

Housing Assignment with Endogenous Supply: Theory and Application to Rent Control

Jimmy Ho (Washington University in St. Louis)

Yulin Hong (City University of Hong Kong)

Zhongji Wei (Universitat Pompeu Fabra)

Michael B. Wong (University of Hong Kong)

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Motivation

- ▶ Affordable housing shortages stifle growth and increase inequality (Glaeser & Gyourko 2018; Hsieh & Moretti 2019; Baum-Snow 2023)
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- ▶ What are the effects of rent control on *uncontrolled* rents?
- ▶ Recent evidence is mixed
 - ▶ Wang (2011); Autor et al. (2014): average uncontrolled rents fall
 - ▶ Mense et al. 2023; Kholodilin (2024): average uncontrolled rents rise

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- ▶ Effects can be heterogeneous across quality distribution (Abel, Carrer and Luque 2024)

Limitations of Existing Theory

Standard rent control models: (e.g., Wang 2011)

- ▶ Uncontrolled housing = perfectly divisible
 - ▶ Single shadow price per unit of “housing services”
- ▶ Implication: rent control affects uncontrolled rents *uniformly*

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Housing assignment models:

- ▶ Richer characterization of cross-sectional price-quality patterns (Määttänen and Terviö 2014; Landvoigt et al. 2015; Epple et al. 2020)
- ▶ But housing quality distribution typically treated as *exogenous*
- ▶ Attempts to endogenize supply impose strong restrictions on substitution patterns (Epple et al. 2020; Kang 2023)

This Paper

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2. Provide *theory of rent control* using assignment approach
3. Document stylized facts from Hong Kong (2006–2016) to motivate model

Key Finding

Rent control can have *opposite* price effects in different quality segments

- ▶ E.g. low-quality rents \uparrow , high-quality rents \downarrow
- ▶ Why? Rationing alters household sorting
- ▶ Bad for low-income households outside controlled sector

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Broader implication:

- ▶ Assignment approach helps reconcile estimates, useful for housing analysis

Stylized Facts: Hong Kong

Why Hong Kong?

- ▶ Nearly one-third of the population lives in public rental housing (PRH)
 - ▶ PRH rents heavily subsidized and unresponsive to market conditions

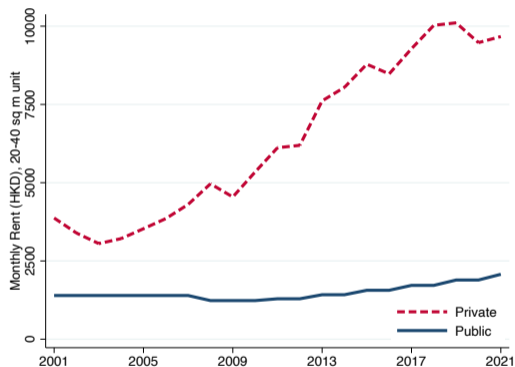
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 - ▶ Widespread subdivided units (SDUs), often smaller than a parking space
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- ▶ Ideal setting to study spillovers from rent regulation to uncontrolled housing

Fact 1: Public and Private Rents Diverged (2006-2016)



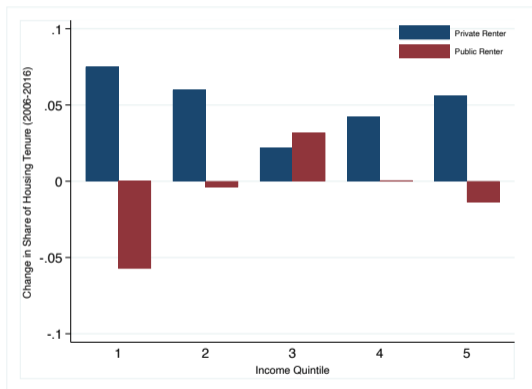
▶ PRH rents

- ▶ Tied to income growth and capped ($\leq 10\%$ every two years)
- ▶ For $\sim 20\text{--}40\text{ m}^2$ units, rose from HK\$1,396 to HK\$1,563/month

