Dissecting the Aggregate Market Elasticity*

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Abstract

We study the aggregate stock market elasticity in a general equilibrium model with heterogeneous investors, passive demand, and financial constraints. Using state-global perturbations, we show that general equilibrium adjustments of the risk-free rate and risk premium govern flow-driven price dynamics. With nonzero cross-price elasticity and efficient risk allocation, markets are infinitely elastic even if individual investors are inelastic. When risk is misallocated and flows shift the consumption-wealth ratio, flows raise prices, with larger effects when active wealth share is low. Heterogeneity can raise elasticity and leverage constraints amplify price impact. A quantitative implementation confirms these mechanisms and explains excess market volatility.

Keywords: Aggregate market elasticity, risk misallocation, demand shocks, demand shifters, excess volatility, asset pricing.

JEL Classification: G12, G11, G21.

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1 Introduction

Why is the stock market so volatile? This question has long been central to asset pricing research. Classic tests of excess volatility attribute the puzzle to unobserved "dark matter" factors.[1] More recent evidence points to a demand-based channel: asset prices react strongly to portfolio flows. Yet, as shown in the left panel of Figure 1, these flows are small in magnitude, implying that markets must be highly *inelastic*, with even small quantity shifts generating large price changes (Gabaix and Koijen, 2023). Moreover, the right panel shows that the "volatility multiplier"—the ratio of return volatility to flows—is time-varying and countercyclical, spiking in periods of financial stress.

These facts raise three questions. Why is the aggregate stock market so inelastic? Can standard frictionless models reproduce such low elasticities? If not, which frictions are essential to explain the observed price impact of flows and the behavior of the volatility multiplier?

This paper develops a general equilibrium model with investor heterogeneity and portfolio frictions to address these questions. The general equilibrium (GE) perspective is essential: the key object of interest is the *aggregate (macro) elasticity*—the change in the total stock market value in response to a $1 flow between bonds and stocks.[2] Unlike micro elasticities, which capture substitution across individual stocks, aggregate elasticity reflects market-wide reallocations between risky and risk-free assets. Its determination therefore requires considering how both the risk-free rate and the risk premium adjust simultaneously to portfolio flows. Understanding these GE adjustments is crucial for characterizing market elasticity and, ultimately, for explaining excess volatility.

Allowing the interest rate to endogenously adjust to portfolio-flow shocks is essential for determining aggregate market elasticity.[3] If investors' cross-price elasticity is zero—meaning that demand for the risky asset is independent of the interest rate—then aggregate elasticity reduces to a weighted average of individual elasticities, as in Gabaix and Koijen (2023). In this case, the

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[1] See LeRoy and Porter (1981), Shiller (1981, 1992), and Cochrane (1992); and Chen, Dou, and Kogan (2024) for a recent discussion.

[2] We use "aggregate elasticity" and "macro elasticity" interchangeably.

[3] A portfolio-flow shock corresponds to an asymmetric shock to risky-asset demand, affecting only a subset of investors and leading to portfolio reallocation in equilibrium.

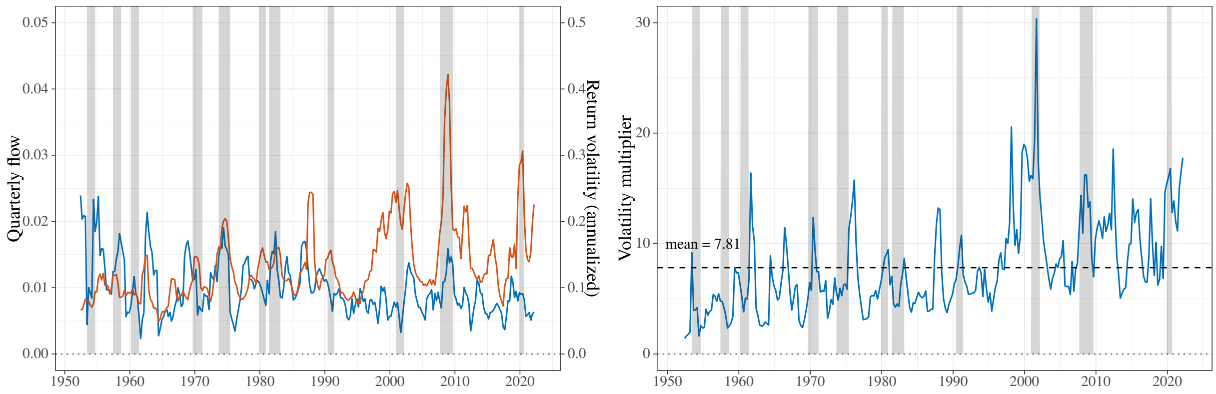

Figure 1. Quarterly flows and return volatility. The left panel plots quarterly flows (in blue) and return volatility (in red). The right panel plots the "volatility multiplier," defined as the ratio of return volatility to flows. Source: Flow of Funds and CRSP.

market remains inelastic whenever investor demand is relatively unresponsive to price changes. Importantly, the behavioral element in Gabaix and Koijen's GE model implies a constant interest rate. By contrast, when cross-price elasticity is nonzero and interest rates are allowed to move, the market can become infinitely elastic, regardless of how price-inelastic individual investors are. In particular, we show that if passive investors initially hold the efficient share of the risky asset, the decline in the interest rate fully offsets the increase in the risk premium, leaving the price of the risky asset unchanged after a portfolio-flow shock.[4]

This mechanism shows that individual investors being price-inelastic is not sufficient to generate inelastic markets. Instead, aggregate elasticity depends on the initial allocation of risk. A portfolio-flow shock reallocates risk between passive and active investors. If risk is initially allocated efficiently, small portfolio adjustments do not affect aggregate savings behavior, which ultimately determines the price-dividend ratio. In this case, the decline in the interest rate fully offsets the rise in the risk premium, leaving asset prices unchanged. By contrast, when risk is misallocated, portfolio reallocation alters aggregate savings behavior, preventing the interest rate from fully offsetting changes in the risk premium and generating finite elasticity.

We establish this result first in a two-period model and then extend the intuition to a dynamic heterogeneous agent framework with multiple investor types, capturing reallocations across

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[4] Johnson (2006) studies equilibrium price changes in response to shifts in risky-asset supply. While his definition of liquidity differs from ours, it also allows for interest rate adjustments when stock prices change.

households, intermediaries (e.g., banks, broker-dealer, hedge funds), and long-term investors (e.g., insurance companies and pension funds). We incorporate two key frictions: passive investment, consistent with household inertia and target-date fund allocations, and leverage constraints on active investors, consistent with evidence on margin requirements and intermediary behavior.[5] These frictions shape aggregate elasticity and, together with flow shocks, determine market volatility. Inelasticity alone does not generate excess volatility, nor do flows in infinitely elastic markets. Both are required.

Studying aggregate elasticity in models with frictions is challenging because closed-form solutions are rarely available. To overcome this, we develop a tractable analytical approach that delivers closed-form characterizations of elasticity using perturbations that are global in the state variables. This method allows us to show that aggregate elasticity is state-dependent and time-varying, governed by both the distribution of wealth between active and passive investors and how risk is allocated within the active sector. We find that passive investors amplify price impacts, heterogeneity increases elasticity via risk misallocation, and binding leverage constraints further amplify price impact of flows.

We then decompose aggregate elasticity into components driven by the effect of passive flows on the risk premium, the risk-free rate, and the drift of the risky asset. This decomposition highlights the general equilibrium channel through which interest rate and risk premium responses move elasticity in opposite directions. Together, these forces determine how passive flows translate into asset-price dynamics and market volatility.

Lastly, we calibrate the model and assess its implications for aggregate elasticity and market volatility. This requires solving and estimating a high-dimensional dynamic asset pricing model. A key contribution of our work is showing how to computationally manage this complexity. In particular, while the perturbation approach delivers tractable analytical insights, we solve the full dynamic model numerically to evaluate the quantitative importance of different frictions.

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[5] See, e.g., Brunnermeier and Pedersen (2009), Garleanu and Pedersen (2011), Chabakauri (2013), and Adrian and Shin (2014).

Related literature

Our work relates to the literature on macro demand elasticity that studies the impact of asset flows on the aggregate stock market price. The papers closest to ours are Johnson (2006) and Gabaix and Koijen (2023). In a representative agent model, Johnson (2006) perturbs the risky asset supply and finds finite elasticity even in the frictionless Lucas economy. Gabaix and Koijen (2023) introduce a behavioral element that fixes interest rates and generates large price impacts in a setting with representative households investing in constrained funds. We build on this literature by developing a general equilibrium model with heterogeneous investors and financial frictions, showing that both GE effects and constraints are central for obtaining inelastic markets.[6]

The macro elasticity literature contrasts with the larger body of work on micro elasticity, which examines relative price changes across individual stocks (e.g., Shleifer, 1986; Harris and Gurel, 1986; Chang, Hong, and Liskovich, 2015; Pavlova and Sikorskaya, 2023; Schmickler, 2020). Evidence suggests that micro elasticities are much larger than aggregate elasticities, consistent with the fact that different stocks are closer substitutes for each other than are equities and bonds.

A further strand of work is demand-system asset pricing pioneered by Koijen and Yogo (2019). Their framework estimates asset-level demand slopes from flows and prices. We extend this approach by incorporating cross-elasticities across asset classes, following Fuchs, Fukuda, and Neuhann (2023) and Haddad, He, Huebner, Kondor, and Loualiche (2025). Even small substitution between equities and bonds can produce large effects on macro elasticities, since portfolio rebalancing amplifies the aggregate price response to flows. This highlights that aggregate inelasticity depends not only on own-asset demand but also on cross-asset substitution.

In addition, our framework extends the analysis to dynamic settings. Binsbergen, David, and Opp (2025) show that static IO-style estimators can be biased when shocks are persistent, while He, Kondor, and Li (2025) emphasize that supply shocks affect both expected returns and the endogenous risk structure. Building on these insights, we show that persistence and the evolution

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[6] For more on macro elasticity, see Johnson (2008, 2009), Deuskar and Johnson (2011), Li, Pearson, and Zhang (2020), Hartzmark and Solomon (2025), among others.

of premia are crucial for aggregate elasticity, shaping both short-run substitution and long-run adjustments in volatility and covariance.

We also connect to the literature on intermittent rebalancing, most notably Chien, Cole, and Lustig (2012). Their model shows how infrequent portfolio adjustments slow demand responses and create persistent deviations from frictionless benchmarks. In our setting, passive investors play a similar role: incomplete rebalancing prolongs adjustment to shocks, reinforcing the persistence of price impacts. This mechanism complements the dynamic considerations above by showing that sluggish rebalancing is itself an important source of aggregate inelasticity.

Finally, our paper relates to preferred-habitat models such as Vayanos and Vila (2021), along with recent contributions by Kekre, Lenel, and Mainardi (2025) and Ray, Droste, and Gorodnichenko (2024). These studies emphasize intermediary frictions and market segmentation as sources of inelasticity. By contrast, our framework highlights wealth effects: shifts in aggregate wealth directly alter portfolio demand and thereby macro elasticities. This mechanism, absent in the preferred-habitat literature, shows that even without intermediary frictions, wealth effects alone can generate powerful amplification of asset price movements.

2 A Simple Model of the Aggregate Market Elasticity

In this section, we analyze the determination of the aggregate market elasticity in a simple general equilibrium, demand-based asset pricing model. To keep the discussion transparent, we specify investor demands directly, rather than deriving them from utility functions. A micro-founded version of these demands, embedded in a dynamic heterogeneous-agents model, will be presented in Section 3.

Environment. We consider a two-period economy with two assets: a risky asset and a riskless bond. There are two types of investors: passive (p) and active (a). A fraction ω_j of the population is of type $j \in \{a, p\}$. The risky asset pays a random dividend Y' in the second period. Its price is denoted by P, while the price of the riskless asset is R_f^{-1}.

The budget constraints of investor j are

$$PQ_j + R_f^{-1} B_j + C_j = W_j, \qquad C'_j = Y' Q_j + B_j,$$

where Q_j is the investor's holdings of the risky asset, B_j denotes holdings of the riskless bond, C_j is first-period consumption, and C'_j is second-period consumption. Initial wealth is given by $W_j = (P+Y)Q_{j,-1} > 0$.

We define investor j's portfolio share in the risky asset as

$$\alpha_j \equiv \frac{PQ_j}{W_j - C_j}.$$

Passive investors maintain a fixed risky asset share $\alpha_p = \overline{\alpha}_p \geq 0$. Active investors' portfolio share, in contrast, depends on the risk premium

$$\alpha_a = \overline{\alpha}_a(\pi),$$

for some increasing function $\overline{\alpha}_a(\cdot)$, where $\pi \equiv \log \frac{1}{R_f} \mathbb{E}\left[\frac{Y'}{P}\right]$ denotes the risk premium.

Consumption is specified as

$$C_j = \overline{c}_j(r, \pi) W_j,$$

where $r \equiv \log R_f$ is the log risk-free rate. The consumption-wealth ratio $\overline{c}_j(\cdot)$ depends on both r and π. If $\overline{c}_j$ decreases with r, investors save more when the interest rate is high (a substitution effect); if $\overline{c}_j$ increases with r, investors save less (an income effect). Similar logic applies to the risk premium. We assume that $\overline{c}_j(\cdot)$ does not depend on the passive portfolio share $\overline{\alpha}_p$.

The market clearing conditions are

$$\sum_{j\in\{p,a\}} \omega_j C_j = Y, \qquad \sum_{j\in\{p,a\}} \omega_j Q_j = 1, \qquad \sum_{j\in\{p,a\}} \omega_j B_j = 0,$$

with initial risky asset endowment $\sum_{j\in\{p,a\}} \omega_j Q_{j,-1} = 1$.

Market for risky assets. Let $p = \log \frac{P}{Y}$ denote the log price-dividend ratio, and let $\mu \equiv \log \frac{\mathbb{E}[Y']}{Y}$ denote the log dividend growth. The risk premium is

$$\pi = \mu - p - r. \tag{1}$$

Investor j's demand for the risky asset is

$$Q_j = \alpha_j \left[1 - \overline{c}_j(r, \pi)\right] \frac{W_j}{P}.$$

Using Equation (1) to eliminate π, we can express demand as a function of price of the risky asset, the risk-free rate, and, for passive investors, the exogenous portfolio share: $Q_j = \overline{Q}_j(p, r, \overline{\alpha}_p)$.

The market clearing condition for the risky asset is therefore

$$\underbrace{\omega_a \overline{Q}_a(p, r, \overline{\alpha}_p)}_{\text{active demand}} = \underbrace{1 - \omega_p \overline{Q}_p(p, r, \overline{\alpha}_p)}_{\text{net supply}}. \tag{2}$$

Equilibrium in the risky asset market requires that active investors absorb the net supply, defined as total supply minus the amount held by passive investors. Since active demand does not depend on $\overline{\alpha}_p$, changes in the passive portfolio share affect only the net supply curve.

Market for goods. The response of the consumption-wealth ratio to the interest rate and the risk premium is central to our analysis. In the dynamic model of Section 3, the average consumption-wealth ratio depends only on the sum $(r + \pi)$, so its sensitivity to interest rates equals its sensitivity to risk premia. Assumption 1 imposes the same structure in this two-period setting, ensuring consistency with standard micro-founded models.

Assumption 1. *The average consumption-wealth ratio is a function of the expected return on the risky asset,* $(r + \pi)$*,*

$$\sum_{j \in \{p, a\}} x_j \overline{c}_j(r, \pi) = \overline{c}(r + \pi),$$

for some function $\overline{c}(\cdot)$*, where* $x_j \equiv \frac{\omega_j W_j}{\omega_a W_a + \omega_p W_p}$ *denotes investor* j*'s wealth share.*

Intuitively, since bonds are in zero net supply, the return on investors' overall portfolios equals the return on the risky asset. From goods market clearing and Equation (1), we obtain

$$\overline{c}(\mu - p) = \frac{1}{1 + e^p}, \tag{3}$$

using the fact that $r + \pi = \mu - p$ and $\sum_j x_j \overline{c}_j = \frac{Y}{Y+P}$.

Assumption 1 implies that the system of equations determining p and r has a useful *recursive property*: condition (3) depends only on p, while condition (2) depends on both p and r.

2.1 General equilibrium implications of portfolio flows

We now examine how the price-dividend ratio p and the riskless return r respond to a portfolio flow from the riskless bond into the risky asset. Let $\overline{\alpha}_p^\star$ denote the passive portfolio share in the initial equilibrium, with $(p^\star, r^\star)$ the corresponding equilibrium prices.

For a small perturbation, $\overline{\alpha}_p = \overline{\alpha}_p^\star e^{\hat{\alpha}_p}$, we linearize demand around the initial equilibrium. Defining $\hat{p} \equiv p - p^\star$, $\hat{r} \equiv r - r^\star$, and $q_j \equiv \log(Q_j / Q_j^\star)$, we obtain

$$q_j \;=\; -\zeta_{j,p}\,\hat{p} \;-\; \zeta_{j,r}\,\hat{r} \;+\; f_j, \tag{4}$$

where the own-price and cross-price elasticities are given by $\zeta_{j,p} \equiv -\frac{\partial \log \overline{Q}_j}{\partial p}$ and $\zeta_{j,r} \equiv -\frac{\partial \log \overline{Q}_j}{\partial r}$. The *flow shock* is given by

$$f_j \;\equiv\; \frac{\partial \log \overline{Q}_j}{\partial \log \overline{\alpha}_p} \times \hat{\alpha}_p,$$

capturing an exogenous reallocation from bonds to stocks. Since active demand does not depend on $\overline{\alpha}_p$, we have $f_a = 0$. Formally, a flow shock is an asymmetric demand shock affecting some investors but not others, inducing equilibrium portfolio rebalancing.

An important implication of (4) is that demand for the risky asset depends on both the price-dividend ratio p and the bond price through the risk-free rate r. Since active investors respond to

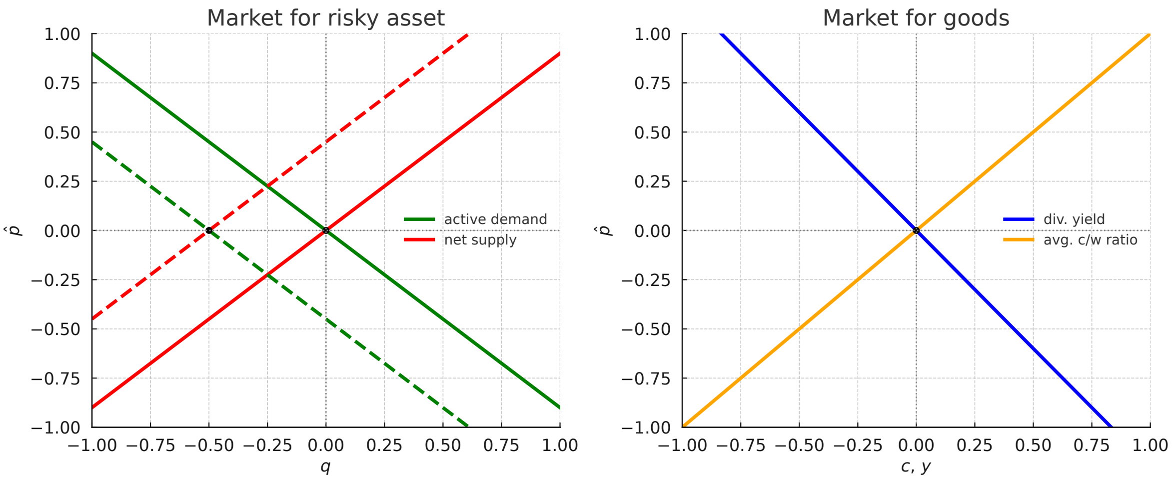

Figure 2. Equilibrium in the risky asset (left) and goods (right) markets.

the risk premium $\pi = \mu - p - r$, their demand depends on both prices. Therefore, the prices of the risky and risk-free assets must be jointly determined in equilibrium.

Equilibrium. Let $s_j \equiv \omega_j Q_j^\star$ denote the equilibrium quantity share of the risky asset held by type j. Linearizing the market-clearing condition $\sum_j \omega_j Q_j = 1$ gives

$$s_a\left(-\zeta_{a,p}\,\hat{p} - \zeta_{a,r}\,\hat{r}\right) \;=\; s_p\left(\zeta_{p,p}\,\hat{p} + \zeta_{p,r}\,\hat{r} - f_p\right), \tag{5}$$

as $s_a q_a + s_p q_p = 0$.

The left panel of Figure 2 shows the equilibrium in the risky-asset market. The downward-sloping curve represents active demand, while the upward-sloping curve is net supply, i.e., total supply minus passive holdings.[7]

Linearizing the goods-market condition $\bar{c}(\mu - p) = 1/(1 + e^p)$ around $p^\star$ yields

$$\chi_p\,\hat{p} \;=\; -\frac{e^{p^\star}}{(1 + e^{p^\star})^2}\,\hat{p}, \qquad \text{where} \qquad \chi_p \equiv -\bar{c}'(\mu - p^\star). \tag{6}$$

The right panel of Figure 2 shows the goods market. We focus on the case $\chi_p > 0$, so the average

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[7] Passive demand is $Q_p \;=\; \bar{\alpha}_p\,[1 - c_p(r, \pi)]\,\frac{W_p}{P}$, given $W_p = (P + Y)Q_{p,-1}$ and $\pi = \mu - p - r$, so its linearized contribution $-s_p q_p$ makes net supply upward sloping in p provided $\frac{\partial c_p}{\partial p}$ is not too large.

consumption-wealth ratio is increasing in p (investors save less when expected returns are low). The right-hand side in (6) is the (local) slope of the dividend-yield curve, which is downward sloping in p. By the recursive property, only p enters the goods market clearing condition.

The inelastic markets hypothesis. Consider a *partial equilibrium* version of the model in which we hold the interest rate fixed at $r = r^\star$ and ignore goods market clearing. Using the linearized risky asset market condition (5) with $\hat{r} = 0$, the price of the risky asset satisfies

$$\hat{p} = \underbrace{\frac{1}{\zeta_p}}_{\substack{\text{inverse market}\\ \text{elasticity}}} \times \underbrace{f}_{\text{flow shock}}, \tag{7}$$

where

$$\zeta_p \equiv s_a \, \zeta_{a,p} + s_p \, \zeta_{p,p}, \qquad f \equiv s_p \, f_p, \qquad s_j \equiv \omega_j Q_j^\star.$$

Equation (7) shows that the price-dividend ratio responds to flow shocks, with price impact given by the inverse of the (aggregate) market elasticity—an average of investors' own-price elasticities. This is the core implication of the *inelastic markets hypothesis* of Gabaix and Koijen (2023), who estimate large price responses to stock-market flows, suggesting relatively inelastic demand.

