Product Purchasing Contest[*][†][‡]

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November 27, 2025

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Abstract

Loyalty-driven industries increasingly adopt contests in which consumers compete for rewards through repeated purchases, raising concerns about excessive consumption. Unlike monetary bids, consumption-based investment generates utility while incurring production costs. I develop a framework that extends contest design from allocation rules to competing investment. As the firm can extract all welfare generated by competitive consumption, the profit-maximizing design uniquely implements the first-best outcome while fully appropriating consumer surplus. The conventional equivalence across mechanisms breaks down since welfare now depends on who spends how much—not just total spending. This structure creates a trade-off between efficiency and distribution, revealing a designer-regulator tension.

Keywords: consumption-based investment, firm-designed contest, consumer protection, unintended consequence, profit inequivalence

JEL Classification: D18, D42, D91, L12, L59

[*]I am deeply grateful to my advisors Bertan Turhan and Donghyuk Kim for their invaluable guidance, generous support, and enduring encouragement. I thank GianCarlo Moschini, Jian Li, Wei Zhang, Boli Xu, and participants at many seminars and conferences for their helpful comments and insightful discussions. All remaining errors are mine.

[†]Earlier versions of this paper were presented under the title "Product Purchasing Contest: A Strategy to Leverage Competition among Consumers."

[‡]Please click here for the Online Appendix

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1 Introduction

Contests are competition for rewards through non-refundable investments, and many public allocation mechanisms adopt this structure to collect revenue.[1] Recently, however, firms have increasingly adopted contests as profit-collecting mechanisms in which competition is based on consumption rather than monetary investment. While consumption differs from money in welfare implications, profit-oriented mechanisms may misalign with public interests, such as welfare. Yet the strategic, design, and welfare implications of consumption-based investment in firm-designed contests remain underexplored.

In *product purchasing contest*, firms motivates consumers to repeatedly purchase firm's products (*the regular goods*) to improve winning chance of an exclusive reward (*the premium good*). For example, in the music industry, meet-and-greet invitations are often limited to fans who purchase large quantities of physical albums. Despite the rise of digital streaming, Korea, where this practice is prominent, saw a 34.9% annual increase in top 400 album sales between 2013 and 2022, in contrast to the global 2.8% decline.[2] This strategy is prevalent, observed across regions[3] and across industries driven by consumer loyalty or status programs, such as luxury, fashion, online gaming, the movie theater industry, live-streaming services, and crowdfunding platforms.

The success of this strategy has raised concern over excessive consumption of regular goods. First, the induced purchases prompts worries about exploitative spending and reduced consumer surplus.[4] Second, the over-consumption has sparked criticism of its wastefulness.[5] Third, the outcome often leaves heavy spenders empty-handed despite substantial purchases, fostering relative deprivation. These concerns have led to discussions about whether such practices apply to an abuse of market power that calls for intervention.[6]

[1]Similar mechanisms are commonly observed in the allocation of scarce resources, such as land, housing, education, business licenses, or immigration rights, where financial expenditures play a critical role in determining access. For example, many immigration programs require applicants to deposit funds or invest in businesses as a condition for residency, public housing programs often mandate upfront security deposits or long-term savings commitments, and elite educational institutions may consider financial contributions as part of their admission process, and radio spectrum auctions frequently require bid deposits.

[2]A more detailed industrial background is provided in Appendix A (with the luxury fashion industry).

[3]American artists can reference marketing strategies from Korean artists in targeting the Asian market (Knopper, 2020).

[4]In 2024, Hermès faced a class-action lawsuit for allegedly employing a similar mechanism in the sale of its iconic Birkin bags (Scribner and Tashjian, 2024).

[5]Some fans have reportedly spent thousands of dollars purchasing the identical physical albums (KOCCA, 2020). Here, the term "identical" album refers to albums that contain the identical songs. However, significant changes have occurred in the industry in this regard, elaborated in Appendix A and B.

[6]The controversy over this strategy likely arose because it departed from established industry practices and conflicted with conventional consumer expectations. The music industry, however, has continuously adapted its norms, institutional structures, and monetization strategies in response to technological disruption. The rise of digitization and the spread of streaming services transformed the industry landscape and may have

Nevertheless, the challenge is not merely about firms adopting a contest, but a fundamentally different kind of one.[7] Standard contests rely on monetary investment that yields no utility to consumers and imposes no production cost on the contest designer. By contrast, in product purchasing contests the investment is consumption itself. This shift in the investment method has three implications: (i) it changes welfare effects since investment generates both utility for consumers and incurs production cost, (ii) it introduces an additional design lever, the attributes of the regular good (price, and in extensions, quality), and (iii) it integrates markets that would otherwise be separate, making consumers' demands strategically interdependent through competition. This distinctive nature of consumption-based investment reshapes the mechanism along welfare, design, and strategic dimensions. As a result, policy design becomes nontrivial and raises new questions.

Are product purchasing contests socially productive or wasteful? Do they harm consumer surplus, and if so, would a full prohibition improve overall welfare? If not, what policy alternatives—such as fine-tuned regulation of allocation rules—might preserve efficiency while addressing the ex-post welfare gap that underlies non-winning consumers' relative deprivation? Theoretically, how does consumption-based investment reshape canonical contest results?

Answering these questions requires a formal framework that can dissect the mechanisms driving firms' and consumers' strategic decisions. Meanwhile, capturing the whole market effect should integrate numerous factors in a complex way: dynamically interrelated pricing and contest choices of multi-product firms, their interaction with consumers' competitive consumption, and industry-specific norms and other frictions, while much about the practice itself remains unknown. I therefore first focus on a parsimonious approach rather than a fully general theory, to isolate two stylized structutres—firm-designed contests and consumption-based investment. Within this framework, I analyze a static monopoly using the classic Tullock contest format. In this setting, the designer retains flexibility over the allocation rules, and the investment function is endogenously shaped by the firm's pricing, with its curvature determined by consumption utility; these two design dimensions interact in determining market outcomes.

This setup supports two policy analyses. First, I compare the profit-maximizing product

encouraged firms to employ contests to sustain physical album sales.

[7] Although public discussions and legal debates have offered preliminary interpretations—such as "tying" or "indirect auction"—these views provide only partial insights and fail to capture the full economic structure of product purchasing contests. These interpretations often appear in public discussions and media coverage. Two common interpretations have emerged: "indirect auction," is used by a Fair Trade Commission (the fair trade authority in Korea) official (Hong, 2018), and "tying," a concept applied to in legal cases involving Hermes' sales strategies with Birkin bag (Danziger, 2024; Williams, 2024). More detailed discussions are offered in the Online Appendix.

purchasing contest with its full prohibition. This contrast not only informs the welfare implications of banning the practice but also provides a focused comparison between consumption-based and monetary bidding, clarifying how designing investment technology shapes outcomes. Second, I study partial regulation that preserves the product-purchasing structure but constrains the allocation rule. This approach evaluates incremental, practically relevant interventions and allows disciplined comparison among commonly employed mechanisms (e.g., winner-pay, all-pay auctions) within the product purchasing contest, highlighting how their strategic and welfare implications diverge from conventional contests.

Building on this baseline, I progressively enrich the model by embedding the contest in a managerial design context. This extension endogenizes key strategic decisions—such as the attributes of the regular good and the supply of premium goods—and brings the framework closer to institutional reality while revealing novel implications for the intersection of contest theory and industrial organization.

The main findings inform both policy design and theory: Banning product purchasing contests can harm social welfare. This potentially unintended consequences imply an inherent tension between total welfare and its distribution. Even fine-tuned regulation faces the trade-off, indicating the breakdown of the traditional equivalence across institutions, due to the differentiated nature of consumption-based investment.

The first main result concerns the impact of product purchasing contests. In equilibrium, the contest dominates the Prohibition Outcome[8] in both profits and welfare. Although designed to maximize profits, it achieves a first-best outcome, but not in a Pareto-improving way since it fully extracts consumer surplus.

Intuitively, this dominance arises from their ability to sustain excessive investment (over-dissipation) in the premium good through its inherent bundling structure of regular and premium goods. From the consumer's perspective, the regular good functions as an in-kind investment subsidy: purchases yield consumption utility, making additional investment toward the premium good less costly, and encouraging more aggressive participation. Whereas willingness to invest is conventionally limited by the expected value of the reward, consumers here are willing to spend beyond the expected value of the premium good because part of their spending is compensated by regular good utility. From the firm's perspective, integrating the two markets through a product purchasing contest enables surplus from regular good consumption to spill over into the premium good contest. The firm can now internalize the welfare generated by regular good consumption, which is

[8] The prohibition scenario assumes the premium goods are allocated through a plain, non-product-purchasing contest while the regular good market operates under a typical monopoly.

unattainable under simple separate sales. This internalization incentivizes the firm to design the contest so as to fully extract the surplus generated by regular good consumption. This mechanism resembles the surplus extraction of a non-linear pricing, but here it is achieved using only linear pricing integrated with a contest. As a result, prohibiting product purchasing contests to protect consumers can unintendedly sacrifice total welfare.

The first main result has three key implications: First, it challenges the prevailing welfare intuition behind public concerns: what appears to be "excessive" consumption under product purchasing contests is actually welfare-enhancing. Second, its higher profitability provides an economic rationale for the widespread use of product purchasing contests, reinforcing the relevance of the framework. Third, a full prohibition generates a stark trade-off between efficiency and firm-consumer welfare distribution, prompting more nuanced policy options. Drawing on the conventional equivalence across mechanisms, I examine whether partial regulation—adjusting allocation rules while preserving the product-purchasing structure—could mitigate ex-post welfare inequality among consumers (winners vs. non-winners) while preserving total efficiency. This motivation leads to the second main result, which examines the impact of such fine-tuned regulations.

The second main result establishes that the profit-maximizing Tullock contest is the unique global optimum—among all allocation rules, including mixed strategies—not only in profits but also in total welfare under consumption-based investment. This finding breaks the conventional expected-revenue equivalence across allocation mechanisms, showing that the consumption-based investment fundamentally reshapes the profit and welfare outcomes. Specifically, the winner-pay auction is inefficient because only winners can contribute to welfare, while the all-pay auction is suboptimal due to dispersed investments that deviate from the optimal individual consumption.[9]

This profit–welfare inequivalence arises because allocation rules interact with the investment technology under consumption-based investment. In monetary contests, investments are mere transfers from contestants to the designer; given a fixed total investment, how it is collected does not affect social welfare. Here, by contrast, both the reward (premium good) and the investment (regular good) have their own welfare effects. As a result, social welfare depends not only on the total investment but also on who spends how much.

From a policy perspective, this inequivalence implies that interventions should be tai-

[9]The second main result also highlights how competition intensity shapes profits and welfare under product purchasing contests. All-pay auctions, with their deterministic winner rule, induce socially inefficient competition: some consumers overinvest, wasting resources, while others underinvest, missing potential welfare gains. The optimal Tullock contest instead uses a stochastic rule to sustain intermediate competition, mitigating both overspending and underinvestment. This theoretical insight aligns with practice: in music and luxury industries, firms often appear to introduce strategic randomness to keep competition between lotteries and auctions, based largely on consumer reports and media coverage.

lored to the regulator's objective—efficiency or equity. A utilitarian regulator would avoid regulation, as the profit-maximizing design is already efficient. However, if distributional fairness among consumers sufficiently matters—particularly the gap between winners and non-winners—then partial regulation (e.g., winner-pay or all-pay auctions) may be desirable, though it entails some loss in total welfare.

Beyond the baseline model, I embed the contest in the firm's broader managerial design context—endogenizing premium good supply and regular good quality, and introducing consumer heterogeneity and behavioral primitives such as the value of winning and the "event ticket pricing puzzle." These extensions show that the core implications of product purchasing contests largely persist across the richer environments. At the same time, bringing contest theory and industrial organization together produces distinct implications that do not arise when each is considered alone: strategic over- or underproduction of premium goods, exclusion of lower value consumers, and two departures from canonical bundling—profitability no longer hinges on consumer heterogeneity, and the standard correlation result reverses.

This paper studies contests as a profit-oriented firm strategy, contributing to both the contest design and industrial organization literatures.[10] (i) Mechanism level: incorporating consumption-based investment turns the investment technology itself into an additional design variable shaping strategy and welfare, beyond the classic focus on allocation rules. (ii) Institutional level: the model places a decentralized contest designer operating under potential regulatory intervention, uncovering an efficiency–distribution trade-off and a resulting designer–regulator tension. By doing so, this paper sheds light on the market-driven mechanism innovations in today's evolving economy, where firms increasingly experiment with monetization strategies and new institutional forms. Understanding how such mechanisms spontaneously emerge, gain acceptance in the marketplace, and interact with regulation is crucial for both policy and future research.

The paper proceeds as follows. Section 2 reviews the releted literature and Section 3 presents the model. Section 4 examines full prohibition, while Section 5 explores fine-tuned alternatives. Section 6 considers extensions. Section 7 concludes. The Appendix provides industry background, and the Online Appendix contains proofs and further discussions.

2 Related Literature

Since Tullock's seminal work (Tullock, 1967), contest theory has provided a fundamental framework for modeling competition in nature and society. Extensive research has focused on theoretical foundations, equilibrium outcomes and conditions. Comprehensive reviews

[10]Web-scraping data are used to provide descriptive statistics that motivate the study.

are available in Nitzan (1994), Konrad et al. (2009), Vojnović (2015), Corchón and Serena (2018), Fu and Wu (2019), and Beviá and Corchón (2024).

Building on this foundation, a rich literature examines optimal contest design by endogenizing institutional features, such as the number of winners, prize profiles (Clark and Riis, 1998; Schweinzer and Segev, 2012; Fang et al., 2020; Letina et al., 2023), prize values (Kaplan et al., 2002, 2003),[11] winner-selection rules (Michaels, 1988; Epstein and Nitzan, 2006; Wang, 2010; Schweinzer and Segev, 2012; Epstein et al., 2013; Polishchuk and Tonis, 2013; Letina et al., 2023), bid caps (Letina et al., 2020), strategic exclusion (Baye et al., 1993), and contestant biases. Optimal contest mechanisms are often achieved through joint adjustments of these institutional features.[12] While existing models have primarily focused on designing the rules of competition, the technology of investment has been treated as a fixed structure shaped by ability or preference. In contrast, this study expands the scope of contest design to include investment technology by introducing consumption-based investment. By designing product attributes —such as price or quality—the firm can endogenize investment technology, thereby making the competition process itself a central dimension of contest design.

This study also contributes to the literature on the relative performance of mechanisms. While Tullock contests converge to an all-pay auction as randomness diminishes under standard assumptions(Alcalde and Dahm, 2010; Ewerhart, 2017), relative performance depends on the design of institutional factors[13](Fang, 2002; Epstein et al., 2013; Franke et al., 2014). The premise in these comparisons is that they hold the investment technology fixed across mechanisms. However, once the competition process is designable, such an assumption is necessarily relaxed, and thus the optimal investment technology becomes mechanism-specific. This study therefore compares mechanisms in a more general yet disciplined setting where the investment technology varies endogenously—its slope and curvature responding to institutional design.[14] This work also offers a new perspective

[11]Despite a similarity in the concept of the potential positive effects of sunk-costs, product purchasing contests are distinct from the bid-dependent reward contests. In the endogenous reward contests, the welfare contribution derived from efforts is realized through the increasing value of the reward (corresponding to the premium good in product purchasing contests), which is ultimately given to the winner of the contest. In contrast, in product purchasing contests, the utility and cost derived from the regular good is guaranteed to both sides regardless of the allocation outcomes. Furthermore, the welfare contribution through regular goods in product purchasing contests is influenced not only by contestants' efforts but also by the endogenously determined marginal cost of investments, the price of the regular goods.

[12]For instance, Letina et al. (2023) show that optimal marginal returns to investments and the number of prizes can be simultaneously determined with given contestants' risk preferences.

[13]Such as contestant heterogeneity or designer's bias.

[14]Two contrasts follow. First (prohibition), I compare a product-purchasing contest with consumption-based investment to a conventional contest with monetary investment. Second (fine-tuning), while investment remains consumption-based, different allocation rules are compared.

on evaluation of mechanisms by introducing a regulatory layer. Conventional analyses typically adopt the designer's viewpoint, ranking mechanisms based on how well they serve the designer's objective (e.g., maximizing revenue). However, in product purchasing contest—an IO setting in which firms design contests while a public authority shapes the institutional environment, these objectives are not necessarily aligned. This misalignment introduces an additional strategic dimension, in which firm-side mechanism design and regulator-side policy design interact. This paper thus highlights a designer-regulator tension and introduces an additional institutional layer to mechanism evaluation.

This study further extends the application of contest frameworks to new market contexts. Prior work has explored contests in areas such as advertising competition (Friedman, 1958; Schmalensee, 1976), R&D races (Fullerton and McAfee, 1999), sales force incentives (Kalra and Shi, 2001; Lim et al., 2009), public goods fundraising (Morgan, 2000; Goeree et al., 2005; Liu et al., 2022), and crowdsourcing (Archak and Sundararajan, 2009; Chawla et al., 2019). This paper contributes to the novel intersection of contest and industrial organization by reframing contest design as an integral component of firm strategy rather than a stand-alone mechanism. In product purchasing contests, designable elements such as the number of rewards and the investment technology interact with the managerial decisions, including the supply of premium goods and the pricing and quality of regular goods. This new connection highlights the strategic embedding of contest design within broader firm-level strategies. Furthermore, product purchasing contests embeds a bundling structure in which a chance of access to the premium goods is tied to consumption of regular goods. While prior work has considered stochastic provision of a component within bundles (Manelli and Vincent, 2006), this paper departs from that setting by endogenizing the probability of winning: it is no longer exogenously assigned, but earned through investments relative to others. This structural shift generates distinct implications that challenge conventional bundling theory and extend the scope of contest-based incentives in market design.

Finally, this study complements the literature on the event ticket allocation mechanism under limited supply, motivated by the so-called Event Ticket Pricing Puzzle, where event tickets are generally underpriced with relatively coarse pricing schemes. Existing research explores interactions between non-market-clearing uniform pricing and the arbitrage-motivated rent-seeking behaviors in the primary market and the secondary market, respectively. Leslie and Sorensen (2014) discuss welfare implications by considering both the costs due to rent-seeking behaviors and the improvements in allocative efficiency through resales. Budish and Bhave (2023) examine positional auctions as a price discovery mechanism. Product purchasing contest lies somewhere between these two approaches, internalizing rent-seeking behavior through a contest mechanism that still enables collective price discovery. Meanwhile, Courty (2019) suggested “Centralized Exchange”, an alternative mechanism for ticket allocation. This study introduces a novel framework that enables price discovery while remaining within a uniform pricing system.

3 Model

3.1 Market Environment

I consider a market in which a firm monopolizes two goods: a premium good and a regular good. There are $n \in \mathbb{N}$ homogeneous consumers who are rational, risk-neutral, and have complete information. While the supply of the indivisible premium good is fixed ($k \in \mathbb{N}, k < n$), the divisible regular good can be supplied unlimitedly at a constant marginal cost ($c \geq 0$). For simplicity, the premium good is produced at zero cost.[15][16] The model adopts a single-period monopoly framework in which all strategic variables are simultaneously determined, unless stated otherwise. For the solution concept, this study focuses on symmetric Nash equilibria, allowing for both pure and mixed strategies.

3.2 Consumers

Let $v \in \mathbb{R}_+$ denote the consumer's valuation of the premium good. The utility function for the regular good, denoted $u(x)$, is concave ($u''(x) \leq 0$ for all $x \geq 0$) with $u(0) = 0$.[17] and $u'(0) < \infty$,[18] ensuring the incentive compatibility. Reservation utility is normalized to zero. Consumers have quasi-linear preferences in money (no wealth effects), and no budget constraints. Resale of the premium good is infeasible.[19][20] Additive preferences over bundled (tied) goods are assumed, implying neither complementarity nor substitutability.[21]

[15]Section 6.2 relaxes this assumption, endogenizing the supply of premium goods.

[16]When supply is limited, constant marginal cost does not qualitatively affect the model's outcomes (Banciu et al., 2010), aside from its potential impact on entry and exit decisions.

[17]Beyond strictly increasing concave utilities, I allow concave functions that need not be monotonically increasing (e.g., quadratic). Negative marginal utility may reflect either the disutility from an additional regular good or utility falling below the normalized outside option.

[18]This assumption is plausible when consumers have access to potential outside options (e.g., streaming services as imperfect substitutes to physical albums).

[19]If the premium good is a service or an experience rather than a tangible product, then it is inherently non-resalable. For example, a personal meet-and-greet moment with musicians cannot be resold. Additionally, invitations to exclusive services are non-transferable, particularly when product purchasing contests operate within a membership program or require contestants to provide identification upon entering the contest. For instance, even in the virtual meet-and-greets (using a video call), which became common during the pandemic, winners are typically required to verify their identity beforehand, such as by presenting a passport.

[20]Moreover, under complete information and with identical consumers, no arbitrage opportunities exist unless the premium good's valuation is higher in the secondary market than in the primary market.

[21]Complementarity means the bundle is valued more than the sum of its parts, irrespective of any positive correlation between individual valuations. Conversely, substitutability means the bundle is valued less than the sum, independent of any negative correlation.

3.3 Market Institution and Monopolist as an Institution Setter

As an institution setter, the monopolist either employs a product purchasing contest or not. Each winner in contests receives at most one unit of the premium good. The relative profitability of the product purchasing contest depends on the benchmark in its absence. Figures 1 and 2 visualize the structural difference in market institutions.

3.3.1 Market under Prohibition Benchmark

Before introducing the product purchasing contest, I first discuss Prohibition Outcome benchmark, as it is close to the standard multi-product monopoly.

Definition: Prohibition Outcome (A Plain Contest for Premium Goods with a Monopoly on Regular Goods)

As a counterfactual benchmark, premium and regular goods are assumed to be sold separately, with a plain (non–product-purchasing) contest allocating the premium good (the Prohibition Outcome).[22] This benchmark is preferable to the alternative of direct sales at a fixed price: When comparing with the product purchasing contest, this approach controls for contest-specific factors,[23] isolates the role of using regular goods as the investment medium, and keeps the analysis grounded in contest theory. Expected profits under this Prohibition Outcome are identical to those from direct sales.

In the plain contest, consumers make monetary investments (e.g., lottery tickets) that carry no intrinsic consumption value and impose no cost on the firm.

As a benchmark, when the monopolist uses a plain contest for the premium good while selling regular goods separately, the profit maximization problem is formulated as (1):

$$max_{p,R}\Pi(p,R) = \underbrace{\sum_{j=1}^{n} X_j(R)}_{\text{Profits from the contest}} + \underbrace{(p-c)\sum_{j=1}^{n} Y_j(p)}_{\text{Profits from the market for regular good}} \tag{1}$$

where X_j denotes the monetary investment, and Y_j represents the quantity of the regular good purchased, both made by consumer j. The parameter R denotes the type[24] of the contest mechanism (discussed in Section 3.4), and, if endogenized, acts as a strategic design variable together with the regular good price p. Under this market institution, the

[22] This approach is also observed in real world, such as charity auctions (e.g., the Warren Buffet lunch auction), where high-value experiences are allocated through bidding mechanisms. While real-world implementations often use auctions, the simple market environment ensures the equivalence expected revenue between the optimal Tullock contest and auctions.

