Cows and Trees*

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December 2, 2025

Abstract

The Brazilian Amazon plays a crucial role in regulating global climate and preserving biodiversity, yet it faces mounting pressures from deforestation, driven primarily by cattle ranching. The expansion of pastureland is shaped by cattle's dual role as both output and capital stock, leading to nontrivial dynamic patterns. We develop a structural empirical model of ranchers' cattle management and land use decisions that accounts for deforestation costs, herd dynamics, and price expectations. The model estimates reveal that deforestation is inelastic to temporary shocks to beef prices but highly elastic to persistent price changes, rationalizing existing estimates in the literature. Finally, we simulate various policies and discuss the implications of highly price-elastic deforestation.

Keywords: Cattle, Dynamics, Deforestation, Amazon, Carbon Emissions

JEL Codes: C13, Q12, Q15, Q18, R14

*We would like to thank Victor Aguirregabiria, Phil Haile, Ariel Pakes, Lisa Tarquinio, and various conference and seminar participants for helpful comments and discussions. We also thank Richard Chen, Antonio Cozzolino, and Marguerite Obolensky for outstanding research assistance. Financial support from the NYU Stern Center for Sustainable Business and University of Toronto Mississauga are gratefully acknowledged. All remaining errors are our own. The views expressed in this article are those of the authors. They do not necessarily represent those of Amazon nor the Federal Trade Commission or any of its Commissioners.

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1 Introduction

Deforestation is responsible for over 10% of global greenhouse gas emissions (Sims, Gibbs and Harris, 2025). Numerous policies have been proposed to curb deforestation, including encouraging more efficient cattle management, providing incentives for forest protection, and beef taxes. Evaluating the effectiveness of these policies requires understanding how deforestation responds to prices—for example, highly elastic deforestation would lead to high rates of indirect land use change (Searchinger et al., 2008; Roberts and Schlenker, 2013) and undermine the effectiveness of payments for forest protection (Gan and McCarl, 2007). However, the evidence on deforestation-price elasticities is seemingly contradictory: intertemporal analysis suggests that deforestation is highly price inelastic (Hargrave and Kis-Katos, 2013; Harding et al., 2021) while cross-sectional analysis indicates that it is highly price elastic (Souza-Rodrigues, 2019).

In the context of deforestation in the Brazilian Amazon, we can reconcile these seemingly contradictory findings by modeling of the root cause of deforestation: cattle ranching.[1] Cattle serve a dual role in ranchers' production decisions, as both consumption goods and a capital stock (Jarvis, 1974; Rosen, 1987). In this setting, a transitory price increase leads ranchers to cull more of their herd to take advantage of the high price. On the other hand, a persistent price increase encourages ranchers to cull fewer (female) cattle, increasing their herd size and reducing beef output in the short term in order to earn higher profits in the future (Rosen, Murphy and Scheinkman, 1994).

Deforestation dynamics largely track these cattle dynamics. As herd size grows, ranchers face increasing costs to manage their herd on a fixed amount of land, and deforestation occurs to create more pasture. Given that a transitory price increase leads to a smaller herd size, it reduces deforestation pressure. Given that a persistent price increase triggers herd growth, it increases deforestation pressure. Consequently, elevated prices can be associated with increased or decreased deforestation rates, depending on the nature of price variation.

We estimate our model using panel data on deforestation and cattle herds for the Brazilian Amazon from 2000-2020. Our estimation procedure accounts for ranchers' dynamic decision making; we use the Euler equations from the model as estimating equations. We also incorporate pasture land prices for estimation, relying on the assumption that land prices equal ranchers' marginal value of land on average.

While there are many ways to define a long-run elasticity in the context of a dynamic model, our estimates indicate that deforestation is highly price elastic in the long run. For example, we find that deforestation policy functions have an elasticity with respect to the mean beef price of around 3. Similarly, in equilibrium simulations, a policy that (persistently) decreases mean beef price received by ranchers by 17% leads to more than 77% reduction in deforestation rates, indicating an elasticity over 4. However, consistent with Rosen, Murphy and Scheinkman (1994), we find that that there can be a very weak partial correlation between deforestation and beef prices—that is, deforestation can appear to be very price inelastic when these correlations are mapped to elasticities. We propose that this explains why

[1] Almeida et al. (2016) indicate that pasture land conservatively accounts for 62% of deforested land in Brazilian Amazonia, or almost 90% of deforested land with an identified human use.

regression analysis on actual data suggest that deforestation is price inelastic (Hargrave and Kis-Katos, 2013; Harding et al., 2021), while the cross-sectional of Souza-Rodrigues (2019) points to highly price-elastic deforestation.

We perform policy simulations to explore the implications of this high long run elasticity for Pigouvian deforestation taxes (or other policy initiatives) and policy design more broadly. A deforestation tax set equal to some estimates of the social costs of deforestation would be sufficient to stop deforestation entirely. (In these estimates, the social value of carbon in a hectare of Amazonian rainforest is much larger than the value of a hectare of pastureland). Furthermore, these taxes will lead to higher beef prices for producers due to the increased scarcity of pastureland. Many have discussed the political challenges of implementing policies that impose costs on voters (and farmers in particular), but our results suggest that the incumbent ranchers may potentially benefit from policies that make deforestation more costly. While higher prices and profit per hectare would benefit incumbent ranchers, the reduced area of pasture land would eventually lead to lower total ranching income. We estimate an estimated loss of ranching income to Brazil of $1331.6 M/yr. Given that forest preservation programs like the REDD+ already receive hundreds of millions of dollars in funding per year, it may be feasible for other countries to compensate Brazil for this loss of income.

More broadly, the finding that deforestation is highly price elastic in the long run has important implications for policy design. For example, policies such as those supporting biofuels tend to increase agricultural commodity prices, potentially inducing agricultural expansion around the world. Additionally, policies that aim to protect forests by paying landowners for conservation can be undermined by equilibrium effects; reducing the amount of land available for agricultural production (or potential production) can lead to higher commodity prices, inducing other land to be converted into agricultural production. These "leakage" effects are more pronounced when land use is highly elastic. Back-of-the-envelope calculations using our estimates suggest that a policy that decreases deforestation locally, may have only 20% of its impact globally due to this "leakage".

Related Literature. Early studies, such as Jarvis (1974) and Rosen (1987), examined the dual role of cattle as both consumption and capital goods, highlighting their influence on supply responses and production dynamics. Subsequent work by Favaro (1990), Rosen, Murphy and Scheinkman (1994), Mundlak and Huang (1996), and Aadland (2004) analyzed the time-series properties of cattle production across countries with varying technologies, including Argentina, the United States, and Uruguay, to explore the role of cattle management in shaping industry dynamics. Our study builds on this literature by integrating land-use decisions within the critical context of Amazonian deforestation, offering new insights into the interaction between cattle cycles and environmental impacts.

An emerging literature has been emphasizing the importance of dynamic considerations in understanding land-use changes. This literature builds upon the contributions in industrial organization, including but not limited to Hendel and Nevo (2006) and Kalouptsidi (2014). Scott (2013) demonstrates the importance of incorporating dynamic models to accurately estimate land-use elasticities, focusing on crop decisions in the United States. Sant'Anna (2024) examines the expansion of sugarcane cultivation in Brazil in response to biofuel policies. Hsiao (2024) explores the role of policy coordination and commitment in shaping the palm oil industry in Indonesia and Malaysia. Building on the dynamic frameworks of Scott (2013) and Kalouptsidi, Scott and Souza-Rodrigues (2021), Araujo, Costa and Sant'Anna (2024) model land-use choices among crops, pasture, and forest using a logit framework, where the crops/pasture margin helps them to estimate deforestation elasticities and the carbon-efficient forest cover in the Brazilian Amazon. Similarly, Assunção et al. (2023) address Amazonian deforestation, with a particular focus on the uncertainty of location-specific productivities when assessing the impacts of carbon pricing. Our study complements these contributions by introducing cattle management dynamics, a crucial yet underexplored margin for understanding deforestation patterns in the Amazon.

A third strand of literature has concentrated on deforestation and its interaction with commodity prices based on static models. Souza-Rodrigues (2019) uses cross-sectional variation to examine the demand for deforestation in private properties and the long-run implications of permanent conservation policies. Static models that incorporate general equilibrium effects include Pellegrina (2022), Dominguez-Iino (2024), and Barrozo (2024).[2] These models emphasize the spatial distribution of land use and the associated equilibrium effects in a tractable way but abstract from forward-looking behavior, which is the central focus of our paper.

Several recent studies have investigated the impact of monitoring and the role of institutions, including Gandour, Souza-Rodrigues and Assunção (2019), Burgess, Costa and Olken (2019), Harding et al. (2021), and Assunção et al. (2022). Payments for ecological services programs have also been analyzed in various contexts by Alix-Garcia, Sims and Yañez Pagans (2015), Jayachandran et al. (2017), Jack and Jayachandran (2018), and Simonet et al. (2019). Similar to the static models discussed earlier, these studies abstract from the role of forward-looking agents in shaping deforestation decisions.

A common feature of the literature on Amazonian deforestation, including Hargrave and Kis-Katos (2013), Assunção and Rocha (2019), and Assunção et al. (2022), is that deforestation regressions often yield coefficients on beef prices that are small (inelastic), statistically insignificant, and sometimes negative. Ultimately, we will argue that such regression do not capture the long-run price responsiveness of deforestation.

