Imperfect Exchange Rate Pass-through: Empirical Evidence and Monetary Policy Implications

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October 24, 2025

Abstract

This paper studies the conduct of monetary policy in an open economy setting using optimized Taylor-type rules. To that end, we construct a small open economy (SOE) model interacting with the rest of the world (ROW) nesting two different pricing paradigms: local currency pricing (LCP), equivalent to dominant currency pricing in our two-country setup, alongside producer currency pricing (PCP). Moreover, we incorporate traded intermediate inputs and investment goods, incomplete international asset markets and, in order to capture distributional effects of policy, a TANK framework with a proportion of 'rule of thumb' consumers without access to assets markets. The main results are: first, using US and Canadian data, we find that LCP easily beats PCP in a likelihood race; second, for the closed economy ROW the price-level rule closely mimics the optimized general inflation-output rule, whereas for the SOE the corresponding result requires a nominal income rule.

Keywords: Imperfect Exchange Rate Pass-Through; Currency Pricing; Monetary Policy; Zero-Lower-Bound.

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The authors are grateful to participants at seminars at the Bank of England, Surrey, Victoria, King's College and the CEF 2019 conference in Ottawa for useful comments on earlier versions of the paper.

Contents

1 Introduction

The aftermath of the global financial crisis led to a challenging of the conventional Mundell-Fleming view that monetary policy is able to offset the effects of foreign shocks and that therefore international spillovers are negligible. A major contributing factor has been the fact that several economies have been experiencing a liquidity trap, in which the zero-lower-bound (ZLB) on nominal interests has limited the ability of central banks to conduct (conventional) monetary policy. Moreover, recent literature on currency pricing emphasizes the importance of understanding the main sources of price rigidities: export prices may be set in the producer's currency (Producer Currency Pricing, PCP henceforth), in the buyer's currency (Local Currency Pricing, LCP henceforth), or in a local currency that is neither the buyer's nor seller's (Dominant Currency Pricing, DCP henceforth). Direct evidence on currency invoicing is provided by Boz *et al.* (2022) for 115 countries since 1990.

Each of these pricing models will have different implications for the transmission of monetary policy, the magnitude of international spillovers and whether or not central banks should target exchange rate fluctuations. Consider a small open economy (SOE) trading with the rest of the world (ROW), which in our paper is modelled as Canada and the US. In the basic Mundell-Fleming model, which assumes PCP and the law of one price, a ROW monetary expansion will lead to a depreciation of the dollar, a rise in the price of the SOE exported goods in ROW currency (i.e. full exchange rate pass-through) and therefore resulting in expenditure switching. In contrast, under a LCP regime, export prices from the SOE are sticky in the destination country's currency. In this case, shocks to the nominal exchange rate will not affect the prices in the ROW currency, such that in the short-run there will be a deviation from the law of one price and the exchange rate pass-through into imported prices is imperfect.

As for a DCP regime, the work of Gopinath *et al.* (2020) provides a framework to explain why a significant proportion of bilateral trade worldwide is carried out in US dollars (the 'dominant' currency), emphasizing strategic complementarities in pricing and the role of traded intermediate goods. Predictably, spillovers from US monetary policy are amplified under this pricing regime, while the US dollar exchange rate pass-through will be sizeable and considerably more muted for bilateral (non-US dollar) exchange rates. In this context, domestic (SOE) monetary policy should attempt to correct exchange rate 'mis-alignments' vis-à-vis the US dollar.

In this paper we bring all these ingredients together by modelling and investigating empirically the interaction between US monetary policy and that of its neighbour Canada under these different currency pricing regimes. Typically, a comparison of all three pricing regimes requires a n-country framework where $n \geq 3$ and two countries may then price their bilateral exports in the currency of a third (the dominant) country. But for $n = 2$ we have only three possibilities: both countries follow PCP, both follow LCP, or one country follows LCP and the other (the dominant one) PCP. We consider the first and last of these possibilities with the ROW (the US) following PCP and the SOE either PCP or LCP. So in this particular instance, the SOE LCP regime aligns perfectly with the DCP paradigm. Since most of Canada's trade is with the US *and* is carried out in US dollars, we expect this export pricing model to perform better empirically and indeed this turns out to be the case.

We use a 2-bloc ROW-SOE setup such that, in the limit, as the ROW becomes very large, the

ROW bloc (fitted to US data) becomes a closed NK economy that is not affected by shocks from the other countries. Canada, our choice of SOE trading partner, is in turn modelled as a SOE that is affected by shocks and monetary policy in the ROW. Moreover, and following Gopinath *et al.* (2020), we incorporate additional trade channels in the form of traded intermediate inputs produced domestically and abroad, as well as incomplete international asset markets. To increase the importance of imperfect exchange rate pass-through, we also allow for trade in investment goods. Finally, and, in order to capture distributional effects of policy, we employ a TANK framework with a proportion of 'rule of thumb' consumers without access to assets markets.

Having estimated the model for both pricing regimes, we then study the policy implications in terms of optimized simple Taylor-type nominal interest-rate rules in both the SOE and the ROW. In order to compute welfare-optimized simple rules (OSR), an important feature of the model set-up is that it is non-linear; this enables us to compute second-order welfare effects using a perturbation solution of order two about a non-stochastic steady state determined by the policy choice of the trend inflation rate. The criterion for ranking policy rules is the inter-temporal household welfare at the stochastic steady state, maximized on average over all realizations of the shocks driving the exogenous stochastic processes.

Our main contribution to the literature is twofold. First, we add to the empirical literature on PCP vs LCP by comparing the data fit for Canada-US in terms of a likelihood race and second-moment validation for these two pricing regimes. Second, we contribute to the optimal policy literature by examining the implications of these two paradigms for the conduct of monetary policy using welfare-optimized Taylor-type nominal interest rate rules. These are subject to a 'soft' zero-lower-bound constraint, in the sense described below, on the nominal interest rate with a particular focus for the SOE on consumer price versus producer price inflation targeting, and price level versus nominal income targeting.

Our specific findings can be summarized as following. First, the striking result from our Bayesian estimation is that in a likelihood race LCP beats PCP with large marginal likelihood differences: allowing for imperfect exchange rate pass-through is empirically relevant and improves model fit remarkably. Second, we find that there are important propagation channels active in the SOE that must be taken into account for any policy-related study. Indeed, our analysis of welfare-optimized interest rate rule shows that the more muted response of output to a monetary policy shock under LCP compared with PCP calls for a far more aggressive response of the nominal interest rate to its inflation and output growth targets. In these rules, in either PCP or LCP currency pricing regimes, producer-price inflation targeting is significantly welfare-superior to its consumer-price counterpart and there is no role for exchange rate targeting, which contrasts with the optimal policy predictions in Gopinath *et al.* (2020). In both cases, optimized rules are 100% inertial.

Additional results address old debates regarding price-level and nominal income targeting in either closed or small open economies. For the closed economy, the welfare-optimized price-level rule closely mimics the optimized general inflation-output rule, whereas for the SOE the corresponding result requires a nominal income rule. We believe this is a new result for the open economy literature. The intuition is as follows: the monetary policy transmission takes place largely through its effect on the exchange rate that results in consumption switching between imports and exports in response to all the shocks. A feedback in the nominal interest rate that responds to changes in the exchange rate would interfere with this mechanism and is avoided in the welfare optimization. However, exchange rate volatility *does* impact on output, consumption, hours supplied and therefore utility. The optimized nominal income rule then accommodates exchange rate flexibility but respond more strongly to the *direct* determinant of welfare, namely the change in output, that would be the case for the SOE economy.

But in both these cases the LCP rule is again far more aggressive than its PCP counterpart. In our ROW-SOE framework, the level of the long-run steady state inflation target is set by the ROW central bank and is adopted by the SOE. Its optimal value, chosen the achieve a given low value of the nominal interest rate hitting the ZLB (the 'soft' constraint), is much lower than the typical target annual inflation of 2%; indicating that the inflation target is too blunt an instrument to efficiently reduce the costs of zero-bound episodes. These results then clearly demonstrate important implications of both openness and the currency pricing regime for monetary policy conducted using Taylor-type simple nominal interest rate rules.

Our paper is closely related to the works of Bodenstein *et al.* (2017) and Jones *et al.* (2020), which also employ a 2-country framework, the former studying the effects of foreign shocks under a liqiuidity trap, the latter focusing on the identification of forward guidance shocks. Our study is also related to the literature on models with limited asset market participation (LAMP), which provide a very tractable way of introducing a financial friction into macro-models and for examining the distributional effects of policy. The focus has been largely limited to closed economies as in Bilbiie (2008), but more recent research by Boerma (2014) and Levine *et al.* (2021) suggests that trade openness and LAMP have important consequences for the design of monetary rules.

Finally, our paper builds upon the optimal policy literature reviewed in Corsetti *et al.* (2010) and Gali (2015). Here a distinction between 'targeting rules' and 'implementable' nominal interest rate rules should be emphasized. Corsetti *et al.* (2010), Corsetti *et al.* (2018), Corsetti *et al.* (2020) and Senay and Sutherland (2019) focus on the former, with inflation rules that target welfare-relevant gaps created by the absence of risk-sharing. Devereux and Yetman (2014), and our paper, conducts policy analysis in terms of the latter in terms of implementable nominal interest rules. Unlike the former, we introduce a soft ZLB constraint that avoids ZLB episodes and Section 5 elaborates this aspect. Senay and Sutherland (2019) adopt an incomplete markets two-country framework with many sources of shocks in a general setting of multiple assets. Our ROW-SOE model which has two bonds in the SOE, domestic and foreign, and multiple shocks in both the ROW and the SOE is then a special case of their model. We introduce a risk-premium as in Benigno (2009) which can be viewed as deriving from portfolio adjustment costs. Welfare-optimal simple rules, first introduced into the macroeconomic literature by Levine and Currie (1987), are welfare inferior to the Ramsey policy adopted in much of this literature. However, in general they closely mimic the welfare outcome of the latter (see, for example, Schmitt-Grohe and Uribe, 2007 whose computational methodology we follow closely).

The rest of the paper's structure is organized as follows: Section 2 sets out the model. Section 3 describes the estimation procedure and Section 4 the empirical results. Section 5 then uses the estimated model to compute optimized simple rules. These impose a soft ZLB constraint on the nominal interest rate in the form of a delegation game between the government and an instrument-independent central bank. Section 6 of the paper concludes and an online Appendix provides further details of the model, its solution and estimation.

2 Model Description

The model economy is a New Keynesian two-bloc dynamic general equilibrium model with sticky prices. We first set out a general model of two economies of different population sizes and then consider as a limiting case a small open economy interacting with the ROW but with no policy strategic interdependence.

We depart from the standard SOE model exemplified by Gali and Monacelli (2005) and Gali (2015) along the following dimensions. First, for the SOE, we nest two different pricing paradigms: local currency pricing (LCP) alongside producer currency pricing (PCP). For the ROW in the non-limiting case we assume PCP, so up to that point we have the dominant currency pricing regime. Second, the production function uses not just labour but also capital and intermediate inputs produced domestically and abroad. Trade in investment and intermediate goods adds to the significance of imperfect exchange rate pass-through. Third, we study strategic complementarities in pricing. These first three features mimic those in Gopinath *et al.* (2020) and extends traded inputs to capital accumulation. Fourth, we include a rudimentary commodity sector with fixed exogenous output and an exogenous price AR1 process which is estimated separately. Fifth, we adopt an incomplete markets two-bond set-up with foreign bonds subject to a risk premium that depends on the total exposure of the SOE to foreign debt as in Benigno (2009). This feature is then a special case of an open economy incomplete market models with multiple assets as studied in Senay and Sutherland (2019), referred to in the literature background above. Finally we study distribution effects of currency pricing and monetary policy in a simple way using a two-agent New Keynesian (TANK) framework: a proportion of 'rule of thumb' households have limited asset market participation (LAMP). LAMP is a well documented feature of both developing and developed economies and its relevance for the US and Canada is explored when we come to the estimation.

Details of the optimization problems facing agents including policymakers and general equilibrium conditions now follow. Full details of the first order conditions, equilibrium and deterministic steady state about which the perturbation solution is obtained is provided in the online appendices of the paper.

2.1 Households

Households in the H bloc hold both domestic and foreign bonds, but those in the F bloc only hold domestic bonds. In order to accommodate financial frictions in a simple way and examine distributional effects, households are divided in those who participate in the financial sector and can lend or borrow to each other. These are *Ricardian consumers*. The remaining rule-of-thumb consumers are credit-constrained and must consume out of wage income net of tax.

In a stochastic environment, household j of both types maximizes

$$\Omega_0^j = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U_t^j \right] \tag{1}$$

where, following Smets and Wouters (2007), the single-period utility is of the general form proposed by King *et al.* (1988):

$$U_t^j = \frac{\left[C_t^j - \chi C_{t-1}^j \right]^{1-\sigma}}{1-\sigma} \exp \left[(\sigma - 1) \frac{(H_t^j)^{1+\psi}}{1+\psi} \right] \tag{2}$$

where C_t^j is real consumption, H_t^j is hours supplied, β is the discount factor, χ controls habit formation, σ is the inverse of the elasticity of intertemporal substitution (for constant labour), and ψ is the inverse of the Frisch labour supply elasticity. Note that, unlike in the original Smets-Wouters model, we use internal instead of external habit formation.[1]

There are $(1 - \lambda)$ non-credit-constrained Ricardian ($j = R$) consumers. The R household solves

$$\max_{C_t^R, L_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}^R, H_{t+s}^R) \right] \tag{3}$$

subject to a nominal budget constraint given by

$$P_t^B B_{H,t} + P_t^{B*} S_t B_{F,t}^* = B_{H,t-1} + S_t B_{F,t-1}^* + P_t W_t \left(1 - \tau_t^w\right) H_t^R - P_t C_t^R + \Gamma_t \tag{4}$$

with nominal profits given by Γ_t and a proportional labour tax given by τ_t^w. $B_{H,t}$ and $B_{F,t}^*$ are domestic and foreign bonds respectively, bought at nominal prices P_t^B and P_t^{B*} and denominated in the respective currencies. P_t is the CPI index that includes an imported component (see (11) below) and S_t is the nominal exchange rate.