[23] As this study is rooted in real-world outcomes, controlling for these contest-specific factors, including behavioral elements such as the utility of winning (discussed in Section 6), enhances the rigor of the analysis.

[24] In this context, 'type' refers to allocation mechanisms or winner-selection rules within a contest.

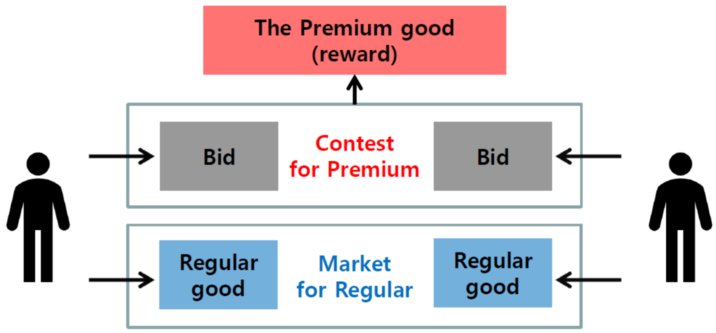


Figure 1: Graphic Illustration of counterfactual Prohibition Outcome.

consumers' utility maximization problem is defined as (2):

$$max_{X_i,Y_i} \underbrace{M(X_i;X_{-i},R,k)v - x_i}_{\text{Contest for premium good}} + \underbrace{u(Y_i) - pY_i}_{\text{Market for regular good}} \tag{2}$$

where $M(X_i;X_{-i},R,k)$ denotes the conditional winning probability, reflecting the allocation mechanism in Section 3.4 below. It depends on X_i, consumer i's investment, and X_{-i}, the investments of all other consumers. Each consumer makes two separate decisions (X_i and Y_i) on each market for premium and regular goods, respectively.

3.3.2 Market under a Product Purchasing Contest

Definition: Product Purchasing Contest

In a product purchasing contest, consumers invest through consumption, so a single purchasing decision determines both demand for the regular good and access to the premium good. This unification of consumers' decision-making integrates the previously separate markets and gives rise to three key implications.

First, unlike conventional money bids, this consumption-based investment has its own welfare effects. While monetary investment is welfare-neutral, consumption-based investment simultaneously generates utility and incurs production cost.

Second, firms can influence consumers' investment decision within a contest even by adjusting the attributes of the regular goods, such as quality and price. This provides the firm with greater control over the contest structure than in traditional contests, enabling the designer to optimize the performance of contest as profit generating device.

Third, using regular goods as an investment medium fundamentally alters how demand for regular good is determined. Due to the competitive interactions, it is no longer solely driven by consumer preferences but also other consumers' relative consumption through competition, becoming interdependent across consumers.

Building on this conceptualization, if the monopolist implements a product purchasing contest, its profit maximization problem is formulated as (3):

$$max_{p,R}\Pi(p,R) = (p-c)\sum_{j=1}^{n} x_j(p,R) \tag{3}$$

Unlike (1) in the Prohibition Outcome, the conditional demand x_j depends on both R and p. Under this alternative market structure, the consumers' problem is defined as (4):

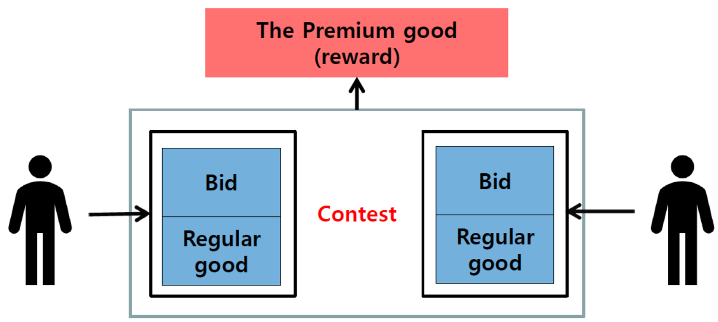


Figure 2: Graphic Illustration of how the two markets for premium and regular good are integrated through product purchasing contest.

$$max_{x_i}\ M(x_i; x_{-i}, R, k)v - px_i + u(x_i) \tag{4}$$

Problem (4) differs from Problem (2) in that consumer i has only one strategic variable, x_i, simultaneously representing both regular good purchases and the bids for the premium good. Regular good demand is now interdependent through $M(x_i; x_{-i}, R, k)$, capturing the strategic interactions introduced by competition.

3.4 Allocation Mechanism

I employ the standard Tullock contest model (ratio form) as the allocation mechanism. The conditional winning probability M, commonly referred to as "contest success function" is defined as (5) for the case where only 1 unit of the premium good is available.[25]

$$M(x_1, ..., x_n; R, k=1) = \begin{cases} \frac{x_i^R}{\sum_{j=1}^{n} x_j^R} & \text{if } \ max(x_1, ..., x_n) > 0 \\ \frac{1}{n} & \text{otherwise} \end{cases} \tag{5}$$

In the ratio-form contest success function, it is relative, not absolute, investment that determines M. The use of a power function in the ratio-form is justified by its axiomatic foundation (Skaperdas, 1996), which demonstrates that the power function is the only continuous function that satisfies the nice properties including positivity, monotonicity,

[25]I first present the functional form of M with 1 premium good as (5) for its intuitive clarity. The more general case of the contest success function M with multiple premium goods is provided as (6) below.

symmetry, similarity, independence from irrelevant alternatives, and homogeneity.

In (5), the parameter R measures the marginal return on investment. A higher R indicates greater marginal returns to investment, and therefore stronger incentives to invest more aggressively. R governs how winners are chosen, with typical examples including random allocation ($R = 0$), a lottery ($R = 1$), and an auction ($R = \infty$). Beyond these special cases, the Tullock contest framework accommodates a continuous spectrum of allocation rules as R varies. Although abstract, these mechanisms cover the full range of allocation rules that can induce any investment levels from nothing to the reward's expected value. This flexibility ensures that contest designers retain broad discretion to tailor contest structures to specific environments. The abstract representation also reflects asymmetric information between firms and and regulators in the implementation of contests.[26][27]

When multiple premium goods ($k > 1$) are available, an additional assumption is required concerning the procedure for allocating multiple prizes (Nitzan, 1994). Following Clark and Riis (1996), the analysis assumes that winners are chosen sequentially:[28] The k-prize contest can be understood as repeating the one-prize contest k times with fixed investments, removing each winner sequentially from subsequent selections. The corresponding contest success function for consumer i is given as (6). Let $|m|$ denote the number of active consumers who purchase a positive quantity of the regular good:

$$M_i(x_1, ..., x_n; R, k) = \begin{cases} P_i^1 + \sum_{j=1}^{k-1} \left[\Pi_{s=1}^{j} (1 - P_i^s) P_i^{j+1} \right] & \text{if} \quad |m| \geq k \\ 1 & \text{if} \quad |m| < k \text{ and } x_i > 0 \\ \frac{k-|m|}{n-|m|} & \text{if} \quad |m| < k \text{ and } x_i = 0 \end{cases} \tag{6}$$

where P_i^l represents the probability that consumer i wins the lth premium good, conditional on not winning in any of the previous $(l - 1)$ rounds.[29]

[26]The flexibility of contest mechanisms underscores the focus of this study—examining regulatory implications in decentralized environments rather than prescribing particular designs.

[27]The main analysis treats R as endogenous, while the exogenous case is discussed in the online Appendix.

[28]This mechanism is proven to be optimal in Letina et al. (2023) when multiple prizes are available despite a bit different economic environment, making it a reasonable candidate for selecting multiple winners.

[29]For example, assuming every consumer makes identical purchases with $k = 3$, the winning probability for each consumer is determined as $(3/n)$, as demonstrated by (7):

$$\underbrace{\frac{1}{n}}_{\text{(a)}} + \underbrace{\left[1 - \frac{1}{n}\right]}_{\text{(b)}} \underbrace{\frac{1}{(n-1)}}_{\text{(c)}} + \underbrace{\left[1 - \frac{1}{n} - \left(1 - \frac{1}{n}\right) \frac{1}{(n-1)}\right]}_{\text{(d)}} \underbrace{\frac{1}{(n-2)}}_{\text{(e)}} = \frac{3}{n} \tag{7}$$

The terms in (7) are defined as follows: (a) represents the probability of winning in the first round of the 1-winner contest; (b) denotes the probability of not winning in the first round; (c) represents the probability of winning in the second round, conditional on not winning in the first; (d) denotes the probability of not winning in both the first and second rounds; and (e) represents the probability of winning in the third round, To conclude the model description, three key assumptions are highlighted below.

Assumption 1 (No Preference Over Market Institution) *Consumers are indifferent to the choice of market institution itself, provided that their expected consumer surplus remains the same.*[30]

Assumption 2 (Prohibition Outcome Also Uses a Contest for Premium Goods) *In the counterfactual scenario, the premium goods are allocated through a contest rather than direct sales.*[31]

Assumption 3 (Contest Success Function and Multiple Prizes) *The firm is assumed to freely choose the allocation mechanism within the Tullock contest framework (Tullock, 1980), where the contest success function takes the standard ratio form. When multiple prizes ($k > 1$) are available, the allocation follows the sequential selection mechanism (Clark and Riis, 1996).*

3.5 Implications of Product Purchasing Contests

This subsection focuses on the theoretical and strategic implications of the product purchasing contest framework relative to the conventional model through the lens of contest theory. To do so, I incorporate the contest success function defined in Section 3.4 into the consumer's problems (2) and (4), formulating the model of interest, as (8) and (9). In (8), let $\bar{u}$ denote the consumer surplus from the regular good market in the prohibition benchmark. Since $\bar{u}$ does not affect the optimal choice of X_i, it can be omitted from the optimization problem.

$$max_{X_i} \left(\frac{X_i^R}{\sum_{j=1}^{n} X_j^R} \right) v - X_i + \bar{u} \tag{8}$$

$$max_{x_i} \left(\frac{px_i^R}{\sum_{j=1}^{n} px_j^R} \right) v - px_i + u(x_i) \tag{9}$$

Implementing the product purchasing contest transforms the consumer's problem from (8) to (9). Monetary investment X_i is replaced by consumption-based investment px_i, and the utility function $u(x_i)$ is incorporated. Since the Tullock contest success function is homogeneous of degree zero, the price p cancels from both numerator and denominator.

conditional on not winning in either the first or second rounds.

[30]While consumers may have psychological or contextual preferences regarding different market institutions, consumer preferences are solely determined by the utility derived from the goods they consume and the costs they incur, rather than by the structure of the allocation mechanism in this model

[31]This assumption ensures that the comparison between the product purchasing contest and the counterfactual scenario isolates the effects of the fundamental structural difference—consumption-based investment versus monetary investment—without being confounded by behavioral contest-specific factors, such as the value of winning associated with competitive allocation.

Hence, (9) reduces to (10).

$$max_{x_i} \left(\frac{x_i^R}{\sum_{j=1}^n x_j^R}\right) v - px_i + u(x_i) \tag{10}$$

Moving to consumption-based investment in (10) brings two fundamental changes. First, the investment technology is no longer fixed: it becomes a designable structure, endogenously shaped by the firm's pricing p. Second, the utility $u(x)$ affects the curvature of investment, creating a qualitatively different strategic environment. As a result, despite consumers' exogenously given quasi-linear money preferences, the effective investment function $px_i - u(x_i)$ is now endogenous[32] and strictly convex, reshaping the nature of investment incentives.

These fundamental changes entail two novel strategic implications.

First, within a given allocation rule, the pricing of the regular good directly affects profits and welfare. A notable departure from the conventional model is that market outcomes no longer depend solely on total investment px, but also on its composition, the combination of p and x. Even with the same total outlay, different price–quantity mixes lead to different profits and welfare outcomes, which is because marginal utility and production cost depend only on quantity, not price. As a result, the contest designer can strategically set the regular good price to implement most favorable outcome among the possible combinations.

Second, this strategic pricing decision also interact with the choice of allocation mechanisms. This evaluation naturally compares mechanisms paired with their own profit-maximizing regular good price. This additional dimension introduces a fundamentally different comparative framework from the standard approach. Conventionally, such cross-mechanism comparisons assume a common, exogenously fixed investment technology. In contrast, the product purchasing contest framework treats the investment technology as part of the designable market environment through regular good pricing. Mechanisms are thus compared under mechanism-specific, endogenously chosen investment functions.

This comparative approach applies not only to the design problems faced by contest designers but also to regulators evaluating policy interventions. For example, assessing the effect of introducing or prohibiting a product purchasing contest requires comparing the market outcomes of (8) and (10), each with its own optimally chosen allocation mechanism and regular good pricing decision.

Turning to the firm's problem, a critical distinction arises between Problem (1) and (3). Unlike (1), Problem (3) explicitly incorporates the price and cost of the regular good, which play a direct role in determining profits under a product purchasing contest.

[32] Alternative interpretation is that the coefficient of X_i in (9) is exogenously given at 1 while the coefficient of x_i in (10) is p that is determined by the monopolist.

4 Full Prohibition Analysis

Section 4 compares the equilibrium outcomes of the product purchasing contest with its prohibition counterfactual. Three cases are considered: SO (Social Optimum), PO (Prohibition Outcome), and PP (Product Purchasing Contest). Section 4.1 introduces the equilibrium outcome for SO. Section 4.2 characterizes PO and PP, and Section 4.3 synthesizes the findings, discussing their implications and motivating the subsequent analysis in Section 5. Throughout Section 4, the analysis focuses on pure strategy equilibria. Section 5 then proves that this equilibrium is uniquely optimal among all mixed strategy equilibria. All results build on the primitives introduced in Section 3.

4.1 The Socially Optimal Outcome (SO)

The following conditions, (1) and (2), characterize the Socially Optimal outcome (SO). Let y^*_{SO} and, SW^*_{SO} represent the equilibrium individual quantity of the regular good and social welfare, respectively: (1) $u'(y^*_{SO}) = c$, (2) $SW^*_{SO} = kv + n\int_0^{y^*_{SO}}[u'(y) - c]dy = kv + n[u(u'^{-1}(c)) - cu'^{-1}(c)]$.

Proof. Presented in the Online Appendix. ■

4.2 Prohibition Outcome (PO)

Prohibition Outcome is defined as a plain Tullock contest for premium good with a monopoly on regular good market. R^*_{PP} and R^*_{CF} denote the optimal, profit-maximizing values of R, each corresponding to the scenario that yields the highest profits. The optimal R is determined at the highest level where any pure strategy Nash equilibrium exists.

The symmetric CF equilibrium is characterized by the following conditions (1)–(5). The solutions y^*_{PO}, p^*_{PO}, R^*_{PO}, Π^*_{PO}, CS^*_{PO}, and SW^*_{PO} denote the equilibrium individual regular good quantity, the regular good price, the allocation mechanism, profits, consumer surplus, and social welfare, respectively: (1) $p^*_{PO} = u'(y^*_{PO}) = c - y^*_{PO}u''(y^*_{PO}) > c$, (2) $R^*_{PO} = \frac{k}{\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right]}$, (3) $\Pi^*_{CF} = kv + n[u'(y^*_{PO}) - c]y^*_{PO}$, (4) $CS^*_{PO} = 0 + \int_0^{y^*_{PO}}[u'(y) - u'(y^*_{PO})]dy > 0$, (5) $SW^*_{PO} = kv + n[u'(y^*_{PO}) - c]y^*_{PO} + \int_0^{y^*_{PO}}[u'(y) - u'(y^*_{PO})]dy$.

Proof. Presented in the Online Appendix. ■

The PO equilibrium is characterized by lower regular good consumption, leading to lower social welfare than under the SO outcome. his welfare loss arises because the regular good price is set at the profit-maximizing level (PO) rather than the socially optimal level (SO), generating the conventional deadweight loss of monopoly pricing.

4.3 Product Purchasing Contest (PP)

The following conditions (1)-(6) characterize the symmetric PP equilibrium. Here, x^*_{PP}, p^*_{PP}, R^*_{PP}, Π^*_{PP}, CS^*_{PP}, and SW^*_{PP} denote the equilibrium individual regular good quantity, the regular good price, the allocation mechanism, profits, consumer surplus, and social welfare, respectively: (1) $x^*_{PP} = u'^{-1}(c)$, (2) $p^*_{PP} = \frac{1}{u'^{-1}(c)}\left[\frac{kv}{n} + u(u'^{-1}(c))\right]$, (3) $R^*_{PP} = \frac{\frac{n}{v}[\frac{kv}{n}+\{u(u'^{-1}(c))-cu'^{-1}(c)\}]}{[\frac{k(n-1)}{n}-\sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}]}$, (4) $\Pi^*_{PP} = nx^*_{PP}(p^*_{PP}-c) = kv+n[u(u'^{-1}(c))-cu'^{-1}(c)]$, (5) $CS^*_{PP} = \frac{kv}{n}+u(u'^{-1}(c))-\frac{u'^{-1}(c)}{u'^{-1}(c)}\left[\frac{kv}{n}+u(u'^{-1}(c))\right] = 0$, (6) $SW^*_{PP} = \Pi^*_{PP}+CS^*_{PP} = kv+n[u(u'^{-1}(c))-cu'^{-1}(c)]+0 = SW^*_{SO}$.

Proof. Presented in the Online Appendix. ■

Comparing Results SO, PO, and PP yields Theorem 1, the first main result of the paper.

Theorem 1 (Profit-Maximizing Product Purchasing Contest) *The profit-maximizing product purchasing contest (PP) achieves the first-best outcome (SO), while fully extracting consumer surplus.*

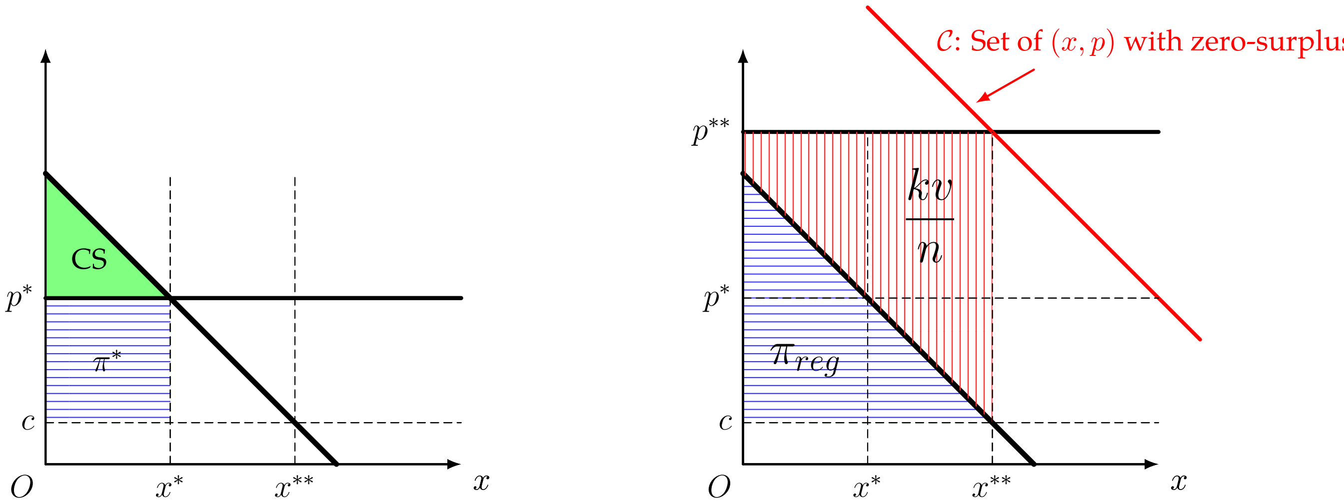
Proof. Presented in the Online Appendix. ■

What makes this difference between PP and PO is $c = u'(x^*_{PP}) = u'(y^*_{SO}) < u'(y^*_{PO})$.

Compared to the benchmark case, the PP equilibrium is characterized by a higher competition intensity, higher regular good consumption, increased profits, and reduced consumer surplus. As a result, because the firm can extract all welfare generated by competitive consumption, the profit-maximizing design implements the first-best outcome, but not in a Pareto-improving way, because it fully appropriates consumer surplus.

Intuitively, the dominance of PP arises from its ability to sustain excessive investment (over-dissipation) in the premium good through its inherent bundling structure of regular and premium goods. The regular good functions as an in-kind investment subsidy in that purchases yield consumption utility, making additional investment less burdensome, and thus encouraging more aggressive monetary spending. In contests with monetary bids, willingness to invest is limited by the expected value of reward. In contrast, consumption-based investment relaxes this bound because part of consumers' investment is compensated by utility derived from regular goods they purchase. Furthermore, the bundling structure integrates previously separate two markets, enabling the firm to internalize the surplus generated by the regular good consumption.

Figure (3) illustrates how the internalization works by contrasting the standard monopoly pricing of the regular good with the profit-maximizing product purchasing contest. Panel (a) shows that under monopoly, the marginal utility of the regular good exceeds the price



(a) Monopoly

(b) Product Purchasing Contest

Figure 3: Illustration of the Internalization: p^* and x^* denote the profit-maximizing price and quantity under a monopoly, and p^{**} and x^{**} denote the product purchasing contest counterpart.

for $x < x^*$, leaving a green area of consumer surplus because the monopolist cannot internalize it ($p^* < u'(x)$). The plain contest counterpart for the premium good is omitted since it does not affect this surplus under separate sales. Panel (b) illustrates how a product purchasing contest ties each unit of the regular good to an incremental chance of winning the premium good. By designing the winning probability to rise ($p^{**} - u'(x)$) as the marginal utility of the regular good falls ($u'(x) - c$),[33] the firm flattens the effective marginal willingness to pay (p^{**}) for the bundle and can fully extract it through linear pricing. The mechanism thus internalizes the surplus that would otherwise remain with consumers. This flattening mechanism resembles the homogenizing effect of profitable bundling: by pairing goods whose willingness to pay is negatively correlated, a seller can smooth heterogeneity and capture more surplus.

Meanwhile, the red curve $\mathcal{C}$ in Figure (3b) is not a standard demand curve, but the zero-surplus indifference curve under the product purchasing contest, defined as (11):

$$\mathcal{C} = \{(x, p) \mid u(x) - px + M(x_i, x^*_{-i}; k, R^*)v = 0, x \geq 0, p > 0\} \tag{11}$$

Hence, the area between $\mathcal{C}$ and the price cannot be interpreted as consumer surplus.

This internalization gives the contest designer an incentive to enlarge the overall welfare pie created from the purchased regular goods, knowing that the surplus can now be captured through the contest. This is accomplished by pushing more regular good sales

[33]In terms of mechanism structure, closely related study is Liu et al. (2022), who proposes a decreasing marginal pricing to incentivize contributions in a fundraising setting. The mechanism here instead creates increasing marginal benefits through premium good access with linear pricing.

through price adjustments while simultaneously intensifying the competition ($R^*_{PP} > R^*_{PO}$). This expand-and-extract approach resembles that of the non-linear pricing, but achieves the same outcome with a linear pricing integrated with a contest. Rather than imposing a fixed fee, the designer ties the regular good to non-linearly increasing stochastic access to the premium good.[34] As shown in Figure (3b), the profit-maximizing p^{**} induces the efficient consumption x^{**} satisfying $u'(x^{**}) = c$, thus realizing the full welfare potential generated by regular good consumption. This strategy drives consumers to become more aggressive in investment, enhancing the contest's profitability. Consequently, total welfare is maximized because the designer's profit motive is now perfectly aligned with expanding the welfare generated by regular good consumption.