2 Institutional Background

The Amazon Rainforest is an immense region nearly ten times the size of California, two-thirds of which is in the Brazilian territory. Before the 1960s, the Brazilian Amazon was sparsely populated, characterized by open access and subsistence-based local economies centered on rubber and Brazil nut extraction (see,

[2]Dominguez-Iino (2024) and Barrozo (2024) examine the market power of agricultural supply chains and its impact on deforestation. Farrokhi et al. (2024) extends trade general equilibrium models to include dynamic land use change and deforestation, albeit with myopic agents.

e.g., Souza-Rodrigues (2019) for further details). Cattle ranching in the area began only after Brazil's military dictatorship launched policies in the 1960s and 1970s to encourage settlement in the region, with the dual objectives of securing national borders and promoting regional development. It was not until the late 1980s that environmental concerns began to influence policy.

Land Use, Tenure and Regulation. Approximately 20 percent of the Brazilian Amazon has been deforested, totaling over 700,000 square kilometers—an area larger than Texas. The majority of cleared land is used for agriculture, with more than 70% designated as pasture and roughly 8 percent allocated to crop cultivation (Almeida et al., 2016).

About half of the Amazon is protected through various legal designations, including indigenous territories and conservation units such as national parks, extractive reserves, and areas of ecological interest. Deforestation in these areas is either strictly regulated or entirely prohibited. The remainder of the Amazon consists of undesignated public lands, where deforestation is forbidden, and private lands, which make up approximately 20 percent of the region's total area, according to the 2006 Agricultural Census (IBGE, 2006). Although deforestation on private land can be legal if authorized and in accordance with the Forest Code, empirical evidence suggests limited compliance with these regulations, with much of the deforestation in the Amazon occurring illegally (Borner et al., 2014; Rajão et al., 2020).

Over the past 25 years, two key policy initiatives have significantly influenced deforestation control in the region: the introduction of satellite monitoring and the Priority List, both integral to the Action Plan for the Prevention and Control of Deforestation in the Legal Amazon (PPCDAm) launched in 2004. The DETER satellite system, developed by the Brazilian Institute for Space Research (INPE), employs high-frequency remote sensing to monitor forest loss, enabling the almost-real-time regular processing of land-use images and generating deforestation alerts for law enforcement. This innovation has been instrumental in curbing deforestation rates (Assunção, Gandour and Rocha, 2023). Introduced in 2008, the Priority List targets municipalities with high deforestation levels, focusing regulatory efforts on these "blacklisted" areas. Within its first two years, this policy reduced deforestation by approximately 40% (Assunção et al., 2022). These changes fundamentally reshaped conservation policies in the region: Prior to the mid-2000s, the Brazilian Environmental Protection Agency's (IBAMA) monitoring operations in the Amazon relied primarily on data collected by its headquarters and regional offices, with land and air patrols being limited in effectiveness due to the vast size of the region and the risks faced by law enforcement.

Cattle Industry Structure. Cattle ranching in the Amazon is characterized by extensive grazing, with feedlots comprising only 1.5% of the cattle stock in the region (IBGE, 2006). Cattle reach adulthood at around three years old, which is the slaughter age for males and the breeding age for females. The production process consists of two main stages: breeding, where calves are raised, and fattening, where cattle are prepared for slaughter.[3] While some farms specialize in one stage, others—referred to as

[3]Cattle used for milk production account for only about 4% of the total Amazonian herd (IBGE, 2006, 2017).

complete-cycle farms—manage both. Trade between breeding and fattening farms is common but limited by the costs and risks of transporting live animals, particularly younger ones, who are more susceptible to stress and injury during transit.

The Amazonian cattle industry is composed of nearly half a million farms (IBGE, 2006, 2017), most of which are price takers selling cattle on the spot market. These cattle are typically sold to local slaughterhouses, which are owned by meatpackers, part of a concentrated meatpacking industry. Once processed, the beef is either exported—approximately 22% of the total production—or sold to domestic markets, accounting for about 78% of the supply. Domestic sales are primarily directed to supermarkets outside the Amazon region. The price of beef is largely determined by international market conditions, adjusted for transportation costs to the port.

3 Data and Descriptive Evidence

3.1 Data Sources

The data used in this study comes from various sources. Land use and deforestation data are obtained from Mapbiomas, an annual panel data at a 30-meter resolution from 1985 to 2023, including classifications for primary forest, pasture (good vs. degraded), cropland, secondary forest, and other land types. Our primary variable of interest from Mapbiomas is the deforestation rate by municipality and year, which we calculate as the area of gross forest loss in a given year divided by the total municipality area.

Our second primary dataset comes from IBGE's Municipal Livestock Survey (PPM-IBGE), which provides cattle counts by municipality and year.[4]

Beef prices, exchange rates, and price index are taken from the CME Group, B3, Brazilian Central Bank, and IBGE. Local prices are obtained by subtracting transportation cost from the international prices. To calculate transport costs, we follow a strategy similar to Souza-Rodrigues (2019) in using data on transportation networks from the Brazilian Ministry of Transportation, information on ports used for exporting beef from the Ministry of Industry, Foreign Trade, and Services, and freight values and gas prices from SIFRECA and the Brazilian National Agency for Petroleum, Gas, and Biofuels.

In addition to these primary data sources, several other datasets are used. Municipality-level land price data for various types of agricultural land (2000–2012) comes from ANUALPEC. Agricultural yield data comes from FAO-GAEZ. Zootechnical indices are provided by Amigos da Terra (2020). Geographic boundaries are provided by IBGE. Consumer price indices for the US and Brazil are obtained from the World Bank.

Our final dataset includes 754 municipalities in the Legal Amazon. All variables are available for 2002–2020. Table 1 presents descriptive statistics for the sample used for estimation; the coverage of this sample is 2002-2018 because our estimating equations use two-year leads of some variables. On average (unweighted averages across municipalities), the annual deforestation rate across municipality-years is

[4] Cattle data was downloaded from `https://sidra.ibge.gov.br/tabela/3939`.

about 0.98% of the total municipality area. Pasture accounts for an average of 33.39% of municipal land area, while crops cover approximately 4.27%.

The average spot price of beef during the sample period is $3.48/kg (live weight), with transportation costs accounting for about 4.25% of the beef price. Appendix B.1 explains the calculation of transportation costs in detail. Pastureland is priced at an average of $1059.5 per hectare. For most estimates of the social cost of carbon, the pasture land values we observe are far lower than the value of carbon released by deforestation, which is typically estimated as having a value of over $10,000 per hectare.[5] The largest pasture land values in the data are just over $3,500 per hectare.

Table 1: Summary Statistics

	count	mean	sd	min	max
Deforestation rate (prop of total area)	12773	0.010	0.009	0.0001	0.084
Share of area in pasture	12773	0.334	0.242	0.0013	0.939
Share of area in crops	12773	0.043	0.087	0.0000	0.649
Stocking density (head/ha)	12773	0.934	0.577	0.0000	3.833
Stocking density capacity (head/ha)	12773	4.021	0.398	3.1239	5.092
Spot price at port (USD/kg live weight)	12773	3.480	0.540	2.5383	4.495
Transportation cost to port (USD/kg live wt)	12773	0.144	0.086	0.0321	0.434
Price of pasture land (USD/ha)	902	1059.495	675.890	29.9642	3557.450
Year	12773	2010.018	4.895	2002.0000	2018.000

Notes: Means are unweighted across municipalities and years. Dollar values are based on deflating BRL to 2012 levels and then converting to USD using the 2012 exchange rate of .5473 USD per BRL.

Figure 1 illustrates the evolution of international beef prices over time. It also highlights the range of net prices, which are calculated by subtracting transportation costs from the international prices. The figure clearly shows that the time variation in international prices far outweighs the cross-sectional variation caused by transportation costs.

Figure 2 illustrates the decline in deforestation rates between 2004 and 2012, following a peak in 2004, despite rising prices during the same period. While other factors, including policy changes, also influence these trends, this suggests a negative correlation between price shocks and deforestation, a point we explore further in the next subsection.

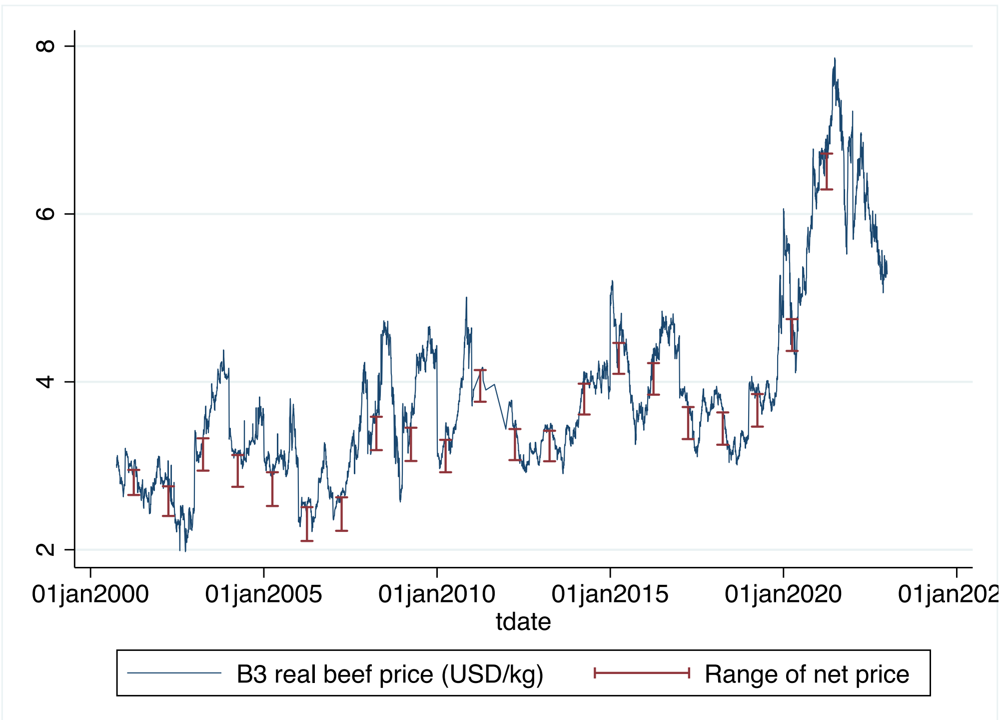
Focusing on the Amazonian states of Mato Grosso and Pará, Figure 3 illustrates that the Brazilian herd is gradually increasing, with the exception of a pronounced dip from 2007 to 2008. This dip coincides with a the late-2000s agricultural commodity price spike, and it is suggestive that ranchers may have responded to the temporary price increase by increasing slaughter rates, a dynamic response that will be part of our analysis below.

