Identifying Relationship-level Effects Using Covariance Restrictions

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Match decomposition using fixed effects

Fixed effects are often used to decompose the product of a match.

- **Corporate credit** (Amiti and Weinstein 2018; Khwaja and Mian 2008)
- **Workers/firms** (Abowd et al. 1999) (AKM)
- **Import/export** (Kramarz et al. 2020)

More generally, many-to-many bipartite networks (e.g., Bonhomme 2020).

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$$\Delta l_{fb} = d_f + s_b + \epsilon_{fb} \ \ (\ldots + \ \Gamma X_{fb})\,.$$

Fixed effects identify homogeneous demand and supply shocks (worker/firm effects).

Homogeneity assumption rules out key policy questions.

AKM approach is potentially **biased** in realistic settings.

A bivariate model with relationship-specific effects

We study the **bivariate** model

$$\eta_{fb} \equiv \begin{pmatrix} \Delta r_{fb} \\ \Delta l_{fb} \end{pmatrix} = A \begin{pmatrix} u^d_{fb} \\ u^s_{fb} \end{pmatrix} \; (\ldots + \; \Gamma X_{fb}) \, .$$

Changes in price and quantity (match outcomes) are driven by relationship-specific demand and supply shocks.

Identify A**:** supply and demand coefficients of P/Q.
Identify u_{fb}**:** shocks themselves.

Key assumption: A is fixed across relationships (within period/sub-sample).

Our generalisation

We replace AKM homogeneity assumption with much weaker **correlation** assumption: u_{fb} vector is *correlated*, not *constant* across f and b dimensions.

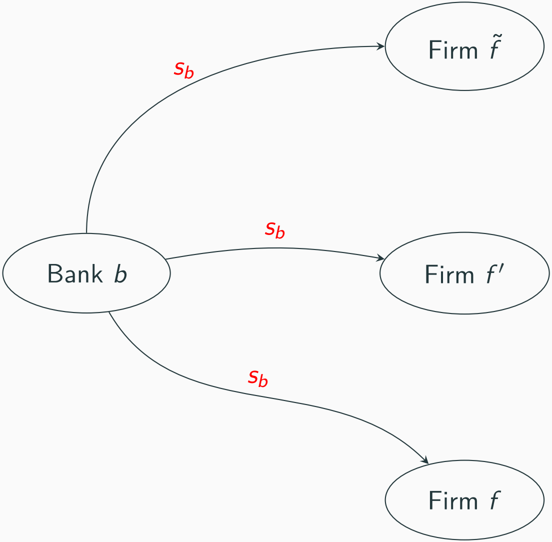
We identify from those correlations using **covariance restrictions**.

Can be interpreted as an IV approach under simplifying assumptions.

Propose a simple test of the AKM assumptions.

Modest assumptions on degree of agents (Jochmans and Weidner 2019).

Fixed effects model



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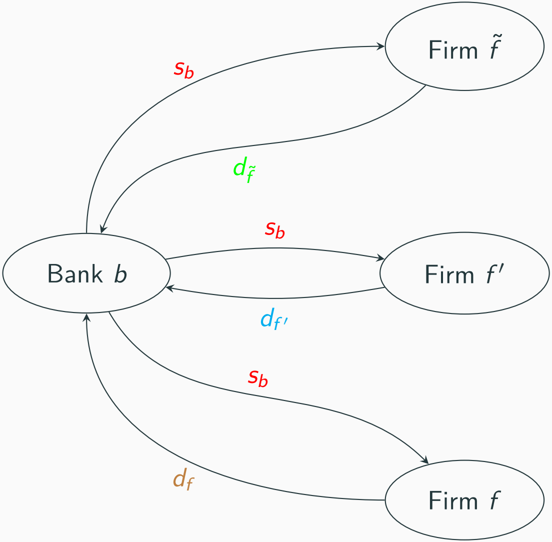
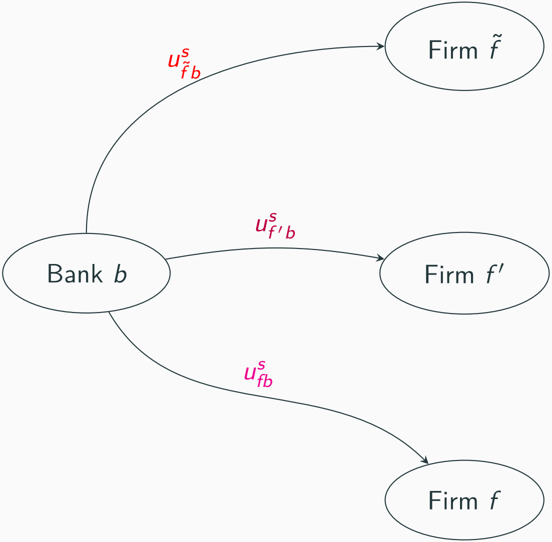