▶ Comparable private rents

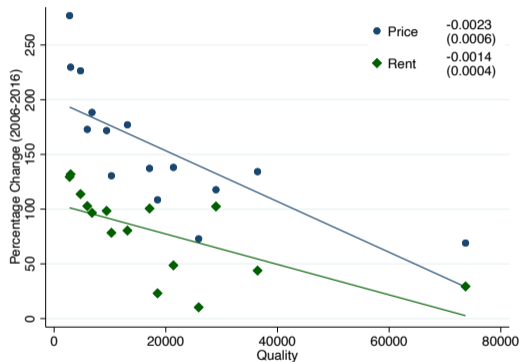
- ▶ Rose from HK\$5,715 to HK\$10,495/month
- ▶ Nearly nearly 7x PRH rents by 2016

Fact 2: Public Housing Misallocation Increased



- ▶ Public housing is rationed via wait lists and eligibility check at entry
- ▶ After entry, weak income re-testing and modest rent increases
- ▶ In 2006-2016, low-income households increasingly pushed into private low-quality rentals
 - ▶ In lowest income quintile, public renter share *fell* by ~ 5.5 p.p.
 - ▶ In middle income quintile, public renter share *rose* by ~ 3 p.p.

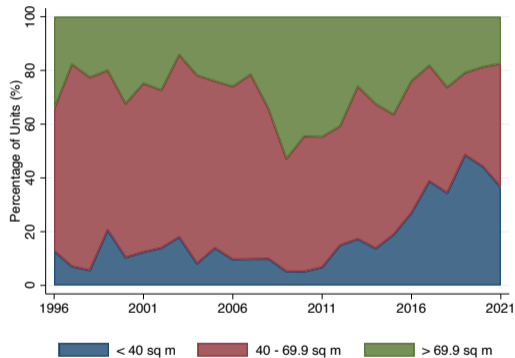
Fact 3: Low-Quality Rents Rose Disproportionately



RVD rental indices and census data for 15 quality segments (region \times size)

- ▶ Between 2006 and 2016:
 - ▶ Lowest-quality private units: rent rose by $\approx 114\%$
 - ▶ Highest-quality private units: rent rose by $\approx 29\%$
- ▶ Contradicts the “law of one price” implicit in existing rent control models.

Fact 4: New Construction Shifted Toward Small Units



- ▶ Share of newly completed private units $< 40 \text{ m}^2$:
 - ▶ $\lesssim 10\%$ in 2006.
 - ▶ $\approx 27\%$ in 2016.
- ▶ Share of $40\text{--}69.9 \text{ m}^2$ units fell from 64% to 49%.
- ▶ Indicates developers shift toward lower-quality, smaller units.

Fact 5: Landlord Subdivision Became a Key Supply Channel

Tight squeeze



- ▶ Subdivided units (SDUs):
 - ▶ Created by partitioning existing flats into multiple micro-units
 - ▶ Concentrated in older urban buildings, often unsafe and poorly regulated
- ▶ 2021: ~3% of population (215,700) live in units < 13.5 m²
- ▶ Subdivision is a key response of landlords to high demand for ultra-low-quality housing.

SCMP

Implications of the Facts

- ▶ Facts 1–2: consistent with traditional rent-control models:
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- ▶ Facts 3–5: *not* reconciled by existing models:
 - ▶ Heterogeneous rent growth across quality segments
 - ▶ Endogenous construction of small units and landlord subdivision
- ▶ We need a housing assignment model that:
 1. Relaxes perfect divisibility
 2. Endogenizes developer construction across qualities
 3. Allows landlords to transform existing stock across qualities at a cost

Two-Quality Assignment Model

Baseline Two-Quality Model: Households

- ▶ Measure n of households, each with income I draws utility

$$u(c, j), \quad j \in \{L, H\},$$

with $u_c, u_j, u_{cj} > 0$ and $u_{cc} < 0$, subject to budget

$$c + r_j \leq I, \quad j \in \{L, H\}.$$

- ▶ In equilibrium, there is an income cutoff I_{HL} such that:
 - ▶ $I > I_{HL}$ choose H
 - ▶ $I \leq I_{HL}$ choose L

Baseline Model: Developers

- ▶ Representative developer problem:

$$\max_{h_H^D, h_L^D} (r_H - k_H - p\theta_H)h_H^D + (r_L - k_L - p\theta_L)h_L^D.$$