[5]For example, Ometto et al. (2023) estimate that the average above-ground biomass in the Brazilian Amazon is 174 Mg C per hectare. At .434 tonnes of carbon per Mg biomass, we end up with over 270 tonnes of CO2 released per hectare:

$$\frac{\text{174 tonnes biomass}}{\text{ha}}\frac{\text{.434 tonnes C}}{\text{tonne biomass}}\frac{\text{44 tonnes CO2}}{\text{12 tonnes C}} = \frac{\text{276.9 tonnes CO2}}{\text{ha}}.$$

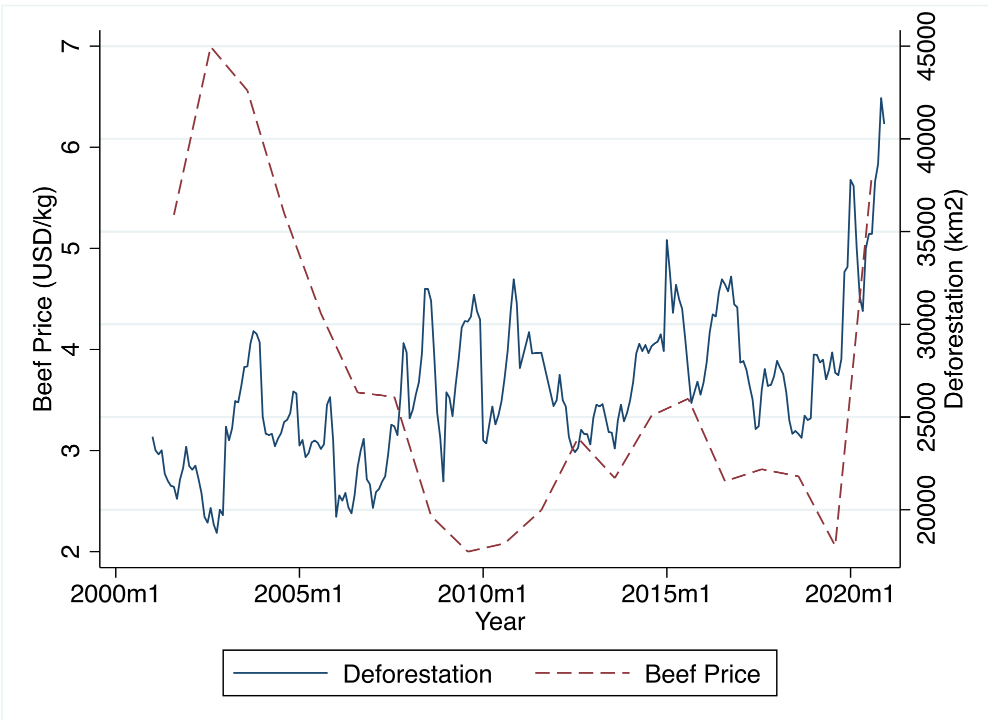
At the Biden administration's carbon price of $51 per tonne of CO2, this implies a social cost of carbon of over $14,000 per

Figure 1: Beef Prices (net of transport cost) over time



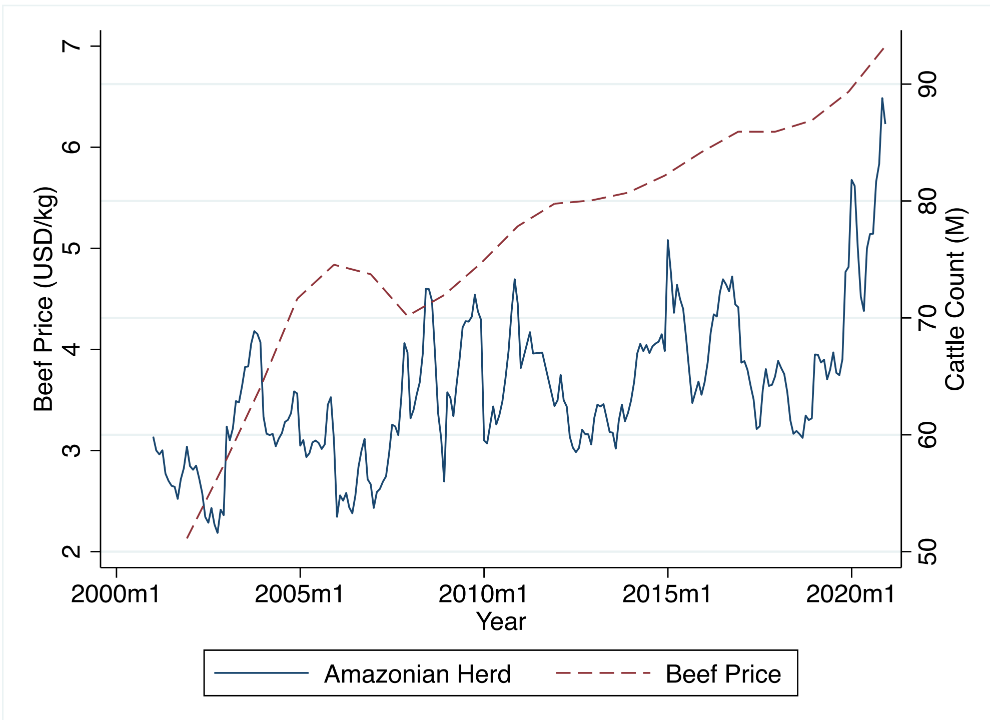
Notes: The solid blue line represents the price of B3's soonest expiring live cattle futures contract, adjusted to 2012 BRL using the Brazilian CPI, and then using the 2012 exchange rate of .5473 USD per BRL. The intervals show the range of net prices after subtracting transportation costs.

Figure 2: Amazonian deforestation and commodity prices



Notes: The solid blue line represents the price of B3's soonest expiring live cattle futures contract, adjusted to 2012 BRL using the Brazilian CPI, and then using the 2012 exchange rate of .5473 USD per BRL. The dashed line represents gross forest loss in the Brazilian Amazon calculated using Mapbiomas data.

Figure 3: Amazonian deforestation and cattle herd



Notes: The solid blue line represents the price of B3's soonest expiring live cattle futures contract, adjusted to 2012 BRL using the Brazilian CPI, and then using the 2012 exchange rate of .5473 USD per BRL. The dashed line represents the total cattle herd within Brazilian Amazonia calculated using IBGE data.

3.1.1 Reduced-Form Regressions

We now present some descriptive regressions. We begin by regressing the log of deforestation rates on the log of beef prices, along with control variables and municipality fixed effects, to have a sense of the potential magnitude of the deforestation elasticity with respect to these prices. The results are shown in Table 2. In the first column, the estimated relationship is negative but statistically insignificant, suggesting that prices may have little to no effect on reducing deforestation. This is a counterintuitive finding, particularly the negative sign, that is yet common in the literature; see e.g., Hargrave and Kis-Katos (2013), Assunção and Rocha (2019), Harding et al. (2021), and Assunção et al. (2022).

Table 2: Descriptive Deforestation Regressions

	(1) ln(DF)	(2) ln(DF)	(3) ln(DF)	(4) ln(DF)
ln(Spot Price)	-0.179	-1.471*	1.150	
	(0.207)	(0.741)	(0.853)	
ln(1 year Ahead Price)		1.404	-1.506	
		(0.882)	(0.882)	
PriorityList	-0.425***	-0.327***	-0.232*	-0.229*
	(0.137)	(0.113)	(0.114)	(0.116)
First 6 Months Rain	-0.000258*	-0.000221	-0.000278**	-0.000188
	(0.000133)	(0.000129)	(0.000119)	(0.000114)
Dry Season Length	-2.41e-05	5.74e-05	3.99e-05	-8.74e-05
	(0.000209)	(0.000196)	(0.000192)	(0.000177)
Cattle Density	-0.000790***	-0.000821***	-0.000829***	-0.000848***
	(0.000155)	(0.000162)	(0.000165)	(0.000168)
Observations	16,910	16,910	16,910	16,910
R-squared	0.771	0.775	0.783	0.803
Municipality FEs	Yes	Yes	Yes	Yes
Time Trend	-	-	Yes	-
Year FEs	-	-	-	Yes
Annual Clusters	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

