable: | | | | | |
|---|---|---|---|---|---|---|
| | ln(Citations) | | | ln(Patents) | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| ln(R&D Spending) | 0.800*** | 0.795*** | 0.813*** | 0.997*** | 1.046*** | 0.976*** |
| | (0.08) | (0.09) | (0.06) | (0.10) | (0.08) | (0.08) |
| ln(Cross-Technology Spillovers) | 3.111*** | 3.079*** | 2.924*** | 2.552*** | 2.678*** | 2.369*** |
| | (0.37) | (0.33) | (0.35) | (0.55) | (0.30) | (0.48) |
| ln(Downstream Spillovers) | | -0.269 | | | 1.668** | |
| | | (0.81) | | | (0.81) | |
| Specification | OLS | OLS | IV | OLS | OLS | IV |
| IV 1st Stage F-Stat | | | 983.84 | | | 907.66 |
| $R^2$ | 0.886 | 0.886 | 0.969 | 0.907 | 0.913 | 0.976 |
| Obs | 176 | 176 | 144 | 176 | 176 | 144 |

$$\ln\left(pats_{jt}\right) = \alpha_j + \alpha_t + \mu_1 \ln\left(R\&D_{jt}\right) + \mu_2 \ln\left(spill_{jt}\right) + \varepsilon_{jt}$$

Return

# Did a Foreign Big Push cause Solar Prices to Fall?




◂ Return

# Nested CES Substitution Matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \tilde{\boldsymbol{\Sigma}}_1 & 0 & \dots & 0 \\ 0 & \tilde{\boldsymbol{\Sigma}}_2 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \tilde{\boldsymbol{\Sigma}}_{\Theta-1} \end{pmatrix}$$

where

$$\tilde{\boldsymbol{\Sigma}}_\theta = \begin{pmatrix} \lambda + (\sigma - \lambda)\varepsilon^E_{\theta d} & (\lambda - \sigma)\varepsilon^E_{\theta d} \\ (\lambda - \sigma)\varepsilon^E_{\theta c} & \lambda + (\sigma - \lambda)\varepsilon^E_{\theta c} \end{pmatrix}$$

- $\varepsilon^E_{\theta e}$ are sectoral elasticities (income shares)
- $\sigma = \lambda \Rightarrow \boldsymbol{\Sigma} = \sigma \mathbf{I}$ (standard CES)

◂ Return

# Impact of Policy Reform

| | No Spillovers | Calibrated Spillovers | Double Spillovers |
|---|---|---|---|
| **Long-Run Impacts** | | | |
| *Relative Clean Technology by Sector* | | | |
| $\%\Delta \bar{B}_{car}$ | 0% | $+92.3\%$ | $+29.6\%$ |
| $\%\Delta \bar{B}_{elec}$ | $+\infty\%$ | $+98.5\%$ | $+33.8\%$ |
| *Emissions Intensity* | | | |
| $\%\Delta\bar{\omega}$ | -71.5% | -67.7% | -52.3% |
| **Transitional Impacts** | | | |
| *Half-Lives of Convergence by Sector* | | | |
| $t_{car}^{(1/2)}$ | – | 83 years | 18 years |
| $t_{elec}^{(1/2)}$ | – | 97 years | 22 years |
| *Carbon Emissions by Year* | | | |
| $\%\Delta\mathcal{E}_{2035}$ | -39.4% | $-47\%$ | -49.7% |
| $\%\Delta\mathcal{E}_{2060}$ | -42.4% | $-53\%$ | -50.8% |
| **Degree of Increasing Returns to Innovation** | | | |
| *Spectral Radius* | | | |
| $\max\lvert\kappa_j\rvert$ | 1.01 | 0.992 | 0.974 |

◂ Return | Linearization vs. Simulation | Determinants of Long-Run Impact
Determinants of Increasing Returns | Half-Lives | Size of Policy Reform

