Allocative Efficiency of Green Finance Instruments [*]

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Abstract

This paper investigates the allocative efficiency of green finance instruments through a general equilibrium model with heterogeneous firms and financial frictions. We emphasize the impact of the timing of financial instruments—'ex-post', such as carbon taxes, versus 'ex-ante', like green credit schemes—on the distribution of dirty capital and its environmental implications. Our study reveals that ex-post instruments inadvertently direct dirty capital towards financially constrained firms with higher emission intensity, potentially exacerbating economy-wide emission. Conversely, ex-ante instruments yield beneficial redistributions. The study emphasizes the significance of incorporating the distributive effects of green finance tools into their design and advocates for a general equilibrium viewpoint to evaluate their effectiveness comprehensively, highlighting the pivotal role of instrument timing.

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1 Introduction

Green finance has witnessed a remarkable upswing, fueled by hope that it could influence firms' decisions to adopt green technology and serve as a key driver in the shift towards a sustainable economy. However, recent debates have cast doubt on the effectiveness of current green finance instruments. Concerns have been raised about whether these instruments are capable of inducing significant improvements in firm behavior (Berk and Van Binsbergen, 2021). The aggregate impact of these tools could manifest in various ways: it could be positive (Green and Vallee, 2022), neutral (Arnold, 2023, and Duchin et al., 2022), or even counterproductive (Hartzmark and Shue, 2023).

Our paper aims to delve deeper into this issue by providing a unified framework to analyze the impact of green finance instruments on firms' adoption of green technology. In particular, we explore the differential effects of these green instruments on financially constrained versus unconstrained firms. We reveal that this distinction leads to notable distributional outcomes of dirty capital allocation, which in turn critically influence the overall effectiveness of such instruments. Our study highlights the nuanced interaction of green finance instruments and financial constraints, offering valuable insights for policymakers and stakeholders in sustainable development.

We present a tractable general equilibrium model that incorporates financial frictions and heterogeneous enterprises, similar to Lanteri and Rampini (2023), together with an imperfect elastic supply of capital. This model explores the decision-making process of enterprises in choosing between environmentally sustainable ('green') and pollutant ('dirty') technologies under different green finance instruments. Our analysis reveals that green finance instruments can lead to varied distributional outcomes, significantly influencing their overall effectiveness.

Specifically, green instruments that reward or penalize based on the retrospective environmental impact, which we refer to as *'ex-post instruments'* (like carbon taxes), tend to redirect dirty capital towards financially constrained firms. In contrast, *'ex-ante instruments,'* which offer upfront subsidies for green financing or penalties for dirty financing (such as green credit schemes), tend to shift dirty capital towards less financially constrained firms.

A critical consideration is the emission intensity of constrained firms: if these firms emit more pollutants per unit of dirty capital (often due to higher capital utilization and lower maintenance), then the redistributional impact of these green instruments becomes crucial. Ex-post instruments might inadvertently increase total emissions by channeling dirty capital to more polluting, constrained firms. Conversely, ex-ante instruments effectively encourage these firms to transition to green technology, potentially enhancing their environmental impact. Therefore, the overall effectiveness of green finance instruments in this model is significantly influenced by the distributional shift of dirty capital and the pollution intensity of financially constrained firms.

The underlying rationale for the distinct redistributive impacts of these green instruments lies in the varying financial conditions of firms. Financially constrained firms, lacking in internal capital, prioritize upfront cash requirements (or down payments) when making investment decisions; conversely, financially unconstrained firms, with ample cash reserves, focus more on the frictionless cost of capital usage (user cost of capital).

The influence of green instruments on the relative attractiveness of green versus dirty capital can be dissected into two effects: a direct effect on either the down payment or the user cost of capital, and an indirect effect stemming from equilibrium changes in capital price, which affects both down payment and user cost.

Ex-post instruments directly increase the user cost of dirty capital while leaving the down payment unchanged. However, given the limited elasticity of capital supply, a general equilibrium effect emerges: a decrease in the demand for dirty capital leads to a lower market price. This indirectly lowers both the user cost and down payment for dirty capital. In this context, the outcome is an increase in the user cost of capital due to direct effects, while the initial down payment decreases due to indirect market adjustments. Consequently, this dual effect causes a redistribution of dirty capital, shifting it from financially unconstrained firms to constrained firms, owning to the simultaneous increase in user cost and reduction in down payment for dirty capital.

In contrast, ex-ante instruments directly increase the down payment for dirty capital while maintaining the user cost unchanged. On the other hand, the indirect effect from capital price adjustments reduces both the user cost and down payment for dirty capital.

This combined influence of increased down payment and decreased user cost for dirty capital results in a redistribution of dirty capital from constrained to unconstrained firms.

The distributional effect of green finance instruments becomes particularly significant considering dirty capital generates more emissions in financially constrained firms than unconstrained firms. Constrained firms often resort to over-utilizing the same amount of dirty capital to maximize output, leading to increased emissions. This is a direct consequence of their limited access to capital; they need to extract as much value as possible from their existing assets. In contrast, unconstrained firms, with better access to capital, are more likely to use their resources efficiently and responsibly, leading to lower emissions from the same amount of dirty capital. This variance in emission intensity, when coupled with the distributional effects of green finance instruments, plays a critical role in shaping both the aggregate environmental outcome and the effectiveness of these instruments.

The prevailing approach in sustainable investing typically involves channeling investments towards companies with a positive environmental impact and avoiding those with a negative impact. Although this strategy is somewhat linked to external financing, it aligns more closely with 'ex-post instruments' like carbon taxes rather than upfront green financing models like green credit. This is because it rewards environmental friendliness ex-post, offering financing benefits for demonstrated greenness rather than providing upfront funding for green initiatives.

For financially constrained firms, this strategy poses a challenge. These firms, lacking sufficient internal capital, are unlikely to increase their investment in green capital for a future financial benefit. This is due to their immediate capital limitations. Additionally, the general equilibrium effect of this investing strategy, which tends to lower the price of dirty capital, inadvertently channels more of dirty capital towards financially constrained firms. While this investment approach may effectively reduce the overall amount of dirty capital at a social level, it could also be counterproductive. It might unintentionally increase total emissions due to the higher emission intensity of financially constrained firms.

Our model offers a potential bridge over the existing gaps in green finance literature. For instance, Duchin et al. (2022) find that in response to ESG investing pressures, large firms divest polluting assets, selling them off to smaller, private firms – a phenomenon that exactly echoes our model's predictions. This suggests that the prevailing sustainable investment strategies, due to their ″ex-post″ nature, might be counterproductive (Hartzmark and Shue, 2023). Conversely, the coal lending bans, because of their ″ex-ante″ nature, have successfully compelled firms that rely on bank financing to retire their power plants, proving to be effective (Green and Vallee, 2022).

Our analysis of ″ex-ante″ versus ″ex-post″ green financial instruments also offers broader insights into the practical application of green policies and sustainable investment strategies. In Table 1, we organize current green financial tools into ″ex-ante″ and ″ex-post″ classifications. Additionally, Section 6.1 provides an in-depth examination of each instrument, explaining the basis for their categorization into these distinct groups.

[Place Table 1 about here]

The rest of the paper is structured as follows: Section 2 reviews relevant literature, setting the stage for our analysis. Section 3 introduces a simple two-period model to elucidate how different green finance instruments influence the investment decisions in green and dirty capital among financially constrained and unconstrained firms. Section 4 conducts a numerical exercise to demonstrate the equilibrium impacts of these green instruments on total emissions and allocative efficiency. In Section 5, we extend our model to a dynamic framework, allowing for more realistic calibration of firm financial constraints and a broader range of policy scenarios. Section 6 engages in an in-depth discussion on the practical application of green finance instruments, weighing the advantages and disadvantages of ″ex-ante″ versus ″ex-post″ approaches beyond our model. The paper concludes in Section 7.

2 Related Literature

In exploring the complex dynamics between green finance instruments and firm investment in green technologies, our study draws upon and contributes to several strands of academic literature.

First, we build on quantitative general equilibrium models of climate change, i.e. the dynamic integrated climate-economy (DICE) model of William Nordhaus, as in Nordhaus (2014). Recent contributions extending the DICE model include Acemoglu et al. (2012), Acemoglu et al. (2016), Golosov et al. (2014) and Hassler and Krusell (2012), which study optimal carbon taxation and directed technological change. These studies offer a crucial understanding of the broader economic impacts of climate policies but often neglect the detailed implications of financial constraints faced by firms. Our research seeks to fill this gap by integrating these constraints into the general equilibrium framework, thereby enhancing the model's applicability to real-world scenarios.

Another significant aspect of our study is examining how financial constraints influence corporate environmental responsibility. Empirical studies by Hong et al. (2012), Goetz (2018), and Xu and Kim (2022) have demonstrated that financially unconstrained firms tend to engage more in social and environmental responsibilities. Lanteri and Rampini (2023) further elucidates the theory behind firms' choices between dirty and clean technology under financial constraints. Our work builds on these foundations to explore the implications of green finance instruments on these choices, incorporating factors like heterogeneous emission intensity and inelastic capital supply which have significant implications for aggregate outcomes.

We also contribute to the literature evaluating the effects of sustainable investing strategies on firm investment decisions and overall economic outcomes. Notable works in this realm include those by Heinkel et al. (2001), Davies and Van Wesep (2018), Pástor et al. (2021), Berk and Van Binsbergen (2021), Broccardo et al. (2022), Edmans et al. (2022), and Pedersen (2023). Recent empirical research indicates that sustainable investing has succeeded in increasing the cost of capital for environmentally harmful ('brown') firms compared to their 'green' counterparts (see, e.g., Chava (2014),Van der Beck (2021), Kacperczyk and Peydró (2022), Pástor et al. (2022), Aron-Dine et al. (2023), Green and Vallee (2022), and Gormsen et al. (2023)). Nonetheless, research by Akey and Appel (2021), Bartram et al. (2022), and Hartzmark and Shue (2023) highlights the potential unintended consequences of green finance schemes, questioning their overall effectiveness. In particular, the findings of Hartzmark and Shue (2023) suggest that reducing financing costs for green firms may result in minimal environmental improvements compared to the effects on brown firms. Our paper contributes to this discourse by offering a unified framework to assess the effectiveness of various green instruments within the context of financial constraints, focusing on both 'ex-ante' and 'ex-post' instruments and their distributional effects.

Lastly, our study also extensively relies on the theories of constrained efficiency in macroeconomics, drawing on seminal works like those of Diamond (1967), Stiglitz (1982), and Davila et al. (2012). These theories are pivotal in understanding the interactions between climate externalities, financial frictions, and optimal policy making, as reviewed in Nuño and Moll (2018).

3 Two-period Model

In this section, we present a simple two-period model that integrates key aspects that is needed to convey the basic intuition, including financial frictions, heterogeneous enterprises, and a capital supply characterized by imperfect elasticity. Our goal is to shed light on the ways in which various green finance instruments influence investment choices in green and dirty capital. Special attention is given to the redistribution of dirty capital between financially constrained and unconstrained firms, highlighting the allocative impact of these green instruments on the total emission.

3.1 Model Setup

Time is discrete and there are two periods for the economy, called time 0 and 1. A representative household is risk-neutral, with a discount factor β for the utility from the second period. The economy is a small open economy, and private agents can have access to the world financial market with gross interest rate fixed at β^{-1} unless additional restrictions in financial transactions are specified.

3.1.1 Firms' Problem

There are heterogeneous firms with different initial net worth and different productivity. The number of firms are infinite with a measure of one. The representative household owns all the firms in the economy.

Production Technology. There are two types of capital that can be used for firm production, called dirty and green (denoted k^d and k^g, respectively). Firms utilize a combination of dirty and green capital for their production processes. The composite capital goods are denoted by $k = g(k^d, k^g)$, in which g is the function combining dirty and green capital into composite capital goods.

Firms' production is characterized by the function $y = zk^\alpha$, in which z is firm's productivity level and $\alpha \in (0, 1]$ controls firms' return to scale. Utilizing capital incurs costs, which differ based on the type of capital. The cost for using one unit of dirty and green capital are denoted as c^d and c^g, respectively. Moreover, the utilization of dirty capital results in emissions, an aspect that will be discussed in more detail later in the analysis.

Firms purchase capital goods k^d and k^g at $t = 0$ at market prices of q^d and q^g, respectively. Production occurs at $t = 1$, after which all capital is fully depreciated. We assume there are no uncertainty in productivity going to the next period for simplicity. But we do allow for different levels of productivity, so that firms will have different marginal product for their capital inputs.

Financial Frictions. Firms are endowed with some initial net worth w. They can engage in borrowing within the credit, subject to Kiyotaki and Moore (1997) type collateral constraints. Specifically, firms can borrow up to a θ fraction of their purchased capital's value.[1] We consider an extreme form of financial friction in equity financing: in the first period, firms are unable to issue external equity.

Optimization Problem. For a typical individual firm i with an initial net worth of w_i (where $w_i > 0$) and a productivity level z_i, its optimization problem in the absence of any

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[1] This rule can be motivated by the limited enforcement scenario where, at the end of the first period, the manager has the opportunity to default and divert a $1 - \theta$ fraction of the capital, but cannot divert any capital in the second period.

policy intervention is formulated as follows (omitting subscript i for simplicity):

$$\max\ d_0 + \beta d_1 \tag{1}$$

subject to the constraints:

$$\begin{aligned} w + b - d_0 &\geq q^d k^d + q^g k^g, &(2)\\ z k^\alpha - c^d k^d - c^g k^g &\geq d_1 + \beta^{-1} b, &(3)\\ \theta q^d k^d + \theta q^g k^g &\geq b, &(4)\\ d_0 &\geq 0 &(5) \end{aligned}$$

The firm aims to maximize its discounted dividends over two periods by choosing dividend payout d_0, capital investments k^d and k^g and external debt b. Emissions are not considered in this optimization. Constraint (2) is the firm's budget constraint at time 0, indicating the use of internal capital w and debt b for dividend payouts d_0 and capital purchases. Constraint (3) is the budget constraint at time 1, where the firm's revenue is allocated for debt repayment and dividend distribution. Constraint (4) is the firm's borrowing constraint, which captures that firms are only able to borrow up to a θ fraction of their capital's value. Finally, Constraint (5) specifies that firms cannot issue external capital, as indicated by $d_0 > 0$.

3.1.2 Capital Goods Market

For the supply of capital, we assume the representative household is endowed with a technology to supply capital goods: it requires $\chi^j(k) = a^j \left(q^j\right)^{\varepsilon^j}$, for $j \in \{d, g\}$ units of final goods (also the consumption goods, serving as the numeraire) to produce k units of type j capital goods. a^j controls the relative expensiveness of dirty and green capital while ε^j controls the elasticity of capital supply curve. We assume dirty and green capital have the same supply elasticity, $\varepsilon^d = \varepsilon^g = \varepsilon$, for simplicity.

The capital market is competitive, and the representative household takes the capital prices, q^d and q^g, as given when supplying capital goods. Market equilibrium for the two types of capital goods is maintained through the following market-clearing conditions:

$$\int k_i^d \, di = K^d, \quad \int k_i^g \, di = K^g. \tag{6}$$

This indicates that the total demand of each type of capital good from all firms (k_i^d and k_i^g for each firm i) equals the aggregate supply of each capital good in the market (K^d and K^g).

3.1.3 Emissions and Climate Goods

The use of dirty capital in the economy results in climate damage. Specifically, emissions are directly proportional to the utilized dirty capital. For an individual firm i, its emissions are given by $e_i = z_i^\iota \times k_i^d$ with $\iota \geq 0$. To calculate the total emissions, we integrate individual firm emissions across all firms, represented as $E = \int e_i \, di$. This model setup links a firm's marginal emission of dirty capital to its productivity. Consequently, two firms with the same amount of dirty capital may have different emission levels if their financial constraint vary. This aspect is crucial for analyzing the allocative efficiency of green finance instruments. A more detailed discussion and microfoundation of this reduced-form setup are provided in Section 3.4.

Households, in our model, derive utility not only from consumption goods but also from climate goods. The utility from climate goods is inversely affected by the aggregate emission E, which results from using dirty capital in production. For simplicity, we assume that climate damage directly influences utility. An alternative approach, as explored in studies like Golosov et al. (2014) and Acemoglu et al. (2016), considers the overall productivity being negatively impacted by aggregate emissions. However, the fundamental insights of this paper are not contingent on the specific nature of this assumption. The critical aspect is that emissions exert a negative externality on the aggregate economy, and these externalities are not internalized by private firms.

Emissions may include a variety of pollutants, such as carbon dioxide, sulfur dioxide, nitrogen dioxide, as well as the release of chemical contaminants, toxic substances, particulate matter, among others. The principles discussed here are broadly applicable to any negative externality. Nonetheless, it's important to recognize that in practical scenarios, for certain pollutants and emissions, alternative policy approaches are more commonly adopted. In these cases, rather than leveraging green finance instruments, regulatory mechanisms like quantity limits, legal penalties, and other enforcement methods are frequently employed.

3.1.4 Green Finance Instruments

We categorize green instruments into two types: 'ex-post instruments' that reward or penalize based on the retrospective environmental impact, like carbon taxes; and 'ex-ante instruments,' that offer upfront subsidies for green financing or penalties for dirty financing, like green credit schemes.

The firm's optimization problem incorporating these two green instruments is formulated as:

$$\max\ d_0 + \beta d_1 \tag{7}$$

subject to

$$\begin{aligned} w + b - d_0 &\geq q^d k^d + q^g k^g, && (8) \\ zk^\alpha - c^d k^d - c^d k^g &\geq d_1 + \beta^{-1} b + \tau^d z^\iota k^d + \tau^g k^g, && (9) \\ \xi^d \theta q^d k^d + \xi^g \theta q^g k^g &\geq b, && (10) \\ d_0 &\geq 0. && (11) \end{aligned}$$

In the context of 'ex-post instruments', we examine carbon tax as an illustrative example. Carbon tax is a form of environmental tax levied on carbon emissions. It is intended to encourage the reduction of greenhouse gas emissions by penalizing the use of carbon-intensive capital while incentivizing cleaner alternatives. In this framework, the government can impose a tax $\tau^d > 0$ on carbon emissions to discourage dirty capital use. Conversely, a subsidy $\tau^g < 0$ can be applied to the utilization of green capital.

Regarding 'ex-ante instruments', we consider green credit schemes as an example. These schemes are designed to support and incentivize environmentally friendly projects, primarily through preferential loan conditions for green projects. In our framework, the emphasis is on the differential collateralizability of green and dirty capital. Specifically, the government can adjust collateral requirements of debt financing, making them less favorable for dirty capital ($\xi^d < 1$) and more favorable for green capital ($\xi^g > 1$).

Furthermore, we can explore the external equity financing of firms based on the environmental impact of their projects. We modify the last dividend equation as $d_0 \geq \kappa^d q^d k^d + \kappa^g q^g k^g$. In this case, the government can impose $\kappa^d > 0$ to penalize external equity financing of firms using dirty capital and $\kappa^g < 0$ to incentivize financing with green capital. In this simplified two-period model without risk and default, the instruments for external debt and equity financing are isomorphic. Therefore, for simplicity, we only focus on the green credit scheme.

3.2 Capital Choice without Green Instruments

In this subsection, we analyze the firm's optimal allocation between dirty and green capital in the absence of green instruments. Our focus is particularly on the differing choices made by financially constrained and unconstrained firms.

For clarity in exposition, we assume that firms have the same marginal emission rate (i.e., $\iota = 0$) when analyzing capital choices and the distributive effects of green finance instruments.[2] We further assume that using dirty capital is more costly than using green capital, i.e., $c^d > c^g$. We define the output function as $F = zk^\alpha$, where F' represents the marginal product of capital. We introduce the multipliers η for the firm's dividend constraint, and λ for the collateral constraint, respectively. The firm's first-order conditions

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[2] The patterns of firms' capital choices and the distributive effects of green instruments are largely independent of marginal emissions. To simplify the analysis, we abstract from heterogeneous marginal emissions. However, variations in firms' marginal emissions will impact overall emissions, which is addressed in Section 3.4 in the context of the aggregate effects of green instruments.

are derived as follows:

$$\begin{aligned} d_0 &: \quad \mu = 1 + \eta, && (12) \\ k^d &: \quad \mu q^d - \lambda\theta q^d = \beta F' g_1 - \beta c^d, && (13) \\ k^g &: \quad \mu q^g - \lambda\theta q^g = \beta F' g_2 - \beta c^g, && (14) \\ b &: \quad \mu = 1 + \lambda. && (15) \end{aligned}$$

Adopting the framework of Jorgenson (1963), we use U to denote the frictionless user cost of capital and Φ for the down payment, which is the minimum internal funds required per unit of capital. The user cost and down payment for dirty (U^d and Φ^d) and green (U^g and Φ^g) capital are defined as:

$$\begin{aligned} U^d &= q^d + \beta c^d, && (16) \\ U^g &= q^g + \beta c^g, && (17) \\ \Phi^d &= q^d(1-\theta), && (18) \\ \Phi^g &= q^g(1-\theta). && (19) \end{aligned}$$

These costs reflect the immediate (*down payment*) and total frictionless (*user cost*) financial commitments required to invest in each type of capital.

We employ a general CES (Constant Elasticity of Substitution) composite capital goods function to model the combination of dirty and green capital:

$$g(k^d, k^g) = \left[\gamma \left(k^d\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(k^g\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \quad (20)$$

In this function, σ is the elasticity of substitution between the two types of capital, while γ determines their respective shares in the composite capital.