Our model



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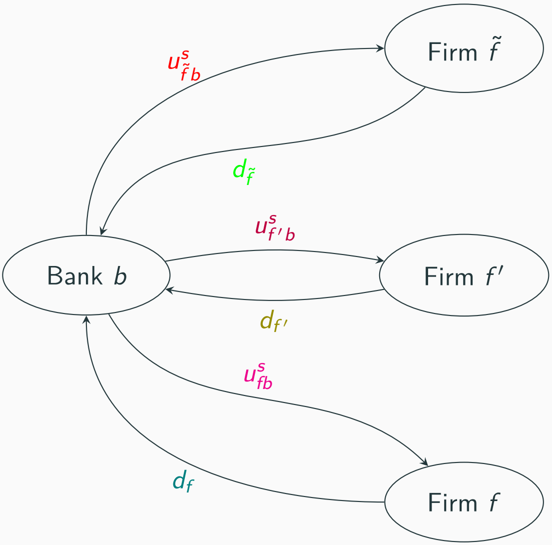
Firm $\tilde{f}$
$u^s_{\tilde{f}b}$
$d_{\tilde{f}}$
$u^s_{f'b}$
Bank b
Firm f'
$d_{f'}$
u^s_{fb}
d_f
Firm f

Empirical contribution

We apply our method to the **Anacredit** dataset – 9 countries, 18 quarters, near-universe of corporate credit.

AKM assumptions are **rejected** for nearly all country-periods.

We show that Khwaja and Mian (2008) and Amiti and Weinstein (2018) FE "shocks" are biased: interest rates robustly decreasing in "demand shock".

In contrast, our shocks have theoretically consistent effects.

We document role of firms' credit composition in monetary policy transmission.

Roadmap

1. Methodological contribution
   - **Identification**
   - **Estimation and Inference**
2. Simulations
   - **Bias**
   - **Size**
3. Application to AnaCredit
   - **Evidence for heterogeneity**
   - **Evidence of AKM bias**
   - **Monetary policy transmission at relationship level**

Assumptions for identification

The model:

$$\eta_{fb} = D_{fb} \times Au_{fb} \ \left(\ldots + \ \Gamma X_{fb} \right), f = 1, \ldots, F, b = 1, \ldots B.$$

η_{fb}, u_{fb} are 2×1 vectors.

Assumption 1

The following hold

1. *A is invertible and constant across firm-bank pairs,*
2. $E\left[u_{fb} | D_{fb} = 1, \bar{D}\right] = 0,$
3. $E\left[u^d_{fb} u^s_{f'b} | D_{fb} = 1, D_{f'b} = 1, \bar{D}\right] = 0,$
   $E\left[u^d_{fb} u^s_{fb'} | D_{fb} = 1, D_{fb'} = 1, \bar{D}\right] = 0, \ b' \neq b, \ f' \neq f.$

Henceforth drop D; understood that equations relate to observed quantities.

Identification result

We exploit the novel moments

$$\text{cov}(\eta_{fb}, \eta_{f'b}) \equiv \Sigma_{FF} = A\Lambda_{FF}A', f' \neq f$$
$$\text{cov}(\eta_{fb}, \eta_{fb'}) \equiv \Sigma_{BB} = A\Lambda_{BB}A', b' \neq b,$$

where $\Lambda_{FF}, \Lambda_{BB}$ diagonal by Assumption 1.

Bank's supply is correlated over firms, as is demand to that bank, vice versa.

Proposition 1

If $\Lambda_{FF} \neq c\Lambda_{BB}$ for any scalar c, then the solution to

$$\Sigma_{FF} - A\Lambda_{FF}A' = 0$$
$$\Sigma_{BB} - A\Lambda_{BB}A' = 0$$

is unique up to scale, sign, and column ordering.

Solution in closed form: eigenvectors of $\Sigma_{FF}\Sigma_{BB}^{-1}$. See Rigobon (2003).

Example: corporate credit

Paravisini et al. (2023): **heterogeneity** in demand and supply due to specialisation.