All preceding analyses are conducted under the interior-solution assumption, where the equilibrium regular good price is high enough to exceed $u'(0)$.

4.4 Discussion of Findings and Their Implications

The findings in Section 4 yield three key implications.

First, the results overturn the prevailing welfare intuition behind public concerns: what appears to be "excessive" consumption under the product purchasing contests is actually welfare-increasing although it indeed raises regular good consumption and reduces consumer surplus. Second, the higher profitability of the profit-maximizing product purchasing contest provides an economic rationale for the widespread use of the practice, underscoring the relevance of the framework. Third, from a policy perspective, prohibiting product purchasing contests to protect consumers may have unintended consequences, as it undermines social welfare.

The analysis shows the trade-off between efficiency and distribution calls for more nuanced policy tools than full prohibition. At the same time, one dimension of the public concerns remains unaddressed-the perceived unfairness and frustration among heavy spenders not selected as winners. Drawing on the conventional equivalence in revenue across mechanisms, it may be possible to implement a more equitable ex-post distribution without sacrificing efficiency. Moreover, the multi-dimensional design space of product purchasing contests enables partial and incremental regulation: adjusting allocation rules while preserving the product-purchasing structure instead of a simple ban-or-allow approach.

In this regard, the all-pay and winner-pay auctions are natural candidates. They yield equivalent expected revenue but deliver more desirable ex-post welfare distributions (among consumers) in conventional setting. These mechanisms are also understandable

[34] Non-linear pricing is theoretically effective but often impractical when not established as a market norm; a product purchasing contest may indirectly circumvent such resistance.

and implementable. These considerations motivate the analysis in Section 5.

5 Fine-Tuned Regulation Analysis

Building on Section 4, this section examines all-pay and winner-pay auctions within the product purchasing contest framework and assess their effects as fine-tuned regulations.

To evaluate these mechanisms as potential policy instruments, I compare their market outcomes with those of the optimal Tullock contest. The mechanisms can be classified along two dimensions (Figure 4). (i) payment structure—whether only the winner pays or all participants pay regardless of the outcome; and (ii) discrimination—whether the allocation is fully based on investment or determined stochastically. This classification produces four familiar forms: the Tullock contest (all-pay, stochastic), the all-pay auction (all-pay, fully discriminatory), the winner-pay auction (winner-pay, fully discriminatory), and the winner-pay contest (winner-pay, stochastic). Notably, the winner-pay contest is weakly dominated by the winner-pay auction.

Given the aforementioned equivalence among the mechanisms in terms of expected profits and welfare,[35] this section investigates whether this classical equivalence still holds when extended to the product purchasing contest framework. Specifically, I aim to explore how the consumption-based investment disrupts this established equivalence.

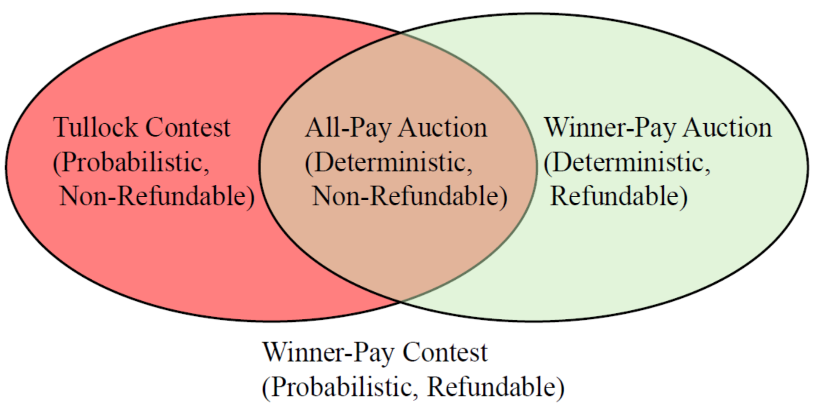


Figure 4: Graphic Illustration of How the Candidate Mechanisms are Categorized.

To begin with, I illustrate how to characterize the market equilibria with those mechanisms under product purchasing contests. Section 5.1 discusses an all-pay auction and Section 5.2 illustrates a winner-pay auction.[36] Throughout this section, the analysis is based on the economic environment defined by in Section 3.

[35] In this context, ”profits” can be used interchangeably with ”revenue,” assuming no costs are associated with lottery tickets in a traditional contest setting.

[36] These implications may not extend to corner solution cases, another unconventionally introduced feature.

5.1 All-Pay Auctions

All-pay auctions corresponds to the case of a Tullock contest (5) where $R = \infty$. As a starting point of characterizing the equilibria, Proposition 1 discusses the existence of any pure strategy Nash equilibria.

Proposition 1 *Suppose the economic environment defined in Section 3 are satisfied with $R = \infty$. Then there exists no pure strategy Nash equilibria for the all-pay auction under product purchasing contest.*

Proof. Presented in the Online Appendix. ■

Then all-pay auctions under product purchasing contest may have mixed strategy Nash equilibria as in the plain contest. Proposition 2 considers a broader range of R in the Tullock contest success function where a pure strategy Nash equilibrium does not exist, which makes Proposition 1 be a corollary of Proposition 2 with $R = \infty > R^*_{PP}$.

Proposition 2 *For $R > R^*_{PP}$, no symmetric pure strategy Nash equilibria exist under product purchasing contest.*

Proof. Presented in the Online Appendix. ■

The optimal Tullock contest under product purchasing contest is determined at the maximum level of R, where pure strategy Nash equilibria exist, the same as those of a standard Tullock contest. This prompts inquiries into the existence of equilibria when $R > R^*$ and their economic implications. To address these questions, I follow the approach of Alcalde and Dahm (2010), adopting the discrete framework of Baye et al. (1994) and applying Dasgupta and Maskin (1986). Initially, I define a discrete strategic space for consumers, establishing the existence of symmetric mixed strategy Nash equilibria. Subsequently, I demonstrate that symmetric Nash equilibria converge to a mixed strategy Nash equilibrium upon a continuous strategic variable space as its limiting equilibrium. Finally, I discuss the economic implications of these findings, particularly in comparison to the optimal Tullock framework under product purchasing contest.

With the discrete strategic space and the applicable discontinuity of the expected payoff functions, we can establish Lemma 3 below: under the defined environment for the product purchasing contest, there exist mixed strategy Nash equilibria with the discrete strategic space, and its limiting equilibria coincide with the mixed strategy Nash equilibria with the continuous strategic space.

Proposition 3 *Under a product purchasing contest, there exists symmetric mixed strategy Nash equilibria with both finite and continuous strategic variable space. Moreover, there exists* $\overline{\mu} = lim_{G\to\infty}\overline{\mu}^G$ *and* $\overline{\mu}$ *is a mixed strategy to the continuous contest.*

Proof. Presented in the Online Appendix. ■

With the established existence of a symmetric Nash equilibria under a general environment, we now turn to the properties of the equilibria.

Theorem 2 *Any mixed strategy Nash equilibrium under product purchasing contest has lower profits than the optimal Tullcok product purchasing contest.*

Proof. Presented in the Online Appendix. ■

Expected-revenue equivalence across mechanisms breaks down under consumption-based investment. This profit–welfare inequivalence arises because the endogenous allocations rules interact with the consumers' investment technology once the regular good is used as an investment medium. In standard contests, monetary spending is welfare-neutral; it merely transfer of money from contestants to the designer. When the investment carries its own welfare effects, welfare depends not only on the total investment, but also on its distribution across consumers.

Following Proposition 3, I formulate the mixed strategy as a probability distribution $F(x)$ according to x, the choice variable.[37] With normalized outside option with zero-payoff, for every x in the mixed strategy Nash equilibria, (12) is the general form of the mixed strategy Nash equilibria with n consumers and $k(< n)$ rewards.

$$u(x) - px + v\left[\sum_{j=0}^{k-1}\binom{n-1}{j}\{F(x)^{n-j-1}\}\{1-F(x)\}^j\right] = 0 \qquad (12)$$

Proposition 4 (All-Pay Auctions are not optimal) *With arbitrary* $k < n$ *rewards, an all-pay auction is suboptimal in a symmetric mixed strategy Nash equilibrium, dominated by the optimal Tullock contest under product purchasing contest.*

Proof. The proof is straightforward and can be completed by combining Proposition 2 and Theorem 2. ■

Consequently, all-pay auctions are suboptimal for both profits and welfare relative to the optimal Tullock contest. Theorem 2 establishes that product purchasing contests breaks

[37] Potentially, there might be a continuum of asymmetric mixed strategy Nash equilibria under multiple prize all-pay auction, Barut and Kovenock (1998).

the equivalence: the equivalence between all-pay auctions and anonymous Tullock contests with sufficiently low randomness (Alcalde and Dahm, 2010; Ewerhart, 2017) no longer holds in this setting.

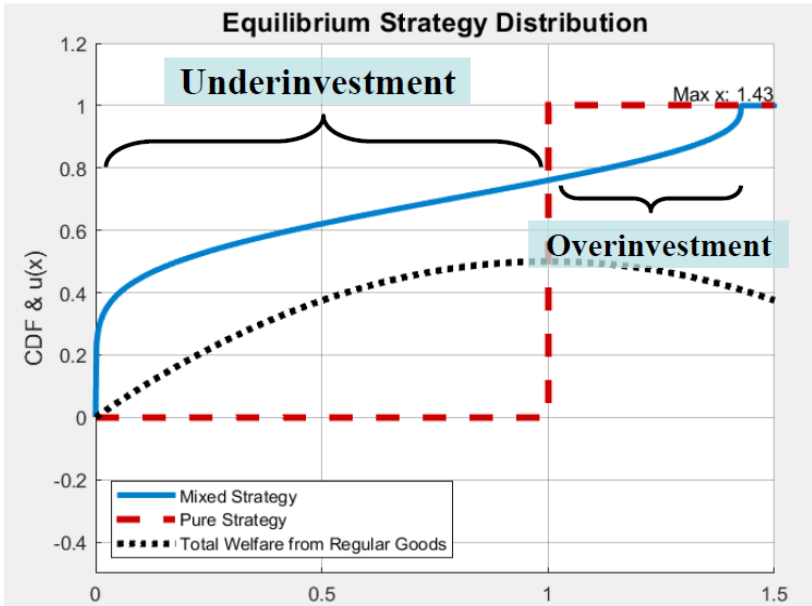


Figure 5: Equilibrium Strategy Distributions and Welfare Implications: Optimal Product Purchasing Contests vs. All-Pay Auctions. Notes: The horizontal axis represents individual purchases of the regular good, and the vertical axis indicates both the cumulative distribution (CDF) of mixed strategies and the welfare (utility) generated by the regular good.

Figure 5 illustrates the suboptimality of equilibrium mixed strategies, using the all-pay auction as a salient example. It compares the auction's equilibrium strategy distribution with that of the optimal Tullock contest in the product purchasing contest.[38] Although the figure depicts the all-pay auction case, the same reasoning applies to any equilibrium mixed strategy relative to the optimal Tullock contest.[39]

The dashed curve shows a pure strategy that degenerates at $x = 1$, where the hump-shaped welfare function peaks, attaining the first-best outcome. By contrast, the solid curve illustrates an equilibrium mixed strategy that deviates from this optimum. When realized investment falls below 1, potential gains from producing welfare-enhancing regular goods are left unrealized; when it exceeds 1, production becomes excessive and welfare-reducing. Although marginal welfare turns negative beyond the optimum, consumers

[38]In Figure 5, I choose a non-monotonically increasing quadratic utility function $u(x) = x - (1/2)x^2$ with $c = 0$. With this specification, the marginal utility of regular goods remains positive for $0 < x < 1$ but turns negative $x > 1$. The solid curve illustrates the equilibrium mixed strategy in the all-pay auction under product purchasing contest, where the equilibrium price for regular goods is determined to be 2.39.

[39]While this graph is based on the case of three premium goods and ten consumers, the implications of this illustration generally apply to arbitrary number of premium goods and consumers.

still buy additional regular goods because each extra unit sufficiently raises their chance of winning the premium prize. This overly intense competition in all-pay auctions drives socially inefficient regular good production, reducing both profits and total welfare.[40]

5.2 Winner-Pay Auctions

With the winner-pay auction within the product purchasing contest framework, the consumers' problem is given as (13) with $M(x_1, ..., x_n; R, k)$ defined as (14).

$$max_x\{v + u(x) - px\}M(x_1, ..., x_n; R, k) \tag{13}$$

$$M_i(x_1, ..., x_n; R, k) = \begin{cases} P_i^1 + \sum_{j=1}^{k-1} \left[\Pi_{s=1}^{j}(1 - P_i^s)P_i^{j+1}\right] & \text{if} \quad |m| \geq k \\ 1 & \text{if} \quad |m| < k \text{ and } x_i > 0 \\ \frac{k-|m|}{n-|m|} & \text{if} \quad |m| < k \text{ and } x_i = 0 \end{cases} \tag{14}$$

Unlike the utility maximization problem (4), $u(x) - px$, the surplus derived from purchased regular goods is within the bracket multiplied by the contest success function M, a defining feature of the 'winner-pay' mechanism. Again, the winning probability follows Clark and Riis (1996) in distributing multiple premium goods.

Theorem 3 (Winner-pay auctions are suboptimal) *With arbitrary $k < n$ rewards, an winner-pay auction is less profitable than the optimal Tullock contest under purchasing product contest.*

Proof. Presented in the Online Appendix. ■

The intuition of Theorem 3 is that while the optimal Tullock contest can fully extract from all active consumers, the winner-pay auction, by construction, allows only winners to contribute to social welfare. Non-winners, despite their active participation, add nothing to welfare, revealing a fundamental inefficiency of the mechanism.

5.3 Numerical Comparisons: Profits and Welfare

Sections 4 and 5 characterize the equilibria under each mechanism, but comparing their relative performance remains difficult because the all-pay auction has only have mixed strategy equilibria and lacks a closed-form solution. Numerical comparisons are therefore provided under parametric assumptions.[41] Details on the numerical implementation and resolution of related challenges appear in the Online Appendix.

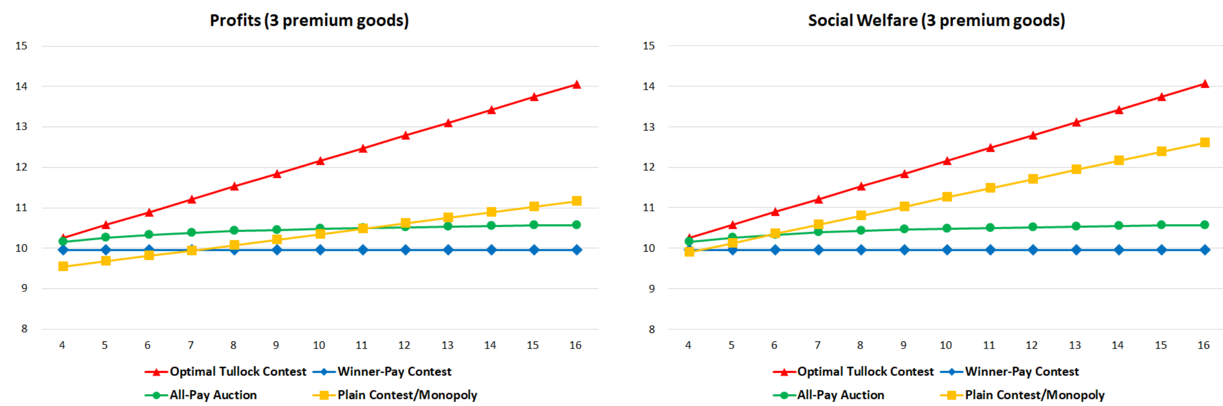
Figures 6a and 6b summarize the results. **Profits:** The optimal Tullock contest consistently delivers the highest profits, while the winner-pay auction yields the lowest. The

[40]The same logic applies even with the monotonically increasing concave utility for regular goods and a constant marginal cost.

[41]I assume a log utility function $u(x) = \log(1 + x)$, with a marginal cost of 0.4 for the regular good, 3 premium goods, and a valuation of $v = 3$.

ranking between the all-pay auction and the Prohibition Outcome depends on market size: when the number of consumers is small, the all-pay auction generates higher profits, but beyond a threshold (e.g., $n = 7$ in Figure 6a) the Prohibition Outcome yields higher profits.

Welfare: The qualitative pattern is similar. The optimal Tullock contest still attains the highest welfare and the winner-pay auction remains least efficient. However, because the Prohibition Outcome preserves positive consumer surplus (unlike in product purchasing contest, where surplus is fully dissipated), it overtakes the all-pay auction at a smaller market size (e.g., $n = 5$ in Figure 6b). Hence, in welfare terms, the Prohibition Outcome dominates the all-pay auction over a broader range of consumer numbers.



(a) Profits

(b) Welfare

Figure 6: Numerical Comparisons

5.4 Distribution of Welfare

Thus far, the analysis has evaluated aggregate expected social welfare, defined as the sum of expected profits and expected consumer surplus. While this utilitarian criterion is standard, its distributional implications may also matter. Welfare distribution can be viewed in two ways: (i) between the firm and consumers and (ii) among consumers, distinguishing winners from non-winners. This section focuses on the latter.[42] A skewed ex-post distribution may raise fairness concerns, since all consumers invest similar amounts of the regular good in a product-purchasing contest.[43] Winner-pay mechanisms can mitigate such ex-post inequality, thereby promoting a fairer allocation among consumers. The policymaker's objective function incorporating this distributional concerns is:

Definition: The Policy Maker's Objective Function with Value on Welfare Distribution

$$SW - \lambda(n, v) \tag{15}$$

[42] The firm-consumer distribution is discussed in a behavioral context in the Online Appendix.

[43] Cohen et al. (2022) propose four fairness criteria; in product purchasing contests, the consumer-level distribution aligns with their Surplus Fairness criterion.

Let $\lambda(m, w)$ denote a disutility function where m is the number of consumers and w is the maximum gap in the ex-post consumer surplus. Under the same individual expenditure, w should be v. $\lambda(w, v)$ is defined as $\lambda(m, w) > 0$ for any $m > 0$ and $w > 0$, $\lambda(m, 0) = \lambda(0, w) = 0$, $\frac{\partial \lambda(m,w)}{\partial m} > 0$, and $\frac{\partial \lambda(m,w)}{\partial w} > 0$. Under a winner-pay auction, social welfare under the utilitarian criterion coincides with function (15) since $\lambda = 0$.

Not surprisingly, when $\lambda(m, w)$ is sufficiently large, a policymaker concerned with ex-post distribution may prefer a winner-pay auction to leaving the product-purchasing contest unrestricted. This regulatory concern raises a crucial policy question: does imposing a winner-pay auction format actually better achieve the regulator's objective (15) compared with banning the practice? If the policymaker's objective under a winner-pay product-purchasing contest does not exceed that under full prohibition, mandating the winner-pay format would be inferior to a simple ban.

Importantly, prohibition does not eliminate ex-post inequality among consumers: even a plain contest that survives after banning product-purchasing contests generates distributional imbalance. To focus the comparison, we consider the optimal Tullock contest and the winner-pay auction, restricting attention to cases where the monopolist prefers the latter to the Prohibition Outcome. Proposition 5 shows that the winner-pay auction delivers a better outcome in terms of the policymaker's objective, indicating that mandating a winner-pay auction format can be an effective intervention when ex-post distributional concerns are sufficiently salient.

Proposition 5 *If the winner-pay auction is preferred to the optimal product purchasing contest according to the objective function, then it is also preferred to the benchmark Prohibition Outcome.*

Proof. Presented in the Online Appendix. ■

In conclusion, a fine-tuned winner-pay auction under product purchasing contest is justified if ex-post welfare distribution is sufficiently valued.

6 Extensions

Building on the baseline framework, this section progressively enriches the model by embedding in the contest in a various managerial design contexts. Section 6.1 introduces consumer heterogeneity. Section 6.2 allows the endogenous supply of premium good. Section 6.3 considers endogenous quality of regular goods. Section 6.4 explores an alternative counterfactual benchmark in the "event ticket pricing puzzle" context. Section 6.5 considers the value of winning within a contest.

6.1 Heterogeneous Consumers

6.1.1 Heterogeneity in Preferences

To extend the analysis, I consider heterogeneity in consumer valuations for both premium and regular goods. The simplest form is considered, wherein consumers have either high and low valuation for each good. This two-type valuations for two goods necessarily leads to two polar cases: perfectly positive and negative correlation of valuations. While there are potentially multiple Nash equilibria can exist (Wang, 2010), this analysis focuses on symmetric pure strategy equilibria.

Four consumer groups are defined by whether they place high or low value on each good. For instance, HL consumer group has high value for the premium good and low value for the regular one. Thus, in the positive (negative) correlation analysis, the relevant groups reduce to HH and LL (HL and LH).

The implicit solution can be given as the following system of linear equations.

$$u'_x(x) - p + \left[\frac{rx^{r-1}\{(n-1)x^r + my^r\}}{(nx^r + my^r)^2}\right] v_x = 0 \tag{16}$$

$$u'_y(y) - p + \left[\frac{ry^{r-1}\{(m-1)y^r + nx^r)\}}{(nx^r + my^r)^2}\right] v_y = 0 \tag{17}$$

where x and y are the optimal quantities of regular good for the two consumer types, and $n, m \in \mathbb{N}$ denote the numbers of consumers in the respective groups.

Two countervailing forces drive the monopolist's decision. (i) The *internal margin effect* pushes toward excluding low-valuation consumers, since their presence weakens incentives of high-valuation consumers. (ii) The *external margin effect* favors broader participation, since sales of regular goods increase with more consumers. Profitability therefore depends on which margin dominates.

When consumers are sufficiently heterogeneous, the internal (external) margin effect dominates over the external (internal) counterpart, and thus frustrating (encouraging) participations of low-value consumers becomes more profitable. This exclusion (inclusion) is achieved via the mixture of setting a higher (lower) price and minimizing (widening) the randomness of winning.