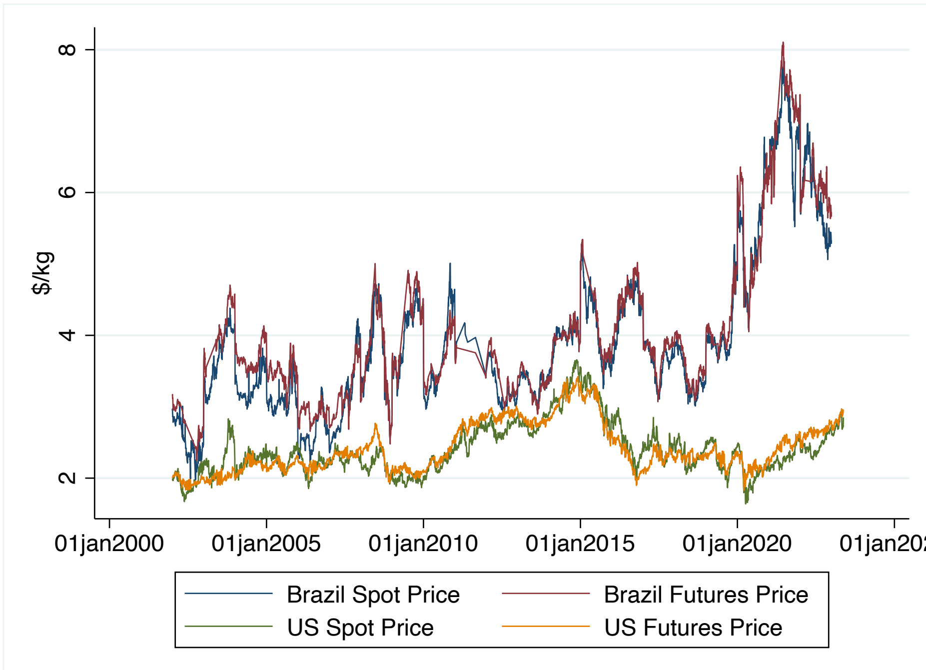
In column two, we add one-year-ahead beef futures prices to the regression, which increases the magnitude of the coefficient on spot prices substantially and makes it statistically significant. The coefficient on future prices is negative, with a similar magnitude but statistically insignificant. This seemingly suggests that while current prices may reduce deforestation, expected price increases may lead to more deforestation. However, in column three, when we include a time trend, the signs of both coefficients reverse, and they become statistically insignificant. These results highlight the fragility of the reduced-form regressions in identifying the elasticity of deforestation with respect to international prices. The last column includes time dummies, which prevent identification of the impacts of international prices due to collinearity.

In principle, a straightforward approach to disentangle short- and long-run price elasticities would be explore distinct short- and long-run price variation in the data. However, as illustrated in Figure 4,

hectare. This does not include below-ground carbon released by deforestation.

there are no clear instances where spot and futures prices move in opposing directions. Even if we had more independent variation in spot and futures prices, our interest in long-run elasticities comes from wanting to understand in the impacts of effectively permanent changes in the economic environment, and one-year ahead futures prices may not be a good proxy for permanent price changes. Futures contracts for agricultural commodities typically have delivery dates no more than two years in the future.

Figure 4: Futures and spot prices



Notes: US prices come from the CME; Brazil prices from B3. Spot prices are defined as the soonest expiring live cattle futures contract. Futures prices are defined as the soonest expiring live cattle futures contract with a delivery date at least nine months in the future. All prices are converted to 2012 levels using their country's respective CPI.

4 Model of Agriculture in Amazonia

We develop our model at the municipality level, assuming the presence of a representative rancher-deforester in each municipality (henceforth referred to as simply as a 'rancher'). We could equivalently assume that there is a representative continuum of ranchers in each municipality (see Appendix A.2). The rancher engages in both pasture preparation and the complete cattle production cycle, encompassing breeding, fattening, and preparing cattle for slaughter. We follow Araujo, Costa and Sant'Anna (2024) in assuming that the decision maker may either formally own, lease, or hold informal property rights to the land. In essence, the requirement is that they are residual claimants to the net discounted cash flow generated by their farming operations.

4.1 Ranching

We let K_{mt} denote the adult-equivalent cattle herd size in municipality m and year t. B_{mt} is the amount of beef produced in municipality m and year t. Ultimately, the cost of holding cattle will depend on the stocking rate—cattle per hectare—which is given by

$$k_{mt} \equiv \frac{K_{mt} - B_{mt}}{A_{rmt}},$$

where A_{rmt} is the area of land used for pasture—we will explain below how this is endogenously determined.

Ranching flow payoffs are given by

$$\pi_{rmt}\left(B_{mt}, k_{mt}, A_{rmt}\right) = P^B_{mt} Y \cdot B_{mt} - k_{mt} A_{rmt} \cdot C^r_{mt}\left(k_{mt}\right), \tag{1}$$

where P^B_{mt} is the local price of beef (\$/kg live weight), i.e., the international price net of transportation costs to ports, Y is the live weight per animal (403.226 kg), and $C^r_{mt}\left(k_{mt}\right)$ is the per-head cost of raising cattle.

As the density of cattle increases, the capacity of the land to nourish the cattle through grass becomes strained, and it may be necessary to purchase supplementary feed. High stocking rates also pose other costs—for example, screwworm parasites become a larger threat and must be monitored more carefully at higher stocking rates. In addition to the stocking rate, holding costs depend on the productivity of the land. Specifically, we assume that the relevant factor is the ratio of the stocking rate to a measure of the productivity of the land, which we denote by Y_{gm}. Y_{gm} represents the annual grass yield (in kilograms per hectare) obtained from FAO-GAEZ data.[6] We model per-head holding costs as follows:

$$C^r_{mt}\left(k_{mt}\right) = \theta_{r0} + \theta_{r1}\frac{k_{mt}}{Y_{gm}}.$$

θ_{r0} represents a fixed cost component, θ_{r1} controls the sensitivity of holding costs to congestion.

The parameter θ_{c0} can also absorb per-unit processing costs and slaughterhouse markups. That is, the rancher receives $P^B_{mt}Y - \theta_{c0}$ per animal slaughtered (not counting congestion costs), and so θ_{c0} can also capture the gap between the net price of an animal $P^B_{mt}Y$ and the wholesale price ranchers actually receive, in addition to any actual fixed costs that ranchers incur.

4.1.1 Herd Composition and Dynamics

Underlying the aggregate stock variable K_{mt} are cattle stocks of different ages: K_{1mt} represents calves, K_{2mt} represents yearlings, and K_{3mt} represents adults. The aggregate herd size is given by $K_{mt} = \gamma_1 K_{1mt} + \gamma_2 K_{2mt} + K_{3mt}$, where γ_1 and γ_2 are weights that convert calves and yearlings into adult-equivalent units. We assume $\gamma_1 = .33$ and $\gamma_2 = .67$.

[6] We take municipality-level averages of FAO-GAEZ's high-input, rain-fed grass yield.

Following RMS, we employ a three-period maturation model, based on the idea that cattle take roughly two years to mature. Formally, the cattle stock evolves according to the following equations:

$$K_{1,m,t+1} = g\left(K_{3mt} - B_{mt}\right) \tag{2}$$

$$K_{2,m,t+1} = K_{1mt} \tag{3}$$

$$K_{3,m,t+1} = (1-\delta)\left(K_{3mt} - B_{mt}\right) + K_{2mt} \tag{4}$$

Equation 2 indicate that cows successfully give births to calves at rate g. It can be understood that maturing steers are always slaughtered, so $\left(K_{3mt} - B_{mt}\right)$ corresponds to the number of adult cows that are not culled. Equation 3 indicates that calves mature into yearlings. Equation 4 indicates that yearlings mature into adults, and adult cows are subject to natural mortality at rate δ.

We calibrate $g = .6256$ and $\delta = .02$ based on zootechnical studies from the Brazilian Amazon (Amigos da Terra, 2010).[7]

We let $\mathbf{K}_{mt} = \left(K_{1mt}, K_{2mt}, K_{3mt}\right)$ denote the vector of cattle stocks. Note that we previously defined rancher's payoffs as a function of the stocking rate and area of pasture land: $\pi_{rmt}\left(B_{mt}, k_{mt}, A_{rmt}\right)$. We will alternatively express these payoffs as a function of the full cattle stock vector and area of pasture land, which of course implies the stocking rate: $\pi_{rmt}\left(B_{mt}, \mathbf{K}_{mt}, A_{rmt}\right)$.

4.2 Deforestation

Let $\bar{A}_m$ denote the area of available land in municipality m. Available land can either be in forest or pasture. Other land uses, such as urban development and mining, are excluded from $\bar{A}_m$ by assumption. In year t, available land is either in agricultural land, denoted by A_{mt}, or in forested land. The amount of forested land is therefore given by $\bar{A}_m - A_{mt}$.

We assume that all deforestation during year t converts forested land directly to agricultural land, which becomes available for use in the following year. The deforestation rate in year t and municipality m is given by

$$D_{mt} = A_{m,t+1} - A_{mt}.$$

The deforestation cost per hectare deforested is given by:

$$C^d_{mt}\left(D\right) = \theta_D \frac{D}{\bar{A}_m} + \theta_X X_{mt}.$$

This function implies that per-hectare deforestation costs are increasing with respect to the share of the municipality being deforested. This reflects the scarcity of local inputs, particularly labor. Deforestation typically occurs during the dry season (July-September), and clearing a large portion of a municipality in

[7] We assume beef quality is invariant to age and breeding history, so mature cows are treated the same as adult males. This simplification allows us to model all adults as neoclassical capital with an exponential death rate. These assumptions are necessary because the stock data consist of total head counts, without classification by age or sex, thus eliminating the need to track the adult age distribution.

a single year would require a substantial amount of short-term labor. Increasing average costs may also arise because the deforestation process likely begins in less costly areas and progresses to more costly ones over time. Furthermore, these rising costs can serve to rationalize the likely presence of credit constraints, which can lead to incremental deforestation over the years.