The first order conditions for the Ricardian households for holdings of domestic and foreign bonds and labour supply are;

$$P_t^B = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] \tag{5}$$

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t = R_t^* \phi \left(\frac{S_t B_{F,t}^*}{P_{H,t} Y_t} \right) \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \Pi_{t,t+1}^S \right] \tag{6}$$

$$\lambda_t = \frac{(1 - \sigma_c^R) U_t^R}{C_t^R - \chi C_{t-1}^R} - \beta \chi \frac{(1 - \sigma_c^R) U_{t+1}^R}{C_{t+1}^R - \chi C_t^R} \tag{7}$$

$$W_t (1 - \tau_t^w) = \frac{\left[C_t^R - \chi C_{t-1}^R \right] H_t^{R^{\psi^R}}}{1 - \beta \chi \frac{U_{t+1}^R}{U_t^R} \frac{C_t^R - \chi C_{t-1}^R}{C_{t+1}^R - \chi C_t^R}} \tag{8}$$

[1]External habit means that consumption is an externality which leads to an equilibrium where consumption and output is too high relative to the first-best. This has the implausible consequence that increasing inflation in the steady state, as we do in our normative analysis to impose a ZLB constraint on the nominal interest rate, can be welfare-improving.

where the stochastic discount factor, single-period utility and marginal utility are given by

$$\Lambda_{t,t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$$
$$U_t^R = \frac{\beta^t}{1-\sigma_c^R}\left[C_t^R - \chi C_{t-1}^R\right]^{-\sigma_c^R+1} \exp\left[(\sigma_c^R - 1)\frac{H_t^{R\,1+\psi^R}}{1+\psi^R}\right]$$
$$\lambda_t = \frac{(1-\sigma_c^R)U_t^R}{C_t^R - \chi C_{t-1}^R} - \beta\chi\frac{(1-\sigma_c^R)U_{t+1}^R}{C_{t+1}^R - \chi C_t^R}$$

Equation (6) then gives a UIP condition modified to allow for risk. $\Pi_{t,t+1}^S \equiv \frac{S_{t+1}}{S_t}$ is the rate of change of the nominal exchange rate over the interval $[t, t+1]$ (i.e., the depreciation rate). Nominal return on home bonds is by definition $R_t = \frac{1}{P_t^B}$, where R_t is set by the central bank. We assume foreign bonds are subject to a risk premium that depends on the exposure to foreign debt, $R_t^* = \frac{1}{P_t^{B^*}\phi\left(\frac{S_t B_{F,t}^*}{P_{H,t}Y_t}\right)}$. Additionally, we assume $\phi(0) = 0$ and $\phi' < 0$.

The remaining λ consumers are rule of thumb credit-constrained ($j = C$) and have no income from monopolistic retail firms. They must consume out of wage income and their consumption is given by

$$C_t^C = W_t\,(1-\tau_t^w)\,H_t^C \tag{9}$$

Liquidity-constrained consumers now choose C_t^C and $L_t^C = 1 - H_t^C$, to maximize an analogous welfare function to (3) subject to (9). An analogous equilibrium condition for labour supply as for R households plus (9) describes their behaviour given the real wage.[2]

Total consumption and labour supply by non-Ricardian and Ricardian is then $\lambda C_t^C + (1-\lambda)C_t^R$ and $\lambda H_t^C + (1-\lambda)H_t^R$ respectively. In the welfare analysis later in the paper the policymaker is utilitarian and adopts a single-period welfare measure $\lambda U_t^C + (1-\lambda)U_t^R$.

2.2 Consumption Demand for Domestic and Imported Goods

In each bloc, domestically produced and imported goods are consumed with prices denominated in the country's currency with notation summarized in Table 2

< Table 2 here >

For given aggregate consumption $C_t = C_t^R, C_t^C$ for both Ricardian and credit-constrained consumers, household demand for consumption goods from domestic retailers (C_H) and foreign retailers (C_F, i.e. imports) is chosen to maximise the Dixit-Stigitz quantity aggregator

$$C_t = \left[\mathrm{w}_C^{\frac{1}{\mu_C}} C_{H,t}^{\frac{\mu_C-1}{\mu_C}} + (1-\mathrm{w}_C)^{\frac{1}{\mu_C}} C_{F,t}^{\frac{\mu_C-1}{\mu_C}}\right]^{\frac{\mu_C}{\mu_C-1}} \tag{10}$$

[2]Households's conditions in the ROW are derived under the same assumptions, see the online Appendix for full details.

The corresponding Dixit-Stigitz price index is given by

$$P_t = \left[\mathrm{w}_C (P_{H,t})^{1-\mu_C} + (1-\mathrm{w}_C)(P_{F,t})^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \tag{11}$$

Analogous aggregates apply to the ROW.

Now define CPI, domestic and imported inflation rates over the time interval $[t-1,t]$ by $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$ respectively. Then from (11) we have

$$\Pi_t = \left[\mathrm{w}_C \left(\Pi_{H,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1-\mathrm{w}_C) \left(\Pi_{F,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \tag{12}$$

Parameter μ_C is the elasticity of substitution between home and foreign goods, while parameter w_C is related to the degree of home-bias in preferences and plays a critical role in this paper. In turn, $1-\mathrm{w}_C$ is interpreted as an index of openness to international trade in final goods: when $\mathrm{w}_C = 1$, the share of foreign goods in the composite consumption index approaches zero. The degree of openness $1-\mathrm{w}_C$ is identical across economies and $\mathrm{w}_C = 1$ denotes an economy in autarky, i.e. a closed economy. In contrast, if $\mathrm{w}_C = 0$, there is no home-bias in consumption. Note also that there is international trade in intermediate goods which enter into production (see Section 2.4 below).

Maximizing total consumption (10) subject to a given aggregate expenditure $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ yields

$$C_{H,t} = \mathrm{w}_C \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t \quad \text{and} \quad C_{F,t} = (1-\mathrm{w}_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t \tag{13}$$

In our general model setup we assume that retail firms either set prices in home currency (i.e. PCP) or local pricers (i.e. LCP). For now, however, we assume PCP. Define the real exchange rate as the relative aggregate consumption price $RER_t \equiv \frac{P_t^* S_t}{P_t}$, where S_t is the nominal exchange rate. With PCP, because the home country is small, the law of one price (LOP), i.e. perfect exchange rate pass-through for imports, implies that $P_t^* = P_{F,t}^*$, $S_t P_t^* = P_{F,t}$, so $RER_t = \frac{P_{F,t}}{P_t}$ and *terms of trade* for the home country are defined as $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$, i.e. the price of imported goods relative to domestic ones. It follows that $\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}}$. .

2.3 Capital Producers

Capital producers[3] purchase investment goods from home and foreign retail firms at real price $\frac{P_t^I}{P_t}$ selling at real price Q_t to maximize expected discounted profits

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[Q_{t+k} (1 - \mathcal{S}(I_{t+k}/I_{t+k-1})) I_{t+k} - \frac{P_t^I}{P_t} I_{t+k} \right]$$

[3] In our model investment and capital accumulation decisions can be included in those of the household by allowing it to own the capital and rent to firms without changing the equilibrium. Separating out the decisions, as in our paper, is a useful modelling device for incorporating a banking sector as in Gertler and Kiyotaki (2010). This will be an avenue for future work.

where total capital accumulates according to

$$K_t = (1-\delta)K_{t-1} + (1-\mathcal{S}(X_t))I_t IS_t \tag{14}$$

IS_t is an investment shock.

Dixit-Stiglitz aggregators over home and imported investment are:

$$I_t = \left[\mathrm{w}_I^{\frac{1}{\mu_I}} I_{H,t}^{\frac{\mu_I-1}{\mu_I}} + (1-\mathrm{w}_I)^{\frac{1}{\mu_I}} I_{F,t}^{\frac{\mu_I-1}{\mu_I}} \right]^{\frac{\mu_I}{\mu_I-1}} \tag{15}$$

$$P_{I,t} = \left[\mathrm{w}_I (P_{H,t})^{1-\mu_I} + (1-\mathrm{w}_I)(P_{F,t})^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}} \tag{16}$$

and analogous demand for home and imported investment goods to (13) are

$$I_{H,t} = \mathrm{w}_I \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_H} I_t \quad \text{and} \quad I_{F,t} = (1-\mathrm{w}_I) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_I} I_t \tag{17}$$

For the FOCs, we define the gross real return on capital R_t^K as

$$R_t^K = \frac{r_t^K \left(1-\tau_t^k\right) + (1-\delta)Q_t}{Q_{t-1}} \tag{18}$$

such that the right-hand-side is the gross return to holding a unit of capital from $t-1$ to t, while the left-hand-side is the gross return from holding bonds and the opportunity cost of capital. τ_t^k is a tax on corporate profits assumed exogenous in the model. We further define investment adjustment costs and the rate of change of investment as

$$\begin{aligned} \mathcal{S}(X_t) &\equiv \phi_X (X_t - X)^2 \\ X_t &\equiv \frac{I_t}{I_{t-1}} ;\ S',\, S'' \geq 0 ;\ S(1) = S'(1) = 0 \end{aligned}$$

where ϕ_X is the elasticity of investment adjustment costs.

2.4 Traded Intermediate Goods

We now introduce capital and trade in investment goods, and allow for intermediate inputs in production. Both are important channels for the effect of exchange rate changes on the supply side, but the intermediate goods channel is more direct. Moreover a large proportion of trade is in intermediate goods.

Modelling trade in intermediate inputs, M_t, is analogous to investment goods with

$$M_t \equiv \left[\mathrm{w}_M^{\frac{1}{\mu_M}} M_{I,t}^{\frac{\mu_M-1}{\mu_M}} + (1-\mathrm{w}_M)^{\frac{1}{\mu_M}} M_{F,t}^{\frac{\mu_M-1}{\mu_M}} \right]^{\frac{\mu_M}{\mu_M-1}} \tag{19}$$

$$P_t^M \equiv \left[\mathrm{w}_M P_{H,t}^{1-\mu_M} + (1-\mathrm{w}_M) P_{F,t}^{1-\mu_M} \right]^{\frac{1}{1-\mu_M}} \tag{20}$$

Minimizing total intermediate input (19) subject to a given aggregate expenditure $P_t M_t = P_{H,t} M_{H,t} + P_{F,t} M_{F,t}$ yields

$$M_{H,t} = \mathrm{w}_M \left(\frac{P_{H,t}}{P_t^M} \right)^{-\mu_M} M_t \quad \text{and} \quad M_{F,t} = (1 - \mathrm{w}_M) \left(\frac{P_{F,t}}{P_t^M} \right)^{-\mu_M} M_t \tag{21}$$

2.5 Firms

There are wholesale and retail sectors. The former act in perfect competition producing a homogeneous intermediate good, the latter in monopolistic competition producing differentiated final goods.

2.5.1 Wholesale sector

Production technology in the wholesale sector uses aggregate labour supply, capital and intermediate inputs in a Cobb-Douglas production function:

$$Y_t^W = F(A_t, H_t, K_{t-1}, M_t) = (A_t H_t)^{\alpha_H} M_t^{\alpha_M} (K_{t-1})^{1-\alpha_H-\alpha_M} \tag{22}$$

where A_t is labour augmenting productivity. Wholesale firms sell at nominal price P_t^W to retailers, so profit maximisation gives first order conditions equating marginal products with factor prices:

$$\begin{aligned}
F_{H,t} &= \alpha_H \frac{Y_t^W}{H_t} \frac{P_t^W}{P_t} \frac{P_{H,t}}{P_t} = W_t \\
F_{K,t} &= (1 - \alpha_H - \alpha_M) \frac{Y_t^W}{K_{t-1}} \frac{P_t^W}{P_t} \frac{P_{H,t}}{P_t} = r_t^K \\
F_{M,t} &= \frac{\alpha_M P_t^W Y_t^W}{M_t} = \frac{P_t^M}{P_t^W}
\end{aligned}$$

where P_t is price index of final consumption goods given by (11).

2.5.2 Retail Sector with Complete or Incomplete Exchange Rate Pass-through

We consider two current pricing regimes for exports, PCP and LCP. For PCP there is complete exchange rate pass through and the law of one price holds. Each home retailer $m \in (0,1)$ purchases output from the intermediate good sector at price $P_{H,t}^W$ and converts into a differentiated good sold at price $P_{H,t}(m)$ to households, capital good producers and governments who use the technology

$$C_{H,t} = \left(\int_0^1 C_{H,t}(m)^{(\zeta-1)/\zeta} dm \right)^{\zeta/(\zeta-1)} \tag{23}$$

to combine into baskets, where ζ is the elasticity of substitution. Similarly for $I_{H,t}$ and G_t.

For each m, the consumer chooses $C_{H,t}(m)$ at a price $P_{H,t}(m)$ to maximize (23) given total expenditure $\int_0^1 P_{H,t}(m) C_{H,t}(m) dm$. This results in a set of consumption demand equations for each differentiated good m with price $P_{H,t}(m)$ of the form

$$C_{H,t}(m) = \left(\frac{P_{H,t}(m)}{P_{H,t}}\right)^{-\zeta} C_{H,t} \tag{24}$$

where $P_{H,t} = \left[\int_0^1 P_{H,t}(m)^{1-\zeta} dm\right]^{\frac{1}{1-\zeta}}$. $P_{H,t}$ is the aggregate price index. Note that $C_{H,t}$ and $P_{H,t}$ are Dixit-Stiglitz aggregators – see Dixit and Stiglitz (1977). Demand for investment, government services and exports takes the same form so in aggregate

$$Y_t(m) = \left(\frac{P_{H,t}(m)}{P_{H,t}}\right)^{-\zeta} Y_t \tag{25}$$

Following Calvo (1983), we now assume that there is a probability of $1 - \xi$ at each period that the price of each retail good m is set optimally to $P_t^0(m)$. If the price is not re-optimized, then it is held fixed.[4] For each retail producer m, given its real marginal cost

$$MC_t = \frac{P_t^W}{P_{H,t}} \tag{26}$$

For a LCP retail firm we must distinguish the price setting in domestic and foreign markets. Denote the price for domestically marketed home-produced goods in home currency by $P_{H,t}^{\ell}$ and foreign-marketed goods in foreign currency by $P_{H,t}^{*\ell}$. The superscript ℓ now stands for the 'LCP regime' whereas $P_{H,t}$ previously now refers to the 'PCP regime'.

The optimal price for *home-produced goods* the domestic market $P_{H,t}^{\ell O}$ is given by maximizing real profits:

$$\max_{P_{H,t}^{\ell O}(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{H,t+k}^{\ell}} (Y_{t+k} - EX_{t+k})(m) \left[P_{H,t}^{\ell}(m) - P_{H,t+k}^{\ell} MC_{t+k}^{\ell} MS_{t+k}\right]$$

where under LCP the real marginal cost for each home-produced good sold at home is given by

$$MC_t^{\ell} = \frac{P_t^W}{P_{H,t}^{\ell}} \tag{27}$$

Note that real profits use the price of domestically-produced goods, $P_{H,t}^{\ell}$ as the numeraire.