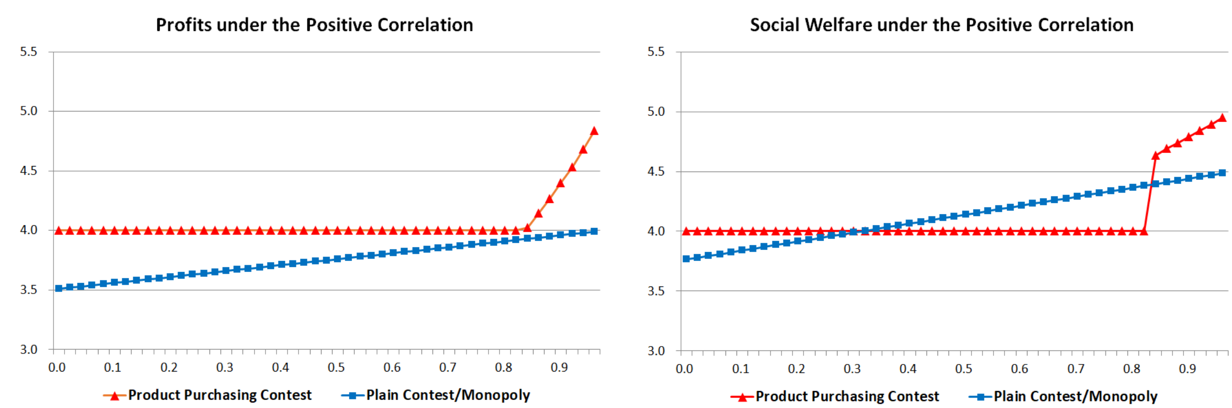
Obtaining a closed-form solution is challenging, even when considering only two types of valuations. Consequently, a numerical approach is employed to explore the equilibrium properties, under particular parametric conditions.[44]

[44] I establish heterogeneity in preferences as follows: I fix the high value for the premium good as $v = 3$, and the high value for the regular good is given by the function $u(x) = x - \frac{1}{2}x^2$. The degree of heterogeneity is introduced through α, so that the low value for the premium good is denoted as αv, and for the regular

6.1.1.1: Positive Correlation

When valuations are positively correlated, the internal margin dominates the external margin, giving the contest designer an incentive to exclude LL consumers and induce intense competition among HH consumers. As Figure 7a shows, the product purchasing contest (red triangles) consistently yields higher profits than Prohibition Outcome (blue squares). The kink in the PP curve marks the point where the valuation gap narrows enough for LL consumers to enter. Once both types participate, LL consumers are fully extracted while HH consumers retain some positive surplus. When only HH consumers are active, they are fully extracted regardless of α, so profits remain flat. By contrast, in the Prohibition Outcome profits rise monotonically with α as demand from LL consumers expands. Under positive correlation, consumption-based investment allows the firm to profitably discourage the LL consumers and intensify competition among HH consumers, outperforming a standard monopoly contest.

Figure 7b shows mixed welfare effects. When heterogeneity is moderate, the product-purchasing contest can generate lower social welfare than a plain contest under monopoly. The difference turns on whether LL consumers contribute to welfare. In product purchasing contest, welfare rises with α only if LL consumers actively participate. When only HH consumers are active, welfare equals profits, whereas in the Prohibition Outcome LL consumers always expand demand as α grows, raising both profits and consumer surplus. Thus, welfare gains from consumption-based investment hinge on broad participation: if LL consumers are excluded, product purchasing contests may underperform the prohibition counterfactual despite their higher profits.



(a) Profits (b) Welfare

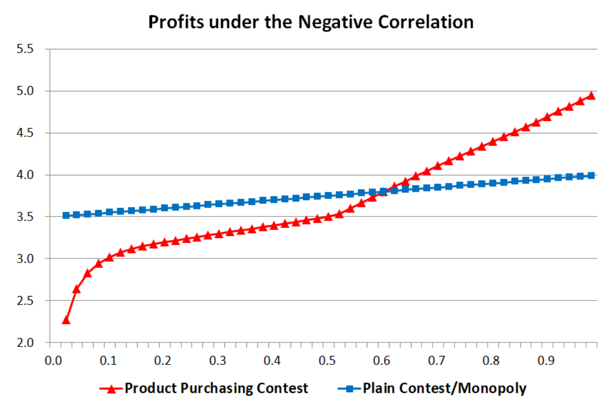
Figure 7: Heterogeneous Consumers: Positive Correlation

6.1.1.2: Negative Correlation

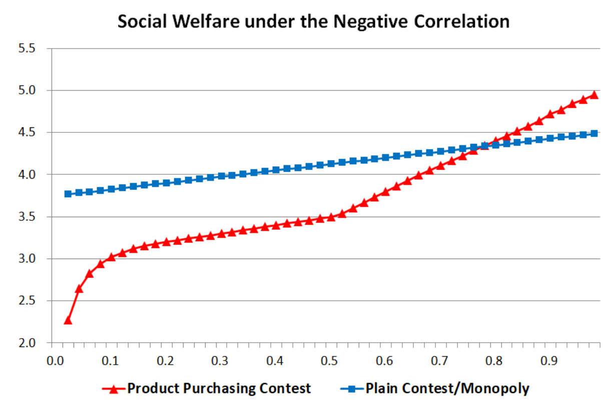
good, it is given by the function $u(x) = x - \frac{1}{2\alpha}x^2$. The higher α is, the less heterogeneous consumers are.

Under negative correlation, the contest designer faces a different dilemma. Excluding any group now forgoes the side with the higher valuation for at least one good, making the opportunity cost of exclusion higher than under positive correlation.[45] However, including both groups can weaken competition.

In Figure 8a, unlike under positive correlation, the product purchasing contest (red triangles) yields lower profits than the Prohibition Outcome when heterogeneity is large. This occurs because HL consumers place less value on the regular good, while the excluded LH consumers value it more, making exclusion costlier. Moreover, unlike the positive correlation case, profits under the product purchasing contest increase with α regardless of whether LH consumers are excluded, because HL demand for the regular good increases with α. The kink in the red curve marks the point where the valuation gap narrows enough for LH consumers to enter.



(a) Profits



(b) Welfare

Figure 8: Heterogeneous Consumers: Negative Correlation

Figure 8b shows mixed welfare effects, as in Figure 7b but through a different way. A product purchasing contest raises social welfare only when preferences are sufficiently homogeneous. Compared with Figure 8a, the range of α where the Prohibition Outcome dominates widens. The difference reflects welfare distribution: a Prohibition Outcome preserves more consumer surplus, while the product purchasing contest largely extracts it.

6.1.1.3: Discussion

Two main insights emerge. First, a product purchasing contest is always more profitable when correlation is positive, while under negative correlation profitability hinges on consumer heterogeneity. Second, when consumers are sufficiently homogeneous, the contest outperforms the Prohibition Outcome regardless of the correlation's sign.

These implications depart from the standard bundling theory, where the consumer

[45] In this parameterization, the LH group is typically excluded because their maximum surplus from the regular good is only 0.5, far below $v = 3$.

heterogeneity plays a central role. Profits rises under heterogeneous consumers with negatively or mildly positively correlated values (Belleflamme and Peitz, 2015). With homogeneous consumers, separate pricing already extracts the full surplus, leaving bundling no advantage. In product purchasing contests, however, greater homogeneity intensifies competition and lets the monopolist capture more profit.

The difference also originates from the demand structure. Whereas standard bundling theory assumes unit demand, the product purchasing contest naturally allows for multi-unit demand. In this setting, profitability depends on the optimal joint decision of competition intensity (R) and the regular good price (p), and is maximized when consumers are homogeneous and values are perfectly positively correlated.

This contrast highlights a novel mechanism and, in doing so, provides a new context in which bundling strategies can be economically justified.[46] Furthermore, this extension supports the implications under homogeneous consumers, particularly when low-value consumers are strategically excluded by the firm.

6.1.2 Discussion on Heterogeneity in Wealth

This section discusses heterogeneity in wealth and how product purchasing contests function as a profitable screening device. The implications parallel the analysis of heterogeneous preferences. By leveraging regular goods as an investment medium, firms can differentiate consumers by financial capacity: those with tighter budget constraints face greater marginal pressure—for example, higher opportunity costs from liquidity constraints—relative to their wealthier counterparts. This mechanism may allow firms to exclude financially constrained consumers and thereby intensify competition among wealthier consumers. Such

[46] The condition under which bundling is more profitable than non-bundling (separate sales) has been of great interest to economists. Bundling is effective when it is used as a device for price discrimination (Adams and Yellen, 1976), product differentiation (Chen, 1997), entry deterrence or foreclosure (Whinston, 1990; Nalebuff, 2004; Choi and Jeon, 2021) exploiting the complementarity (Matutes and Regibeau, 1988), and for homogeneizing the distribution of consumer valuations on the products (Schmalensee, 1984). Unlike the commonly used framework introduced by Adams and Yellen (1976), I introduce the strategic interactions (due to the existence of the contest) between consumers in demand for products. Conventionally, the demand for each component is assumed to be independent (even when the valuations are correlated), but in this study, the quantity demanded can be endogenously determined through the contest, which is why multi-unit demand for each consumer needs to be allowed. This study is also related to some variations of bundling in which the supply of components is limited. Banciu et al. (2010) investigate how the optimal bundling scheme is determined when two quantity-constrained bundled goods are vertically differentiated. Cao et al. (2015) study the implication of the optimal bundling strategy when the supply of products in a bundle is limited with various extensions. This study considers a particular application of a pure bundling strategy with a limited quantity of a component. Because the component with limited capacity (premium) is allocated through a contest, the bundled component is a chance of obtaining the indivisible reward, not the reward itself or its fraction. Due to the nature of the contest, the ratio between the components is endogenously determined, unlike a typical bundle where the bundling ratio is exogenously given as constant.

exclusion may also serve other strategic goals, including blocking resale-motivated purchases and maintaining the exclusivity of premium goods (as in industries like luxury and fashion).[47]

6.2 Endogenous Supply of Premium Good

Endogenizing the supply of premium goods can be achieved by introducing a cost function and treating quantity as an additional strategic variable, subject to the feasibility constraint $0 \leq k \leq n$. While the formal analysis above extends trivially to interior solutions, the more interesting cases arise at the corners, where the firm has an incentive to either under- or oversupply premium goods.

Proposition 6 *Let k^* be the welfare-maximizing quantity of premium goods with strictly convex cost functions. Then the profit-maximizing and welfare-maximizing supply of premium goods coincide at the interior solution $k^* \in (0, n)$. When the welfare-maximizing solution is at the boundary ($k^* = 0$ or $k^* = n$), the firm may instead profitably oversupply (one unit) or undersupply ($n-1$ units) the premium good, thereby sustaining the contest. While such corner solutions reduce consumer surplus yet may raise total welfare through induced consumption of regular goods.*

Proof. Presented in the Online Appendix. ■

More detailed discussion and the foundation of functional forms are provided in the Online Appendix.

6.3 Endogenous Quality of Regular Good

This section turns to the design of the regular good itself by relaxing the fixed-preference assumption. Particularly, producers are now allowed to invest in its quality. Such investments are central because the marginal utility of regular goods, shown in the main analysis, is directly influenced by quality. Moreover, the profitability of such investments depends critically on whether product purchasing contests are permitted, implying that regulation affects welfare not only through the contest institution but also through induced changes in product quality. A striking illustration comes from the Korean music industry, where the surge in album sales reflects not only the inducement of contests but also the enrichment of album packages beyond traditional levels (Herman, 2020). These forces are mutually reinforcing rather than independent. Given this background, I analyze the optimal investment

[47] This issue aligns with practices observed in reality. For instance, Hermes, when responding to class-action lawsuits regarding tying practices, denied employing such strategies but acknowledged its efforts to encourage sales to the "right clients" (Williams, 2024). This expression may sound that the direct objective of the screening mechanism may operate independently of the firm's explicit profit motives. Still, this method can work as a way to deter arbitrageurs who purchase iconic products for resale purposes. As such, it can be viewed as a tool for achieving intermediate objectives, such as ensuring premium goods are sold to desirable consumers, thereby maintaining the brand's prestige and exclusivity.

in regular-good quality, comparing the equilibrium outcomes under product purchasing contests with those in the Prohibition Outcome.

Let θ denote the level of investment in the quality of regular good. So let $u(x;\theta)$ denote the utility function for regular good, where $u(x;\theta)$ is increasing in θ. $u_x(x;\theta)$ is increasing in θ as well, which guarantees the demand functions with different θ do not cross each other. Let θ^* and $\tilde{\theta}$ are equilibrium level of investment under product purchasing contest and Prohibition Outcome. Making investment necessarily incurs both variable $c(\theta)$ and fixed cost $F(\theta)$, both of which are strictly increasing in θ.

Proposition 7 *Let θ^* and $\tilde{\theta}$ denote the optimal level of investment on the quality of the regular good under the optimal Tullock contest and a plain contest for premium good with a monopoly on regular good, respectively. Then the following relationship holds in equilibrium.*

$$\theta^* > \tilde{\theta}$$

Proof. Presented in the Online Appendix. ∎

Further details on the ongoing evolution of the music industry are provided in the Appendix.

6.4 Alternative Counterfactual: Event Ticket Underpricing Puzzle

The longstanding puzzle of event ticket pricing suggests that many entertainment event tickets may not be optimally priced according to theoretical predictions: tickets are often underpriced and employ only crude price discrimination.

The puzzle's underlying causes lie beyond the scope here;[48] this section simply assumes that consumers incur disutility when prices exceed their reference level.

Definition: Price Resistance Outcome

Due to some behavioral factors (fairness concern, social pressure, and so forth), there exists the price ceiling, which keeps prices below market-clearing level, the consumers' highest willingness to pay. Premium goods are allocated on a first-come, first-served basis, generating rent-seeking competition[49] similar to an all-pay auction.

In this setting, consumers anticipate positive surplus from underpriced tickets and therefore engage in costly rent-seeking behavior.[50] Assuming homogeneous rent-seeking technology among consumers, the equilibrium is the standard mixed-strategy Nash equilibrium of an all-pay auction.

[48] More detailed discussion of this puzzle is provided in the Online Appendix

[49] Racing toward premium goods Either online or offline.

[50] Leslie and Sorensen (2014) investigate event ticket market, discussing rent-seeking behaviors in the primary market, particularly in relation to the secondary market.

While both this race for the premium good and product purchasing contest involve competition among consumers, they differ in how consumers' investments are allocated. In the race for the premium good, efforts are just wasted like burning money. As a result, the entire surplus from the premium good is dissipated. By contrast, in a product purchasing contest, consumer investments are not wasted but appropriated by the monopolist as profit. From this perspective, product purchasing contests internalize the welfare loss of queueing competition by commercializing such rent-seeking behaviors. Under this counterfactual, product purchasing contest is more likely to be economically rationalized and feature higher welfare. The framework can be extended to cases where premium goods are sold at a positive price, rather than provided for free. Proposition 8 formalizes this argument.

Proposition 8 *Product purchasing contest is more profitable than Price Resistance Outcome when the highest feasible price of premium good is sufficiently lower than v, the highest surplus for the premium good.*

Proof. Presented in the Online Appendix. ■

6.5 The Value of Winning Itself in a Contest

The results of many experiments on all-pay contest often are inconsistent with the theory. One commonly observed pattern is overbidding compared to theoretical predictions (Dechenaux et al., 2015). That is, in all-pay format contests people often tend to invest more than expected in theory. A plausible interpretation is that subjects value winning itself, beyond the tangible rewards.[51]

Now, suppose the value from winning is composed of two parts, the intrinsic value of the premium good v and the joy of winning $w \in \mathbb{R}_+$ (Sheremeta, 2010),

$$\Rightarrow V \equiv v + w$$

This replacement V instead of v can directly apply to the all analyses with the same implications, indicating the main results are robust to this behavioral extension.

7 Conclusion

Motivated by public debate, this paper reframes a controversial market practice as a product purchasing contest with a novel consumption-based investment. Formalizing this mechanism provides a rationalized framework that clarifies the welfare trade-offs behind prevailing intuition and shows how profit-maximizing design departs from the standard monopoly-contest benchmark.

[51] Some subjects invest a positive amount of bid for even zero monetary value of reward (Sheremeta, 2010).

Building on this framework, two findings shape the welfare and design implications. First, outright prohibition can sacrifice total welfare to preserve consumer surplus, revealing a trade-off between efficiency and distribution. Second, the standard equivalence across contest formats breaks down under consumption-based investment, making policy design dependent on the regulator's welfare objective.

This paper makes three contributions at the intersection of contest theory and industrial organization. First, it embeds contests into firms' managerial decisions—such as pricing and quality—by incorporating consumption-based investment, thereby extending contest design from allocation rules to investment technology. Second, it bridges contest theory and industrial organization, opening new contexts and revealing novel results, including altered consumer and designer strategies, reshaped welfare implications of allocation rules, and new profitability conditions for pricing and bundling. Third, it analyzes firm-designed contests in regulated environments, highlighting designer-regulator tensions arising from misaligned objectives.

Discussing implications and limitations of the study opens up new avenues for future study: First, while this paper develops a general framework for product purchasing contests, future research can ground it in more specific markets where institutional features fundamentally affect the analysis. For example, in experience goods industries such as movies, social learning and consumption spillovers introduce new strategic dimensions to contest design, warranting theoretical extension. Because these externalities shape how competing firms' contests interact, analyzing such settings reveals novel competitive outcomes with relevance for competition policy, motivating empirical evaluation.

A second avenue concerns the acceptance and durability of market institutions. This paper rationalizes product purchasing contests by profitability, yet their very success contrasts with the short-lived Ticketmaster's positional auction—abandoned not for lack of revenue but because consumers disliked the format. This divergence points to the possibility that long-term survival may hinge not only on profitability but also on consumer acceptance. Consumer preferences for institutions such as perceived fairness and even enjoyment (e.g., gamification)— factors often overlooked in standard approaches — can constrain or sustain market mechanisms. Future work could examine how behavioral, cultural, and historical factors shape a mechanism's perceived legitimacy and endurance. This question is especially salient in the digital economy, where platforms continually experiment with new monetization and institutional forms, and where understanding consumer acceptance can inform competition policy and guide sustainable market design.

A third direction extends the mechanism to public-sector challenges. Existing research on fundraising and charitable giving largely examines lotteries with monetary rewards, whereas this paper highlights how differentiated in-kind rewards - both premium and regular goods- can reshape participation incentives and their broader market outcomes. Potential extensions include designing donation schemes that leverage such goods; for example, in Korea and Japan, local governments offer regional products to donors, yet this practice remains underexplored. More broadly, studying market-driven mechanisms that have emerged spontaneously can inform the design of public mechanisms by complementing the rationalized top-down approaches.

A Appendix: Industrial Background

A.1 The Music Industry

Since the mid-2000s, physical album sales have declined worldwide (Figure 9), a trend largely attributed to technological innovations that provided economical yet satisfying substitutes—mp3 files in the mid-2000s and streaming services from the mid-2010s.

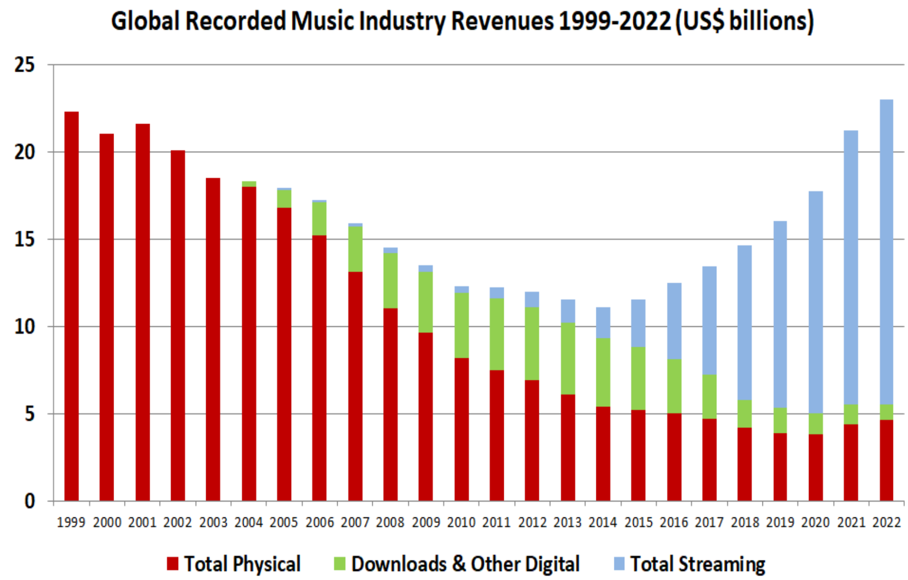


Figure 9: Global Recorded Music Industry Revenues 1999-2022, Source: International Federation of the Phonographic Industry Global Music Report 2023

By contrast, Korea's album sales have surged despite this global trend (Figure 10). A key driver is a marketing practice that restricts entry to exclusive autograph-signing events to contest winners, with winning probability increasing in the number of albums purchased (Herman, 2020).[52] Limited event capacity intensifies competition, leading some fans to purchase hundreds of albums to secure invitations (Herman and Rashid, 2020).

Descriptive evidence indicates that the recent surge in physical album sales reflects intensive-margin growth—individual fans buying multiple copies—rather than a broader consumer base. Survey data from the Music Industry White Paper show that the share of respondents who purchased any physical albums fell from 25.7% in 2016 to 20.2% in 2020.[53] By contrast, high per-album spending became common: in the 2022 White Paper, 33.4%

[52] The same article also highlights the strategy of releasing the same album in multiple versions to encourage additional purchases as collectibles, considering it as another significant factor. Appendux B documents the interconnected nature of this 'multiple version' strategy and the Product Purchasing Contest. Section 6.3 demonstrates that the simultaneous application of both strategies creates synergies, amplifying each other.

[53] Survey design changes after 2021 prevent direct comparison with later years (KOCCA, 2022, 2023, 2024).

of respondents reported spending more than 30,000 KRW on a single album. Average monthly outlays aligns well this pattern—the share spending over 30,000 KRW per month on albums rose from 12.6% to 33.2% between 2016 and 2020 (KOCCA, 2017, 2018, 2019, 2020, 2021).[54] A 2022 Korea Consumer Agency survey further shows that among fans buying albums to attend meet-and-greet autograph events (25.4% of CD purchasers), 84% spent more than 50,000 KRW, with the top decile exceeding 500,000 KRW (Lee, 2022). Overall, physical album growth appears to stem from repeat purchases by existing buyers (intensive margin) rather than an expanding buyer pool (extensive margin). This shift has prompted policy debates about whether authorities should regulate such sales practices to protect consumers (Hong, 2018).

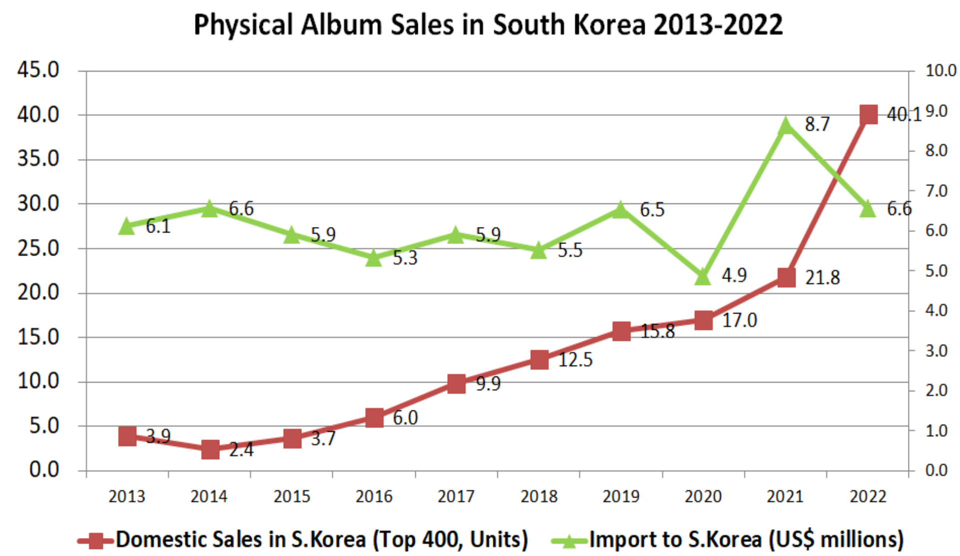


Figure 10: Physical Albums Sales in Korea 2013-2022, Note: The values are estimated using by the author using (i) aggregate sales volume of the annual top 400 albums from the Korean Circle Chart Annual Review (domestic and export sales are mixed), (ii) the dollar value of annual exports and imports of physical albums from the Korea Customs Service, and (iii) total domestic and foreign sales volumes in 2019 and 2020.