1. P & Q responses to supply/demand are linear & constant *within-sample*.
   - By country-time period, but also slice further (industry, region, firm characteristics)
2. $E\left[u^d_{fb} u^s_{f'b}\right] = 0$
   - Firms are atomistic: firm f demand does not impact bank b supply.
   - No spillovers: firm f' supply does not impact f demand.
   - Put info on large exposures, bank fundamentals or supply chains in X_{fb}.
3. $E\left[u^d_{fb} u^s_{fb'}\right] = 0$
   - Reorientation delay: firm f demand from b unimpacted by b' supply.
   - Shocks are causal: firm f's outlook can't be both supply *and* demand.

Omit granular firms and control for bank's exposure to upstream/downstream firms.

If event triggers *simultaneous* supply/demand *responses*, condition on it in X_{fb}.

Estimation

The sample counterparts are

$$S_{FF} = \frac{1}{N_{FF}} \sum_{b=1}^{B} \sum_{f' \neq f} \eta_{fb} \eta'_{f'b}$$

$$S_{BB} = \frac{1}{N_{BB}} \sum_{f=1}^{F} \sum_{b' \neq b} \eta_{fb} \eta'_{fb'},$$

where $N_{FF} = \frac{1}{2} \sum_{b=1}^{B} F_b(F_b - 1), N_{BB} = \frac{1}{2} \sum_{f=1}^{F} B_f(B_f - 1)$, and F_b is the number of firms connected to bank b and B_f the banks connected to firm f.

Minimum distance estimator:

$$q(\boldsymbol{\eta}, \theta) = \begin{pmatrix} \text{vech}\left(S_{FF} - A\Lambda_{FF}A'\right) \\ \text{vech}\left(S_{BB} - A\Lambda_{BB}A'\right) \end{pmatrix},$$

θ vectorises $A, diag(\Lambda_{FF}), diag(\Lambda_{BB})$.

The framework for inference

- Data has a complicated dependence structure.
  - These challenges are common in the networks literature
- The key to asymptotics is:
  - Slightly more structure on demand and supply shocks.
  - A non-vanishing share of firms is well-connected.
  - No need to assume that all firms/banks heavily connected.
  - Neither F nor B grows too fast relative to the other.

Extensions

1. **Multiple time periods**
    - So far, only considered single time period - can also pool across periods.
    - Consistency and asymptotic normality extend, cluster over firms/banks and time for robust variance estimate.
2. **Including covariates**
    - Under cond. mean indep. assumption on X_{fb}, can partial out covariates.
    - Mirrors AKM/existing approaches.
3. **Shocks as dependent variables**
    - Shocks are generated regressors and induce dependence in regressions.
    - Show asymptotic normality with adjusted variance estimator.

Simulations: Summary

Simulate data for networks of different sizes calibrated to Italian data from 2022Q3-2023Q4.

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Percent Bias:

$T = 1$	$B = 10$	$B = 25$	$B = 100$	$B = 500$
A_{11}	-0.04	-0.02	-0.01	0.00
A_{21}	-0.32	-0.09	-0.02	-0.00
A_{12}	-0.13	-0.08	-0.02	-0.00
A_{22}	-0.10	-0.06	-0.01	-0.00
$T = 4$	$B = 10$	$B = 25$	$B = 100$	$B = 500$
A_{11}	-0.02	-0.01	-0.00	0.00
A_{21}	-0.05	0.03	-0.01	-0.01
A_{12}	-0.05	-0.01	-0.00	-0.00
A_{22}	-0.04	-0.01	-0.00	-0.00

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Empirical Size:

$T = 1$	$B = 10$	$B = 25$	$B = 100$	$B = 500$
A_{11}	10.7	8.1	6.0	5.7
A_{21}	10.7	9.1	6.5	5.4
A_{12}	15.4	8.9	4.9	5.3
A_{22}	18.5	14.2	5.9	5.0
$T = 4$	$B = 10$	$B = 25$	$B = 100$	$B = 500$
A_{11}	5.1	4.7	5.8	6.1
A_{21}	5.6	4.4	5.2	6.6
A_{12}	7.6	5.9	5.6	5.4
A_{22}	11.3	6.0	5.3	4.9

Simulations: Summary

Simulate data for networks of different sizes calibrated to Italian data from 2022Q3-2023Q4.