For the *foreign market*, exports are sold at a price $P_{H,t}^{*\ell}$ in units of foreign currency which give real profits (again using $P_{H,t}^{\ell}$ as the numeraire) $\frac{S_t P_{H,t}^{*\ell} EX_t}{P_{H,t}^{\ell}}$. Then the optimal price in units of foreign currency is now

$$\max_{P_{H,t}^{*\ell}(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{H,t+k}^{\ell}} EX_{t+k}(m) \left[P_{H,t}^{*\ell}(m) S_{t+k} - P_{H,t+k}^{\ell} MC_{t+k}^{*\ell} MS_{t+k}\right]$$

[4]Thus we can interpret $\frac{1}{1-\xi}$ as the average duration for which prices are left unchanged.

where, using (27), the real marginal cost for each exported home-produced good is given by

$$MC_t^{*\,\ell} \equiv \frac{P_t^W}{S_t P_{H,t}^{*\,\ell}} = \frac{MC_t^\ell \, P_{H,t}^\ell}{S_t \, P_{H,t}^{*\,\ell}} \tag{28}$$

and for home and foreign markets respectively we have

$$\begin{aligned}(Y_t - EX_t)(m) &= \left(\frac{P_{H,t}^\ell(m)}{P_{H,t}^\ell}\right)^{-\zeta} (Y_t - EX_t) \\ EX_t(m) &= \left(\frac{P_{H,t}^{*\,\ell}(m)}{P_{H,t}^{*\,\ell}}\right)^{-\zeta} EX_t\end{aligned}$$

Substituting in this demand schedule, taking first-order conditions with respect the relevant price, dropping the m index as all firms face the same marginal cost leads to

$$\textbf{Home Market}: \frac{P_{H,t}^{\ell O}}{P_{H,t}^\ell} = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{k=0}^\infty \xi^k \Lambda_{t,t+k} \Pi_{H,t,t+k}^\zeta (Y_{t+k} - EX_{t+k}) MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^\infty \xi^k \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}^\ell\right)^{\zeta-1} \left(\frac{P_{H,t+k}^\ell}{P_{H,t+k}}\right) (Y_{t+k} - EX_{t+k})} \tag{29}$$

$$\textbf{Foreign Market}: \frac{P_{H,t}^{*\,\ell O}}{P_t^{*\,\ell}} = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{k=0}^\infty \xi^k \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}^{*\,\ell}\right)^{\zeta} \left(\frac{S_{t+k} P_{H,t+k}^{*\,\ell}}{P_{H,t+k}}\right) EX_{t+k} MC_{t+k}^{*\,\ell}}{\mathbb{E}_t \sum_{k=0}^\infty \xi^k \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}^{*\,\ell}\right)^{\zeta-1} \left(\frac{S_{t+k} P_{H,t+k}^{*\,\ell}}{P_{H,t+k}}\right) EX_{t+k}} \tag{30}$$

The summations in these expressions can be expressed in recursive form - see online Appendix for further details.

Using the aggregate price indices $P_{H,t}^\ell$ and $P_{H,t}^{*\,\ell}$ for home-consumed and exported goods respectively, and the fact that all resetting firms will choose the same price, by the Law of Large Numbers we can find the evolution of the price index as given by

$$\begin{aligned}\left(P_{H,t}{}^\ell\right)^{1-\zeta} &= \xi \left(P_{H,t-1}^\ell\right)^{1-\zeta} + (1 - \xi) \left(P_t^{\ell O}\right)^{1-\zeta} \\ P_{H,t}^{*\,\ell\;1-\zeta} &= \xi \left(P_{H,t-1}^{*\,\ell}\right)^{1-\zeta} + (1 - \xi) \left(P_{H,t}^{*\,\ell O}\right)^{1-\zeta}\end{aligned}$$

which can be written in the form required

$$1 = \xi \left(\Pi_{H,t-1,t}^\ell\right)^{\zeta-1} + (1 - \xi) \left(\frac{P_{H,t}^{\ell O}}{P_{H,t}^\ell}\right)^{1-\zeta} \tag{31}$$

$$1 = \xi \left(\Pi_{H,t-1,t}^{*\,\ell}\right)^{\zeta-1} + (1 - \xi) \left(\frac{P_{H,t}^{*\,\ell O}}{P_t^{*\ell}}\right)^{1-\zeta} \tag{32}$$

Using the demand schedules, we can write the price dispersion that gives the average loss in output as

$$\begin{aligned}\Delta_{H,t}^{\ell} &= \frac{1}{M}\sum_{m=1}^{M}\left(\frac{P_{H,t}^{\ell}(m)}{P_{H,t}^{\ell}}\right)^{-\zeta}\\ \Delta_{H,t}^{*\ell} &= \frac{1}{M}\sum_{m=1}^{M}\left(\frac{P_{H,t}^{*\ell}(m)}{P_{H,t}^{*\ell}}\right)^{-\zeta}\end{aligned}$$

for firms $m = 1, ..., M$. It is not possible to track all $P_t(m)$ but as it is known that a proportion $1-\xi$ of firms will optimise prices in period t, and from the Law of Large Numbers, that the distribution of non-optimised prices will be the same in as the overall distribution. Therefore, price dispersion can be written as a law of motion

$$\Delta_{H,t}^{\ell} = \xi(\Pi_{H,t-1,t}^{\ell})^{\zeta}\Delta_{H,t-1}^{\ell} + (1-\xi)\left(\frac{P_{H,t}^{\ell O}}{P_{H,t}^{\ell}}\right)^{-\zeta}. \tag{33}$$

$$\Delta_{H,t}^{*\ell} = \xi(\Pi_{H,t-1,t}^{*\ell})^{\zeta}\Delta_{H,t-1}^{*\ell} + (1-\xi)\left(\frac{P_{H,t}^{*\ell O}}{P_{t}^{*\ell}}\right)^{-\zeta} \tag{34}$$

Using this, aggregate final output is divided between exports invoiced in foreign currency EX_t and domestic production invoiced in domestic currency $Y_t - EX_t$. Then allowing for dispersion we have

$$Y_t = \left(\frac{\frac{EX_t}{Y_t}}{\Delta_{H,t}^{*\ell}} + \frac{\left(1-\frac{EX_t}{Y_t}\right)}{\Delta_{H,t}^{\ell}}\right)Y_t^W \tag{35}$$

Note that (35) implies a loss of output due to dispersion in prices.

Exporters from the foreign bloc are PCPers, so $S_t P_{F,t}^* = P_{F,t}$ and therefore we have imported inflation as

$$\Pi_{F,t} = \Pi_t^S \Pi_{F,t}^* \tag{36}$$

With LCP in the SOE we have that

$$\mathcal{T}_t^* \equiv \frac{P_{H,t}^*}{P_{F,t}^*} = \frac{P_{H,t}^{*\ell}}{P_{F,t}^*} = \frac{P_{H,t}^{*\ell}}{\frac{P_{F,t}}{S_t}} \tag{37}$$

It follows from $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ and from (26) and (27) that

$$\mathcal{T}_t\mathcal{T}_t^* = \frac{MC_t^{\ell}}{MC_t^{*\ell}} \tag{38}$$

where the r.h.s is the quotient of the real marginal cost of home-produced goods sold at home and exported. In the perfect exchange rate pass-through case (PCP), $MC_t^{*\ell} = MC_t^{\ell}$, $\Pi_{H,t-1,t}^* = \Pi_{H,t-1,t}$

and we have $\mathcal{T}_t\mathcal{T}_t^* = 1$. Table 3 summarizes this PCP-LCP distinction in terms of its effect on the law of one price and the online Appendix gives full details.

< Table 3 here >

2.5.3 Price Setting with the Kimball Aggregator

We now allow for strategic complementarity in pricing that gives rise to variable, as opposed to constant, mark-ups on real marginal costs. We model this using a *Kimball aggregator* as in Kimball (1995) and Klenow and Willis (2016) to generalize the Dixit-Stigliz aggregator used up to now. Here we focus on the closed economy ROW and the pricing decision. PCP and LCP pricing generalizes in the same way as do the Dixit-Stigliz aggregators used for investment and intermediate goods.

We assume that buyers of final goods choose differentiated goods subject to Kimball's (1995) homothetic demand aggregator

$$\int_0^1 \Upsilon\left(\frac{Y_t\,(m)}{Y_t}\right) dm = 1 \tag{39}$$

which is used to combine differentiated goods indexed by $m \in (0,1)$, where $\Upsilon(1) = 1$, $\Upsilon'(\xi) > 0$ and $\Upsilon''(\xi) < 0 \;\forall \xi \geq 0$. The buyer then maximises aggregate quantity of purchased goods subject to equation (39). Further details of the solution and the functional form used for $\Upsilon(x)$ in the computations are given in the online Appendix.

2.6 The Commodity Sector and Trade Balance

We introduce a commodity sector (e.g. oil) in the Home country treating output as an exogenous constant endowment Y^O. Revenues are then driven only by the price of the commodity $P^*_{O,t}$ denominated in foreign currency, which is an AR1 exogenous process. The commodity is entirely exported and the only channel through which oil production and price affects the model is via the trade balance and the government budget constraint.[5] A tax rate τ^o_t applies to this sector.

The nominal trade balance then is given by

$$P_tTB_t = S_tP^*_{O,t}Y^O + P_{H,t}Y_t - P_tC_t - P_{I,t}I_t - P_{M,t}M_t - P_{H,t}G_t \tag{40}$$

is the difference between output, commodity revenue, private and public consumption, and investment.

2.7 Bloc Size Effects and the SOE

In our representative agent model, all variables such as Y_t and Y_t^* are *per capita* quantities and can differ for example because labour productivity in the steady state $A \neq A^*$. The implication up to now is that population sizes are the same in both blocs. We now let the F bloc have a population

[5]Given the empirical case we study below, we focus on an oil sector, but this setup can easily be applied to capture commodity dependence on energy, minerals or agricultural exports.

n times that of the H bloc. In the limit as $n \to \infty$ we get to a SOE-ROW model used in the rest of the paper.

$$\begin{aligned} Y_t &= C_{H,t} + n\,C^*_{H,t} + I_{H,t} + n\,I^*_{H,t} + M_{H,t} + n\,M^*_{H,t} + G_t \\ &= C_{H,t} + I_{H,t} + M_{H,t} + G_t + EX_t \qquad (41) \\ n\,Y^*_t &= n\,C^*_{F,t} + C_{F,t} + n\,I^*_{F,t} + I_{F,t} + +n\,M^*_{F,t} + M_{F,t} + n\,G^*_t \\ &= n\,(C^*_{F,t} + I^*_{F,t} + M^*_{F,t} + G^*_t + EX^*_t) \qquad (42) \end{aligned}$$

where *per capita* non-oil exports by the Home and Foreign Country are respectively given by

$$\begin{aligned} EX_t &\equiv nC^*_{H,t} + nI^*_{H,t} + n\,M^*_{H,t} \\ &= n(1-\mathrm{w}^*_C)\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\mu^*_C} C^*_t + n(1-\mathrm{w}^*_I)\left(\frac{P^*_{H,t}}{P^{I^*}_t}\right)^{-\mu^*_I} I^*_t + n(1-\mathrm{w}^*_M)\left(\frac{P^*_{H,t}}{P^{M^*}_t}\right)^{-\mu^*_M} M^*_t \end{aligned}$$

$$\begin{aligned} EX^*_t &\equiv C_{F,t}/n + I_{F,t}/n + M_{F,t}/n \\ &= (1-\mathrm{w}_C)/n\left(\frac{P_{F,t}}{P_t}\right)^{-\mu_C} C_t + (1-\mathrm{w}_I)/n\left(\frac{P_{F,t}}{P^I_t}\right)^{-\mu_I} I_t + (1-\mathrm{w}_M)/n\left(\frac{P_{F,t}}{P^M_t}\right)^{-\mu_M} M_t \end{aligned}$$

Nominal trade balance in the Home and Foreign blocs are now respectively

$$\begin{aligned} P_t\,TB_t &= S_t P^*_{O,t} Y^O + P_{H,t}\,n\,(C^*_{H,t} + I^*_{H,t} + M^*_{H,t}) - P_{F,t}(C_{F,t} + I_{F,t} + M_{F,t}) \\ &= P_{H,t}\,EX_t - P_{F,t} n\,EX^*_t \qquad (43) \\ P^*_t\,TB^*_t &= P^*_{F,t}(C_{F,t} + I_{F,t} + M_{F,t})/n - P^*_{H,t}(C^*_{H,t} + I^*_{H,t} + M^*_{H,t}) \\ &= P^*_{F,t} EX^*_t - P^*_{H,t} EX_t/n \qquad (44) \end{aligned}$$

denominated in units of H and F currency respectively for the two blocs. For any $1 \leq n < \infty$ we can set up the Home model with the output and trade balance equilibria given by (41) and (43) with TB_t given by

$$\begin{aligned} TB_t &= RER_t \frac{P^*_{O,t}}{P^*_t} Y^O + \frac{P_{H,t}}{P_t}\,EX_t - \frac{P_{F,t}}{P_t} n\,EX^*_t \\ &= \frac{P_{H,t}}{P_t}\,n\,(1-\mathrm{w}^*_C)\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\mu^*_C} C^*_t + \frac{P_{H,t}}{P_t}\,n\,(1-\mathrm{w}^*_I)\left(\frac{P^*_{H,t}}{P^{I^*}_t}\right)^{-\mu^*_I} I^*_t \\ &+ \frac{P_{H,t}}{P_t}\,n\,(1-\mathrm{w}^*_M)\left(\frac{P^*_{H,t}}{P^{M^*}_t}\right)^{-\mu^*_M} M^*_t \\ &- \frac{P_{F,t}}{P_t}\,(1-\mathrm{w}_C)\left(\frac{P_{F,t}}{P_t}\right)^{-\mu_C} C_t - \frac{P_{F,t}}{P_t}\,(1-\mathrm{w}_I)\left(\frac{P_{F,t}}{P^I_t}\right)^{-\mu_I} I_t \\ &- \frac{P_{F,t}}{P_t}\,(1-\mathrm{w}_M)\left(\frac{P_{F,t}}{P^M_t}\right)^{-\mu_M} M_t \end{aligned}$$

$$(45)$$

where $RER_t \equiv \frac{S_t P_t^*}{P_t}$ is the real exchange rate. (See online Appendix for full details).