The term $\theta_X X_{mt}$ captures other factors that may influence deforestation costs. In particular, we include a dummy variable indicating years from 2006 and onward: the enforcement era. Additionally, we include a dummy variable that indicates whether a municipality was included on Brazil's Priority List ("blacklist"), which began in 2008. See Assunção et al. (2023) for a study of the impact of this policy.

For a continuum of agents to aggregate to a representative agent in this context, each agent's per-hectare deforestation cost must depend on the total amount of deforestation in their municipality (not only their own deforestation).

4.3 Rancher's Problem

Every year, the representative rancher for municipality m chooses two variables the maximize expected discounted payoffs: the slaughter rate B_{mt}, and the deforestation rates D_{mt}.

The representative rancher's expected discounted payoffs at time t can be expressed as follows:

$$E_t \left[\sum_{s=t}^{\infty} \beta^s \left(\pi_{r,m,t+s} \left(B_{mt}, s_{cms}, \mathbf{K}_{mt}, A_{ms} \right) - C^d_{mt}(D_{mt}) \right) \right]. \tag{5}$$

The corresponding Bellman equation is

$$\begin{aligned} V_{mt} \left(\mathbf{K}_{mt}, A_{mt} \right) = \max_{B_{mt}, D_{mt}, s_{cmt}} \; & \pi_{rmt} \left(B_{mt}, s_{cmt}, \mathbf{K}_{mt}, A_{mt} \right) - C^d_{mt}(D_{mt}) \\ & + \beta E_t \left[V_{m,t+1} \left(\mathbf{K}_{m,t+1}, A_{m,t+1} \right) \right], \end{aligned} \tag{6}$$

with $A_{m,t+1} = A_{mt} + D_{mt}$, and $\mathbf{K}_{mt}$ and B determining $\mathbf{K}_{m,t+1}$ through equations 2-4.

4.4 First-Order Conditions

In this section, we lay out the first-order conditions to the rancher's problem that we will use for estimation below. Each of these first-order conditions will be used for estimation in Section 5.

4.4.1 Cattle Euler Equation

In Appendix A.1.1, we show that the first-order condition with respect to the slaughter rate B_{mt} can be expressed as follows:

$$
\begin{aligned}
\underbrace{Y \times P^B_{mt}}_{\text{Revenue from selling cow today}} &= \underbrace{\frac{d}{dB_{mt}}\left[(K_{mt} - B_{mt})\, C^r_{mt}\left(\frac{K_{mt} - B_{mt}}{A_{rmt}}\right)\right]}_{\text{Cost of holding cow one more period}} \\
&+ \underbrace{\beta(1-\delta)E_t\left[Y \times P^B_{m,t+1}\right] + g\beta^3 E_t\left[Y \times P^B_{m,t+3}\right]}_{\text{Revenue from selling cow and offspring in the future}} \\
&+ \underbrace{g\beta E_t\left[\gamma_1 \frac{d}{dB_{m,t+1}}\left[\left(K_{m,t+1} - B_{m,t+1}\right) C^r_{m,t+1}\left(\frac{K_{m,t+1} - B_{m,t+1}}{A_{r,m,t+1}}\right)\right]\right]}_{\text{Cost of holding offspring as calf}} \\
&+ \underbrace{g\beta^2 E_t\left[\gamma_2 \frac{d}{dB_{m,t+2}}\left[\left(K_{m,t+2} - B_{m,t+2}\right) C^r_{m,t+2}\left(\frac{K_{m,t+2} - B_{m,t+2}}{A_{r,m,t+2}}\right)\right]\right]}_{\text{Cost of holding offspring as yearling}}
\end{aligned}
\tag{7}
$$

This equation captures the trade-off between slaughtering and holding a cow to slaughter it next period. If slaughtered in time t, the cow will generate revenues $Y \times P^B_{mt}$. If held for one period, it will generate expected discounted revenues of $\beta\,(1-\delta)\,E_t\left[P^B_{m,t+1}\right]$, and its progeny will generate discounted revenues $g\beta^3 E_t\left[P^B_{m,t+3}\right]$. Holding the cow a year longer also implies extra holdings costs, both from the cow itself and its offspring.

4.4.2 Deforestation Euler Equation

In Appendix A.1.2, we show that the first-order condition with respect to the deforestation rate D_{mt} can be expressed as follows:

$$
\underbrace{\frac{d}{dD_{mt}} C^d_{mt}\left(D_{mt}\right)}_{\text{Current DF costs}} = \beta E_t\left[\underbrace{\frac{d}{dA_{r,m,t+1}}\left((K_{m,t+1} - B_{m,t+1}) C^r_{m,t+1}\left(\frac{K_{m,t+1} - B_{m,t+1}}{A_{r,m,t+1}}\right)\right)}_{\text{Future holding costs}} + \underbrace{\frac{d}{dD_{m,t+1}} C^d_{m,t+1}\left(D_{m,t+1}\right)}_{\text{Future DF costs}}\right].
\tag{8}
$$

This Euler equation expresses the idea that there should be indifference (in expectation) between incurring deforestation costs now and delaying them one period, accounting for the extra holding costs next period associated with delayed pasture expansion.

4.5 Land Values

To complete the model, we note that land prices can provide important information about the model parameters. Specifically, suppose pasture is competitively supplied to the rancher. In this case, the value of the marginal land determines the land price. This implies that the derivative $\frac{d}{dA_{mt}}V_{mt}$ equals the price of pasture land, denoted as P^L_{mt}. It is important to emphasize that the rancher's full value function, $V_{mt}(\mathbf{K}_{mt}, A_{mt})$, should not simply be equated with land values in municipality m in year t. The full value function reflects not only the value of land but also the value of the cattle stock. Instead, it is more appropriate to equate the rancher's marginal value of land with observed land prices, which will hold as an equilibrium condition if land markets are competitive. Under this assumption, we obtain the following equation:

$$\begin{aligned} P^L_{mt} = \frac{dV_{mt}}{dA_{mt}} = &- \frac{d}{dA_{mt}} \left[(A_{mt+1} - A_{mt}) \cdot C^d_{mt} \left(A_{mt+1} - A_{mt} \right) \right] \\ &- \frac{d}{dA_{rmt}} \left[(K_{mt} - B_{mt}) \cdot C^r_{mt} \left(\frac{K_{mt} - B_{mt}}{A_{rmt}} \right) \right], \end{aligned} \tag{9}$$

where we rely on the envelope theorem to take the derivative of the value function with respect to the state variable A_{mt}.

Intuitively, land prices help identify the fixed cost parameter θ_{r1}. The fixed cost influences the profitability of pasture land, and the price of pasture land provides information about its profitability.

5 Estimation

At a high level, our estimation strategy is based on the first-order conditions described above (equations (7), and (8)) and the land value equation (9). We use the generalized method of moments (GMM) to estimate the cost parameters θ_D, θ_{r0}, and θ_{r1} We let $\theta = (\theta_D, \theta_{r0}, \theta_{r1})$ denote the vector of parameters to be estimated. With three equations and three unknown parameters the model is exactly identified, so we do not need to choose a weighting matrix for the GMM objective function. Following standard practice, we impute $\beta = .9$ for the discount factor.

For the Euler equations, equations (7), and (8), we replace the expectation terms by the realized future values of the variables and forecast errors, e_{dmt} and e_{rmt}. That is, the deforestation equation becomes

$$\begin{aligned} e_{dmt}(\theta) = &\frac{d}{dD_{mt}} C^d_{mt}(D_{mt}; \theta) \\ &- \beta \left(\frac{d}{dA_{r,m,t+1}} \left(K_{m,t+1} C^r_{m,t+1} \left(\frac{K_{m,t+1}}{A_{r,m,t+1}}; \theta \right) \right) + \frac{d}{dD_{m,t+1}} C^d_{m,t+1}\left(D_{m,t+1}; \theta \right) \right), \end{aligned} \tag{10}$$

where we have now made the cost functions' dependence on the parameters explicit. Formally, the forecast error $e_{dmt}(\theta)$ is defined as the expectation term on the right-hand side of equation (8) minus its realization:

$$\begin{aligned} e_{dmt}(\theta) =& \beta E_t \left[\frac{d}{dA_{r,m,t+1}} \left(K_{m,t+1} C^r_{m,t+1} \left(\frac{K_{m,t+1} - B_{m,t+1}}{A_{r,m,t+1}}; \theta \right) \right) + \frac{d}{dD_{m,t+1}} C^d_{m,t+1} \left(D_{m,t+1}; \theta \right) \right] \\ & - \beta \left[\frac{d}{dA_{r,m,t+1}} \left(K_{m,t+1} C^r_{m,t+1} \left(\frac{K_{m,t+1} - B_{m,t+1}}{A_{r,m,t+1}}; \theta \right) \right) + \frac{d}{dD_{m,t+1}} C^d_{m,t+1} \left(D_{m,t+1}; \theta \right) \right]. \end{aligned} \tag{11}$$

Equation (10) is then equation (8) with equation (11) substituted in.