Incorporating this composite capital goods function into the first order conditions, the optimal ratio of dirty to green capital in a firm's investment is given by:

$$\frac{k^d}{k^g} = \left(\frac{\gamma}{1-\gamma}\right)^{\sigma} \left(\frac{U^d + \Phi^d\lambda}{U^g + \Phi^g\lambda}\right)^{-\sigma}. \quad (21)$$

Here, $U^j + \Phi^j\lambda$ represents the firm's *total cost* for each type of capital, integrating not just the frictionless user cost but also the shadow cost related to the down payment. This shadow cost, emerging from financial constraints, varies by firm and hinges on each firm's financing condition. For financially unconstrained firms ($\lambda = 0$), this total cost simplifies to the basic user cost, indicating indifference towards down payments. Conversely, for financially constrained firms (characterized by a high λ), the significance of down payments increases.

Equation (21) reveals that the ratio of dirty to green capital depends on their relative total costs: $U^d + \Phi^d\lambda$ for dirty capital and $U^g + \Phi^g\lambda$ for green capital. Financially unconstrained firms prioritize the relative user cost of capital, U. In contrast, financially constrained firms place more emphasis on the relative down payment requirement, Φ. This distinction in focus drives the different capital allocation choices between the two types of firms.

Given our assumptions that the utilization cost of dirty capital is higher than that of green capital, $c^d > c^g$, an equilibrium condition emerges: $\frac{U^d}{U^g} > \frac{\Phi^d}{\Phi^g}$. This implies that in equilibrium, dirty capital is characterized by a lower initial down payment but incurs higher total costs during usage. Conversely, green capital requires a higher down payment but is cheaper to use over time.

This equilibrium dynamic has significant implications for investment choices between financially unconstrained and constrained firms. Financially unconstrained firms, which are less sensitive to initial down payments, tend to invest more in green capital due to its lower user costs. On the other hand, financially constrained firms, which are more affected by the initial down payment, tend to invest more in dirty capital. This investment pattern aligns with the theoretical framework proposed by Lanteri and Rampini (2023) and is consistent with empirical findings documented in Hong et al. (2012) and Xu and Kim (2022), which observe similar behaviors in capital allocation decisions.

3.3 Impact of Green Finance Instruments

In this subsection, we explore the influence of green finance instruments on firms' decisions regarding dirty and green capital, and the resultant distributional effects.

3.3.1 Capital Choice with Green Instruments

Incorporating the effects of carbon tax and green credit schemes, the firm's first-order conditions are modified as follows:

$$\begin{aligned} d_0 &: \quad \mu = 1 + \eta, && (22) \\ k^d &: \quad \mu q^d - \lambda \xi^d \theta q^d = \beta F' g_1 - \beta \left(c^d + \tau^d\right), && (23) \\ k^g &: \quad \mu q^g - \lambda \xi^g \theta q^g = \beta F' g_2 - \beta \left(c^g + \tau^g\right), && (24) \\ b &: \quad \mu = 1 + \lambda. && (25) \end{aligned}$$

These conditions reflect the additional costs or benefits associated with the carbon tax (τ^d and τ^g) and the modified collateral requirements ($\xi^d \theta$ and $\xi^g \theta$) under green credit.

The user cost and down payment for dirty and green capital under these green instruments are recalculated as:

$$\begin{aligned} U^d &= q^d + \beta \left(c^d + \tau^d\right), && (26) \\ U^g &= q^g + \beta \left(c^g + \tau^g\right), && (27) \\ \varPhi^d &= q^d (1 - \xi^d \theta), && (28) \\ \varPhi^g &= q^g (1 - \xi^g \theta). && (29) \end{aligned}$$

which take into account the additional factors introduced by green finance instruments.

The optimal ratio of dirty to green capital, as derived in equation (21), still applies but with the updated user cost and down payment values. This equation continues to be a critical factor in determining how firms allocate their resources between dirty and green capital, particularly under the influence of green finance instruments.

3.3.2 Direct and Indirect impact of Green Instruments

The influence of green instruments on the relative costs of capital can be decomposed into two channels: a direct effect and an indirect effect resulting from equilibrium changes in capital prices. Green instruments typically exert direct effects on U^j or $\varPhi^j$, and induce indirect effects on both U^j and $\varPhi^j$ through equilibrium changes in capital price q^j.

If the capital goods market is characterized by fixed prices (perfectly elastic supply), then green instruments primarily exert direct effects. For example, a carbon tax impacting U^j and green credit affecting $\varPhi^j$ will encourage both financially unconstrained and constrained firms to shift towards green capital. However, a carbon tax is more effective for unconstrained firms, while green credit is more effective for financially constrained firms due to variations in the tightness of constraints, represented by λ.

The dynamics become more nuanced when the supply of capital goods is not perfectly elastic. Under these conditions, the effects of green instruments diverge significantly between financially constrained and unconstrained firms.

3.3.3 Distributional Effects of Green Instruments

Ex-post Instruments such as Carbon Tax: The introduction of a carbon tax targets the component U^d, causing a direct increase, while leaving $\varPhi^d$ unchanged. This policy also triggers general equilibrium effects. A decrease in the overall demand for dirty capital reduces its market price q^d, indirectly reducing both the user cost (U^d) and the down payment ($\varPhi^d$) for dirty capital. The direct impact of the tax predominates in the case of user cost, whereas the indirect effect, through the change in q^d, is more significant for the down payment.

Consequently, a carbon tax raises the user cost of dirty capital (U^d), while reducing its down payment ($\varPhi^d$). This results in differing responses from firms: financially unconstrained firms, more sensitive to user cost, are likely to decrease their investment in dirty capital, favoring green capital instead. On the other hand, financially constrained firms, influenced more by down payments, may increase their investment in dirty capital due to the lowered capital price. This leads to a shift in the allocation of dirty capital towards financially constrained firms.

Ex-ante Instruments such as Green Credit: Green credit schemes directly increase $\varPhi^d$ while keeping U^d unchanged.[3] The general equilibrium effect, however, decreases dirty

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[3] When discussing the effects of green credit, we assume that these instruments penalize dirty investment. This is a reasonable simplification as the choice between dirty and clean capital is based on relative costs.

capital price q^d, reducing both U^d and Φ^d. In this context, the indirect effect on the user cost of capital is more pronounced, whereas the direct effect is more significant for the down payment.

Consequently, green credit schemes tend to lower the user cost of dirty capital (U^d), while increasing its down payment (Φ^d). Financially unconstrained firms, focusing more on user cost, may shift their investment away from green capital towards dirty capital. In contrast, financially constrained firms, influenced more by down payments, might find the financing advantages of green capital more appealing, leading to an increased investment in green capital. This dynamic causes a redistribution of dirty capital towards financially unconstrained firms.

Other Green Instruments and a General Principle: Real-world green instruments might be multifaceted and impact both the user cost of capital and the down payment. Under an imperfectly elastic capital supply, these instruments can generate varying distributional effects depending on the relative strengths of their direct and indirect influences on user cost and down payment. A key principle is that green instruments influencing the user cost tend to be more effective for unconstrained firms, potentially redistributing dirty capital to constrained firms. In contrast, green instruments impacting the down payment are more beneficial for constrained firms, possibly reallocating dirty capital to unconstrained firms. The timing of cash flows plays a crucial role in determining whether a green instrument exerts a stronger influence on the user cost or the down payment. For instance, prevailing green investment strategies, which provide financing advantages based on past environmental performance, are analogous to carbon tax in our two-period model. This similarity arises because the financing benefits are deferred to a later period rather than provided upfront. Therefore, a financially constrained firm, with limited internal capital, is less likely to invest in green capital in anticipation of future financing benefits.

Table 2 summarizes the impact of these instruments on financially constrained and unconstrained firms.

[Place Table 2 about here]

3.4 From Distributional Effect to Aggregate Outcome

The distributional effect of green finance instruments becomes particularly significant when dirty capital generates higher levels of emissions in financially constrained than unconstrained firms (i.e., $\iota > 0$). This difference can significantly influence the aggregate environmental impact. Specifically, ”ex-ante” instruments direct dirty capital towards firms that are unconstrained and less emission-intensive, thereby being allocatively efficient and effectively reducing total emissions. In contrast, ”ex-post” instruments divert dirty capital towards more constrained and emission-intensive firms, resulting in allocative inefficiency and can potentially be counter-productive in reducing emission.

One primary reason for this disparity is the differing utilization intensity between these firms. Constrained firms often resort to over-utilizing the same amount of dirty capital to maximize output, leading to increased emissions. This is a direct consequence of their limited access to capital; they need to extract as much value as possible from their existing assets. In contrast, unconstrained firms, with easier access to capital, can afford to use their resources more efficiently and responsibly, resulting in lower emissions for the same amount of capital. In the Appendix A.1, we extend the simple two-period model to include variable capital utilization. Under certain conditions, the equilibrium marginal emissions from dirty capital align precisely with our reduced-form emission setup.

There might be other factors contributing to higher emissions in financially constrained firms. For instance, financially constrained firms tend to implement cost-cutting strategies. These firms often reduce their expenditure on crucial areas such as pollution control, environmental management systems, and regular maintenance. Such reductions can inadvertently lead to increased emissions due to less efficient and poorly maintained equipment and processes.

The impact of these strategies is further intensified by enforcement challenges in smaller firms. The success of green instruments largely relies on effective enforcement. However, smaller and financially constrained firms often escape the attention of regulatory bodies, resulting in a gap in policy implementation. Consequently, perceiving a lower risk of regulatory action or penalties, these firms might not strictly adhere to environmental standards, leading to heightened emissions.

In light of these differences, it becomes evident that the allocation of dirty capital to financially constrained firms is less ’efficient’ than to unconstrained firms. As a result, green finance instruments that inadvertently shift dirty capital towards constrained firms can lead to unintended environmental consequences. For instance, a carbon tax, while intended to discourage the use of dirty capital, might paradoxically increase total emissions if it leads to a higher concentration of dirty capital in constrained firms. On the other hand, green credit schemes, by reallocating some dirty capital to unconstrained firms, could effectively reduce overall emissions. These nuanced effects underscore the importance of considering firm-specific characteristics and behaviors in policy design and implementation.

4 Numerical Analysis

In this section, we conduct a numerical analysis to assess the allocative effects of various green instruments, as discussed in our preceding theoretical analysis. Specifically, we demonstrate how ex-post instruments redirect dirty capital towards constrained firms, whereas ex-ante instruments channel dirty capital towards unconstrained firms, leading to significantly different aggregate outcomes.

Our analysis is structured as follows. In Section 4.1, we begin with a simple scenario featuring fixed capital supply, constant returns to scale production ($\alpha = 1$), and homogeneous marginal emissions ($\iota = 0$) to clarify firms’ capital choices. We further examine a carbon tax (ex-post) and dirty lending ban (ex-ante) to investigate their redistributive impacts. In Section 4.2, we extend our analysis to scenarios with heterogeneous marginal emissions ($\iota > 0$) to explore the aggregate impacts of green instruments on total emissions. Finally, in Section 4.3, we introduce scenarios with variable capital supply. We break down the aggregate change in emissions into an allocative effect and a capital-supply effect, and discuss the allocative efficiency of each instrument. Appendix B provides additional results, including: (i) decreasing-returns settings ($\alpha < 1$) and their connection to the impact elasticity in Hartzmark and Shue (2023), and (ii) sensitivity analyses on marginal emissions and capital-supply elasticity.

Table 3 reports the baseline calibration for the two-period model. The elasticity of substitution between dirty and green capital is set to 20, reflecting the significant potential for substitution between these two inputs. Apart from utilization costs, 0.10 for dirty versus 0.05 for green, both capital types are symmetric. Each is supplied inelastically at 1 unit and shares the same collateralizability ($\theta = 0.4$). Firms start with net worth equal to 1 and draw productivity from a uniform distribution on $[1, 10]$.

[Place Table 3 about here]

4.1 Capital Choices and the Distributive Effects of Green Instruments

In this subsection, we analyze a simplified case characterized by a fixed capital supply, constant returns to scale in production ($\alpha = 1$), and homogeneous marginal emissions ($\iota = 0$). This setup allows us to clarify firms' capital allocation decisions and the redistributive effects of different green instruments on capital distribution.[4]

Figure 1 presents firms' optimal capital choice without any policy intervention. The panels (a) through (d) detail the firm's multiplier λ, the relative importance of down payment, the relative total cost, and the dirty to green capital ratio as a function of productivity, respectively. In this simplified case, all firms begin with identical net worth; variations arise from differences in productivity. Consequently, firms with higher productivity face greater financial constraints. Therefore, as shown in Figure 1 Panel (a), a firm's shadow price of constraints, λ, increases with firm productivity. When firms are less financially constrained, the user cost predominates their cost considerations; as constraints tighten, the down payment's significance grows (Panel b). Due to the higher user cost but lower down payment requirement of dirty capital, financially unconstrained firms show a preference for green capital, whereas financially constrained firms prefer dirty capital (Panel c). This preference is visually captured in Panel (d) by an upward-sloping curve of the dirty to green capital ratio.

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[4] Under constant returns to scale, variations among firms arise solely from differences in productivity, which helps isolate and emphasize the reallocation effects. Specifically, in this context, the marginal product of capital is given by $F' = z$, and there exists a threshold productivity level z^* below which firms do not produce. For the precise form and detailed derivation of z^*, please refer to Appendix A.2.

[Place Figure 1 about here]

Next, we turn to the different distributive consequences of implementing ex-ante and ex-post green instruments. In particular, we compare their direct partial equilibrium (PE) effect without equilibrium price change and their general equilibrium effect (GE) with equilibrium price change. We employ a carbon tax ($\tau^d = 0.2$) as an example of an ex-post instrument and a dirty capital lending ban ($\xi^d = 0.8$) as an example of an ex-ante instrument. The results are illustrated in Figure 2 and Figure 3, with each figure's Panel (a) to (d) showing relative total cost, the dirty to green capital ratio, and the percentage change in dirty and green capital, respectively. The solid red line represents the GE effect, the dashed red line the PE effect, and the blue line the no-policy benchmark.

The carbon tax's direct PE effect raises the user cost of dirty capital without altering its down payment. Figure 2 demonstrates that this increases the relative total cost of dirty capital for all firms, with a more pronounced effect on unconstrained firms, for whom the user cost is a more critical component of total cost. This PE effect prompts a shift away from dirty capital towards green capital for all firms, with unconstrained firms showing a stronger response. However, once GE price adjustments are allowed, the picture becomes more nuanced. The decline in dirty capital's equilibrium price, caused by reduced demand, lowers both its user cost and down payment. The direct PE effect predominates for the user cost, thereby still increasing the relative total cost for unconstrained firms versus the benchmark, albeit to a lesser degree. Conversely, for down payments, the indirect GE effect prevails, reducing the relative total cost for constrained firms compared to the benchmark and thus inversely affecting their investment behaviors. This leads to a decrease in investments in dirty capital for unconstrained firms and an increase in investments in dirty capital for constrained firms. That is, the GE effect of the carbon tax reallocates dirty capital from unconstrained to constrained firms.

[Place Figure 2 about here]

In stark contrast, the ex-ante instrument—dirty capital lending ban—exhibits an inverse pattern to the ex-post instrument. Its direct PE effect increases the down payment required for dirty capital while leaving the user cost unchanged. As illustrated in Figure 3, this effect raises the relative total cost of dirty capital for all firms, with a more significant impact on constrained firms. This effect also drives a universal reduction in dirty capital investment, more so among constrained firms. Incorporating the GE effect, which accounts for the decline in dirty capital price, the outcome reverses for unconstrained firms and becomes attenuated for constrained firms. This leads to a reduced dirty capital investment by constrained firms but an increased investment by unconstrained firms. That is, the GE effect of the lending ban shifts dirty capital from constrained to unconstrained firms. Intriguingly, the substantial impact of the lending ban reverses the preference for dirty versus green capital for both types of firms. As a result, unconstrained firms exhibit a higher dirty to green capital ratio compared to constrained firms.

[Place Figure 3 about here]

Appendix B.1 confirms that our distributional findings are robust to the introduction of decreasing returns to scale. It further shows that the impact-elasticity pattern documented by Hartzmark and Shue (2023) holds under the baseline or under an ex-post instrument, but reverses once a sufficiently stringent ex-ante instrument is imposed.

4.2 Aggregate Impacts on Total Emission

The impact of ”ex-ante” and ”ex-post” green instruments on aggregate emissions can differ markedly. Notably, ex-post green instruments may inadvertently lead to increased emissions. This unintended consequence occurs because these instruments can redistribute dirty capital to financially constrained firms, which typically exhibit higher marginal emission rates, thereby increasing overall emissions.

To examine the aggregate impacts on emissions, we extend our analysis to scenarios with heterogeneous marginal emissions ($\iota = 0.5$). The outcomes of implementing ”ex-post” and ”ex-ante” green instruments are illustrated in Figures 4 and 5. Panels (a) and (b) display total emissions (E) and emission intensity (E/Y), respectively.

In Figure 4, we increase the carbon tax parameter from 0 to 0.4. As the carbon tax rises, both total emissions and emission intensity increase. This pattern suggests that ex-post instruments may unintentionally exacerbate emissions rather than mitigate them.

The reason lies in the redistribution of dirty capital towards financially constrained firms, which have higher marginal emission rates.

[Place Figure 4 about here]

The allocative effects are the opposite for ex-ante instruments. In Figure 5, we reduce the collateralizability parameter for dirty capital, ξ^d, from a benchmark value of 1 to 0.6, reflecting a stricter lending restriction on dirty capital. As a result, both total emissions and emission intensity consistently decline. This occurs because, under ex-ante instruments, dirty capital is reallocated to financially unconstrained firms, which tend to have lower marginal emission rates.

[Place Figure 5 about here]

Firm-level heterogeneity in marginal emissions is pivotal for the aggregate consequences of capital reallocation. In the main text we fix $\iota = 0.5$ for clarity. Appendix B.2 relaxes this assumption and provides a more detailed analysis of how varying ι shapes reallocation and the overall impact of green instruments.

4.3 Variable Capital Supply: Level vs. Allocative Channels

In the benchmark analysis, we examine the scenario with a fixed capital supply, where green instruments do not influence the overall usage of dirty capital. This allows us to isolate the effects of capital allocation. In this subsection, we extend our analysis to consider a variable capital supply. We decompose the impact of green instruments on total emissions into two components: the ”allocative effect” and the ”capital-supply effect.” We demonstrate that the elasticity of capital supply is a critical factor in determining the relative strength of these effects, thereby influencing the overall emission outcome.

We explore a case where the capital supply elasticity is set at $\varepsilon = 0.5$. To assess the efficiency of capital allocation, we calculate the average emission rate of dirty capital, defined as the total emissions divided by the aggregate dirty capital, $H^d = E/K^d$. An increase in the average emission rate suggests higher emissions for a given amount of dirty capital. Using this framework, we decompose the change in total emissions into two components: the change due to shifts in the allocation of dirty capital among firms (allocative effect) and the change due to variations in the overall amount of dirty capital (capital-supply effect), as shown below:

$$K^d H^d - K^d_{bench} H^d_{bench} = \underbrace{K^d \left(H^d - H^d_{bench}\right)}_{\text{Allocative Effect}} + \underbrace{\left(K^d - K^d_{bench}\right) H^d_{bench}}_{\text{Capital-Supply Effect}} \tag{30}$$

The results of applying ”ex-post” and ”ex-ante” green instruments are illustrated in Figures 6 and 7, with Panels (a) through (d) presenting total emissions, emission intensity, average dirty capital emission rate, and the decomposition of emission changes in response to policy interventions, respectively.

In Figure 6, we increase the carbon tax parameter from 0 to 0.4. As the carbon tax rises, both total emissions and emission intensity follow a hump-shaped pattern, initially increasing before decreasing. This unintended increase of emission arises from the reallocation of dirty capital towards financially constrained firms, which raises the average emission rate of such capital. Panel (c) highlights this dynamic, showing a consistent rise in the average emission rate as the carbon tax increases. Moreover, Panel (d) decomposes the emission changes into two distinct components. It becomes evident that while the carbon tax reduces the aggregate amount of dirty capital—potentially lowering total emissions—the decline in allocative efficiency, conversely, increases emissions. When this adverse allocative effect outweighs the benefits, ex-post instruments can become counterproductive.

[Place Figure 6 about here]

The allocative effect behaves differently for ex-ante instruments. In Figure 7, we adjust the collateralizability parameter for dirty capital, ξ^d, reducing it from a benchmark value of 1 to 0.6. Here, both total emissions and emission intensity show a consistent downward trend. The average emission rate also decreases as dirty capital is reallocated towards financially unconstrained firms. This effect is further clarified in Panel (d), revealing that the economy benefits from both a reduction in the overall amount of dirty capital and improved allocative efficiency. Together, these factors lead to a significant decrease in total emissions.

[Place Figure 7 about here]

In Appendix B.3, we further show that the allocative component is essentially invariant to capital-supply elasticity—staying positive for ex-post instruments and negative for ex-ante instruments. Changes in elasticity affect emissions mainly through the capital-supply channel. Consequently, ex-ante instruments consistently improve allocative efficiency by shifting dirty capital toward lower-emission, unconstrained firms, whereas ex-post instruments worsen it by reallocating dirty capital to higher-emission, constrained firms.

5 Dynamic Model

Our two-period model allows for more tractable analysis, however, firms' productivity and net worth are exogenously given. In this section, we study a fully dynamic quantitative model.

Compared with the two-period model, the dynamic model has the following additional features: (1) firms' productivity changes stochastically over time, and their investment and saving behavior will take future dynamics into account; (2) firms experience entry and exit and may have incentives to accumulate net worth endogenously over time. These firm dynamics affect the endogenous equilibrium distribution of constrained and unconstrained firms. In contrast, the two-period model assumes an exogenous distribution of different types of firms. In addition to the endogenous wealth distribution, the dynamic model allows for the introduction and analysis of a richer set of green instruments, including green credit policies, carbon taxes, and historical performance-based sustainable investments.