In the risky asset panel of Figure 2, an increase in the portfolio share of passive investors shifts the net supply curve to the left. The new equilibrium lies further up the active demand curve, implying lower expected returns and a higher price-dividend ratio. When demand is inelastic, even small inflows to stocks generate a sizable increase in $\hat{p}$.

The crucial role of cross-elasticity. The partial equilibrium analysis abstracts from cross-elasticity. In general equilibrium, however, both r and p adjust, and the aggregate market elasticity is fundamentally different, as the next proposition shows.

Proposition 1 (Infinite market elasticity)**.** *Define the aggregate cross-elasticity* $\zeta_r \equiv s_a \, \zeta_{a,r} + s_p \, \zeta_{p,r}$*. If* $\zeta_r \neq 0$*, then a portfolio-flow shock has* no *price impact:* $\hat{p} = 0$*. The shock is absorbed* *by offsetting movements in the interest rate and the risk premium:*

$$\hat{r} = \frac{1}{\zeta_r} f, \qquad \hat{\pi} = -\frac{1}{\zeta_r} f,$$

where $\hat{\pi} \equiv \pi - \pi^\star$.

Proof. From Equation (6), we obtain $\hat{p} = 0$. If $\zeta_r \neq 0$, then $\hat{r} = \frac{1}{\zeta_r} f$ from Equation (5). Since $\hat{\pi} = -\hat{p} - \hat{r}$ with $\hat{p} = 0$, it follows that $\hat{\pi} = -f/\zeta_r$. □

With any nonzero cross-elasticity $\zeta_r \neq 0$, the aggregate market becomes infinitely elastic, regardless of how inelastic individual demands may be. Even if $\zeta^q_{j,p} \approx 0$ for all j, a small but positive ζ_r is sufficient to make aggregate elasticity unbounded.

The disconnect between individual and aggregate elasticities arises from a general equilibrium effect. In the goods market (right panel of Figure 2), the price-dividend ratio is determined by consumption behavior. This mechanism is well known in the macro-finance literature, most clearly in models with unit EIS, where p is tied directly to the discount rate. Our framework generalizes this logic. To restore equilibrium in the risky asset market (left panel), the interest rate adjusts so that active demand equals net supply at the *original price*. Graphically, this appears as the dashed active-demand curve shifting to intersect the new net-supply curve at the initial price.

Consider a positive flow shock to the risky asset (e.g., funds shift toward stocks, so $f > 0$). Lower bond demand increases r, while increased demand for the risky asset reduces the risk premium. In equilibrium these effects offset exactly, leaving the price-dividend ratio unchanged. If the rise in r were smaller than the fall in the risk premium, there would be excess demand for goods (equivalently, excess bond supply). Simultaneous clearing of risky and risk-free asset markets therefore requires infinite aggregate elasticity in this model.

The need for a micro-founded demand system. The analysis underscores the central role of cross-price elasticity. The aggregate market can be infinitely elastic—even if individual investors are inelastic—so long as the average consumption-wealth ratio is unaffected by the flow shock f.

When the flow shock shifts both the net-supply curve and the average consumption-wealth ratio, prices adjust and aggregate elasticity becomes finite.[8]

Understanding aggregate elasticity therefore requires a framework that links portfolio reallocation shocks to both risky asset demand and bond demand (i.e., savings behavior). To this end, we develop a dynamic heterogeneous-agent asset pricing model and introduce a new methodology for deriving investor demand within this setting.

3 A Dynamic Asset Pricing Model with Passive Investors

We now develop a dynamic heterogeneous-agent asset pricing model that provides a micro-founded demand system. This framework allows us to trace how portfolio reallocation shocks affect both stock and bond demand, and thereby determine aggregate market elasticity.

3.1 Environment

Financial markets. The aggregate endowment Y_t follows a geometric Brownian motion:

$$\frac{dY_t}{Y_t} = \mu\, dt + \sigma\, dZ_t, \tag{8}$$

where $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^d$ are constants. The process $Z = \{Z_t\}_{t\geq 0}$ is a standard d-dimensional Brownian motion on the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ satisfying the usual conditions. The multidimensional structure of Z allows arbitrary correlation between endowment shocks and portfolio-flow shocks (introduced below).

Investors trade a unit-supply risky asset, which is a claim to the aggregate endowment Y_t. Let P_t denote the ex-dividend price and $Y_t\, dt$ the dividend flow. The instantaneous return process is

$$dR_t \;=\; \frac{dP_t + Y_t\, dt}{P_t} \;\equiv\; \mu_{R,t}\, dt + \sigma_{R,t}\, dZ_t, \tag{9}$$

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[8] An analogous argument applies to uncertainty. With unit EIS, the consumption–wealth ratio is fixed, so higher uncertainty leaves prices unchanged. For uncertainty to reduce prices, EIS must exceed one so that higher uncertainty lowers the consumption–wealth ratio (see Bansal and Yaron, 2004).

where $\mu_{R,t}$ is the expected return and $\sigma_{R,t} \in \mathbb{R}^{1\times d}$ is the vector of exposures.

Investors also have access to a risk-free asset paying the rate r_t. In equilibrium, prices depend on an aggregate state vector $X_t \in \mathbb{R}^N$, so we write $r_t = r(X_t)$, $\mu_{R,t} = \mu_R(X_t)$, and $\sigma_{R,t} = \sigma_R(X_t)$. The state vector evolves as

$$dX_t \;=\; \mu_{X,t}\,dt + \sigma_{X,t}\,dZ_t,$$

with drift $\mu_{X,t} \in \mathbb{R}^N$ and exposure matrix $\sigma_{X,t} \in \mathbb{R}^{N\times d}$, both determined in equilibrium.

Demography. The economy consists of $(J{+}1)$ investor types, indexed by $j = 0, 1, \ldots, J$, with population mass ω_j. Investors die with Poisson intensity κ, and each instant a mass $\kappa\,\omega_j$ of type-j agents are born, so the total population remains constant and normalized to one. Newborns inherit parental wealth. Mortality risk ensures a nondegenerate stationary wealth distribution.

Preferences are recursive as in Duffie and Epstein (1992), with type-j risk aversion γ_j and a common elasticity of intertemporal substitution (EIS) ψ. Each investor chooses consumption $C_{j,t}$ and a portfolio share $\alpha_{j,t}$ in the risky asset, subject to portfolio constraints specified below.

Passive investors. Agents of type $j = 0$ are *passive investors*, who maintain a fixed portfolio share $\alpha_{0,t} = \overline{\alpha}_{p,t}$. The share $\overline{\alpha}_{p,t}$ evolves according to an Ornstein-Uhlenbeck process:

$$d\overline{\alpha}_{p,t} \;=\; \theta_p\big(\overline{\alpha} - \overline{\alpha}_{p,t}\big)dt \;+\; \sigma_{\alpha_p}\,dZ_t, \tag{10}$$

with long-run mean $\overline{\alpha} > 0$, mean-reversion $\theta_p > 0$, and loading $\sigma_{\alpha_p} \in \mathbb{R}^{1\times d}$ on the d-dimensional Brownian motion Z_t. Innovations to $\overline{\alpha}_{p,t}$ represent *portfolio flow shocks*, i.e., fluctuations in the fraction of wealth passive investors allocate to the risky asset. As in Section 2, a flow shock is an asymmetric demand shock that affects only the passive investors and triggers portfolio reallocation.

Portfolio flows may comove with cash–flow shocks or arise independently. For example, if $d = 2$, take $\sigma = (\sigma_1, 0)$ for endowment risk and $\sigma_{\alpha_p} = (\sigma_{\alpha_p 1}, \sigma_{\alpha_p 2})$ for the passive-share process: the first factor generates correlation with fundamentals, while the second delivers portfolio–flow shocks orthogonal to cash-flow risk.

The exposure of the passive share to aggregate shocks can reflect, for instance, *imperfect rebalancing* or *performance-induced flows*. Without rebalancing, a positive return on the risky asset raises the passive share when $\overline{\alpha}_{p,t} < 1$ (and lowers it when $\overline{\alpha}_{p,t} > 1$); similarly, investors may follow a rule-of-thumb behavior where they increase their risky exposure after good performance. In both cases, the passive portfolio share responds to aggregate shocks.

This specification aligns with evidence on passive behavior: households exhibit substantial portfolio inertia (Ameriks and Zeldes, 2004; Brunnermeier and Nagel, 2008); flows chase performance (Chevalier and Ellison, 1997; Dannhauser and Pontiff, 2024); and regulatory changes can sharply increase equity allocations via TDF adoption (Parker, Schoar, Cole, and Simester, 2022), illustrating portfolio flow shocks not tied to cash flow fundamentals.

Active investors. Investors of type $j = 1, \ldots, J$ are *active investors*, who rebalance portfolios subject to state-dependent leverage constraints:

$$\alpha_{j,t} \leq \frac{\overline{\sigma}}{\|\sigma_{R,t}\|},$$

where $\|\sigma_{R,t}\|$ corresponds to return volatility, the Euclidean norm of the vector of risk exposure.

This constraint resembles a Value-at-Risk (VaR) limit, often applied to banks and leveraged institutions (see, e.g., Adrian and Shin 2014 and Brunnermeier and Pedersen 2009).

Investors' problem. An investor of type $j = 0, \ldots, J$ solves

$$V_{j,t} = \max_{[C_j, \alpha_j]} \mathbb{E}_t \left[\int_t^{\infty} f_j(C_{j,s}, V_{j,s}) \, ds \right], \qquad s.t. \tag{11}$$

$$dW_{j,t} = \left[(r_t + \pi_t \alpha_{j,t}) W_{j,t} - C_{j,t} \right] dt + \alpha_{j,t} W_{j,t} \sigma_{R,t} \, dZ_t,$$

with $W_{j,t} \geq 0$ and $\alpha_{j,t} \in \Omega_{j,t}$. For passive investors ($j = 0$), $\Omega_{0,t} = \{\alpha_0 : \alpha_0 = \overline{\alpha}_{p,t}\}$, where $\overline{\alpha}_{p,t}$ follows (10). For active investors ($j = 1, \ldots, J$), $\Omega_{j,t} = \{\alpha_j : \alpha_j \leq \overline{\sigma}/\|\sigma_{R,t}\|\}$. The risk premium is $\pi_t = \mu_{R,t} - r_t$.

Preferences are recursive with aggregator

$$f_j(C,V) = \rho \frac{(1-\gamma_j)V}{1-\psi^{-1}} \left[\left(\frac{C}{\left((1-\gamma_j)V\right)^{\frac{1}{1-\gamma_j}}} \right)^{1-\psi^{-1}} - 1 \right],$$

where the discount factor $\rho \equiv \hat{\rho} + \kappa$ reflects both time impatience $\hat{\rho}$ and the death probability κ.

Market clearing and equilibrium. We define equilibrium as follows:

Definition 1. *A competitive equilibrium consists of stochastic processes adapted to the filtration generated by Z_t: the aggregate endowment Y, the price of risky asset P, the risk-free rate r; and, for each investor $j \in \{0, \ldots, J\}$, wealth W_j, consumption C_j, and portfolio share α_j, such that*

(i) The aggregate endowment evolves according to (8)*, with $Y_0 > 0$.*

(ii) Given $\{P_t, r_t\}$, the choices (C_j, α_j) solve investor j's problem in (11).

(iii) Markets clear for goods, risky assets, and bonds:

$$\sum_{j=0}^{J} \omega_j C_{j,t} = Y_t, \qquad \sum_{j=0}^{J} \omega_j Q_{j,t} = 1, \qquad \sum_{j=0}^{J} \omega_j B_{j,t} = 0,$$

$Q_{j,t} = \alpha_{j,t} W_{j,t}/P_t$ and $B_{j,t} = (1-\alpha_{j,t})W_{j,t}$ denote holdings of the risky and riskless assets.

3.2 Equilibrium characterization

In this section, we characterize equilibrium by defining a Markov equilibrium in terms of the wealth distribution and the portfolio share of passive investors.

Investors' problem. With homothetic preferences, investor j's value function can be written as

$$V_{j,t} = \left(\frac{c_{j,t}}{\rho^{\psi}} \right)^{\frac{1-\gamma_j}{1-\psi}} \frac{W_{j,t}^{1-\gamma_j}}{1-\gamma_j}, \qquad \text{where} \qquad \frac{dc_{j,t}}{c_{j,t}} = \mu_{c_j,t}\, dt + \sigma_{c_j,t}\, dZ_t,$$

and $c_{j,t} = c_j(X_t)$ corresponds to the consumption-wealth ratio: $\frac{C_{j,t}}{W_{j,t}} = c_{j,t}$.

The optimal portfolio weight for an active investor is

$$\alpha_{j,t} = \min\left\{\frac{\pi_t}{\gamma_j \|\sigma_{R,t}\|^2} + \varsigma_{j,t},\ \frac{\overline{\sigma}}{\|\sigma_{R,t}\|}\right\}, \tag{12}$$

where $\varsigma_{j,t} \equiv \frac{1-\gamma_j^{-1}}{\psi-1}\frac{\sigma_{c_j,t}\sigma_{R,t}'}{\|\sigma_{R,t}\|^2}$ is the hedging component. The risk exposure of unconstrained investors thus combines a myopic demand, $\frac{\pi_t}{\gamma_j\|\sigma_{R,t}\|^2}$, with a hedging demand reflecting the covariance between $c_{j,t}$ and risky returns. The leverage constraint caps exposure at $\overline{\sigma}/\|\sigma_{R,t}\|$.

From the Hamilton-Jacobi-Bellman equation, the consumption-wealth ratio is

$$c_{j,t} = \psi\rho + (1-\psi)\left[r_t + \pi_t\alpha_{j,t} - \frac{\gamma_j}{2}\|\sigma_{R,t}\|^2\alpha_{j,t}^2\right] + \xi_{j,t}, \tag{13}$$

where

$$\xi_{j,t} \equiv \mu_{c_j,t} + (1-\gamma_j)\sigma_{c_j,t}\sigma_{R,t}'\alpha_{j,t} + \frac{\psi-\gamma_j}{1-\psi}\frac{\|\sigma_{c_j,t}\|^2}{2},$$

is a forward-looking term capturing how expected shifts in $c_{j,t}$ affect consumption choices.

Thus, the consumption–wealth ratio depends both on current investment opportunities, summarized by $r_t + \pi_t\alpha_{j,t} - \frac{\gamma_j}{2}\alpha_{j,t}^2\|\sigma_{R,t}\|^2$, and on future opportunities, summarized by $\xi_{j,t}$.

Pricing condition. Let $p_t \equiv P_t/Y_t$ denote the price-dividend ratio of the risky asset. Since the expected return is $r_t + \pi_t = \frac{1}{p_t} + \mu_{P,t}$, the price-dividend ratio satisfies

$$\frac{1}{p_t} = r_t + \pi_t - (\mu + \mu_{p,t} + \sigma\sigma_{p,t}'), \tag{14}$$

where $\sigma_{R,t} = \sigma + \sigma_{p,t}$ and $(\mu_{p,t}, \sigma_{p,t})$ follow from Itô's lemma.

Aggregate state variable. Let $x_{j,t}$ denote the wealth share of type-j investors:

$$x_{j,t} \equiv \frac{\omega_j W_{j,t}}{P_t}.$$

The aggregate state is $X_t = (x_t, \overline{\alpha}_{p,t})$, where $x_t = (x_{1,t}, x_{2,t}, \ldots, x_{J,t})$ and $\overline{\alpha}_{p,t}$ evolves as in (10). By Itô's lemma, the law of motion for $x_{j,t}$ is

$$\frac{dx_{j,t}}{x_{j,t}} = \left[r_t + \pi_t \alpha_{j,t} - c_{j,t} - \mu_{P,t} + (1 - \alpha_{j,t}) \|\sigma_{R,t}\|^2 + \kappa \frac{\omega_j - x_{j,t}}{x_{j,t}} \right] dt + (\alpha_{j,t} - 1)\, \sigma_{R,t}\, dZ_t.$$

Risk premium and interest rate. Let $\mathcal{J}_t^u$ and $\mathcal{J}_t^c$ denote the sets of unconstrained and constrained active investors at time t, with wealth shares $x_{u,t} \equiv \sum_{j\in\mathcal{J}_t^u} x_{j,t}$ and $x_{c,t} \equiv \sum_{j\in\mathcal{J}_t^c} x_{j,t}$. From market clearing for the risky asset, demand from unconstrained investors satisfies

$$\underbrace{\sum_{j\in\mathcal{J}_t^u} x_{j,t}\alpha_{j,t}}_{\text{unconstrained active demand}} = \underbrace{1 - x_{0,t}\overline{\alpha}_{p,t} - x_{c,t}\overline{\alpha}_{c,t}}_{\text{net supply}}, \tag{15}$$

where $\overline{\alpha}_{c,t} \equiv \overline{\sigma}/\|\sigma_{R,t}\|$ is the portfolio share of constrained agents.

Combining this condition with the optimal portfolios yields the risk premium:

$$\pi_t = \frac{\gamma_{u,t}\|\sigma_{R,t}\|^2}{x_{u,t}} \left[1 - x_{0,t}\, \overline{\alpha}_{p,t} - x_{c,t}\, \overline{\alpha}_{c,t} - x_{u,t}\, \varsigma_t\right], \tag{16}$$

where $\gamma_{u,t} \equiv [\sum_{j\in\mathcal{J}_t^u} \frac{x_{j,t}}{x_{u,t}} \frac{1}{\gamma_j}]^{-1}$ is the average risk aversion and $\varsigma_t \equiv \sum_{j\in\mathcal{J}_t^u} \frac{x_{j,t}}{x_{u,t}} \varsigma_{j,t}$ the average hedging demand of unconstrained investors. Equation (16) shows that the risk premium rises with higher risk aversion, lower passive demand, tighter leverage constraints, or weaker hedging motives.

Goods market clearing requires

$$\sum_{j=0}^{J} x_{j,t} c_{j,t} = \frac{1}{p_t}.$$

Combining this with (13) gives the equilibrium interest rate:

$$r_t = \rho + \psi^{-1}(\mu_{P,t} + \xi_t) + (1 - \psi^{-1}) \sum_{j=0}^{J} x_{j,t} \frac{\gamma_j \alpha_{j,t}^2}{2} \|\sigma_{R,t}\|^2 - \pi_t, \tag{17}$$

where $\xi_t \equiv \sum_{j=0}^{J} x_{j,t}\xi_{j,t}$. The first two terms reflect impatience and intertemporal substitution, while the third term captures precautionary savings.

Endogenous volatility. Return volatility has both an exogenous and an endogenous component, $\sigma_{R,t} = \sigma + \sigma_{p,t}$, where σ reflects cash-flow volatility and $\sigma_{p,t}$ arises from movements in the price-dividend ratio p_t. By Itô's lemma,

$$\sigma_{p,t} = \frac{p_x(X_t)}{p(X_t)}\sigma_{x,t} + \frac{p_{\overline{\alpha}_p}(X_t)}{p(X_t)}\sigma_{\alpha_p}. \tag{18}$$

Equation (18) shows that endogenous volatility depends on (i) the sensitivity of p_t to changes in the wealth distribution $x_{j,t}$ and the response of wealth shares to shocks, $\sigma_{x,t}$, and (ii) the sensitivity of p_t to changes in passive portfolios and the response of $\overline{\alpha}_{p,t}$ to shocks. The first term is standard in heterogeneous-agent models (see Panageas, 2020), while the second arises only with exogenous portfolio flow shocks and is quantitatively relevant when markets are sufficiently inelastic (i.e., when $\frac{p_{\overline{\alpha}_p}(X_t)}{p(X_t)}$ is large). Thus, (18) links market elasticity to return volatility $\|\sigma + \sigma_{p,t}\|$.

Markov equilibrium. Equations (16) and (17) allow us to express π_t and r_t in terms of $c_{j,t}$, p_t, and their derivatives, once $(\mu_{c_j,t}, \mu_{p,t})$ and $(\sigma_{c_j,t}, \sigma_{p,t})$ are written as functions of $(\mu_{X,t}, \sigma_{X,t})$ via Itô's lemma. Substituting these into (13) and (14), together with the laws of motion for the state variables, yields a system of $J + 2$ PDEs in $\{c_j(X_t)\}_{j=0}^{J}$ and $p(X_t)$. These functions depend on $J + 1$ state variables: the J wealth shares $x_{j,t}$ for $j = 1, \ldots, J$ and the passive portfolio share $\overline{\alpha}_{p,t}$.

4 The Determinants of the Aggregate Market Elasticity

In this section, we analyze how the *aggregate market elasticity* governs the price impact of portfolio flows. The elasticity is determined by the coupled PDE system (13)–(14), which generally has no closed-form solution. To study its determinants, we extend the perturbation method of Kogan and Uppal (2001) and derive asymptotic closed-form approximations.

State-global perturbations Our perturbation approach studies economies in a neighborhood of a benchmark that admits a closed-form solution. A convenient benchmark arises when preferences are homogeneous, $\gamma_j = \gamma$, and passive investors are fully invested in the risky asset, $\overline{\alpha}_{p,t} = 1$. In this case, the economy collapses to a Lucas economy, as shown in Lemma 1. Throughout, a superscript $\star$ denotes benchmark values.

Lemma 1 (Benchmark economy)**.** *Suppose* $\gamma_j = \gamma$*,* $\overline{\alpha}_{p,t} = 1$*, and* $\rho > \left(1 - \psi^{-1}\right)\left(\mu - \frac{\gamma\|\sigma\|^2}{2}\right)$*. Then,*

$$\pi^\star = \gamma\|\sigma\|^2, \qquad r^\star = \rho + \psi^{-1}\mu - \left(1 + \psi^{-1}\right)\frac{\gamma\|\sigma\|^2}{2}, \qquad p^\star = \left[\rho - \left(1 - \psi^{-1}\right)\left(\mu - \frac{\gamma\|\sigma\|^2}{2}\right)\right]^{-1},$$

$c_j^\star = (p^\star)^{-1}$*, and* $\alpha_j^\star = 1$ *for* $j = 0, \ldots, J$*.*

We analyze small deviations from this benchmark. Formally, consider a family of economies indexed by $\epsilon > 0$, where ϵ scales departures from the benchmark along the following dimensions.

First, investors have heterogeneous risk aversion:

$$\gamma_j = \gamma\left(1 + \hat{\gamma}_j \epsilon\right), \qquad \sum_{j=0}^{J} \omega_j \hat{\gamma}_j = 0, \tag{19}$$

so that γ is the population-weighted average and $\hat{\gamma}_j$ measures deviations. When $\epsilon = 0$, preferences are homogeneous; when $\epsilon = 1$, we recover the heterogeneous-investor economy.