Similarly, the cattle Euler equation becomes

$$\begin{aligned} e_{rmt}(\theta) \quad &= Y \times P^B_{mt} - \tfrac{d}{dB_{mt}} \left[(K_{mt} - B_{mt}) C^r_{mt} \left(\tfrac{K_{mt} - B_{mt}}{A_{rmt}}; \theta \right) \right] \\ & - \beta(1-\delta) Y \times P^B_{m,t+1} - g\beta^3 Y \times P^B_{m,t+3} \\ & - g\beta\gamma_1 \tfrac{d}{dB_{m,t+1}} \left[\left(K_{m,t+1} - B_{m,t+1} \right) C^r_{m,t+1} \left(\tfrac{K_{m,t+1} - B_{m,t+1}}{A_{r,m,t+1}}; \theta \right) \right] \\ & - g\beta^2\gamma_2 \tfrac{d}{dB_{m,t+2}} \left[\left(K_{m,t+2} - B_{m,t+2} \right) C^r_{m,t+2} \left(\tfrac{K_{m,t+2} - B_{m,t+2}}{A_{r,m,t+2}}; \theta \right) \right], \end{aligned} \tag{12}$$

where the forecast error $e_{rmt}(\theta)$ is defined as the right-hand side of equation (7) (which contains expectations) minus its realization.

We introduce an error term, $e_{lv,mt}(\theta)$, into the land value equation, (9), which becomes

$$\begin{aligned} e_{lv,mt}(\theta) \equiv & P^L_{mt} - \frac{d}{dA_{mt}} \left[(A_{mt+1} - A_{mt}) \cdot C^d_{mt} \left(A_{mt+1} - A_{mt}; \theta \right) \right] \\ & + \frac{d}{dA_{rmt}} \left[(K_{mt} - B_{mt}) \cdot C^c_{mt} \left(\frac{K_{mt} - B_{mt}}{A_{rmt}}; \theta \right) \right]. \end{aligned} \tag{13}$$

In this case, the error term should be interpreted as measurement error in observed land prices.

We can estimate the model imposing that the error terms be mean zero—that is, that the first-order conditions and land value equation hold on average. In other words, the moments we use for estimation are

$$E \begin{pmatrix} e_{dmt}(\theta) \\ e_{rmt}(\theta) \\ e_{lv,mt}(\theta) \end{pmatrix} = 0. \tag{14}$$

For specifications with covariates X_{mt} in the deforestation cost function, we also include the moments $E\left[e_{dmt}(\theta) X_{mt}\right] = 0$.

Each of these moments has an economic interpretation. Equation (10) relates the marginal cost of deforestation to holding costs. In a static model, deforestation should proceed until its marginal cost corresponds to the marginal benefit of reducing holding costs. Equation (10) is the dynamic version of such a condition, and deforestation moment, $E\left[e_{dmt}(\theta)\right] = 0$, requires the parameter to make this optimality condition hold on average. Equation (12) relates the revenue from culling cattle to the value of holding cattle for future sale. The moment condition $E\left[e_{rmt}(\theta)\right] = 0$ requires that this optimality condition hold on average. The land value moment, $E\left[e_{lv,mt}(\theta)\right] = 0$, requires that, on average, the observed land values correspond to the marginal value of pasture land in the model.

Our moments are appealing in that they do not rely on any particular exclusion restrictions or other assumptions regarding the correlations of variables. We assume only that agents' forecast errors average out to zero. The specification of the model in itself arguably represents a strong assumption; however, conditional on the model being correct, the assumption that forecast errors average out to zero represents a mild assumption about agents' expectations. If agents have rational expectations, then the agents' expectations correspond to the actual expectations given the equilibrium data generating process, and because forecast errors are mean zero by construction, our moments would be valid. While the assumption of rational expectations is sufficient for the validity of our moments, it is not necessary. All we need is that agent's expectations of the realized variables in the above equations are correct on average, whether or not they have fully rational expectations.

That said, care should be taken when constructing moments from Euler equations. Consider equation (12), and observe that the equation includes observed realizations of variables dated after time t. It would not be appropriate to use, for example, a moment like $E\left[e_{rmt}(\theta)\, P^B_{m,t+1}\right] = 0$. The forecast error $e_{rmt}(\theta)$ captures the difference between a time-t expectation (right-hand side of equation (7)) and its realization. One of the things that that influences this forecast error is the realization of $P^B_{m,t+1}$, so $P^B_{m,t+1}$ is naturally correlated with the forecast error.

5.1 Identification

We do not estimate a supply curve or supply elasticities directly. Instead, we recover the cost function parameters that underlie supply decisions. We can then compute supply responses based on this cost function. That is, we can solve and simulate the rancher's problem within various economic environments. Thus, supply responses are not primitives, but a function of the cost parameters and the economic environment.

We now provide some intuition for how the cost function is identified. It is simplest to begin with the Euler equation for cattle management, and consider the simplified version of the model where cattle mature in one period (what we say here applies equally well to the full model, but the notation is far more cumbersome):

$$P^B_{m,t}Y = \beta\left(1-\delta+g\right)E_t\left[P^B_{m,t+1}Y\right] + \frac{d}{dB_{m,t}}\left[(K_{m,t}-B_{m,t})C^r_m\left(\frac{K_{m,t}-B_{m,t}}{A_{m,t}};\theta\right)\right]. \tag{15}$$

This equation states that a rancher should be indifferent between selling a cow today or holding it for one more period, incurring extra holding costs and selling it and its offspring next period. After substituting in the expectational error term and rearranging, we have The right hand side of the equation corresponds to the net present value of holding a cow for one more. In general, as long as the discount rate isn't too large, holding the cow will lead to more discounted revenues. That is, it is natural to expect that

$$\beta\left(1-\delta+g\right)E_t\left[P^B_{m,t+1}Y\right]>P^B_{m,t}Y.$$

The extra holding costs must then be sufficiently large to cancel out the extra revenue associated with holding the cow, keeping the rancher indifferent between selling now or later along the optimal path.

Recall that we introduce a forecast error term capturing the difference between expected and realized values in the Euler equation, which in the context of this simplified model becomes

$$e_{rmt}=\beta\left(1-\delta+g\right)Y\left(E_t\left[P^B_{m,t+1}\right]-P^B_{m,t+1}\right),$$

and equation (15) becomes

$$e_{rmt}\left(\theta\right)=P^B_{m,t}Y-\beta\left(1-\delta+g\right)P^B_{m,t+1}Y-\frac{d}{dB_{m,t}}\left[(K_{m,t}-B_{m,t})C^r_m\left(\frac{K_{m,t}-B_{m,t}}{A_{m,t}};\theta\right)\right]. \tag{16}$$

Therefore, the moment $e_{rmt}\left(\theta\right)=0$ calls for finding holding cost parameters that make the marginal holding costs equal to the extra revenue associated with holding the for one more period. We can think of this as identifying the congestion cost parameter, θ_{r1}, although strictly speaking it provides information about both holding cost parameters.

We can then turn to the deforestation Euler equation, equation (10). If deforestation rates aren't increasing quickly, then the terms

$$\frac{d}{dD_{mt}}C^d_{mt}\left(D_{mt};\theta\right)-\beta\frac{d}{dD_{m,t+1}}C^d_{m,t+1}\left(D_{m,t+1};\theta\right)$$

reflect a temptation to defer deforestation to the next period. Along the optimal path, this must be balanced with the extra holding costs associated with delaying pasture expansion, captured by the term

$$\beta\frac{d}{dA_{r,m,t+1}}\left(K_{m,t+1}C^r_{m,t+1}\left(\frac{K_{m,t+1}}{A_{r,m,t+1}};\theta\right)\right),$$

which is intuitively related to the holding costs identified by the cattle Euler equation above. Having already identified the holding cost parameters, imposing the moment condition $E\left[e_{dmt}\left(\theta\right)\right]=0$ requires that the level of deforestation costs be such the savings associated with delayed deforestation balance with holdings costs.

Finally, the land value equation tells us about the expected discounted stream of payoffs to the rancher. This provides information about the fixed cost parameter, θ_{r0}, which controls the average cost of holding cattle, and therefore the average profitability of pasture land.

Table 3: Cost Estimates: Simplified Cattle Dynamics

	(1)	(2)	(3)	(4)
θ_{r0}	119.2	206.014	163.066	219.858
	(−83.51,352.77)	(2.1889,370.36)	(2.5142,362.8)	(36.41,371.39)
θ_{r1}	1742.79	1450.07	1594.88	1403.39
	(1186.4,2331.8)	(1119.4,1969.6)	(1089.8,2117.4)	(1104.3,1841.8)
θ_d	65464.3	35312.4	58659.9	34175.6
	(47606.,98678.)	(22773.,56755.)	(43587.,86015.)	(22233.,54371.)
Post-2006		766.743		720.299
		(419.79,972.22)		(327.92,943.23)
Blacklist			1079.56	470.119
			(215.51,1831.5)	(31.549,1168.4)
Mean holding cost	377.637	421.044	399.57	427.965
	(227.27,543.74)	(281.94,541.01)	(275.02,547.5)	(298.15,545.14)
Mean DF cost	643.109	934.34	614.551	904.263
	(467.67,969.4)	(770.92,1087.6)	(464.52,869.93)	(743.19,1061.7)
Mean marginal DF cost	1286.22	1281.24	1190.82	1240.
	(935.35,1938.8)	(1096.1,1527.5)	(892.37,1714.9)	(1079.,1462.1)
Observations	12773	12773	12773	12773

Notes: The table presents cost parameter estimates and derived statistics for specifications with simplified cattle dynamics (i.e., no calf or yearling stocks). Mean cost statistics average over municipalities and years in our sample. The Post-2006 and Blacklist dummy variables are included in the deforestation cost function. Standard errors, shown in parentheses, are clustered by year and calculated using the block bootstrap procedure. Observations are by municipality-year.