This marketing practice extends well beyond Korea. In Japan, it is also commonly used so that regulators require consumers to decide how many albums they intend to buy, with payment triggered only if they secure a winning slot. In the United States—the world's largest music market—artist such as Taylor Swift adopt loyalty-based ticketing system in which fans earn points by purchasing merchandise or volunteering, and those points raise their chances of getting a ticket (Krueger, 2019).

[54] Among the top 50 best-selling albums annually from 2018–2022 (250 total), only 4 (1.6%) had regular prices above 30,000 KRW; the mean regular and discounted prices were 20,142 and 16,784 KRW, respectively. Price data were scraped from Aladin (https://www.aladin.co.kr/) on August 1, 2024.

A.2 The Luxury Industry

In the high-end luxury industry, it is commonly known that top-spending clients spend much more than average clients, which motivates the industry to provide more and more exclusive benefits to such "Very Important Clients (VIC)" (Sherman, 2016). Examples include invitations to private fashion shows (Sherman, 2016) and offering exclusive personalized shopping experiences (Williams, 2022). A distinctive privilege is offering access to a chance to purchase the most popular items itself because the quantity produced of the items is strategically constrained and always in excess demand (Solca, 2020).

For example, Hermes is believed to not allow ordinary clients to access the most sought-after item, "Birkin Bag" or "Kelly Bag" (Sherman, 2015). So many aspirational consumers desire to be recognized as a VIC who can have a better chance to access the items. It is common knowledge among luxury consumers that to obtain such an opportunity, the first thing they need to do is establish solid relationships with sales associates by building up an impressive purchasing history of other items at the store. Sales managers and associates in each Hermes outpost have a broad discretion in managing the demand and the inventory of such popular items (Sherman, 2015). Recently in China, luxury consumers are feeling rising pressure to purchase less-popular items only to get access to the most desired items, expressing their concern that it is too exploitative. Some consumers even develop strategies to improve their chances of obtaining luxury products, specifying the type of items that are more appealing to sales associates of Hermes (Hall, 2022).

B Appendix: Evolution of Quality of Regular Good in the Music Industry

To motivate the analysis, I document how the *variety* of physical albums in the Korean music market has evolved over time. The sample consists of albums ranked in the top 30 of the Circle Chart annual sales from 2013–2020, which together account for roughly 70% of the sales volume among the top 100 albums.

Variety is measured as the minimum number of copies a consumer must purchase to obtain all distinct collectible components in an album. For instance, an album with three cover versions and seven independent photo cards requires seven purchases to secure every collectible. Although album composition is highly idiosyncratic, it has become standard for releases to feature multiple versions[55] and randomized photo cards (Lee, 2022). I therefore focus on these two elements when quantifying variety.[56]

[55] This "versioning" is horizontal rather than vertical (quality).

[56] Alternatively, variety can be defined as the maximum number of distinct packages a consumer could purchase without receiving an identical set of components. This view recognizes that randomized inserts

To compute the variety measure, album-level component data were required. Existing databases document musical attributes (e.g., track titles, genres, composers), but no systematic source tracks the physical components of each album. I therefore collected this information by web-scraping Korea's major online bookstore Aladin (https://www.aladin.co.kr/), which lists album contents through text and descriptive graphics. An album was defined as containing "random" components only when the term "random" appeared in these descriptions. Nine albums were excluded for insufficient detail: although labeled random, they did not specify the details of distinct random variants.[57] The final sample used to calculate the minimum-quantity variety measure consists of 231 albums.

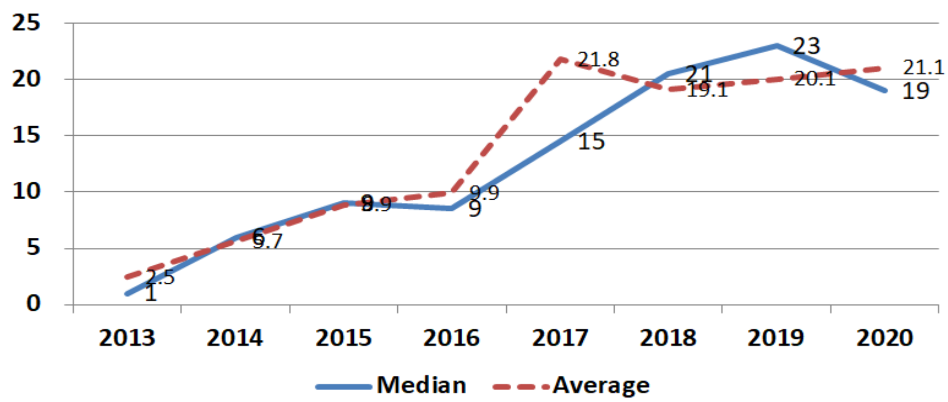


Figure 11: The median and average of the variety, defined with different versions of the covers and photocards of the annual top 30 albums on the Circle Chart from 2013 to 2020.

Figure 11 plots the median (blue solid) and mean (red dashed) variety of sampled albums, 2013–2020. Both measures rose sharply between 2013 and 2017 and then stabilized. In 2013 the mean variety was 2.5, indicating that random components were already present, but the median was only 1, implying that most albums still followed the traditional single-version format. By 2014 the median jumped to 6. The share of single-version albums fell from 18 of 27 in 2013 (66.7%) to only 1 of 120 from 2017–2020 (0.8%), marking a structural shift in physical album packaging. Median variety correlates strongly with domestic sales of the top-400 albums (0.915 for median; 0.889 for mean), suggesting that firms' ability to supply differentiated regular goods is closely linked to higher sales, highlighting the

do not guarantee full collection at the minimum quantity: buyers may repeatedly receive components they already own yet still value additional purchases if some chance of novelty remains. For instance, if two packages share the same CD and cover but contain different photo cards, owning one may still give positive marginal utility for the other. Under this interpretation, the maximum measure is the smallest number of packages from which a consumer would gain positive incremental utility, assuming no package is perfectly identical.

[57] These nine albums—three each from 2013, 2014, and 2015—are concentrated early in the sample period, likely reflecting that randomized components were not yet standard practice and that disclosure conventions had not fully developed.

importance of modeling quality choice in product purchasing contests.

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Supplemental Appendix for "Product Purchasing Contest"

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November 27, 2025

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Contents

A Additional Discussions

A.1 A More Detailed Discussion on Indirect Auction, Tying, and Product Purchasing Contest

The "Indirect Auction" idea highlights that consumers make indirect payments by purchasing regular goods rather than bidding directly for premium goods. However, the real distinction goes beyond just the payment format; it lies in how this practice reshapes the economic implications of the payment. Unlike in traditional contests (auctions), consumers derive utility from the purchased regular goods, while firms bear production costs. Additionally, in conventional contests, the contest designer has no direct control over resources like money, time, or effort. In product purchasing contests, however, the firm can design the investment medium, its regular goods, by setting their value, cost, and price. This implies that firms have more subtle control over the contest structure than traditional contest designers do. All these factors are not taken into account in the standard theory.

Meanwhile, the concept of "Tying" highlights a qualitative difference that the "indirect auction" concept overlooks. By using regular goods as an investment medium, each regular good is tied to a chance of obtaining a premium good, thereby integrating previously separated markets for premium and regular goods. However, "tying" alone fails to fully convey the profound shift in consumer demand that results from this market integration. In product purchasing contests, consumers are no longer making isolated purchasing decisions based solely on preferences; instead, their choices are now shaped additionally by the strategic consideration within the contest. This transformation implies demand for regular good is not independent across consumers: it becomes mutually influenced by the purchasing strategies of other consumers. This interdependent demand is a key feature that fundamentally distinguishes product purchasing contests from typical markets, where demand is determined without consideration of competitive interactions.

Given the limitations of these concepts like "indirect auction" and "tying", a more sophisticated framework is necessary to properly define the problem and develop effective solutions. My research addresses this gap, providing a new perspective for analyzing these market behaviors and offering a more precise approach to understanding their economic implications.

A.2 A More Detailed Discussion on Contest

An alternative contest success function often used is the difference-form. Unlike the Tullock contest, which implies no chance of winning with zero investments, the difference-form model may be more suitable in contexts where zero investment does not necessarily lead to complete defeat (Hirshleifer, 1989). However, in a product purchasing contest, where participation requires purchasing regular goods, the Tullock contest is a more suitable framework.

In the ratio-form contest success function, R is a key parameter that measures the marginal return on investment, which reflects the winner-selection rules of the contest. A higher R indicates greater marginal returns to effort, leading contestants in symmetric equilibrium to increase their bids as R rises. The parameter R represents the winner-selection rule, with typical examples including random allocation when $R = 0$, a simple lottery when $R = 1$, and an auction when $R = \infty$. Beyond these common forms, the Tullock contest framework accommodates a wide range of nameless allocation mechanisms. Although these mechanisms are abstract and lack specificity, they comprehensively cover possible allocation rules in the profit dimension. This flexibility ensures that contest designers retain broad discretion over the winner-selection rule, a critical feature when tailoring contests to specific goals. Such flexibility is crucial for decentralized contest design, where the policymaker's objectives shape the environment in which contests are designed.

This broad adaptability aligns well with the direction of this research, which focuses not on prescribing specific contest designs, but rather on examining the regulatory implications of decentralized contest environments. In other words, this study seeks to understand how the legal and institutional frameworks surrounding contest design influence designers' choices, indirectly shaping the contests themselves. The flexibility offered by the Tullock model, including nameless and non-traditional allocation rules, complements this focus by enabling a wide range of design possibilities within a decentralized setting. This approach also works well in addressing potential asymmetric information between firms and government agencies regarding how a contest is practiced and what winner-selection rule is used.

While the abstraction of winner-selection rules may be seen as a limitation due to its lack of specificity, it opens up broad possibilities by covering a wide spectrum of potential allocation mechanisms. When R is finite, for instance, the highest investments do not always guarantee victory, introducing an element of randomness.[1] This randomness can stem from

[1] As a matter of fact, whatever the specific reasons may be, there are many reported cases in the Korean music industry where fans, despite investing extremely high amounts in purchases, did not receive invitations to the exclusive fan meetings.

institutional factors or intentional design choices. Additionally, there may be simple errors in aggregating and processing information on consumers' monetary investments. If these mistakes can be influenced by effort, they could themselves be rationally intended. One way to interpret such outcomes is as an auction involving noise.[2] This 'noise' often arises when the contest designer has multidimensional preferences beyond profit, and the contestants have incomplete information about these preferences (Corchón and Dahm, 2010). In many real-world settings, this incomplete information is common. For example, in industries such as luxury industry, clients often establish long-term relationships directly with sales associates. Given that sales associates have discretion in offering rewards (Sherman, 2015; Hall, 2022), clients' identities or relative social skills can influence their decisions to provide premium goods. Similarly, in the popular music industry, certain fans may be deliberately excluded from winning contests due to their over-enthusiasm or other non-quantifiable traits, as determined by the artist's management. In general, firms have mechanisms to identify individuals in practice.

Regarding the implication of R in the framework, how a particular level of R is determined in a contest can be a subtle issue. Either of whether it is exogenously given (due to institutional factors or random errors), or endogenously chosen (intentional selection by the firm) is possible. Considering it is directly related to the allocation rule, assuming that R can be freely chosen by the firm may be rationalized. Under endogenous R assumption, the monopolist firm is allowed to choose the format of the winner selection along the parameter R in the Tullock contest framework, thereby maximizing profits. Nevertheless, there is a potential concern regarding the firm's ability to fine-tune randomness. Regarding this ability issue, it may not be appropriate to assume the government agency to have the ability to fine-tune the randomness either. As a result, I assume the regulatory authority can only suggest R corresponding to a most commonly used mechanisms such as a simple lottery ($R = 1$) and an auction ($R = \infty$). Consequently, I consider both cases in the analysis. In either case, no additional cost incurs for adjusting the parameter R.

A.3 When R is Exogenously Given

Section A.3 analyzes the scenario where R is exogenously determined, restricting the monopolist's ability to set the allocation rule. It is assumed that the exogenous value of R remains constant regardless of whether a product purchasing contest is implemented. Accordingly, this section compares the market outcomes between the product purchasing contest and Counterfactual Scenario 1, under the assumption that both scenarios share a common value of R, where a pure strategy Nash equilibrium exists.

[2] In fact, when the parameter R is determined endogenously, the pure strategy effort-maximizing equilibrium R is set at a level that yields qualitatively the same results as the all-pay auction equilibrium.

Lemma A-1 *Suppose setups in Section 3 are satisfied and R is exogenously given. Then the profits under product purchasing contest is equivalent to those of a plain contest for premium good with a monopoly on regular goods as long as $R \leq R^*_{PO}$.*

Proof. Presented in Appendix. ■

The following corollary examines the potential profitability of using a simple lottery as an allocation mechanism under the product purchasing contest, in comparison to its use for allocating the premium good in Counterfactual Scenario 1. The simple lottery's effects are of particular interest due to its common application as an allocation mechanism.

Corollary A-1 *Suppose setups in Section 3 are satisfied. Then considering $R^*_{PO} > 1$, the product purchasing contest with the simple lottery $(R = 1)$ as its allocation mechanism has the same equilibrium property with that of a plain contest for premium good with a monopoly on regular goods.*

Proof. Presented in Appendix. ■

Corollary A-1 can be easily deduced from Lemma A-1, but the explicit proof is also provided in Appendix. Consequently, the simple lottery does not yield additional profits under the product purchasing contest compared to Counterfactual Scenario 1.

Proposition A-1 is one of the main results of this study, which is deduced by combining Theorem 1 and Lemma A-1. This result has a general implication on the relative performance of the product purchasing contest versus Counterfactual Scenario 1, considering both endogenous and exogenous scenarios for R. Proposition A-1 demonstrates the superiority of product purchasing contest in profitability and efficiency, despite its negative impact on consumer surplus.

Proposition A-1 *Under the economic environment satisfying setups made in Section 3, the product purchasing contest achieves (strictly/weakly) higher profits and social welfare compared to Counterfactual Scenario 1 when the contest allocation rule (R) is determined (endogenouly/exogenously) when focusing on the pure strategy Nash equilibria.*[3]

Proof. Presented in Appendix ■

[3]The scope of the analysis is extended to mixed strategy Nash equilibria in Section 5. The pure strategy Nash equilibria with endogenous R turns out to be the unique optimum when considering general mixed strategy Nash equilibria.

A.4 Detailed Sketchy of Discrete Contest

The assumption of discrete strategic variables aligns with real-world scenarios where consumers are expected to purchase indivisible regular goods, which is also commonly adopted in experimental settings in the contest literature (Millner and Pratt, 1989), as highlighted by Baye et al. (1994) and Alcalde and Dahm (2010) as well. Following the terminology of Alcalde and Dahm (2010) and Dasgupta and Maskin (1986), I refer to this game as the finite contest. In the contest with given $G \in \mathbb{N}_+$, the consumer's strategy space is specified as $\{0, \overline{x}/G, 2\overline{x}/G, ..., (G-1)\overline{x}/G, \overline{x}\}$, where $\overline{x}$ is determined by $p\bar{x} - u(\bar{x}) = v$. Obviously, it is not rational for consumers to buy more than $\overline{x}$ units of regular goods, so that $\overline{x}$ can be taken as the highest feasible value of the strategy. Due to the concavity assumption on $u(x)$, the left hand side of the equation $p\bar{x} - u(\bar{x}) = v$ is an increasing function in x as $p > u'(x)$. Note that the solution of the optimal Tullock contest under product purchasing contest is given as x^* such that $px^* - u(x^*) = \frac{k}{n}v < v$. Considering the optimality condition of x^* such that $u'(x^*) = c$ and the concavity of $u(x)$, it can be deducted that $u'(\bar{x}) < c$. Thus, if any $x > x^*$ is a part of equilibrium strategy, then the transaction of x-th unit in this contest does not contribute to improving social welfare, as the marginal cost exceeds the corresponding marginal utility. Additionally, if the transaction of $x(< x^*)$-th unit is not made in the equilibrium, such an equilibrium is sub-optimal since $u'(x) > c$.

This discussion pertains specifically to product purchasing contests, wherein consumers' expenditure in a contest generates both utility and production costs through purchased regular goods. Importantly, the discussion presented does not apply to plain contests which lack the utility and costs associated with regular goods. This distinction underscores why the traditional equivalence among different mechanisms (the optimal Tullock contest, all-pay, and winner-pay auction) does not hold within the context of product purchasing contests. Furthermore, despite employing a similar approach, the limiting equivalence between plain Tullock contests with sufficiently high R and all-pay auctions (Alcalde and Dahm, 2010) is not upheld in the context of product purchasing contests. These findings are elaborated in Lemma 3, Proposition 2, and Proposition 4 below.

To ensure the existence theorem (Lemma 6 and Theorem 6) in Dasgupta and Maskin (1986) can be well applied to the product purchasing contest, it is necessary to discuss the continuity of the consumers' payoff function in the game. It is sufficient to consider the contest success function defined in (6) since px and $u(x)$ are continuous. Clearly, Tullock contest success function is not continuous everywhere. To meet the conditions suggested by Dasgupta and Maskin (1986), the set of such discontinuity is sufficiently "small",[4] that is, has measure zero. This can be demonstrated by showing its dimension

[4]This expression is quoted from Alcalde and Dahm (2010).

is less than n, the number of consumers. Observe that under Tullock contest (including all-pay auction), the contest success function is continuous everywhere when the number of consumers (except the consumer) who actively participate in the contest is greater or equal to the number of premium good. If the number of active consumers are less than the number of premium good, however, consumer i's payoff function is discontinuous at $D = \{(x_1, x_2, \ldots, x_m, x_{m+1}, \ldots, x_i, \ldots, x_{-n}) | X_m = (x_1, x_2, \ldots, x_m) > 0, X_{-m} = (x_{m+1}, \ldots, x_i, \ldots, x_n) = 0\}$ with given m active consumers. Without loss of generality, those active consumers are numbered from 1 to m. Observe that the set of points where the expected payoff function is discontinuous has at most m dimension ($m < k < n$ by construction) Clearly, at the set of discontinuities D, $x_i = 0$ cannot be a best response to the given other consumers' strategy because consumer i can win the contest with certainty only by actively participating the contest by deviating to an arbitrarily small $x_i > 0$.

For all-pay auction as a special case of Tullock contest, a similar logic can be applied when the number of active consumers are smaller than the number of premium goods at D defined above. Additionally, even when the number of active consumers are greater than that of premium good, discontinuity of the expected payoff function can arise when there are tied consumers. Even in that case, however, a similar logic can be applied as well, with obviously smaller dimension than n.

A.5 Alternative Counterfactual Benchmarks: Internalizing Rent-seeking and Event Ticket Underpricing Puzzle

The implications of product purchasing contests highly depends on the hypothetical counterfactuals. In the positive analysis comparing market performance with and without product purchasing contests, a key assumption is that the premium good is allocated through a contest[5] even under the counterfactual analysis. This common allocation rule assumption is useful to ensure a fair comparison of relative performances between the traditional and a novel format contest by controlling for any contest-specific factors. Still, given that a contest is not a unique way to distribute a premium good, examining different counterfactuals is beneficial for a deeper understanding of product purchasing contests. Relaxing the common allocation rule assumption between product purchasing contest and its counterfactuals, this section discusses the implications of Section 3 under alternative, still rationalizable counterfactual benchmarks.

What alternative counterfactual scenarios can we consider beside a plain contest for premium goods? The candidate counterfactuals can be considered from two angles: how to set the price of the premium good and how to allocate the premium good (especially in

[5] Regardless of whether it is a plain, non-product-purchasing or a product purchasing contest.

situations in which there is potential excessive demand for the premium good under the price regime).

9.1.2.1 Counterfactual Scenario 2

Regarding pricing, a good starting point is uniform pricing for the premium good, which is likely the most common way of selling a product. With the same assumptions in Setup 1 through 4 from Section 3, setting the uniform price at the highest willingness to pay is the profit-maximizing strategy. For analytical purposes, suppose there is a sufficiently small but still positive transaction cost, ϵ. Introducing this transaction cost eliminate the allocation issue as well. The formal definition of the counterfactual scenario is provided as follows.

Definition: Counterfactual Scenario 2

Suppose Setup 1 through 4 in Section 3 are satisfied. Now, the monopolist sells the premium good directly at a profit-maximizing uniform price, not allocating it through a simple, non-product-purchasing contest.

Then, a class of multiple pure strategy Nash equilibria can be established where arbitrarily k consumers purchase the premium good at the price of $(v - \epsilon)$, and the remaining $(n - k)$ consumers remain outside of the market, failing to purchase premium good. In this case, the profit is as high as the profit under counterfactual scenario 1 with endogenously determined R in the plain contest for premium goods. On the other hand, if R is exogenously given, then the profits under this counterfactual scenario is weakly greater than the Counterfactual Scenario 1.

9.1.2.2 Counterfactual Scenario 3

Counterfactual Scenario 2 is simple and clear. Nevertheless, this assumption may be an oversimplification and not easily extend to real-world contexts such as event ticket pricing, which is also relevant to the music industry motivation.[6] Under this particular context, a driver of the so-called "Event Ticket Pricing Puzzle" may kick in, and therefore it forces some constraint on the firm's ability to set a uniform price for premium good. The longstanding puzzle of event ticket pricing suggests that many entertainment event tickets may not be optimally priced according to theoretical predictions. Two key dimensions to consider are: firstly, prices are often set below the market-clearing level, and secondly, the

[6]Concerts and meet-and-greets are similar in that they provide an opportunity for fans to meet the musician directly. However, beyond the differences in product characteristics between the two driven by the presence or absence of a music performance, the economic characteristics from a production technology perspective are entirely different. Generally speaking, music services are non-rivalrous and benefit from economies of scale. In contrast, personal moments with the musician at meet-and-greets are inherently rivalrous.

degree of price discrimination is too crude to be optimal.[7]

Researchers have proposed various explanations for this phenomenon. Some argue that musicians may prefer a specific distribution of well-being around events (Courty and Pagliero, 2014), while others suggest that commercializing certain products can alter their perceived value (Sandel, 2013). Behavioral economists point to consumers' antipathy against to for-profit price increases not justified by rising costs as a significant constraint on profit maximization (Kahneman et al., 1986). This aversion could be linked to consumers' preference for fairness, where price increases are seen as unfair if they exceed historically formed expectations (Cohen et al., 2022). Indeed, the live concert industry has seen a significant surge in ticket price since the end of the 1990s, with VIP prices escalating even more dramatically (Krueger, 2005, 2019). Furthermore, certain markets might be considered repugnant, even when buyers and sellers agree on transactions (Roth, 2007). In line with this, musicians may also seek to avoid being labeled as greedy by the media (Courty and Pagliero, 2014).