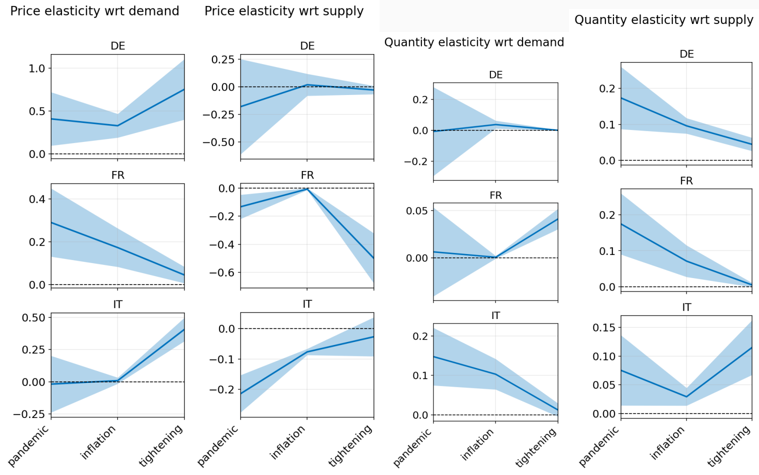
- Bias **falls quickly** with B – excellent performance for $B \geq 25$.
- **Pooling multiple time periods** dramatically improves MSE.
- Tests for elements of A well-sized.
- *Average estimated shocks* outperform estimated fixed effects.

▸ Simulations

Sample: period and countries

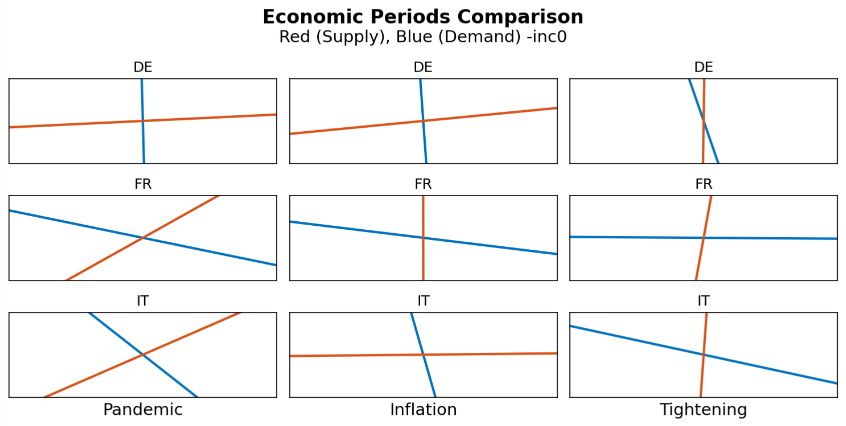
- We study supply and demand dynamics in 9 euro area credit markets,
- ... leveraging the **AnaCredit** database.
- Credit Types: Revolving credit, credit lines, and term loans.
- Measurement:
  - Δl_{fb}: "Midpoint" growth in committed amount
  - Δr_{fb}: Change in value-weighted interest rate
  - Both metrics are winsorized and demeaned.
  - X_{fb} contains lagged relationship specific characteristics.
- Three 6-quarter periods:
  - 2019Q3–2020Q4: Pandemic
  - 2021Q1–2022Q2: Inflationary build-up
  - 2022Q3–2023Q4: Monetary tightening

Coefficients Over Time



(a) $\partial P / \partial u^{d}_{fb}$ **(b)** $\partial P / \partial u^{s}_{fb}$ **(c)** $\partial Q / \partial u^{d}_{fb}$ **(d)** $\partial Q / \partial u^{s}_{fb}$

The Evolution of Supply and Demand Curves



AKM assumptions are not compatible with the data

The AKM model can be tested via over-identifying restrictions!

AKM requires that: $\Lambda_{FF} = \text{diag}(0, 1)$ and $\Lambda_{BB} = \text{diag}(1, 0)$

The AKM assumptions are rejected at the **5%** level for **25 out of 27** country-periods! **1%** level for **24 out of 27**.

Critical values are 4.61 and 5.99, respectively:

quantile	min	0.1	0.25	0.50	0.75	0.90	max
test stat.	2.95	5.91	12.19	75.86	222.86	404.14	923.31

Failures to reject: pandemic period in Portugal and tightening in Netherlands.