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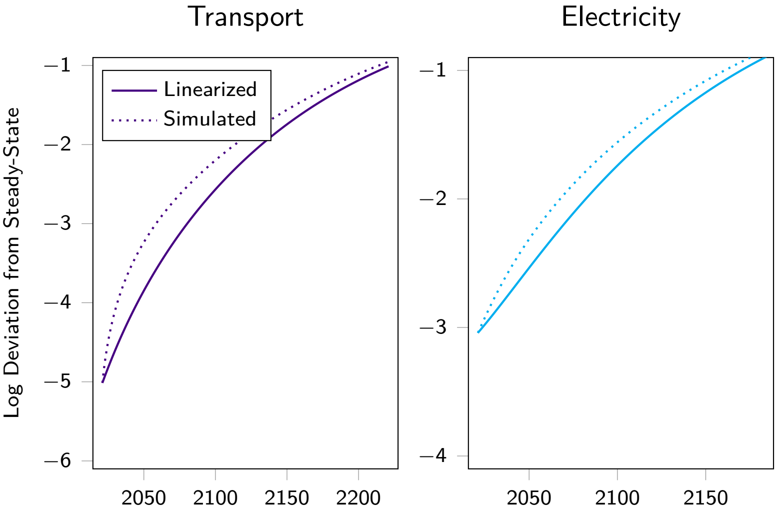
Return Linearization vs. Simulation Determinants of Long-Run Impact

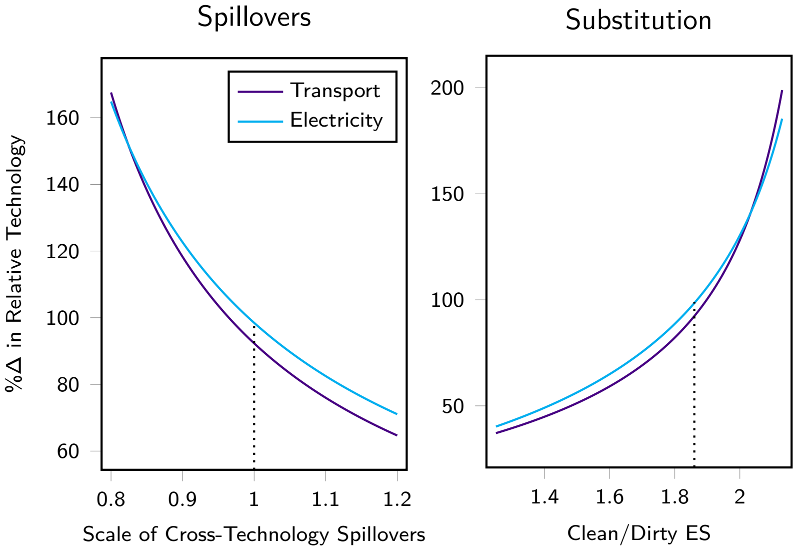
Determinants of Increasing Returns Half-Lives Size of Policy Reform