5.2 Parameter Estimates

Tables 3 and 4 present the estimated cost parameters for specifications with simplified and full cattle dynamics, respectively. Standard errors, shown in parentheses in the table, are clustered by year to capture spatial dependence in the data, and calculated using the block bootstrap procedure.

The implied average cost of deforestation is on the order of $600 per hectare in most specifications (although the cost of the marginal hectare is somewhat larger). For comparison, Almeida and Uhl (1995) report slash and burn clearing costs of $291 per hectare in 1993; which is a comparable cost to the average deforestation cost estimated in our model after adjusting for inflation.

Table 4: Cost Estimates: Full Model

	(1)	(2)	(3)	(4)
θ_{r0}	-220.531	-121.598	-171.062	-106.332
	$(-433.39,-30.268)$	$(-298.1,2.5445)$	$(-322.73,-11.828)$	$(-271.15,10.992)$
θ_{r1}	2101.84	1741.42	1921.62	1685.81
	(1434.8,2812.8)	(1346.2,2347.1)	(1316.1,2550.)	(1327.7,2207.3)
θ_d	65465.7	34681.6	58591.1	33574.
	(47607.,98680.)	(22332.,55261.)	(43503.,85904.)	(21883.,52911.)
Post-2006		782.816		736.225
		(449.51,985.26)		(357.55,956.2)
Blacklist			1090.7	466.416
			(222.86,1834.7)	(36.45,1131.)
Mean holding cost	133.281	236.964	185.125	252.963
	$(-112.13,369.69)$	$(-4.1984,465.4)$	$(-14.186,376.12)$	(55.32,465.65)
Mean DF cost	643.123	940.458	614.27	910.423
	(467.69,969.42)	(783.02,1093.2)	(464.4,869.23)	(749.79,1070.4)
Mean marginal DF cost	1286.25	1281.16	1189.86	1240.25
	(935.37,1938.8)	(1093.4,1517.5)	(891.67,1713.1)	(1080.7,1445.1)
Observations	12773	12773	12773	12773

Notes: The table presents cost parameter estimates and derived statistics for specifications with the full cattle dynamic model (i.e., with time to build). Mean cost statistics average over municipalities and years in our sample. The Post-2006 and Blacklist dummy variables are included in the deforestation cost function. Standard errors, shown in parentheses, are clustered by year and calculated using the block bootstrap procedure. Observations are by municipality-year.

6 Short- and Long-Run Price Responses

In this section, we solve and simulate the rancher's problem to show that simulated data from the model can replicate empirical patterns in the data. In the simulated model, despite the fact that the long-run elasticity of deforestation is large, the correlation between contemporaneous deforestation and beef prices can be effectively zero, or even negative.

We solve the rancher's problem given an exogenous process for beef prices. When we compute policy simulations in section 7, prices will be endogenous, but here, we are only interested in how a rancher responds to price shocks in the short and long run.

For these simulations, we assume that beef prices follow a simple AR(1) process that matches the degree of persistence observed in the data:

$$P_t^B = (1 - 0.72)\,\mu_p + 0.72 P_{t-1}^B + \varepsilon_{B,t},$$

where $\varepsilon_{B,t}$ is i.i.d. normal with variance 0.50. The degree of autocorrelation and variance imputed here match those observed in Brazilian spot prices during our sample years.

We impute different values for the mean price μ_p, and solve and simulate the model separately for each value of μ_p. Given a mean price, we can think about responses to P_t^B (conditional on μ_p) as short-run price responsiveness. We will interpret differences in behavior for different values of μ_p as long-run price responsiveness.

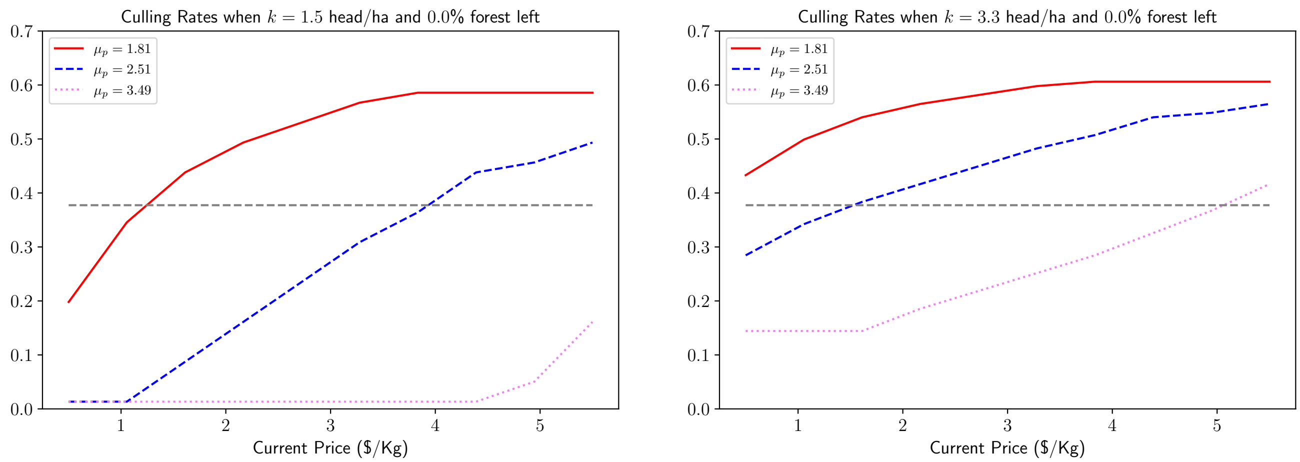
We solve the model using a form of backward induction. Since deforestation is irreversible, we start by solving the model assuming full deforestation, i.e., $A_{mt} = \bar{A}_m$. With full deforestation, the A_{mt} state variable can no longer change, and the agent's problem simplifies to a cattle management problem.

We discretize the state space for A_{mt}, and having solved the model for $A_{mt} = \bar{A}_m$, we then consider the next lower value of A_{mt}—say, $A_{mt} = \bar{A}_m - \epsilon$, where the rancher can either remain at this level of deforested land or deforest the last ϵ hectares of forest to arrive at the fully deforested state. Having solved the value function for the $\bar{A}_m$ and $\bar{A}_m - \epsilon$ states, we can then consider the $A_{mt} = \bar{A}_m - 2\epsilon$ state, where the rancher can remain at the current level of deforested land, deforest ϵ more, or deforest 2ϵ to reach the fully deforested state. We can proceed like this, backward inducting through the forest cover variable (rather than backward inducting through time). In our implementation, ϵ corresponds to .05% of the municipality's land area.

In what follows, we illustrate the model solution for a post-2006 municipality that has grass productivity equal to the sample mean and is not on the blacklist.

Figure 5 illustrates some culling policy functions conditional on $A_{mt} = \bar{A}_m$–that is, behavior for the cattle management problem when there is no deforestation left to be done. The left panel shows the culling rate on the vertical axis and the current beef price on the horizontal axis when the cattle stock density is 1.5 heads per hectare, while the right panel shows the same for a stock density of 3.3 heads per hectare. The red line represents the culling rate when the long-run mean price is \$1.85/kg; the blue dotted line corresponds to a mean price of \$2.52/kg (similar to the mean in the data); and the purple dotted line, a mean price of \$3.48/kg.

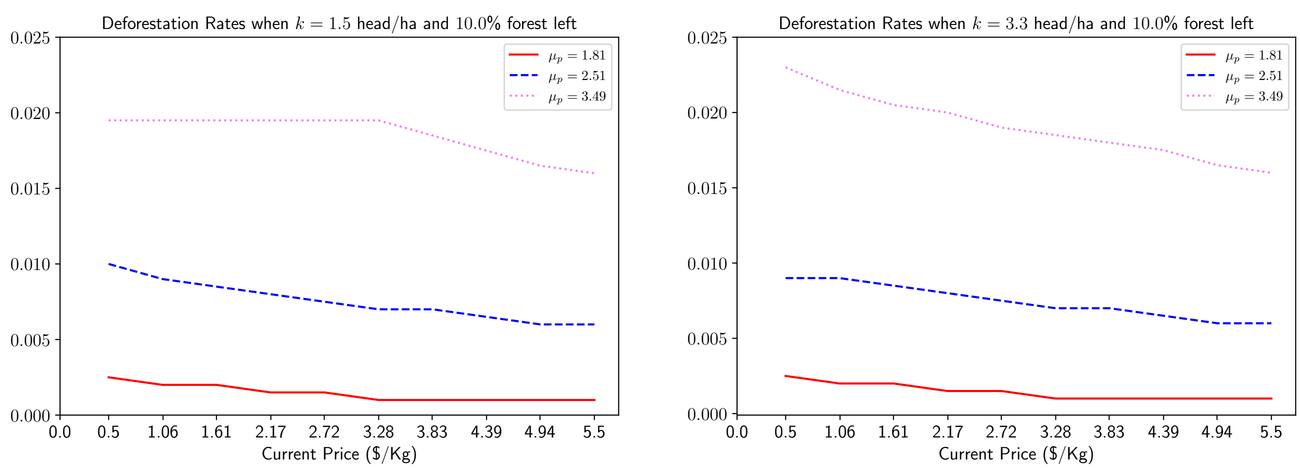
Figure 5: Cattle Management Policy Functions



Notes: The different lines correspond to policy functions for different mean prices. The horizontal axis maps out the current price of beef. The culling rate is measured as the fraction of the adult cattle stock slaughtered in a given year. The dashed gray line indicates the culling rate that would keep the herd size constant in the long run.