Then, using trade data, we *calibrate* w_C, w_I, w_M, w_C^*, w_I^* and w_M^* in the steady state as follows: In the steady state without loss of generality by a choice of output units we can put all prices equal to unity. Then we can write the steady state of (45) in terms of observable non-dimensional quantities as

$$\begin{aligned} \frac{TB}{Y} &\equiv tb = \frac{Y^O}{Y} + \left(n\,(1-\mathrm{w}_C^*)\frac{C^*}{Y^*} + n\,(1-\mathrm{w}_I^*)\frac{I^*}{Y^*} + n\,(1-\mathrm{w}_M^*)\frac{M^*}{Y^*} \right) \frac{Y^*}{Y} \\ &- (1-\mathrm{w}_C)\frac{C}{Y} - (1-\mathrm{w}_I)\frac{I}{Y} - (1-\mathrm{w}_M)\frac{M}{Y} \\ &\equiv \mathrm{exo} + \mathrm{exc} + \mathrm{exi} + \mathrm{exim} - \mathrm{imcs} - \mathrm{imi} - \mathrm{imim} \end{aligned} \tag{46}$$

letting $\mathrm{tb} = \frac{TB}{Y}$. The first term on the rhs of (46) is oil output as a proportion of home GDP; the second term to the fourth terms are exports of consumption, investment and intermediate goods as a proportion of home GDP. Given trade data for these fractions and letting n be the relative population size, we can then calibrate the weight w_C^* to hit exc. Similarly, we can calibrate weights w_I^* and w_M^* to hit exi and excms. The fifth, sixth and seventh terms on the rhs of (46) are imports of consumption, investment and intermediate goods as shares of home GDP. Let these be imcs, imis and imims respectively and they can be used to calibrate w_C, w_I and w_M in a similar fashion.

Closed ROW-SOE Special Case

But now consider the ROW-SOE case as $n \to \infty$ and $w_C^* \to 1$, $w_I^* \to 1$ and $w_M^* \to 1$. Then for the F-bloc:

$$\begin{aligned} EX_t^* &= \frac{C_{F,t}}{n} + \frac{I_{F,t}}{n} + \frac{M_{F,t}}{n} \to 0 \\ TB_t^* &= P_{F,t}^* EX_t^* - P_{H,t}^* EX_t/n \to 0 \text{ as } n \to \infty \end{aligned}$$

Hence the ROW becomes a *closed economy* bloc, but $TB_t \neq 0$. Then in (46) in the home country $n(1-w_C^*)$ is replaced with $\left(\frac{exc}{C^*}\right)Y$, $n(1-w_I^*)$ is replaced with $\left(\frac{exi}{I^*}\right)Y$ and $n(1-w_M^*)$ is replaced with $\left(\frac{exim}{M^*}\right)Y$. Therefore, exports (43), driven by consumption, investment and intermediate goods demand in the ROW, are equal to:

$$EX_t = \left(exc \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\mu_C^*} \frac{C_t^*}{C^*} + exi \left(\frac{P_{H,t}^*}{P_{I,t}^*}\right)^{\mu_I^*} \frac{I_t^*}{I^*} + exim \left(\frac{P_{H,t}^*}{P_{M,t}^*}\right)^{\mu_M^*} \frac{M_t^*}{M^*} \right) Y \tag{47}$$

Equation (47) shows how export demand for the SOE depends on fluctuations in consumption, investment and intermediate goods in the ROW and on steady state trade shares for the SOE; exc, exi and exim. The later are calibrated using trade data - see Table 4.

2.8 Financial Intermediation

Efficient financial intermediation for Ricardian savers and capital producers within the Home country implies the zero arbitrage condition:

$$\mathbb{E}_t\left[\Lambda_{t,t+1}R^K_{t+1}\right] = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}}\right]R_t = 1$$

which we take as the equilibrium equation for Q_t.

2.9 Central Bank, Foreign Assets and Monetary Policy

The nominal interest rate R_t is a policy variable, typically given in the literature by a standard Taylor-type rule[6] that includes an exchange rate depreciation term:

$$\begin{aligned}\log\left(\frac{R_t}{R}\right) &= \rho_r\log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\Big[\theta_\pi\log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \theta_s\log\left(\frac{\Pi_{S,t-1,t}}{\Pi_S}\right) \\ &+ \theta_y\log\left(\frac{Y_t}{Y}\right) + \theta_{dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\Big] + \epsilon_{M,t}\end{aligned} \tag{48}$$

Foreign bond holdings evolves according to home country nominal terms

$$P^{B^*}_t S_t B^*_{F,t} = S_t B^*_{F,t-1} + P_t TB_t$$

Now define $B_{F,t} \equiv \frac{S_t B^*_{F,t}}{P_t}$ to be the stock of foreign bonds in home country consumption units. Then

$$P^{B^*}_t B_{F,t} = \frac{\Pi^S_{t-1,t}}{\Pi_{t-1,t}}B_{F,t-1} + TB_t \tag{49}$$

Finally a government nominal balanced budget constraint gives

$$P_{H,t}G_t = P_tW_tH_t\tau^w_t + (1-\alpha)Y^W_t P_{H,t}MC_t\tau^k_t + RER_t\,P^{*O}_t\,Y^O\,\tau^O \tag{50}$$

recalling that $MC_t \equiv \frac{P^W_t}{P_{H,t}}$. This requires that the wage income tax is set such that

$$\tau^w_t = \frac{\frac{P_{H,t}}{P_t}G_t - (1-\alpha)Y^W_t MC_t\tau^k_t - RER_t\,P^{*O}_t\,Y^O\,\tau^O}{W_tH_t} \tag{51}$$

and τ^o_t is a tax on oil revenue.

We assume that tax rates $(\tau^k_t, \tau^k_t, \tau^o_t)$ are held fixed at the steady state value of τ^K. The fiscal stabilization instrument G_t follows a Taylor-type rule

$$\log G_t - \log G = \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{52}$$

[6] In a closed-economy NK model with credit-constrained consumers, Bilbiie (2008) shows that an inversion of the Taylor principle occurs with a sufficient high proportion of such households.

where $\epsilon_{G,t}$ is a fiscal policy shock process.

Monetary Policy in ROW Block

The ROW nominal interest rate is given by the following Taylor-type rule

$$\log\left(\frac{R_t^*}{R*}\right) = \rho_r \log\left(\frac{R_{t-1}^*}{R^*}\right) + (1-\rho_r^*)\left[\theta_{\pi^*}\log\left(\frac{\Pi_{t-1,t}^*}{\Pi^*}\right) + \theta_{y*}\log\left(\frac{Y_t^*}{Y*}\right) + \theta_{dy*}\log\left(\frac{Y*_t}{Y*_{t-1}}\right)\right] + \epsilon_{M^*,t} \tag{53}$$

This completes the specification of the two-bloc open-economy model given six domestic shock processes, A_t, G_t, IS_t, MCS_t, ToT_t; four ROW shock processes, A_t^*, G_t^*, IS_t^*, MCS_t^* and an oil price shock process P_t^{*O}. We assume these are all AR1. The complete equilibrium and the steady state about which the perturbation solution is computed is set out in the supplement to this paper.

3 Estimation

3.1 Data

To estimate the model, we use quarterly information on seven key variables for Canada: GDP, consumption, investment, consumer price index (CPI), nominal exchange rate, nominal interest rate (including Wu-Xia-type shadow rates computed in MacDonald and Popiel, 2017), oil prices and five US variables: output, consumption, investment, consumer price index (CPI) and the Wu and Xia (2016) nominal shadow Federal Funds rates for the sample period from 1991Q1 to 2019Q4 as to capture the inflation targeting mandate of the Bank of Canada. Quarterly crude oil prices were obtained from FRED Economic Data and is deflated with US CPI, all other quarterly data are from IMF's International Financial Statistics. All real variables are in log-differences and seasonally adjusted. In addition, we calibrate share of the trade targets using data from World Integrated Trade Solution (WITS).

3.2 Calibrated Parameters

We use a combination of estimated and calibrated parameters, with Table 4 summarizing the calibration of parameters and the steady state values of selected endogenous variables, matching, as accurately as possible, the empirical evidence and available (quarterly) data on key statistics of these economies.

< Table 4 here >

As in much of the literature, the depreciation rate of capital, δ, is set at 10% per annum, implying a quarterly value of 0.025. The home discount rate is set at $\beta = 0.99$, while we assume the value of 2.00 for Ricardian risk aversion. The substitution elasticity between imported and home goods (μ_C) is calibrated at 1.50 and following Medina *et al.* (2005), Chang *et al.* (2015) and

Adler *et al.* (2016), the export elasticities μ_C^* and μ_I^* is set to 1.50. In terms of the elasticity of substitution among different retail varieties, we adopt a mean of 7 for (ζ).

Using IMF and World Bank data on broad aggregates, we calibrate the government share of production (g_y) at 21%. On the other hand, the trade weights, foreign productivity and foreign discount factor are calibrated to hit the trade targets explained in section 2. Trade shares (consumer, investment and intermediate goods in exports) are calibrated using World Bank and WITS data, suitably normalised to account for the fact that GDP does not include intermediate inputs by construction. Regarding the oil targets, we calibrate share of oil production in GDP by taking the value of average oil production divided by GDP, resulting in a value of 0.0568 for Canada. For the oil taxation rate, we use the value of 15%.

Turning to the use of Kimball versus Dixit-Stiglitz aggregators, we need to calibrate or estimate both a price and a 'super price elasticity' from which two parameters are obtained, γ and η, defining the function form we adopted given in the online Appendix. Concerning micro-econometric evidence on the super-elasticity, Klenow and Willis (2016) suggest a value at most equal to 2, as opposed to value of 10 in much of the macro-literature (including Smets and Wouters, 2007). However, we find this value results in a very small difference in the equilibrium of the model using the Kimball as opposed to the Dixit-Stiglitz aggregator, so we only report results for the latter. This suggests modellers should seek other sources of real nominal rigidities in models such as ours.[7]

3.3 Bayesian Estimation: prior distributions

We estimate the model by Bayesian methods, which entails retrieving the posterior distribution of the model's parameters, say Θ, conditional on the data Y^T. In our two-bloc setup, the ROW bloc does not depend on interactions with the SOE, so it can be estimated separately, which is carried out using US data.[8] The price of oil ($P_{O,t}^*$) is an exogenous process in the model, so we have estimated the standard deviation of this shock separately by fitting an AR(1) process and then use the estimated coefficient and standard deviation in the SOE model.

In order to implement Bayesian estimation, prior distributions must be defined for the parameters and the structural shocks. This choice is usually guided by inherent theoretical restrictions and evidence from previous studies. We use normal distributions as priors for unbounded parameters when more informative priors seem to be necessary, while beta distributions are used for all parameters bounded between 0 and 1, i.e., fractions or probabilities. We use inverse gamma distributions as priors when non-negativity constraints are necessary. All priors are assumed to be the same across specifications.

Tables 5 and 6 list the prior distribution along with the prior mean and standard deviation of all the estimated parameters. We assume a normal distribution centred at 2 and a standard deviation of 0.25 for the risk aversion parameter of non-Ricardian households (σ), in line with the literature on (Ricardian) risk aversion. Turning to the calibration of the proportion of constrained consumers, we use the prior of $\lambda = 0.2$ for US and Canada.

Regarding the monetary policy rule, we assign a normal prior with a mean of 2.00 and standard

[7]We return to this point in the concluding section.

[8]Full results are available in the online Appendix.

deviation of 0.25 to θ_Π, while for the feedback parameter on GDP (θ_y), GDP growth (θ_{dy}) and depreciation rate (θ_{ds}), we assume normal distribution of mean 0.10 and standard deviation of 0.05. In turn, a beta distribution with mean of 0.75 and standard deviation of 0.1 is assigned to ρ_M.

As for the shock processes, we use a beta distribution for the persistence of all shocks with a mean of 0.5 and a standard deviation of 0.10. Given the uncertainty regarding the sources of business cycle fluctuations, we adopt uninformative gamma distributions for the standard deviations of all shocks, with a prior mean of 0.10 along with a standard deviation of 0.20. For input shares, we follow Gopinath *et al.* (2020) in using 0.67 and 0.33 for the means of intermediate and labour shares, respectively.

4 Empirical Results

4.1 Posterior Estimates, Model Comparisons and Variance Decompositions

Tables 5 and 6 display the posterior means of the Bayesian estimation along with the 95% confidence intervals for the PCP and LCP cases. Parameter estimates are generally plausible, with the posterior estimates much more sharply peaked than the prior distribution for most parameters, implying that the data is reasonably informative, away from the priors.[9] The volatility of shocks is high, which is consistent with the literature on open economies. The standard deviations of investment, terms of trade and markup shocks are large relative to other shocks, with the government spending shock having a lesser role in driving fluctuations in Canada. Note the higher estimated volatility of the terms of trade shock in the PCP model compared to the LCP version, suggesting that without the appropriate modelling of imperfect exchange rate pass-through setting mis-specifications in the rest of the models are magnified, as important channels in the economy are otherwise shut down.

< Tables 5 and 6 here >

Turning to Calvo price stickiness, there is a significant difference between the two price regimes, with LCP estimated to be more flexible than PCP, a result that highlights the importance of the imperfect exchange rate pass-through setting for policy considerations. Regarding the policy parameters, we observe a high degree of policy inertia ($\rho_M = 0.93$) and that the central bank responds strongly to inflationary pressures, with θ_Π larger than 2, but only modestly to output fluctuations, with a slightly stronger feedback on exchange rate movements. Tables 5 and 6 also provide a formal Bayesian likelihood race comparison, suggesting that setting the imperfect exchange rate pass-through in the model is very relevant and does improve model fit remarkably.

We then investigate the contribution of each of the structural shocks to the fluctuations of the endogenous variables in the model, as summarized in Tables 7 and 8. The most significant observation is the considerable role of foreign shocks, specifically markup and investment shocks, in explaining the dynamics of the model - these are slightly larger in the LCP setting. The second

[9]See online Appendix; the procedures of Ratto and Iskrev (2011) provide more formal checks and indicate that all parameters are reasonably well identified.

noticeable result is the dominant share of disturbances from the terms of trade shock, followed by price markup and investment shocks, with relatively smaller contribution from other domestic shocks. In particular, terms of trade shocks drive movements in exchange rates, exports, imported consumption and investment goods in the case of PCP, while contributing more to variations in inflation and imported intermediate inputs in the LCP setting.