5.1 Model Setup

Firm Optimization Problem. A model period can be divided into two stages. In the first stage, with realized productivity and previously determined capital stocks, firms produce output, repay debt (hence no debt default). If any ex-post instruments are imposed, they also pay penalties and receive rewards based on past green activities. In the second stage, firms decide on current dividend payments, borrowing or saving in the credit market, and capital investment for the next period. Additionally, if any ex-ante instruments are imposed, they pay penalties and receive rewards based on their committed future green activities, fully defined by the type of capital they choose to invest in.

In particular, let us start from the second stage of time t. At this stage, the firm's net worth is denoted as w_t, and the firm's factor-neutral productivity for production is given by z_t. The firm needs to choose dividend d_t, borrowing b_{t+1}, capital investment k_t^d, k_t^g for the next period $t+1$. q_t^d is the capital price for k_t^d, and q_t^g is for the price of k_t^g. When productivity is low enough, it could be possible that the firm does not choose any positive capital investment, but instead it only chooses dividend payments or saving.

The firm is subject to a set of constraints. In the credit market, the firm faces a collateral constraint, where its debt borrowing b_t cannot exceed a fraction of the value of its capital, given by $\xi_t^d \theta q_t^d k_t^d + \xi_t^g \theta q_t^g k_t^g$ (equation 34). Here, θ, $0 \leq \theta \leq 1$, is a common collateral constraint parameter across different types of capital. The parameter ξ_t^d ($0 \leq \xi_t^d \leq 1$), is specific to dirty capital, reducing its collateralizability if a dirty lending ban is imposed. Similarly, ξ_t^g ($\xi_t^g \geq 1$), increases the collateralizability of green capital when green credit policies are implemented.

In addition, the firm faces an external equity financing constraint: dividends must be non-negative, $d_t \geq 0$ (equation 35). This assumption rules out external equity issuance, restricting the firm's financing to internal net worth and collateralized debt. The firm's budget constraint (equation 32) requires that the sum of internal net worth w_t and new debt b_t must cover dividend payouts and new capital purchases.

In the next period, $t+1$, the firm's production is determined by the factor-neutral productivity z_t and the capital investments k_t^d and k_t^g from the previous period. The firm's capital stock is represented by a composite good, $g(k_t^d, k_t^g)$, which is assumed to follow a CES function in k_t^d and k_t^g. After completing production, debt repayment, dividend payment, and tax payment, the firm's net worth at time $t+1$, w_{t+1}, is defined as the residual value of its operations and financial activities, as shown in equation (33).

The firm incurs a cost using dirty capital, $c^d k_t^d$, where c^d is a cost parameter. Additionally, there may be regulation costs, $\tau_{t+1}^d e(z_t) k_t^d$, where τ_{t+1}^d is the carbon tax parameter, and $e(z_t) k_t^d$ represents the firm's total emissions. The term $e(z_t)$ is an increasing function of firm productivity z_t, such as $e(z_t) = z_t^{\iota}$ with $\iota > 0$, as in the two-period model.

In the first stage of period $t + 1$, the firm's investors or shareholders agree that a portion of the firm's net worth, Γ_{t+1}, must be distributed as dividends, effectively a partial forced liquidation. The payment of Γ_{t+1} is distributed to shareholders, which is why the term $\beta E_t \Gamma_{t+1}$ appears in the firm's objective function. We assume Γ_{t+1} increases with k_t^d and decreases with k_t^g, capturing the essence of historical performance-based sustainable investments. For instance, we define $\Gamma_{t+1} = \gamma^d k_t^d - \gamma^g k_t^g$, where $\gamma^d \geq 0$ and $\gamma^g \geq 0$. This reflects the idea that investors adjust their disinvestment or investment decisions based on the firm's past green investment behavior.

At the end of $t + 1$, the firm will exit with some exogenous probability of ρ, in which case the firm pays out all of its net worth w_{t+1} as its dividends. In such a case, the exiting firm will be replaced by a new firm, and we assume a new firm begins with initial net worth w_0 and productivity z_0. In the case of not exiting, the firm draws a new productivity z_{t+1}.

In summary, the firm's optimization problem can be formulated recursively as shown in equation (31). Here, $V(z_t, w_t; S_t)$ represents the firm's value function, with its individual state variables including productivity z_t and net worth w_t at the end of time t. We use s_t denote individual state variable, $s_t = (z_t, w_t)$, and we use S_t and S_{t+1} to denote aggregate state variables. These aggregate variables may encompass changes in green credit enforcement ($\xi_t^j, j \in \{d, g\}$), carbon tax rates (τ_t^d), equity financing conditions ($\mathit{\Phi}_t$), and historical performance-based sustainable investments (Γ_{t+1}) over time. This formulation enables the model to study both steady-state outcomes and transitional dynamics effectively.

$$V(z_t, w_t; S_t) = \max_{\{d_t, b_t, k_t^d, k_t^g\}} d_t + \beta(1-\rho) E_t V(z_{t+1}, w_{t+1}; S_{t+1}) + \beta \rho E_t w_{t+1} + \beta E_t \Gamma_{t+1} \tag{31}$$

subject to

$$
\begin{aligned}
w_t + b_t &= d_t + q_t^d k_t^d + q_t^g k_t^g, && (32)\\
w_{t+1} &= z_t g(k_t^d, k_t^g) - \left(c^d + \tau_{t+1}^d e(z_t)\right) k_t^d \\
&+ (1-\delta)(q_{t+1}^d k_t^d + q_{t+1}^g k_t^g) - R b_{t+1} - \Gamma_{t+1}\left(k_t^d, k_t^g\right), && (33)\\
b_t &\leq \xi_t^d \theta q_t^d k_t^d + \xi_t^g \theta q_t^g k_t^g, && (34)\\
d_t &\geq 0, && (35)\\
k_t^d &\geq 0, && (36)\\
k_t^g &\geq 0. && (37)
\end{aligned}
$$

Aggregate Capital Supply and Dynamics. The production cost of new capital is specified by the following convex functional form:

$$\chi^j(I^j) = \frac{(a^j\delta)^{-\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} \left(I^j\right)^{1+\frac{1}{\varepsilon}}, \quad \text{for } j \in \{d, g\},$$

where $\varepsilon > 0$ denotes the price elasticity of capital supply, and a^j governs the relative production cost for each type of capital ($j = d$ for dirty, $j = g$ for green).

From the first-order condition of capital producers, the inverse supply function is:

$$I^j(q^j) = a^j \delta \left(q^j\right)^{\varepsilon}, \quad \text{for } j \in \{d, g\}.$$

Capital stocks evolve over time according to:

$$K_{t+1}^j = (1-\delta)K_t^j + I_t^j, \quad \text{for } j \in \{d, g\},$$

In steady state, where $K_{t+1}^j = K_t^j = K^j$, the capital stocks satisfy:

$$K^j = a^j \left(q^j\right)^{\varepsilon}, \quad \text{for } j \in \{d, g\}.$$

Market Clearing. In equilibrium the dirty- and green-capital markets satisfy

$$K_t^j = \int k_t^j(i)\, di, \qquad j \in \{d, g\},$$

so the prices q_t^d and q_t^g adjust until aggregate firm demand equals the supply provided by capital producers. Because the economy is open, firms borrow or lend internationally at an exogenous world risk-free rate R that satisfies $\beta R = 1$. The goods market clears each period when aggregate consumption equals total output minus investment.

5.2 Calibration on the Quantitative Model

To conduct quantitative exercises, we calibrate the parameters in our dynamic model. We assume that one model period corresponds to one year. For production, similar to the previous two-period model, we assume the production technology exhibits constant returns to scale, and the capital composite $g(k^d, k^g)$ is defined as:

$$g(k^d, k^g) = \left[\gamma \left(k^d\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(k^g\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{38}$$

where γ denotes the share parameter for dirty capital in the technology, and σ represents the constant elasticity of substitution between dirty and green capital, with $\sigma > 1$. We assume a relatively large value, $\sigma = 20$, implying that dirty and green capital are highly substitutable. This assumption aligns with other studies, such as Lanteri and Rampini (2023). The parameter γ is set to 0.5 in the benchmark model, indicating that the relative importance of dirty and green capital in production is assumed to be equal. For example, this could represent a scenario where a bus service provider uses either fuel-powered buses or electric buses as its vehicles for daily operations. This symmetry also allows us to focus on the implications arising from the differential cost structures of dirty and green capital.

Firm-level productivity z follows an AR(1) process, where the persistence parameter ρ_z is set to 0.7, and the standard deviation σ_z is set to 0.25. These values are consistent with estimates from several studies, including Hennessy and Whited (2005), Hennessy and Whited (2007), Asker et al. (2014), and Moll (2014).

The discount factor β for households (and firm owners) is set to 0.9, and the risk-free interest rate R is $1/\beta$. The probability for firms' death shock ρ is assumed to be 0.15 (e.g., see Bernanke et al. (1999) and also Arellano et al. (2019)).

The collateral constraint parameter in the credit market, θ, is set to 0.30, consistent with the literature (e.g., Quadrini (2000), Buera and Shin (2013), and Moll (2014)). Both the capital pledgeability parameters for dirty capital (ξ_d) and green capital (ξ_g) are normalized to 1.0 in the baseline model. In subsequent experiments, we vary ξ_d and ξ_g to analyze the effects of different green credit instruments. Finally, we normalize the initial net worth of new firms (w_0) to 1.0.

We now calibrate two additional parameters in our model: the elasticity of emissions with respect to firm-level productivity when using dirty capital (ι) and the unit cost of dirty capital (c_d). We normalize the unit cost of green capital (c_g) to 0, focusing on c_d as the relative cost of dirty capital. A higher ι implies that financially constrained firms emit more per unit of dirty capital than unconstrained firms. Meanwhile, raising c_d reduces the equilibrium price of dirty capital relative to green capital, which leads financially constrained firms to choose even more dirty capital. Although both parameters affect the relative emissions of constrained vs. unconstrained firms, they do so through subtly different channels.

To calibrate ι and c_d, we use firm-level data that combines emissions records from the U.S. Environmental Protection Agency (EPA) with Compustat data on firms' financial and production characteristics. The EPA dataset covers various pollutants (e.g., carbon dioxide, nitrogen dioxide, sulfur dioxide, particulates, and chemical compounds), while Compustat provides financial information. Details on the datasets, data cleaning procedures, and the merging process are found in Section D of the Appendix.

We define each firm's emission intensity (emissions per output) in a given year as total emissions divided by sales, and emissions per capital as total emissions divided by Net Property, Plant, and Equipment (Compustat item PPENT). Within each year and broad industry classification (two-digit SIC), we sort firms into five groups based on their Whited and Wu (2006) index (WW index). We winsorize the emissions variables at the 5% level and then compute group-specific means.

In our model, we similarly sort producing firms into five groups according to their financial constraint tightness (λ) and compute average emissions per output and emissions per capital for each group. For comparability, we normalize these group averages by the overall sample mean both in the data and the model.

We calibrate $\iota = 0.85$ and $c_d = 0.05$. Figure 8 illustrates that, in both the data and the model, financially constrained firms exhibit higher emission intensity (Panel A) and higher emissions per capital (Panel B) than do unconstrained firms. The difference in emissions per capital is especially pronounced, while the difference in emissions intensity is less pronounced, because constrained firms have higher productivity. Overall, our model-implied patterns closely align with the empirical observations. In Appendix E, we further explore the robustness of this calibration by examining how changes in ι and c_d affect the model's implied relationship between financial constraints and emissions.

[Place Figure 8 about here]

We set the capital depreciation rate δ to 0.1 and the elasticity of capital supply ε to 1, which are standard in the macroeconomic literature. We then calibrate the relative production cost parameters a^d and a^g such that, in the benchmark equilibrium, the ratio of aggregate green capital to total capital used in production is approximately 30%. Measuring the actual green capital share in the economy is challenging, so we rely on two empirical proxies. First, using our micro-level dataset, we examine firms' adoption of pollution-reducing or emission-preventing technologies (e.g., recycling, recovery, and treatment technologies). For each firm-year, we compute the ratio of emissions reduced by such technologies to total emissions. The median value of this ratio across all firms and years is approximately 28.3%, which we interpret as a proxy for the share of green capital in production. Second, from a broader perspective, approximately 41% of U.S. electricity generation came from zero-carbon sources (e.g., wind, solar, and other renewable energies) in 2022, according to recent estimates.[5] Interpreting zero-carbon electricity as a broader proxy for green capital suggests an upper bound of around 41%. Overall, our model

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[5] `https://www.weforum.org/agenda/2023/03/us-electricity-energy-carbon-renewables/`.

calibration targets a green capital share that falls within the range suggested by these two proxies.

The full set of dynamic model parameters is summarized in Table 4.

[Place Table 4 about here]

5.3 Quantitative Experiments

We analyze the quantitative effects of various green finance instruments. First, we examine carbon taxes and green credit as examples of ”ex post” and ”ex ante” instruments, and compare their effectiveness in a dynamic setting. Next, we study the impact of a historical performance-based sustainable investment strategy, which is widely used in sustainable finance and investment practices.

To structure our analysis, we proceed in three steps. First, we assess each policy’s impact on aggregate output (Y), total emissions (E), and emission intensity (E/Y). Second, we decompose the overall emission change into an allocative effect and a capital-supply effect. As shown in equation (30), the change in emissions per unit of dirty capital (E/K^d) captures the allocative effect, while the remainder is attributed to the capital-supply effect. Finally, to track shifts in investment between dirty and green technologies, we report the dirty–green capital ratio (K^d/K^g), both in aggregate and separately for more- and less-constrained firms.

5.3.1 Quantitative Impact of Ex-post and Ex-ante Instruments

Ex-post Instruments. Panel A of Table 5 presents the effects of a carbon tax, τ^d, raised from 0 to 5%, on key macroeconomic aggregates and firm-level outcomes. Firms are classified by the tightness of their financial constraints, measured by the parameter λ: those falling in the top 30th percentile are designated as “highly constrained,” while the remainder are considered less constrained.

[Place Table 5 about here]

At the aggregate level, the carbon tax reduces both output and emissions. Specifically, aggregate output declines by 4.08%, while emissions fall by 6.55%, resulting in a 2.58% reduction in emissions intensity (E/Y). The increase in emission per dirty capital (E/K^d) imply that the reallocation effect alone would have increased emissions by 2.37%. Therefore, the observed reductions in emissions under the carbon tax are primarily attributable to a contraction in the overall supply of dirty capital. This is further reflected in the decline of the aggregate dirty-to-green capital ratio (K^d/K^g).

The reallocation effect is clearly reflected in the disaggregated firm-level outcomes. Highly constrained firms, despite facing a carbon tax, modestly expand their operations. Their output increases by 0.55%, and emissions rise by 1.27%, alongside a 1.96% increase in the K^d/K^g ratio—indicating a greater reliance on dirty capital. Emissions intensity (E/Y) in this group rises by 0.71%, consistent with their shift toward more emissions-intensive production. By contrast, less constrained firms contract sharply. Their output and emissions fall by 6.46% and 10.82%, respectively, and their K^d/K^g ratio drops by 12.84%, reflecting a retreat from dirty capital usage. Emissions intensity in this group declines by 4.66%, consistent with a move toward cleaner production.

Ex-ante Instruments. Panel B of Table 5 examines the effects of a green credit policy that raises the pledgeability of green capital (ξ^g) from 1.0 to 1.25 and simultaneously reduces that of dirty capital (ξ^d) from 1.0 to 0.75.

At the aggregate level, the green credit policy leads to a substantial reduction in emissions, which fall by 8.32%, while aggregate output declines modestly by 1.24%, resulting in a 7.16% decrease in emissions intensity (E/Y). Changes in emissions per unit of dirty capital (E/K^d) indicate that the reallocation of capital accounts for a 5.49% reduction in emissions. The remaining reductions in emissions (2.83%) are primarily driven by a contraction in the aggregate supply of dirty capital.

The reallocation effect is more pronounced when we examine firm-level heterogeneity. Highly constrained firms substantially reduce their reliance on dirty capital, as reflected in a 66.19% drop in the K^d/K^g ratio. Emissions in this group fall by 33.34%, while output declines only marginally by 0.60%, resulting in a sharp 32.93% reduction in emissions intensity (E/Y). In contrast, less constrained firms respond differently. Dirty capital is reallocated toward this group, leading to a 31.80% increase in their K^d/K^g ratio and a 5.33% rise in emissions.

Instruments Comparison. Both ex-post and ex-ante instruments achieve emissions reductions, but they differ markedly in economic cost and allocative efficiency. The carbon tax reduces emissions by 6.55% but at the cost of a 4.08% decline in output. In contrast, the green credit policy achieves a larger emissions reduction of 8.32% while limiting the output loss to just 1.24%. Measured in terms of cost-effectiveness, the ex-ante instrument delivers 6.7% of emissions reduction per point of output loss—substantially outperforming the carbon tax, which achieves only 1.6%.

The key distinction lies in how each policy interacts with firm heterogeneity and capital reallocation. The carbon tax lowers the relative price of dirty capital, which unintentionally encourages financially constrained, emissions-intensive firms to expand. This reallocation of dirty capital offsets part of the intended environmental gains, working against the contraction in overall dirty capital supply. In contrast, the green credit policy reshapes the financing environment by making green capital more accessible and dirty capital less so. This leads to a productive reallocation that redirects dirty capital away from emission-intensive producers.

The Output-Emissions Frontier. To test whether the efficiency advantage holds beyond a single calibration point, we examine a broader policy range and construct an *output–emission frontier*. The carbon-tax rate τ^d is varied from 0 to 0.15, while the collateral parameters for the green-credit scheme move from $\xi^g = 1$ to 2 and $\xi^d = 1$ to 0. For each setting we solve for the general-equilibrium steady state and plot the resulting output–emissions pairs in Figure 9. The green-credit curve lies everywhere above and to the left of the carbon-tax curve: at any given emissions target it yields higher output, and at any given output target it achieves lower emissions. Hence the ex-ante instrument dominates the ex-post carbon tax throughout the entire policy range whenever social welfare values both economic activity and environmental quality.

5.3.2 The impact of Sustainable Investments.

Sustainable investment has grown rapidly in recent years, gaining widespread popularity as a key strategy for promoting environmentally responsible economic activities. Most sustainable investment practices are based on a firm's historical green performance, making them an ex-post policy instrument. Our dynamic setting is particularly suited for analyzing such historical performance-based sustainable investing. Unlike the two-period model, where firms are always unconstrained in the second period, the dynamic model allows for financial constraints in every period. This feature enables us to study how ex-post external equity financing shapes firms' investment decisions and sustainability outcomes over time. In particular, our analysis provides a theoretical framework for understanding the empirical findings of Hartzmark and Shue (2023), who document that sustainable investment strategies can generate unintended consequences through equilibrium effects on capital allocation.

In Table 6, we examine the general equilibrium effects of a sustainable investment policy that reallocates external equity financing based on firms' capital composition. In practice, sustainable investment strategies tend to favor firms with a larger share of green assets, while limiting funding to those with higher reliance on carbon-intensive capital. To capture this mechanism, we model financing decisions at the capital level: increasing γ^g represents enhanced equity inflows linked to green capital, while raising γ^d reflects a reduction in equity financing—or increased forced liquidation—associated with dirty capital. Specifically, we increase γ^d from 0 to 0.075 and γ^g from 0 to 0.175 in the forced liquidation term $\Gamma_{t+1} = \gamma^d k_t^d - \gamma^g k_t^g$. Given that the benchmark economy-wide dirty-to-green capital ratio is approximately 7:3, we choose γ^d and γ^g in a 3:7 ratio so that the net financing effect is roughly neutral at the aggregate level. This setup allows us to isolate the effect of redirecting financial resources toward greener capital holdings, without altering the overall size of equity financing in the economy.

[Place Table 6 about here]

At the aggregate level, the policy results in a 5.67% reduction in output and a 5.34% reduction in emissions. Emissions intensity (E/Y) rises slightly by 0.35%, suggesting that emissions fall less proportionally than output. This pattern is due to a reallocation of resources toward less constrained, less productive firms. Financially constrained firms—typically more productive but more reliant on dirty capital—face increasing difficulty in accumulating wealth due to higher forced liquidation requirements. As a result, their ability to invest and grow is restricted, leading to a contraction in their production share. This shift reduces both aggregate output and emissions, but causes a slight increase in emissions intensity.

Disaggregated firm-level outcomes reveal that the reallocation occurs almost proportionally between dirty and green capital. Among highly constrained firms, the K^d/K^g ratio increases by 0.72%. This mild increase suggests that the relative price of dirty capital has fallen sufficiently to induce a limited reallocation toward dirty capital, despite the policy's penalizing design (with q^d decreasing by 3.58% and q^g decreasing by 3.46%). This outcome is consistent with the general intuition for ex-post instruments: because the equity flow incentive is based on historical green performance and thus determined ex-post, highly constrained firms—who prioritize minimizing relative downpayment costs—respond primarily to immediate price changes rather than long-term financing incentives, leading them to marginally increase dirty capital holdings. In contrast, less constrained firms experience a slight decline in their K^d/K^g ratio by 0.51%.

However, compared to the carbon tax, the change in capital composition is much weaker under sustainable investment. This difference arises because external equity financing incentives have disproportionate effects on financially constrained and unconstrained firms. These incentives are only effective if the firm remains financially constrained in the next period; if a firm becomes unconstrained, internal and external capital are valued equally, and equity injections do not create additional value. Given that productivity is persistent in our model, financially constrained firms are more likely to remain constrained in the future. As a result, sustainable investment policies generate stronger financing incentives for constrained firms, which attenuates their incentive to shift aggressively toward dirty capital even when its relative price declines.