Assessing and exploring the latent reasons behind the puzzle is beyond the scope of this paper. For practical purpose, this section just takes the fact that consumers derive disutility from the higher price than their expectation itself. With the given additional assumption, an alternative counterfactual scenario is suggested with the simplest form of constraint on the uniform price of premium good. Then specifying the allocation mechanism is now necessary with the sufficiently large gap between the highest willingness to pay(v) and the upper bound on the price for premium good. For simplicity, the "First Come, First Served" mechanism which entails a race[8] toward the premium good. This is a more traditional way to induce competition among consumers than product purchasing contest. Suppose premium goods are distributed using this method. The formal definition of the counterfactual scenario is provided as follows.

Definition: Counterfactual Scenario 3

Suppose setups in Section 3 are satisfied. Due to some behavioral factors (fairness concern, social pressure, and so forth), there exists the price ceiling, which results in the lower price than the consumers' highest willingness to pay. The premium good is allocated by "First Come, First Served" method, which corresponds to all-pay auction-like competition.

[7]From another direction, some studies suggest that such a strategy can be economically rationalized due to its higher profitability. For instance, Loertscher and Muir (2022) discuss the profitability of conflating (a rough way of price discriminating) in the ticket pricing.

[8]Either online or offline.

Under this counterfactual scenario, there exists a rent-seeking incentives among consumers since the highest feasible price is lower than the market-clearing level. Anticipating a guaranteed positive consumer surplus from the premium good, consumers are willing to engage in a race, which is a typical form of rent-seeking behaviors. Leslie and Sorensen (2014) investigate event ticket market, discussing rent-seeking behaviors in the primary market, particularly in relation to the secondary market. For simplicity, this section assumes consumers are homogeneous even in the rent-seeking technology dimension.

Analytically, these behaviors result in a common mixed strategy Nash equilibrium in the all-pay auction for obtaining the premium good. While both this race for the premium good and product purchasing contest involve competition among consumers, they differ in how consumers' investments are allocated. In the race for the premium good with "First Come, First Served" mechanism, investments made to win the race are just wasted like burning money values. Hence, economic welfare derived from the premium good is fully dissipated in equilibrium. In contrast, in a product purchasing contest, investments are transferred to the monopolist as profits. Thus, from this perspective, product purchasing contests internalize of the welfare loss due when allocating scarce resources by commercializing such rent-seeking behaviors. Under this counterfactual, product purchasing contest is more likely to be economically rationalized and feature higher welfare. Nevertheless, premium good can also be sold at some price, not just freely granted as a reward. The assumption that premium good is provided just as a reward free of charge can be additionally relaxed. Proposition A-2 below formally demonstrates this reasoning.

Proposition A-2 *Suppose setups in Section 3 are satisfied. Product purchasing contest is more likely to be more profitable than Counterfactual Scenario 2 when the highest feasible price of premium good is sufficiently lower than the consumer's highest willingness to pay for the premium good under product purchasing contest.*

Proof. For a simple illustration, consider the situation defined in Section 3 with fixed 1 premium good supply. Suppose $\overline{p_p}$ is the highest feasible level of premium good price ($\overline{p_p} < v$) when it is sold as an individual product, untied with any regular goods. But under product purchasing contest, as the premium good is sold as a(an) (endogenously determined according to the contest) fraction tied with 1 unit of regular good, no typical social norm is assumed to exist for the novel way of packaging (and therefore, no technical constraints are assumed here).[9] Anticipating $(v - \overline{p_p})$ surplus, consumers are willing to

[9]One piece of indirect evidence supporting this view is the contrasting outcomes observed in different market institutions. While Ticketmaster's auctions for event tickets were discontinued after several years, contest-based invitations to exclusive meet-and-greets have persisted despite public complaints and concerns

engage in a rent-seeking toward the premium good. Directly applying the well-known full dissipation result of all-pay auctions, the expected surplus is wasted as welfare loss and the expected consumer surplus is zero in equilibrium. Then both profits and welfare are reduced by the difference between consumer valuation on the premium good and the highest feasible premium good price. No arbitrage opportunity exists under the fully rational consumer assumption. If $\overline{p_p}$ is sufficiently low, unlike the conclusion in Corollary 1, even the simple lottery under product purchasing contest may outperform the profits potentially gained under Counterfactual Scenario 3. Specifically, if $\overline{p_p} < \frac{(n-1)}{n}v$ where $\frac{(n-1)}{n}v$ is the expected revenue when a simple lottery is implemented, then the product purchasing contest with a simple lottery is more profitable than the counterfactual uniform pricing benchmark.

■

Under Counterfactual Scenario 3, the price of the premium good should coincide with the highest level of feasible price. Then the consumers' rent seeking behaviors result in a common mixed strategy Nash equilibrium in the all-pay auction for the underpriced premium good. From the monopolist's perspective, such a problem is even more difficult to cope with, considering the possibility of resales in the secondary market. To mitigate such market extortion driven by uniform pricing, an alternative approach has been taken by Ticketmaster in live concert ticket sales. Ticketmaster implements position auctions as a price-discovery mechanism that can address too-low uniform pricing and the accompanying arbitrage opportunity. It performs well as Budish and Bhave (2023) demonstrates both theoretically and empirically the advantages of position auctions in event ticket pricing. Nevertheless, it was not able to continue, possibly due to the tendency where consumers

about potential harm. This persistence might reflect distinctive features of the Korean pop music market, shaped by cultural or social norms. Although this explanation has its own merit, attributing the phenomenon solely to its idiosyncratic cultural factors may not provide a fully comprehensive account. A more compelling hypothesis can be drawn from behavioral economics. A key distinction between purchasing a premium good through uniform pricing and through a product purchasing contest lies in the "framing" of expenditure: specifically, how consumers perceive the per-unit price of the regular goods they purchase. Under uniform pricing, consumers pay a single price for one unit of the premium good, whereas in a contest, they distribute their spending across multiple regular goods. Even if total expenditure or consumer surplus remains the same, the per-unit price of regular goods in a product purchasing contest can be much lower than the price of the premium good under uniform pricing. For instance, a consumer might spend \$200 on a premium good with uniform pricing, paying \$200 for the item itself. In contrast, the same consumer could purchase ten regular goods at \$20 each in a product purchasing contest. If consumers' mental accounting is more sensitive to the per-unit price than to the total expenditure, the \$200 premium good might be perceived as more expensive than spending the same amount in a contest. This disproportionate perception of proportionate changes in economic variables can be better understood through the lens of biases arising from framing effects, as discussed in the works on focusing by Kőszegi and Szeidl (2013) or salience by Bordalo et al. (2022).

often prefer uniform pricing over auctions (Einav et al., 2018). Faced with the behavioral resistance to the event ticket priced higher than a certain threshold, product purchasing contests can be suggested as a way to circumvent these behavioral constraints with the given structural change in the industry along with the evolution of the industry toward the digitization.[10]

A.6 Discussion on the Strictly Convex Cost Function for Premium Good

Before moving on to the next section, whether the motivating industrial backgrounds align well with sufficiently strictly convex cost function assumption is discussed.

In the music industry, superstar musicians may face exceptionally high opportunity costs for each unit of time. For instance, consider exclusive fan events such as meet-and-greets, where fans highly value personal interactions with their idols, even if brief. These moments are inherently rivalrous consumption, and therefore cannot be shared with other fans. Hence, exclusive events like meet-and-greets make it challenging for musicians to benefit from economies of scale, unlike concerts. Consequently, from the musicians' perspective, the marginal cost for each additional invitation to the events may sharply escalate.

Similarly, within the luxury industry, firms may encounter sharply increasing opportunity cost in the form of potentially forgone future profits. More specifically, by the inherent nature of luxury goods, more quantity supplied in the market may make the luxury goods look less fascinating. Demand for luxury goods often stems from conspicuous consumption motivations and the limited availability of these goods serves as a signal for individuals to display their high social status (Bagwell and Bernheim, 1996). Notably, intentionally limiting the quantity of goods is a well-established strategy within the luxury industry (Kapferer and Bastien, 2012).

Rather than taking the strictly convex cost function including the notion of opportunity cost, we can even explicitly incorporate the conspicuous consumption motivation in the model. For illustration, I assume that the consumers' willingness to pay for the premium good can be decomposed by two parts (A1): a constant part v and variable part $f(k)$, which depends on the quantity of premium goods sold.

$$V(k) \equiv v + f(k) \tag{A1}$$

where $f(k)$ satisfies $f(k) > 0$ $f'(k) < 0$ and $f''(k) > 0$, convexely decreasing.[11] Such functions are useful to capture the notion of a trade-off between additional revenues from extra premium good (in the sense of the external margin) and decreased consumers'

[10] Still, as illustrated in Introduction, product purchasing contest is also facing some resistance, which motivates this study.

[11] An example of such a function is $f(k) = \frac{1}{k}$.

willingness to pay for the premium goods (in the sense of the internal margin).

$$\Rightarrow \Pi^*_{Tullock} = k(v + f(k)) - C_p(k) + nA(c_r)$$

where $A(c_r) = [u(u'^{-1}(c_r)) - c_r u'^{-1}(c_r) - [u'(y^*) - c_r]y^*]$. Intuitively, $A(c_r)$ is the increase in profits generated from the sale of the regular good when transitioning from a plain contest with a monopoly on the regular good to the optimal Tullock contest under product purchasing contest framework.

Additionally, I assume that $kf(k)$ is concave so that $\frac{\partial kf(k)}{\partial k} > 0$ but $\frac{\partial^2 kf(k)}{\partial k^2} < 0$. Then the interior solution for the optimal number of premium good is determined as k such that the following the first order condition and the second order condition hold ($k < n$).

$$\frac{\partial \Pi^*_{Tullock}}{\partial k} = V'(k) - C_p'(k) = 0$$

$$\frac{\partial^2 \Pi^*_{Tullock}}{\partial k^2} = V''(k) - C_p''(k) < 0$$

Also, this results can extend to a case with a strictly convex cost function. Thus, if an interior solution exists, then the analysis in Section A.3 directly applies.

A.7 Corner Solution Cases: When the Optimality Condition $p \geq u'(0)$ is Binding in Equilibrium

The first order condition (Expression A12) should be restated as follows:

$$p = max\left\{u'(x) + \frac{vr}{nx}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right], u'(0)\right\}$$

Suppose the price is binding since $u'(x) + \frac{vr}{nx}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] < u'(0)$ and let $\underline{x}$ is the implicit solution of the following Equation (A2).

$$\Rightarrow \underline{p} = u'(0) = u'(\underline{x}) + \frac{vr}{n\underline{x}}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] \tag{A2}$$

To solve for $\bar{x}$, we can use the consumer surplus extracted through product purchasing contest and the profit gained outside of regular good, then equating them. When each consumer buy $\bar{x}$ units of regular good for the contest, the consumer surplus is given as (A3).

$$\Rightarrow CS(\bar{x}) = \frac{kv}{n} + \int_0^{\bar{x}}[u'(t) - u'(\bar{x})]dt = \frac{kv}{n} + u(\bar{x}) - u'(\bar{x})\bar{x}$$

$$\Rightarrow CS(x) = \frac{kv}{n} + u(\bar{x}) - u'(\bar{x})\bar{x} \tag{A3}$$

And the profit gained outside of regular goods is given as (A4).

$$(\underline{p} - c)\bar{x} = (u'(\underline{x}) + \frac{vr}{n\underline{x}}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] - c)\bar{x}$$

$$\Rightarrow (\underline{p} - c)\bar{x} - (u'(x) - c)\bar{x} = \frac{vr}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] \tag{A4}$$

Equating (A3) and (A4) leads to (A5). To solve for $\bar{x}$, substituting (A5) into (A2) results in (A6), which presents $\bar{x}$ as the implicit solution.

$$\frac{vr}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] = \frac{kv}{n} + u(\bar{x}) - u'(\bar{x})\bar{x}$$

$$\Rightarrow \frac{vr}{n} = \frac{\frac{kv}{n} + u(\bar{x}) - u'(\bar{x})\bar{x}}{\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right]} \tag{A5}$$

$$\Rightarrow u'(0) = u'(\bar{x}) + \left[\frac{kv}{n} + u(\bar{x}) - u'(\bar{x})\bar{x}\right]\frac{1}{\bar{x}}$$

$$\Rightarrow u'(0) = \left[\frac{kv}{n} + u(\bar{x})\right]\frac{1}{\bar{x}}$$

$$\Rightarrow u'(0)\bar{x} - u(\bar{x}) = \frac{kv}{n} \tag{A6}$$

Note that $u'(0)0 - u(0) = 0$ when $x = 0$ and $u'(0) - u'(\bar{x}) > 0$ as $\bar{x}$ increases. Since $u'(0)$ is constant, and $u'(0)$ is always greater than $u'(x)$ because the utility function $u(x)$ is assumed to be concave, so $u'(x)$ is decreasing in x. Note that this function $u'(0)\bar{x} - u(\bar{x})$ is convex in x. Hence, when the right hand side is increasing, then $\bar{x}$ is also increasing, which is demonstrated by the following comparative statics analysis.

Policy Implications when the Optimality Condition is Binding

For illustration, the numerical specification assumed in Section A.3 are fixed except the marginal cost, which is now 0.1. A smaller marginal cost leads to lower equilibrium regular good price, which is more likely to result in a binding solution. With these specification, the regular price in the optimal Tullock contest is binding at 1, which equals the $u'(0)$. In Figure A1, nevertheless, the optimal Tullock contest (red triangle) exhibits the highest profits among the four different mechanisms. Figure A2 displays social welfare; when the

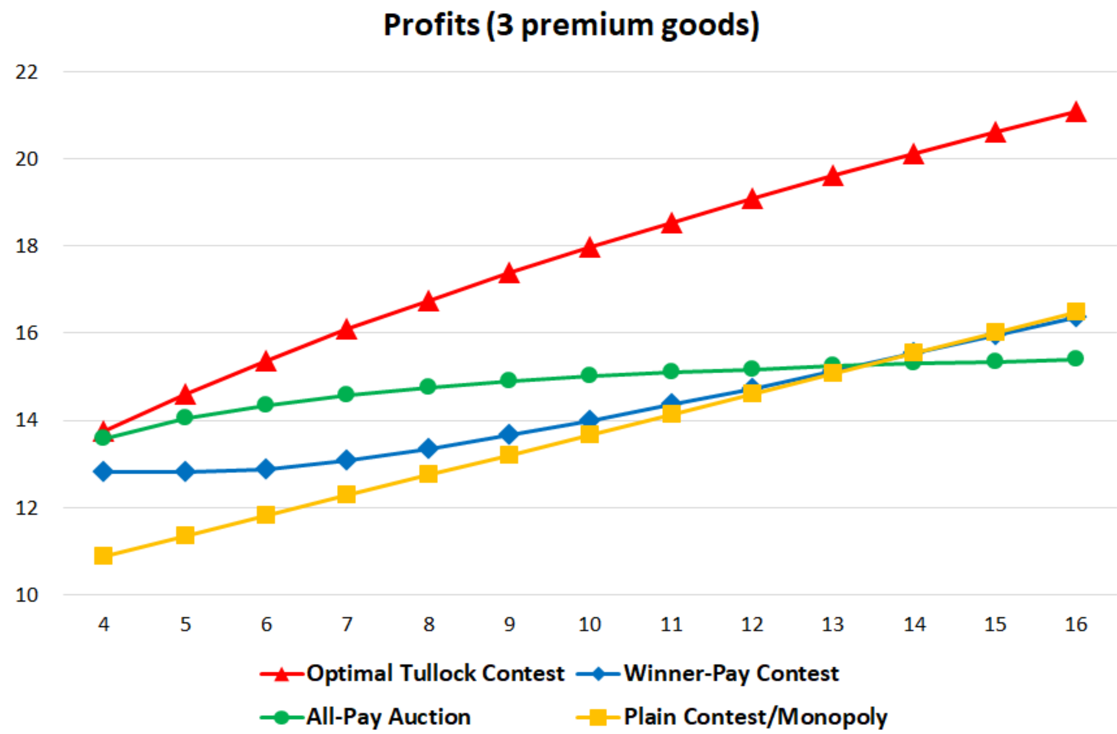

Figure A1: The vertical axis represents the profits and the horizontal axis represents the number of consumers in the market with 3 premium goods

number of consumers is relatively large, the winner-pay contest gains (blue diamond) the highest social welfare, and even the plain contest with a monopoly dominates over the optimal Tullock contest. While the optimal Tullock contest fails to accomplish the first best outcome when the regular good price is binding at the highest level of the marginal utility for regular good for incentive compatibility to hold, both the winner-pay contest and the plain contest with a monopoly can have a lower regular good price than the highest level of marginal utility. They, therefore, can result in higher social welfare by allowing consumers to buy and enjoy a higher quantity of regular goods.

Comparative Statics

Firstly, with respect to n,

$$\Rightarrow u'(0)\frac{dx}{dn} - u'(x)\frac{dx}{dn} = -\frac{kv}{n^2}$$

$$\Rightarrow \frac{dx}{dn}[u'(0) - u'(x)] = -\frac{kv}{n^2}$$

$$\Rightarrow \frac{dx}{dn} = -\frac{(kv/n^2)}{[u'(0) - u'(x)]} < 0$$

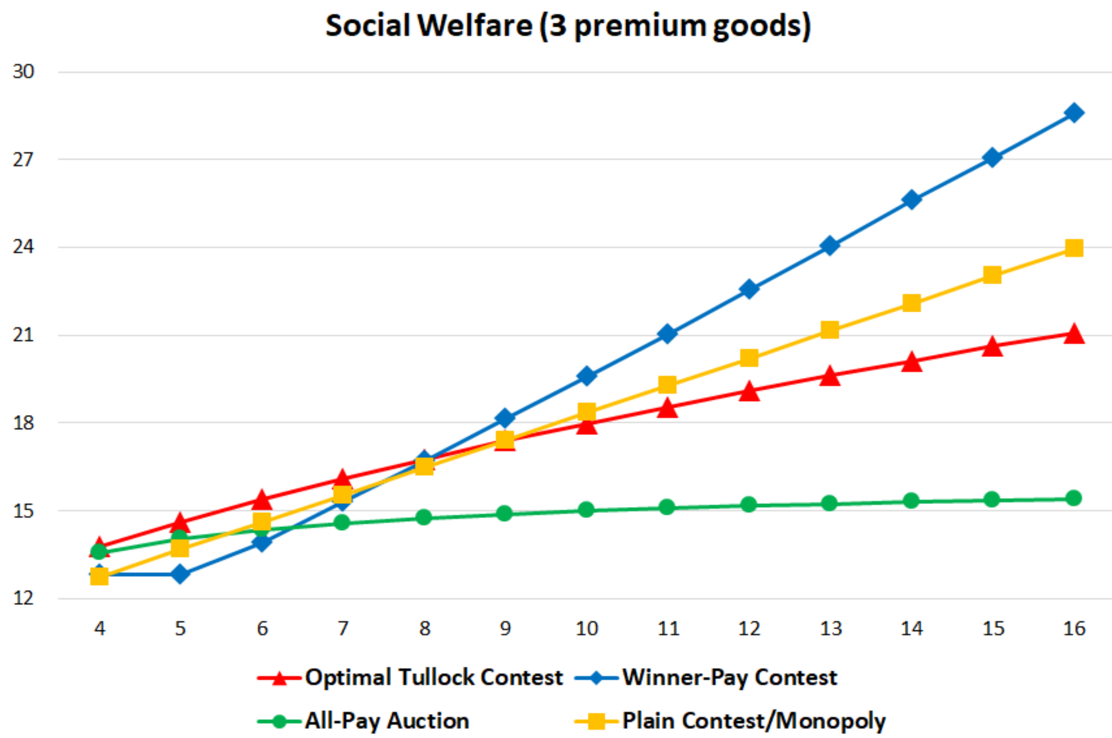

Figure A2: The vertical axis represents the profits and the horizontal axis represents the number of consumers in the market with 3 premium goods

Since $u'(0) > u'(x)$ for any x. Hence increasing n leads to decrease in $\bar{x}$

Next, with respect to v

$$\Rightarrow u'(0)\frac{dx}{dv} - u'(x)\frac{dx}{dv} = \frac{k}{n}$$

$$\Rightarrow \frac{dx}{dv}[u'(0) - u'(x)] = \frac{k}{n}$$

$$\Rightarrow \frac{dx}{dv} = \frac{(k/n)}{[u'(0) - u'(x)]} > 0$$

Since $u'(0) > u'(x)$ for any x. Hence increasing n leads to decrease in $\bar{x}$

Lastly, with respect to k

$$\Rightarrow u'(0)\frac{dx}{dk} - u'(x)\frac{dx}{dk} = \frac{v}{n}$$

$$\Rightarrow \frac{dx}{dk}[u'(0) - u'(x)] = \frac{v}{n}$$

$$\Rightarrow \frac{dx}{dk} = \frac{(v/n)}{[u'(0) - u'(x)]} > 0$$

Since $u'(0) > u'(x)$ for any x. Hence increasing n leads to decrease in $\bar{x}$

Proposition A-3 *The optimal Tullock contest under product purchasing contest may not accomplish the first best outcome and even be worse than other mechanisms (e.g., all-pay auctions) when* $p = u'(0)$ *in equilibrium*

Proof.

A counterexample can be provided to demonstrate the claim. Let x^*, $\bar{x}$, and $\tilde{x}$ represent the following outcomes: x^* denotes the solution corresponding to the first best outcome, where $u'(x^*) = c$; $\tilde{x}$ represents the monopoly on regular good outcome, characterized by $u'(\tilde{x})+u''(\tilde{x})\tilde{x} = c$; and $\bar{x}$ corresponds to the binding case outcome, satisfying $u'(0)\bar{x}-u(\bar{x}) = \frac{kv}{n}$. It is possible that $\bar{x} < \tilde{x}$, and in such a case, social welfare is higher under $\tilde{x}$ than under $\bar{x}$.

To suggest a specific example, assume $u(x) = \log(1 + x)$, $n = 10$, $k = 1$, $u'(0) = 1$, and $c = 0.2$. The equilibrium outcomes are then determined as follows: $x^* = 4$ for the first-best outcome, $\tilde{x} \approx 1.236$ for the monopoly on regular good outcome, and $\bar{x} \approx 0.9863$ for the binding case outcome. The corresponding social welfare values are $SW(\bar{x}) \approx 7.89$ and $SW(\tilde{x}) \approx 8.5751$. Numerical values are rounded to the fourth decimal place.