< Table 7 here >

< Table 8 here >

4.2 Impulse Response to a Monetary Policy and Terms of Trade Shocks

Figures 1- 3 contrasts impulse responses to (domestic and foreign) monetary policy shocks, as well as to terms of trade shocks.[10] In addition to previously defined variables, we also consider a 'law of one price wedge' (LOOP) variable, defined as $\mathcal{T}_t^* \mathcal{T}_t$, which is unity for PCP, but less than (greater than) unity for LCP when an exchange rate appreciation (depreciation) occurs.

We consider a monetary tightening in either the SOE and the ROW for both PCP and LCP regimes, and a negative terms of trade shock. In the SOE and the ROW there is a direct (fall in) demand effect; but for the SOE there is an expenditure switching effect arising from changes in the nominal exchange rate and terms of trade.

A Positive Monetary Shock in the SOE.
A positive shock to the nominal interest rate rule increases the actual interest rate dominating countervailing effects from the rule itself. For PCP (with complete exchange rate pass-through), from Figure 1 the nominal exchange depreciation rate (defined as $\Pi_t^S \equiv \frac{S_t}{S_{t-1}}$) falls, which represents an appreciation. This increases the price of exports in dollars and the respective quantity, EX_t, falls. The terms of trade $\mathcal{T} \equiv \frac{P_{F,t}}{P_{H,t}}$ falls and works against the direct demand effect, but with PCP the latter dominates and imports fall. Taken together, the direct demand and expenditure switching effects then result in a fall in output and consumption for both Ricardian and RoT consumers. Demand for labour falls resulting in a fall in the real wage.

For LCP there is no exchange rate pass-through and the terms of trade fall by more. Hence, the terms of trade effect on imports dominates the direct demand effect and imports rise, and in equilibrium exports now rise slightly. Thus, the configuration of changes in imports and exports is the complete opposite of PCP. Overall, direct and indirect demand effects on output and the real wage under LCP are more muted than PCP.

Differences in the outcome for Ricardian (R) and RoT consumers reveal interest distributional effects of a monetary tightening in the SOE. The R consumers can use savings to smooth out the effects of the monetary (and indeed any) shock, consequently their consumption and hours supplied fall. But for RoT agents, consumption is constrained by $C_t^C = (1 - \tau_w) W_t H_t^C$ and must reflect

[10] We use the estimated ROW model, but to ensure comparability for the two SOE settings, we employ the respective priors; the impulse response functions (irf) are percentage deviations from the steady state; thus for a variable X_t with steady state X, the irf to a shock process Y_t is given to first order by $100 \log(X_t/X) - \log(Y_t/Y) \approx 100 \frac{Y}{X} \frac{(X_t - X)}{(Y_t - Y)} = 100 \frac{DX_t/X}{DY_t/Y}$ or, in other words, the elasticity in percentage terms.

the fall in the real wage W_t. The burden of adjustment then falls on hours supplied, which rise. Since the real wage falls by less under LCP, both consumption and labour supply responses are less than under PCP. Inter-temporal welfare as a percentage change relative to the steady state falls for R consumers, but rises slightly for RoT consumers. However, the steady state welfare is less for RoT compared with R consumers owing to the latter owning economy's stock of capital (through ownership of capital producers) and receiving monopolistic profits (through ownership of retail firms). So absolute welfare remains higher for the R group: unconstrained optimization must be beneficial.

< Figure 1 here >

Spillover effects of a ROW Positive Monetary Policy Shock on the SOE.

In Figure 1, a ROW positive monetary policy shock results in an exchange rate depreciation and an increase in the terms of trade. Indeed, we observe that the spillover expenditure switching effect from exchange rate depreciation are quite significant under both PCP and LCP, highlighting the importance of currency fluctuations for the conduct of monetary policy in a small open economy. Both domestic and imported inflation (and therefore CPI) rise, the latter as a result of a rise in the terms of trade. The SOE nominal interest rate rule responds positively to increases in CPI inflation, output and the depreciation rate. ROW output falls (a standard closed NK economy effect), but for PCP EX_t responds to the expenditure switching effect and rises. For LCP, the latter effect is closed down and exports respond only to the negative ROW demand effect and fall. The terms of trade rises, bringing about a fall in imports for both PCP and LCP. Aggregate outcomes under PCP and LCP are quite similar with the latter responses to the shock slightly muted. But there are again significant distributional effects with contrasting responses of hours supplied by R and RoT consumers and significant welfare difference for the latter.

< Figure 2 here >

A Negative Terms of Trade Shock in the SOE

We model a terms of trade shock, ToT_t as an AR1 shock to the equilibrium relative price $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$. The latter implies that $\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}}$ so with a shock we have

$$\frac{\mathcal{T}_t \, ToT_t}{\mathcal{T}_{t-1} \, ToT_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} \Rightarrow \log\left(\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}}\right) = \log\left(\frac{\Pi_{F,t}}{\Pi_{H,t}}\right) - \log\left(\frac{ToT_t}{ToT_{t-1}}\right) \tag{54}$$

In Figure 3 we see similar responses as for the negative ROW monetary policy shock, the main difference being a direct effect on imports that now rise. LCP responses for output, consumption and hours supplied are more muted as in the other shocks, an outcome that will have implications for welfare-optimized policy rules considered later.

< Figure 3 here >

5 Optimized Simple Rules with a ZLB Constraint

Up to now we have estimated the ROW-SOE model in the limit as the ROW becomes very large and can, from its own viewpoint, be considered as a closed economy. The estimated Taylor-type nominal interest rate rules in the SOE have been defined in terms of targets for CPI inflation, output, the growth of output with persistence in the interest rate. The same applies to the closed ROW model where there is no distinction between CPI and GDP deflator inflation. Moving from a positive to a normative exercise, we now consider and rank alternative rules given the *same estimated* remaining component of the model environment.

As observed in the introduction, policy analysis can be conducted in terms of optimal policy or simple rules. An acknowledged problem with the former (the 'Ramsey problem') is that such rules make unrealistic observability assumptions regarding macro-economic variables such as the output gap, the natural real rate of interest and even shock processes. Simple rules by contrast make the policy instrument a function of observable variables only. These are then both easy to implement and monitor by the public. Since simple rule are forms of commitment (and time-inconsistent), the latter feature is important for credibility; or, in other words, establishing a reputation for such commitment.

Following Deak *et al.* (2020), we adopt a mandate framework for implementing simple nominal interest rate rules that consists of four components: (i) a conditional welfare objective delegated to the central bank discussed below (ii) a form of Taylor-type instrument rule that responds to specified observable macroeconomic variables including, for the SOE, the depreciation rate to capture 'exchange rate targeting'. (iii) a 'soft' zero lower bound constraint on the nominal interest rate in the form of specified low probability of a zero-bound episode[11] and (iv) a long-run (steady-state) inflation rate. With these four features the mandate makes the central bank goal-dependent, but instrument-independent in the sense that it remains free to choose the strength of its response to the targets in the rule.

There are two central banks given mandates of this form for the rest ROW and the SOE in question. We consider these in turn.

5.1 The Optimized Rule in the ROW

The computation of optimized simple rules in an estimated model is central to this paper. We first make some general points before turning to the rule for the ROW. We follow Schmitt-Grohe and Uribe (2007) quite closely, but with some important differences.

First, recall the estimated nominal interest rate rule for the ROW of the form:

$$\log\left(\frac{R^*_{n,t}}{R^*_n}\right) = \rho^*_r \log\left(\frac{R^*_{n,t-1}}{R^*_n}\right) + (1-\rho^*_r)\left(\theta^*_\pi \log\left(\frac{\Pi^*_t}{\Pi^*}\right) + \theta^*_y \log\left(\frac{Y^*_t}{Y^*}\right) + \theta^*_{dy} \log\left(\frac{Y^*_t}{Y^*_{t-1}}\right)\right) + \epsilon^*_{M,t}$$

Unlike rules studied in the NK literature which respond to the output gap and therefore a flexi-price version of the model, this rule makes no such demands on the policymaker and rational agents; it only requires knowledge of the model itself and its deterministic steady state. Schmitt-Grohe and

[11] To implement this we add a penalty term to the policy-maker's welfare criterion taking the form of the variance of the nominal interest rate (see Chapter 6, Section 4.2 of Woodford (2003), Levine *et al.* (2008) and Levine *et al.* (2012)). The general idea is that instead of truncating the state space of the model at the constraint, we introduce a welfare penalty if the constraint is violated; this penalty acts as a soft constraint while maintaining differentiability of the policy functions. More precisely, we allow the nominal interest rate hit the zero bound with a small probability which can be interpreted as the tightness level of the ZLB constraint.

Uribe (2007) refer to such rules as 'implementable'.

For optimal policy purposes, we remove the policy shock $\epsilon^*_{M,t}$ and re-parameterize this rule as

$$\log\left(\frac{R^*_{n,t}}{R^*_n}\right) = \rho^*_r \log\left(\frac{R^*_{n,t-1}}{R^*_n}\right) + \alpha^*_\pi \log\left(\frac{\Pi^*_t}{\Pi^*}\right) + \alpha^*_y \log\left(\frac{Y^*_t}{Y^*}\right) + \alpha^*_{dy} \log\left(\frac{Y^*_t}{Y^*_{t-1}}\right) \quad (55)$$

which allows for the possibility of an integral rule with $\rho^*_r = 1$. We now set out to find welfare optimal rules within the family of simple rules defined (55) seeking a combination $\rho^* \equiv [\rho^*_r, \alpha^*_\pi, \alpha^*_y, \alpha^*_{dy}]$ that maximize a welfare measure discussed below. Two forms of rule found in the literature are special cases of (55). First, put $\alpha^*_{dy} = \alpha^*_y = 0$ to get:

$$\log\left(\frac{R^*_{n,t}}{R^*_n}\right) = \log\left(\frac{R^*_{n,t-1}}{R^*_n}\right) + \alpha^*_\pi \log\left(\frac{\Pi^*_t}{\Pi^*}\right)$$

which integrating gives

$$\log\left(\frac{R^*_{n,t}}{R^*_n}\right) = \alpha^*_\pi \log\left(\frac{P^*_t}{\bar{P}^*_t}\right) \quad (56)$$

which is a *price-level rule* with the trend price-level given by $\frac{\bar{P}_t}{\bar{P}_{t-1}} = \Pi$. The benefits of price-level targeting versus inflation targeting have been studied in the literature for some time now - see, for example, Svensson (1999), Schmitt-Grohe and Uribe (2000), Vestin (2006), Gaspar *et al.* (2010), Giannoni (2014), Deak *et al.* (2019). These papers examine the good determinacy/stability and robustness properties of price-level targeting. Holden (2016) shows these benefits extend to a ZLB setting. Our paper shows that these results for a closed economy carry over to an open-economy setting. The intuition for the benefits of price-targeting is as follows: faced with of an unexpected temporary rise in inflation, price-level stabilization commits the policymaker to bring inflation below the target in subsequent periods. In contrast, with inflation targeting, the drift in the price level is accepted.

The second form of rule puts $\alpha^*_{dy} = \alpha^*_\pi$ and $\alpha^*_y = 0$. Integrating as before we then arrive at

$$\log\left(\frac{R^*_{n,t}}{R^*_n}\right) = \alpha^*_\pi \left(\log\left(\frac{P^*_t}{\bar{P}^*_t}\right) + \log\left(\frac{Y^*_t}{Y^*}\right)\right) = \alpha^*_\pi \log\left(\frac{P^*_t Y^*_t}{\bar{P}^*_t Y^*}\right) \quad (57)$$

which is an *nominal income rule* with a zero trend growth rate.

We impose two requirements on the optimized rule: first, it is saddle-path stable (that is, it satisfies the Blanchard-Kahn stability conditions for local uniqueness). Second, it satisfies an approximate ZLB constraint on the nominal interest rate that requires a given low frequency of lower bound episodes. This requirement is implemented in the context of a delegation mandate discussed below.[12]

The welfare criterion is as follows. As mentioned above, the policymaker is utilitarian and adopts a single-period welfare measure $\lambda U^C_t + (1-\lambda)U^R_t$ in both the ROW and the SOE.[13] Let

[12]Schmitt-Grohe and Uribe (2007) impose a further requirement consisting of an upper bound on the α^*_i. by imposing a given low frequency of hitting the lower bound we are able to avoid this arbitrary choice of upper bound.

[13]For the latter from the impulse responses in the SOE in Section 4.2 we have seen significant distributional effects

x_t^* be a vector of all ROW state variables (including shock processes such as A_t^* and G_t^*) at time t. The *conditional* inter-temporal welfare at time t, computed as a perturbation approximation of order at least 2, depends on both x_t and its zero-growth steady state x. For the model with only monetary policy the steady state depends only on the steady state gross inflation rate Π^*. Therefore we can write the conditional welfare measure as $\Omega_t^* = \Omega_t^*(x_t, x) = \Omega_t^*(x_t, \Pi^*)$, where

$$\Omega_t^*(x_t^*, \Pi^*) = \mathbb{E}_t\left[(1-\beta^*)\sum_{\tau=0}^{\infty}\beta^\tau U_{t+\tau}^*(x_{t+\tau}^*, \Pi^*)\right] = (1-\beta^*)U_t^*(x_{t+\tau}^*, \Pi^*) + \beta^*\mathbb{E}_t\left[\Omega_{t+1}^*\right] \quad (58)$$

in Bellman recursive form.

The conditional welfare criterion immediately leads to a time-inconsistency which is well-known for the Ramsey problem, but less so for optimized simple rules. Time-inconsistency arises because in a forward-looking rational expectations environment the policymaker can affect the dynamic equilibrium via both current and future (promised) actions. The latter influences current outcomes through the rational expectations channel, whereas current actions have only contemporaneous effects. The consequence of time-inconsistency is that, merely with the passage of time, as x_t^* varies there exits an incentive to re-optimize and for simple rules to choose different values of ρ^*. In other words, $\rho^* = \rho_t^*(x_t^*, \Pi^*)$. Note there is a further form of inconsistency, namely to choose a different steady state x^* and therefore Π^* for our monetary model. Our choice of welfare measure avoids the first of these two time-inconsistency problems by choosing the conditional welfare measure $\Omega_t^*(x_t^*, \Pi^*) = \Omega_t^*(x^*, \Pi^*) = \Omega_t^*(\Pi^*, \Pi^*)$, which we write simply as $\Omega(\Pi^*)$. In what follows below we refer simply to Ω_t^*.[14] It follows that the rule implies a commitment mechanism to ensure the credibility of a given announced long-run inflation target. But commitment to a given steady state is already implied in the implementable Taylor of the form (55), so in that sense there is nothing new about that assumption.