# Accurate Linearized Path



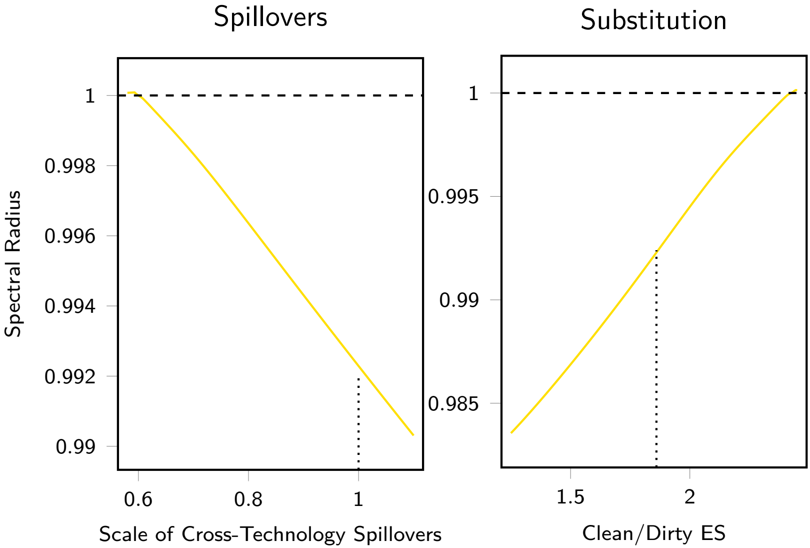

Return

# Determinants of Long-Run Impact



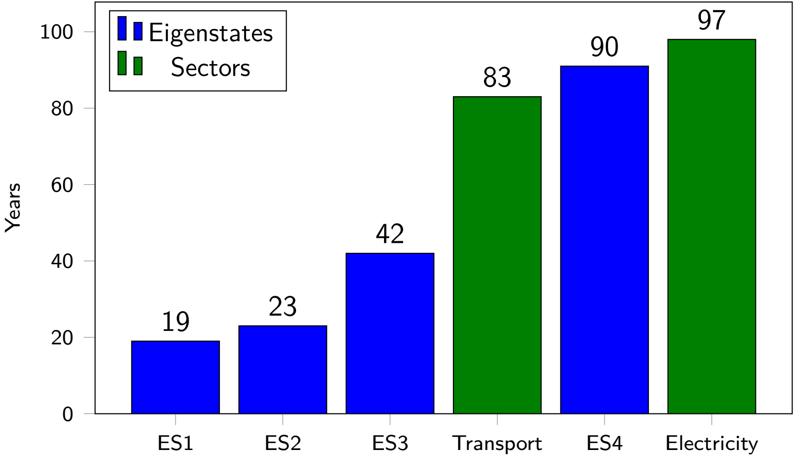

◂ Return

# Determinants of Increasing Returns



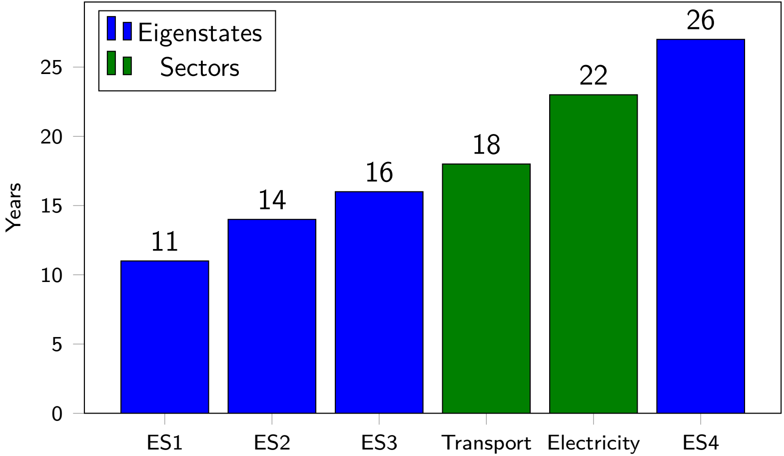

Return

# Half-Lives of Convergence



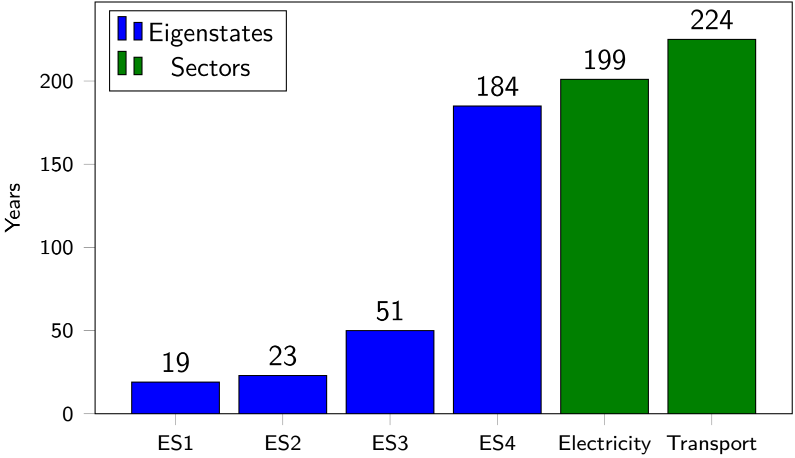

Return | Double Spillovers | No Within-Sector Spillovers

# Half-Lives of Convergence (Double Spillovers)



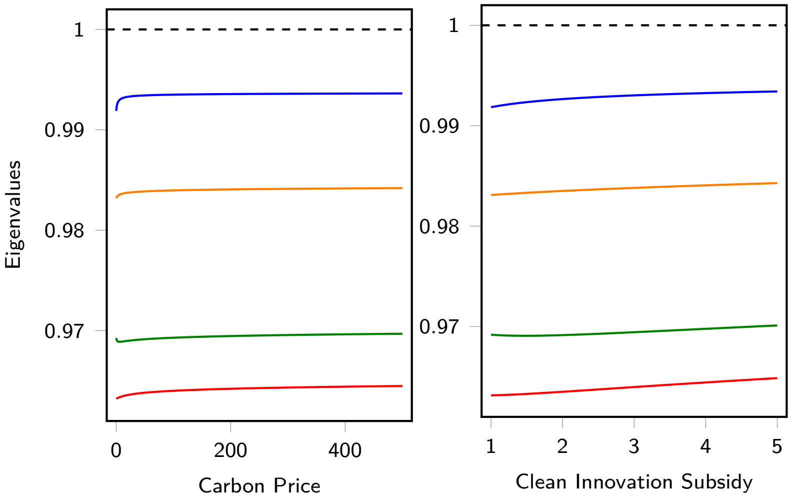

Return

# Half-Lives of Convergence (No Within-Sector Spillovers)




Return

# Increasing Returns Don't Depend on Size of Policy Reform




Return

# Second-Best Planner's Problem

$$\max_{\{c_t,\{A_{jt},s_{jt}\}\}} \sum_{t\geq 0} \frac{1}{(1+\rho)^t} u(c_t) \;\; s.t.$$

$$\Omega_t F(\{Y_{jt}\}) = c_t + \sum_j r_j \Lambda_{jt}$$

$$A_{jt} = \gamma^{z_{jt}} A_{jt-1}$$

$$\mathcal{S} = \sum_j s_{jt}$$

$$\{\Lambda_{jt}, \{\ell_{j\iota t}\}\} = \text{argmax}\left[\mathcal{Y}_t - \sum_j r_j \Lambda_{jt} - \tau_t \mathcal{E}_t\right] \;\; s.t. \;\; L = \sum_j \int_0^1 \ell_{j\iota t} d\iota$$

- External Carbon Price $\tau_t \Rightarrow$ IC Constraint

Return

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- External Carbon Price $\tau_t \Rightarrow$ IC Constraint