Second, passive investors need not be fully invested in the risky asset. Their portfolio share is

$$\overline{\alpha}_{p,t} = 1 + \hat{\alpha}_{p,t}\,\epsilon, \tag{20}$$

where $\hat{\alpha}_{p,t}$ measures deviations from full investment benchmark and evolves as an Ornstein-Uhlenbeck process, consistent with Equation (10).

Third, the leverage constraint coefficient is perturbed according to

$$\overline{\sigma} = \|\sigma\|\left(1 + \hat{\sigma}\,\epsilon\right), \tag{21}$$

so that the tightness of the constraint is of order $O(\epsilon)$, matching the order of leverage demand. Finally, we assume a small mortality rate, $\kappa = \hat{\kappa}\,\epsilon$.

Equilibrium objects now depend on both the state X_t and the parameter ϵ. We expand them to second order:

$$p(X,\epsilon) = p_0(X) + p_1(X)\epsilon + p_2(X)\epsilon^2 + O(\epsilon^3),$$
$$c_j(X,\epsilon) = c_{j,0}(X) + c_{j,1}(X)\epsilon + c_{j,2}(X)\epsilon^2 + O(\epsilon^3),$$

where $p_k(X)$ and $c_{j,k}(X)$, $k \in \{0,1,2\}$, are functions of the state X.

Unlike standard DSGE perturbations, which linearize in both X and ϵ, our method is *state-global*: we solve directly for functions of X rather than coefficients around a steady state.[9] We refer to this approach as a *state-global perturbation*. This approach builds on the perturbation method of Kogan and Uppal (2001) and, more closely, the extension by Silva (2020). This method allows us to characterize how aggregate market elasticity varies with the state of the economy.

We proceed in two steps. First, we compute the first-order corrections $p_1(X)$ and $c_{j,1}(X)$. Second, we solve for the second-order terms $p_2(X)$ and $c_{j,2}(X)$. Zeroth-order terms are given by Lemma 1, i.e., $p_0(X) = p^\star$ and $c_{j,0}(X) = c_j^\star$.

4.1 The first-order demand system

Demand for risky asset. Investor j's demand for the risky asset is

$$Q_j(X;\epsilon) = \alpha_j(X;\epsilon)\,\frac{x_j}{\omega_j},$$

where x_j is the wealth share. Define the proportional deviation from the benchmark as

$$q_{j,t} \;\equiv\; \frac{Q_j(X_t;\epsilon) - Q_j(X_t;0)}{Q_j(X_t;0)} \;=\; \alpha_{j,1}(X_t)\,\epsilon + O(\epsilon^2),$$

where $\alpha_{j,1}(X)$ is the first-order correction to the portfolio weight.

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[9] See Kargar, Passadore, and Silva (2023) for a comparison with standard perturbation methods.

Three results determine $\alpha_{j,1}(X)$. First, the volatility of the price-dividend ratio is second order:

$$\sigma_{p,t} = \frac{p_X(X_t)}{p(X_t)}\,\sigma_X(X_t) = O(\epsilon^2),$$

since $p_{X,0} = \sigma_{X,0} = 0$ (Lemma 1). Second, hedging demand is also $O(\epsilon^2)$ because $\sigma_{c_j}(X_t)$ is second order. Third, expanding the pricing condition (14) gives

$$\hat{\pi}_t = -\,\hat{r}_t - \frac{1}{p^\star}\hat{p}_t + O(\epsilon^2),$$

where $\hat{\pi}_t \equiv \pi_t - \pi^\star$, $\hat{r}_t \equiv r_t - r^\star$, and $\hat{p}_t \equiv \frac{p_t - p^\star}{p^\star}$, using $\mu_{p,t} = O(\epsilon^2)$.

Substituting into (12), the first-order demand system is

$$q_{j,t} = -\,\zeta_{j,p}\,\hat{p}_t - \zeta_{j,r}\,\hat{r}_t + f_{j,t} + O(\epsilon^2), \tag{22}$$

where, for unconstrained investors,

$$\zeta_{j,p} = \frac{1}{p^\star}\frac{1}{\gamma\|\sigma\|^2}, \qquad \zeta_{j,r} = \frac{1}{\gamma\|\sigma\|^2}, \qquad f_{j,t} = -\,\hat{\gamma}_j\,\epsilon.$$

For passive and constrained investors, own- and cross-price elasticities vanish, $\zeta_{j,p} = \zeta_{j,r} = 0$, with demand shifters $f_{j,t} = \hat{\sigma}\,\epsilon$ for constrained investors and $f_{0,t} = \hat{\alpha}_{p,t}\,\epsilon$ for passive investors.

Equation (22) shows that the first-order approximation to the dynamic heterogeneous agent model in Section 3 provides microfoundations for the demand system in Section 2. As in the simple model, unconstrained active investors exhibit nonzero own- and cross-price elasticities, here explicitly linked to structural parameters. Shocks to $\overline{\alpha}_{p,t}$ move the passive investors' demand shifter $f_{0,t}$—the dynamic analog of the comparative statics in Section 2. In addition, the model generates further demand shifts from occasionally binding leverage constraints faced by active investors.

Consumption-wealth ratio. A first-order expansion of (13) yields

$$c_{j,t} \;=\; \psi\rho \;+\; (1-\psi)\left[r_t + \pi_t - \frac{\gamma_j \|\sigma\|^2}{2}\right] \;+\; O(\epsilon^2).$$

Consistent with Assumption 1, the risk-free rate and the risk premium enter symmetrically at first order. Substituting the expansion $\hat{\pi}_t = -\hat{r}_t - p^{\star -1}\hat{p}_t + O(\epsilon^2)$ gives

$$\hat{c}_{j,t} \;\equiv\; c_{j,t} - c_j^\star \;=\; \chi_{j,0} \;+\; \chi_{j,p}\,\hat{p}_t \;+\; O(\epsilon^2),$$

with coefficients

$$\chi_{j,p} = (\psi - 1)\,\frac{1}{p^\star}, \qquad \chi_{j,0} = (\psi - 1)\,\frac{\gamma\|\sigma\|^2}{2}\,\hat{\gamma}_j\,\epsilon.$$

Hence, conditional on p_t, the consumption-wealth ratio is independent of r_t, as in Section 2. Its slope with respect to p_t depends on the EIS: it rises with p_t when $\psi > 1$, the standard assumption in macro-finance models. Finally, to first order, the consumption–wealth ratio is unaffected by $\overline{\alpha}_{p,t}$, a property that will be central for aggregate market elasticity.

Aggregate market elasticity. Given the demand system, equilibrium prices follow from market clearing in the risky-asset and goods markets:

$$\sum_{j=0}^{J} x_{j,t}\, q_{j,t} = 0, \qquad \sum_{j=0}^{J} x_{j,t}\, \hat{c}_{j,t} = -\frac{1}{p^\star}\,\hat{p}_t.$$

Aggregating across investors, and dropping the j subscript on coefficients that are common across types, yields the linear system

$$\begin{bmatrix} \zeta_p & \zeta_r \\ \chi_p + \frac{1}{p^\star} & 0 \end{bmatrix} \begin{bmatrix} \hat{p}_t \\ \hat{r}_t \end{bmatrix} = \begin{bmatrix} f_t \\ -\chi_{0,t} \end{bmatrix},$$

where $f_t \equiv \sum_{j=0}^{J} x_{j,t} f_{j,t}$ and $\chi_{0,t} \equiv \sum_{j=0}^{J} x_j \chi_{j,0}$.

Definition 2 (Aggregate market elasticity)**.** *The aggregate market elasticity* $\epsilon_{M,t}$ *is defined as the inverse of the proportional change in the price of the risky asset in response to a flow shock* f_t*:*

$$\epsilon_{M,t} \;\equiv\; \left[\frac{\partial \hat{p}_t}{\partial f_t}\right]^{-1}.$$

Proposition 2 (First-order impact of portfolio flows)**.** *Suppose* $\rho > \left(1-\psi^{-1}\right)\left(\mu - \frac{\gamma\|\sigma\|^2}{2}\right)$*. Then:*

(i) The price impact is zero to first order: $\epsilon_{M,t}^{-1} = \frac{\partial \hat{p}_t}{\partial f_t} = 0$.

(ii) The interest rate and risk premium responses satisfy

$$\frac{\partial \hat{r}_t}{\partial f_t} = \frac{1}{\zeta_r}, \qquad \frac{\partial \hat{\pi}_t}{\partial f_t} = -\frac{1}{\zeta_r}.$$

Proposition 2 shows that the main lesson from the simple model in Section 2 extends to the dynamic economy: portfolio inflows into the risky asset reduce the risk premium, while the corresponding shift out of bonds raises the risk-free rate. To first order, these effects offset exactly, leaving the price-dividend ratio unchanged. Hence, the aggregate market is infinitely elastic, regardless of the individual investors' own-price elasticities $\zeta_{j,p}$.

Why is the market infinitely elastic? As shown in Figure 2, price impact is absent as long as the consumption-wealth ratio does not respond directly to the portfolio-flow shock. Otherwise, price changes would generate excess demand or supply in the goods market.

To see why flows leave consumption unchanged to first order, differentiate (13) with respect to $\alpha_{j,t}$:

$$\frac{\partial c_{j,t}}{\partial \alpha_{j,t}} \;=\; (1-\psi)\,\gamma_j\,\|\sigma_{R,t}\|^2\left[\frac{\pi_t}{\gamma_j\|\sigma_{R,t}\|^2} + \frac{1-\gamma_j^{-1}}{\psi-1}\,\frac{\sigma_{c_{j,t}}\,\sigma_{R,t}^{\top}}{\|\sigma_{R,t}\|^2} - \alpha_{j,t}\right].$$

The term in brackets is equal to zero at the privately optimal portfolio, as implied by the investor's first-order condition. By the envelope theorem, small changes in $\alpha_{j,t}$ around the optimum have negligible welfare effects on welfare and hence do not affect $c_{j,t}$.

In the first-order approximation, this derivative is evaluated at the benchmark Lucas economy, where all investors hold optimal portfolios. The derivative is thus zero, and by Proposition 2, price impact is negligible for small (first-order) deviations from the frictionless allocation: the consumption-wealth ratio does not react to flow shocks. In the simple model of Section 2, this was imposed as an *assumption*; here it emerges as a *result* of expanding around the efficient allocation.

This reasoning also clarifies when aggregate elasticity becomes finite: when the initial allocation of risk is *inefficient*. To study such economies requires larger departures from the benchmark, captured by a second-order approximation of the demand system.

4.2 Inefficient passive demand

We next turn to a second-order approximation of the demand system to study how risk allocation affects aggregate market elasticity. To focus on the mechanism most clearly, consider the special case without preference heterogeneity or leverage constraints.

The role of risk misallocation. In this setting, demand for the risky asset is still given by (22) up to second order. Without heterogeneity, the relevant demand shifter is

$$f_t \equiv x_{0,t} \left(\overline{\alpha}_{p,t} - 1\right),$$

where $x_{0,t}$ denotes the wealth share of passive investors.

The key difference relative to the benchmark arises from the consumption-wealth ratio. Aggregating (13) across investors, letting $c_t \equiv \sum_{j=0}^{J} x_{j,t} c_{j,t}$, and imposing market clearing $\sum_{j=0}^{J} x_{j,t}\alpha_{j,t} = 1$, yields

$$c_t \;=\; \psi\rho \;+\; (1-\psi)\left[r_t + \pi_t - \frac{\gamma\|\sigma\|^2}{2}\sum_{j=0}^{J} x_{j,t}\alpha_{j,t}^2\right] \;+\; O(\epsilon^3).$$

Under the efficient allocation $\alpha_{j,t} = 1$, so $\sum_j x_{j,t}\alpha_{j,t}^2 = 1$. Any deviation from this efficient allocation creates dispersion in portfolios, leading to $\sum_j x_{j,t}\alpha_{j,t}^2 > 1$ by Jensen's inequality, which lowers the risk-adjusted expected return $r_t + \pi_t - \frac{\gamma\|\sigma\|^2}{2}\sum_j x_{j,t}\alpha_{j,t}^2$. When $\psi > 1$, this reduction weakens incentives to save and raises the consumption-wealth ratio for a given $r_t + \pi_t$.

Movements in the passive portfolio therefore shift the aggregate consumption–wealth ratio. Its second-order expansion is

$$\hat{c}_t \;\equiv\; c_t - c^\star \;=\; \chi_{0,t} \;+\; \chi_p\, \hat{p}_t \;+\; O(\epsilon^3),$$

with coefficients

$$\chi_p = (\psi - 1)\,\frac{1}{p^\star}, \qquad \chi_{0,t} = (\psi - 1)\,\frac{\gamma\|\sigma\|^2}{2}\,\frac{f_t^2}{x_{0,t}(1 - x_{0,t})}.$$

Because $\chi_{0,t}$ is quadratic in the flow f_t, it is $O(\epsilon^2)$. For example, when passive investors are initially underexposed to the risky asset ($\overline{\alpha}_{p,t} < 1$ and $\psi > 1$), an inflow into stocks reduces the consumption-wealth ratio:

$$\frac{\partial \chi_{0,t}}{\partial f_t} \;=\; -\,(\psi - 1)\,\gamma\|\sigma\|^2\,\frac{1 - \overline{\alpha}_{p,t}}{1 - x_{0,t}}.$$

Note that $\chi_{0,t}$ is locally insensitive to portfolio flow changes when passive portfolio is efficient ($\overline{\alpha}_{p,t} = 1$), consistent with the first-order analysis.

Figure 3 illustrates the effects of portfolio flows when passive investors are initially underexposed to risk, i.e., $\overline{\alpha}_{p,t} < 1$. An inflow into the risky asset shifts both the net supply curve (left panel) and the average consumption-wealth ratio schedule (right panel) leftward. The interest rate adjusts so that active demand meets net supply at the goods market clearing price. In this case, portfolio flows have a positive price impact and the aggregate elasticity is *finite*.

Proposition 3 (Aggregate elasticity: inefficient passive demand)**.** *Without preference heterogeneity*

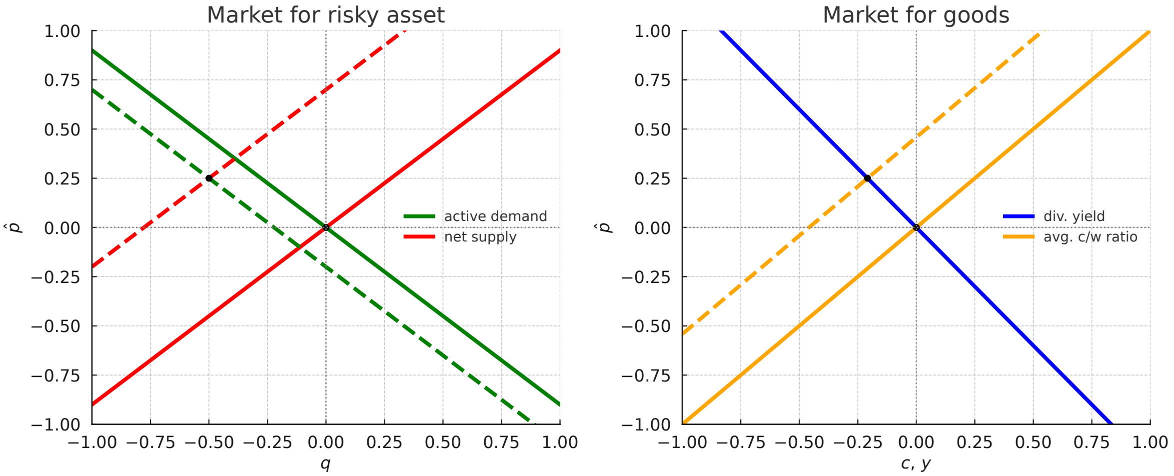


Figure 3. Equilibrium with inefficient risk allocation: risky asset (left) and goods (right) markets.

or leverage constraints, the inverse aggregate elasticity is

$$\varepsilon_{M,t}^{-1} = -(\chi_p + y^\star)^{-1}\frac{\partial \chi_{0,t}}{\partial f_t} = \left(1-\psi^{-1}\right)\frac{\gamma\|\sigma\|^2}{y^\star}\frac{1-\overline{\alpha}_{p,t}}{x_{a,t}} + \mathcal{O}\left(\epsilon^2\right),$$

where $x_{a,t} = 1 - x_{0,t}$ *is the wealth share of active investors and* $y^\star = 1/p^\star$ *is the dividend-price ratio in the benchmark economy.*

Proposition 3 shows that, unlike in Gabaix and Koijen (2023), the risky-asset elasticities ζ_p and ζ_r are not the direct drivers of aggregate elasticity. Because the demand system is recursive (interest rates only enters the risky asset demand) the overall price impact is governed instead by how portfolio flows impact the consumption-wealth ratio.

The macro elasticity is finite whenever $\overline{\alpha}_{p,t} \neq 1$ and $\psi \neq 1$. In the unit EIS case, the price-dividend ratio is pinned down by the subjective discount rate, so portfolio flows generate offsetting movements in the risk premium and the risk-free rate. With efficient risk allocation ($\overline{\alpha}_{p,t} = 1$), there is again no price impact.

The price impact is positive when $\psi > 1$ and $\overline{\alpha}_{p,t} < 1$: interest rate adjustments no longer fully offset the risk premium response, so the latter dominates. This mirrors representative agent models in which uncertainty shocks affect prices only when $\psi > 1$ (e.g., Bansal and Yaron, 2004).

The state-global perturbation analysis also makes clear that elasticity is state dependent. Figure 4

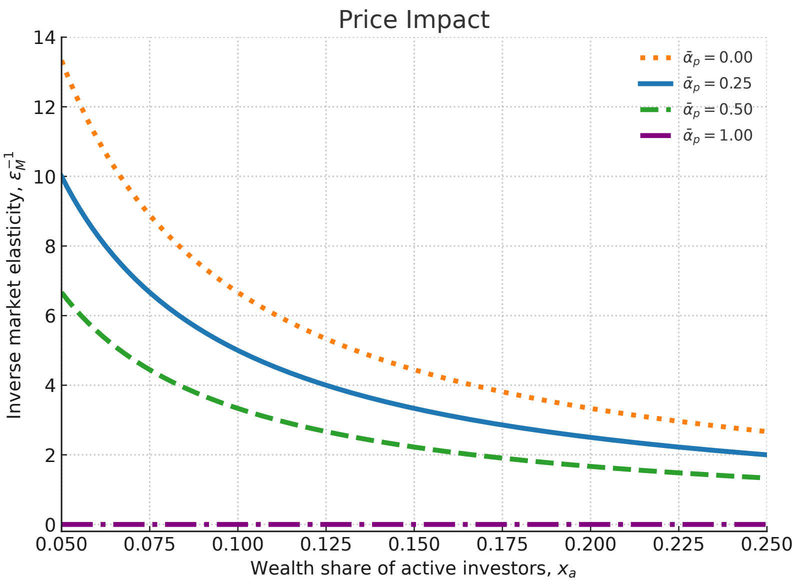


Figure 4. Price impact: inefficient passive demand.

plots the price impact as a function of active investors' wealth for different passive-share levels. Price impact rises when active investors are under capitalized, reflecting their limited risk-bearing capacity, and when the passive share deviates further from its efficient level. In the extreme case where passive investors hold no equities ($\overline{\alpha}_{p,t} = 0$), we recover a version of Basak and Cuoco (1998) in which price impact is maximized.

4.3 Preference heterogeneity and leverage constraints

We now reintroduce preference heterogeneity and leverage constraints into the model. These features shift the conditions under which passive investors' portfolios are efficient and therefore alter the aggregate market elasticity. The next proposition provides the main result.

Proposition 4 (Aggregate elasticity: heterogeneity and leverage constraints)**.** *The inverse aggregate market elasticity is*

$$\varepsilon_{M,t}^{-1} = \left(1-\psi^{-1}\right)\frac{\gamma\,\|\sigma\|^2}{y^\star}\,\frac{\overline{\alpha}_{p,t}^{\mathrm{opt}}-\overline{\alpha}_{p,t}}{x_{a,t}-x_{c,t}}\left(1-x_{c,t}\right) + O\left(\epsilon^2\right), \tag{23}$$

where $x_{c,t}$ is the wealth share of constrained active investors and the passive investors' optimal *portfolio share is*

$$\overline{\alpha}_{p,t}^{\text{opt}} \;=\; 1 \;-\; \frac{x_{a,t} - x_{c,t}}{1 - x_{c,t}} \, \frac{\gamma_0 - \mathbb{E}_t^u[\gamma_j]}{\gamma} \;-\; \frac{x_{c,t}}{1 - x_{c,t}} \Big(\frac{\overline{\sigma}}{\|\sigma\|} - 1 \Big), \tag{24}$$

with $x_{u,t} \equiv x_{a,t} - x_{c,t}$ *the wealth share of unconstrained active investors and*

$$\mathbb{E}_t^u[\gamma_j] \;\equiv\; \sum_{j \in \mathcal{J}^u} \frac{x_{j,t}}{x_{u,t}} \gamma_j .$$

Proposition 4 characterizes aggregate market elasticity in the full model. As in the frictionless benchmark, the market remains infinitely elastic when passive investors hold their optimal portfolio share, $\overline{\alpha}_{p,t} = \overline{\alpha}_{p,t}^{\text{opt}}$. The optimal portfolio share corresponds to the value of $\overline{\alpha}_{p,t}$ such that the passive investor would have no incentive to change their portfolio. Under homogeneous preferences ($\gamma_j = \gamma$) and slack leverage constraints ($x_{c,t} = 0$), this condition reduces to $\overline{\alpha}_{p,t} = 1$. More generally, preference heterogeneity and binding leverage constraints shift the efficient passive share away from unity, with its level determined by the wealth distribution and the relative risk aversion of different investor types.