Now let $\rho^* \equiv [\rho_r^*, \alpha_\pi^*, \alpha_y^*, \alpha_{dy}^*]$ be the equilibrium policy choice of feedback parameters for the ROW that defines the form of the rule. The equilibrium is solved by backward induction in the following two-stage *delegation game*.

1. **Stage 1**: The policymaker chooses a per period probability of hitting the ZLB, a trend inflation rate consistent with this probability and designs the optimal loss function in the mandate.

2. **Stage 2**: The CB receives the mandate in the form of a welfare criterion, a trend inflation rate and a rule of the form (55). Welfare is then optimized with respect to ρ resulting in an optimized rule.

This delegation game is then solved by backwards induction as follows:

Stage 2: The CB Mandate

of monetary policy ,so this assumption is important.

[14] This is another departure from Schmitt-Grohe and Uribe (2007), who choose a welfare conditional on the Ramsey steady state: for our model this is $\Pi* = 1$ (zero net inflation), but we allow for $\Pi^* > 1$ chosen to lower a given probability of hitting the lower bound in a welfare-optimal equilibrium.

Given a steady state inflation rate target, Π^*, the ROW Central Bank (CB) receives a mandate to implement the rule (55) and to maximize with respect to ρ^* a modified welfare criterion that penalises the variance of the interest rate with a weight $w-r^*$. In recursive form, it can be written

$$\begin{aligned}(\Omega_t^*)^{mod} &\equiv \mathbb{E}_t\left[(1-\beta^*)\sum_{\tau=0}^{\infty}(\beta^*)^{\tau}\left(U_{t+\tau}^* - 100w_r^*\left(R_{n,t+\tau}^* - R_n^*\right)^2\right)\right] \\ &= (1-\beta^*)\left(U_t^* - 100w_r^*\left(R_{n,t}^* - R_n^*\right)^2\right) + \beta\mathbb{E}_t\left[(\Omega_{t+1}^*)^{mod}\right] \end{aligned} \quad (59)$$

This results in a probability of hitting the ZLB[15]

$$p^* = p(\Pi^*, \rho^*(\Pi^*, w_r^*))$$

where $\rho^*(\Pi^*, w_r^*)$ is now the *optimized form* of the rule given the steady state target Π^* and the weight on the interest rate volatility, w_r^*. Then given a target low probability $\bar{p}^*$ and given w_r^*, Π^* is now chosen so satisfy

$$p^*(R_{n,t}^* \leq 1) \equiv p^*(\Pi^*, \rho^*(\Pi^*, w_r^*)) \leq \bar{p}^*$$

This then achieves the 'soft' ZLB constraint

$$R_{n,t}^* \geq 1 \text{ with high probability } 1-\bar{p}^* \quad (60)$$

Stage 1: Design of the Mandate

The policymaker first chooses a per period probability $\bar{p}^*$ of the nominal interest rate hitting the ZLB (which defines the tightness of the ZLB constraint). Then it maximizes the *actual* household inter-temporal welfare again in recursive form

$$\Omega_t^* = \mathbb{E}_t\left[(1-\beta^*)\sum_{\tau=0}^{\infty}\beta^{\tau}U_{t+\tau}^*\right] = (1-\beta^*)U_t^* + \beta^*\mathbb{E}_t\left[\Omega_{t+1}^*\right] \quad (61)$$

with respect to w_r^*.

This two-stage delegation game defines an equilibrium in choice variables w_r^*, ρ^* and Π^* that maximizes the true household welfare subject to the ZLB constraint (60).

< Table 9 here >

Table 9 shows the results for the ROW using the general rule (55). The probability per period of hitting the nominal interest rate ZLB is reported, as are the consumption equivalent variations (CEV), which are calculated as follows:

$$CEV(w_r^*, \Pi^*, \rho_1^*) = \frac{\Omega^*(w_r^*, \Pi^*, \rho_1^*) - \Omega^*(0, 1, \rho_2^*)}{CE^*} \quad (62)$$

[15]We assume that the nominal interest rate, $R_{n,t} \sim N(R_n, VAR(R_n))$, has a normal distribution with mean at its steady state value and variance as its theoretical variance, then we can pin down $p(R_{n,t} \leq 1)$ with this normal PDF.

CE^* is the consumption equivalent at the steady state, which represents the utility gain when consumption increases by 1% keeping hours fixed; that is $CE_t^* = U_t^*(1.01C_t^*, H_t^*) - U_t^*(C_t^*, H_t^*)/(1-\beta) \times 100$.[16] Hence, the CEV is the welfare gain (loss) for different settings of the rule $\rho^* = \rho_1^*, \rho_2^*$ with different values on the weight of nominal interest rate's variation in our penalty function and on trend inflation to household welfare. In the table, ρ_2^* is the optimized rule with the weight and net inflation set at zero ($w_r^* = 0$, $\Pi* = 1$).[17]

The bottom three rows of the Table compares the welfare outcomes for the optimized rule of this form with the estimated rule obtained from the Bayesian estimation of the model. The CEV loss for the rule with the estimated trend is 1.80% but this is largely the consequence of an estimated trend net inflation of 0.47% per quarter or almost 2% annually. When this contribution is removed the loss becomes 0.43% per quarter, which falls further to 0.42% when the estimated monetary shock $\epsilon_{M,t}$ is removed. These are then the pure business cycle costs in the model under the estimated rule compared with the optimized rule in the middle row of the Table. Total business cycle costs of shocks, which can be found by comparing the stochastic and deterministic welfare outcomes, are higher and much higher than those reported in the seminal study by Lucas (1987) and updated in Lucas (2003), which are less than 0.01%. The reason for this is his choice of utility function which, unlike our NK-type models, excludes hours (and therefore leisure) in these original studies.

So, the optimized simple rule with a zero inflation trend will improve household utility by a CEV of 1.8%. However, it comes at the cost of a very high probability of a ZLB of 0.369 per quarter or, in other words, spending over a third of the time with ZLB episodes. The upper three rows of the Table compute the revised policy rules and welfare costs of reducing this probability. This involves reducing the very aggressive optimized rule in the middle row and shifting the inflation distribution to the right. Thus, if the aim is to reduce the probability 0.05, 0.025, 0.01 then the rule becomes increasingly less aggressive in response to the mandate with higher weights w_r^* at a CEV welfare cost of 0.21%, 0.28%, 0.38%. These then are means and the welfare costs by imposing the ZLB constraint using our penalty function approach.

Now turn to the price-level rule. We have seen from Table 9 that the OSR with ZLB is very close to a price-level rule. So, what are the welfare costs of using an optimized form of such a rule? Table 10 provides the answer. First, the welfare costs of the optimized price-level rule compared with the general one without the ZLB constraint is quite low; namely around 0.08% CEV, but seen a lower probability of hitting the ZLB. The costs of imposing the same low probabilities of ZLB episodes are only slightly higher to those before. Thus, overall the price-level targeting rule brings low welfare costs, which should be set against the greater simplicity the rule brings.

< Table 10 here >

[16] For the steady state in the estimated we have $CE_s s = 45.04$.

[17] This is slightly different from the calculation of the widely used certainty equivalence measure proposed by Schmitt-Grohe and Uribe (2007) that compares the original and perturbed dynamic equilibria and solves for CEV. Our procedure is much easier to implement and provides a very good approximation.

5.2 The Optimized Rule in the SOE

A well-known traditional recommendation to monetary policy makers in open economies is that an optimal monetary policy in an open economy requires exchange rate flexibility. However, the argument relies in part on the notion that exchange rate movements have a large immediate impact on aggregate demand, by allowing instantaneous adjustment of relative prices in export markets. However, this only occurs under the PCP regime and not under LCP where the "expenditure-switching effect" is absent.

How do we model flexibility in our framework? To answer this, consider the following SOE counterpart of the ROW rule (55)

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + \alpha_\pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \alpha_s \log\left(\frac{\Pi_{S,t-1,t}}{\Pi_S}\right) + \alpha_y \log\left(\frac{Y_t}{Y}\right) + \alpha_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)$$

which includes an interest rate response to a nominal exchange depreciation captured by the magnitude of α_s. Exchange rate non-flexibility is then captured by the size of α_s. This was the form of the rule in the estimation, but without a monetary policy shock process. As for the ROW, we also provide results for the price-level rule ($\alpha_s = \alpha_y = \alpha_{dy} = 0$ and $\rho_r = 1$) and for the nominal income rule ($\alpha_s = \alpha_y = 0$ and $\rho_r = \alpha_{dy} = 1$). For the SOE, we investigate a further possible rule of interest with a domestic price inflation target $\Pi_{H,t}$ that replaces the CPI inflation target. This adds to exchange rate flexibility by eliminating the contribution to the inflation target originating from exchange rate fluctuations.

One further constraint is important for the SOE: we impose a constant nominal exchange rate in the steady state, $\Pi_S = 1$ which requires the steady state inflation rate in the SOE to be the same as the ROW, i.e., $\Pi = \Pi^*$. This removes Stage 2 of the delegation game for the ROW, leaving the policymaker with only one means of imposing the ZLB constraint, namely the choice of the weight on the variance of the nominal interest rate w_r. The rest of the delegation game is as before with unstarred variables replaces starred ones.

< Table 12 here >

< Table 13 here >

Tables 12 and 13 now repeat the exercise in Table 9 for the SOE with two currency-price-setting regimes, PCP and LCP. We adopt a welfare criterion that ranks policy according to a utilitarian weighted sum of the inter-temporal welfare of credit-constrained and Ricardian consumers, introduced in Section 2.1, with weights λ and $1 - \lambda$ respectively. For the impulse responses in Section 4.2, in order to compare like with like, we imposed the *same* parameter values set at the priors in the SOE. Now we use the estimated SOE for the PCP and LCP cases, so the difference between the optimized rules will depend on both the differences in modelling and the estimates. For the latter there are two areas to emphasize: first, the Calvo price parameter is significantly lower for PCP compared with LCP indicating more price flexibility and a smaller real effect of monetary policy. Second, the poorer fit for PCP has the effect of increasing the volatility of the estimated model. For instance, consider two shock processes for domestic shocks that make important contributions to the variance decomposition in Tables 7 and 8: the mark-up and technology shocks, MS_t and A_t, respectively. The long-run standard deviation for shock process i, taking into account the persistence, is given by $\frac{\sigma_i}{\sqrt{1-\rho_i^2}}$, $i = MS, A$. These are given respectively by 0.082, 0.029 for PCP and 0.066, 0.025 for LCP. These estimated shock processes are carried into the optimized rule along with all parameter estimates (apart from the rule itself).

A number of results stand out. First, *producer-price targeting dominates consumer-price targeting.* The welfare-optimizing rule is one with producer-price rather than CPI inflation targeting for both PCP and LCP currency pricing regimes, with or without ZLB considerations. We choose the optimized producer-price inflation targeting rule without a ZLB constraint as the benchmark relative to which the CEV is calculated.

Second, our results *favour exchange rate flexibility.* In either the PCP and LCP case, the response of the optimized rule to exchange rate inflation is almost zero. This result, and the significant welfare superiority of the producer price over CPI inflation targeting rule, gives strong support for exchange flexibility since the latter implies a response to imported inflation, which is partly driven by exchange rate depreciation.

Third, all *optimized rules are inertial* ($\rho_r = 1$) and this enables us to compare them easily. In particular, optimal monetary policy under LCP is *far more aggressive than under PCP*. The response of the nominal interest rate to changes in inflation and output are stronger for the LCP regime compared with PCP. This is in accordance with our results from impulse responses in Section 4.2 and from the fact that prices are more flexible under LCP, which makes monetary policy more muted. This result is also consistent with the findings of Betts and Devereux (2000).

Fourth, the *welfare costs of achieving a soft ZLB constraint are higher under PCP*. This is a consequence of the higher volatility of estimated shock processes, which results in a far higher probability of hitting the ZLB. For PCP, the welfare costs of imposing the ZLB relative to the optimal policy with no ZLB constraint are far higher and in the region of 0.5-1.0% CEV, whereas under LCP they are very small.

Fifth, from the bottom three rows *the welfare costs associated with non-optimized estimated rule are also very high.* Finally, whereas for the ROW the optimized rule under a ZLB constraint was close to a *price-level rule*, for the SOE under both PCP and LCP pricing regimes it is close to a *nominal-income regime.* In Tables 14 and 15 we constrain the rules to be optimized price-level and nominal interest rate rules, respectively. Both rules achieve welfare outcomes very close to the optimized hybrid rules, confirming the earlier discussion. The intuition for these contrasting results for the closed and open economies is as follows: we have seen in the open economy that monetary policy transmission takes place largely through its effect on the exchange rate. A feedback in the nominal interest rate that responds to changes in the exchange rate would interfere with this mechanism. However, exchange rate volatility does impact on output, consumption, hours supplied and therefore utility. The optimized nominal interest rate rule allows for exchange rate flexibility by responding more strongly to the *direct* determinants of welfare by reacting to output changes more than for the closed economy.

< Table 14 here >

< Table 15 here >

6 Conclusions

In this study, we present an analysis of the empirical relevance of imperfect exchange rate pass-through and the consequences for domestic and foreign monetary policy. We build a comprehensive small open economy New Keynesian model interacting with the rest of the world with traded intermediate and investment goods and with limited asset market participation. Two currency pricing regimes for the exports of the SOE are considered - PCP and LCP (equivalent to DCP in our 2-country setting). We estimate our proposed models for different economies, the ROW model on US data and SOE model on Canadian data using Bayesian estimation techniques. This provides not only a better understanding about the consequences of shocks that generate fluctuations in the exchange rate on different small open economies, but also an insight into spillover effects of monetary policy as well. Our findings have been summarized in the Introduction and the previous Section.

Future work could extend our research along the following dimensions: First, our paper has worked largely within the framework of a SOE or 2-country setup exemplified by Gali and Monacelli (2005), but with some important extensions. Future work could usefully extend the analysis to a n-country model with $n \geq 3$ thus allowing for a comparison between PCP, LCP and dominant currency pricing (DCP). Second, in our TANK model with LAMP and Ricardian consumers, we have confined ourselves to the case where the proportion of rule-of-thumb consumers is below that where the inverted aggregate demand logic of Bilbiie (2008) and the inverted Taylor rule applies (see also Levine *et al.*, 2021). This is relevant for Canada, but not applicable to an emerging economy such as Mexico, with a similar SOE relationship to the US. A model estimated on Mexico data would provide a 'tale of two countries' narrative within the NAFTA. Moreover, asset market participation could also be endogenized as in Davenport and Mann (2021). Third, it is of interest to revisit strategic complementarity alongside state-contingent price contracts as in Dotsey and King (2005). This can be linked to the endogenous choice of currency pricing as in Mukhin (2022). Fourth, our results suggest a weak role for exchange rate stabilization, but it would be interesting to study optimized rules with international trade in multiple assets as in Senay and Sutherland (2019), as this setup may enhance the gains in correcting exchange rate misalignments. Fifth, the introduction of wage stickiness (highlighted by Gali and Monacelli (2016) for the open economy) and capacity utilization would make our model an open-economy counterpart of Smets and Wouters (2007). Finally, regarding appropriate Taylor-type rules for the open economy, extending the policy analysis to other targeting rules including the nominal wage (as in Levine *et al.* (2008) for the closed economy) would be of interest.