Return

# Supporting Innovation Subsidies

$$\xi_{jt}\Pi_{jt} = (1-\alpha)S_{jt}\mathcal{Y}_t - \sum_{\hat{t}\geq t}\prod_{\hat{s}=1}^{\hat{t}-t}\frac{1}{R_{t+\hat{s}}}(SCC_{\hat{t}} - \tau_{\hat{t}})\mathcal{E}_{\hat{t}}\frac{\partial \ln(\mathcal{E}_{\hat{t}})}{\partial \ln(A_{jt})} + \frac{1}{R_{t+1}}\left[\xi_{jt+1}\Pi_{jt+1} + \sum_i \xi_{it+1}\Pi_{it+1}g_{it+1}\varphi_{ijt+1}\right]$$

- Pollution term: Wedge $\times$ impact on equilibrium emissions
- Corrects spillover externality sent via spillover network

Return SCC

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$$\xi_{jt}\Pi_{jt} = (1-\alpha)S_{jt}\mathcal{Y}_t - \sum_{\hat{t}\geq t}\prod_{\hat{s}=1}^{\hat{t}-t}\frac{1}{R_{t+\hat{s}}}(SCC_{\hat{t}} - \tau_{\hat{t}})\mathcal{E}_{\hat{t}}\frac{\partial \ln(\mathcal{E}_{\hat{t}})}{\partial \ln(A_{jt})}$$
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- Pollution term: Wedge $\times$ impact on equilibrium emissions
- Corrects spillover externality sent via spillover network

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- Separation of externalities; exclusive focus on spillover creation

## Second-Best

- Now compromise between two externalities
- Reward clean technologies for not polluting