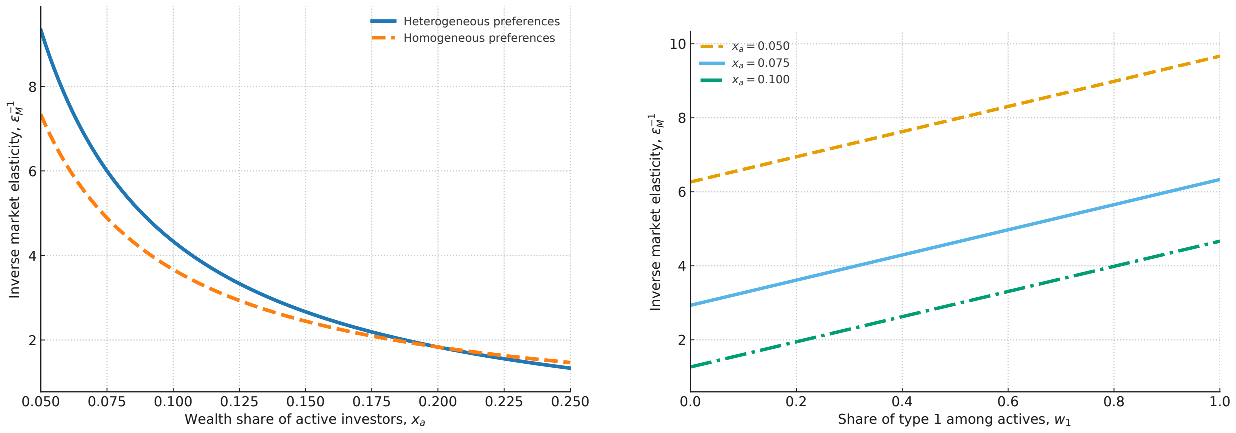
Preference heterogeneity. With slack leverage constraints ($x_{c,t} = 0$), (24) simplifies to

$$\overline{\alpha}_{p,t}^{\text{opt}} = 1 - x_{a,t} \, \frac{\gamma_0 - \mathbb{E}_t^u[\gamma_j]}{\gamma},$$

so, unlike the homogeneous preference case, the efficient passive share $\overline{\alpha}_{p,t}^{\text{opt}}$ differs from unity whenever $\gamma_0 \neq \mathbb{E}_t^u[\gamma_j]$. The inverse elasticity in (23) becomes

$$\varepsilon_{M,t}^{-1} = \left(1 - \psi^{-1}\right) \frac{\gamma \|\sigma\|^2}{y^\star} \, \frac{\overline{\alpha}_{p,t}^{\text{opt}} - \overline{\alpha}_{p,t}}{x_{a,t}} \;+\; O(\epsilon^2).$$

Optimal risk sharing implies that when passive investors are more risk averse than the average



(a) Homogeneous vs. heterogeneous preferences **(b)** Role of wealth distribution of active investors

Figure 5. Price impact comparisons. Panel (a) compares heterogeneous and homogeneous preferences. Panel (b) shows the sensitivity of the price impact to wealth distribution among active investors when $J = 2$ for different passive portfolio shares.

unconstrained active ($\gamma_0 > \mathbb{E}^u_t[\gamma_j]$),

$$\frac{\partial \overline{\alpha}^{\text{opt}}_{p,t}}{\partial x_{a,t}} = -\frac{\gamma_0 - \mathbb{E}^u_t[\gamma_j]}{\gamma} < 0,$$

so the efficient passive equity share rises as the active wealth share falls, typically after adverse aggregate shocks that erode active wealth that is more exposed to risk. If passive investors do not rebalance in such episodes, market inelasticity increases because $x_{a,t}$ declines (reducing risk-bearing capacity) while the gap $\overline{\alpha}^{\text{opt}}_{p,t} - \overline{\alpha}_{p,t}$ widens (moving the economy farther from the efficient allocation). Consequently, price impact rises as $x_{a,t}$ falls, as shown in the left panel (a) of Figure 5: the heterogeneous- and homogeneous-preference curves coincide at $x_a = 0.20$, but only the former steepens markedly as x_a declines.

Panel (b) of Figure 5 shows how the distribution of wealth across active investors shapes price impact. With two active types ($J = 2$) and $\gamma_1 > \gamma_2$, the average risk active aversion is

$$\mathbb{E}^u_t[\gamma_j] \;=\; w_{1,t}\gamma_1 + \big(1 - w_{1,t}\big)\gamma_2, \qquad w_{1,t} \equiv \frac{x_{1,t}}{x_{a,t}}.$$

When leverage constraints are slack ($x_{c,t} = 0$), the efficient passive share satisfies $\overline{\alpha}^{\text{opt}}_{p,t} = 1 -$

$x_{a,t}\,(\gamma_0 - \mathbb{E}_t^u[\gamma_j])/\gamma$, so holding $x_{a,t}$ fixed,

$$\frac{\partial\,\overline{\alpha}_{p,t}^{\text{opt}}}{\partial w_{1,t}} = \frac{x_{a,t}}{\gamma}\,(\gamma_1 - \gamma_2) \;>\; 0.$$

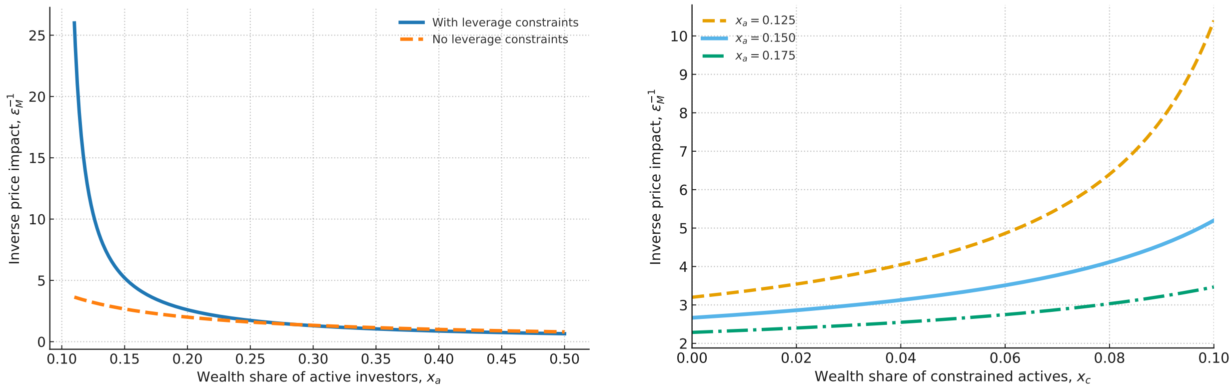
Thus, as the wealth share of the more risk averse active type rises, $\mathbb{E}_t^u[\gamma_j]$ increases and the passive sector should optimally hold more of the risky asset. If the passive portfolio is not adjusted, the deviation $\overline{\alpha}_{p,t}^{\text{opt}} - \overline{\alpha}_{p,t}$ widens, worsening risk allocation and raising price impact, consistent with the upward shift in the panel (b) of Figure 5.

Leverage constraints. When some active investors are constrained ($x_{c,t} > 0$), the marginal investors are the *unconstrained* active agents, and their risk-bearing capacity,

$$x_{u,t} \;\equiv\; x_{a,t} - x_{c,t},$$

becomes the key determinant of price impact. The panel (a) of Figure 6 plots price impact against the active wealth share $x_{a,t} = 1 - x_{0,t}$, with and without binding constraints. To isolate the role of constraints, we set $\gamma_0 = \mathbb{E}_t^u[\gamma_j]$, removing pure preference heterogeneity effects. As $x_{a,t}$ declines, e.g., after a negative aggregate shock when $\overline{\alpha}_{p,t} < 1$, price impact rises more sharply under binding constraints because $x_{u,t}$ shrinks and the risk-bearing capacity of marginal investors is depleted. Panel (b) holds $x_{a,t}$ fixed and varies the constrained share $x_{c,t}$. The price impact increases monotonically with $x_{c,t}$. This is intuitive: price impact is larger when a larger share of active investors is constrained.

Taking stock. Proposition 4 clarifies the main determinants of aggregate market elasticity. The elasticity is *finite* only when risk is misallocated: small portfolio flows around the frictionless allocation have no first-order price impact. Preference heterogeneity amplifies price impact in downturns because, as the active wealth share falls, the efficient passive share rises, widening the gap between efficient and actual portfolios. Binding leverage constraints further increase price



(a) Role of leverage constraints *(assumes* $x_{c,t} = 0.10$*)* **(b)** Varying the wealth share of constrained actives, $x_{c,t}$

Figure 6. Price impact under leverage constraints. Panel (a) compares the cases with and without leverage constraints (holding $x_{c,t} = 0.10$). Panel (b) plots the price impact as a function of $x_{c,t}$ for different values of $x_{a,t}$.

impact by shrinking the marginal (unconstrained) risk-bearing capacity, $x_{u,t}$. Taken together, these forces imply rich state dependence of aggregate elasticity in economies with heterogeneous investors and financial frictions.

5 Quantitative Implications

In this section, we turn to the quantitative implications of the model described in Section 3. We begin by describing our calibration strategy before presenting numerical results.

5.1 Calibration strategy

To calibrate the model, we process Flow of Funds (FoF) data to construct $J = 6$ major sectors: household mutual fund holdings, household non-mutual fund holdings, hedge funds, broker-dealers (L.130), the rest of the world (L.133), and the insurance-pension sector. Figure 7 plots the fraction of the U.S. stock market held by each of these sectors since 2012, the year when hedge fund holdings became separately available in the FoF. The legend reports the average fraction held by each sector. Households are the largest holders, accounting for 17.3% of the market through mutual funds and 43.3% through other vehicles. Foreign investors and the insurance-pension sector hold 17.5%

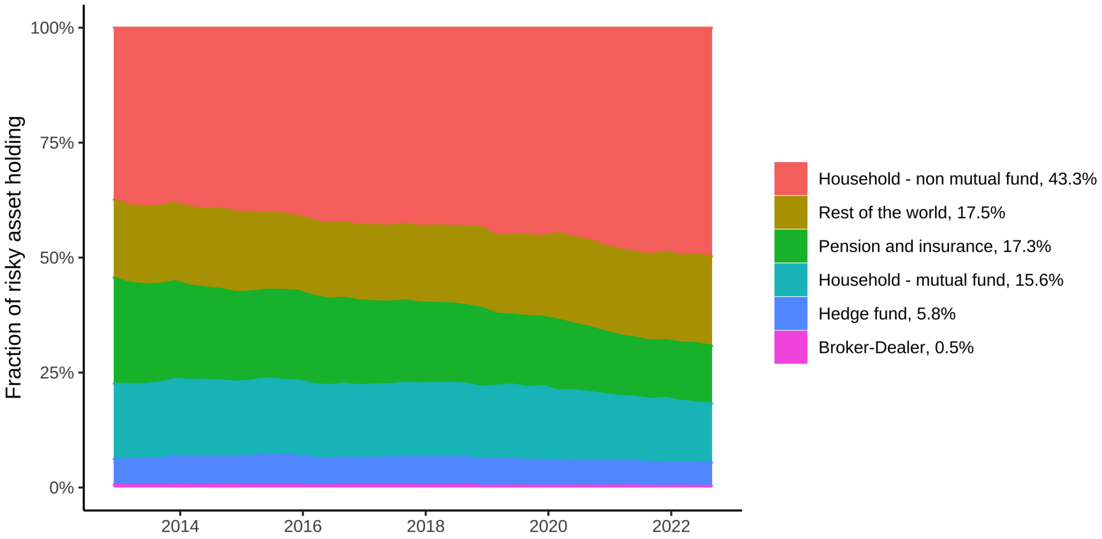


Figure 7. Fraction of stock market held by different sectors from 2012 to 2022. The average fraction held is reported in the legend. Source: Flow of Funds.

and 17.3%, respectively. Hedge funds and broker-dealers are comparatively small, with average holdings of 5.8% and 0.5%, consistent with Figure 1 in Koijen, Richmond, and Yogo (2024).

We exclude sectors with negligible stock market exposure, such as nonfinancial business (L.102), general government (L.105), the monetary authority (L.109), private depository institutions (L.110), money market funds (L.121), government-sponsored enterprises (L.125), agency- and GSE-backed mortgage pools (L.126), asset-backed security issuers (L.127), finance companies (L.128), REITs (L.129), holding companies (L.131), and other financial businesses (L.132). We then aggregate pass-through sectors back to their ultimate end users. For example, closed-end funds (L.123) and exchange-traded funds (L.124) are consolidated into the household (L.101) balance sheet, while mutual funds (L.122) are allocated across end investors based on the FOF breakdown. Defined contribution pensions (Tables L.118.c, L.119.c, and L.120.c) are also aggregated into the household sector.

We separately construct the hedge fund sector, since both households (L.101) and foreign investors (L.133) include hedge funds in their reported holdings. Domestic hedge funds are identified from Table B.101.f, while foreign hedge funds are obtained from the Enhanced Financial Accounts. We subtract hedge fund holdings from both the household and foreign sectors and treat them as a distinct category. Finally, we combine the remaining insurance and pension subsectors, including property-casualty insurance (L.115), life insurance (L.116), and defined benefit pensions and retirement plans (L.118.b, L.119.b, L.120.b).

We assign sectoral masses ω_j and risk aversion coefficients γ_j to match two sets of moments. First, average sectoral wealth shares are calibrated to match the empirical distribution of stock market holdings. Second, we match sectoral betas from time-series regressions of risky asset holdings on aggregate risky asset supply. These regression coefficients provide an empirical analog to the model-implied relationship between sectoral and aggregate risky positions.

A key parameter in the calibration is the average passive portfolio share, $\overline{\alpha}$. Estimates in the literature vary widely. Chinco and Sammon (2024) infer a passive share of about one third from index reconstitution events, while Koijen et al. (2024) report a much higher estimate of roughly two thirds in 2016Q4, based on measures of active share (Cremers and Petajisto, 2009). We calibrate the passive share by aggregating total passive wealth and dividing by total risky holdings.

The process for passive demand is modeled as the exogenous CIR process in Equation (10). Its parameters are calibrated as follows. The mean of the process, $\overline{\alpha}$, is chosen to match the average passive portfolio share described above. The volatility σ_p is disciplined by observed fluctuations in equity flows, which we measure by scaling quarterly flows by lagged market capitalization, following Appendix D3 of Gabaix and Koijen (2023). We use indirect inference to align model-implied aggregate flows with those in the data (blue line, left panel of Figure 1), in line with their approach. The persistence parameter θ_p is informed by evidence on household portfolio inertia. Brunnermeier and Nagel (2008) document substantial inertia using PSID data, finding that household portfolios change little absent major events, consistent with slow rebalancing. Their regression analysis suggests an inertia coefficient near 0.75, indicating strong persistence.[10] Finally, we calibrate the correlation between passive flow shocks and aggregate shocks by regressing quarterly flows (again, blue line in the left panel of Figure 1) on aggregate market returns, with the estimated beta providing the implied correlation.

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[10] Relatedly, Parker, Schoar, and Sun (2023) show that the adoption of target-date funds has contributed to highly stable household portfolio shares.

Table 1. Parameter values

This table reports the parameter values used in calibrating the model.

	Parameter	Choice
	Preferences & distribution	
ψ	EIS	1.5
γ_0	Risk aversion of passive investors	10
γ_j	Risk aversion of active investors	(9.368, 4.925, 3.212, 1.552)
ρ	Rate of time preference	0.01
	Technology	
μ	Endowment growth rate	0.022
σ	Endowment volatility	0.035
	Passive demand	
$\overline{\alpha}$	Mean	0.25
θ_p	Mean reversion parameter	0.9
	Leverage constraints	
$\overline{\sigma}$	Tightness of the leverage constraint	0.05

The parameter values are summarized in Table 1.

5.2 Quantitative results

To evaluate the quantitative implications of the model, we solve the full dynamic system numerically rather than relying on the perturbation approach of Section 4. A global solution method is necessary given the dimensionality of the state space, which includes $J+1$ state variables corresponding to one passive sector and six active sectors. We adopt the neural network-based method of Duarte, Duarte, and Silva (2024), which is well suited for high-dimensional problems. Details of the solution method are provided in the appendix.

6 Conclusion

This paper develops a general equilibrium model with heterogeneous investors, passive demand, and financial constraints to study the determinants of aggregate market elasticity. We show how general equilibrium adjustments, particularly the joint movement of the risk-free rate and the risk premium, fundamentally shape the price impact of portfolio flows.

A central insight is the role of cross-price elasticity. When demand for risky assets responds to interest rates, shifts in the risk premium are offset by changes in the risk-free rate. In this case, the market can be infinitely elastic even if individual investors are highly inelastic. Aggregate elasticity becomes finite only when risk is initially misallocated: portfolio flows then alter aggregate savings behavior, preventing interest rates from fully offsetting risk-premium movements.

Macro elasticity is both state-dependent and time-varying. Passive investors amplify price impacts, leverage constraints further tighten risk-bearing capacity, while preference heterogeneity increases elasticity by improving risk allocation. Inelasticity alone does not generate excess volatility, nor do flows in infinitely elastic markets; both elements are needed to explain observed fluctuations and the countercyclical volatility multiplier.

Our analytical results rely on a state-global perturbation method that provides closed-form characterizations of elasticity, while our quantitative analysis solves the full dynamic model numerically. The calibrated model highlights how the wealth distribution across households, intermediaries, and long-term investors shapes aggregate elasticity and volatility.

Overall, our findings suggest that market inelasticity is best understood as a symptom of inefficient risk allocation, rather than a structural feature of financial markets. Improving the allocation of risk across sectors may therefore be more effective for reducing excess volatility than targeting individual investor behavior. Future work could extend this framework to study asset pricing anomalies, monetary policy transmission, and the role of market structure in shaping aggregate elasticity.

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Appendix

A Derivations for Section 2

A.1 The bond demand view

Let $b_j \equiv \frac{B_j}{W_j}$ denote the bond-to-wealth ratio. The market clearing condition for bonds can be expressed as follows

$$\underbrace{x_a b_a}_{\textbf{active bond demand}} = \underbrace{-x_p b_p}_{\textbf{net bond supply}}$$

The linearized bond demand for a passive investor is given by

$$b_j - b_j^\star = \left[r - r^\star + \frac{c_j'(\mu - p^\star)}{1 - c_j(\mu - p^\star)}(p - p^\star) \right] b_j^\star - \alpha_j^\star e^{r^\star}(1 - c_j(\mu - p^\star))\hat{\alpha}_j$$

For simplicity, focus on the case $\overline{\alpha}_p = 1$, so $b_p^\star = b_a^\star = 0$. The passive bond demand is then given by

$$x_p b_p = f^b,$$

where $f^b \equiv -x_p[1 - c_p(\mu - p^\star)]\hat{\alpha}_p$. The active bond demand is given by

$$x_a b_a = -\zeta_p^b(p - p^\star) - \zeta_r^b(r - r^\star),$$

where $\zeta_p^b = \zeta_r^b = -x_a g_a'(\mu - p^\star - r^\star)[1 - c_a(\mu - p^\star)]$.

The demand system can be written as follows:

$$\begin{bmatrix} \zeta_p^q & \zeta_r^q \\ \zeta_p^b & \zeta_r^b \end{bmatrix} \begin{bmatrix} p - p^\star \\ r - r^\star \end{bmatrix} = \begin{bmatrix} f^q \\ f^b \end{bmatrix},$$

where we denote here $f^q \equiv f$ for symmetry.

Inverting the system above, we obtain

$$\begin{bmatrix} p - p^\star \\ r - r^\star \end{bmatrix} = \frac{1}{\zeta_p^q \zeta_r^b - \zeta_r^q \zeta_p^b} \begin{bmatrix} \zeta_r^b & -\zeta_r^q \\ -\zeta_p^b & \zeta_p^q \end{bmatrix} \begin{bmatrix} f^q \\ f^b \end{bmatrix}.$$

The price is given by

$$p - p^\star = \frac{\zeta_r^b f^q - \zeta_r^q f^b}{\zeta_p^q \zeta_r^b - \zeta_r^q \zeta_p^b} = x_p \hat{\alpha}_p \frac{\zeta_r^b + \zeta_r^q(1 - c_p(\mu - p^\star))}{\zeta_p^q \zeta_r^b - \zeta_r^q \zeta_p^b} = 0,$$

using the fact that $\zeta_r^q = x_a g_a'(\mu - p^\star - r^\star)$ and $c_a(\mu - p^\star) = c_p(\mu - p^\star)$.

B Derivations for Section 3

B.1 Investors' problem

The Hamilton-Jacobi-Bellman (HJB) equation for investor j can be written as

$$0 = \max_{C_j, \alpha_j} f_j(C_j, V_j) + V_{j,W}\left[rW_j + (\mu_R - r)\alpha_j W_j - C_j\right] + V_{j,X}\mu_X$$
$$+ \frac{1}{2}V_{j,WW}W_j^2\alpha_j^2\|\sigma_R\|^2 + V_{j,WX}W_j\sigma_X\sigma_R'\alpha_j + \frac{1}{2}\sum_{k=1}^{d}\sigma_{X,k}'V_{j,XX}\sigma_{X,k},$$

subject to $\alpha_j \in \Omega_j$. For ease of notation, we dropped time subscripts. Note that $V_{j,X}$ and $V_{j,WX}$ are $1 \times N$ vectors, $V_{j,XX}$ is a $N \times N$ matrix, and both $V_{j,W}$ and $V_{j,WW}$ are scalars. The drift μ_X is a $N \times 1$ vector, the diffusion σ_X is a $N \times d$ matrix, while σ_R is a $1 \times d$ vector. The notation $\sigma_{X,k}$ denotes the k−th column of σ_X, that is, the exposure to the k−th Brownian motion.

The optimal consumption is given by

$$C_j = \rho^\psi((1-\gamma_j)V_j)^{\frac{1-\gamma_j\psi}{1-\gamma_j}}V_{j,W}^{-\psi}.$$

The optimal portfolio share for an active investor is given by

$$\alpha_j = \min\left\{-\frac{V_{j,W}(\mu_R - r)}{V_{j,WW}W\|\sigma_R\|^2} - \frac{V_{WX}}{V_{WW}W}\frac{\sigma_X\sigma_R'}{\|\sigma_R\|^2}, \frac{\overline{\sigma}}{\|\sigma_{R,t}\|}\right\}.$$

Given the homotheticity of preferences, the value function for investor j can be written as

$$V_{j,t} = \left(\frac{\xi_{j,t}}{\rho^\psi}\right)^{\frac{1-\gamma_j}{1-\psi}}\frac{W_{j,t}^{1-\gamma_j}}{1-\gamma_j}. \tag{B.1}$$

This particular parametrization of the value function implies that the consumption-wealth ratio is given by

$$\frac{C_{j,t}}{W_{j,t}} = \xi_{j,t}.$$

The optimal portfolio share for active investors is given by

$$\alpha_{j,t} = \min\left\{\frac{\mu_{R,t} - r_t}{\gamma_j\|\sigma_{R,t}\|^2} - \frac{1-\gamma_j^{-1}}{1-\psi}\frac{\sigma_{\xi_j,t}\sigma_{R,t}'}{\|\sigma_{R,t}\|^2}, \frac{\overline{\sigma}}{\|\sigma_{R,t}\|}\right\}.$$

It is convenient to consider the investor's risk exposure $\sigma_j \equiv \alpha_j \|\sigma_R\|$, which is then given by

$$\sigma_{j,t} = \min\left\{\frac{\eta_t}{\gamma_j} - \frac{1-\gamma_j^{-1}}{1-\psi}\frac{\sigma_{\xi_j,t}\sigma'_{R,t}}{\|\sigma_{R,t}\|}, \overline{\sigma}\right\},$$

where $\eta_t \equiv \frac{\mu_{R,t}-r_t}{\|\sigma_{R,t}\|}$ denotes the Sharpe ratio of the risky asset.