■

B Proofs

B.1 Socially Optimal Outcome

Proof. SW^*_{SO} is given as (2) in Result 1 with n symmetric consumers whose the utility function of the regular good is given $u(x)$, k fixed premium goods, with y^*_{SO} such that $u'(y^*_{SO}) = c$, and equivalently, $u'^{-1}(c) = y^*_{SO}$ as $u'(.)$ is monotonic (the strict concavity assumption), and therefore there exists the inverse function of $u'(.)$, and $u'^{-1}(.)$. With given number of premium goods, the SO outcome is determined at the unique quantity of the regular good, y^*_{SO}, which maximizes social welfare where the marginal utility equals the marginal cost. ■

B.2 Prohibition Outcome

Proof. I solve two separate markets one by one. First, I consider the market for the premium good. Suppose the monopolist considers a plain contest to allocate the k premium goods to n contenders. To characterize consumers' demand function for the lottery tickets in the contest for the premium good, I first consider the consumer's utility maximization problem. The allocation mechanism (the conditional probability of winning) function M is given as (6).

$$max_{x_i}\ M(x_i; X_{-i}, R, k)v - x_i \text{ and} \tag{A1}$$

When considering the symmetric equilibrium(x=x_i for all i), the first order condition for the demand for the lottery tickets is given as,

$$\Rightarrow x^* = \frac{vr}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] \tag{A2}$$

With the conditional demand for x, the monopolist's optimal mechanism choice problem with respect to R becomes

$$Max_r\ \Pi = nx^* = vr\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right]$$

$$\Rightarrow \frac{\partial x^*}{\partial R} = \frac{v}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] > 0 \tag{A3}$$

$$\Rightarrow \frac{\partial \Pi}{\partial R} = v\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] > 0 \tag{A4}$$

Considering both (A3) and (A4) above, the monopolist has an incentive to raise R as high as possible unless the incentive compatibility is violated. In a symmetric equilibrium, the expected payoff for each consumer with optimally chosen R^* satisfies,

$$\Rightarrow \frac{k}{n}v - x(R) = 0 \Rightarrow \frac{kv}{n} = \frac{vR}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] \tag{A5}$$

$$\Rightarrow R^* = \frac{k}{\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right]} \tag{A6}$$

With the optimal chosen mechanism r^* in (A6), the expected consumer surplus is zero.

$$\Rightarrow x^* = \frac{kv}{n} \tag{A7}$$

$$\Rightarrow \Pi^* = kv \tag{A8}$$

Note that if the monopolist sells the k premium goods to the randomly chosen k consumers, the monopolist gains the same profits.

Now we turn to the market for the regular good. Based on the utility maximization problem (2), the first order condition for y_i is,

$$u'(y_i) = p$$

Then the monopolist's profit maximization problem in the regular good is (with the symmetric demand for all consumers),

$$max_y \; n[u'(y) - c]y$$

The first order condition is

$$u'(y^*) = c - y^* u''(y^*) \equiv \bar{p} > c \tag{A9}$$

due to the concavity assumption on the utility function $u(.)$ (and therefore, it is the global max as well). The optimal y^* is smaller than the socially optimal level of $\tilde{y}$ such that $u'(\tilde{y}) = c$.

$$\Rightarrow \Pi^* = kv + n[u'(y^*) - c]y^* \tag{A10}$$

$$\Rightarrow CS^* = 0 + \int_0^{y^*} [u'(y) - u'(y^*)]dy > 0 \tag{A11}$$

$$\begin{aligned} SW^* = \Pi^* + CS^* &= kv + n[u'(y^*) - c]y^* + \int_0^{y^*} [u'(y) - u'(y^*)]dy \\ &= kv + \int_0^{y^*} [u'(y) - c]dy \\ &< kv + \int_0^{\tilde{y}} [u'(y) - c]dy = S\bar{W} \end{aligned}$$

because $y^* < \tilde{y}$ (Expression A9)

Thus, the optimal Tullock without bundling case fails to attain the efficient outcome due to the price in the regular good market which is higher than the constant marginal cost. ■

B.3 Proof of Theorem 1: Product Purchasing Contest accomplishes the first best outcome (Proof of Product Purchasing Contest is also included)

Proof. For a market outcome where consumers are fully extracted to be sustained as a Nash equilibrium, the equilibrium price needs to be at least as large as the highest possible marginal utility of the regular good, i.e., $p \geq u'(0)$, in order to satisfy the incentive compatibility condition. In this analysis, I assume an interior solution for equilibrium is established such that $p > u'(0)$. The corner solution case, where $p = u'(0)$, is discussed in Section A.7. Under the given contest, the first order condition of the consumers' problem leads to the following inverse demand function (A12) when focusing on the symmetric equilibrium.

$$p = u'(x) + \frac{vR}{nx}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] \tag{A12}$$

Now, turning to the monopoly firm's profit maximization problem:

$$\Rightarrow max_{x,R}\Pi(x,R) = \{p(R)-c\}x = \{u'(x)-c\}x + \frac{vR}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] \tag{A13}$$

Assuming that the monopolist simultaneously choose x and R, I simplify the analysis by characterizing R as a function of x. Since the partial derivative with respect to R (Expression A14) is always positive, it is profitable for the monopolist to set R as high as possible, until the expected consumer surplus is driven to zero. Otherwise, that is, if R is so high that the expected consumer surplus becomes negative, the incentive compatibility condition is violated, and thus consumers would not participate in the contest.

$$\Rightarrow \frac{\partial \Pi}{\partial R} = \frac{v}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] > 0 \tag{A14}$$

The expected consumer surplus is given by Expression (A13). The monopolist has an incentive to set R appropriately to fully extract the consumer surpluses:

$$\frac{vr}{n}\left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right] = CS(x) = \frac{kv}{n} + \int_0^x [u'(t)-u'(x)]dt \tag{A15}$$

$$\Rightarrow CS(x) = \frac{kv}{n} + [u(x) - u'(x)x] \tag{A16}$$

Then, rearranging (A15), R can be expressed as a function of x:

$$\Rightarrow R = \frac{\frac{n}{v}[\frac{kv}{n} + \{u(x)-u'(x)x\}]}{[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}]} \tag{A17}$$

Substituting R from Expression (A17) into the monopolist's profit function (A13), the optimization problem becomes:

$$\Rightarrow max_x\Pi(x) = \{u'(x)-c\}x + \frac{kv}{n} + \int_0^x [u'(t)-u'(x)]dt \tag{A18}$$

$$\Rightarrow max_x \Pi(x) = (p-c)x = -cx + \frac{kv}{n} + u(x) \tag{A19}$$

Taking the derivative, the first order condition is:

$$\Rightarrow max_x \Pi'(x) = -c + u'(x) = 0 \tag{A20}$$

The implicit solution of x,

$$\Rightarrow u'(x) = c \tag{A21}$$

By the concavity assumption on $u(x)$, the first order condition is both necessary and sufficient condition for optimality. Since $u'(x)$ is monotonic, its inverse function $u'^{-1}(x)$ exists. Therefore, the optimal x is determined by (A22).

$$\Rightarrow x^* = u'^{-1}(c) \tag{A22}$$

$$\Rightarrow nx^* = X^* = nu'^{-1}(c) \tag{A23}$$

$$\Rightarrow R^* = \frac{\frac{n}{v}[\frac{kv}{n} + \{u(u'^{-1}(c)) - cu'^{-1}(c)\}]}{[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1} \frac{(k-j)}{(n-j)}]} \tag{A24}$$

$$\Rightarrow p^* = \frac{1}{u'^{-1}(c)} \left[\frac{kv}{n} + u(u'^{-1}(c)) \right] \tag{A25}$$

$$\Rightarrow \Pi^* = nx^*(p^* - c) = kv + n[u(u'^{-1}(c)) - cu'^{-1}(c)] \tag{A26}$$

$$\Rightarrow CS^* = \frac{kv}{n} + u(u'^{-1}(c)) - \frac{u'^{-1}(c)}{u'^{-1}(c)} \left[\frac{kv}{n} + u(u'^{-1}(c)) \right] = 0 \tag{A27}$$

$$SW^* = \Pi^* + CS^* = kv + n[u(u'^{-1}(c)) - cu'^{-1}(c)] + 0 \tag{A28}$$

$$= \bar{SW} \text{(Equation 12)} \tag{A29}$$

Since the quantity of regular good is produced at marginal cost, $u'(x) = c$, the first best, efficient outcome is achieved. Therefore, in terms of social welfare (from the utilitarian perspective), the optimal Tullock contest under product purchasing contest is more desirable than the plain contest for premium good with a monopoly on regular good, the benchmark case. From a consumer surplus perspective, however, consumers are worse off under this mechanism. Thus, the optimal Tullock contest under product purchasing contest is socially more efficient but not Pareto improving. ∎

B.4 Proof of Lemma A-1

Proof. Then with the given demand function $x(p)$ (or the inverse demand function $p(x)$), the profit function is organized as follows (a positive marginal cost c for the regular good is assumed):

$$\Pi = (p - c)x = \left[\frac{Rv}{4x} + u'(x) - c\right] x$$
$$= \frac{Rv}{4} + \{u'(x) - c\}x$$

To maximize $\{u'(x) - c\}x$, the first order condition is:

$$\Rightarrow u'(x) - c + xu''(x) = 0 \tag{A30}$$

Note that the first order condition (A30) is equivalent to that of consumers' problem on the regular good under a plain contest with a conventional monopoly.[12] This does not necessarily mean the two problems are the same because of the implicit constraint in the product purchasing contest, the individual rationality (a non-negative expected payoff for consumers should be guaranteed), which is never binding under monopoly market for the regular good.

Applying the symmetric pure strategy Nash equilibrium, the expected consumer payoff is given as Expression (A31).

$$\frac{k}{n}v - px + u(x) \tag{A31}$$

Substituting the price (A12) into the expected payoff (A31),

$$\frac{k}{n}v - \left[u'(x) + \frac{vR}{nx}\left\{\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right\}\right] x + u(x)$$

$$\Rightarrow \frac{k}{n}v - \left[\frac{vR}{n}\left\{\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right\}\right] - u'(x)x + u(x)$$

$$\Rightarrow \frac{v}{n}\left[k - R\left\{\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right\}\right] - u'(x)x + u(x) \geq 0$$

[12] Note that under a plain contest with a monopoly, the demand function for the regular good is determined by solving $\max_x u(x) - px$, which results in the demand function as the first order condition $u'(x) = p$. With the demand function, the profit maximization problem becomes $\max_x (p - c)x = (u'(x) - c)x$, then the first order condition of the profit maximization problem with regard to the quantity of regular good is given as $u'(x) - c + xu''(x) = 0$.

Rearranging the individual rationality constraint leads to the following,

$$\Rightarrow R \leq \frac{k - \frac{n}{v}\{u'(x)x - u(x)\}}{\left\{\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right\}}$$

, which implies the individual rationality holds as long as R is low enough. Let R^* denote the level of R where the the weak inquality holds with equality. Note that $R^*_{PP} > R^* > R^*_M$. Then the regular good price(and therefore, quantity) is determined at the monopoly level unless the the individual rationality holds, which concludes that product purchasing contest and a plain contest with a conventional monopoly has the same market outcome. From the comparative statics perspective, it is noteworthy that within the range of $R^*_M < R < R^*$, as R increases, there is only adjustment in the distribution of social welfare, going from the consumers to the producer with fixed social welfare since the equilibrium quantity of regular good does not depend on R.

■

B.5 Proof of Corollary A-1

Proof. For a simple illustration, let $k = 1$ and there are n homogeneous consumers. With the simple lottery $(R = 1)$, the utility maximization problem is given as Expression (A32).

$$max_{x_i}\left[\frac{x_i}{x_i + \sum_{j\neq i}^{n} x_j}\right]v - px_i + u(x_i) \tag{A32}$$

Restricting attention to the symmetric equilibrium, the first order condition of the consumers problem leads to the following inverse demand function (A33).

$$p = u'(x) + \frac{(n-1)v}{n^2 x} \tag{A33}$$

Now we turn to the monopoly firm's profit maximization problem (A34).

$$\Rightarrow max_x \Pi(x) = n(p-c)x = n\{u'(x) - c\}x + \frac{(n-1)}{n}v \tag{A34}$$

Then on the right hand side, the revenue generated from the simple lottery $(n-1)v/n$ is independent of x and the remaining part $(n\{u'(x) - c\}x)$ is the same as the maximization problem 20. Thus, the market equilibrium is the same as the counterfactual benchmark above. ■

B.6 Proof of Proposition A-1

Proof.

When R is endogenously determined, comparing Result 2 and Result 3 proves that product purchasing contest has strictly higher profits and social welfare than a plain contest with a monopoly.

When R is exogenously given and $R \leq R^*_{PO}$, Lemma A-1 demonstrates that product purchasing contest has weakly higher profits and social welfare than a plain contest under monopoly. Then the only remaining possibility is when R is exogenously given and $R > R^*_{PO}$.

Continued from Lemma A-1, then starting from the point $R > \frac{k-\frac{n}{v}\{u'(x)x-u(x)\}}{\left\{\frac{k(n-1)}{n}-\sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right\}}$, the implication begins to change, the individual rationality constraint does not hold. With exogenously given $R, k, n,$ and v, there still exists a pure strategy equilibrium by relaxing the individual rationality constraint through incorporating a appropriate lower bound on x, to increase the highest possible consumer surplus, which is necessarily accompanied with lower regular good price. Still, the pure strategy Nash equilibrium is satisfied at the point where the individual rationality constraint is binding because it still ensures the highest profits from the regular good side with the individual rationality constraint. This implication is valid while $R^* < R \leq R^*_{PP}$ where R^*_{PP} coincides with the highest level of R specified above when allowing endogenously determined R. Under a plain contest, for $R > R^*$ only mixed strategy Nash equilibria exist, and the expected revenue from such a plain contest is equivalent among every $R^* < R$ (Alcalde and Dahm, 2010). Consequently, in that regard, within the range of $R \leq R^*_{PP}$, product purchasing contest has at least as high profits as a plain contest with a monopoly, and has a strictly higher profits at a certain parametric range.

■

B.7 Proof of Proposition 1

Proof. Although product purchasing contest has an adjusted structure, the standard approach can still be directly applicable to show non-existence of equilibria under all-pay auction. Suppose $x = x_1, ..., x_n$ is a pure strategy Nash equilibrium where $x_i \geq 0$ for arbitrary $i \in N$. Suppose first there exists i such that $x_i = max_{j \neq i} x_j$ and let m the number of consumers buying x_i unit of regular goods. In other words, the number of highest bidding consumers are larger than that of the premium good. If $px_i - u(x_i) = v$, then x cannot be sustained as a pure strategy Nash equilibrium as consumer i obtains negative expected payoff $\frac{v}{m} - px_i + u(x_i) = -\frac{(m-1)v}{m} < 0$, so consumer i can be better off by unilaterally deviating to $x_i = 0$ where zero expected payoff is ensured. Keeping the assumption that there exists i such that $x_i = max_{j \neq i} x_j$, let's turn to the case where $px_i - u(x_i) < v$. This case still cannot be sustained as a pure strategy Nash equilibrium because consumer i can be better off by increasing the purchased regular good by sufficiently small amount of $\epsilon > 0$

such that $v - p(x_i + \epsilon) + u(x_i + \epsilon) > \frac{v}{m} - px_i + u(x_i) \Rightarrow \frac{(m-1)v}{m} > p\epsilon + u(x_i) - u(x_i + \epsilon)$. Next, consider there exists i such that $x_i > max_{j \neq i} x_j$. This case also cannot be sustained as a pure strategy Nash equilibrium because consumer i can be better off by unilaterally decreasing the purchased regular good by ϵ such that $(x_i - \epsilon) > max_{j \neq i} x_j$. ■

B.8 Proof of Proposition 2

Proof. Suppose x is a symmetric pure strategy Nash equilibrium under product purchasing contest with given n, k, and R. Then the expected consumer surplus is given as,

$$\Rightarrow \frac{kv}{n} - \frac{vR}{n} \left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1} \frac{(k-j)}{(n-j)} \right] - u'(x)x + u(x) \geq 0$$

, which should be nonnegative for the individual rationality to be satisfied. Rearranging this,

$$\Rightarrow R \leq \frac{n}{v \left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1} \frac{(k-j)}{(n-j)} \right]} \left[\frac{kv}{n} - u'(x)x + u(x) \right] = R^*$$

where $x = u'^{-1}(c)$ (Equation A21). Thus, under $R > R^*$, purchasing x units of regular good is strictly dominated strategy to purchasing 0 units, which ensures at least zero expected payoff. In other words, $R \leq R^*$ is the necessary condition for a symmetric pure strategy Nash equilibrium to exist. ■

B.9 Proof of Proposition 3

Proof.

The economic environment is symmetric as consumers have common value in both premium and regular good and their payoffs are determined by the equivalently applied contest success function.[13] Hence, Lemma 6 of Dasgupta and Maskin (1986) ensures the existence of a symmetric equilibrium for the contest defined on the finite grid G. Following the approach of Alcalde and Dahm (2010), I show that the required conditions in Dasgupta and Maskin (1986) are also satisfied. Four conditions must be satisfied for their theorem to apply. First, the sum of payoffs must be upper semi-continuous. $\sum_{i=1}^{n} E\Pi_i(x) = kv - \sum_{i=1}^{n} \{px_i - u(x_i)\}$ is continuous, and the continuity is a sufficient condition for upper semi-continuity. Second, the expected payoff function $E\Pi_i(x)$ has to be bounded, which holds as $-v \leq E\Pi_i(x) \leq v$ for $x_i \in [0, \overline{x}]$ and $i = 1, 2, \ldots, n$. Third, there must exist a function that defines the set of the expected payoff function being discontinuous, which is used to "relate" the consumers' strategies. The contest success functions we are

[13]This symmetric property corresponds to the Anonymity condition in Alcalde and Dahm (2010).

considering are discontinuous only when at least a subset of consumers purchase the same number of regular goods, especially zero units. In this case, therefore, we can take the identity function to define the set of discontinuities. Fourth, a stricter version of weakly lower semi-continuity at the set of discontinuity, which is called "Property (α)" has to be satisfied. Let m denote the number consumers who are active participants in the contest already. Let $\overline{x_i}$ denote the consumer i's purchased quantity of regular good that is at the point where the expected payoff is discontinuous. Similarly, let $\overline{x_{-i}}$ denote the vector of the quantity of regular good that is at the point where the expected payoff is discontinuous purchased by all of the other consumers except i. Property (α) is fulfilled, since

$$\begin{aligned}\lim \inf_{x_i \mapsto + \overline{x}} E\Pi_i\left(x_i, \overline{x_{-i}}\right) &= v - px_i + u(x_i) \\ &> \frac{k - |m|}{\{n - |m|\}} v \\ &= E\Pi_i\left(\overline{x_i}, \overline{x_{-i}}\right)\end{aligned}$$

holds. Putting these together, Theorem 6 in Dasgupta and Maskin (1986) can be applied. ■

B.10 Proof of Theorem 2

Proof. Let $\bar{\mu}^G = \left(\bar{\mu}_1^G, .., \bar{\mu}_n^G\right)$ denote an equilibrium to the contest with finite grid G. Let $\bar{\mu}_{g_i}^G$ denote the probability mass that consumer i assign to the pure strategy $g_i\overline{x}/G$ when playing the mixed strategy $\bar{\mu}_i^G$. Then social welfare with this equilibrium when k units of the premium good is provided is given below (Without loss of generality, fix an arbitrary consumer i for a notational purpose, but social welfare is just n times of per capita social welfare since we consider a symmetric mixed strategy Nash equilbrium).

$$\Rightarrow n \sum_{g_i=0}^{G} \left[\int_0^{(g_i\overline{x}/G)} \{u'(x) - c\} dx \right] \bar{\mu}_{g_i}^G + kv$$

where the probability weight put on the strategy space $\bar{\mu}_g^G$ is provided with

$$\Rightarrow \sum_{g_i=0}^{G} \bar{\mu}_{g_i}^G = 1$$

For any p, the price for the regular good, the expected social welfare can be split into expected consumer surplus and profit.

$$n\sum_{g_i=0}^{G}\left[\int_0^{(g_i\overline{x}/G)}\{u'(x)-c\}dx\right]\bar{\mu}_{g_i}^G+kv=n\sum_{g_i=0}^{G}\left[\int_0^{(g_i\overline{x}/G)}\{u'(x)-p+p-c\}dx\right]\bar{\mu}_{g_i}^G+kv \tag{A35}$$

$$=n\sum_{g_i=0}^{G}\left[\int_0^{(g_i\overline{x}/G)}\{u'(x)-p\}dx+\int_0^{(g_i\overline{x}/G)}(p-c)dx\right]\bar{\mu}_{g_i}^G+kv \tag{A36}$$

$$=n\left\{\sum_{g_i=0}^{G}\left[\int_0^{(g_i\overline{x}/G)}\{u'(x)-p\}dx+\int_0^{(g_i\overline{x}/G)}(p-c)dx\right]\bar{\mu}_{g_i}^G+\frac{kv}{n}\right\} \tag{A37}$$

The mathematical expression (A37) right above can be further expanded as follows:

$$\Rightarrow n\left\{\sum_{g_i=0}^{G}\left[\int_0^{(g_i\overline{x}/G))}\{u'(x)-p+\Omega(g_i;\bar{f}_{g_{-i}}^G)\}dx+\int_0^{(g_i\overline{x}/G)}(p-c)dx\right]\bar{\mu}_{g_i}^G\right\}$$

where

$$\Omega(g_i;\bar{\mu}_{g_{-i}}^G)=\sum_{g_1=0}^{G}\cdots\sum_{g_{i-1}=0}^{G}\sum_{g_{i+1}=0}^{G}\cdots\sum_{g_n=0}^{G}\prod_{j\neq i}^{n}\bar{\mu}_{g_j}^G\Psi_i\left(\frac{g_i\overline{x}}{G},\frac{g_{-i}\overline{x}}{G}\right)$$

and

$$\Psi_i\left(\frac{g_i\overline{x}}{G},\frac{g_{-i}\overline{x}}{G}\right)=\frac{(g_i\overline{x}/G)^R}{(g_i\overline{x}/G)^R+\sum_{i\neq j}^{n}(g_j\overline{x}/G)^R}$$

and

$$\sum_{g_i=0}^{G}\Omega(g_i;\bar{\mu}_{g_{-i}}^G)\bar{\mu}_{g_i}^G=\frac{k}{n}$$

Consequently, the expected social welfare as the sum of expected consumer surplus and profits are given as follows:

$$\Rightarrow n\left\{\underbrace{\sum_{g_i=0}^{G}\left[\int_0^{(g_i\overline{x}/G)}\{u'(x)-p+\Omega(g_i;\bar{\mu}_{g_{-i}}^G)\}dx\right]\bar{\mu}_g^G}_{\text{expected consumer surplus}}+\underbrace{\sum_{g=0}^{G}\left[\int_0^{(g\overline{x}/G)}(p-c)dx\right]\bar{\mu}_{g_i}^G}_{\text{expected profits}}\right\}$$

By the individual rationality condition, the non-negative expected consumer surplus is necessary for any Nash equilibria,

$$\Rightarrow \left\{ \underbrace{\sum_{g_i=0}^{G} \left[\int_0^{(g_i \overline{x}/G)} \{u'(x) - p + \Omega(g_i; \bar{\mu}^G_{g_{-i}})\} dx \right] \bar{\mu}^G_{g_i}}_{\text{expected consumer surplus}} \right\} \geq 0$$

Hence, the expected social welfare is at least as high as the expected profits,

$$\Rightarrow \underbrace{n \sum_{g_i=0}^{G} \left[\int_0^{(g_i \overline{x}/G)} \{u'(x) - c\} dx \right] \bar{\mu}^G_{g_i} + kv}_{\text{expected social welfare}} \geq \underbrace{n \sum_{g_i=0}^{G} \left[\int_0^{(g_i \overline{x}/G)} (p - c) dx \right] \bar{\mu}^G_{g_i}}_{\text{expected profits}}$$

Note that the following is uniquely maximized at $\hat{x}$ where $u'(\hat{x}) = c$, due to the concavity assumption on $u(x)$. $u'(x) < c$ when $x > \hat{x}$ and therefore $u'(x) > c$ when $x < \hat{x}$ When $x > \hat{x}$, $u'(x) < c$ and

$$\Rightarrow \int_0^{(g_i \overline{x}/G)} \{u'(x) - c\} dx$$

Thus, the welfare maximizing equilibrium mixed strategy where

$$\bar{\mu}^G_{g_i} = \begin{cases} 1 & \text{if } \; u'(g_i \overline{x}/G) = c \\ 0 & \text{otherwise} \end{cases} \tag{A38}$$

is the unique mixed strategy where social the maximum social welfare is accomplished, assuming p is determined according to x (the first order condition above). If there does not exist g_i such that $u'(g_i \overline{x}/G) = c$, then the condition can be easily replaced with an alternative condition where $|u'(g_i \overline{x}/G) - c|$ is minimized with given G.