- ▶ Fixed cost k_j
 - ▶ θ_j units of generic input (e.g., land), variable cost p
-
- ▶ Input market clearing:

$$Q = \theta_H h_H + \theta_L h_L, \quad p = p(Q), \quad p'(\cdot) > 0.$$

Baseline Model: Landlords and Transformation

- ▶ Landlord can transform units across qualities with costly adjustment
 - ▶ One L unit $\rightarrow \theta_L/\theta_H$ units of H (and vice versa).
- ▶ Representative landlord problem:

$$\max_{h_H^L, h_L^L} r_H h_H^L + r_L h_L^L - \phi(h_L^L - \bar{h}_L),$$

subject to:

$$\theta_H h_H^L + \theta_L h_L^L = \theta_H \bar{h}_H + \theta_L \bar{h}_L,$$

$$\phi'' > 0, \phi(0) = \phi'(0) = 0.$$

Baseline Model: Equilibrium

- ▶ Total supply in each segment:

$$h_j(r_L, r_H) = h_j^L(r_L, r_H) + h_j^D(r_L, r_H), \quad j \in \{L, H\}.$$

- ▶ Market clearing:

$$h_H + h_L = n, \quad h_L = nF(l_{HL}),$$

where F is the income CDF.

Proposition 1: Under mild assumptions on costs and $p(\cdot)$, there exists a unique competitive equilibrium (r_L, r_H, l_{HL}, p) .

Balanced Population Growth

Consider an increase in n (balanced growth, same F):

- ▶ Both equilibrium rents r_L and r_H increase
- ▶ Supply of both qualities rises
- ▶ Relative rent responses depend on:
 - ▶ Income distribution F and demand parameters
 - ▶ Supply elasticity of the input ($p'(\cdot)$)

Proposition 2: r_H can respond less than r_L to population growth if the input supply is sufficiently elastic (i.e. p' is sufficiently large).

Rent Control in the Two-Quality Model

Introducing Rent Control

- ▶ Add a controlled sector of share μ of total housing, all *low quality* (L).
 - ▶ Controlled rent $r_c < r_L$ is exogenous.
- ▶ Timing:
 1. Government sets μ and r_c .
 2. Households with income $\leq I_{HC}$ apply for controlled units.
 3. Units randomly rationed among applicants.
 4. Rejected applicants choose between uncontrolled L and H .
- ▶ Normalize $n = 1$ for simplicity.

Comparative Statics: Lower Controlled Rent

Proposition 3: $r_c \downarrow$ with μ fixed \implies

▶ $r_L \uparrow$, and $h_L \uparrow$.

▶ $r_H \downarrow$, and $h_H \downarrow$.

Comparative Statics: Lower Controlled Rent

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Intuition:

- ▶ Lower r_c attracts higher-income applicants.
- ▶ More failed applicants spill into uncontrolled L .
- ▶ Excess demand in low-quality uncontrolled segment pushes r_L up.
- ▶ High-quality demand falls, as some high-income households shift into the controlled segment, reducing r_H .
- ▶ Landlords/developers respond: more low-quality, less high-quality supply

Comparative Statics: Rent Control Expansion

Experiment: Increase μ (expand controlled sector) holding r_c fixed.

- ▶ Implemented by re-designating some initial low-quality units as controlled:

$$\Delta h_L^0 = -\Delta\mu.$$

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Proposition 4: $\mu \uparrow$ with r_c fixed \implies

- ▶ $r_L \uparrow$ and $h_L \uparrow$ (in remaining uncontrolled market).
- ▶ $r_H \downarrow$ and $h_H \downarrow$.

Intuition: Rent Control Expansion

- ▶ Higher μ has two effects:
 1. Mechanically reduces *uncontrolled* low-quality supply.
 2. Increases acceptance probability in the controlled sector.
- ▶ Some households who would have rented high-quality units now occupy controlled low-quality units:
 - ▶ Demand for uncontrolled H falls.
 - ▶ Demand for uncontrolled L falls as well, but not by as much as supply
- ▶ Result: Excess demand in uncontrolled L , excess supply in H :
 - ▶ r_L rises, incentivizing more low-quality construction/subdivision.
 - ▶ r_H falls, reducing high-quality construction.

Continuous-Quality Assignment Model

Continuous-Quality Model: Overview

▶ Quality $q \in [\underline{q}, \bar{q}]$ is continuous.

▶ Households:

$$\max_{c, q} u(c, q) \quad \text{s.t.} \quad c + r(q) \leq I.$$

▶ First-order condition:

$$r'(q) = \frac{u_q(c, q)}{u_c(c, q)}.$$

▶ Each q maps to an income level $I(q)$ and vice versa; market clearing implies:

$$F(I(q)) = H(q),$$

where H is the CDF of housing quality.