Plugging the consumption-wealth ratio into the HJB equation and using Equation (B.1), we obtain

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\rho^{-1}\xi_j - 1\right] + r + \eta\sigma_j - \xi_j + \frac{1}{1-\psi}\left[\frac{\xi_{j,X}}{\xi_j}\mu_X + \frac{1}{2}\sum_{k=1}^{d}\sigma'_{X,k}\frac{\xi_{j,XX}}{\xi_j}\sigma_{X,k}\right]$$
$$-\frac{\gamma_j}{2}\sigma_j^2 + \frac{1-\gamma_j}{1-\psi}\frac{\xi_{j,X}}{\xi_j}\sigma_X\frac{\sigma'_R}{\|\sigma_R\|}\sigma_j + \frac{1}{2}\frac{\psi-\gamma_j}{(1-\psi)^2}\sum_{k=1}^{d}\sigma'_{X,k}\frac{\xi'_{j,X}}{\xi_j}\frac{\xi_{j,X}}{\xi_j}\sigma_{X,k}.$$

Rearranging the expression above, we obtain

$$\xi_{j,t} = \psi\rho + (1-\psi)\left[r_t + \eta_t\sigma_{j,t} - \frac{\gamma_j}{2}\sigma_{j,t}^2\right] + \mu_{\xi_j,t} + (1-\gamma_j)\sigma_{\xi_j,t}\frac{\sigma'_{R,t}}{\|\sigma_{R,t}\|}\sigma_{j,t} + \frac{\psi-\gamma_j}{1-\psi}\frac{\|\sigma_{\xi_j,t}\|^2}{2},$$

where the law of motion of $\xi_{j,t}$ is given by

$$\frac{d\xi_{j,t}}{\xi_{j,t}} = \mu_{\xi_j,t}dt + \sigma_{\xi_j,t}dZ_t,$$

and the drift and diffusion of $\xi_{j,t}$ are given by Ito's lemma:

$$\mu_{\xi_j,t} = \frac{\xi_{j,X}}{\xi_j}\mu_{X,t} + \frac{1}{2}\sum_{k=1}^{d}\sigma'_{X,k,t}\frac{\xi_{j,XX,t}}{\xi_{j,t}}\sigma_{X,k,t}, \qquad \sigma_{\xi_j,t} = \frac{\xi_{j,X}}{\xi_j}\sigma_{X,t}.$$

B.2 Pricing condition

Let $y_t \equiv Y_t/P_t$ denote the dividend yield on the risky asset. From Equation (9), we can write the expected return on the risky asset as:

$$r_t + \eta_t\|\sigma_{R,t}\| = y_t + \frac{1}{dt}\frac{d(Y_t/y_t)}{(Y_t/y_t)} = y_t + \mu - \mu_{y,t} + \|\sigma_{y,t}\|^2 - \sigma\sigma'_{y,t}.$$

Rearranging the expression above, we obtain

$$y_t = r_t + \eta_t\|\sigma_{R,t}\| - \mu + \mu_{y,t} - \|\sigma_{R,t}\|^2 + \sigma\sigma'_{R,t}, \tag{B.2}$$

where $\sigma_{R,t} = \sigma - \sigma_{y,t}$ and $(\mu_{y,t}, \sigma_{y,t})$ are given by Ito's lemma:

$$\mu_{y,t} = y_{X,t}\mu_{X,t} + \frac{1}{2}\sum_{k=1}^{d} \sigma'_{X,k,t}\, y_{XX,t}\, \sigma_{X,k,t}, \qquad \sigma_{y,t} = y_{X,t}\sigma_{X,t}.$$

B.3 Aggregate state variable

Define the share of wealth of type-j investors as follows

$$x_{j,t} \equiv \frac{\omega_j W_{j,t}}{P_t}.$$

We define the aggregate state variable as $X_t = (x_t, \overline{\alpha}_{p,t})$, where $x_t \equiv (x_{1,t}, x_{2,t}, \ldots, x_{J,t})$. The law of motion of $\overline{\alpha}_{p,t}$ is given by (10). To compute the law of motion of $x_{j,t}$, first, note that the law of motion of wealth for a type-j investor can be written as

$$\frac{dW_{j,t}}{W_{j,t}} = \left[r_t + \eta_t\sigma_{j,t} - \xi_{j,t}\right] dt + \sigma_{j,t}\frac{\sigma_{R,t}}{\|\sigma_{R,t}\|} dZ_t$$

From Ito's lemma, the law of motion of $x_{j,t}$ is given by

$$\begin{aligned}\frac{dx_{j,t}}{x_{j,t}} = &\left(r_t + \eta_t\sigma_{j,t} - \xi_{j,t} - \mu + \mu_{y,t} + \sigma\sigma'_{R,t} - \sigma_{j,t}\|\sigma_{R,t}\| + \kappa\frac{\omega_j - x_{j,t}}{x_{j,t}}\right) dt \\ &+ (\sigma_{j,t} - \|\sigma_{R,t}\|)\frac{\sigma_{R,t}}{\|\sigma_{R,t}\|} dZ_t,\end{aligned}$$

using $\mu_{P,t} = \mu - \mu_{y,t} + \|\sigma_{R,t}\|^2 - \sigma\sigma'_{R,t}$.

B.4 Asset prices

Let $\mathcal{J}_t^u \subset \{1, 2, \ldots, J\}$ denote the set of unconstrained active investors at period t, that is, the set of investors such that $\sigma_{j,t} < \overline{\sigma}$. Let $\mathcal{J}_t^c \subset \{1, 2, \ldots, J\}$ denote the set of constrained active investors, that is, the set of investors such that $\sigma_{j,t} = \overline{\sigma}$. From the market clearing condition for the risky asset in Equation (15), we obtain

$$\eta_t = \frac{\gamma_{u,t}}{x_{u,t}}\left[(1 - \overline{\alpha}_{p,t}x_{0,t})\|\sigma_{R_t}\| - \overline{\sigma}x_{c,t} + \sum_{j\in\mathcal{J}_t^u} x_{j,t}\frac{1 - \gamma_j^{-1}}{1 - \psi}\frac{\sigma_{\xi_j,t}\sigma'_{R,t}}{\|\sigma_{R,t}\|}\right],$$

where $x_{u,t} \equiv \sum_{j\in\mathcal{J}_t^u} x_{j,t}$, $x_{c,t} \equiv \sum_{j\in\mathcal{J}_t^c} x_{j,t}$, and $\gamma_{u,t} \equiv \left[\frac{1}{x_{u,t}}\sum_{j\in\mathcal{J}_t^u}\frac{x_{j,t}}{\gamma_j}\right]^{-1}$ is the aggregate risk aversion of the unconstrained investors.

From the market clearing condition for goods, the first term in Equation (15), we obtain

$$
\begin{aligned}
y_t = \psi\rho + (1-\psi)&\left[r_t + \eta_t\|\sigma_{R,t}\| - \sum_{j=0}^{J} x_{j,t}\frac{\gamma_j}{2}\sigma_{j,t}^2\right] \\
&+ \sum_{j=0}^{J} x_{j,t}\left[\mu_{\xi_j,t} + (1-\gamma_j)\sigma_{\xi_j,t}\frac{\sigma'_{R,t}}{\|\sigma_{R,t}\|}\sigma_{j,t} + \frac{\psi-\gamma_j}{1-\psi}\frac{\|\sigma_{\xi_j,t}\|^2}{2}\right].
\end{aligned}
$$

Using the pricing condition (B.2), we obtain the expression for the risk-free rate

$$
\begin{aligned}
r_t = \rho - \eta_t\|\sigma_{R,t}\| + \psi^{-1}(\mu - \mu_{y,t} + \|\sigma_{R,t}\|^2 - \sigma\sigma'_{R,t}) + \left(1-\psi^{-1}\right)\sum_{j=0}^{J} x_{j,t}\frac{\gamma_j}{2}\sigma_{j,t}^2 \\
+ \psi^{-1}\sum_{j=0}^{J} x_{j,t}\left[\mu_{\xi_j,t} + (1-\gamma_j)\sigma_{\xi_j,t}\frac{\sigma'_{R,t}}{\|\sigma_{R,t}\|}\sigma_{j,t} + \frac{\psi-\gamma_j}{1-\psi}\frac{\|\sigma_{\xi_j,t}\|^2}{2}\right].
\end{aligned}
$$

B.5 The system of PDEs

To compute the equilibrium, one needs to solve a system of $J+2$ partial differential equations (PDEs), involving the consumption-wealth ratio $\xi_j(X)$ for the $J+1$ type of investors and the dividend yield $y(X)$. These functions depend on $J+1$ state variables, the $J-$dimensional vector x_t and the portfolio share of passive investors $\overline{\alpha}_{p,t}$.

The differential equation for the consumption-wealth ratio is given by

$$
\begin{aligned}
\xi_{j,t} = \psi\rho + (1-\psi)\left[r_t + \eta_t\sigma_{j,t} - \frac{\gamma_j}{2}\sigma_{j,t}^2\right] + \frac{\xi_{j,X}}{\xi_j}\mu_{X,t} + \frac{1}{2}\sum_{k=1}^{d}\sigma'_{X,k,t}\frac{\xi_{j,XX,t}}{\xi_{j,t}}\sigma_{X,k,t} \\
+ (1-\gamma_j)\frac{\xi_{j,X}}{\xi_j}\sigma_{X,t}\frac{\sigma'_{R,t}}{\|\sigma_{R,t}\|}\sigma_{j,t} + \frac{\psi-\gamma_j}{1-\psi}\frac{1}{2}\left\|\frac{\xi_{j,X}}{\xi_j}\sigma_{X,t}\right\|^2.
\end{aligned}
$$

Plugging the expressions for interest rate and the Sharpe ratio (r_t, η_t), the risk exposure $\sigma_{j,t}$, the drift and diffusion of the aggregate state variables $(\mu_{X,t}, \sigma_{X,t})$, and the aggregate volatility $\|\sigma_{R,t}\|$, we can express the condition above in terms of $\xi_{j,t}$ and y_t and their derivatives.

Similarly, we can write the condition for the dividend yield:

$$
\begin{aligned}
y_t = r_t + \eta_t\left\|\sigma - \frac{y_{X,t}}{y_t}\sigma_{X,t}\right\| - \mu + \frac{y_{X,t}}{y_t}\mu_{X,t} + \frac{1}{2}\sum_{k=1}^{d}\sigma'_{X,k,t}\frac{y_{XX,t}}{y_t}\sigma_{X,k,t} \\
- \left\|\sigma - \frac{y_{X,t}}{y_t}\sigma_{X,t}\right\|^2 + \sigma\left(\sigma - \frac{y_{X,t}}{y_t}\sigma_{X,t}\right)',
\end{aligned}
$$

which again can be expressed only in terms of $\xi_{j,t}$ and y_t and their derivatives.

C Derivations for Section 4

C.1 Proof of Lemma 1

Proof. The assumption $\epsilon = 0$ implies that there is no preference heterogeneity and passive investors are fully invested in the risky asset. We guess and verify that in this benchmark economy, there are no variation in expected returns. In particular, the wealth distribution plays no role in the economy. This implies that $\mu_{c_j,0}(X) = \sigma_{c_j,0}(X) = \mu_{p,0}(X) = \sigma_{p,0}(X) = 0$. In this case, the risk premium is given by

$$\pi_0(X) = \frac{\gamma}{x_{u,t}} \left[1 - x_{0,t} - x_{c,t}\overline{\alpha}_{c,t}\right] \|\sigma_{R_t}\|^2,$$

using the fact that $\varsigma_t = 0$, as $\sigma_{c_j,t} = 0$, and $\overline{\alpha}_{p,t} = 1$.

Given that $\sigma_{y,t} = 0$, we have that $\sigma_{R,0}(X) = \sigma$. Using the fact that $\overline{\sigma} = \|\sigma\|$ and $x_{u,t} = 1 - x_{0,t} - x_{c,t}$, we obtain the risk premium

$$\pi_0(X) = \gamma\|\sigma\|^2,$$

using $\alpha_{c,t} = 1$. Using $\sigma_{c_j,t} = 0$ and the expression for $\pi_0(X)$, we obtain that $\alpha_{j,0}(X) = 1$, for $j = 1, \dots, J$, from Equation (16).

The interest rate is given by

$$r_0(X) = \rho + \psi^{-1}\mu - \gamma(1 + \psi^{-1})\frac{\|\sigma\|^2}{2}.$$

The consumption-wealth ratio $c_{j,0}(X)$ is given by

$$c_{j,0}(X) = \psi\rho + (1 - \psi)\left[r_0(X) + \pi_0(X)\alpha_{j,0}(X) - \frac{\gamma}{2}\alpha_{j,0}(X)^2\|\sigma\|^2\right].$$

Plugging in the expression for $r_0(X)$, $\eta_0(X)$ and $\sigma_{j,0}$, we obtain

$$c_{j,0}(X) = \rho - \left(1 - \psi^{-1}\right)\left(\mu - \frac{\gamma\|\sigma\|^2}{2}\right),$$

where we assume $\rho > \left(1 - \psi^{-1}\right)\left(\mu - \frac{\gamma\|\sigma\|^2}{2}\right)$.

From the market clearing condition for goods, we obtain:

$$\frac{1}{p_0(X)} = \rho - \left(1 - \psi^{-1}\right)\left(\mu - \frac{\gamma\|\sigma\|^2}{2}\right).$$

The drift and diffusion of the wealth shares are given

$$\mu_{X,j,0}(X) = x_j\left[r_0(X) + \pi_0(X)\alpha_{j,0}(X) - c_{j,0}(X) - \mu\right]$$
$$\sigma_{X,j,0}(X) = x_j(\alpha_{j,0}(X) - 1)\sigma_{R,0}(X),$$

where $\mu_{X,j,0} = \sigma_{X,j,0} = 0$, using the expression for returns, portfolio share, and consumption-wealth ratio. The result $\mu_{X,j,0} = 0$ uses the fact that $\kappa = 0$. ☐

C.2 Proof of Proposition 2

Proof. We consider next the first-order correction terms. Note that the diffusion terms for c_j and p are both equal to zero up to the first order, since $\sigma_{c_j,t} = O(\epsilon^2)$ and $\sigma_{p,t} = O(\epsilon^2)$. From Ito's lemma:

$$\sigma_{c_j,t} = \underbrace{\frac{c_{j,X}}{c_j}}_{O(\epsilon)}\underbrace{\sigma_{X,t}}_{O(\epsilon)} = O(\epsilon^2).$$

We have $c_{j,X} = O(\epsilon)$, because $c_{j,X,0} = 0$, as $c_{j,0}(X)$ does not depend on X. Also, $\sigma_{X,t} = O(\epsilon)$ because $\sigma_{X,0}(X) = 0$. This implies that $\sigma_{c_j,1}(X) = 0$. An analogous argument applies to p_t, so that we have $\sigma_{p,1}(X) = 0$ and $\|\sigma_{R,1}(X)\| = 0$.

Risk exposure of active investors. The risk exposure for active investors can be written as

$$\sigma_j(X,\epsilon) = \min\left\{\frac{\eta(X,\epsilon)}{\gamma_j} + \frac{1-\gamma_j^{-1}}{\psi - 1}\frac{\sigma_{c_j}(X,\epsilon)\sigma_R'(X,\epsilon)}{\|\sigma_R(X,\epsilon)\|}, \|\sigma\| + \hat{\sigma}\epsilon\right\},$$

where $\sigma_j(X;\epsilon) \equiv \alpha_j(X;\epsilon)\|\sigma_R(X;\epsilon)\|$ $\eta(X;\epsilon) \equiv \frac{\pi(X;\epsilon)}{\|\sigma_R(X;\epsilon)\|}$

Expanding the first term inside brackets in ϵ, we obtain

$$\sigma_j(X,\epsilon) = \min\left\{\frac{\eta_0(X)}{\gamma} + \left(\frac{\eta_1(X)}{\gamma} - \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j\right)\epsilon + \frac{1-\gamma^{-1}}{\psi-1}\frac{\sigma_{c_j,2}(X)\sigma'}{\|\sigma\|}\epsilon^2 + O(\epsilon^3), \|\sigma\| + \hat{\sigma}\epsilon\right\},$$

Adding and subtracting $\|\sigma\| + \hat{\sigma}\epsilon$, and using $\frac{\eta_0(X)}{\gamma} = \|\sigma\|$, we obtain

$$\sigma_j(X,\epsilon) = \min\left\{\left(\frac{\eta_1(X)}{\gamma} - \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j - \hat{\sigma}\right)\epsilon + \frac{1-\gamma^{-1}}{\psi-1}\frac{\sigma_{c_j,2}(X)\sigma'}{\|\sigma\|}\epsilon^2 + O(\epsilon^3), 0\right\} + \|\sigma\| + \hat{\sigma}\epsilon. \quad \text{(C.1)}$$

Consider first the case where the following condition is satisfied:

$$\frac{\eta_1(X)}{\gamma} - \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j - \hat{\sigma} = O(1), \quad \text{(C.2)}$$

If this is the case, then

$$\left(\frac{\eta_1(X)}{\gamma} - \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j - \hat{\sigma}\right)\epsilon \gg \left|\frac{1-\gamma^{-1}}{\psi - 1}\frac{\sigma_{c_j,2}(X)\sigma'}{\|\sigma\|}\right|\epsilon^2,$$

for small ϵ. So, the sign of the term inside the min operator in (C.1) is determined by the first term.

We can then write $\sigma_j(X,\epsilon)$ as follows:

$$\sigma_j(X,\epsilon) = \|\sigma\| + \min\left\{\frac{\eta_1(X)}{\gamma} - \frac{\eta_0(X)}{\gamma}\hat{\gamma}, \hat{\sigma}\right\}\epsilon + O(\epsilon^2).$$

In the region of the state space where condition (C.2) holds, one can determine whether an investor is constrained or unconstrained only based on the first-order terms. Suppose now that the following condition holds

$$\frac{\eta_1(X)}{\gamma} - \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j - \hat{\sigma} = O(\epsilon). \tag{C.3}$$

This condition states that, up to the first order, the leverage constraint is either always binding or slack by just a tiny amount parameterized by ϵ and ϵ^2 terms inside the min operator in (C.1). In this case, we can write $\sigma_j(X,\epsilon)$ as follows:

$$\sigma_j(X,\epsilon) = \|\sigma\| + \hat{\sigma}\epsilon + \min\left\{\left(\frac{\eta_1(X)}{\gamma} - \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j - \hat{\sigma}\right)\epsilon + \frac{1-\gamma^{-1}}{\psi-1}\frac{\sigma_{c_j,2}(X)\sigma'}{\|\sigma\|}\epsilon^2, 0\right\} + O(\epsilon^3).$$

In this region of the state space where condition (C.3) is satisfied, we need the second-order term to determine whether an investor is constrained. This distinction will be relevant when computing the second-order correction.

For the first-order correction terms here, we focus on the case where condition (C.2) holds.

Aggregate risk aversion. The aggregate risk aversion of unconstrained investors, defined above, is given by

$$\gamma_u(X,\epsilon) = \frac{x_u}{\sum_{j\in\mathcal{J}^u}\frac{x_j}{\gamma(1+\hat{\gamma}\epsilon)}} = \gamma + \gamma\sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j\,\epsilon + O(\epsilon^2)$$

Market price of risk. The market price of risk can be written as

$$\begin{aligned}\eta(X,\epsilon) &= \frac{\gamma_u(X,\epsilon)}{1-x_0-x_c}\left[(1-(1+\hat{\alpha}_p\epsilon)x_0)\|\sigma\| - (\|\sigma\|+\hat{\sigma}\epsilon)x_c\right] + O(\epsilon^2),\\ &= \eta_0(X) + \gamma\|\sigma\|\left[\sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j - \left(\frac{\hat{\alpha}_p x_0 + \frac{\hat{\sigma}}{\|\sigma\|}x_c}{1-x_0-x_c}\right)\right]\epsilon + O(\epsilon^2).\end{aligned}$$

In the region of the state space where all active investors are unconstrained, we have

$$\eta(X,\epsilon) = \eta_0(X) + \gamma\|\sigma\| \left(\sum_{j\in\mathcal{J}^u} \frac{x_j}{x_u}\hat{\gamma}_j - \frac{\hat{\alpha}_p\, x_0}{1-x_0} \right)\epsilon + O(\epsilon^2).$$

The expression above shows the impact of fluctuations in the aggregate risk aversion and the effect of portfolio inflows in the market price of risk. If the average risk aversion in state X is lower than its level at $\epsilon = 0$, then the market price of risk will be lower than its level at $\epsilon = 0$, everything else constant.

Interest rate. The interest rate is given by

$$\begin{aligned} r(X,\epsilon) &= r_0(X) + \left[-\eta_1(X)\|\sigma\| + \left(1-\psi^{-1}\right)\left(\sum_{j=0}^{J} x_{j,t}\frac{\gamma}{2}2\|\sigma\|\sigma_{j,1}(X) + \sum_{j=0}^{J} x_{j,t}\frac{\gamma}{2}\|\sigma\|^2\hat{\gamma}_j \right)\right]\epsilon + O(\epsilon^2) \\ &= r_0(X) + \gamma\|\sigma\|^2 \left[-\frac{\eta_1(X)}{\gamma\|\sigma\|} + \left(1-\psi^{-1}\right)\sum_{j=0}^{J} x_{j,t}\frac{\hat{\gamma}_j}{2} \right]\epsilon + O(\epsilon^2) \\ &= r_0(X) - \gamma\|\sigma\|^2 \left[\left(\sum_{j\in\mathcal{J}^u} \frac{x_j}{x_u}\hat{\gamma}_j - \sum_{j=0}^{J} x_{j,t}\frac{\hat{\gamma}_j}{2} \right) - \frac{\hat{\alpha}_p x_0 + \frac{\hat{\sigma}}{\|\sigma\|}x_c}{1-x_0-x_c} + \psi^{-1}\sum_{j=0}^{J} x_{j,t}\frac{\hat{\gamma}_j}{2} \right]\epsilon + O(\epsilon^2), \end{aligned} \tag{C.4}$$

where we use the fact that $\sum_{j=0}^{J} x_{j,t}\sigma_{j,1}(X) = 0$ from the first-order correction of the risky asset market clearing condition.