Thus, the two result (the individual rationality condition and the result that the unique welfare maximizing mixed strategy is the pure strategy at x such that $u'(x) = c$), the expected profits can be also maximized under the pure strategy Nash equilibrium defined above. The expected profits under any of other mixed strategy Nash equilibria has lower profits. ■

B.11 Proof of Theorem 3

Proof. This result can be obtained by directly comparing the closed form solution between the two regime. So, I characterize the optimal profits of winner-pay auctions. As this is a special case of a winner-pay contest discussed above, it is equivalent to solving the same problem with $x_{lose} = 0$. Then we automatically get $\int_0^{x_{lose}} \{u'(x) - p\} dx = 0$. Then the consumer surplus constraint is now restated as follows:

$$\Rightarrow \{u(x) + v - u'(x)x\}\frac{RA(k,n)}{(k + RA(k,n))} = v + u(x) - u'(x)x \tag{A39}$$

To equate the left and right hand side of Equation (A39), $\frac{RA(k,n)}{(k+rA(k,n))} = 1$ needs to be satisfied. This can be accomplished by taking $r \to \infty$, which corresponds to a winner-pay auction. This result coincides well with the implication of the implicit solution of R provided in the winner-pay contest case above.

Substituting this into the profit function, then

$$\Rightarrow \Pi = k\{u'(x) - c\}x + k\{u(x) + v - u'(x)x\}$$

$$\Rightarrow \Pi = k\{u(x) + v - cx\}$$

$$\Rightarrow \frac{\partial \Pi}{\partial x} = u'(x) - c = 0$$

Thus, the equilibrium quantity x^*_{win} and price p^* is determined as follows (note that both are independent on k):

$$\Rightarrow u'(x^*_{win}) = c$$

$$\Rightarrow p^* = \frac{[u(x^*_{win}) + v]}{x^*_{win}}$$

Substituting to the profit function again,

$$\Rightarrow \Pi_{winner} = kv + k\{u(u'^{-1}(c)) - cu'^{-1}(c)\}$$

$$\Rightarrow \Pi_{Tullock} = kv + n[u(u'^{-1}(c)) - cu'^{-1}(c)]$$

Since $n > k$, by the assumption,

$$\Rightarrow \Pi_{Tullock} > \Pi_{winner}$$

for arbitrary k and n. ■

Remark 1

The first order condition with respect to x_{win} under the symmetric equilibrium gives,

$$\Rightarrow p = u'(x_{win})\frac{k}{(k + RA(k,n))} + [u(x_{win}) + v]\frac{RA(k,n)}{x_{win}(k + RA(k,n))}$$

where

$$\Rightarrow A(k,n) \equiv \left[\frac{k(n-1)}{n} - \sum_{j=1}^{k-1}\frac{(k-j)}{(n-j)}\right]$$

Then the profit function gained from the winning consumers side becomes

$$k(p-c)x_{win} = k\left[u'(x_{win})\frac{k}{(k+RA(k,n))} + [u(x_{win})+v]\frac{RA(k,n)}{x_{win}(k+RA(k,n))} - c\right]x_{win}$$

Rearranging the profit function leads to

$$\Pi = k\{u'(x_{win}) - c\}x_{win} + k\{u(x_{win}) + v - u'(x_{win})x_{win}\}\frac{RA(k,n)}{(k+RA(k,n))}$$

the conditional consumer surplus when winning the auction when not considering the loser is given as $v + u(x) - u'(x)x$ (Cf. the consumer surplus in a Tullock contest is $\frac{k}{n}v + u(x) - u'(x)x$) in a symmetric equilibrium. Hence, the second constraint requires the following equality to hold.

$$\{u(x_{win})+v-u'(x_{win})x_{win}\}\frac{RA(k,n)}{(k+RA(k,n))} = v+u(x_{win})-u'(x_{win})x_{win} - \int_0^{x_{lose}}\{u'(x)-p\}dx$$

An implicit solution for R is expressed as follows:

$$R = \frac{k\left[v + u(x_{win}) - u'(x_{win})x_{win} - \int_0^{x_{lose}}\{u'(x)-p\}dx\right]}{A(k,n)\int_0^{x_{lose}}\{u'(x)-p\}dx}$$

It is observed that as x_{loser} goes to zero, the denominator of the equilibrium R goes to ∞ which results in a winner-pay auction, a special case of winner-pay contests. Then optimal R and p are simultaneously determined with satisfying another constraint $u(x_{lose}) \leq p$. Therefore, unlike the optimal Tullock contest discussed above, the winners can enjoy positive expected consumer surpluses if the seller finds it more profitable to set the regular good price sufficiently low to accommodate even losing contenders (incentive compatibility). Otherwise, the optimal contest format is a winner-pay 'auction' where every winners are fully extracted without allowing losing consumers to purchase any regular goods (by setting sufficiently high regular good price).

B.12 Proof of Proposition 5

Proof.

Suppose the sufficient condition for the winner-pay auction to be preferred to the product purchasing contest is satisfied as follows:

$$kv + k\int_0^{x^*_{PP}}[u'(x)-c]dx > kv + n\int_0^{x^*_{PP}}[u'(x)-c]dx - \lambda(n,v)$$

$$\Rightarrow \lambda(n,v) > (n-k)\int_0^{x^*_{PP}} [u'(x)-c]dx \tag{A40}$$

as λ function is large, that is, the policy maker cares sufficiently about the welfare distribution.

The equivalent condition for the winner-pay auction is preferred to counterfactual benchmark is,

$$kv + k\int_0^{x^*_{PP}} [u'(x)-c]dx > kv + n\int_0^{x^*_{PO}} [u'(x^*_{PO})-c]dx - \lambda(n,v)$$

$$\Rightarrow \lambda(n,v) > n\int_0^{x^*_{PO}} [u'(x^*_{PO})-c]dx - k\int_0^{x^*_{PP}} [u'(x)-c]dx \tag{A41}$$

$$\Rightarrow \lambda(n,v) > (n-k)\int_0^{x^*_{PP}} [u'(x)-c]dx - \underbrace{n\left(\int_0^{x^*_{PP}} [u'(x)-c]dx - \int_0^{x^*_{PO}} [u'(x^*_{PO})-c]dx\right)}_{\text{Increased welfare in the regular good market}} \tag{A42}$$

A key observation linking (A41) and (A40) is that both expressions include the same λ. This is due to the assumption of homogeneous consumers, where the difference in ex-post welfare in a symmetric pure strategy equilibrium is the value of the premium good, v. Since the increased welfare in the regular good market in (A42) is positive, the right-hand side of (A42) is smaller than that of (A40). Therefore, (A40) serves as a sufficient condition for (A42). Consequently, if the winner-pay auction under the product purchasing contest is preferred, it will automatically be preferred over Counterfactual Scenario 1.

■

B.13 Proof of Proposition 6

Proof.

With a strictly convex cost function $c_p(k)$ for producing k units of premium goods, we consider two cases: (1) $v \leq c_p(1)$ and (2) $v > c_p(n)$.

(1) $v \leq c_p(1)$**:**

In this scenario, it is clearly optimal for the producer to refrain from supplying any premium goods under a monopoly. Under a product purchasing contest, however, we need to account for the extra profits gained from consumer competition, even though the producing the premium good itself results in negative profits.

$$\Rightarrow \Pi^*_{Tullock} = (v - c_p(1)) + n[u(u'^{-1}(c_r)) - c_r u'^{-1}(c_r)]$$

$$\Rightarrow \Pi^*_{Monopoly} = n[u'(y^*) - c_r]y^*$$

$$\Rightarrow \Pi^*_{Tullock} - \Pi^*_{Monopoly} = (v - c_p(1)) + n[u(u'^{-1}(c_r)) - c_r u'^{-1}(c_r) - [u'(y^*) - c_r]y^*]$$

I define the difference in profits gained from regular good under efficient and monopoly outcome as follows:

$$\Rightarrow A(c_r) = [u(u'^{-1}(c_r)) - c_r u'^{-1}(c_r) - [u'(y^*) - c_r]y^*]$$

Thus,

$$\Rightarrow \Pi^*_{Tullock} - \Pi^*_{Monopoly} = v + nA(c_r) - F$$

$$\Rightarrow \Pi^*_{Tullock} - \Pi^*_{Monopoly} > 0 \text{ if } nA(c_r) > (c_p(1) - v)$$

Therefore, if either n or $A(c_r)$ is sufficiently large, the producer has an incentive to implement the contest despite the negative marginal profit from additional unit of premium good.

Thus, considering the impact of the marginal cost in the regular good, as the marginal cost of producing regular goods increases, the contest becomes less profitable than the monopoly.

(2) $v > [c_p(n) - c_p(n-1)]$

In this case, it is clearly optimal for the producer to fully supply all premium goods under a monopoly (without a product purchasing contest). However, under a product purchasing contest, we need to consider whether the additional profits from the contest might exceed the marginal profits from producing more premium goods.

$$\Rightarrow \Pi^*_{Tullock} = (n-1)v - c_p(n-1) + n[u(u'^{-1}(c_r)) - c_r u'^{-1}(c_r)]$$

$$\Rightarrow \Pi^*_{Monopoly} = vn - c_p(n) + n[u'(y^*) - c_r]y^*$$

$$\Rightarrow \Pi^*_{Tullock} - \Pi^*_{Monopoly} = (c_p(n) - c_p(n-1) - v) + n[u(u'^{-1}(c_r)) - cu'^{-1}(c_r) - [u'(y^*) - c_r]y^*]$$

Again,

$$\Rightarrow \Pi^*_{Tullock} - \Pi^*_{Monopoly} = (c_p(n) - c_p(n-1) - v) + nA(c_r) > 0 \tag{A43}$$

The inequality in (A43) holds if $A(c_r)$ is sufficiently large.

Although the cost function is strictly convex, if $v > C'(n)$, then it is evidently more profitable to supply and sell all n units of premium good directly, rather than restricting the supply and implementing a contest.

Now, suppose we have an interior solution. For simplicity, I allow k to take any real number. The optimality conditions are given as the standard first and the second order conditions, which are satisfied by the assumptions.[14]

The first order condition is:

$$\Rightarrow \frac{\partial \Pi^*_{Tullock}}{\partial k} = v - C'(k) = 0$$

The second order condition is:

$$\Rightarrow \frac{\partial^2 \Pi^*_{Tullock}}{\partial k^2} = -C''(k) < 0$$

In this case, the profit-maximizing strategy is to implement a product purchasing contest with k premium goods, rather than selling as a monopoly. This strategy is equivalent to the basic model in Section 3. Consequently, the product purchasing contest is superior to the plain contest with a monopoly, Counterfactual Scenario 1.

■

Let's define the difference in profits gained from regular good under efficient outcome and monopoly outcome as follows:

$$\Rightarrow \Pi^*_{Tullock} - \Pi^*_{Monopoly} = (v - c_p) + n[u(u'^{-1}(c)) - cu'^{-1}(c) - [u'(y^*) - c]y^*]$$

$$\Rightarrow A(c) = [u(u'^{-1}(c)) - cu'^{-1}(c) - [u'(y^*) - c]y^*]$$

Let $A(c_r)$ the difference between the per capita profits gained from the first-best outcome case and the monopoly case, which is a function of c_r, the marginal cost of the regular good. $A(c_r)$ is a crucial factor to determine if a product purchasing contest is profitable or not, and therefore, the marginal cost of regular good plays a key role in determining whether the contest is profitable.

The increasing profits gained from the regular good by inducing a contest is decreasing in the marginal cost. Hence, the higher marginal cost for regular good makes a product purchasing contest less likely to be optimal. This is comparable to the well-known result that higher marginal cost makes bundling less likely to be more profitable than separate sales.

Remark 2 *Comparative statics suggests $A'(c_r) < 0$, which implies the difference between the per capita profits gained from the first-best outcome case and the monopoly case is decreasing in the marginal cost of regular good. Hence, the higher marginal cost for regular good makes a product*

[14] If we take the discrete number of k, this condition is equivalent to there being some $k < n$ such that $v > C'(k)$ and $v < C'(k+1)$.

purchasing contest less likely to be more profitable than the counterfactual benchmark without product purchasing contest.

Proof.

$$A(c) = [u(u'^{-1}(c_r)) - c_r u'^{-1}(c_r) - [u'(y^*) - c_r]y^*]$$

By applying the derivative of the inverse function,

$$\Rightarrow A'(c_r) = \frac{u'(u'^{-1}(c_r))}{u''(c_r)} - u'^{-1}(c_r) - c\frac{1}{u''(c_r)} + y^*$$

We get the result above by applying the envelope theorem,

$$\Rightarrow \frac{d[u'(y^*) - c_r]y^*}{dc_r} = \frac{\partial[u'(y^*) - c_r]y^*}{\partial c_r} = -y^*$$

$$\Rightarrow A'(c_r) = \frac{c}{u''(c_r)} - u'^{-1}(c_r) - \frac{c_r}{u''(c_r)} + y^*$$

$$\Rightarrow A'(c_r) = -u'^{-1}(c_r) + y^*$$

Because $u'^{-1}(c_r)$ is the quantity where $u'(x) = c_r$, which is necessarily larger than y^* which is the optimal quantity under the monopoly under the decreasing marginal utility $(u'^{-1}(c_r) > y^*)$. Hence,

$$\Rightarrow A'(c_r) = -u'^{-1}(c_r) + y^* < 0$$

■

B.14 Proof of Proposition 7

Proof. Firstly, I solve the optimal investment decision for the regular good in the optimal Tullock contest under a product purchasing contest framework. The optimal profit of the contest is given as shown in Expression (A26), with adjustments to incorporate $c(\theta)$ and $F(\theta)$.

$$\Rightarrow \Pi^* = kv + n[u(x(\theta);\theta) - c(\theta)x(\theta)] - F(\theta) \tag{A44}$$

Differentiating (A44) with respect to θ yields the first order condition (A45).

$$\frac{d\Pi^*}{d\theta} = n\left[u_x x'(\theta) + u_\theta - c'(\theta)x(\theta) - c(\theta)x'(\theta)\right] - F'(\theta) = 0$$

$$\Rightarrow n\left[\{u_x - c(\theta)\}x'(\theta) + u_\theta - c'(\theta)x(\theta)\right] = F'(\theta) \tag{A45}$$

Equation (A21, the first order condition) implies that the optimal θ, denoted by θ^*, is implicitly determined as follows:

$$\Rightarrow u_x(x^*(\theta^*); \theta^*) = c(\theta^*) \tag{A46}$$

Substituting (A46) into the first order condition (A45) leads to the expression (A47).

$$\Rightarrow n\left[u_\theta - c'(\theta)x^*(\theta)\right] = F'(\theta)$$

$$\Rightarrow u_\theta = c'(\theta)x^*(\theta) + \frac{1}{n}F'(\theta) \tag{A47}$$

Next, consider the counterfactual plain contest with a monopoly on the regular good. The profit function with θ is given as (A48):

$$\Pi^* = kv + n[u_x(\tilde{x}(\theta); \theta) - c(\theta)]\tilde{x} - F(\theta) \tag{A48}$$

The first order condition implies that the optimal θ, denoted by $\tilde{\theta}^*$, is implicitly determined by:

$$\Rightarrow n[u_{xx}(\tilde{x}^*(\tilde{\theta}^*); \tilde{\theta}^*)x'(\theta) + u_{x\theta} - c'(\tilde{\theta}^*)]\tilde{x}^* + n[u_x(\tilde{x}(\theta); \theta) - c(\theta)]x'(\theta) = F'(\theta)$$

$$\Rightarrow n[u_{xx}x'(\theta) + u_{x\theta} - c'(\tilde{\theta}^*)]\tilde{x}^* + n[u_x(\tilde{x}(\theta); \theta) - c(\theta)]x'(\theta) = F'(\theta)$$

$$\Rightarrow n[u_{xx}x(\theta) + u_x - c(\theta)]x'(\theta) + n[u_{x\theta} - c'(\theta)]x(\theta) = F'(\theta) \tag{A49}$$

The first order condition for the profit-maximizing quantity of the regular good x (after θ is determined in the first stage) under the plain contest with a monopoly is given as (A50):

$$u_{xx}x(\theta) + (u_x - c(\theta)) = 0 \tag{A50}$$

Substituting (A50) into the first order condition (A49) above results in (A51).

$$\Rightarrow u_{x\theta}x(\theta) = c'(\theta)x(\theta) + \frac{1}{n}F'(\theta) \tag{A51}$$

To compare θ^* to $\tilde{\theta}$, note that the right hand side of both Equation (A47) and (A51) have the same functional form in θ. Expressions (A47) and (A51) can thus be rewritten as follows (note that $u_{x\theta}(\tilde{x}; \theta)$ is constant):

$$u_{x\theta}(\tilde{x}; \theta)\tilde{x}(\theta) = \int_0^{\tilde{x}} u_{x\theta}(\tilde{x}; \theta)dx \tag{A52}$$

$$u_\theta(x^(\theta); \theta) = \int_0^{x^*} u_{x\theta}(x, \theta)dx \tag{A53}$$

Considering the following (A54) holds for any $x \in [0, \tilde{x}]$, it results in (A55). Specifically, since $x(\theta) < \tilde{x}(\theta)$ for any $x \in [0, \tilde{x}]$ and the marginal utility decreases in x, so the inside of the integral in (A53) is greater pointwisely than that of (A52).

$$u_{x\theta}(x(\theta), \theta) > u_{x\theta}(\tilde{x}(\theta), \theta) \tag{A54}$$

$$\Rightarrow u_{\theta}(x^*; \theta) > u_{x\theta}(\tilde{x}; \theta)\tilde{x}(\theta) \tag{A55}$$

Thus, under the assumption that the right hand side of both (A47) and (A51) is strictly increasing in θ, the following relationship (A56) between θ^* (the optimal θ in the optimal Tullock contest under product purchasing contest) and $\tilde{\theta}$ (the optimal θ in the plain contest with a monopoly) is established.

$$\theta^* > \tilde{\theta} \tag{A56}$$

This concludes the proof. ■

C A Brief Sketch on the Numerical Analysis to Solve for Mixed Strategy Nash Equilibria under All-Pay Auction Mechanism

To conduct a numerical comparison of the all-pay auction mechanism against other mechanisms, accurate calculation of expected profits is necessary. This entails characterizing consumers' mixed strategy for the product purchasing contest with given specific assumptions on the number of rewards, the number of consumers, their preferences, as well as the marginal cost and the price for each regular good. The mixed strategy is represented by a cumulative distribution function (cdf) $F(x)$ defined over potential choice values of x. Utilizing this cdf, I derive a probability density function (pdf) for each price through numerical differentiation. Then, by numerically integrating $xf(x)$ over the x values with positive pdf, we can obtain the individual expected expenditure for each price. Subsequently, the optimal price for the regular good, from the seller's perspective, can be determined at the price level that maximizes profits over all feasible prices. Thus, our initial challenge lies in numerically solving the following equation.

$$u(x) - px + v\left[\sum_{j=0}^{k-1}\binom{n-1}{j}\{F(x)^{n-j-1}\}\{1-F(x)\}^j\right] = 0 \tag{A1}$$

Firstly, I determine $F(x)$ for each price level. This requires specifying the parametric functional form of the utility function. To explore implications across a range of reasonable functions, I consider three types: quadratic, logarithmic, and square root functions. Given the concavity assumption of the utility function and the equilibrium condition $p > u'(x)$, a positive solution for $0 \leq F(x) \leq 1$ is guaranteed. The bisection method is employed in numerical operations to solve for $F(x)$. By doing so, I acquire numerical data representing the cdf for each price. The process for obtaining the pdf for each price from this cdf information is outlined below (Heath, 2018).

Suppose we have n data points for the cdf denoted as $\{x_i, F(x_i)\}$ for $i = 1, 2, 3, ..., n$. For $i = 2, 3, ..., (n-1)$, the pdf for each point is computed utilizing the centered difference formula for the first derivative.

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}} \tag{A2}$$

For the first term $(i = 1)$, the pdf is calculated using the forward difference formula for the first derivative.

$$f'(x_1) \approx \frac{f(x_2) - f(x_1)}{x_2 - x_1} \tag{A3}$$

Conversely, for the last term $(i = n)$, the pdf is determined using the backward difference formula for the first derivative.

$$f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \tag{A4}$$

Now that we have the pdf for each x, we proceed to calculate the expected expenditure by numerically integrating $xf(x)$. This process does not use any information about the parametric functional form of the cdfs.

Nevertheless, it is challenging to conduct numerical integration. The cdf of the mixed strategies often fluctuates too rapidly in both tails, making it difficult to accurately capture their behavior, no matter how small the unit of x is. This often leads to non-monotonic cdfs or values significantly less than 1 when integrating cdfs. Despite placing x with a width of 0.0001 from 0 to 5, the performance was not very satisfactory, especially as the number of premium goods increases. A simple way to deal with this is to get the increment of x to be finer, which entails a substantial computational burden.

Consequently, rather than directly solving the above equation to obtain more detailed cdf information, I chose a method that involves obtaining broad information about the cdf and then interpolating it for direct integration of the function. I utilized Hermite Cubic Interpolation, ensuring a monotonically increasing functional form. Although cubic splines are a better option in smoothness, they present a potential concern of the cdf being locally non-monotonic due to overshooting, even when the base data points are monotonically aligned (Heath, 2018).

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