Shocks are characterised by heterogeneity

Collapse at the firm-time level

	p10	p25	p50	p75	p90	SD
Avg. demand innovation	-0.677	-0.253	0.000	0.171	0.677	0.646
SD demand innovation	0.019	0.063	0.225	0.863	1.681	0.780

Collapse at the bank-time level

	p10	p25	p50	p75	p90	SD
Avg. supply innovation	-0.218	-0.088	0.009	0.095	0.231	0.399
SD supply innovation	0.267	0.485	0.712	0.952	1.266	0.511

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- For $>$50% of firms, within-firm SD is $>$30% of between-firm SD.
- For nearly 75% of banks, within-bank SD larger than between-bank SD.
- Considerable variation cannot be studied using FE!

AKM-type estimates exhibit bias

	Change in Interest Rate			
Demand innovation (f,b,t)	0.219***			0.261***
	(0.008)			(0.012)
Supply innovation (f,b,t)	-0.187***		-0.259***	
	(0.007)		(0.009)	
Khwaja-Mian FT		-0.483***	1.151***	
		(0.054)	(0.084)	
Khwaja-Mian BT		-0.751***		-1.260***
		(0.096)		(0.104)
Khwaja-Mian Resid		-0.470***	1.150***	-1.549***
		(0.054)	(0.082)	(0.111)

Regression at firm-bank-time level. Relationship-specific and Khwaja-Mian shock estimates. 9 countries, 18 quarters, country-time FEs.

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Credit markets and monetary policy transmission

	Demand innovation	Supply innovation
Share fixed rate loans	0.014	-0.018**
	(0.015)	(0.008)
Monetary Policy	-0.538***	0.692***
× Share fixed rate loans	(0.195)	(0.195)
Central Bank Information	0.960***	-0.392***
× Share fixed rate loans	(0.223)	(0.150)
Share collateralised loans	0.015**	-0.009
	(0.007)	(0.011)
Monetary Policy	-0.064	0.431***
× Share collateralised loans	(0.094)	(0.086)
Central Bank Information	0.087	-0.028
× Share collateralised loans	(0.106)	(0.088)

9 countries, 18 quarters, FT & BILT FEs, 1-quarter lagged regressors, and Jarociński and Karadi (2020) shocks.

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ΔQ and ΔP vs. Demand and Supply Innovations

	Credit growth	Change in Interest Rate	Demand innovation	Supply innovation
Share fixed rate loans	-0.004	0.042***	0.014	-0.018**
	(0.003)	(0.012)	(0.015)	(0.008)
Monetary Policy × Share f.r.l.	0.086	-1.091***	-0.538***	0.692***
	(0.057)	(0.143)	(0.195)	(0.195)
CB Info × Share f.r.l.	-0.084	1.082***	0.960***	-0.392***
	(0.053)	(0.159)	(0.223)	(0.150)
Share collateralized loans	0.006	0.025***	0.015**	-0.009
	(0.010)	(0.005)	(0.007)	(0.011)
Monetary Policy × Share c.l.	0.131**	-0.324***	-0.064	0.431***
	(0.066)	(0.078)	(0.094)	(0.086)
CB Info × Share c.l.	0.048	0.271***	0.087	-0.028
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Monetary Policy × Share f.r.l.	0.086	-1.091***	-0.538***	0.692***
	(0.057)	(0.143)	(0.195)	(0.195)
CB Info × Share f.r.l.	-0.084	1.082***	0.960***	-0.392***
	(0.053)	(0.159)	(0.223)	(0.150)
Share collateralized loans	0.006	0.025***	0.015**	-0.009
	(0.010)	(0.005)	(0.007)	(0.011)
Monetary Policy × Share c.l.	0.131**	-0.324***	-0.064	0.431***
	(0.066)	(0.078)	(0.094)	(0.086)
CB Info × Share c.l.	0.048	0.271***	0.087	-0.028
	(0.063)	(0.088)	(0.106)	(0.088)

9 countries, 18 quarters, FT & BILT FEs, 1-quarter lagged regressors, and Jarociński and Karadi (2020) shocks.

Conclusion

Relaxing **homogeneity** assumption identifies relationship-specific effects.

Wide range of potential applications – finance, labour, trade.

Simple identification, estimation, inference – Stata/Python routine soon.

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Next step: impact on Khwaja-Mian type firm-level outcome regressions

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Discipline models, motivate identification assumptions, inform policy.

Thank you!

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Approach for consistency

First, show that the variance of S_{FF} is vanishing as $F, B \to \infty$: *Within banks*, there are $O(BF^4)$ non-zero covariances, but $N_{FF}^2 = O(B^2 F^4) \Rightarrow O(B^{-1})$.

Across different b, there are $O(B^2 F^2)$ non-zero covs $\Rightarrow O(F^{-2}) < O(B^{-1})$.