The term in the square brackets in Equation (C.4) captures the first-order effect of frictions on the interest rate, $r_1(X)$. First, we see that the interest rate is decreasing in the difference between the average risk aversion of unconstrained investors and the average risk aversion of all investors in the economy. These two averages can differ for two reasons. First, passive investors may have a risk aversion different from the average unconstrained investor, the first term in parentheses in $r_1(X)$. Second, constrained investors are exactly the ones with low risk aversion, so unconstrained investors are on average more risk averse than all investors in the economy, which include the low risk aversion ones.

Consumption-wealth ratio. The consumption-wealth ratio for investor j is given by

$$\begin{aligned} c_j(X,\epsilon) &= c_{j,0}(X) + (1-\psi)\Big[r_1(X) + \eta_0(X)\sigma_{j,1}(X) + \eta_1(X)\sigma_{j,0}(X) \\ &\qquad -\gamma\|\sigma\|\sigma_{j,1}(X) - \frac{\gamma}{2}\hat{\gamma}_j\|\sigma\|^2\Big]\epsilon + O(\epsilon^2) \\ &= c_{j,0}(X) + (1-\psi)\gamma\|\sigma\|^2\left[\frac{r_1(X)}{\gamma\|\sigma\|^2} + \frac{\eta_1(X)}{\gamma\|\sigma\|} - \frac{\hat{\gamma}_j}{2}\right]\epsilon + O(\epsilon^2) \\ &= c_{j,0}(X) + (1-\psi)\gamma\|\sigma\|^2\left[\left(1-\psi^{-1}\right)\sum_{k=0}^{J} x_{k,t}\frac{\hat{\gamma}_k}{2} - \frac{\hat{\gamma}_j}{2}\right]\epsilon + O(\epsilon^2). \end{aligned}$$

Dividend yield. The dividend yield is given by

$$y(X,\epsilon) = y_0(X) + \left(1-\psi^{-1}\right)\gamma\|\sigma\|^2\sum_{j=0}^{J} x_{j,t}\frac{\hat{\gamma}_j}{2}\epsilon + O(\epsilon^2).$$

Notice that portfolio flows do not affect the dividend yield up to first order. The reason is that the interest rate and risk premium effects exactly cancel each other out. To derive the effect of portfolio flows on asset prices, we need to consider the second-order correction.

The price-dividend ratio is then given by

$$p(X,\epsilon) = p_0(X) - p_0(X)^2\left(1-\psi^{-1}\right)\gamma\|\sigma\|^2\sum_{j=0}^{J} x_{j,t}\frac{\hat{\gamma}_j}{2}\epsilon + O(\epsilon^2).$$

State dynamics. The diffusion of the wealth share of investor j, $j = 1, \ldots, J$, is given by

$$\begin{aligned} \sigma_{X,j}(X) &= x_j\sigma_{j,1}(X)\frac{\sigma}{\|\sigma\|}\epsilon + O(\epsilon^2) \\ &= x_j \min\left\{\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_j - \left(\frac{\hat{\alpha}_p x_0 + \frac{\hat{\sigma}}{\|\sigma\|}x_c}{1-x_0-x_c}\right) - \hat{\gamma}_j,\ \frac{\hat{\sigma}}{\|\sigma\|}\right\}\sigma\epsilon + O(\epsilon^2). \end{aligned}$$

The drift of the wealth share of investor j, $j = 1, \ldots, J$, is given by

$$\begin{aligned} \mu_{X,j}(X,\epsilon) &= x_j\left[r_1(X) + \eta_1(X)\sigma_{j,0}(X) + \eta_0(X)\sigma_{j,1}(X) - \xi_{j,1}(X) - \sigma_{j,1}(X)\|\sigma\|\right]\epsilon + \kappa(\omega_j - x_j) + O(\epsilon^2) \\ &= x_j\left[(\psi-1)\gamma\|\sigma\|^2\left(\sum_{k=0}^{J} x_{k,t}\frac{\hat{\gamma}_k}{2} - \frac{\hat{\gamma}_j}{2}\right) + (\gamma-1)\|\sigma\|\sigma_{j,1}(X)\right]\epsilon + \kappa(\omega_j - x_j) + O(\epsilon^2) \end{aligned}$$

The law of motion of $\hat{\alpha}_p$ can be written as follows:

$$d\hat{\alpha}_p = \theta_p(\overline{\hat{\alpha}} - \hat{\alpha}_p)dt + \sigma_p dZ_t,$$

so the drift of $\overline{\alpha}_p$ is $\mu_{\overline{\alpha}_p} = \theta_p(\hat{\overline{\alpha}} - \hat{\alpha}_p)\epsilon$ and the diffusion is $\sigma_{\overline{\alpha}_p} = \sigma_p\epsilon$.

□

C.3 Second-order correction

In Proposition 5, we compute the second-order correction for our economy.

Proposition 5 (Second-order correction)**.** *Suppose* $\rho > \left(1 - \psi^{-1}\right)\left(\mu - \frac{\gamma\sigma^2}{2}\right)$*. Then,*

(i) The second-order correction for the consumption-wealth ratio and risk exposure are given by:

$$\begin{aligned}
c_{j,2}(X) &= (1-\psi)\frac{\gamma\sigma^2}{2}\left[\left(1-\psi^{-1}\right)\sum_{k=0}^{J} x_{k,t}\hat{\gamma}_k - \hat{\gamma}_j\right] \\
\sigma_{j,2}(X) &= \frac{\eta_2(X)}{\gamma} - \frac{\eta_1(X)}{\gamma}\hat{\gamma}_j + \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j^2 - (1-\gamma^{-1})\sigma_{y,2}(X),
\end{aligned}$$

where

$$\sigma_{y,2}(X) = \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2y_0(X)}\sum_{k=1}^{J}\left(\hat{\gamma}_k - \hat{\gamma}_0\right)x_k\sigma_{k,1}(X)\frac{\sigma}{\|\sigma\|}.$$

(ii) The second-order correction for the Sharpe ratio, interest rate, and dividend yield are given by:

$$\begin{aligned}
\eta_2(X) &= -\frac{(\gamma-1)x_c + 1 - x_0}{1 - x_0 - x_c}\sigma_{y,2}(X) - \gamma\sigma\mathbb{E}^u[\hat{\gamma}_j]\frac{\hat{\alpha}_p x_0 + \frac{\hat{\sigma}}{\sigma}x_c}{1 - x_0 - x_c} - \gamma\sigma\text{Var}^u[\hat{\gamma}_j] \\
r_2(X) &= -\eta_2(X)\sigma + \eta_0(X)\sigma_{y,2}(X) - \psi^{-1}(\mu_{y,2}(X) + \sigma\sigma_{y,2}(X)) \\
&\quad + \left(1-\psi^{-1}\right)\sum_{j=0}^{J} x_j\gamma\sigma\left[\sigma_{j,2}(X) + \frac{\sigma_{j,1}^2(X)}{2\sigma} + \hat{\gamma}_j\sigma_{j,1}\right] \\
&\quad + \psi^{-1}\sum_{j=0}^{J} x_j\left[\mu_{\xi_j,2}(X) + (1-\gamma)\sigma_{\xi_j,2}(X)\sigma\right] \\
y_2(X) &= \left(1-\psi^{-1}\right)\sum_{j=0}^{J} x_j\gamma\sigma\left(\frac{\sigma_{j,1}^2(X)}{2\sigma} + \hat{\gamma}_j\sigma_{j,1}(X)\right),
\end{aligned} \tag{C.5}$$

where

$$\mathbb{E}^u[\hat{\gamma}_j] \equiv \sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j, \quad \textit{and} \quad \text{Var}^u[\hat{\gamma}_j] \equiv \sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j^2 - \left(\sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j\right)^2.$$

Proof. **Step 1: Laws of motion for c_j and y.**

We start by considering the diffusion terms for c_j and y. The expansion of $\sigma_{c_j,t}$ in ϵ is given by

$$\begin{aligned}
\sigma_{c_j}(X,\epsilon) &= \frac{c_{j,X}(X,\epsilon)}{c_j(X,\epsilon)}\sigma_X(X,\epsilon) \\
&= \frac{c_{j,X,1}(X)}{c_{j,0}(X)}\sigma_{X,1}(X)\epsilon^2 + O(\epsilon^3) \\
&= -(\psi-1)\left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2c_{j,0}(X)}\sum_{k=1}^{J}(\hat{\gamma}_k-\hat{\gamma}_0)\,x_k\sigma_{k,1}(X)\epsilon^2 + O(\epsilon^3),
\end{aligned}$$

where we used the fact that $c_{j,\overline{\alpha}_p,1}(X) = 0$. Notice that $\sigma_{c_j,2}(X)$ does not depend on j, that is, it is the same for all investors. Moreover, $\sigma_{c_j,2}(X) > 0$, as $\sigma_{k,1}(X)$ is inversely related to $\hat{\gamma}_k$.

Similarly, the diffusion for dividend yield y can be written as

$$\begin{aligned}
\sigma_y(X,\epsilon) &= \frac{y_{X,1}(X)}{y_0(X)}\sigma_{X,1}(X)\epsilon^2 + O(\epsilon^2) \\
&= \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2y_0(X)}\sum_{k=1}^{J}(\hat{\gamma}_k-\hat{\gamma}_0)\,x_k\sigma_{k,1}(X)\epsilon^2 + O(\epsilon^3).
\end{aligned}$$

where we used the fact that $y_{\overline{\alpha}_p,1}(X) = 0$. The expression above is negative if $\psi > 1$, which implies that $\sigma_{R,2}(X) = -\sigma_{y,2}(X)$ is positive. In this case, a negative aggregate shock redistribute wealth to more risk averse investors, leading to a rise in the risk premium and a decline in the risk-free rate. If $\psi > 1$, the risk premium effect dominates, so the price-dividend ratio, $1/y$, falls in response to the shock. The movement in the price-dividend ratio amplifies the initial effect of the drop in dividends.

The drift of $c_{j,t}$ is given by

$$\begin{aligned}
\mu_{c_j}(X,\epsilon) &= \frac{c_{j,X}(X,\epsilon)}{c_j(X,\epsilon)}\mu_X(X,\epsilon) + \frac{1}{2}\sigma_X'(Z,\epsilon)\frac{c_{j,XX}(X,\epsilon)}{c_j(X,\epsilon)}\sigma_X(X,\epsilon) \\
&= \frac{c_{j,X,1}(X)}{c_{j,0}(X)}\mu_{X,1}(X)\epsilon^2 + O(\epsilon^3) \\
&= -(\psi-1)\left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2c_{j,0}(X)}\sum_{k=1}^{J}(\hat{\gamma}_k-\hat{\gamma}_0)\,\mu_{X_k,1}(X)\epsilon^2 + O(\epsilon^3).
\end{aligned}$$

The drift for y_t is given by

$$\begin{aligned}
\mu_y(X,\epsilon) &= \frac{y_X(X,\epsilon)}{y(X,\epsilon)}\mu_X(X,\epsilon) + \frac{1}{2}\sigma_X'(Z,\epsilon)\frac{y_{XX}(X,\epsilon)}{y(X,\epsilon)}\sigma_X(X,\epsilon) \\
&= \frac{y_{X,1}(X)}{y_0(X)}\mu_{X,1}(X)\epsilon^2 + O(\epsilon^3) \\
&= \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2y_0(X)}\sum_{k=1}^{J}\left(\hat{\gamma}_k - \hat{\gamma}_0\right)\mu_{X_k,1}(X)\epsilon^2 + O(\epsilon^3),
\end{aligned}$$

where we used the fact that $c_{j,\overline{\alpha}_p,1} = y_{\overline{\alpha}_p,1} = 0$.

Step 2: Risk exposures of investors.

We focus on the *inner region*, that is, the case where all investors are sufficiently far from the constraint boundary (on either side). For a constrained investor, the second-order term is zero. For an unconstrained investor, the second-order term is given by

$$\sigma_{j,2}(X) = \frac{\eta_2(X)}{\gamma} - \frac{\eta_1(X)}{\gamma}\hat{\gamma}_j + \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j^2 + \frac{1-\gamma^{-1}}{\psi-1}\frac{\sigma_{c_j,2}(X)\sigma'}{\|\sigma\|},$$

where investor j is unconstrained if the following condition holds:

$$\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_k - \left(\frac{\hat{\alpha}_p x_0 + \hat{\sigma}x_c}{1-x_0-x_c}\right) - \hat{\gamma}_j < \hat{\sigma}.$$

Step 3: Aggregate risk aversion.

The aggregate risk aversion can be written as

$$\gamma_u(X,\epsilon) = \gamma\left[1 + \mathbb{E}^u\left[\hat{\gamma}_j\right]\epsilon - \mathrm{Var}^u\left[\hat{\gamma}_j\right]\epsilon^2\right] + O(\epsilon^3),$$

where

$$\mathbb{E}^u\left[\hat{\gamma}_j\right] \equiv \sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j, \quad \text{and} \quad \mathrm{Var}^u[\hat{\gamma}_j] \equiv \sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j^2 - \left(\sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j\right)^2.$$

Step 4: Market price of risk.

The market price of risk is given by

$$\eta(X,\epsilon) = \frac{\gamma_u(X,\epsilon)}{x_u}\left[(1-(1+\hat{\alpha}_p\epsilon)x_{0,t})\|\sigma - \sigma_y(X,\epsilon)\| - \|\sigma\|(1+\hat{\sigma}\epsilon)x_{c,t} + \sum_{j\in\mathcal{J}_t^u}x_{j,t}\frac{1-\gamma_j^{-1}}{1-\psi}\sigma_{c_j}(X,\epsilon)\frac{\sigma'}{\|\sigma\|}\right],$$

The second-order term is then given by

$$\eta_2(X) = \frac{\gamma_{u,0}(X)}{x_u}\left[-(1-x_{0,t})\sigma_{y,2}(X)\frac{\sigma'}{\|\sigma\|} + \sum_{j\in\mathcal{J}_t^u} x_{j,t}\frac{1-\gamma^{-1}}{1-\psi}\sigma_{c_j,2}(X)\frac{\sigma'}{\|\sigma\|}\right] + \\ + \frac{\gamma_{u,1}(X)}{x_u}\|\sigma\|\left[-\hat{\alpha}_p x_0 - \hat{\sigma}x_c\right] + \frac{\gamma_{u,2}(X)}{x_u}\|\sigma\|\left[1 - x_0 - x_c\right].$$

The expression above can be written as

$$\eta_2(X) = -\frac{(\gamma-1)x_c + 1 - x_0}{1-x_0-x_c}\sigma_{y,2}(X)\frac{\sigma'}{\|\sigma\|} - \gamma\|\sigma\|\mathbb{E}^u[\hat{\gamma}_j]\frac{\hat{\alpha}_p x_0 + \hat{\sigma}x_c}{1-x_0-x_c} - \gamma\|\sigma\|\text{Var}^u[\hat{\gamma}_j].$$

using the fact that $\sigma_{c_j,2}(X) = (1-\psi)\sigma_{y,2}$.

Step 5: Interest rate.

The interest rate is given by

$$r(X,\epsilon) = \rho - \eta(X,\epsilon)\|\sigma_R(X,\epsilon)\| + \psi^{-1}(\mu - \mu_y(X,\epsilon) - \sigma_y\sigma_R(X,\epsilon)') \\ + \left(1-\psi^{-1}\right)\sum_{j=0}^{J} x_j\frac{\gamma_j}{2}\sigma_j^2(X,\epsilon) \\ + \psi^{-1}\sum_{j=0}^{J} x_{j,t}\left[\mu_{c_j}(X,\epsilon) + (1-\gamma_j)\sigma_{c_j}(X,\epsilon)\frac{\sigma'_{R,t}}{\|\sigma_R\|}\sigma_j(X,\epsilon) + \frac{\psi-\gamma_j}{1-\psi}\frac{\|\sigma_{c_j,t}(X,\epsilon)\|^2}{2}\right].$$

The second-order term is given by

$$r_2(X) = -\eta_2(X)\|\sigma\| + \eta_0(X)\sigma_{y,2}(X)\frac{\sigma'}{\|\sigma\|} - \psi^{-1}(\mu_{y,2}(X) + \sigma_{y,2}(X)\sigma') \\ + \left(1-\psi^{-1}\right)\sum_{j=0}^{J} x_j\gamma\|\sigma\|\left[\sigma_{j,2}(X) + \frac{\sigma_{j,1}^2(X)}{2\|\sigma\|} + \hat{\gamma}_j\sigma_{j,1}(X)\right] \\ + \psi^{-1}\sum_{j=0}^{J} x_j\left[\mu_{c_j,2}(X) + (1-\gamma)\sigma_{c_j,2}(X)\sigma'\right].$$

Step 6: Dividend yield.

The dividend yield, y, is given by

$$
\begin{aligned}
y(X,\epsilon) = \psi\rho + (1-\psi)&\left[r(X,\epsilon) + \eta(X,\epsilon)\|\sigma_R(X,\epsilon)\| - \sum_{j=0}^{J} x_j \frac{\gamma_j}{2}\sigma_j^2(X,\epsilon)\right] \\
&+ \sum_{j=0}^{J} x_j \left[\mu_{c_j}(X,\epsilon) + (1-\gamma_j)\sigma_{c_j}(X,\epsilon)\frac{\sigma_R(X;\epsilon)'}{\|\sigma_R(X;\epsilon)\|}\sigma_j(X,\epsilon) + \frac{\psi-\gamma_j}{1-\psi}\frac{\|\sigma_{c_j}(X,\epsilon)\|^2}{2}\right].
\end{aligned}
$$

The second-order term in the expansion of $y(X,\epsilon)$ is given by

$$
\begin{aligned}
y_2(X) = (1-\psi)&\left[r_2(X) + \eta_2(X)\|\sigma\| + \eta_0(X)\sigma_{R,2}(X)\frac{\sigma'}{\|\sigma\|} - \sum_{j=0}^{J} x_j \frac{\gamma}{2}\left(\sigma_{j,1}^2(X) + 2\sigma_{j,0}\sigma_{j,2}(X) + 2\hat{\gamma}_j\sigma_{j,0}\sigma_{j,1}(X)\right)\right] \\
&+ \sum_{j=0}^{J} x_j \left[\mu_{c_j,2}(X) + (1-\gamma)\sigma_{c_j,2}(X)\sigma'\right].
\end{aligned}
$$

Using the expression for the interest rate, we obtain

$$
\begin{aligned}
y_2(X) = \left(1-\psi^{-1}\right)&\left[\mu_{y,2}(X) + \sigma_{y,2}(X)\sigma' + \sum_{j=0}^{J} x_j\gamma\|\sigma\|\left(\frac{\sigma_{j,1}^2(X)}{2\|\sigma\|} + \sigma_{j,2}(X) + \hat{\gamma}_j\sigma_{j,1}(X)\right)\right] \\
&+ \psi^{-1}\sum_{j=0}^{J} x_j \left[\mu_{c_j,2}(X) + (1-\gamma)\sigma_{c_j,2}(X)\sigma'\right].
\end{aligned}
$$

Given that $\mu_{c_j,2} = (1-\psi)\mu_{y,2}$ and $\sigma_{c_j,2} = (1-\psi)\sigma_{y,2}$, we obtain

$$
\begin{aligned}
y_2(X) &= \left(1-\psi^{-1}\right)\left[\gamma\sigma_{y,2}(X)\sigma' + \sum_{j=0}^{J} x_j\gamma\|\sigma\|\left(\frac{\sigma_{j,1}^2(X)}{2\|\sigma\|} + \sigma_{j,2}(X) + \hat{\gamma}_j\sigma_{j,1}(X)\right)\right] \\
&= \left(1-\psi^{-1}\right)\sum_{j=0}^{J} x_j\gamma\|\sigma\|\left(\frac{\sigma_{j,1}^2(X)}{2\|\sigma\|} + \hat{\gamma}_j\sigma_{j,1}(X)\right),
\end{aligned}
$$

where in the second equality, we use the fact that, from the market clearing condition for the risky asset, we have $\sum_{j=0}^{J} x_j\sigma_{j,2}(X) = -\sigma_{y,2}(X)\frac{\sigma'}{\|\sigma\|}$.

Step 7: Risk premium.

Since the risk premium is given by $\pi_t = \eta_t\|\sigma_{R,t}\|$, we can write

$$
\pi(X,\epsilon) = \eta_0(X)\|\sigma\| + \eta_1(X)\|\sigma\|\epsilon + \left(-\eta_0(X)\sigma_{y,2}(X)\frac{\sigma'}{\|\sigma\|} + \eta_2(X)\|\sigma\|\right)\epsilon^2 + O\left(\epsilon^3\right).
$$

We can then write

$$\pi_2(X) = -\eta_0(X)\sigma_{y,2}(X)\frac{\sigma'}{\|\sigma\|} + \eta_2(X)\|\sigma\|. \tag{C.6}$$

□

D Derivation of the Market Elasticity

Let $p(X,\epsilon) \equiv 1/y(X,\epsilon)$ denote the price-dividend ratio. The second-order expansion of $p(X,\epsilon)$ is given by

$$p(X,\epsilon) = \frac{1}{y_0(X)} - \frac{y_1(X)}{y_0^2(X)}\epsilon + \left[\frac{y_1^2(X)}{y_0^3(X)} - \frac{y_2(X)}{y_0^2(X)}\right]\epsilon^2 + O(\epsilon^3). \tag{D.1}$$

Let $F(X) \equiv \frac{W_0(1+\hat{\alpha}_p\epsilon)-W_0}{P} = \hat{\alpha}_p\epsilon x_0$ denote the flow into the risky asset relative to the benchmark economy.