In contrast to the ex-ante green credit policy, which promotes cleaner reallocation while sustaining output, the ex-post sustainable investment reduces emissions primarily by slowing net worth accumulation and investment among productive firms. Although it lowers total emissions, it does so at the expense of aggregate output and with a marginal increase in emissions intensity. Moreover, it performs worse than the carbon tax, resulting in higher emissions and lower output. This trade-off highlights the limitations of historical performance-based investing, consistent with the empirical findings of Hartzmark and Shue (2023). Our results underscore the importance of forward-looking policy design that proactively reshapes financing incentives at the point of investment, rather than relying solely on firms' past environmental performance.

6 Discussion on "Ex-ante" versus "Ex-post"

6.1 Green Instruments in Practice

Our research presents a novel framework for categorizing green finance instruments into two distinct classifications: "Ex-ante" versus "Ex-post." The fundamental criterion for this classification is the timing of financial incentives or penalties. In this subsection we review current green-finance instruments and assign each to one of these two classifications.

A Carbon tax is a mechanism designed to charge emitters for the amount of carbon dioxide they release into the atmosphere, functioning as a critical tool for encouraging the reduction of greenhouse gas emissions. The implementation of carbon taxes across the globe has become an increasingly prevalent approach to address climate change. Direct carbon pricing instruments now cover almost a quarter of global greenhouse gas emissions. This demonstrates significant progress from a decade ago when only 7% of global emissions were covered by such policies (World Bank, 2023).

A carbon tax is considered an ex-post instrument because it's applied retroactively, based on the actual emissions firms have produced. This method allows for an accurate reflection of the pollution generated, with taxes typically due at set intervals (e.g., annually or quarterly), based on a firm's carbon emissions. The delayed payment system inherent in this approach reinforces its classification as ex-post.

The carbon credits trading system, part of the broader carbon pricing strategy alongside carbon taxes, offers a dynamic approach to managing emissions. It allows firms to buy or sell emission allowances, providing flexibility and incentivizing reductions in greenhouse gases. This system's design enables companies to strategically plan for emissions reductions, which can potentially serve as an ex-ante approach due to its forward-looking nature.

However, the system's flexibility regarding the timing of trades means it often operates effectively as an ex-post instrument in practice. This is because firms, especially those facing higher emissions or financial limitations, frequently opt to buy credits retrospectively. This interplay between prospective planning and retrospective adjustment highlights the real-world complexity in achieving desired allocation outcomes.

Carbon offsets are mechanisms for compensating for emissions by funding equivalent carbon dioxide saving projects. While similar to carbon credits in contributing to emission reduction goals, offsets are usually voluntary and can support projects unrelated to the buyer's direct emissions. This flexibility allows entities to support environmental projects globally. However, the voluntary nature and lack of specificity in project relevance can lead to accusations of greenwashing, where companies claim environmental efforts that are not as impactful as presented. Like carbon credits, offsets could be considered ex-ante for their proactive funding of emission reduction projects, but their voluntary aspect and potential for misuse complicate their impact.

Historical performance-based sustainable investing. Recently, sustainable investing has surged in popularity. Investors committed to sustainability actively reallocate their capital from environmentally harmful firms to those with a positive environmental impact, effectively reducing the cost of capital for green firms and increasing it for dirty firms. The prevalent strategy in this movement involves classifying companies as "green" or "dirty" based on their historical environmental performance indicators, such as past emission intensity and ESG scores. This historical-based approach is often chosen due to the challenges in making credible green commitments in advance and concerns over greenwashing, where firms may overstate their environmental commitments. As such, relying on a firm's track record of environmental performance provides a more reliable measure of its commitment to sustainability. Consequently, given its reliance on historical environmental performance to guide investment decisions, sustainable investing inherently adopts an ex-post approach.

Sustainable investing with commitments. Part of the commitment issue stems from the fact that dominant sustainable investing occurs at the firm level, where it's challenging for investors to verify how their funds are being used—whether for financing environmentally friendly projects or those detrimental to the environment. This uncertainty impedes firms from making credible green commitments in advance. However, project financing within the debt market offers a viable solution to this issue. At the project level, the environmental impact assessment becomes more straightforward, enabling sustainable investors to preferentially support green initiatives with financing advantages from the start. Tools such as green debt, green bonds, and policies like coal lending bans exemplify how this strategy can be applied, proving to be effective in addressing climate change (Green and Vallee, 2022). Due to their forward-looking nature, these instruments are categorized as ex-ante instruments.

6.2 "Ex-ante" vs. "Ex-post" beyond Our Model

Understanding the distinctive distributional impacts of 'ex-ante' and 'ex-post' green finance instruments is essential for assessing their overall effectiveness in enhancing aggregate environmental sustainability. However, the real-world implementation of these instruments often faces challenges, and their impacts must be considered across both intensive and extensive margins.

Implementation Issue. While 'ex-ante' instruments are theoretically more allocatively efficient, their practical implementation faces significant challenges. The primary issue lies in the incomplete contracts and the difficulty for firms to credibly commit to the greenness of their projects. This lack of commitment can be attributed to the inherent uncertainty and complexity in predicting the environmental impact of a project in its nascent stages. The absence of standardized metrics and benchmarks for greenness complicates this process, making it harder to distinguish genuinely sustainable projects from those that are not.

In contrast, 'ex-post' instruments are generally easier to implement due to their reliance on past performance metrics. The retrospective nature of these instruments simplifies the validation process, as it is based on tangible, historical data. This ease of implementation contributes to the prevalence of ’ex-post’ strategies in current green instruments and investment strategies. Nonetheless, project-specific financing, an ’ex-ante’ approach, is gaining more popularity in the green debt market, as evidenced by Green and Vallee (2022).

Intensive Margin vs. Extensive Margin. Under our framework, firms are faced with the choice between existing green and dirty technologies, which is essentially an intensive margin decision. In this context, ’ex-ante’ instruments are more effective as they shift dirty capital from financially constrained firm towards unconstrained firms. By providing upfront incentives, these instruments make green technologies more accessible, especially for financially constrained firms.

However, the potential benefits of ’ex-post’ instruments at the extensive margin are not captured in our model. These green instruments predominantly impact financially unconstrained firms, potentially spurring green innovation. Innovation, typically undertaken by less financially constrained entities, is incentivized under ’ex-post’ instruments. These firms, motivated by the prospect of future benefits based on their environmental performance, are likely to invest in new, cleaner technologies. This innovation not only enhances the firm’s green portfolio but also has the potential for broader spillover effects, contributing to the overall greening of the economy.

In conclusion, while ’ex-ante’ instruments are more efficient in driving immediate technology adoption, especially for constrained firms, ’ex-post’ instruments may play a critical role in fostering long-term green innovation. This underscores the importance of a balanced policy approach that addresses both the immediate adoption needs of existing technologies (intensive margin) and the longer-term innovation incentives (extensive margin) to achieve comprehensive environmental sustainability.

7 Conclusion

Our study reveals that a more thoughtful application of green finance instruments is crucial. ’Ex-ante’ instruments, which provide upfront subsidies for green technologies, are shown to be more allocatively efficient, effectively guiding financially constrained firms towards cleaner technologies. On the other hand, ’ex-post’ instruments tend to be less efficient in this regard, often shifting dirty capital towards these constrained firms and potentially leading to higher overall emissions due to their higher emission intensity. This highlights the necessity of designing green finance instruments that are specifically tailored to the varied financial conditions of firms, ensuring a more effective and sustainable environmental impact.

References

ACEMOGLU, D., P. AGHION, L. BURSZTYN, AND D. HEMOUS (2012): "The environment and directed technical change," *American economic review*, 102, 131–166.

ACEMOGLU, D., U. AKCIGIT, D. HANLEY, AND W. KERR (2016): "Transition to clean technology," *Journal of Political Economy*, 124, 52–104.

AKEY, P. AND I. APPEL (2021): "The limits of limited liability: Evidence from industrial pollution," *The Journal of Finance*, 76, 5–55.

ARELLANO, C., Y. BAI, AND P. J. KEHOE (2019): "Financial frictions and fluctuations in volatility," *Journal of Political Economy*, 127, 2049–2103.

ARNOLD, L. G. (2023): "On the neutrality of socially responsible investing: The general equilibrium perspective," *Theoretical Economics*, 18, 65–95.

ARON-DINE, S., J. BEUTEL, M. PIAZZESI, AND M. SCHNEIDER (2023): "Household Climate Finance: Theory and Survey Data on Safe and Risky Green Assets," .

ASKER, J., A. COLLARD-WEXLER, AND J. DE LOECKER (2014): "Dynamic inputs and resource (mis) allocation," *Journal of Political Economy*, 122, 1013–1063.

BARTRAM, S. M., K. HOU, AND S. KIM (2022): "Real effects of climate policy: Financial constraints and spillovers," *Journal of Financial Economics*, 143, 668–696.

BERK, J. AND J. H. VAN BINSBERGEN (2021): "The impact of impact investing," .

BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of Macroeconomics*, 1, 1341–1393.

BROCCARDO, E., O. HART, AND L. ZINGALES (2022): "Exit versus voice," *Journal of Political Economy*, 130, 3101–3145.

BUERA, F. J. AND Y. SHIN (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121, 221–272.

CHAVA, S. (2014): "Environmental externalities and cost of capital," *Management Science*, 60, 2223–2247.

DAVIES, S. W. AND E. D. VAN WESEP (2018): "The unintended consequences of divestment," *Journal of Financial Economics*, 128, 558–575.

DAVILA, J., J. H. HONG, P. KRUSELL, AND J.-V. RÍOS-RULL (2012): "Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks," *Econometrica*, 80, 2431–2467.

DIAMOND, P. A. (1967): "The Role of a Stock Market in a General Equilibrium Model with

Technological Uncertainty," *The American Economic Review*, 57, 759–776.

DUCHIN, R., J. GAO, AND Q. XU (2022): "Sustainability or greenwashing: Evidence from the asset market for industrial pollution," *Available at SSRN*.

EDMANS, A., D. LEVIT, AND J. SCHNEEMEIER (2022): "Socially responsible divestment," *European Corporate Governance Institute–Finance Working Paper*.

GOETZ, M. R. (2018): "Financial constraints and corporate environmental responsibility," *Available at SSRN 3230344*.

GOLOSOV, M., J. HASSLER, P. KRUSELL, AND A. TSYVINSKI (2014): "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, 82, 41–88.

GORMSEN, N. J., K. HUBER, AND S. OH (2023): "Climate capitalists," *Available at SSRN*.

GREEN, D. AND B. VALLEE (2022): "Can finance save the world? measurement and effects of coal divestment policies by banks," *Measurement and Effects of Coal Divestment Policies by Banks (April 8, 2022)*.

HARTZMARK, S. M. AND K. SHUE (2023): "Counterproductive impact investing: The impact elasticity of brown and green firms," *Available at SSRN 4359282*.

HASSLER, J. AND P. KRUSELL (2012): "Economics and climate change: integrated assessment in a multi-region world," *Journal of the European Economic Association*, 10, 974–1000.

HEINKEL, R., A. KRAUS, AND J. ZECHNER (2001): "The effect of green investment on corporate behavior," *Journal of financial and quantitative analysis*, 36, 431–449.

HENNESSY, C. A. AND T. M. WHITED (2005): "Debt dynamics," *The journal of finance*, 60, 1129–1165.

——— (2007): "How costly is external financing? Evidence from a structural estimation," *The Journal of Finance*, 62, 1705–1745.

HONG, H., J. D. KUBIK, AND J. A. SCHEINKMAN (2012): "Financial constraints on corporate goodness," Tech. rep., National Bureau of Economic Research.

JORGENSON, D. W. (1963): "Capital theory and investment behavior," *The American economic review*, 53, 247–259.

KACPERCZYK, M. T. AND J.-L. PEYDRÓ (2022): "Carbon emissions and the bank-lending channel," *Available at SSRN 3915486*.

KIYOTAKI, N. AND J. MOORE (1997): "Credit cycles," *Journal of Political Economy*, 105, 211–248.

LANTERI, A. AND A. A. RAMPINI (2023): "Financing the Adoption of Clean Technology," .

MOLL, B. (2014): "Productivity losses from financial frictions: Can self-financing undo capital misallocation?" *American Economic Review*, 104, 3186–3221.

NORDHAUS, W. D. (2014): *A question of balance: Weighing the options on global warming policies*, Yale University Press.

NUÑO, G. AND B. MOLL (2018): "Social optima in economies with heterogeneous agents," *Review of Economic Dynamics*, 28, 150–180.

PÁSTOR, L., R. F. STAMBAUGH, AND L. A. TAYLOR (2021): "Sustainable investing in equilibrium," *Journal of Financial Economics*, 142, 550–571.

——— (2022): "Dissecting green returns," *Journal of Financial Economics*, 146, 403–424.

PEDERSEN, L. H. (2023): "Carbon Pricing versus Green Finance," *Available at SSRN 4382360*.

QUADRINI, V. (2000): "Entrepreneurship, saving, and social mobility," *Review of Economic Dynamics*, 3, 1–40.

STIGLITZ, J. E. (1982): "The inefficiency of the stock market equilibrium," *The Review of Economic Studies*, 49, 241–261.

VAN DER BECK, P. (2021): "Flow-driven ESG returns," *Swiss Finance Institute Research Paper*.

WHITED, T. M. AND G. WU (2006): "Financial constraints risk," *The Review of Financial Studies*, 19, 531–559.

WORLD BANK (2023): "State and Trends of Carbon Pricing 2023," Tech. rep.

XU, Q. AND T. KIM (2022): "Financial constraints and corporate environmental policies," *The Review of Financial Studies*, 35, 576–635.

Tables and Figures

Table 1: Classification of Green Finance Instruments into Ex-ante and Ex-post Categories

Green Instruments	Ex-post Instruments	Ex-ante Instruments
Carbon Tax	✓	
Carbon Credits Trading System*		✓
Carbon Offsets*		✓
Historical Performance-Based Sustainable Investing	✓	
Sustainable Investing with Commitments		✓

This table classifies various green finance instruments into either ex-ante or ex-post categories based on the timing of their implementation and impact. Instruments marked with an asterisk (*) are theoretically ex-ante, due to their proactive approach, but can function effectively as ex-post in practice.

Table 2: The Impact of Green Instruments under Financial Constraints

Instruments	Ex-post Instruments		Ex-ante Instruments	
Firm Type	Constrained	Unconstrained	Constrained	Unconstrained
Direct Effect	$U^d \uparrow, \Phi^d \rightarrow, U^g \downarrow, \Phi^g \rightarrow$		$U^d \rightarrow, \Phi^d \uparrow, U^g \rightarrow, \Phi^g \downarrow$	
Indirect Effect	$q^d \downarrow, q^g \uparrow$		$q^d \downarrow, q^g \uparrow$	
Overall Effect on User Cost	$U^d \uparrow, U^g \downarrow$		$U^d \downarrow, U^g \uparrow$	
Overall Effect on Down Payment	$\Phi^d \downarrow, \Phi^g \uparrow$		$\Phi^d \uparrow, \Phi^g \downarrow$	
Relative Total Cost	$\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda} \downarrow$	$\frac{U^d}{U^g} \uparrow$	$\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda} \uparrow$	$\frac{U^d}{U^g} \downarrow$
Capital Investment	$k^d \uparrow, k^g \downarrow$	$k^d \downarrow, k^g \uparrow$	$k^d \downarrow, k^g \uparrow$	$k^d \uparrow, k^g \downarrow$

This table summarizes the impact of green instruments on highly constrained firms (large λ) and unconstrained firms ($\lambda = 0$). ↑ indicates an increase, ↓ indicates a decrease, → indicates unchanged.

Table 3: Parameters for the Two-period Model

Parameters	Meaning	Value
β	Firm discount factor	0.95
R	Gross interest rate	$1/\beta$
σ	Elasticity of substitution between k^d and k^g	20.0
γ	Share of k^d	0.5
w_0	Firm initial net worth	1.0
θ	Fraction of capital as collateral	0.4
c^d	Utilization cost of k^d	0.1
c^g	Utilization cost of k^g	0.05

This table summarizes the parameters for the two-period model.

Table 4: Parameters for the Benchmark Dynamic Model

Parameters	Meaning	Value
β	Firm discount factor	0.9
R	Gross risk-free interest rate	$1/\beta$
γ	Share of k^d in production	0.5
σ	Elasticity of substitution between dirty and green capital	20
δ	Capital depreciation rate	0.1
ε	Elasticity of capital supply	1
ρ	Firm death probability	0.15
θ	Fraction of general capital value as collateral	0.30
ρ_z	Persistence for firm level productivity shock	0.70
σ_z	Std. for firm level productivity shock	0.25
ι	Elasticity of emission to firm productivity	0.85
c_d	Utilization cost of dirty capital	0.05

This table reports the parameters used for our dynamic model in the benchmark case.

Table 5: Comparison of Ex-Post and Ex-Ante Green Instruments

	Y	E	E/Y	E/K^d	K^d/K^g
Panel A: Ex-Post Policy (Carbon Tax)					
All Firms					
Benchmark Model	0.991	1.274	1.286	1.819	2.333
Carbon Tax	0.950	1.190	1.252	1.862	2.119
% Change	-4.08%	-6.55%	-2.58%	2.37%	-9.20%
Highly Constrained Firms					
Benchmark Model	0.337	0.450	1.336	2.478	3.446
Carbon Tax	0.339	0.455	1.345	2.486	3.513
% Change	0.55%	1.27%	0.71%	0.32%	1.96%
Less Constrained Firms					
Benchmark Model	0.654	0.824	1.260	1.589	2.096
Carbon Tax	0.612	0.735	1.201	1.612	1.827
% Change	-6.46%	-10.82%	-4.66%	1.46%	-12.84%
Panel B: Ex-Ante Policy (Green Credit)					
All Firms					
Benchmark Model	0.991	1.274	1.286	1.819	2.333
Green Credit	0.978	1.168	1.194	1.719	2.209
% Change	-1.24%	-8.32%	-7.16%	-5.49%	-5.33%
Highly Constrained Firms					
Benchmark Model	0.337	0.450	1.336	2.478	3.446
Green Credit	0.335	0.300	0.896	2.413	1.165
% Change	-0.60%	-33.34%	-32.93%	-2.63%	-66.19%
Less Constrained Firms					
Benchmark Model	0.654	0.824	1.260	1.589	2.096
Green Credit	0.644	0.868	1.348	1.564	2.763
% Change	-1.57%	5.33%	7.01%	-1.55%	31.80%

This table compares the effects of two green instruments. Panel A presents the impact of a carbon tax (τ^d) increasing from 0 to 5% (Ex-Post Policy). Panel B shows the impact of a green credit policy that increases the pledgeability of green capital (ξ^g from 1.0 to 1.25) and decreases that of dirty capital (ξ^d from 1.0 to 0.75) (Ex-Ante Policy). Reported variables include aggregate output (Y), emissions (E), emissions intensity (E/Y), emissions per unit of dirty capital (E/K^d), and the ratio of dirty to green capital (K^d/K^g). The results are reported for all firms, highly constrained firms (top 30% in financial constraint measure λ), and less constrained firms. Percentage changes are computed relative to the benchmark model. Model parameters are listed in Table 4.

Table 6: Sustainable Investment Policies

	Y	E	E/Y	E/K^d	K^d/K^g
All Firms					
Benchmark Model	0.9907	1.2736	1.2856	1.8192	2.3332
ESG Investment	0.9346	1.2056	1.2900	1.7867	2.3303
Percentage Diff	-5.67%	-5.34%	0.35%	-1.79%	-0.12%
Highly Constrained Firms					
Benchmark Model	0.3366	0.4496	1.3356	2.4778	3.4457
ESG Investment	0.3211	0.4312	1.3428	2.4290	3.4706
Percentage Diff	-4.59%	-4.08%	0.54%	-1.97%	0.72%
Less Constrained Firms					
Benchmark Model	0.6541	0.8241	1.2598	1.5888	2.0964
ESG Investment	0.6134	0.7744	1.2624	1.5573	2.0856
Percentage Diff	-6.22%	-6.03%	0.21%	-1.98%	-0.51%

This table reports the effects of a sustainable investment policy implemented by increasing γ^d and γ^g in the forced liquidation term $\Gamma_{t+1} = \gamma^d k_t^d - \gamma^g k_t^g$, from 0 to 0.075 and 0 to 0.175, respectively. A higher γ^d raises the forced dividend payout associated with holding dirty capital, while a higher γ^g increases capital injection linked to holding green capital. The variables reported include aggregate output (Y), emissions (E), emissions intensity (E/Y), emissions per unit of dirty capital (E/K^d), and the ratio of dirty to green capital (K^d/K^g). The first panel presents results for the entire economy, while the second and third panels report outcomes for highly constrained firms (top 30% in financial constraint measure λ) and less constrained firms, respectively. The final row of each panel shows the percentage change relative to the benchmark model. Model parameters are provided in Table 4.

Figure 1: **Capital Choice without Green Instruments**

This figure presents firm capital choices in the benchmark case without green instruments. It is divided into four panels: Panel (a) shows the firm's multiplier, λ; Panel (b) presents the relative importance of down payment, $\frac{\Phi^d\lambda}{U^d+\Phi^d\lambda}$; Panel (c) depicts the relative total cost of dirty versus green, $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$; and Panel (d) illustrates the dirty to green capital ratio, $\frac{k^d}{k^g}$, all as functions of firm productivity. Detailed parameters for the underlying model are provided in Table 3.