Then, $S_{FF} \xrightarrow{p} \Sigma_{FF}$ (uniformly) by Chebyshev (at rate $\sqrt{B}$).

Same true for S_{BB} by symmetry, and consistency of $\hat{\theta}$ follows from standard minimum distance results.

Approach for asymptotic normality

Non-trivial to apply a CLT, observations are not in general independent.

Trick is to expand each $\eta_{fb,i}\eta_{f'b,j},\ i,j \in \{1,2\}$ based on Assumption 2.

Obtain four components, one of which is **independent** across b, call it $\beta_{b,ff',ij}$. $\sqrt{B}\left(\frac{1}{N_{FF}}\sum_{b=1}^{B}\left(\sum_{f'\neq f}\beta_{b,ff',ij} - \Sigma_{FF,ij}\right)\right)$ satisfies Lyapunov's condition where observations are the inner sums for each b.

Joint normality of $\beta_{b,ff'}$ follows from Cramer-Wold.

Similarly scaled sums of all other terms converge to zero in probability, so normality of S_{FF} follows. S_{BB} by symmetry. $\hat{\theta}$ by minimum distance results.

▸ Back

Assumptions for inference 1

Assumption 2
Demand and supply shocks have the structure

$$u^d_{fb} = e^d_{fb} + v^d_{fb}$$
$$u^s_{fb} = e^s_{fb} + v^s_{fb}.$$

where e^i_{fb} *is mean zero and independent of all innovations except for* $e^i_{fb'}$ *and* v^i_{fb} *is mean zero and independent of all innovations except for* $v^i_{f'b}$*. All innovations have strictly positive variance and finite eighth moments, and* $\lim_{F,B\to\infty} \frac{B}{N^2_{FF}} \sum_{b=1}^{B} \text{var}\left(\sum_{f'\neq f} \text{vech}(v_{fb} v'_{f'b})\right)$ *and* $\lim_{F,B\to\infty} \frac{F}{N^2_{FF}} \sum_{f=1}^{F} \text{var}\left(\sum_{b'\neq b} \text{vech}(e_{fb} e'_{fb'})\right)$ *are symmetric positive definite, where* e_{fb} *and* v_{fb} *stack the bank and firm demand and supply components, respectively.*

Assumptions for inference 2

Assumption 3
The following limits hold:

1. $$\lim_{F,B\to\infty} \frac{N}{FB} = \kappa \in (0,1]\,, \quad N \equiv \sum_{b=1}^{B} F_b = \sum_{f=1}^{F} B_f;$$

2. $$\frac{B}{F^2} \to 0 \text{ as } F, B \to \infty;$$

3. $$\frac{F}{B^2} \to 0 \text{ as } F, B \to \infty.$$

Back

Estimating the asymptotic variance

Define

$$\hat{\mathrm{W}}_{FF} = \frac{B^2}{N_{FF}^2}\frac{1}{B}\sum_{b=1}^{B}\left(\sum_{f'\neq f}\mathrm{vech}\left(\eta_{fb}\eta'_{f'b}\right) - \mathrm{vech}(S_{FF})\right)\left(\sum_{f'\neq f}\mathrm{vech}\left(\eta_{fb}\eta'_{f'b}\right) - \mathrm{vech}(S_{FF})\right)'.$$

Proposition 2

Under Assumptions 1-3 and the identification condition in Proposition 1, $\hat{\mathrm{W}} \xrightarrow{p} \mathrm{W}$.

Looks very much like clustered standard error formula!

So far, we have only considered data from a single time period.

Consistency and asymptotic normality extend to pooled data, and $\hat{W}$ straightforward to adjust for serial correlation.

Back

Simulations: Setup

Simulations are based on estimates for Italy in tightening subsample

$$A = \begin{bmatrix} 0.0761 & -0.0687 \\ 0.0124 & 0.0610 \end{bmatrix}$$

Serially uncorrelated (SU) shocks are generated from:

$$u^i_{fb} = z^i_f + z^i_b + z^i_{fb}, \quad i = \{d, s\}, \; f = 1, \ldots, F, \; b = 1, \ldots, B,$$

z's are independent and normally distributed with mean zero and empirically calibrated variance

Serially-correlated (SC) shocks: z^i_f and z^i_b independent mean-zero AR(1), with autoregressive parameters matching the data

- 1000 Monte Carlo samples
- $B = 10, 25, 100, 500$, with $F = 1000B$
- fraction of connections are non-zero, at random, matching sparsity of network