Proof. From Equation (D.1), the first-order impact of flows on the price-dividend ratio can be written as

$$\frac{\partial p(X,\epsilon)}{\partial F(X,\epsilon)} = -\frac{1}{y_0^2(X)}\frac{\partial y_1(X)}{\partial\hat{\alpha}_p}\frac{1}{x_0} + O(\epsilon). \tag{D.2}$$

Since from Proposition 2, $y_1(X)$ does not depend on $\hat{\alpha}_p$, the right hand side of Equation (D.2) is zero, leading to an infinite aggregate elasticity to the first-order:

$$\varepsilon_M^{-1} = \frac{1}{p(X,\epsilon)}\frac{\partial p(X,\epsilon)}{\partial F(X,\epsilon)} = 0 + O(\epsilon).$$

Given that from Equation (14), we have $y_t = r_t + \pi_t - \mu_{P,t}$, we can write the first-order term for the dividend yield as

$$y_1(X) = r_1(X) + \pi_1(X) - \mu_{P,1}(X),$$

where $\mu_{P,t} = \mu - \mu_{y,t} - \sigma_{y,t}\sigma_{R,t}$ is the drift of the risky asset price P_t, and π_t is the risk premium. As shown in Proposition 2, the first-order term for the dividend yield is constant. This means $\mu_{y,1}(X) = \sigma_{y,1}(X) = 0$, leading to $\sigma_{R,1}(X) = 0$. Therefore, we have $\mu_{P,1}(X) = 0$, and

$$\frac{\partial y_1(X)}{\partial\hat{\alpha}_p} = \frac{\partial r_1(X)}{\partial\hat{\alpha}_p} + \frac{\partial\pi_1(X)}{\partial\hat{\alpha}_p}.$$

From Proposition 2, we have

$$\frac{\partial r_1(X)}{\partial \hat{\alpha}_p} = -\sigma \frac{\partial \eta_1(X)}{\partial \hat{\alpha}_p} = \frac{\gamma\sigma^2}{x_u} x_0,$$
$$\frac{\partial \pi_1(X)}{\partial \hat{\alpha}_p} = \sigma \frac{\partial \eta_1(X)}{\partial \hat{\alpha}_p} = -\frac{\gamma\sigma^2}{x_u} x_0.$$

Thus, up to the first order, the effect of portfolio flows on the risk-free rate is the exact opposite of its impact on the risk premium and portfolio flows do not affect the price-dividend ratio up to first order. Therefore, up to the first-order, the aggregate market elasticity is infinite. □

Using Equation (D.1), the derivative of the price-dividend ratio with respect to flows F is given by

$$\frac{\partial p(X,\epsilon)}{\partial F(X,\epsilon)} = -\frac{1}{y_0^2(X)} \frac{\partial y_2(X)}{\partial \hat{\alpha}_p} \frac{\epsilon}{x_0} + O(\epsilon^2),$$

where $y_1(X)$ does not depend on $\hat{\alpha}_p$, leading to no price impact (infinite elasticity) up to the first-order.

D.1 Proof of Proposition 3

Proof. Consider the case where there is no preference heterogeneity and active investors do not face leverage constraints. In this case, $y_2(X)$ simplifies to

$$y_2(X) = \left(1 - \psi^{-1}\right) \gamma\sigma^2 \sum_{j=0}^{J} x_j \frac{\sigma_{j,1}^2(X)}{2\sigma^2},$$

using the fact that $\sigma_{y,2} = \sigma_{j,2} = 0$ when $\hat{\gamma}_j = 0$ for $j = 0, 1, \ldots, J$ when investors have the same preferences.

Using the expression for $\sigma_{j,1}$ in Proposition 2, the (inverse) aggregate market elasticity is given by

$$\frac{1}{p(X,\epsilon)} \frac{\partial p(X,\epsilon)}{\partial F(X,\epsilon)} = -\frac{1-\psi^{-1}}{2y_0(X)} \gamma\sigma^2 \left(2x_0\hat{\alpha}_p + x_a \frac{2\hat{\alpha}_p x_0^2}{x_a^2}\right) \frac{\epsilon}{x_0} + O(\epsilon^2),$$

where $x_a \equiv 1 - x_0$ denotes the wealth share of active investors. This can be written as

$$\frac{1}{p(X,\epsilon)} \frac{\partial p(X,\epsilon)}{\partial F(X,\epsilon)} = \left(1 - \psi^{-1}\right) \frac{\gamma\sigma^2}{y_0(X)} \frac{1 - \overline{\alpha}_p}{x_a} + O(\epsilon^2),$$

where we use $\overline{\alpha}_p = 1 + \hat{\alpha}_p \epsilon$ from Equation (20). □

From Equation (C.6), the impact of flows of the risk premium can be written as

$$\frac{\partial \pi(X,\epsilon)}{\partial F(X,\epsilon)} = \sigma \frac{\partial \eta_1(X)}{\partial \hat{\alpha}_p}\frac{\epsilon}{x_0} + \frac{\partial \pi_2(X)}{\partial \hat{\alpha}_p}\frac{\epsilon}{x_0}$$

D.2 Proof of Proposition 4

Proof. Consider the case in which active investors have heterogeneous risk aversions, but face no leverage constraints. In this case, the expression for $y_2(X)$ in Equation (C.5) can be written as

$$y_2(X) = \left(1-\psi^{-1}\right)\gamma\sigma^2\left[\sum_{j=1}^{J} x_j\left(\frac{\sigma^2_{j,1}(X)}{2\sigma^2} + \hat{\gamma}_j\frac{\sigma_{j,1}(X)}{\sigma}\right)\right] + \left(1-\psi^{-1}\right)\gamma\sigma^2 x_0\left(\frac{\hat{\alpha}_p^2}{2} + \hat{\gamma}_0\hat{\alpha}_p\right)$$

Note that with unconstrained active investors, the effect of endogenous volatility and hedging demand exactly cancel out. We first compute the derivative of the term involving $\sigma^2_{j,1}$:

$$\begin{aligned}
\sum_{j=0}^{J} x_j \frac{\partial \sigma^2_{j,1}}{\partial \hat{\alpha}_p} &= \sum_{j=0}^{J} 2x_j\sigma_{j,1}\frac{\partial \sigma_{j,1}}{\partial \hat{\alpha}_p} \\
&= 2x_0\sigma^2\hat{\alpha}_p + 2\sigma^2\sum_{j=1}^{J} x_j\left(\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_k - \left(\frac{\hat{\alpha}_p x_0}{1-x_0}\right) - \hat{\gamma}_j\right)\left(-\frac{x_0}{1-x_0}\right) \\
&= 2\sigma^2\left[x_0\hat{\alpha}_p + \frac{\hat{\alpha}_p x_0^2}{1-x_0} + \left((1-x_0)\sum_{k=1}^{J}\frac{x_k}{x_u}\hat{\gamma}_k - \sum_{j=1}^{J} x_j\hat{\gamma}_j\right)\left(-\frac{x_0}{1-x_0}\right)\right] \\
&= 2\sigma^2\frac{\hat{\alpha}_p x_0}{1-x_0}.
\end{aligned}$$

The derivatives of the term involving $\sigma_{j,1}(X)$ with respect to $\hat{\alpha}_p$ are given by:

$$\frac{1}{\sigma}\sum_{j=0}^{J} x_j\hat{\gamma}_j\frac{\partial \sigma_{j,1}(X)}{\partial \hat{\alpha}_p} = \sum_{j=1}^{J} x_j\hat{\gamma}_j\left(-\frac{x_0}{1-x_0}\right) + x_0\hat{\gamma}_0 = x_0\left(\hat{\gamma}_0 - \mathbb{E}^u[\hat{\gamma}_k]\right)$$

Thus, the (inverse) aggregate elasticity is then given by:

$$\frac{1}{p(X,\epsilon)}\frac{\partial p(X,\epsilon)}{\partial F(X,\epsilon)} = \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{y_0(X)}\left[\frac{1-\overline{\alpha}_p}{x_a} - \frac{\gamma_0 - \mathbb{E}^u\left[\gamma_j\right]}{\gamma}\right] + O(\epsilon^2),$$

where $x_a \equiv 1 - x_0$ is the wealth share of active investors, and we use $\gamma_j = \gamma\left(1+\hat{\gamma}_j\epsilon\right)$ from Equation (19). □

D.3 Proof of Proposition ??

Proof. Consider the case in which active investors have heterogeneous risk aversions and also face leverage constraints. We can then write $y_2(X)$ as follows

$$
\begin{aligned}
y_2(X) &= \left(1-\psi^{-1}\right)\gamma\sigma^2\sum_{j=0}^{J} x_j\left[\frac{\sigma_{j,1}^2(X)}{2\sigma^2}+\hat{\gamma}_j\frac{\sigma_{j,1}(X)}{\sigma}\right] \\
&= \left(1-\psi^{-1}\right)\gamma\sigma^2\sum_{j=1}^{J} x_j\left[\frac{\sigma_{j,1}^2(X)}{2\sigma^2}+\hat{\gamma}_j\min\left\{\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_k-\frac{\hat{\alpha}_p x_0+\frac{\hat{\sigma}}{\sigma}x_c}{1-x_0-x_c}-\hat{\gamma}_j,\frac{\hat{\sigma}}{\sigma}\right\}\right] \\
&+ \left(1-\psi^{-1}\right)\gamma\sigma^2 x_0\left[\frac{\hat{\alpha}_p^2}{2}+\hat{\gamma}_0\hat{\alpha}_p\right],
\end{aligned}
$$

The derivative of $y_2(X)$ with respect to $\hat{\alpha}_p$ is given by

$$
\frac{\partial y_2(X)}{\partial\hat{\alpha}_p} = \left(1-\psi^{-1}\right)\gamma\sigma^2\left(\sum_{j=1}^{J} x_j\frac{\sigma_{j,1}(X)}{\sigma^2}\frac{\partial\sigma_{j,1}}{\partial\hat{\alpha}_p}-\mathbb{E}^u[\hat{\gamma}_k]x_0+x_0(\hat{\alpha}_p+\hat{\gamma}_0)\right),
$$

where

$$
\begin{aligned}
\frac{1}{\sigma^2}\sum_{j=0}^{J} x_j\frac{\partial\sigma_{j,1}^2}{\partial\hat{\alpha}_p} &= \frac{1}{\sigma^2}\sum_{j=0}^{J} x_j 2\sigma_{j,1}\frac{\partial\sigma_{j,1}}{\partial\hat{\alpha}_p} \\
&= 2x_0\hat{\alpha}_p+2\sum_{j\in\mathcal{J}^u} x_j\left(\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_k-\left(\frac{\hat{\alpha}_p x_0+\frac{\hat{\sigma}}{\sigma}x_c}{1-x_0-x_c}\right)-\hat{\gamma}_j\right)\left(-\frac{x_0}{1-x_0-x_c}\right) \\
&= 2\left[x_0\hat{\alpha}_p+\frac{\hat{\alpha}_p x_0+\frac{\hat{\sigma}}{\sigma}x_c}{1-x_0-x_c}x_0+\left(x_u\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_k-\sum_{j\in\mathcal{J}^u}x_j\hat{\gamma}_j\right)\left(-\frac{x_0}{1-x_0-x_c}\right)\right] \\
&= 2x_0\left[\frac{\hat{\alpha}_p}{x_u}+\left(\frac{\hat{\sigma}}{\sigma}-\hat{\alpha}_p\right)\frac{x_c}{x_u}\right].
\end{aligned}
$$

The second derivative of $y(X,\epsilon)$ can then be written as

$$
\frac{\partial y_2(X)}{\partial\hat{\alpha}_p} = \left(1-\psi^{-1}\right)\gamma\sigma^2\left[\frac{\hat{\alpha}_p}{x_u}+\left(\frac{\hat{\sigma}}{\sigma}-\hat{\alpha}_p\right)\frac{x_c}{x_u}-(\mathbb{E}^u[\hat{\gamma}_k]-\hat{\gamma}_0)\right]x_0.
$$

Thus, the (inverse) aggregate market elasticity is then given by

$$
\frac{1}{p(X,\epsilon)}\frac{\partial p(X,\epsilon)}{\partial F(X,\epsilon)} = \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{y_0(X)}\left[\frac{(1-\overline{\alpha}_p)(1-x_c)-\left(\frac{\overline{\sigma}}{\sigma}-1\right)x_c}{x_u}-\frac{\gamma_0-\mathbb{E}^u\left[\gamma_j\right]}{\gamma}\right]+\mathcal{O}(\epsilon^2),
$$

where we use $\overline{\sigma} = \sigma + \hat{\sigma}\epsilon$ from Equation (21). $\square$

E Useful Formula

The following are useful for computing the derivatives above:

$$\gamma\sigma\frac{\partial\sigma_{y,2}(X)}{\partial\hat{\alpha}_p} = \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2y_0(X)}\sum_{k=1}^{J}\left(\hat{\gamma}_k-\hat{\gamma}_0\right)x_k\left(-\frac{\gamma\sigma^2 x_0}{1-x_0}\right).$$

Note that we can write $\sigma_{j,2}(X)$ as follows

$$\begin{aligned}\frac{\sigma_{j,2}(X)}{\sigma} &= \frac{\eta_2(X)}{\gamma\sigma} - \frac{\eta_1(X)}{\gamma\sigma}\hat{\gamma}_j + \frac{\eta_0(X)}{\gamma\sigma}\hat{\gamma}_j^2 - (1-\gamma^{-1})\frac{\sigma_{y,2}(X)}{\sigma}\\ &= -\left(1+\frac{x_c}{x_u}\right)\frac{\sigma_{y,2}(X)}{\sigma} - \mathbb{E}^u[\hat{\gamma}_k]\frac{\hat{\alpha}_p x_0 + \frac{\hat{\sigma}}{\sigma}x_c}{1-x_0-x_c} - \mathrm{Var}^u[\hat{\gamma}_k]+\\ &\quad -\hat{\gamma}_j\left[\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_j - \frac{\hat{\alpha}_p x_0}{1-x_0}\right] + \hat{\gamma}_j^2.\end{aligned}$$

$$\begin{aligned}\sigma_{j,1}(X) &= \sigma\min\left\{\sum_{k\in\mathcal{J}^u}\frac{x_k}{x_u}\hat{\gamma}_k - \left(\frac{\hat{\alpha}_p x_0 + \frac{\hat{\sigma}}{\|\sigma\|}x_c}{1-x_0-x_c}\right) - \hat{\gamma}_j, \frac{\hat{\sigma}}{\sigma}\right\}\\ \sigma_{j,2}(X) &= \frac{\eta_2(X)}{\gamma} - \frac{\eta_1(X)}{\gamma}\hat{\gamma}_j + \frac{\eta_0(X)}{\gamma}\hat{\gamma}_j^2 - (1-\gamma^{-1})\sigma_{y,2}(X)\\ \sigma_{y,2}(X) &= \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2y_0(X)}\sum_{k=1}^{J}\left(\hat{\gamma}_k-\hat{\gamma}_0\right)x_k\sigma_{k,1}(X)\\ \eta_1(X) &= \gamma\sigma\left[\sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j - \frac{\hat{\alpha}_p x_0}{1-x_0}\right]\\ \eta_2(X) &= -\frac{(\gamma-1)x_c+1-x_0}{1-x_0-x_c}\sigma_{y,2}(X) - \gamma\sigma\mathbb{E}^u[\hat{\gamma}_j]\frac{\hat{\alpha}_p x_0 + \frac{\hat{\sigma}}{\sigma}x_c}{1-x_0-x_c} - \gamma\sigma\mathrm{Var}^u[\hat{\gamma}_j],\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial \sigma_{y,2}}{\partial \hat{\alpha}_p} &= \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2y_0(X)}\sum_{k=1}^{J}\left(\hat{\gamma}_k-\hat{\gamma}_0\right)x_k\left(-\frac{\sigma x_0}{1-x_0}\right)\\
&= \left(1-\psi^{-1}\right)\frac{\gamma\sigma^2}{2y_0(X)}\sigma\left(\hat{\gamma}_0-\mathbb{E}^u[\hat{\gamma}_j]\right)x_0\\
\sum_{j=0}^{J}x_j\gamma\sigma\hat{\gamma}_j\frac{\partial\sigma_{j,1}(X)}{\partial\hat{\alpha}_p} &= \gamma\sigma^2\sum_{j=0}^{J}x_j\hat{\gamma}_j\left(-\frac{x_0}{1-x_0}\right)
\end{aligned}$$

F Derivation of the perturbed solution

In this section, we compute the first-order and second-order correction of the equilibrium objects. It turns out that the system of equations determining the perturbed solution is block-recursive, so we are able to solve for the equilibrium objects one by one, provided we proceed in the appropriate order.

In contrast to the case considered in the text, we allow for portfolio-flow shocks. In particular, we assume that the portfolio share of the passive investor is given by $\alpha_{0,t} = 1 + \epsilon(\overline{\alpha}_{p,t} - 1)$, where $\overline{\alpha}_{p,t}$ follows the process

$$d\overline{\alpha}_{p,t} = \theta_p(\overline{\alpha} - \overline{\alpha}_{p,t})\epsilon dt + \sigma_p\sqrt{\overline{\alpha}_{p,t}}\epsilon dZ_t.$$

Notice that $\alpha_{0,t} = 1$ and $\overline{\alpha}_{p,t}$ is constant when $\epsilon = 0$. Finally, we assume that the mortality parameter is given by $\kappa = \hat{\kappa}\epsilon$.

F.1 First-order correction

Diffusion and drift terms. The diffusion term for the price-dividend ratio is given by

$$\sigma_{p,t} = \frac{p_x}{p}\sigma_x + \frac{p_{\overline{\alpha}_p}}{p}\sigma_p\sqrt{\overline{\alpha}_{p,t}}\epsilon = O(\epsilon^2).$$

Notice that $p_{x_j} = O(\epsilon)$ and $\sigma_{x_j} = O(\epsilon)$, as p and x_j are constant when $\epsilon = 0$, so the zeroth-order terms for p_{x_j}, $p_{\overline{\alpha}_p}$, and σ_{x_j} are equal to zero. This implies that the first-order correction for $\sigma_{p,t}$ is equal to zero. A similar argument shows that $\sigma_{c_j,t} = O(\epsilon^2)$.

The drift of p is given by

$$\mu_{p,t} = \frac{p_x}{p}\mu_x + \frac{p_{\overline{\alpha}_p}}{p}\theta_p(\overline{\alpha}-\overline{\alpha}_{p,t})\epsilon + \frac{1}{2}\sum_{k=1}^{d}\left[\sigma_{x,k}'\frac{p_{xx}}{p}\sigma_{x,k} + 2\sigma_{p,k}\sqrt{\overline{\alpha}_{p,t}}\epsilon\frac{p_{x\alpha_p}}{p}\sigma_{x,k} + \frac{p_{\overline{\alpha}_p\overline{\alpha}_p}}{p}\sigma_{p,k}^2\overline{\alpha}_{p,t}\epsilon^2\right],$$

where $\sigma_{x,k}$ is the k-th column of the $J \times d$ matrix σ_x. As p_x and $p_{\overline{\alpha}_p}$ are first-order in ϵ, and the same goes for μ_x and σ_x, then $\mu_{p,t} = O(\epsilon^2)$. A similar argument shows that $\mu_{c_j,t} = O(\epsilon^2)$. Notice these facts imply that $\varsigma_j = O(\epsilon^2)$ and $\xi_{j,t} = O(\epsilon^2)$.

Risk premium. The risk premium is given by

$$\pi_1(X) = \gamma||\sigma||^2 \left[\sum_{j \in \mathcal{J}^u} \frac{x_j}{x_u} \hat{\gamma}_j - \frac{x_0 \hat{\alpha}_p + x_c \frac{\hat{\sigma}}{||\sigma||}}{1 - x_0 - x_c} \right],$$

using the fact that $||\sigma_{R,t}|| = ||\sigma|| + O(\epsilon^2)$, and $\hat{\alpha}_{p,t} \equiv \overline{\alpha}_{p,t} - 1$.

Portfolio share. The portfolio share of an unconstrained investor is given by

$$\alpha_j(X, \epsilon) = 1 + \left[\frac{\pi_1(X)}{\gamma||\sigma||^2} - \hat{\gamma}_j \right] \epsilon + O(\epsilon^2),$$

the portfolio share of a constrained investor is given by

$$\alpha_j(X, \epsilon) = 1 + \frac{\hat{\sigma}}{||\sigma||} \epsilon + O(\epsilon^2),$$

and the portfolio share of the passive investor is given by $\alpha_0(X, \epsilon) = 1 + \hat{\alpha}_{p,t}\epsilon$. Notice that $\sum_{j=0}^{J} x_j \alpha_{j,1}(X) = 0$, consistent with market clearing.

Interest rate. The first-order correction for the interest rate is given by

$$r_1(X) = \left(1 - \psi^{-1}\right) \gamma||\sigma||^2 \sum_{j=0}^{J} x_j \left[\frac{\hat{\gamma}_j}{2} + \alpha_{j,1}(X) \right] - \pi_1(X),$$

using the fact that $\xi_t = O(\epsilon^2)$, $\mu_{p,t} = O(\epsilon^2)$, and $\sigma_{p,t} = O(\epsilon^2)$. Given the market clearing for the risky asset, we can write:

$$r_1(X) = \left(1 - \psi^{-1}\right) \gamma||\sigma||^2 \sum_{j=0}^{J} x_j \frac{\hat{\gamma}_j}{2} - \pi_1(X),$$

so $r_1(X) + \pi_1(X)$ is independent of $\overline{\alpha}_p$.

Price-dividend ratio. From the pricing condition, we obtain

$$-\frac{1}{p_0(X)^2} p_1(X) = r_1(X) + \pi_1(X).$$

Rearranging the expression above, and using the expression for the interest rate, we obtain

$$p_1(X) = -p_0(X)^2 \left(1 - \psi^{-1}\right) \gamma ||\sigma||^2 \sum_{j=0}^{J} x_j \frac{\hat{\gamma}_j}{2},$$

which is independent of $\overline{\alpha}_{p,t}$.

Consumption-wealth ratio. The consumption-wealth ratio is given by

$$c_{j,1}(X) = (1 - \psi) \left[r_1(X) + \pi_1(X) + \pi_0(X)\alpha_{j,1}(X) - \frac{1}{2}\gamma |||\sigma|||^2 (\hat{\gamma}_j + 2\alpha_{j,1}(X)) \right].$$

Using the expression for $r_1(X)$, we can write the expression above as follows:

$$c_{j,1}(X) = (1 - \psi) \left[(1 - \psi^{-1}) \sum_{i=0}^{J} x_i \frac{\hat{\gamma}_i}{2} - \frac{\hat{\gamma}_j}{2} \right] \gamma |||\sigma|||^2.$$

Wealth dynamics. The diffusion term of x_j is given by

$$\sigma_{x_j}(X) = x_j \alpha_{j,1}(X) \epsilon \sigma + O(\epsilon^2).$$

The drift of x_j is given by

$$\mu_{x_j}(X) = x_j \left[r_1(X) + \pi_1(X) + \pi_0(X)\alpha_{j,1}(X) - c_{j,1}(X) - \alpha_{j,1}(X)||\sigma||^2 + \hat{\kappa} \frac{\omega_j - x_j}{x_j} \right] \epsilon + O(\epsilon^2).$$

We can write the first-order correction of μ_{x_j} as follows:

$$\mu_{x_j,1}(X) = x_j \left[(\psi - 1) \frac{\gamma |||\sigma|||^2}{2} \left(\sum_{i=0}^{J} x_i \hat{\gamma}_i - \hat{\gamma}_j \right) + (\gamma - 1) |||\sigma|||^2 \alpha_{j,1}(X) \right] + \hat{\kappa}(\omega_j - x_j).$$

F.2 Second-order correction

Diffusion and drift terms. The diffusion term for the price-dividend ratio is given by

$$\sigma_{p,2}(X) = \frac{p_{x,1}(X)}{p_0(X)} \sigma_{x,1}(X) + \frac{p_{\overline{\alpha}_p,1}(X)}{p_0(X)} \sigma_p \sqrt{\overline{\alpha}_p}.$$

We can write the expression above as follows:

$$\sigma_{p,2}(X) = -p_0(X)(1-\psi^{-1})\frac{\gamma\|\sigma\||^2}{2}\sum_{j=1}^{J}\left(\hat{\gamma}_j - \hat{\gamma}_0\right) x_j \alpha_{j,1}(X)\sigma,$$

where we used the fact that $p_{\overline{\alpha}_p,1}(X) = 0$.