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Appendices

A Tables

Table 1: Home and ROW Notation

	Domestic Production	Imported Good	Aggregate
Home Country Quantity	$C_{H,t}$	$C_{F,t}$	C_t
Home Country Price	$P_{H,t}$	$P_{F,t}$	P_t
Foreign Country Quantity	$C^*_{F,t}$	$C^*_{H,t}$	C^*_t
Foreign Country Price	$P^*_{F,t}$	$P^*_{H,t}$	P^*_t

An alternative notation that is easily generalized to more than two countries is as follows. Let $C_t(i,j)$ be the consumption of the good by households in country i produced in country j. This has a price $P_t(i,j)$ in the currency of country i. Let $C_t(i)$ and $P_t(i)$ be aggregate consumption and price respectively in country i. Then the previous notation with the home SOE country 1 and the ROW country 2 becomes:

	Domestic Production	Imported Good	Aggregate
Country 1 Quantity	$C_t(1,1)$	$C_t(1,2)$	$C_t(1)$
Country 1 Price	$P_t(1,1)$	$P_t(1,2)$	$P_t(1)$
Country 2 Quantity	$C_t(2,2)$	$C_t(2,1)$	$C_t(2)$
Country 2 Price	$P_t(2,2)$	$P_t(2,1)$	$P_t(2)$

Table 2: Consumption Notations

Table 3: The Law of One Price under PCP and LCP

Origin of Good	Domestic Market	Export Market (PCP)	Export Market (LCP)
Home	$P_{H,t}$	$P^*_H = \frac{P_H}{S_t}$	$P^*_H \neq \frac{P_H}{S_t}$
Foreign	P^*_F	$P^*_F = \frac{P_F}{S_t}$	$P^*_F = \frac{P_F}{S_t}$

Table 4: Calibrated Parameters

Calibrated/Imposed parameter	Symbol	Value
Depreciation rate	δ	0.025
Risk aversion	σ	2.00
Labour share	α	0.70
Risk premium elasticity	ϕ_B	0.001
Price Substitution elasticity (Others)	ζ	7.00
Substitution elasticity (Home/Foreign goods)	$\mu_C = \mu_I$	1.50
Foreign Substitution elasticity (Export/Foreign goods)	$\mu_{C}* = \mu_{I}*$	1.50
Government spending	g_y	0.21
Oil taxation rate	τ_o	0.15
Home Bloc Exported share of Consumption	$n(1 - w_C^*)$ as $n \to \infty$	calibrated so exc=0.0782
Home Bloc domestic share of Investment	$n(1 - w_I^*)$ as $n \to \infty$	calibrated so exi=0.0580
Home Bloc domestic share of Intermediate goods	$n(1 - w_M^*)$ as $n \to \infty$	calibrated so exim=0.0668
Home Bloc Imported share of Consumption	$1 - w_C$	calibrated so imc=0.0780
Home Bloc Imported share of Investment	$1 - w_I$	calibrated so imi=0.0941
Home Bloc Imported share of Intermediate goods	$1 - w_M$	calibrated so imim=0.0636
Oil output	Y^O	calibrated so exo=0.0568
Home Discount factor	β	calibrated so tb=.019
Home productivity	A	calibrated so Y^*/Y=1.22

Table 5: Estimated shocks and parameter values case of PCP

Estimated Parameter Values		Prior		Posterior	
Log data density is 1499.860387.	Symbole	Dist.	(Mean,Std Dev)	Mean	90% HPD Interval
Technology shock	ϵ_A	IG	$0.001, 0.02$	0.0105	0.0081 , 0.0129
Monetary policy shock	ϵ_M	IG	$0.001, 0.02$	0.0052	0.0041 , 0.0062
Markup shock	ϵ_{MS}	IG	$0.001, 0.02$	0.0647	0.0422 , 0.0865
Government shock	ϵ_G	IG	$0.001, 0.02$	0.0010	0.0002 , 0.0019
Investment shock	ϵ_{IS}	IG	$0.001, 0.02$	0.0297	0.0184 , 0.0405
Terms of trade shock	ϵ_{tot}	IG	$0.001, 0.02$	0.0520	0.0448 , 0.0594
Technology shock persistence	ρ_A	β	0.50,0.20	0.9338	0.9009 , 0.9698
Markup shock persistence	ρ_{MS}	β	0.50,0.20	0.6101	0.3689 , 0.8239
Government shock persistence	ρ_G	β	0.50,0.20	0.5159	0.1933 , 0.8421
Investment shock persistence	ρ_{IS}	β	0.50,0.20	0.8015	0.6697 , 0.9328
Terms of trade shock persistence	ρ_{tot}	β	0.5,0.10	0.9906	0.9860 , 0.9955
Monetary Policy shock persistence	ρ_M	β	0.70,0.10	0.9280	0.9105, 0.9464
Feedback from inflation	θ_π	N	2.00,0.25	2.1579	1.8018 , 2.4959
Feedback from output	θ_y	N	0.10,0.05	-0.0028	-0.0502 , 0.0406
Feedback from output growth	θ_{dy}	N	0.10,0.05	0.1213	0.0482 , 0.2008
Feedback from exchange rate depreciation	θ_{ds}	N	0.10,0.05	0.1276	0.0625 , 0.1912
Calvo price stickiness	ξ	β	0.750,0.10	0.5562	0.4760 , 0.6324
Consumption habit formation	χ	β	0.70,0.10	0.8951	0.8356 , 0.9537
Price index	γ	β	0.50,0.10	0.4736	0.3199 , 0.6371
Elasticity of Investment adjustment cost	ϕ_I	N	2.00,0.75	3.2652	2.2699 , 4.3153
Labour Share	α	β	0.50,0.05	0.4633	0.4061 , 0.5252
Intermediate goods Share	α_M	β	0.30,0.05	0.2218	0.1623 , 0.2766
Share of non-Ricardian consumers	λ	N	0.20,0.05	0.2967	0.2350 , 0.3600
Ricardian Frisch elasticity	ψ_R	N	2.00,0.75	1.3771	0.3082 , 2.4101
Non-Ricardian Frisch elasticity	ψ_c	N	2.00,0.75	1.9324	0.7153 , 3.1308
Ricardian risk aversion	σ_R	N	1.50,0.40	2.1193	1.5121 , 2.7338
Non-Ricardian risk aversion	σ	N	1.50,0.40	0.8268	0.2371 , 1.2810

Table 6: Estimated shocks and parameter values in of LCP

Estimated Parameter Values		Prior		Posterior	
Log data density is 1537.088767	Symbole	Dist.	(Mean,Std Dev)	Mean	90% HPD Interval
Technology shock	ϵ_A	IG	$0.001, 0.02$	0.0099	0.0078 , 0.0121
Monetary policy shock	ϵ_M	IG	$0.001, 0.02$	0.0048	0.0040 , 0.0057
Markup shock	ϵ_{MS}	IG	$0.001, 0.02$	0.0368	0.0233 , 0.0497
Government shock	ϵ_G	IG	$0.001, 0.02$	0.0010	0.0002 , 0.0025
Investment shock	ϵ_{IS}	IG	$0.001, 0.02$	0.0345	0.0225 , 0.0453
Terms of trade shock	ϵ_{tot}	IG	$0.001, 0.02$	0.0547	0.0467 , 0.0621
Technology shock persistence	ρ_A	β	0.50,0.20	0.9182	0.8700 , 0.9646
Markup shock persistence	ρ_{MS}	β	0.50,0.20	0.8275	0.6739 , 0.9438
Government shock persistence	ρ_G	β	0.50,0.20	0.5021	0.1662 ,0.8213
Investment shock persistence	ρ_{IS}	β	0.50,0.20	0.7439	0.5929 , 0.9054
Terms of trade shock persistence	ρ_{tot}	β	0.5,0.10	0.9896	0.9846 , 0.9948
Monetary Policy shock persistence	ρ_M	β	0.70,0.10	0.9323	0.9157 , 0.9485
Feedback from inflation	θ_π	N	2.00,0.25	2.0526	1.6842 , 2.4137
Feedback from output	θ_y	N	0.10,0.05	0.0042	-0.0352 , 0.0424
Feedback from output growth	θ_{dy}	N	0.10,0.05	0.1128	0.0336 , 0.1940
Feedback from exchange rate depreciation	θ_{ds}	N	0.10,0.05	0.1222	0.0583 , 0.1834
Calvo price stickiness	ξ	β	0.750,0.10	0.4223	0.3378 ,0.5067
Consumption habit formation	χ	β	0.70,0.10	0.8537	0.7670 , 0.9356
Price index	γ	β	0.50,0.10	0.4537	0.2923 , 0.6119
Elasticity of Investment adjustment cost	ϕ_I	N	2.00,0.75	3.4306	2.4833 , 4.3328
Labour Share	α	β	0.50,0.05	0.4593	0.3982 , 0.5157
Intermediate goods Share	α_M	β	0.30,0.05	0.1969	0.1458 , 0.2481
Share of non-Ricardian consumers	λ	N	0.20,0.05	0.2966	0.2419 , 0.3453
Ricardian Frisch elasticity	ψ_R	N	2.00,0.75	1.7713	0.9251 , 2.6150
Non-Ricardian Frisch elasticity	ψ_c	N	2.00,0.75	2.1261	0.9302 , 3.3501
Ricardian risk aversion	σ_R	N	1.50,0.40	1.5411	0.9583 , 2.0774
Non-Ricardian risk aversion	σ	N	1.50,0.40	0.6373	0.1477 , 1.0766

Table 7: Variance Decomposition of Estimated Model (%)-PCP case

	ϵ_A	ϵ_G	ϵ_{MS}	ϵ_m	ϵ_{tot}	ϵ_{IS}	$\epsilon_{P_O^*}$	ϵ_{A*}	ϵ_{G*}	ϵ_{MS*}	ϵ_{m*}	ϵ_{IS*}
Output	3.32	0.00	39.61	0.12	7.82	4.61	0.07	3.77	0.30	17.73	0.04	22.62
Consumption	1.32	0.00	10.42	0.01	50.41	1.62	0.35	2.75	0.23	16.47	0.10	16.34
Inflation	5.67	0.01	0.72	36.58	5.42	7.60	0.01	29.63	3.26	1.06	1.01	9.04
Nominal Interest Rate	2.74	0.00	2.44	0.15	5.73	11.19	0.04	16.72	1.61	15.92	0.29	43.18
Depreciation rate	0.22	0.00	1.14	3.94	85.29	0.16	0.01	2.01	0.25	3.18	0.34	3.45
Real Exchange Rate	1.24	0.00	10.10	0.02	19.51	2.93	0.06	2.37	0.18	25.24	0.09	38.26
Exports	1.52	0.00	12.34	0.03	23.86	3.59	0.07	5.65	0.46	18.51	0.01	33.96
Imported Consumption Goods	0.32	0.00	1.21	0.01	44.25	0.42	0.22	2.62	0.20	24.10	0.10	26.55
Imported Investment Goods	0.39	0.00	9.54	0.00	9.90	8.32	0.08	5.01	0.41	17.38	0.06	48.91
Imported Intermediate Inputs	0.84	0.00	39.64	1.44	15.68	1.12	0.07	4.06	0.39	19.36	0.09	17.32
Terms of Trade	1.24	0.00	10.10	0.02	19.51	2.93	0.06	2.37	0.18	25.24	0.09	38.26

Table 8: Variance Decomposition of Estimated Model (%) - LCP case

	ϵ_A	ϵ_G	ϵ_{MS}	ϵ_m	ϵ_{tot}	ϵ_{IS}	$\epsilon_{P_O^*}$	ϵ_{A*}	ϵ_{G*}	ϵ_{MS*}	ϵ_{m*}	ϵ_{IS*}
Output	7.55	0.00	28.60	0.05	10.41	7.10	0.06	4.01	0.32	18.20	0.03	23.65
Consumption	2.90	0.00	7.88	0.01	54.24	1.78	0.35	2.48	0.21	15.52	0.10	14.52
Inflation	7.27	0.03	2.00	31.27	5.13	7.17	0.02	29.81	3.15	1.93	0.54	11.69
Nominal Interest Rate	3.09	0.00	1.36	0.28	10.48	7.91	0.05	12.62	1.16	17.26	0.13	45.66
Depreciation rate	0.32	0.00	0.51	4.17	86.52	0.18	0.01	1.94	0.24	2.82	0.35	2.94
Real Exchange Rate	2.60	0.00	4.93	0.08	25.35	3.84	0.06	1.97	0.15	25.00	0.10	35.91
Exports	3.16	0.00	6.14	0.04	32.13	4.96	0.08	4.80	0.37	18.47	0.01	29.84
Imported Consumption Goods	0.67	0.00	0.49	0.03	48.93	0.61	0.22	2.12	0.16	23.13	0.11	23.51
Imported Investment Goods	0.62	0.00	3.61	0.01	12.54	12.78	0.08	4.98	0.41	15.83	0.06	49.09
Imported Intermediate Inputs	0.90	0.00	29.64	0.28	18.47	1.29	0.10	3.44	0.29	25.06	0.08	20.45
Terms of Trade	2.60	0.00	4.93	0.08	25.35	3.84	0.06	1.97	0.15	25.00	0.10	35.91

Table 9: ROW: Inflation-Output Targeting Rule

	Optimized simple rule with ZLB Mandate									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	sd($\epsilon_{M,t}$)
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.1672	0.0006	0.0590	1.00104	-8731.0503	-0.4259	0.010	250	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.3091	0.0009	0.0772	1.00101	-8714.1086	-0.2928	0.025	140	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.5611	0.0015	0.1250	1.00084	-8701.8282	-0.1964	0.050	65	0.0000
	The Optimized simple rule (without ZLB)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Ω^*	CEV (%)	p_zlb	w_r	sd($\epsilon_{M,t}$)
OSR without ZLB	1.0000	33.4490	0.0002	10.3776	1.0000	-8676.8157	0	0.3792	0	0.0000
	Estimated model									
Regimes	ρ_r^*	$\frac{\alpha_\pi^*}{1-\rho_r^*}$	$\frac{\alpha_y^*}{1-\rho_r^*}$	$\frac{\alpha_{dy}^*}{1-\rho_r^*}$	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	sd($\epsilon_{M,t}$)
Estimated rule	0.8365	1.9477	0.0731	0.1505	1.0047	-8893.8205	-1.7041	0.1079	-	0.0024
Estimated rule	0.8365	1.9477	0.0731	0.1505	1.0000	-8736.1503	-0.4659	0.2079	-	0.0024
Estimated rule	0.8365	1.9477	0.0731	0.1505	1.0000	-8735.5782	-0.4614	0.2039	-	0.0000

Notes: Consumption Equivalent at the steady state is 127.3383.