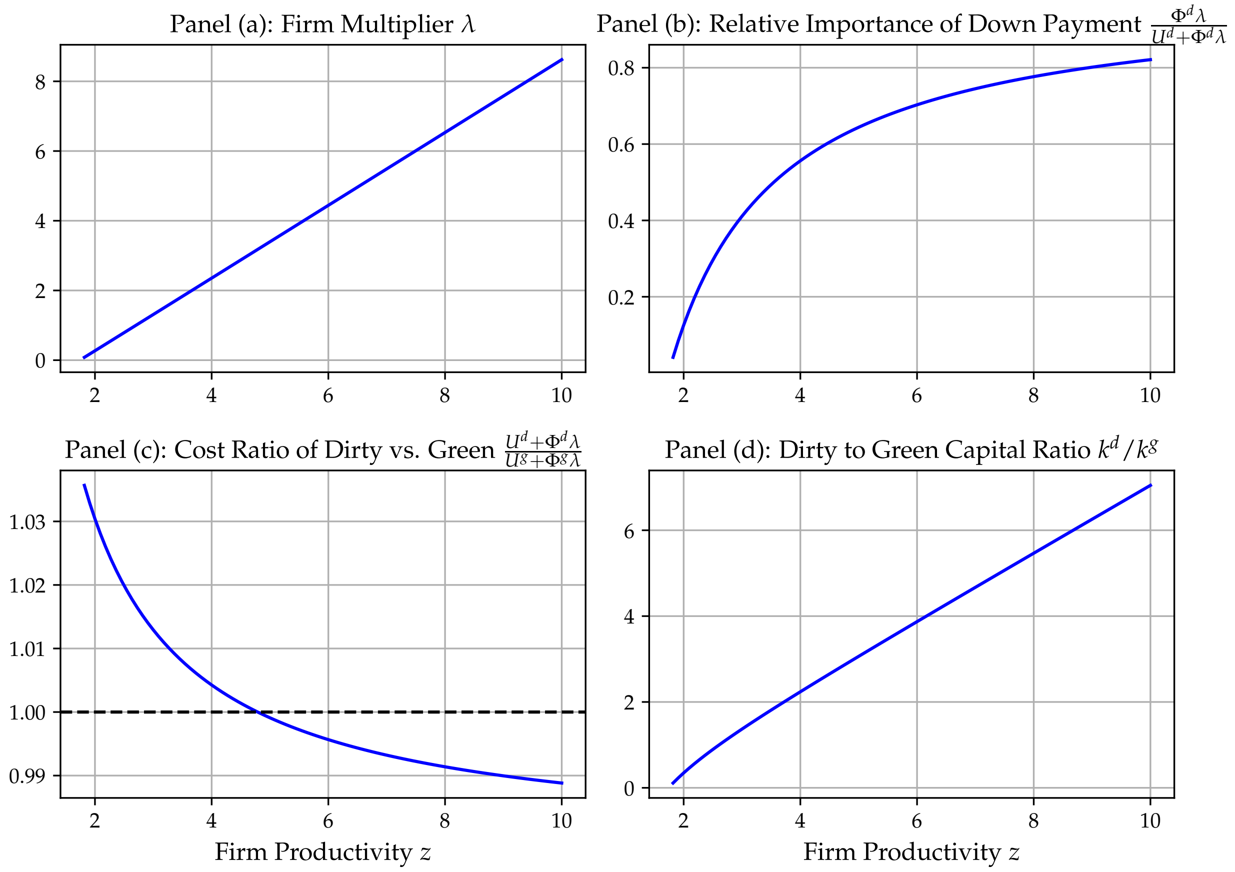
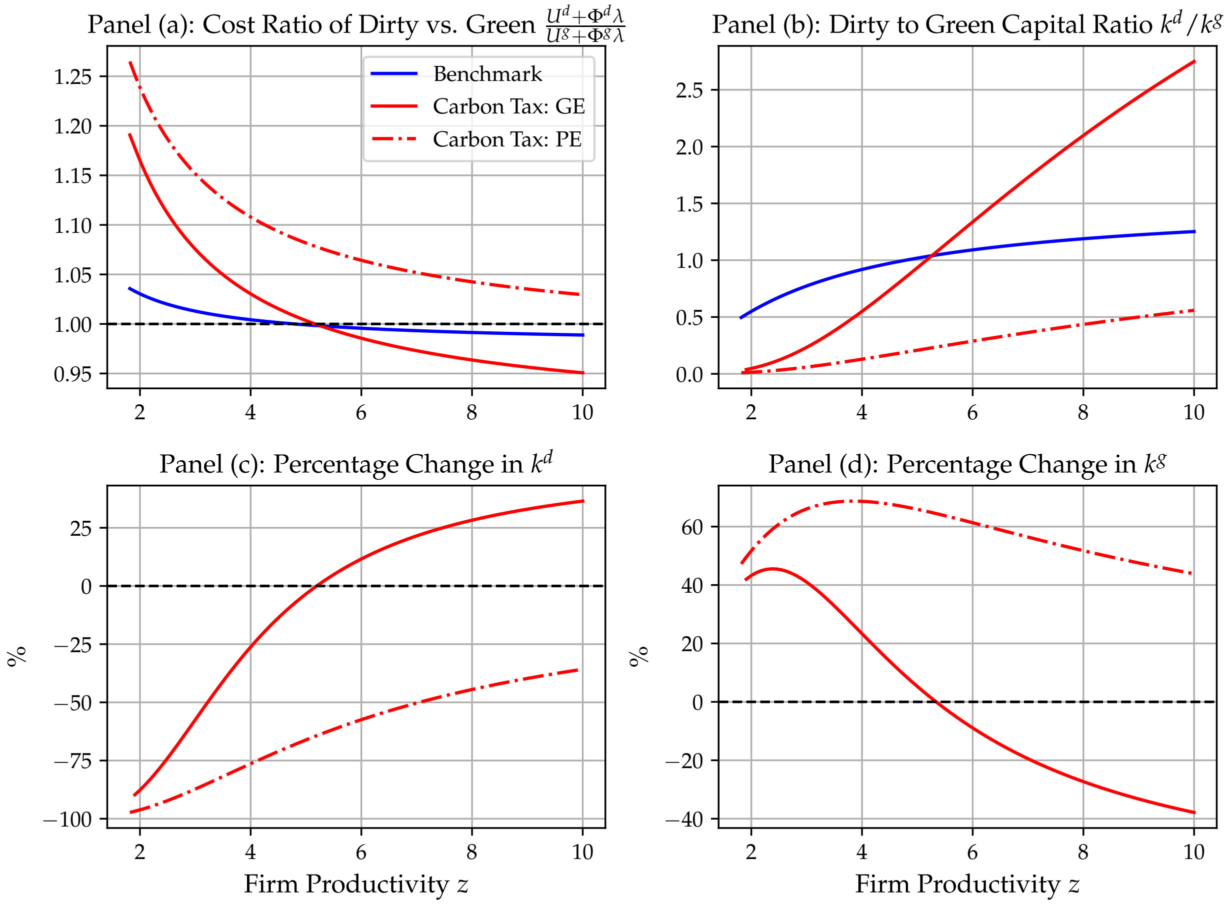


Figure 2: **Distributional Effects: Ex-post Instrument**

This figure illustrates how firm capital choices are influenced by a carbon tax ($\tau^d = 0.2$). It is divided into four panels: Panel (a) compares the relative total costs of dirty versus green capital, expressed as $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$; Panel (b) shows the ratio of dirty to green capital, $\frac{k^d}{k^g}$; Panel (c) depicts the percentage change in dirty capital investment relative to the benchmark; and Panel (d) illustrates the percentage change in green capital investment relative to the benchmark, all as functions of firm productivity. The benchmark case results are represented by a blue curve for comparison. The impact of the carbon tax is shown in two ways: the dashed red curve represents the partial equilibrium effect, assuming constant prices for dirty and green capital as in the benchmark case, while the solid red curve indicates the general equilibrium effect, including changes in capital prices due to the tax. Detailed parameters for the underlying model are provided in Table 3.



Panel (a): Cost Ratio of Dirty vs. Green $\frac{U^d+\Phi^d\lambda}{U^g+\Phi^g\lambda}$

Panel (b): Dirty to Green Capital Ratio k^d/k^g

Panel (c): Percentage Change in k^d

Panel (d): Percentage Change in k^g

Figure 3: **Distributional Effects: Ex-ante Instrument**

This figure illustrates how firm capital choices are influenced by a dirty capital lending ban ($\xi^d = 0.8$). It is divided into four panels: Panel (a) compares the relative total costs of dirty versus green capital, expressed as $\frac{U^d + \Phi^d \lambda}{U^g + \Phi^g \lambda}$; Panel (b) shows the ratio of dirty to green capital, $\frac{k^d}{k^g}$; Panel (c) depicts the percentage change in dirty capital investment relative to the benchmark; and Panel (d) illustrates the percentage change in green capital investment relative to the benchmark, all as functions of firm productivity. The benchmark case results are represented by a blue curve for comparison. The impact of the lending ban is shown in two ways: the dashed red curve represents the partial equilibrium effect, assuming constant prices for dirty and green capital as in the benchmark case, while the solid red curve indicates the general equilibrium effect, including changes in capital prices due to the lending ban. Detailed parameters for the underlying model are provided in Table 3.

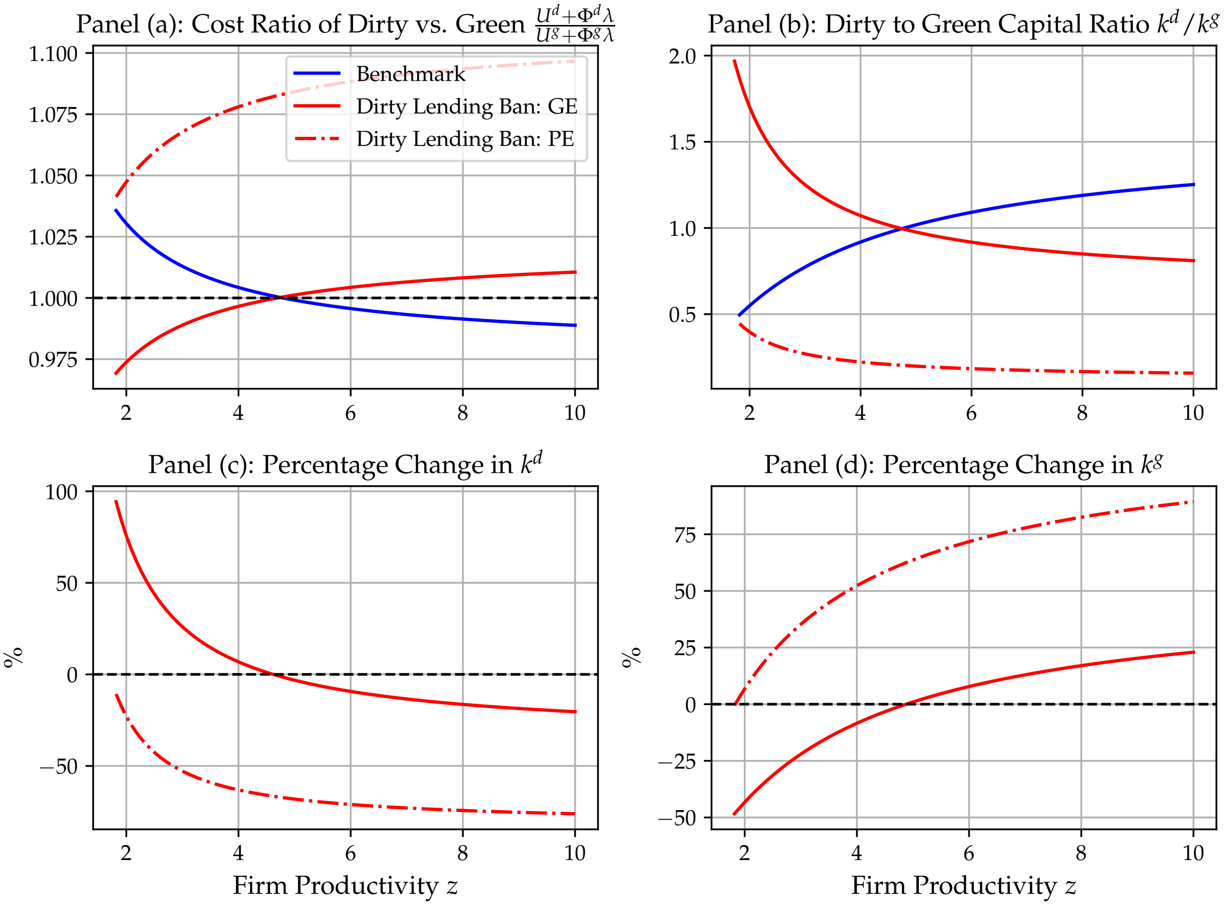
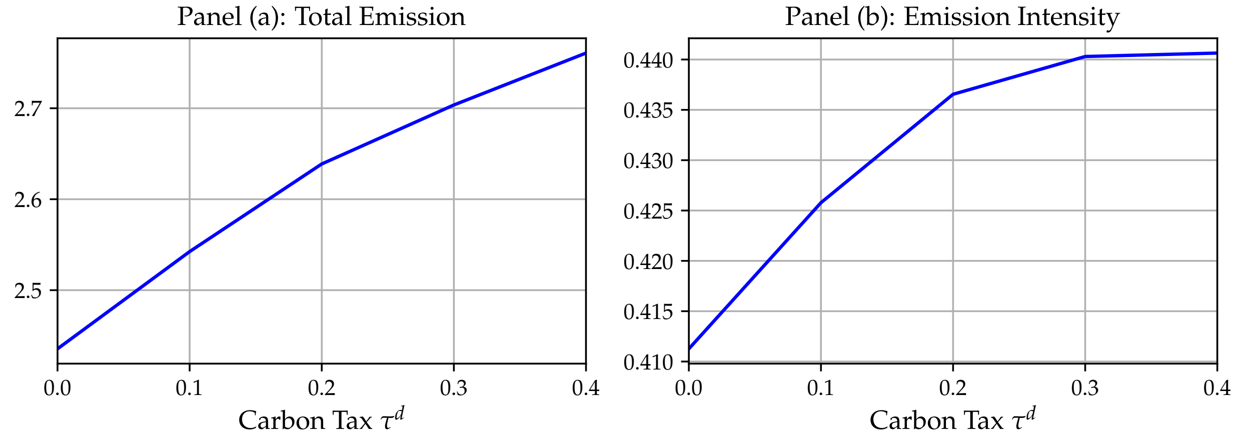


Figure 4: **Aggregate Effect, Ex-post Instrument**

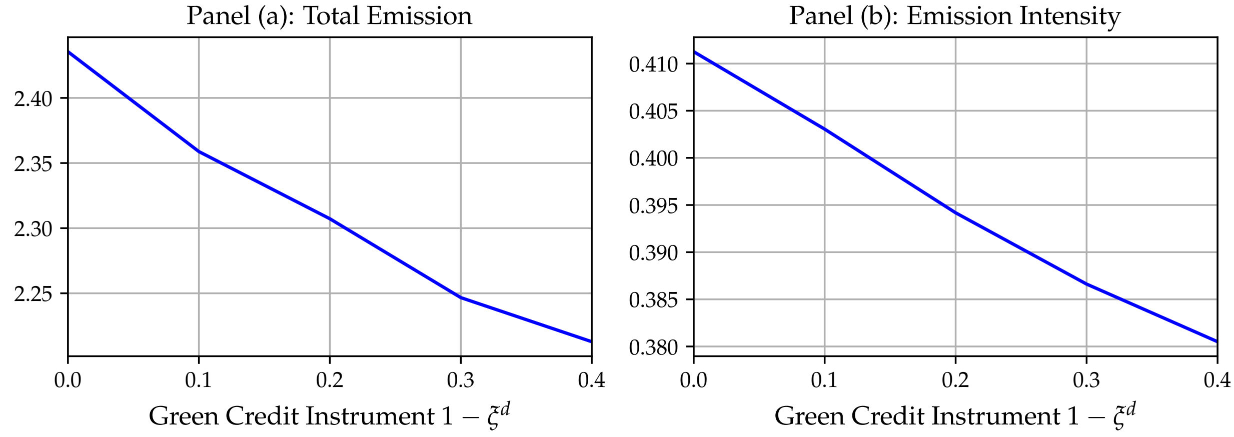
This figure shows the aggregate impact of carbon tax. Panel (a) quantifies total emissions, calculated as the integration of utilized dirty capital, $E = \int z_i^\iota k_i^d$; Panel (b) assesses emission intensity, defined as the ratio of total emissions to aggregate output. Each variable is plotted against the carbon tax rate, τ^d. Detailed parameters for the underlying model are provided in Table 3.


Panel (a): Total Emission

Panel (b): Emission Intensity

Figure 5: **Aggregate Effect, Ex-ante Instrument**

This figure illustrates the aggregate impact of a dirty capital lending ban, represented by $1 - \xi^d$, where a higher value indicates lower collateralizability of dirty capital. Panel (a) quantifies total emissions, calculated as the integration of utilized dirty capital, $E = \int z_i^\iota k_i^d$; Panel (b) assesses emission intensity, defined as the ratio of total emissions to aggregate output. Each variable is plotted against the green credit instrument, $1 - \xi^d$. Detailed parameters for the underlying model are provided in Table 3.


Panel (a): Total Emission

Panel (b): Emission Intensity

Figure 6: **Aggregate Effect, Ex-post Instrument**

This figure shows the aggregate impact of carbon tax with variable capital supply ($\kappa = 2$). It is divided into four panels: Panel (a) quantifies total emissions, calculated as the integration of utilized dirty capital, $E = \int z_i^\iota k_i^d$; Panel (b) assesses emission intensity, defined as the ratio of total emissions to aggregate output; Panel (c) illustrates average dirty capital emission rate, $H^d = E/K^d$; Panel (d) decompose the change in total emissions relative to the benchmark into two components: the allocative effect, calculated as $K^d H^d - K^d H^d_{bench}$, and the level effect, calculated as $K^d H^d_{bench} - K^d_{bench} H^d_{bench}$. Each variable is plotted against the carbon tax rate, τ^d. Detailed parameters for the underlying model are provided in Table 3.

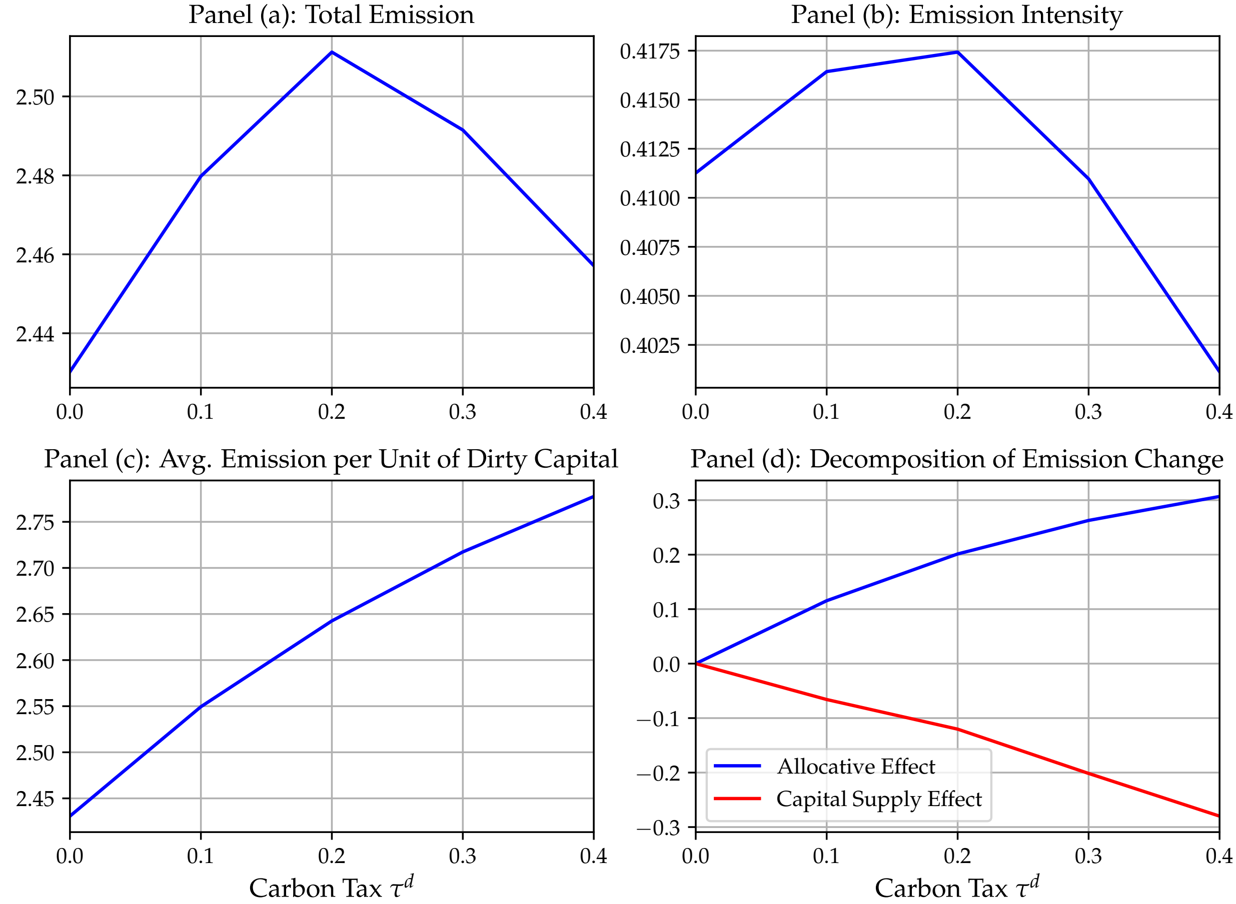


Figure 7: **Aggregate Effect, Ex-ante Instrument**

This figure illustrates the aggregate impact of a dirty capital lending ban with variable capital supply ($\kappa = 2$). The lending ban is represented by $1 - \xi^d$, where a higher value indicates lower collateralizability of dirty capital. It is divided into four panels: Panel (a) quantifies total emissions, calculated as the integration of utilized dirty capital, $E = \int z_i^t k_i^d$; Panel (b) assesses emission intensity, defined as the ratio of total emissions to aggregate output; Panel (c) illustrates average dirty capital emission rate, $H^d = E/K^d$; Panel (d) decompose the change in total emissions relative to the benchmark into two components: the allocative effect, calculated as $K^d H^d - K^d H^d_{bench}$, and the level effect, calculated as $K^d H^d_{bench} - K^d_{bench} H^d_{bench}$. Each variable is plotted against the green credit instrument, $1 - \xi^d$. Detailed parameters for the underlying model are provided in Table 3.