Similarly, the diffusion for c_j is given by

$$\sigma_{c_j,2}(X) = -(\psi - 1)\frac{\gamma\||\sigma\||^2}{2c_{j,0}(X)}(1-\psi^{-1})\sum_{i=1}^{J}\left(\hat{\gamma}_i - \hat{\gamma}_0\right) x_i \alpha_{i,1}(X)\sigma.$$

The second-order correction for the hedging demand is then given by $\varsigma_{j,2}(X) = \frac{1-\gamma^{-1}}{\psi-1}\frac{\sigma_{c_j,2}\sigma'}{\||\sigma\||^2}$.

The second-order correction of the drift of p and c_j are given by

$$\mu_{p,2}(X) = \frac{p_{x,1}(X)}{p_0(X)}\mu_{x,1}(X), \qquad \mu_{c_j,2}(X) = \frac{c_{j,x,1}(X)}{c_{j,0}(X)}\mu_{x,1}(X),$$

which can be written as

$$\mu_{p,2}(X) = -p_0(X)(1-\psi^{-1})\frac{\gamma\|\sigma\||^2}{2}\sum_{j=1}^{J}(\hat{\gamma}_j - \hat{\gamma}_0)\mu_{x_j,1}(X)$$
$$\mu_{c_i,2}(X) = (1-\psi)(1-\psi^{-1})\frac{\gamma\|\sigma\||^2}{2c_{i,0}(X)}\sum_{j=1}^{J}(\hat{\gamma}_j - \hat{\gamma}_0)\mu_{x_j,1}(X).$$

The second-order correction for $\xi_{j,t}$ is then given by $\xi_{j,2}(X) = \mu_{c_j,2}(X) + (1-\gamma)\sigma_{c_j,2}\sigma'$.

Risk premium. The risk premium is given by

$$\pi_2(X) = \gamma\|\sigma\||^2\left[\frac{\gamma_{u,2}(X)}{\gamma} - \frac{\gamma_{u,1}(X)}{\gamma}\frac{x_0\hat{\alpha}_p + x_c\frac{\hat{\sigma}}{\|\sigma\||}}{1-x_0-x_c} + 2\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}}{||\sigma||^2} - \frac{x_c}{1-x_0-x_c}\overline{\alpha}_{c,2}(X) - \varsigma_2(X)\right].$$

Notice that we can write the aggregate risk aversion as follows:

$$\gamma_u(X) = \gamma\left[1 + \mathbb{E}^u[\hat{\gamma}_j]\epsilon - \delta^u[\hat{\gamma}_j]\epsilon^2\right] + \mathcal{O}(\epsilon^3),$$

where $\mathbb{E}^u[\hat{\gamma}_j] \equiv \sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j$ and $\delta^u[\hat{\gamma}_j] \equiv \sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j^2 - \left(\sum_{j\in\mathcal{J}^u}\frac{x_j}{x_u}\hat{\gamma}_j\right)^2$, so $\gamma_{u,2}(X)/\gamma = -\delta^u[\hat{\gamma}_j]$.

Combining the previous two expressions, we obtain

$$\begin{aligned}\frac{\pi_2(X)}{\gamma\|\sigma\||^2} &= -\delta^u[\hat{\gamma}_j] - \mathbb{E}^u[\hat{\gamma}_j]\frac{x_0\hat{\alpha}_p + x_c\frac{\hat{\sigma}}{\|\sigma\||}}{1 - x_0 - x_c} + 2\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}}{\|\sigma\|^2} + \frac{x_c}{1 - x_0 - x_c}\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}}{\|\sigma\||^2} - \varsigma_2(X)\\ &= -\delta^u[\hat{\gamma}_j] - \mathbb{E}^u[\hat{\gamma}_j]\frac{x_0\hat{\alpha}_p + x_c\frac{\hat{\sigma}}{\|\sigma\||}}{1 - x_0 - x_c} + \left(1 + \gamma^{-1} + \frac{x_c}{1 - x_0 - x_c}\right)\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}}{\|\sigma\||^2},\end{aligned}$$

where we used the fact that $\varsigma_2(X) = \frac{1-\gamma^{-1}}{\psi-1}\frac{\sigma_{c_j,2}\sigma'}{\|\sigma\||^2}$, $\sigma_{p,2} = \frac{\sigma_{c_j,2}}{\psi-1}$, and $\overline{\alpha}_{c,2}(X) = -\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}}{\|\sigma\||^2}$.

Portfolio share. The portfolio share of an unconstrained investor is given by

$$\alpha_{j,2}(X) = \frac{\pi_2(X)}{\gamma\||\sigma\||^2} - \frac{\pi_1(X)}{\gamma\||\sigma\||^2}\hat{\gamma}_j + \hat{\gamma}_j^2 - 2\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}(X)}{\|\sigma\|^2} + \varsigma_{j,2}(X),$$

the portfolio share of a constrained investor is $\alpha_{j,2}(X) = -\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}}{\|\sigma\||^2}$, and the portfolio share of the passive investor satisfies $\alpha_{0,2} = 0$.

We can write the expression above as follows:

$$\alpha_{j,2}(X) = \frac{\pi_2(X)}{\gamma\||\sigma\||^2} - \frac{\pi_1(X)}{\gamma\||\sigma\||^2}\hat{\gamma}_j + \hat{\gamma}_j^2 - (1 + \gamma^{-1})\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}(X)}{\|\sigma\|^2}.$$

Notice that $\sum_{j=0}^{J}x_j\alpha_{j,2}(X) = 0$, consistent with market clearing.

Interest rate. The interest rate is given by

$$\begin{aligned}r_2(X) &= \psi^{-1}(\mu_{p,2}(X) + \sigma\sigma_{p,2}(X)') + (1 - \psi^{-1}).\frac{\gamma\|\sigma\||^2}{2}\sum_{j=0}^{J}x_j\left[2\hat{\gamma}_j\alpha_{j,1}(X) + \alpha_{j,1}^2(X) + 2\alpha_{j,2}(X)\right]\\ &\quad + (1 - \psi^{-1})\gamma\|\sigma\|^2\sum_{k=1}^{d}\frac{\sigma_k\sigma_{p,2,k}(X)}{\|\sigma\|^2} - \pi_2(X) + \psi^{-1}\xi_2(X),\end{aligned}$$

where $\xi_2(X) = (\psi - 1)\left[\mu_{p,2}(X) + (1 - \gamma)\sigma_{p,2}\sigma'\right]$

We can write the expression for the portfolio of the unconstrained investor as follows:

$$r_2(X) = \mu_{p,2}(X) + \sigma\sigma_{p,2}(X)' + (1 - \psi^{-1})\gamma\|\sigma\||^2\sum_{j=0}^{J}x_j\left[\hat{\gamma}_j\alpha_{j,1}(X) + \frac{\alpha_{j,1}^2(X)}{2}\right] - \pi_2(X),$$

Price-dividend ratio. The price-dividend ratio is given by

$$\frac{p_1^2(X)}{p_0^3(X)} - \frac{p_2(X)}{p_0^2(X)} = r_2(X) + \pi_2(X) - \mu_{p,2}(X) - \sigma\sigma_{p,2}'.$$

Rearranging the expression above, and using the expression for $r_2(X)$, we obtain

$$\frac{p_2(X)}{p_0(X)} = -p_0(X)(1-\psi^{-1})\gamma\|\sigma\||^2 \sum_{j=0}^{J} x_j \left[\hat{\gamma}_j \alpha_{j,1}(X) + \frac{\alpha_{j,1}^2(X)}{2}\right] + \left(\frac{p_1(X)}{p_0(X)}\right)^2.$$

Consumption-wealth ratio. The second-order correction for the consumption-wealth ratio is given by

$$\begin{aligned} c_{j,2}(X) &= (1-\psi)\left[r_2(X) + \pi_2(X) + \pi_1(X)\alpha_{j,1}(X) + \pi_0(X)\alpha_{j,2}(X)\right] + \xi_{j,2}(X) \\ &\quad - (1-\psi)\gamma\|\sigma\||^2 \left[\alpha_{j,1}(X)\hat{\gamma}_j + \alpha_{j,2}(X) + \frac{\alpha_{j,1}^2(X)}{2} + \sum_{k=1}^{d} \frac{\sigma_k \sigma_{p,k,2}(X)}{\|\sigma\||^2}\right]. \end{aligned}$$

Wealth dynamics. The diffusion term of x_j is given by

$$\sigma_{x_j,2}(X) = x_j \alpha_{j,2} \sigma.$$

The drift of x_j is given by

$$\mu_{x_j,2} = x_j \left[r_2(X) + \pi_2(X) + \pi_1(X)\alpha_{j,1}(X) + \pi_0(X)\alpha_{j,2}(X) - c_{j,2}(X) - \mu_{p,2}(X) - \sigma\sigma_{p,2}(X)' - \alpha_{j,2}(X)\|\sigma\|^2\right].$$

Aggregate market elasticity. The derivative of p with respect to $\overline{\alpha}_p$ is given by

$$\frac{1}{p(X,\epsilon)}\frac{\partial p(X,\epsilon)}{\partial \overline{\alpha}_p} = \frac{1}{p_0(X)}\frac{\partial p_2(X)}{\partial \overline{\alpha}_p}\epsilon^2 + O(\epsilon^3).$$

The market elasticity satisfies the condition

$$\frac{1}{p_0(X)}\frac{\partial p_2(X)}{\partial \overline{\alpha}_p} = -p_0(X)(1-\psi^{-1})\gamma\|\sigma\||^2 \left[x_0(\hat{\gamma}_0 + \hat{\alpha}_p) + \sum_{j\in\mathcal{J}^u} x_j \left(\hat{\gamma}_j + \alpha_{j,1}(X)\right)\left(-\frac{x_0}{x_u}\right)\right]\epsilon^2.$$

From the market clearing for the risky asset, we have $x_0\hat{\alpha}_p + \sum_{j\in\mathcal{J}^u} x_j \alpha_{j,1}(X) + x_c \frac{\hat{\sigma}}{\|\sigma\|} = 0$, so we can write the expression above as follows:

$$\frac{1}{p_0(X)}\frac{\partial p_2(X)}{\partial \overline{\alpha}_p} = p_0(X)(1-\psi^{-1})\gamma\|\sigma\||^2\left[\hat{\gamma}_u(X) - \hat{\gamma}_0 + \frac{1-x_c}{1-x_0-x_c}(1-\overline{\alpha}_p) - \frac{x_c\frac{\hat{\sigma}}{\|\sigma\||}}{1-x_0-x_c}\right]x_0\epsilon^2.$$

F.3 Third-order approximation

F.3.1 Passive demand

Suppose there is no preference heterogeneity and no leverage constraint. Without loss of generality, set $J = 1$. In this case, the price-dividend ratio is given by

$$\begin{aligned}
p(X,\epsilon) &= p^\star - (p^\star)^2(1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{2}\frac{x_0\hat{\alpha}_p^2}{x_1}\epsilon^2 + O(\epsilon^3)\\
\pi(X,\epsilon) &= \pi_0(X) - \gamma\|\sigma\|^2\frac{x_0\hat{\alpha}_p}{x_1}\epsilon + O(\epsilon^3)\\
r(X,\epsilon) &= r_0(X) + \gamma\|\sigma\|^2\frac{x_0\hat{\alpha}_p}{x_1}\epsilon + (1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{2}\frac{x_0\hat{\alpha}_p^2}{x_1}\epsilon^2 + O(\epsilon^3)\\
\alpha_0(X,\epsilon) &= 1 + \hat{\alpha}_p\epsilon + O(\epsilon^3)\\
\alpha_1(X,\epsilon) &= 1 - \frac{x_0}{x_1}\hat{\alpha}_p\epsilon + O(\epsilon^3)\\
c_j(X,\epsilon) &= c_{j,0}(X) + (1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{2}\frac{x_0\hat{\alpha}_p^2}{x_1}\epsilon^2 + O(\epsilon^3)\\
\sigma_{x_1}(X,\epsilon) &= -\frac{x_0}{x_1}\hat{\alpha}_p\sigma\epsilon + O(\epsilon^3)\\
\mu_{x_1}(X,\epsilon) &= \left[(1-\gamma)\|\sigma\|^2(1-x_1)\hat{\alpha}_p + \hat{\kappa}(\omega_j - x_1)\right]\epsilon + O(\epsilon^3),
\end{aligned}$$

where $\mu_p, \sigma_p, \mu_{c_j}$, and σ_{c_j} are all equal to zero up to second order, and $x_0 = 1 - x_1$.

The law of motion of $x_{1,t}$ can be written as

$$\frac{dx_{1,t}}{x_{1,t}} = \left[x_{0,t}(c_{0,t} - c_{1,t}) + x_{0,t}(\alpha_{0,t} - \alpha_{1,t})\left(\|\sigma_{R,t}\|^2 - \pi_t\right) + \kappa\frac{\omega_1 - x_{1,t}}{x_{1,t}}\right]dt + (\alpha_{1,t} - 1)\sigma_{R,t}dZ_t.$$

Diffusion and drift terms. The derivatives of $p(X,\epsilon)$ with respect to x_1 and $\overline{\alpha}_p$ are given by

$$\frac{p_{x_1}(X,\epsilon)}{p^\star} = (1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{2y^\star}\frac{1}{x_1^2}\hat{\alpha}_p^2\epsilon^2 + O(\epsilon^3), \qquad \frac{p_{\overline{\alpha}_p}(X,\epsilon)}{p^\star} = -(1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{y^\star}\frac{x_0}{x_1}\hat{\alpha}_p\epsilon^2 + O(\epsilon^3)$$

The diffusion term for $p(X, \epsilon)$ is then given by

$$\sigma_{p,3}(X) = -(1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{y^\star}\frac{x_0}{x_1}\left[\frac{\hat{\alpha}_p^3}{2x_1^2}\sigma + \hat{\alpha}_p\sigma_p\sqrt{\overline{\alpha}_p}\right].$$

and $\sigma_{c_j,3}(X) = \sigma_{p,3}(X)$. Notice that excess volatility depends on the market elasticity times the volatility of portfolio flows.

The drift of p is given by

$$\mu_{p,3}(X) = (1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{2y^\star}\frac{1}{x_1^2}\hat{\alpha}_p^2\mu_{x_1,1} - (1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{y^\star}\frac{x_0}{x_1}\hat{\alpha}_p\theta_p(\overline{\alpha} - \overline{\alpha}_{p,t}),$$

and $\mu_{c_j,3}(X) = \mu_{p,3}(X)$.

Risk premium. The risk premium is given by

$$\pi(X) = \gamma\|\sigma\|^2\left[1 - \frac{x_0(\overline{\alpha}_p - 1)}{x_1} - \frac{1-\gamma^{-1}}{\psi - 1}\frac{\gamma\|\sigma\|^2}{y^\star}\frac{x_0}{x_1}\left[\frac{(1-\overline{\alpha}_p)^3}{2x_1^2}\sigma + \hat{\alpha}_p\sigma_p\sqrt{\overline{\alpha}_p}\right]\right]$$

G Higher-order perturbations

Suppose we have the $(n-1)$-th order perturbation of $c_j(X, \epsilon) = \sum_{k=0}^{n-1} c_{j,k}(X)\epsilon^k$ and the law of motion of X. Let $lc_j(X, \epsilon) = \sum_{k=0}^{n-1} lc_{j,k}(X)\epsilon^k$ denote the expansion of log $c_j(X, \epsilon)$. Then we can compute $\sigma_j(X)$ up to order n:

$$\sigma_{c_j,n}(X) = \sum_{k=1}^{n-1} lc_{j,k,X}(X)\sigma_{X,n-k}(X),$$

which is independent of the n-th order term in $c_j(X, \epsilon)$ and $\sigma_X(X, \epsilon)$, as $lc_{j,0,X}(X) = 0$. Similarly, we can compute $\sigma_{y,n}(X)$. A similar argument gives $\mu_{p,n}(X)$ and $\mu_{c_j,n}(X)$. We can then compute $\pi_n(X)$ and $\alpha_{j,n}(X)$. The n-th term of the consumption-wealth ratio satisfies the condition

$$c_{j,t} = \psi\rho + (1-\psi)\left[\pi_t(\alpha_{j,t} - 1) + \mu + \mu_{p,t} + \sigma\sigma_{p,t}' - \frac{\gamma_j}{2}\|\sigma_{R,t}\|^2\alpha_{j,t}^2 + \sum_{j=0}^{J} x_j c_{j,t}\right] + \xi_{j,t}$$

We can rewrite the system above in matrix form as follows:

$$[I - (1-\psi)\mathbf{1}_{J+1}x_t']c_t = \zeta_t,$$

where $c_t = [c_{0,t}, \ldots, c_{J,t}]'$, $x_t = [x_{0,t}, \ldots, x_{J,t}]'$, $\mathbf{1}_{J+1}$ is a $(J+1)$-th dimensional vector of ones, and $\zeta_{j,t} \equiv \psi\rho + (1-\psi)\left[\pi_t(\alpha_{j,t}-1) + \mu + \mu_{p,t} + \sigma\sigma'_{p,t} - \frac{\gamma_j}{2}\|\sigma_{R,t}\|^2\alpha^2_{j,t}\right] + \xi_{j,t}$. Applying the Sherman-Morrison formula, we obtain

$$c_t = [I - (1-\psi^{-1})\mathbf{1}_{J+1}x'_t]\zeta_t,$$

or $c_{j,t} = \zeta_{j,t} - (1-\psi^{-1})x'_t\zeta_t$. Notice that ζ_t can be computed at order n based on the coefficients of order $n-1$ and their derivatives.

Computing the derivatives. The derivation above shows that, given the order $n-1$ expansion of $\zeta_j(X,\epsilon)$ and its derivatives, we can compute the expansion of order n. Suppose the expansion of $\zeta_j(X,\epsilon)$ is given by

$$\zeta_j(X,\epsilon) = \sum_{k=0}^{n-1}\zeta_{j,k}(X)\epsilon^k,$$

where $\zeta_{j,k}(X)$ takes the form:

$$\zeta_{j,k}(X) = A_{j,k} + B'_{j,k}(X-\overline{X}) + \frac{1}{2}(X-\overline{X})'C_{j,k}(X-\overline{X}),$$

where $\overline{X}$ is a reference point, $A_{j,k}$ is a scalar, $B_{j,k}$ is a vector, and $C_{j,k}$ is a matrix. Notice that . $\zeta_{j,k}(\overline{X}) = A_{j,k}$, $\zeta_{j,k,X}(\overline{X}) = B_{j,k}$ and $\zeta_{j,k,XX} = C_{j,k}$. Given this expansion, we can compute $c_j(X,\epsilon) = \sum_{k=0}^{n-1} c_{j,k}(X)\epsilon^k$.

G.1 Inner region

Consider the case of no preference heterogeneity and no leverage constraints. Consider the following change of variables: $x_1 = \epsilon\tilde{x}_1$. Define $\tilde{c}_j(\tilde{x}_1, \overline{\alpha}_p) = c_j\left(x_1, \overline{\alpha}_p\right)$, so $c_{j,x_1} = \frac{1}{\epsilon}\tilde{c}_{j,\tilde{x}_1}$ and $c_{j,x_1x_1} = \frac{1}{\epsilon^2}\tilde{c}_{j,\tilde{x}_1\tilde{x}_1}$. This implies the following is true:

$$\sigma_{c_j} = \frac{1}{\epsilon}\frac{\tilde{c}_{j,\tilde{x}_1}}{\tilde{c}_j}\sigma_{x_1},$$

where $\sigma_{x_1} = -\frac{1-\epsilon\tilde{x}_1}{\tilde{x}_1}\hat{\alpha}_p\sigma_R$. Similarly, we can write σ_y

$$\sigma_y = \frac{1}{\epsilon}\frac{\tilde{y}_{x_1}}{y}\sigma_{x_1}.$$

The drift of y is given by

$$\mu_y = \frac{1}{\epsilon}\frac{\tilde{y}_{\tilde{x}_1}}{y}\mu_{\tilde{x}_1} + \frac{1}{2\epsilon^2}\frac{\tilde{y}_{\tilde{x}_1\tilde{x}_1}}{y}\sigma^2_{\tilde{x}_1}.$$

The term of order 0 is the same as before.

H Volatility

Consider the case without preference heterogeneity or leverage constraints. The price-dividend ratio is given by:

$$\hat{p}_t = -(1-\psi^{-1})\frac{\gamma\|\sigma\|^2}{2y^\star}\frac{(1-x_{a,t})(1-\overline{\alpha}_{p,t})^2}{x_{a,t}}.$$

where, given $\overline{\alpha}_{a,t} \equiv \frac{1-(1-x_{a,t})\overline{\alpha}_{p,t}}{x_{a,t}}$, we have

$$\sigma_{x_a,t} = x_{a,t}(\overline{\alpha}_{a,t}-1)\sigma, \qquad \sigma_{\overline{\alpha}_p,t} = \sigma_{\overline{\alpha}_p}$$

The diffusion of $\hat{p}_t$ is given

$$\sigma_{p,t} = \hat{p}_{x_a,t}\sigma_{x_a,t} + \hat{p}_{\overline{\alpha}_p,t}\sigma_{\overline{\alpha}_p}.$$

$$\hat{c}_t \equiv c_t - c^\star = \chi_{0,t} + \chi_p\,\hat{p}_t + O(\epsilon^3),$$

with coefficients

$$\chi_p = (\psi-1)\frac{1}{p^\star}, \qquad \chi_{0,t} = (\psi-1)\frac{\gamma\|\sigma\|^2}{2}\frac{f_t^2}{x_{0,t}(1-x_{0,t})}.$$