Return SCC

# Steady-State Policy

## Corollary

*Let* $\hat{\xi}_{jt} \equiv (\gamma - 1)(1 - \tilde{R}_t^{-1})\xi_{jt} S_{jt}$ *and* $\mathcal{T}_{jt} \equiv \sum_{\hat{t} \geq t} \prod_{\hat{s}=1}^{\hat{t}-t} \tilde{R}_{t+\hat{s}}^{-1} \frac{SCC_{\hat{t}} - \tau_{\hat{t}}}{1-\alpha} \bar{\omega}_{\hat{t}} \frac{\partial \ln (\mathcal{E}_{\hat{t}})}{\partial \ln (A_{jt})}$.
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- Combination of Domar/distortion & centrality (Liu and Ma, 2021)
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  - Myopic Planner ($\tilde{R}^{-1} \to 0$) $\Rightarrow \hat{\xi} = S - \mathcal{T}$
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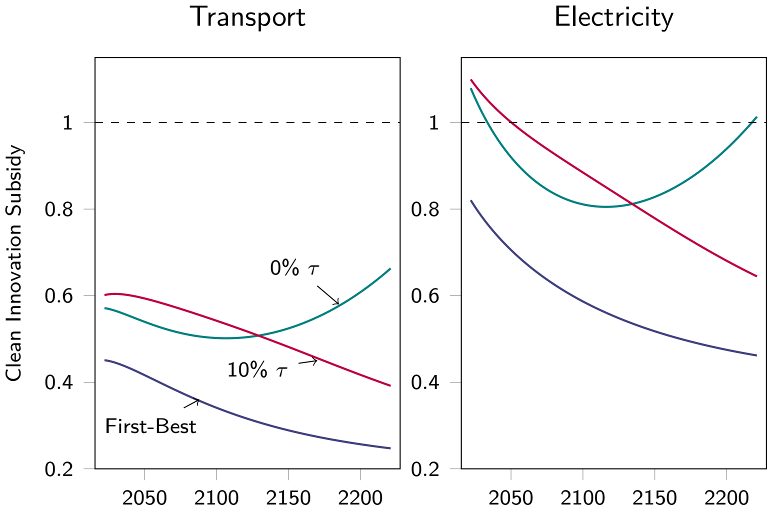
## Big Push is Temporary

- First-best, no spillovers $\Rightarrow$ Long-run laissez-faire
- Spillover network $\Rightarrow$ Permanent intervention

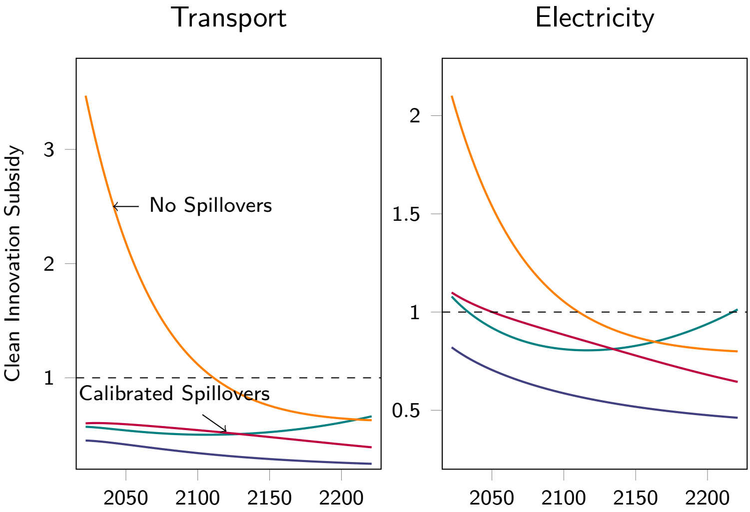
Return

# Optimal Innovation Subsidy Path




Return First-Best High Discounting High Damages Temperature

# Optimal Innovation Subsidy Path (+ No Spillovers)




- No technological lock-in ⇒ No big push

Return

# Big Push is No Longer Optimal

## Profit vs. Spillover Creation

- $\xi \circ \Pi = SMB$
- Clean Transportation
  - Income Share: **4.9%** vs. Centrality: **0.67**
- Clean Electricity Generation
  - Income Share: **3.1%** vs. Centrality: **1**

◂ Return | CES Spillovers | Disaggregated Spillover Network | Second-Best Welfare Loss

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## Why Aren't Second-Best Subsidies Higher?