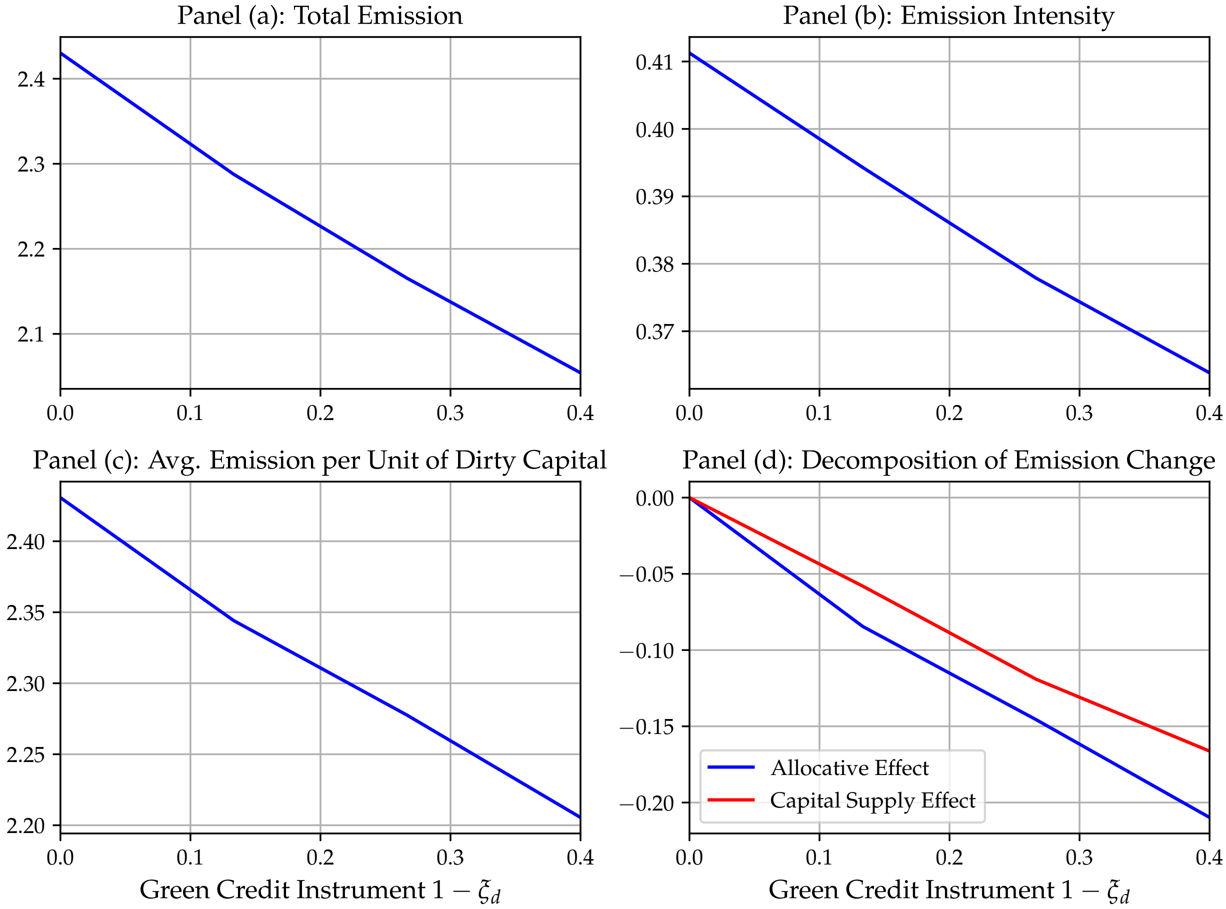
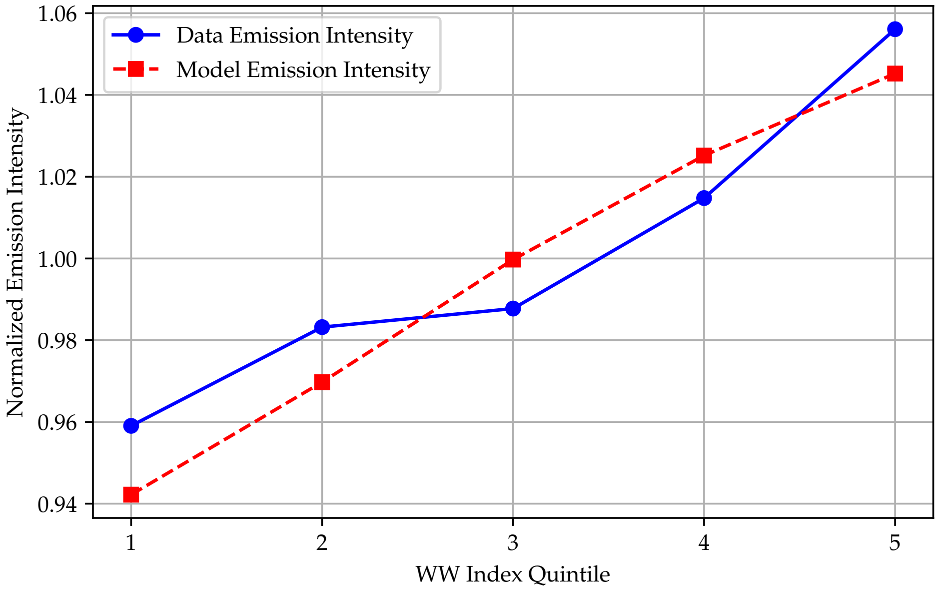


Figure 8: **Emission Intensity and Emission per Capital (Data vs. Model)**

This figure illustrates how emissions vary with financial constraints. Firms are sorted by the Whited and Wu (2006) (WW) index into five quintiles (1–5), from the least to the most financially constrained. Panel A (top) shows normalized emission intensity (emissions per output), while Panel B (bottom) shows normalized emissions per capital (emissions divided by PPENT). Both measures are normalized by the overall sample mean in the data and the model. The solid lines represent the empirical data, and the dashed lines represent the model-implied outcomes under the calibration parameters reported in Table 4 (with $\iota = 0.8$ and $c_d = 0.1$).

Panel (a): Emission Intensity (Data vs. Model)



Panel (b): Emission per Capital (Data vs. Model)

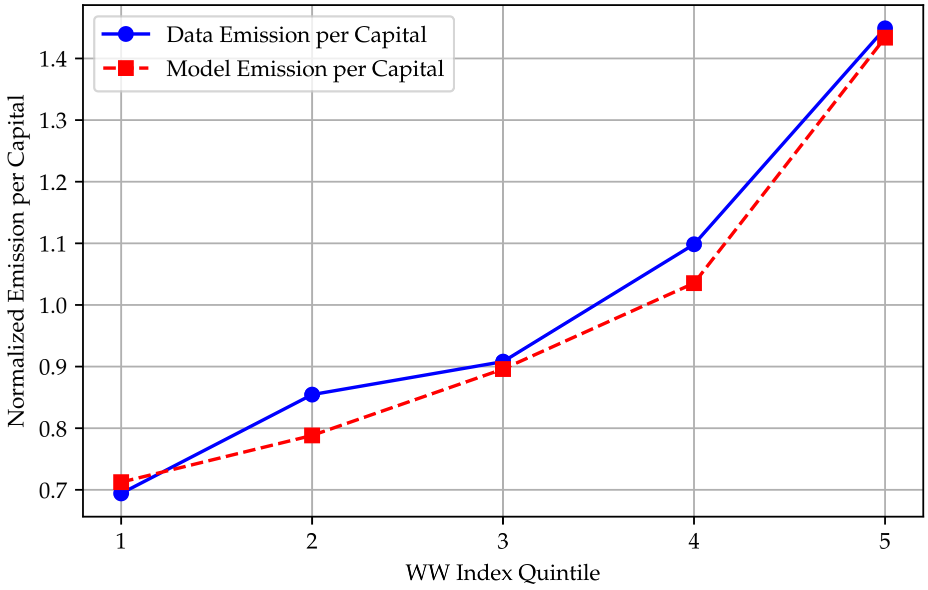
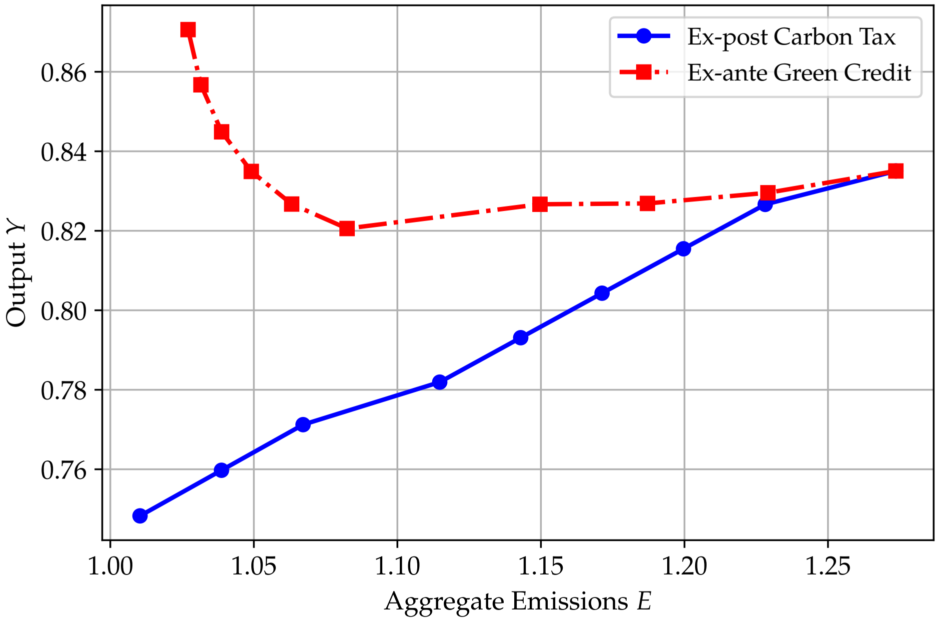


Figure 9: **Output–Emission Frontier**

This figure illustrates the output–emission frontier under two policy designs. The blue dots trace outcomes as the *ex-post* carbon-tax rate rises from $\tau^d = 0$ to 0.15, while the red squares trace outcomes as the *ex-ante* green-credit parameters move from $\xi^g = 1.00 \to 2.00$ and $\xi^d = 1.00 \to 0.00$. Calibration parameters follow Table 4.



Appendix

A Two-period Model: Extensions and Proofs

A.1 Variable Capital Utilization

In this section, we extend the main model by introducing variable capital utilization to provide a microfoundation for the reduced-form emission setup. Firms choose a utilization rate h, which determines the intensity of capital use in production and emissions. The production function is given by $y = \tilde{z}hk$, where $\tilde{z}$ represents productivity, and k is the composite capital combining dirty and green capital. Utilizing capital at rate h incurs a cost of $\frac{1}{\tilde{\iota}}h^{\tilde{\iota}}k$, where $\tilde{\iota} > 1$ determines the curvature of the cost function. Emissions are proportional to the utilized dirty capital, hk^d.

The firm's optimization problem can be written as:

$$\max_{d_0, k^d, k^g, b, h} \; d_0 + \beta d_1, \tag{A.1}$$

subject to the following constraints:

$$w + b - d_0 \geq q^d k^d + q^g k^g, \tag{A.2}$$

$$\tilde{z}hk - c^d k^d - c^g k^g - \frac{1}{\tilde{\iota}}h^{\tilde{\iota}}k \geq d_1 + \beta^{-1}b, \tag{A.3}$$

$$\theta q^d k^d + \theta q^g k^g \geq b, \tag{A.4}$$

$$d_0 \geq 0. \tag{A.5}$$

The first-order condition for the utilization rate h is $\frac{\partial}{\partial h}\left[\tilde{z}hk - \frac{1}{\tilde{\iota}}h^{\tilde{\iota}}k\right] = 0$, which simplifies to $h = \tilde{z}^{\frac{1}{\tilde{\iota}-1}}$. Substituting this result into the production function gives $y = \tilde{z}^{\frac{\tilde{\iota}}{\tilde{\iota}-1}}k$. Similarly, emissions are given by $e = \tilde{z}^{\frac{1}{\tilde{\iota}-1}}k^d$. To align this with the reduced-form emission setup, let $z = \tilde{z}^{\frac{\tilde{\iota}}{\tilde{\iota}-1}}$ and $\iota = \frac{1}{\tilde{\iota}}$. The production function then becomes $y = zk$, and emissions simplify to $e = z^{\iota}k^d$.

This formulation establishes consistency with the reduced-form emission setup in the main model. The dependence of emissions on productivity and dirty capital is derived from the firm's optimal utilization decision. Firms with higher productivity choose higher utilization rates and therefore have greater emissions for the same level of dirty capital. The parameter ι captures the elasticity of emissions with respect to productivity, allowing for heterogeneity in marginal emissions across firms.

A.2 Threshold Productivity under Constant Returns to Scale

The threshold z^* can be computed when $\lambda = 0$. In this case we have

$$k^d \quad : \quad U^d = \beta z^* g_1 \tag{A.6}$$

$$k^g \quad : \quad U^g = \beta z^* g_2. \tag{A.7}$$

From these two equations we have $U^d k^d + U^g k^g = \beta z^* k$. Combing equation (20) and equation (21), we have

$$\begin{aligned} U^d \frac{k^d}{k^g} + U^g &= \beta z^* \frac{k}{k_g} \\ \frac{k}{k_g} &= \left[\gamma \left(\frac{k^d}{k^g}\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\right]^{\frac{\sigma}{\sigma-1}} \\ \frac{k^d}{k^g} &= \left(\frac{\gamma}{1-\gamma}\right)^{\sigma} \left(\frac{U^d}{U^g}\right)^{-\sigma} \end{aligned}$$

$$\begin{aligned} U^d \cdot \left(\frac{\gamma}{1-\gamma}\right)^{\sigma} \left(\frac{U^d}{U^g}\right)^{-\sigma} + U^g = \beta z^* &\left[\gamma \left(\frac{\gamma}{1-\gamma}\right)^{\sigma-1} \left(\frac{U^d}{U^g}\right)^{-(\sigma-1)} + (1-\gamma)\right]^{\frac{\sigma}{\sigma-1}} \\ \text{left:} \quad &\left[\gamma^{\sigma}(U^d)^{1-\sigma} + (1-\gamma)^{\sigma}(U^g)^{1-\sigma}\right] \frac{1}{(1-\gamma)^{\sigma}(U^g)^{-\sigma}} \\ \text{right:} \quad \beta z^* &\left[\gamma^{\sigma}(U^d)^{1-\sigma} + (1-\gamma)^{\sigma}(U^g)^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}} \frac{1}{(1-\gamma)^{\sigma}(U^g)^{-\sigma}} \\ \Rightarrow z^* = &\frac{1}{\beta}\left[\gamma^{\sigma}(U^d)^{1-\sigma} + (1-\gamma)^{\sigma}(U^g)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \end{aligned}$$

B Two-Period Model: Supplementary Numerical Analysis

B.1 Decreasing Return to Scale and the Impact Elasticity

Our main results remain consistent when extending the analysis to include decreasing returns to scale. However, in this case, the financial constraints faced by firms depend not only on idiosyncratic productivity z but also on the firm's net worth w. This extension enables us to explore the model's implications for impact elasticity, as discussed in Hartzmark and Shue (2023).

Hartzmark and Shue (2023) demonstrate that reducing financing costs yields less environmental benefit for green firms compared to dirty firms. Our model echoes this finding, replicating their observed pattern in scenarios without any policy interventions or with ex-post instruments. However, this pattern dissipates when strong enough ex-ante policies are applied. Intuitively, reducing financing costs eases financial constraints more significantly for firms that are already financially constrained than for those that are not. In scenarios without green instruments or with the application of ex-post instruments, financially constrained firms are dirty firms. Thus, easing financial constraints under these conditions tends to shift firm preferences towards greener capital, resulting in a more pronounced improvement for dirty firms than for their greener counterparts. Conversely, with a sufficiently strong ex-ante policy in place, this preference flips, with financially unconstrained firms showing a preference for dirty capital. In such a context, relaxing financial constraints instead leads to increased investment in dirty capital.

Figure A1 uses the model to trace how firms respond to an exogenous net-worth injection across three policy environments. To capture heterogeneity, we assign higher productivity and lower initial net worth to the more financially constrained firms: productivity z spans 1 to 10, while net worth w runs inversely from 2 to 0.5. We compare (i) the benchmark economy without policy, (ii) an economy with a carbon tax of $\tau^d = 0.2$, and (iii) an economy in which the dirty-capital collateral constraint is tightened to $\xi^d = 0.8$. For each case, capital prices are held fixed and every firm's net worth is increased by 0.1. The resulting change in the dirty-to-green capital ratio, k^d/k^g, is plotted in the right-hand column of Figure A1.

[Place Figure A1 about here]

In economies without policy interventions and those employing ex-post instruments, financially constrained firms reduce their dirty capital holdings more than their unconstrained counterparts. The impact is more pronounced under ex-post instruments because these policies increase the disparity in down payments required for dirty versus green capital. This increases the difference in greenness between constrained and unconstrained firms, which in turn amplify the improvement given one unit of net worth injection. Interestingly, in economies with strong ex-ante green instruments, the preference between dirty and green capital reverses. Consequently, easing a firm's financial constraints in such contexts leads to an increase in its dirty capital, contradicting existing literature findings.

Our model predictions are largely consistent with Hartzmark and Shue (2023), noting that the green instruments currently in use are predominantly ex-post. However, careful interpretation of these results is necessary. The findings presented in Hartzmark and Shue (2023) can be partially attributed to the equilibrium effects stemming from the prevalent sustainable investment strategy. This strategy, by reducing the market price of dirty capital, consequently lowers its associated down payment. This effect in turn amplifies the financial disparity between investments in dirty and green capital. As a result, the potential for environmental improvement among financially constrained firms becomes increasingly reliant on their financing conditions. In the absence of this investment strategy, the influence of capital costs on environmental improvements for these constrained firms might not be as significant. This is due to a narrower gap in the down payments required for dirty versus green capital investments, leading to a less pronounced sensitivity to their financial constraints. Furthermore, in scenarios dominated by ex-ante instruments, these findings may no longer apply.

B.2 Marginal Emission Rates and Emission Outcomes

In this section, we explore how varying parameter choices for marginal emission rates, denoted as ι, influence both the reallocation effects and the overall impact on emissions. Specifically, we examine the effects of two policy exercises—a carbon tax and a dirty capital lending ban—on total emissions under different values of ι.

Figure A2 presents the results. Panel (a) shows the change in total emissions following an increase in the carbon tax rate from $\tau^d = 0$ to 0.2. Panel (b) illustrates the change in total emissions resulting from a reduction in dirty capital availability, as indicated by a decrease in ξ^d from 1 to 0.8. Both panels plot these changes against different values of the marginal emission parameter, ι.

The relationship between the marginal emission parameter ι and the effect of a carbon tax is nuanced. The change in total emissions exhibits a hump-shaped pattern with respect to ι: initially, reallocation inefficiencies increase with ι before declining. This can be explained as follows: ex-post policies like a carbon tax tend to shift dirty capital towards financially constrained firms. When these firms have higher marginal emissions per unit of dirty capital compared to unconstrained firms (represented by a higher ι), the reallocation of dirty capital has a more significant impact on aggregate emissions. This channel is dominant when ι is relatively low, as shown in the initial rising part of the curve. However, when ι is sufficiently high, a different channel begins to dominate. Emission quantity-based policies, like a carbon tax, impose different costs per unit of dirty capital on high versus low marginal emission firms ($\tau^d z_h^\iota$ versus $\tau^d z_l^\iota$). When ι is low, the additional tax cost for financially constrained firms does not outweigh their incentive to produce, so these firms opt for cheaper dirty capital, causing a reallocation towards financially constrained firms. Conversely, when ι is high enough, the tax burden becomes so substantial that it outweighs the incentive to use dirty capital, resulting in a reallocation towards unconstrained firms.

The relationship between the marginal emission parameter ι and the effect of a dirty capital lending ban is more straightforward. The change in total emissions consistently decreases with respect to ι. This is because a dirty capital lending ban is a capital-based policy that imposes an equal dollar cost per unit of capital for all firms. Regardless of the value of the marginal emission parameter, dirty capital is reallocated towards unconstrained firms under such a policy. Consequently, a higher ι magnifies the impact of this reallocation on total emissions.

[Place Figure A2 about here]

B.3 Capital-Supply Elasticity and Emission Outcomes

In this section, we examine how changes in supply elasticity affect the impact of green instruments. Specifically, we study the change in total emissions following an increase in the carbon tax rate from $\tau^d = 0$ to 0.2 and the change in total emissions resulting from a reduction in dirty capital lending from $\tau^d = 1$ to 0.8. Both scenarios are analyzed under different levels of capital supply elasticity, ε. The results are presented in Figures A3 and A4, with Panels (a) displaying the change in total emissions and Panels (b) showing the breakdown of emission changes.