Developers and Landlords: Continuous Case

Developers

- ▶ Build density $h^D(q)$, cost per unit of quality q : $k(q) + pq$
- ▶ Zero-profit:

$$r(q) \begin{cases} = k(q) + pq, & \text{if } h^D(q) > 0, \\ \leq k(q) + pq, & \text{if } h^D(q) = 0. \end{cases}$$

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Landlords

- ▶ Initial stock density $h^0(q)$. Choose final stock $h^L(q)$ subject to convex adjustment cost $\Phi(\cdot)$
- ▶ With quadratic costs, we obtain an explicit expression:

$$h^L(q) = h^0(q) + \frac{1}{\kappa q^2} \left(r(q) - \frac{1}{\bar{q} - \underline{q}} \int_{\underline{q}}^{\bar{q}} \frac{r(q')}{q'} dq' \right).$$

- ▶ Landlords shift toward qualities with high $r(q)/q$ relative to average

Equilibrium Conditions

1. Total housing density:

$$h(q) = h^D(q) + h^L(q).$$

2. Assignment and market clearing:

$$f(I(q))I'(q) = h(q),$$

where f is the income density.

3. Household FOC ties $r'(q)$ to $I(q)$ and $r(q)$
 4. Developer and landlord conditions link $h(q)$ to $r(q)$ and p
- ▶ Result: an *integro-differential equation* for the rent schedule $r(q)$

Special Case and Numerical Solution

- ▶ Utility: $u(c, q) = \log c + \theta \log q$. Then:

$$r'(q) = \frac{\theta}{q} [I(q) - r(q)].$$

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where Γ depends on $r(q)$ and the integral $\int r(q')/q' dq'$

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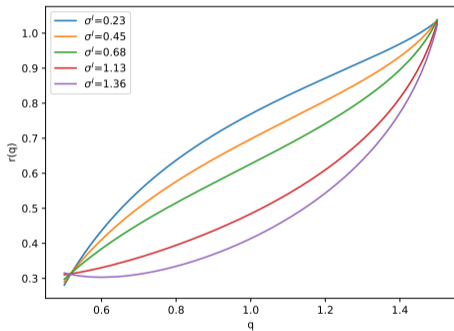
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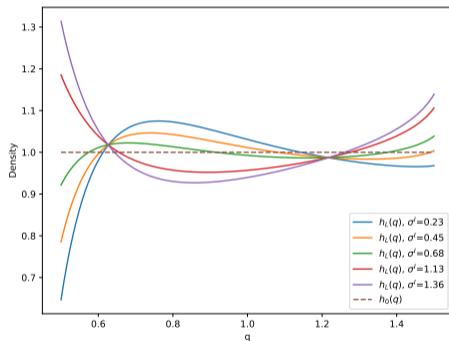
- ▶ Solve for $r(q)$ numerically
 - ▶ Discretize q . Treat $r(q)$ as an unknown vector
 - ▶ Minimize the squared residuals of equilibrium conditions (subject to market-clearing constraints)

Comparative Statics: Income Dispersion

Rents by quality



Density by quality



Mean-preserving spreads in the income distribution F raises rents and supply of units at both ends of the quality distribution

Implications for Hong Kong and Policy

- ▶ Rent control increases income dispersion in low-tail of uncontrolled sector
 - ▶ Increases low-quality rents and supply
 - ▶ Lowers high-quality rents and supply

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- ▶ Rent control increases income dispersion in low-tail of uncontrolled sector
 - ▶ Increases low-quality rents and supply
 - ▶ Lowers high-quality rents and supply
- ▶ Consistent with Hong Kong facts:
 - ▶ Growing housing misallocation (Fact 2).
 - ▶ Fastest rent growth in low-quality segment (Fact 3).
 - ▶ Shift of new construction toward small units (Fact 4).
 - ▶ Proliferation of SDUs (Fact 5).

Conclusion

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- ▶ We build tractable housing assignment models with *endogenous supply*:
 - ▶ Two-quality model with analytic comparative statics.
 - ▶ Continuous-quality model characterized by an integro-differential equation.
- ▶ Main finding: Rent control can exacerbate pressure on low-quality segment, harming households excluded from the controlled sector
- ▶ Stylized facts from Hong Kong align with the model
- ▶ Approach enables more realistic housing policy analysis