- Clean innovation generates a rebound effect
- Innovation subsidies are a poor substitute for carbon prices

Return | CES Spillovers | Disaggregated Spillover Network | Second-Best Welfare Loss

# Social Cost of Carbon

$$SCC_t = -\sum_{\hat{t}\geq t}\prod_{\hat{s}=1}^{\hat{t}-t}\frac{1}{R_{t+\hat{s}}}\mathcal{Y}_{\hat{t}}\frac{\partial \ln\left(\Omega_{\hat{t}}\right)}{\partial \mathcal{E}_t}$$

## Interpretation

- Intertemporal MRS $R_{t+1} \equiv (1+\rho)u'_t/u'_{t+1}$
- Discounted sum of marginal damages

◂ Return

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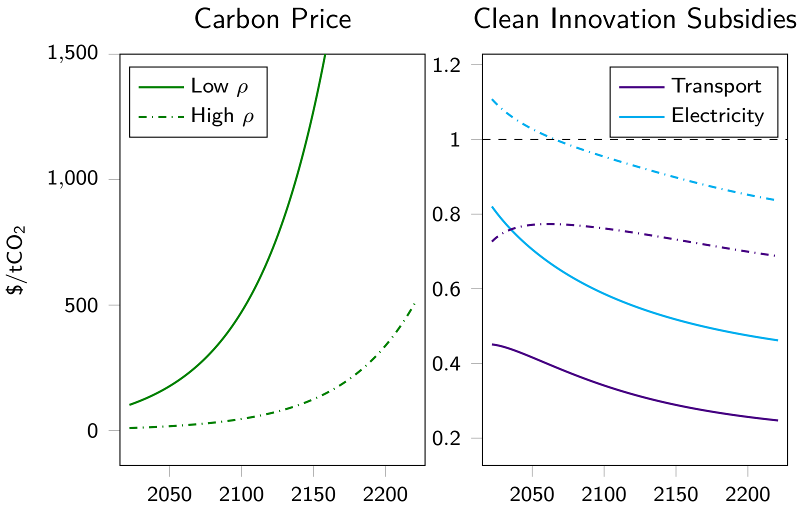
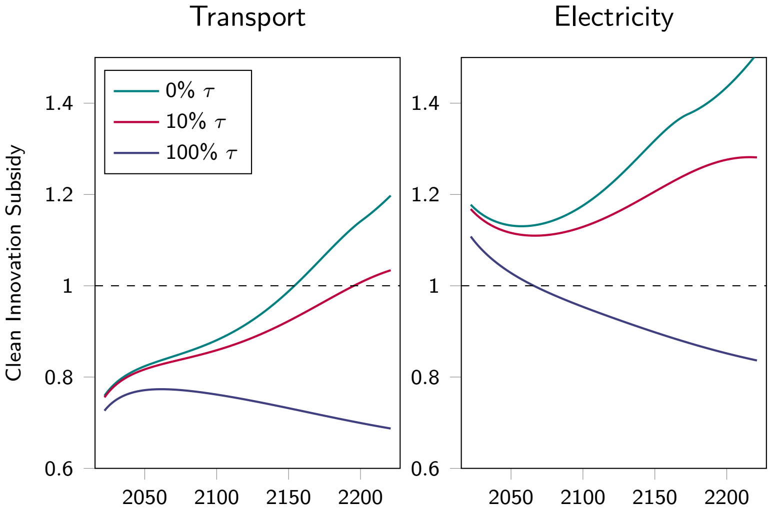
Return