[Place Figure A3 about here]

In Figure A3, we plot the change in total emissions due to a carbon tax as a function of supply elasticity. We observe that the change in total emissions shifts from positive to negative as capital supply becomes more elastic. This occurs because, with more elastic capital supply, the capital-supply effect strengthens and dominates the allocative effect. It is noteworthy, however, that even when the supply is highly elastic ($\kappa = 5$), the allocative effect remains positive and significant. This is because, regardless of supply elasticity, ex-post instruments tend to have a stronger impact on unconstrained firms than on constrained ones. Thus, controlling for total dirty capital usage, constrained firms will always use relatively more dirty capital compared to the no-policy scenario.

[Place Figure A4 about here]

Figure A4 shows the change in total emissions resulting from a dirty capital lending ban as a function of supply elasticity. Similarly, as capital supply becomes more elastic, the reduction in emissions increases, driven by the level effect. Additionally, even when the supply is highly elastic ($\kappa = 5$), the allocative effect is negative and substantial. Therefore, compared to ex-post instruments, ex-ante instruments consistently exhibit greater allocative efficiency, regardless of supply elasticity.

C Data

C.1 Database Introduction

The data in this paper comes from three databases: the Toxic Release Inventory Database, the Pollution Prevention Database, and the Compustat Database.

Toxic Release Inventory Database (TRI) The TRI database tracks the release and management of toxic chemicals that pose certain threats to human health and the environment. Factories and facilities in various industries across the United States are required to report annually on the amount of each chemical substance that is directly released or released through recycling, energy recovery, treatment, and other means into the environment. In particular, *waste recycling (Recycling)* refers to the reuse of emissions, with its value being the sum of on-site recycling emissions (*Recycling On-site*) and off-site recycling emissions (*Recycling Off-site*). *Energy recovery* refers to the process of obtaining energy by burning waste when it is not possible to reuse the waste, with its value being the sum of on-site energy recovery emissions (*Energy Recovery On-site*) energy recovery emissions (*Energy Recovery Off-site*). *Waste treatment (Treatment)* refers to the process of treating the harmful characteristics of waste as much as possible to reduce its impact during emissions, with its value being the sum of on-site treatment emissions (*Treatment On-site*) and off-site treatment emissions (*Treatment Off-site*). Unprocessed direct discharge (*Disposal or Other Releases*) refers to the direct discharge of waste without any treatment. There are three important features for this data set. Firstly, the data in the TRI database is highly accurate and has a broad coverage. According to Section 313 of the Emergency Planning and Community Right-to-Know Act, reporting TRI-related data to the EPA is a mandatory requirement. The EPA requires that facilities meeting the following conditions must record the amount of chemicals listed in the TRI that are released through air, water, soil, and other means each year: (1) The amount of TRI-listed chemicals manufactured, processed, or otherwise used in a particular year exceeds a certain threshold level; (2) The facility has ten or more full-time employees; (3) The facility belongs to industries such as metal mining, utilities, manufacturing, publishing, and hazardous waste. All potentially eligible facilities must conduct a timely self-examination. If they meet the TRI reporting standards, they must submit TRI data to the EPA before July 1st each year. Afterward, the EPA will publish a preliminary dataset, allowing facilities to make final changes. The final version of the data will be generated in September, based on which the EPA's annual analysis report will be produced. The rules, processes, and data quality of the TRI report are closely monitored by the U.S. Environmental Protection Agency (EPA). Secondly, the TRI data provides detailed enterprise emission data. Each entry in the original TRI data corresponds to the emission amount of a specific chemical by a specific factory in a specific year, that is, the granularity is ”year-factory-chemical-emission amount”. Moreover, Section 8 of the original TRI data table provides various sub-items of the emission amount data: recycling emissions, energy recovery emissions, treatment emissions, and direct emissions. The total emission is the sum of the above four types of emissions. We in this paper retain all the indicators that can reflect the enterprise emission structure in the data. Thirdly, the TRI data takes into account the toxicity heterogeneity of chemical substances. Based on the relative toxicity calculated by the EPA's RSEI toxicity model, the TRI can provide emission data weighted by toxicity. This allows the article to consider the differences in the hazards of chemical substances and accurately reflect the differences in potential risks brought by different emissions.

Pollution Prevention Database (P2) Pollution prevention, also known as source reduction, is the most effective emission reduction method advocated by the EPA. During the sample period of this article, the EPA classifies emission reduction actions into eight major categories, which include a total of 73 subcategories. The data on a factory's pollution prevention measures are reported together with the TRI data, located in Section 8.10 of the original TRI data table. If a factory has newly implemented source reduction activities for chemicals listed in the TRI in that year and the activities have taken effect, they must be reported in Section 8.10. For each chemical substance, each factory can report one or more reduction activities and must classify the reported reduction activities. Therefore, the granularity of the original P2 database is ”factory-chemical-substance-year-reduction action”. This article can analyze the enthusiasm of companies in carrying out pollution prevention activities by counting the number of reduction actions of the company. At the same time, combined with the RSEI chemical toxicity data, this article can analyze the degree of pollution prevention weighted by toxicity.

Compustat Database This database is a standard, financial database provided by Wharton Research Data Services (WRDS). It is widely used for academic and business analysis and contains detailed financial statements, market data, fixed income information, ownership structure, and executive compensation data of listed companies worldwide, covering companies from more than 50 countries. The data is highly standardized, facilitating empirical analysis in this article. We use the public firms from US.

C.2 Merging Different Data Sets

Aggregating the data for chemical substances. The original TRI data is at the Year-Facility level for each chemical substance. To merge with US public firms' financial data, we need several steps. Firstly, we add up the various items of emission data located in Section 8 of the TRI table. It sums up the recycling emissions, energy recovery emissions, and treatment emissions to obtain the treated emissions, and then adds the treated emissions to the untreated direct emissions to get the total emissions. Secondly, based on the chemical substance toxicity data, three toxicity weight factors are calculated. The TRI emission data is merged with the RSEI model's toxicity data according to the unique chemical identifier CASRN. Consistent with expectations, all chemical substances that have appeared in the TRI database have corresponding toxicity data in the RSEI data. Subsequently, the emission data adjusted for toxicity is calculated. These are saved as ”Year-Facility-Chemical Substance”. Thirdly, the data of ”Year-Facility-Chemical Substance” is then aggregated to the ”Year-Parent Company” level and merged with the P2 database, which has already been aggregated to the ”Year-Parent Company” level. Many enterprises have not carried out source reduction actions and thus lack data in the P2 database, resulting in no matching values. For these enterprises, the number of source reduction actions will be assigned a zero value. Fourthly, the matching table established by **?** links the names of parent companies with their stock codes (permno). Through this matching table, the TRI data is matched with the stock codes of the parent companies. Fifthly, there are situations where the same stock code (permno) corresponds to multiple parent company names. Therefore, it is necessary to further sum up the emissions of all parent companies to the stock code level. Ultimately, we obtained data at the ”Year-Stock Code” level. In the end, this article obtained an unbalanced panel data set with a time span from 1991 to 2017, including 1869 stock codes.

Merging emission data with financial data. We then combine the emission reduction database processed previously with the Compustat database. The enterprise identifier in the emission reduction database is in the form of stock codes (permno), but the enterprise identifier in the Compustat database is in the form of Global Company Keys (gvkey). Based on the matching table by **?**, we first match the stock codes (permno) of the enterprises existing in the emission reduction database with the Global Company Keys (gvkey), and then matches the emission reduction database with the Compustat database according to the Global Company Keys (gvkey). The matching table by **?** includes the following three situations: First, a one-to-one correspondence between permno and gvkey; second, multiple gvkeys share one permno; third, multiple permnos share one gvkey. The first situation is the ideal case. After excluding observations where both gvkey and permno are repeated at the same time, there are 16 sets in the second situation, and 74 sets in the third situation. We manually examine these duplicate value situations one by one and categorizes them into the following two types: Type A, the company changed its permno (or gvkey) in a certain year during the sample period, and then, while adopting the new permno (or gvkey), keeps the gvkey (or permno) unchanged. This type of sample is retained because it can still be successfully matched one-to-one according to the year when matched with the panel data. Type B, the company underwent mergers, reorganizations, and other events with other companies during the sample period, leading to the occurrence of the second or third situation. The matching situation of this type of sample lacks a consistent pattern, and the sample size is small, so this type of sample is deleted. After matching the matching table with the emission reduction database, we finally obtained 1804 unique gvkeys. Subsequently, we match the emission reduction database with Compustat according to gvkey and year, and the final dataset includes 1592 enterprises.

D Dynamic Model: Firm Optimization and Computation

We first introduce some shorthands that are convenient for the characterization and computation. For the capital composite function $g(k_t^d, k_t^g)$, we assume it is CES:

$$g(k^d, k^g) = \left[\gamma \left(k^d\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(k^g\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

and the marginal products of capital are given as follows:

$$\begin{aligned} g_1 &= \gamma \left(k^d\right)^{\frac{\sigma-1}{\sigma}-1}\left[\gamma \left(k^d\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(k^g\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1}, \\ g_2 &= (1-\gamma)\left(k^g\right)^{\frac{\sigma-1}{\sigma}-1}\left[\gamma \left(k^d\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(k^g\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1}, \end{aligned}$$

or, equivalently, using the ratio of capital stock, $\frac{k^d}{k^g}$, the marginal products are as follows,

$$\begin{aligned} g_1 &= \gamma \left(\frac{k^d}{k^g}\right)^{\frac{\sigma-1}{\sigma}-1}\left[\gamma \left(\frac{k^d}{k^g}\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\right]^{\frac{\sigma}{\sigma-1}-1}, \\ g_2 &= (1-\gamma)\left(\frac{k^g}{k^d}\right)^{\frac{\sigma-1}{\sigma}-1}\left[\gamma + (1-\gamma)\left(\frac{k^g}{k^d}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1}. \end{aligned}$$

and, since g_1 and g_2 only depend on the ratio of capital stock, these expressions are useful later on in simplifying and solving the firm's optimization problem.

For the firm's optimization problem, similarly as the two-period model, we denote the following multipliers: μ_t for the time t budget constraint, λ_t for the time t debt collateral constraint, η_t for non-negative dividend constraint, ν_t^d for $k_t^d \geq 0$, and lastly, ν_t^g for $k_t^g \geq 0$. The details are as follows:

$$V(z_t, w_t; S_t) = \max_{\{d_t, b_{t+1}, k_t^d, k_t^g\}} d_t + \beta(1-\rho)E_t V(z_{t+1}, w_{t+1}; S_{t+1}) + \beta\rho E_t w_{t+1} + \beta E_t \Gamma_{t+1}$$

subject to the following constraints:

$$
\begin{aligned}
\mu_t: \quad & 0 = -d_t + w_t + b_{t+1} - q_t^d k_t^d - q_t^g k_t^g, \\
& w_{t+1} = z_t g(k_t^d, k_t^g) - \left(c^d + \tau_{t+1}^d e(z_t)\right) k_t^d - R b_{t+1} - \Gamma_{t+1}\left(k_t^d, k_t^g\right), \\
\lambda_t: \quad & \xi_{t+1}^d \theta q_{t+1}^d k_t^d + \xi_{t+1}^g \theta q_{t+1}^g k_t^g - b_{t+1} \geq 0, \\
\eta_t: \quad & d_t \geq -\Phi, \\
\nu_t^d: \quad & k_t^d \geq 0, \\
\nu_t^g: \quad & k_t^g \geq 0.
\end{aligned}
$$

Focs for d_t and b_{t+1} give us:

$$
\begin{aligned}
d_t: \quad & \mu_t = 1 + \eta_t, \eta_t \geq 0, \eta_t(d_t + \Phi) = 0, \\
b_{t+1}: \quad & \mu_t = \lambda_t + E_t \widetilde{\eta}_{t+1}, \lambda_t \geq 0,
\end{aligned}
$$

where we denote a shorthand, with $\widetilde{\eta}_{t+1} = 1 + (1-\rho)\eta_{t+1} \geq 1$, and $\widetilde{\eta}_{t+1}$ is the shadow value of firm cash flow in the next period, adjusted by firm exit probability. $\widetilde{\eta}_{t+1}$ will be used frequently in weighting capital returns of the next period in the following analysis.

For capital choices of k_t^d and k_t^g, the F.o.c.s are:

$$
\begin{aligned}
k_t^d: \quad & \left(1 + \eta_t\right) q_t^d - \lambda_t \xi_{t+1}^d \theta q_{t+1}^d \\
= \quad & \nu_t^d + \beta g_1 z_t E_t \widetilde{\eta}_{t+1} + \beta E_t \widetilde{\eta}_{t+1}\left[-\left(c^d + \tau_{t+1}^d e(z_t)\right)\right] - \beta E_t (1-\rho)\eta_{t+1} \tfrac{\partial \Gamma_{t+1}}{\partial k_t^d}, \\
k_t^g: \quad & \left(1 + \eta_t\right) q_t^g - \lambda_t \xi_{t+1}^g \theta q_{t+1}^g = \nu_t^g + \beta g_2 z_t E_t \widetilde{\eta}_{t+1} - \beta E_t (1-\rho)\eta_{t+1} \tfrac{\partial \Gamma_{t+1}}{\partial k_t^g}.
\end{aligned}
$$

Similarly as the two-period model, we denote the down payment costs as:

$$
\begin{aligned}
\Phi_t^d & \equiv \left[q_t^d - \xi_{t+1}^d \theta q_{t+1}^d\right], \\
\Phi_t^g & \equiv \left[q_t^g - \xi_{t+1}^g \theta q_{t+1}^g\right].
\end{aligned}
$$

In turn, we can re-write the Focs for the firm's capital choices as follows, so that the LHS denotes the summation of all marginal costs, and the RHS denotes the total marginal benefits:

$$
\begin{aligned}
&q_t^d E_t\widetilde{\eta}_{t+1} + \lambda_t \Phi_t^d + \beta E_t\left[\widetilde{\eta}_{t+1}\left(c^d + \tau_{t+1}^d e(z_t)\right)\right] + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial \Gamma_{t+1}}{\partial k_t^d} = \nu_t^d + \beta g_1 z_t E_t\widetilde{\eta}_{t+1}, \\
&q_t^g E_t\widetilde{\eta}_{t+1} + \lambda_t \Phi_t^g + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial \Gamma_{t+1}}{\partial k_t^g} = \nu_t^g + \beta g_2 z_t E_t\widetilde{\eta}_{t+1}.
\end{aligned}
$$

For the solution, we can discuss

For firm multiplier λ_t, there are two possible cases:

- if $\lambda_t > 0$, then we have $\mu_t = \lambda_t + E_t\widetilde{\eta}_{t+1}$>1, and this implies that $\eta_t > 0$. So in this case, both equations for dividend constraint and also the constraint for credit bind. We have the following system of equations:

  $$
  \begin{aligned}
  &w_t + b_{t+1} - q_t^d k_t^d - q_t^g k_t^g + \Phi = 0, \\
  &b_{t+1} = \xi_{t+1}^d \theta q_{t+1}^d k_t^d + \xi_{t+1}^g \theta q_{t+1}^g k_t^g,
  \end{aligned}
  $$

  and also, given the fact that the firm has positive capital investment, we have

  $$
  \begin{aligned}
  \nu_t^d &= 0, \\
  \nu_t^g &= 0.
  \end{aligned}
  $$

  The two Focs also imply that for capital ratio of $\frac{k^d}{k^g}$, we can write it as a function of unknown multipliers:

  $$
  \frac{\gamma}{1-\gamma}\left(\frac{k^d}{k^g}\right)^{\frac{-1}{\sigma}} = \frac{q_t^d E_t\widetilde{\eta}_{t+1} + \lambda_t \Phi_t^d + \beta E_t\left[\widetilde{\eta}_{t+1}\left(c^d + \tau_{t+1}^d e(z_t)\right)\right] + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial \Gamma_{t+1}}{\partial k_t^d}}{q_t^g E_t\widetilde{\eta}_{t+1} + \lambda_t \Phi_t^g + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial \Gamma_{t+1}}{\partial k_t^g}}.
  $$

  If there is a solution in this case, then we need to find a solution for these joint equations, and the consistency conditions should also be satisfied : $\lambda_t > 0$,$\eta_t > 0, k^d > 0, k^g > 0$.

- if $\lambda_t = 0$, and the constraint for credit b_{t+1} is not constrained, then there are two possible cases that we need to further discuss:
- (1) if the firm chooses to invest with strictly positive capital: $k^d > 0, k^g > 0$. That is, we have,

  $$b_{t+1} \quad < \quad \xi^d_{t+1}\theta q^d_{t+1}k^d_t + \xi^g_{t+1}\theta q^g_{t+1}k^g_t.$$

  From the two Focs we see that

  $$\begin{aligned}
  &q^d_t E_t\widetilde{\eta}_{t+1} + \lambda_t\Phi^d_t + \beta E_t\left[\widetilde{\eta}_{t+1}\left(c^d + \tau^d_{t+1}e(z_t)\right)\right] + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial\Gamma_{t+1}}{\partial k^d_t} \leq \beta g_1 z_t E_t\widetilde{\eta}_{t+1},\\
  &q^g_t E_t\widetilde{\eta}_{t+1} + \lambda_t\Phi^g_t + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial\Gamma_{t+1}}{\partial k^g_t} \leq \beta g_2 z_t E_t\widetilde{\eta}_{t+1},
  \end{aligned}$$

  In this case, the firm can always increase k^d_t and k^g_t by a sufficiently small amount, $\epsilon \geq 0$, and increase its b_{t+1} by $q^d_t\epsilon + q^g_t\epsilon$, and keep d_t not changed. Then, this is a feasible plan, and since the firm has CRS, its marginal product w.r.t. $g(k^d_t, k^g_t)$ does not change, the firm will never be worse off and will be strictly better off if any of the two Focs has a strict inequality; thus, we can always let the firm borrow to the limit, i.e., b_{t+1} is constrained and we have $\lambda_t > 0$. This implies that this hypothetic case (with $\lambda_t = 0$, $k^d > 0, k^g > 0$) cannot be optimal.
- (2) if the firm chooses to not invest in capital at all, $k^d = 0, k^g = 0$, then the firm only needs to decide on d_t and b_{t+1}. Since we have $\beta R = 1$, and the firm is possibly constrained in the next period, the firm should always delay dividend payout (unless forced to do so) and save for the next period as much as possible. Therefore, we have: $d_t + \Phi = 0$ and $b_{t+1} = -w_t$. This case can be easily checked by inspecting the following conditions that should be simultaneously satisfied:

  $$\begin{aligned}
  &q^g_t E_t\widetilde{\eta}_{t+1} + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial\Gamma_{t+1}}{\partial k^g_t} < \beta g_2 z_t E_t\widetilde{\eta}_{t+1},\\
  &q^d_t E_t\widetilde{\eta}_{t+1} + \beta E_t\left[\widetilde{\eta}_{t+1}\left(c^d + \tau^d_{t+1}e(z_{t+1})\right)\right] + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial\Gamma_{t+1}}{\partial k^d_t} < \beta g_1 z_t E_t\widetilde{\eta}_{t+1}.
  \end{aligned}$$

Based on the previous discussions, given that z_t is an AR(1) process, and that $\Gamma_{t+1}\left(k_t^d, k_t^g\right)$ is linear in k_t^d, k_t^g, we can prove that in general, the multiplier η_t only depends on z_t, the multiplier η_t it will not depend on the firm's current net worth w_t (we can show this by backward induction; suppose at $t+1$ η_{t+1} only depend on $z_{t+1}$and do not depend on firm net worth w_{t+1}. Then we can see the following facts: for the cutoff in z_t (if there exists a cutoff), then it is indepent of current net worth, and it is only affected by current z_t and aggregate variables; if the firm chooses strictly positive invesment, then from the two Focs we can see that this period's endogenous solution on λ_t will be only affected by current z_t and aggregate variables; in sum, the multiplier η_t only depends on z_t at time t) . This result is due to the fact that the firm's problem essentially is a linear problem (in a dynamic setting).