Table 10: ROW: Price Level Rule

	Optimized simple rule with ZLB Mandate								
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.34265	0.0000	0.000	1.00229	-8745.0853	-0.5361	0.010	28
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.5248	0.0000	0.000	1.00156	-8724.568	-0.3750	0.025	14
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.9261	0.0000	0.000	1.00112	-8711.8549	-0.2752	0.050	6
	Optimized simple price-level rule without ZLB								
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π	Ω^*	CEV (%)	p_zlb	w_r
OSR without ZLB	1.0000	10.4683	0.0000	0.000	1.0000	-8698.6345	-0.1713	0.23482	0

Notes: The CEV is measured relative to the optimized simple rule (without ZLB) in Table 9.

Table 11: ROW: Nominal Income Rule

	Optimized simple rule with ZLB Mandate								
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.1126	0.0000	0.1126	1.00114	-8749.925	-0.5741	0.010	14
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.1345	0.0000	0.1345	1.00073	-8738.7971	-0.4867	0.025	9
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.1961	0.0000	0.1961	1.00042	-8731.7825	-0.4317	0.050	5
	Optimized simple price-level rule without ZLB								
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π	Ω^*	CEV (%)	p_zlb	w_r
OSR without ZLB	1.0000	10.4683	0.0000	0.000	1.0000	-8698.6345	-0.1713	0.23482	0

Notes: The CEV is measured relative to the optimized simple rule (without ZLB) in Table 9.

Table 12: SOE: Inflation-Output-Exchange Rate Targeting Rule (PCP)

	Optimized simple rule with ZLB Mandate (targeting domestic inflation $\Pi_{H,t}$)										
Regimes	ρ_r	α_{π_H}	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.1793	0.0033	0.3066	0.0000	1.00104	-1105.6792	-0.6699	0.010	28	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.2338	0.0066	0.4338	0.0003	1.00104	-1102.8197	-0.4511	0.025	17	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.3311	0.0086	0.6961	0.0004	1.00104	-1099.8474	-0.2236	0.050	11	0.0000
	Optimized simple rule without ZLB (targeting domestic inflation $\Pi_{H,t}$)										
Regimes	ρ_r	α_{π_H}	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR without ZLB	1.0000	0.3615	0.0536	0.9936	0.0016	1.00104	-1096.9266	0.0000	0.09613	0	0.0000
	Optimized simple rule with ZLB Mandate (targeting CPI inflation Π_t)										
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.2093	0.0033	0.3110	0.0000	1.00104	-1109.3859	-0.9536	0.010	34	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.2660	0.0052	0.4305	0.0000	1.00104	-1106.6852	-0.7469	0.025	23	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.3517	0.0084	0.6543	0.0000	1.00104	-1103.5079	-0.5037	0.050	15	0.0000
	Optimized simple rule without ZLB (targeting CPI inflation Π_t)										
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR without ZLB	1.0000	0.3786	0.0692	0.9745	0.0023	1.00104	-1099.4048	-0.1897	0.1045	0	0.0000
	Estimated model										
Regimes	ρ_r	$\frac{\alpha_\pi}{1-\rho_r}$	$\frac{\alpha_y}{1-\rho_r}$	$\frac{\alpha_{dy}}{1-\rho_r}$	$\frac{\alpha_{ds}}{1-\rho_r}$	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
Estimated rule	0.9280	2.1579	-0.0028	0.1213	0.1276	1.0047	-1133.0294	-2.7633	0.0020	-	0.0052
Estimated rule	0.9280	2.1579	-0.0028	0.1213	0.1276	1.0000	-1132.7862	-2.7447	0.0220	-	0.0052
Estimated rule	0.9280	2.1579	-0.0028	0.1213	0.1276	1.0000	-1132.7683	-2.7433	0.0227	-	0.0000

Notes: Consumption Equivalent at the steady state is 13.0651.

Table 13: SOE: Inflation-Output-Exchange Rate Targeting Rule (LCP)

	Optimized simple rule with ZLB Mandate (targeting domestic inflation $\Pi_{H,t}$)										
Regimes	ρ_r	α_{π_H}	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω (%)	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.3034	0.0048	0.3674	0.0001	1.00104	-311.1347	-0.0564	0.010	58	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.4581	0.0085	0.5906	0.0005	1.00104	-311.0891	-0.0320	0.025	37	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.8517	0.0154	1.2096	0.0009	1.00104	-311.0459	-0.0089	0.050	21	0.0000
	Optimized simple rule without ZLB (targeting domestic inflation $\Pi_{H,t}$)										
Regimes	ρ_r	α_{π_H}	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR without ZLB	1.0000	8.2266	0.3705	14.0602	0.0361	1.00104	-311.0293	0.0000	0.1271	0	0.0000
	Optimized simple rule with ZLB Mandate (targeting CPI inflation Π_t)										
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω (%)	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.3180	0.0045	0.3632	0.0000	1.00104	-311.3874	-0.1918	0.010	60	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.4565	0.0084	0.5817	0.0004	1.00104	-311.3428	-0.1679	0.025	37	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.6469	0.0155	0.9565	0.0005	1.00104	-311.3063	-0.1483	0.050	12	0.0000
	Optimized simple rule without ZLB (targeting CPI inflation Π_t)										
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR without ZLB	1.0000	0.627	0.0227	0.9845	0.0004	1.00104	-311.3031	-0.1466	0.0562	0	0.0000
	Estimated model										
Regimes	ρ_r	$\frac{\alpha_\pi}{1-\rho_r}$	$\frac{\alpha_y}{1-\rho_r}$	$\frac{\alpha_{dy}}{1-\rho_r}$	$\frac{\alpha_{ds}}{1-\rho_r}$	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
Estimated rule	0.9321	2.0283	0.0027	0.1109	0.1219	1.0047	-311.8554	-0.4424	0.0102	-	0.0048
Estimated rule	0.9321	2.0283	0.0027	0.1109	0.1219	1.0000	-311.8863	-0.4589	0.0179	-	0.0048
Estimated rule	0.9321	2.0283	0.0027	0.1109	0.1219	1.0000	-311.8859	-0.4587	0.0179	-	0.0000

Notes: Consumption Equivalent at the steady state is 1.8674.

Table 14: SOE: Price level Rule

Regimes - PCP	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Act welfare	CEV (%)	p_zlb	w_r
OSR with ZLB (($\bar{p}_{zlb} \leq 0.01$))	1.0000	0.1321	0.0000	0.0000	0.0000	1.00104	-1129.5777	-2.3047	0.0000	0
Regimes -LCP	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Act welfare	CEV (%)	p_zlb	w_r
OSR with ZLB (($\bar{p}_{zlb} \leq 0.01$))	1.0000	0.6393	0.0000	0.0000	0.0000	1.00104	-311.6616	-0.1919	0.0048	1

Notes: The CEV is measured relative to the optimized simple rule (without ZLB) in Tables 12 and 13, respectively.

Table 15: SOE: Nominal Income Rule

	PCP model									
	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.2944	0.0000	0.2944	0.0000	1.00104	-1112.5758	-1.0060	0.010	29
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.5577	0.0000	0.5577	0.0000	1.00104	-1110.2743	-0.8302	0.025	15
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	2.3584	0.0000	2.3584	0.0000	1.00104	-1108.9279	-0.7274	0.050	4
OSR without ZLB	1.0000	6.0929	0.0000	6.0929	0.0000	1.00104	-1108.9115	-0.7262	0.0566	0
	LCP model									
	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.9966	0.0000	0.5858	0.0000	1.00104	-311.3988	-0.0512	0.010	54
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.3636	0.0000	0.3636	0.0000	1.00104	-311.3639	-0.0325	0.025	25
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.6199	0.0000	0.6199	0.0000	1.00104	-311.3431	-0.0214	0.050	6
OSR without ZLB	1.0000	2.0526	0.0000	2.0526	0.0000	1.00104	-311.3429	-0.0213	0.0516	0

Notes: The CEV is measured relative to the optimized simple rule (without ZLB) in Tables 12 and 13, respectively.

Figure 1: Impulse Response to a Domestic Positive Monetary Policy Shock

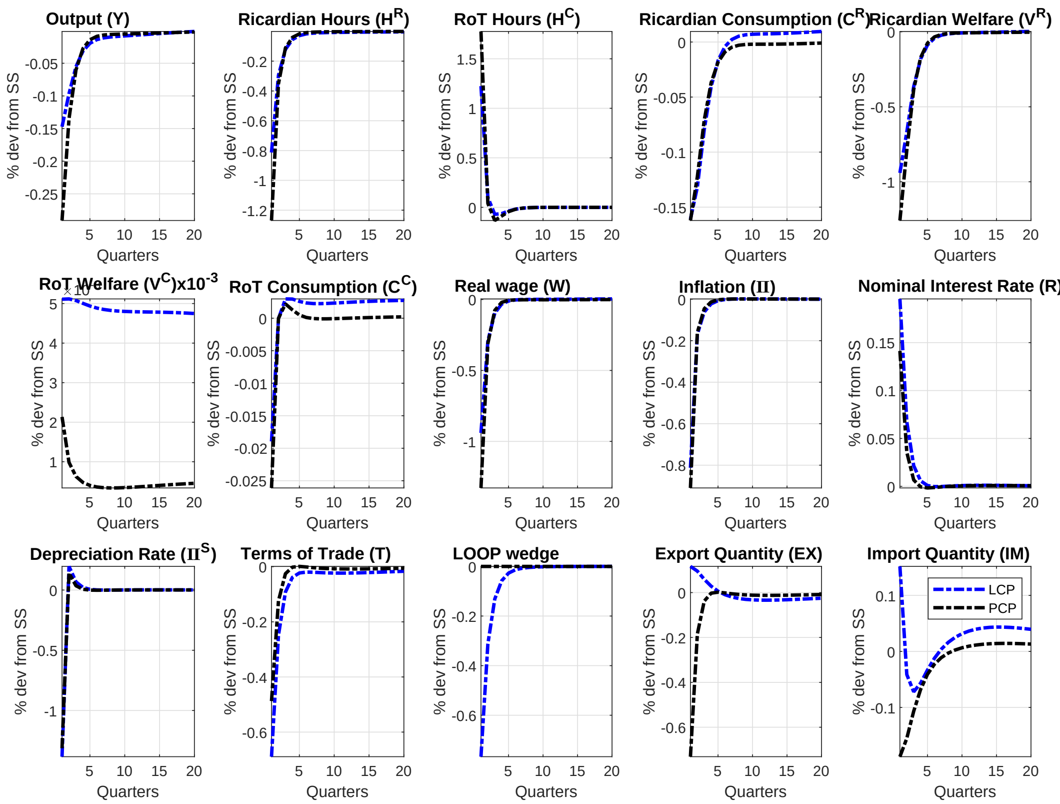

Figure 2: Impulse Response to a ROW Positive Monetary Policy Shock

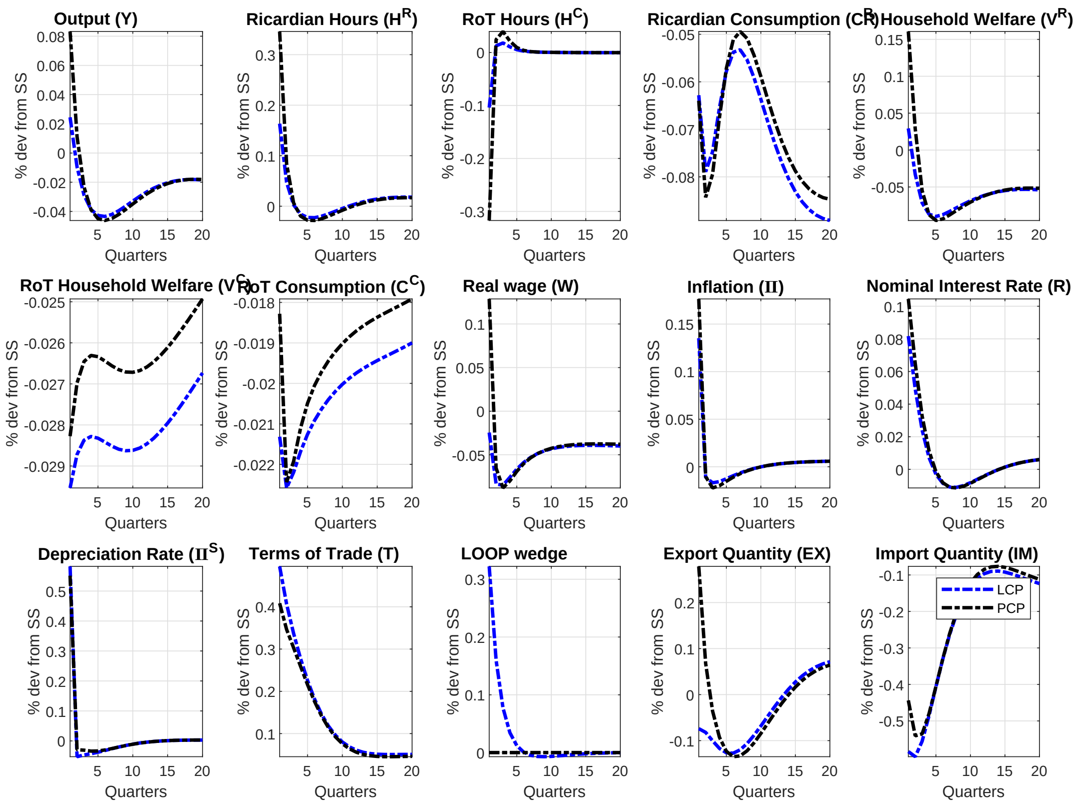

Figure 3: Impulse Response to a Negative ToT Shock