# First-Best Policy Path



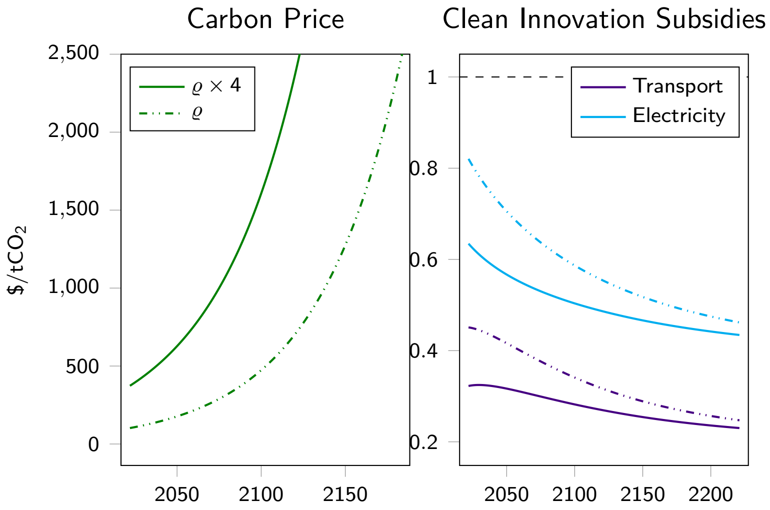

Return

# Optimal Innovation Subsidy Path (High Discounting)



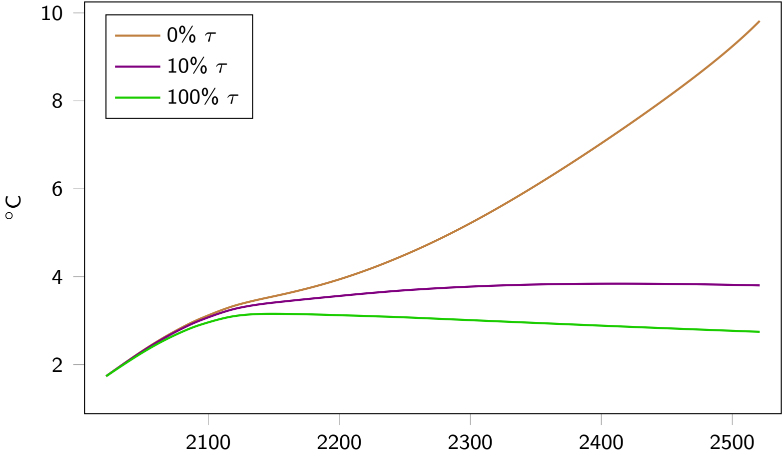

Return

# Optimal Innovation Subsidy Path (High Damages)



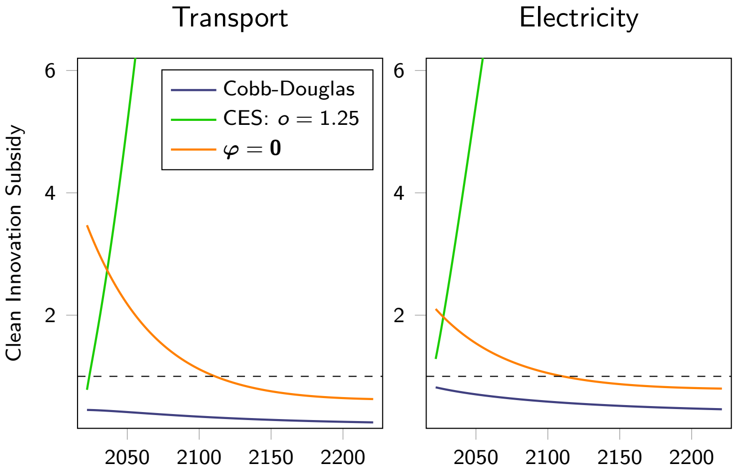

Return

# Temperature Path




Return

# CES Spillovers




$$\phi_{it} = \frac{\left(\sum_j \hat{\varphi}_{ij}^{\frac{1}{o}} A_{jt-1}^{\frac{o-1}{o}}\right)^{\frac{o}{o-1}}}{A_{it-1}}$$

Return

# Disaggregated Spillover Network

## Aggregation of Technologies *Might* Matter for Centrality

- General technology is highly aggregated residual definition
- Consider network disaggregated into 3-digit CPC codes

## Clean Centrality *Not* Sensitive to Aggregation Choice

- Clean Transport: $\mathbf{0.67} \rightarrow \mathbf{0.73}$
- Clean Electricity: $\mathbf{1} \rightarrow \mathbf{1.19}$

Return

# Second-Best Welfare Loss

## Consumption Equivalence

- Small Carbon Price
  - More patient: **1.49%**
  - Less patient: **0.29%**
- No Carbon Price
  - More patient: $\approx$**100%**
  - Less patient: **0.87%**

## A Little Goes a Long Way

- Welfare cost of losing carbon price highly nonlinear
- Clean innovation alone
  - Slow Emission Reductions $\rightarrow$ Rebound Effect
  - Slow Growth $\rightarrow$ Loss of Dirty Spillovers

◂ Return