In sum, the solution for the firm's multiplier $\eta_t(z)$, as a function of firm state z, can be characterized by a system of nonlinear equations. For notional conveinece, let us consiser the case in a stationary economy (for transitional dynamics with aggregate state variable changes, the analysis would be very similar but the solution for the multiplier function $\eta_t(z)$ in general will depend on time and aggregate state variables). In the stationary economy, there exists a threshold z^* , and we have

$$\eta(z) = 0, \text{if } z \leq z^*$$

and if we have $z_t > z^*$, then we the following Focs for η_t:

$$\begin{aligned}
&q_t^d E_t \widetilde{\eta}_{t+1} + \lambda_t \Phi_t^d + \beta E_t\left[\widetilde{\eta}_{t+1}\left(c^d + \tau_{t+1}^d e(z_t)\right)\right] + \beta E_t (1-\rho)\eta_{t+1}\frac{\partial \Gamma_{t+1}}{\partial k_t^d} = \beta g_1 z_t E_t \widetilde{\eta}_{t+1}, \\
&q_t^g E_t \widetilde{\eta}_{t+1} + \lambda_t \Phi_t^g + \beta E_t (1-\rho)\eta_{t+1}\frac{\partial \Gamma_{t+1}}{\partial k_t^g} = \beta g_2 z_t E_t \widetilde{\eta}_{t+1}, \\
&1 + \eta_t = \lambda_t + \beta R E_t \widetilde{\eta}_{t+1}
\end{aligned}$$

Numerical Computation: In numerical exercise, we can proceed as follows:

- we can discretize the space of z using $\{z_i\}_{i=1,\ldots,N}$, and then in the stationary economy, for given aggregate state variables q^d, q^g, Φ^d, Φ^g, the multiplier functions $\eta(z_i)$ and $\lambda(z_i)$, and also $\widetilde{\eta}(z_i) \equiv 1 + (1-\rho)\eta(z_i)$ can be computed as follows (we can ignore time index t and $t+1$ for the aggregate variables) :

$$\eta(z_i) = 0,\text{if } z_i \leq z^*$$

and if we have $z_i > z^*$, for any $i = 1, ..., N$, then we the following Focs for $\eta(z_i)$ and $\lambda(z_i)$:

$$q^d E_{z_i}\widetilde{\eta}(z_{t+1}) + \lambda(z_i)\mathit{\Phi}^d + \beta E_t\left[\widetilde{\eta}(z_{t+1})\left(c^d + \tau^d e(z_t)\right)\right] + \beta E_{z_i}(1-\rho)\widetilde{\eta}(z_{t+1})\frac{\partial\Gamma_{t+1}}{\partial k_t^d} = \beta g_1(k_t^d, k_t^g)z_t \times E$$

$$q^g E_{z_i}\widetilde{\eta}(z_{t+1}) + \lambda(z_i)\mathit{\Phi}^g + \beta E_{z_i}(1-\rho)\widetilde{\eta}(z_{t+1})\frac{\partial\Gamma_{t+1}}{\partial k_t^g} = \beta g_2(k_t^d, k_t^g)z_t \times E_{z_i}\widetilde{\eta}(z_{t+1}),$$

$$1 + \eta(z_i) = \lambda(z_i) + \beta R E_{z_i}\widetilde{\eta}(z_{t+1}),$$

$$\lambda(z_i) \geq 0,$$

$$\eta(z_i) \geq 0.$$

We can iteration methods to find the solution for the multiplier functions $\eta(z_i)$ and $\lambda(z_i)$ (as we know the firm's optimization problem is well defined and it has a unique solution).

- Once we find the solution for the multipliers, we can then find the solutions for all other endogenous choice variavbles: (1) for states with $z_i \leq z^*$, we have $d_t + \mathit{\Phi} = 0$ and $b_{t+1} = -w_t$. (2) for states with $\lambda(z_i) \geq 0$, we can solve for k_t^d, k_t^g from the following equations:

$$w_t + b_{t+1} - q_t^d k_t^d - q_t^g k_t^g + \mathit{\Phi} = 0,$$

$$b_{t+1} = \xi_{t+1}^d \theta q_{t+1}^d k_t^d + \xi_{t+1}^g \theta q_{t+1}^g k_t^g,$$

$$\frac{\gamma}{1-\gamma}\left(\frac{k^d}{k^g}\right)^{\frac{-1}{\sigma}} = \frac{q_t^d E_t\widetilde{\eta}_{t+1} + \lambda_t \mathit{\Phi}_t^d + \beta E_t\left[\widetilde{\eta}_{t+1}\left(c^d + \tau_{t+1}^d e(z_t)\right)\right] + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial\Gamma_{t+1}}{\partial k_t^d}}{q_t^g E_t\widetilde{\eta}_{t+1} + \lambda_t \mathit{\Phi}_t^g + \beta E_t(1-\rho)\eta_{t+1}\frac{\partial\Gamma_{t+1}}{\partial k_t^g}}.$$

- For firm value function $V(z_t, w_t; S_t)$, since we already find solutions for $d_t, b_{t+1}, k_t^d, k_t^g$, we can then use value function iterations but without solving maximization problem to find the converge of V:

$$V(z_t, w_t; S_t) = d_t + \beta(1-\rho)E_t V(z_{t+1}, w_{t+1}; S_{t+1}) + \beta\rho E_t w_{t+1} + \beta E_t \Gamma_{t+1}.$$

E Parameter Sensitivity Analysis

To evaluate the robustness of our main calibration, we conduct a sensitivity analysis focusing on two key parameters: the elasticity of emissions to productivity (ι) and the unit cost of dirty capital (c_d). These parameters shape the interaction between firm-level financial constraints and emissions outcomes in the model.

Figure A6 displays model-implied emissions profiles under different parameter configurations. Firms are sorted into quintiles based on the Whited and Wu (2006) (WW) index. Panel A shows normalized emission intensity (emissions per unit of output), while Panel B presents normalized emissions per unit of capital (emissions divided by PP&E). All values are scaled by their respective sample means in both the model and the data.

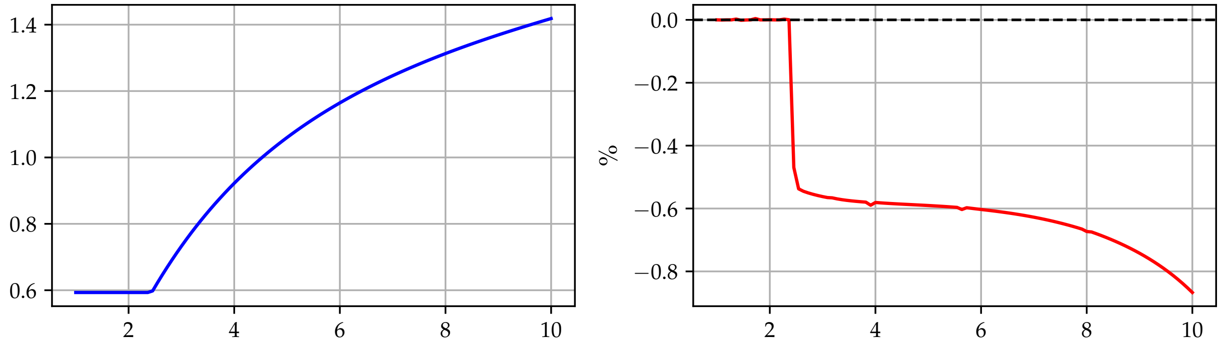
We examine four alternative parameter combinations by varying $\iota \in \{0.6, 1.0\}$ and $c_d \in \{0.0, 0.1\}$. The benchmark case, with $\iota = 0.85$ and $c_d = 0.05$, is included for reference. As expected, increasing ι amplifies the curvature in both emissions measures, as emissions respond more strongly to productivity differences across firms. Likewise, raising c_d lowers the equilibrium price of dirty capital, making it more accessible to financially constrained firms and thereby increasing their emissions.

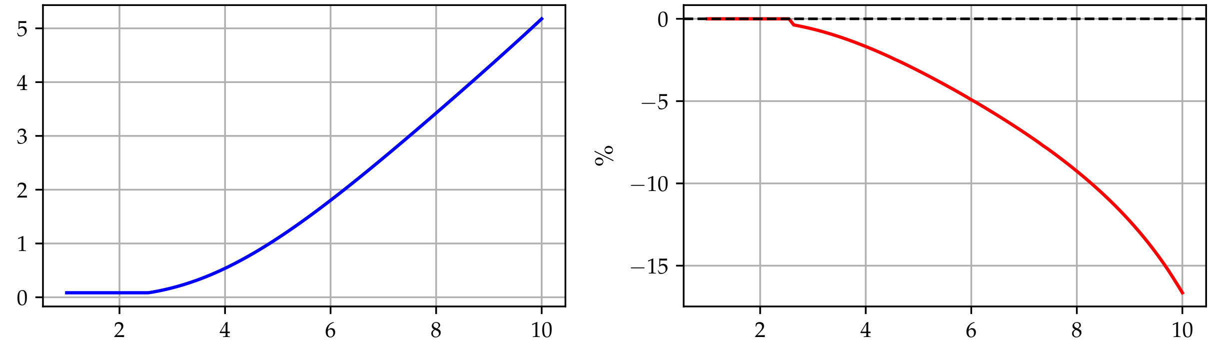
Among the cases considered, the benchmark parameter combination ($\iota = 0.85$, $c_d = 0.05$) produces the closest fit to the empirical curvature observed in the data. This suggests that the baseline calibration closely replicates the cross-sectional variation in emissions patterns associated with financial constraints.

Appendix Tables and Figures

Figure A1: **Impact Elasticity**

This figure illustrates the impact elasticity, defined as the environmental improvement resulting from a one-unit injection of net worth, across various firms in three distinct economies. We explore three scenarios: the benchmark equilibrium without any policy interventions, the equilibrium with carbon tax ($\tau^d = 0.2$), and the equilibrium with dirty capital lending ban ($\xi^d = 0.8$). The left column shows the dirty to green capital ratio for firms within each economy. For each scenario, capital prices are held constant at their original equilibrium values, and a 0.1 increase in firm net worth is introduced. The resulting changes in the ratio of dirty to green capital (k^d/k^g) are depicted in the right column. Detailed parameters for the underlying model are provided in Table 3.


Panel (a) No Policy Benchmark: k^d/k^g Panel (b) No Policy Benchmark: k^d/k^g Changes


Panel (c) Carbon Tax: k^d/k^g Panel (d) Carbon Tax: k^d/k^g Changes

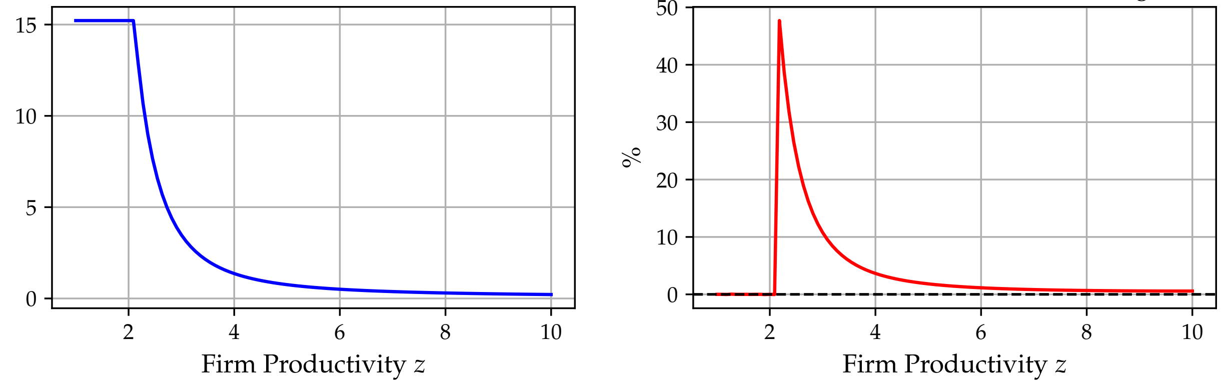

Panel (e) Green Credit: k^d/k^g Panel (f) Green Credit: k^d/k^g Changes

Figure A2: **Effects of Green Instruments Across Marginal Emission Levels**

This figure illustrates the aggregate impact of a carbon tax and a ban on dirty capital lending under different marginal emission parameters, ι. Panel (a) shows the change in total emissions following a carbon tax increase from $\tau^d = 0$ to 0.2. Panel (b) displays the change in total emissions resulting from a coal lending ban, with ξ^d decreasing from 1 to 0.8. Each variable is plotted against the marginal emission parameter, ι. Detailed parameters for the underlying model are provided in Table 3.

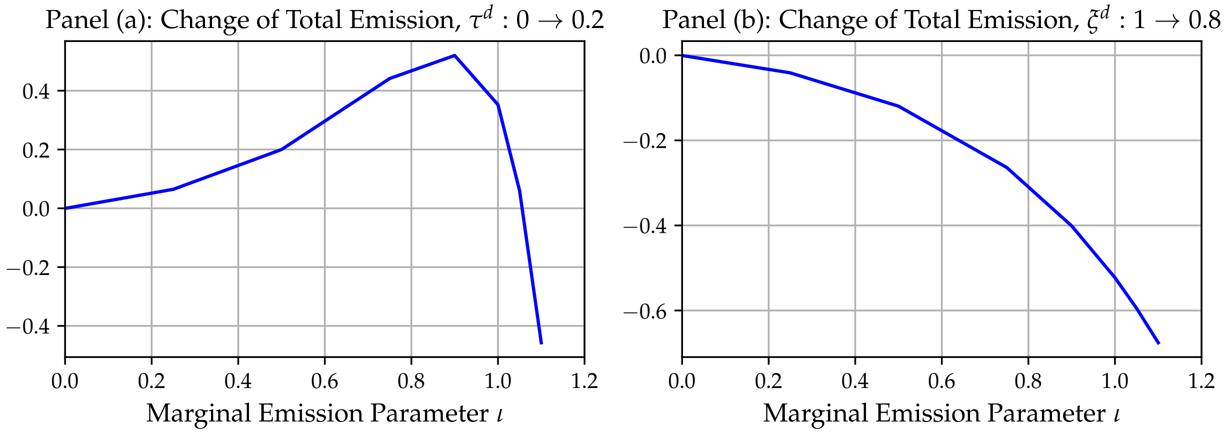


Figure A3: **Aggregate Effect, Ex-post Instrument**

This figure illustrates the aggregate impact of a carbon tax increase from $\tau^d = 0$ to 0.2 under varying capital supply elasticity (ε). Panel (a) depicts the change in total emissions, while Panel (b) decomposes this change relative to the benchmark into two components: the allocative effect and the level effect. Each variable is plotted against capital supply elasticity, ε. Detailed parameters for the underlying model are provided in Table 3.

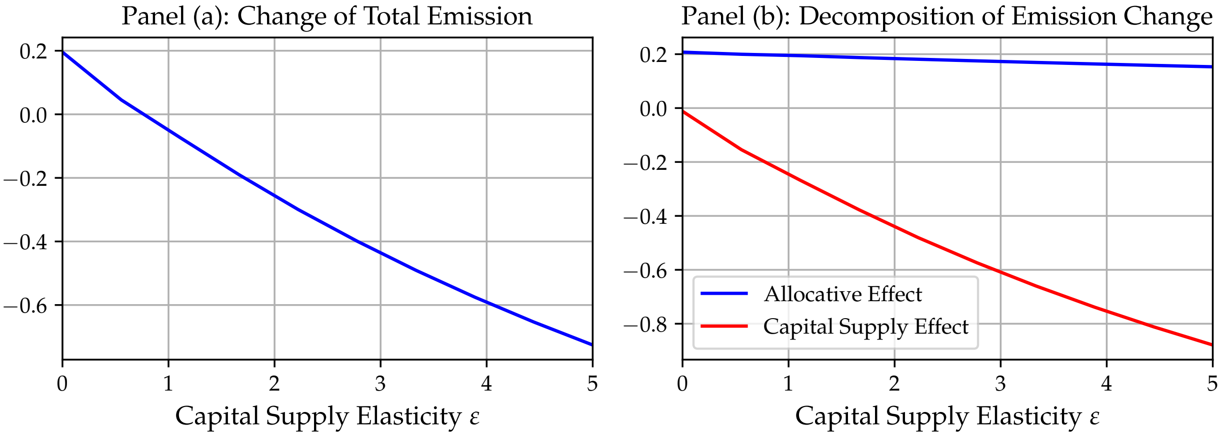


Figure A4: **Aggregate Effect, Ex-ante Instrument**

This figure illustrates the aggregate impact of a coal lending ban, with ξ^d decreasing from 1 to 0.8, under varying capital supply elasticity (ε). Panel (a) depicts the change in total emissions, while Panel (b) decomposes this change relative to the benchmark into two components: the allocative effect and the level effect. Each variable is plotted against capital supply elasticity, ε. Detailed parameters for the underlying model are provided in Table 3.

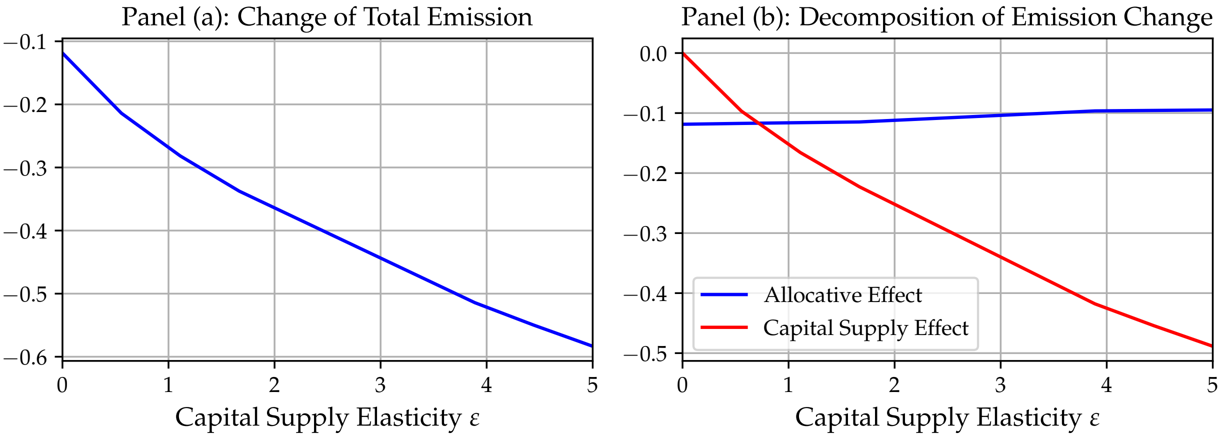


Figure A5: **Emissions Over Time in the US**

This figure plots the emission intensity over time for US public firms. Emission intensity is defined as the total measured emission (computed from EPA micro data sets) scaled by the total firm sales in each given year. For more details of data sets and variable constructions, please see Appendix Section C and also the main text.

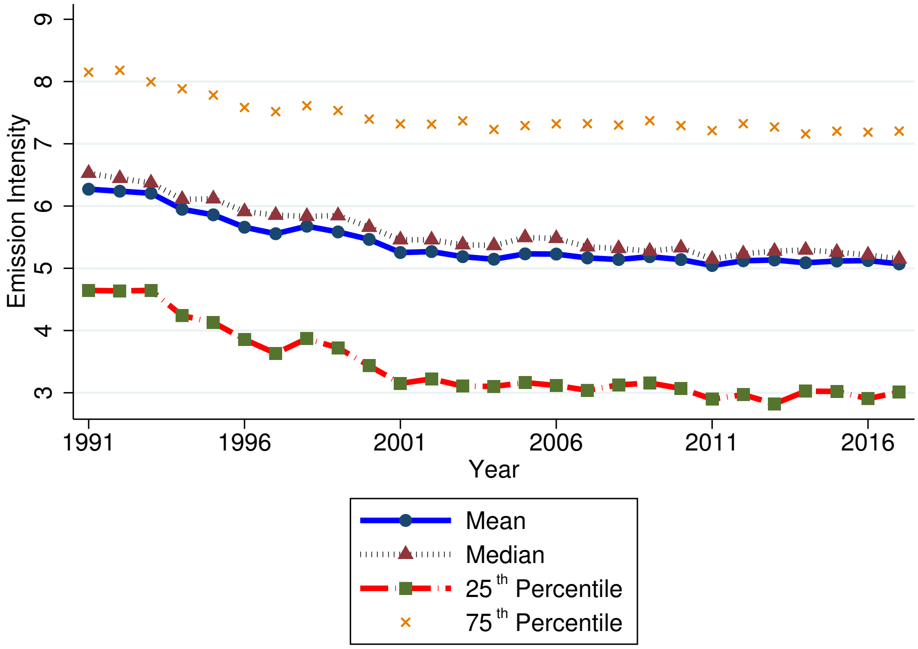
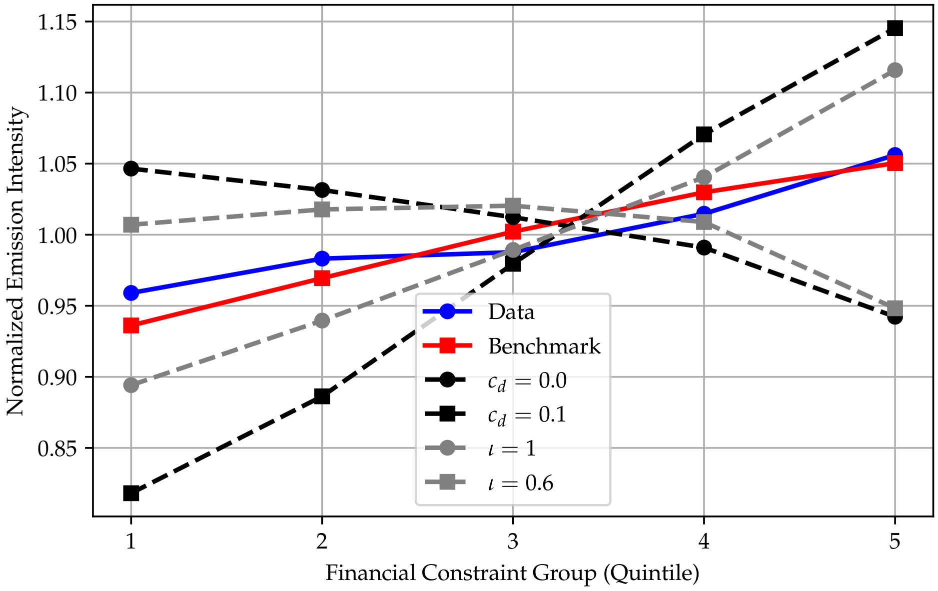


Figure A6: **Emission Intensity and Emissions per Capital: Parameter Sensitivity**

This figure shows how model-implied emissions vary across financially constrained firms under different parameter choices. Firms are sorted by the Whited and Wu (2006) (WW) index into five quintiles (1–5), from the least to the most financially constrained. Panel A displays normalized emission intensity (emissions per output), and Panel B shows normalized emissions per capital (emissions divided by PPENT). Both metrics are normalized by the overall sample mean. Solid lines represent empirical data; dashed lines represent model outcomes under varying parameters ($c_d = 0.0$ or 0.1; $\iota = 0.6$ or 1). The benchmark model corresponds to the case with $c_d = 0.05$ and $\iota = 0.85$. Other parameter values are defined in Table 4.

Panel (a): Emission Intensity: Data vs. Model Cases



Panel (b): Emission per Capital: Data vs. Model Cases