In both panels, the policy function is weakly increasing in P^B. This means that when the rancher observes a higher current price, she responds by slaughtering more cows to take advantage of the favorable price shock. However, when the long-run price increases, from say \$1.85/kg to \$2.52/kg, the rancher's culling rate is lower. These dynamic patterns are also reflected in deforestation policy functions.

Figure 6: Deforestation Policy Functions



Notes: The different lines correspond to policy functions for different mean prices. The horizontal axis maps out the current price of beef. The deforestation rate is measured as the fraction of the municipality's total area.

Figure 6 illustrates deforestation policy functions, conditional on two different stocking densities as in Figure 5, but now we are considering the state where deforested land A_{mt} is equal to 90% of the municipality's available land.

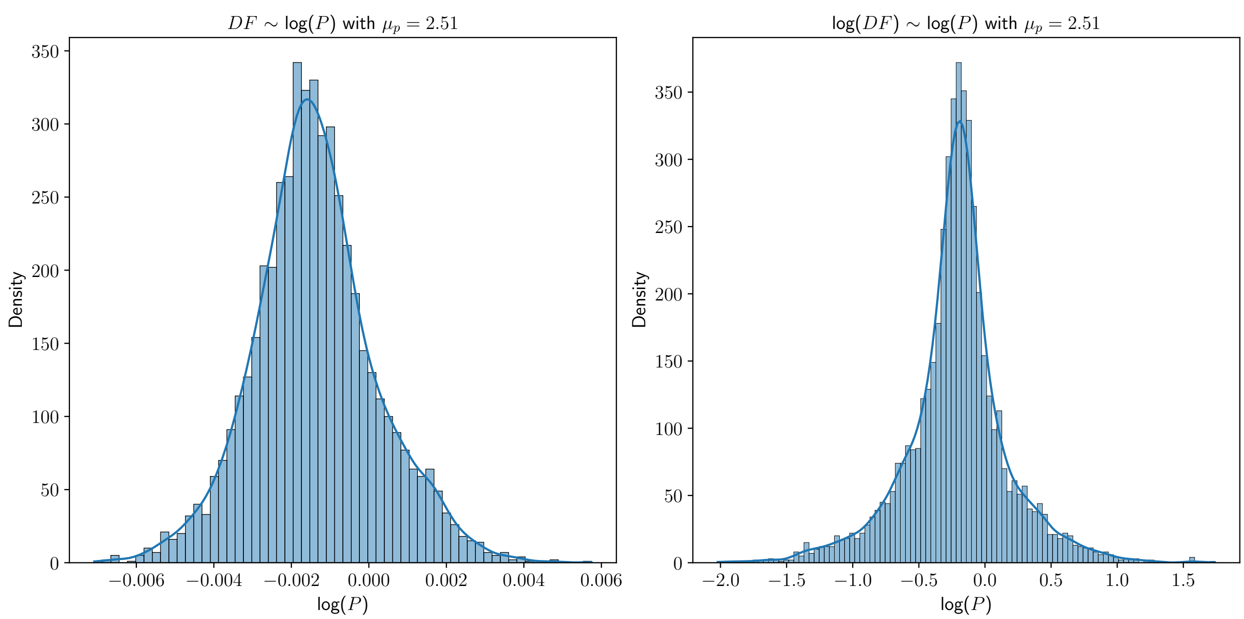
Conditional on the mean price, deforestation rates are decreasing with respect to the current price of beef. This reflects the fact that culling rates are higher and herd growth is lower when current prices are high. Thus, deforestation pressure is low when the current price is high.

However, comparing deforestation rates across mean prices, we have deforestation rates increasing with respect to (mean) price. This reflects the fact that ranching is more profitable in the long run when the mean price of beef is high, making the private returns to deforesting land higher.

These results point to a potential tension between the correlation we should expect to see within a given data generating process and the comparative statics we derive by changing the mean of the data generative process. Next, we confirm using simulations that the correlation we should expect to arise between deforestation and beef prices is likely to be close to zero, and can even be negative.

We simulate data using the solution to the rancher's problem described in this section. We simulate the model 5000 times, generating a sample of 30 years of data each time (after a 20-year burn-in period). For each sample, we regress deforestation rates on the log of the beef price. Figure 7 shows the histogram of the estimated coefficients.

Figure 7: Simulated Regressions



Notes: The figure presents the histogram of estimated coefficients on log price from simulated regressions of deforestation rates.

Figure 7 shows that the estimated elasticities are very small (inelastic) in magnitude, and more likely to be negative than positive. This is consistent with what we saw in our descriptive regressions.

The lesson from this exercise is that even though deforestation appears to be highly inelastic in the data, its long-run elasticity can be large. Comparing deforestation rates in Figure 7 for $\mu_p = 3.48$ and $\mu_p = 2.52$, we see that an approximately 33% increase in mean price is associated with an approximate doubling of the deforestation rate–we could call this a long-run elasticity of about 3.

Typically, when we are concerned about long-run elasticities, we are interested in understanding what will happen in a different policy environment. To that end, the next section performs policy simulations.

7 Policy Simulations

We embed the rancher's problem within a broader equilibrium model to perform policy simulations. We consider two kinds of policy interventions: beef taxes, and deforestation fines.

The model has five components: (1) the model of Brazilian ranchers introduced and estimated above, (2) a global beef demand function, (3) a global crops demand function, (4) a global crops supply function, and (5) a feedlot sector that converts crops into beef.

Our model of ranching consists of a municipality with average parameters that has been scaled up to match aggregate Brazilian production in recent data. This representative municipality has 178M hectares of available land. It is initially 33.5% deforested with a stocking density of .94 head/ha—values corresponding to our sample means.

Global beef demand has the following form:

$$Q_B^D\left(P^B\right) = 173.8 - 31.104P^B,$$

where the parameters were set to match recent consumption levels (when quantities are in millions of metric tons and prices in 2012 USD per kilogram live weight) and have an elasticity of $-.81$. This corresponds to a moderate estimate of the elasticity of demand for beef—see Bouyssou, Jensen and Yu (2024) for a review.

Global crop demand has the following form:

$$Q_C^D\left(P^C\right) = 795.6 - 383.3P^C,$$

where the parameters were set to match recent consumption levels of maize (for food, not counting feed and fuel) and have an elasticity of $-.06$, which corresponds to Roberts and Schlenker's (2013) estimate of the global demand elasticity for grain commodities.

Global crop supply has the following form:

$$Q_C^S\left(P^C\right) = 1365.1(P^C)^{.1},$$

where the parameters were set to match recent production levels of maize and have an elasticity of .1, which corresponds to Roberts and Schlenker's (2013) estimate of the global supply elasticity for grain commodities.

The feedlot consists of a static, linear, and competitively available technology for converting crops into beef. We assume that each kilogram of feedlot beef (live animal weight) requires 5.4 kilograms of crop inputs and 1.53 units of numeraire good. In equilibrium, non-zero feedlot beef production requires a zero profit condition with respect to the prices of crops and beef:

$$P^B = 5.4P^C + 1.53. \tag{17}$$

We denote the amount of feedlot beef produced with Q_B^F. The amount of crops used for feedlot production is $5.4Q_B^F$

Market clearing in beef requires

$$Q_B^D\left(P_t^B\right) = B_t + Q_{B,t}^F, \tag{18}$$

where B_t is pasture beef production.

Market clearing in crops requires

$$Q_C^D\left(P_t^C\right) + 5.4Q_{B,t}^F = Q_C^S\left(P_t^C\right) \tag{19}$$

Equations (17), (18), and (19) define a static equilibrium conditional on the pasture beef production level B_t In each period, the aggregate state variables are the area of pasture land and the associated cattle herd, (A, K). Aggregate behavior will entail a culling rate $B_t^*(A, K)$ and associated level of beef production. We can then write $P_t^*(A, K)$ to denote the equilibrium beef price, which is given by the solution to equations 17-19 given $B_t = B_t^*(A, K)$.

The planner decides on pasture beef production rates B_t and deforestation rates D_t to maximize expected discounted private surplus. That is, the objective function for the planner's problem is consumer surplus (from both beef and crops) plus rancher profits. The social cost of emissions are not included at this stage, for we are solving the planner's problem only for the purpose of solving for competitive equilibrium.

We assume that time T^* is the last period in which deforestation is permitted, with $T^* = 10$ in our baseline specification. The model continues thereafter, but with a fixed pasture area; ranchers only make cattle management decisions from period $T^* + 1$ onward.

Let $W_{mt}(A, K)$ represent the planner's time specific value function. The post-T^* equilibrium will be stationary and we can drop the time subscripts and simply write $W_{m*}(A, K) = W_{mt}(A, K)$ for all $t \geq T^* + 1$. We solve for $W_{m*}(A, K)$ as the fixed point of the Bellman equation

$$W_{m*}(A, K) = \max_B \pi_m(B, 0, K, A, P) + \beta W_{m*}\left(A, K'(B, K), P'\right).$$

Having solved the value function for the post-deforestation world, we can use backward induction to define and solve the value function in previous periods. That is,

$$W_{mT^*}(A,K,P) = \max_{B,D} \pi_m(B,D,K,A,P) + \beta W_{m*}\left(A+D, K'(B,K), P'\right),$$

and

$$W_{mt}(A,K,P) = \max_{B,D} \pi_m(B,D,K,A,P) + \beta W_{m,t+1}\left(A+D, K'(B,K), P'\right)$$

for $t < T^*$.

We can appeal to the Second Fundamental Welfare Theorem to establish that the solution to the planner's problem corresponds to a competitive equilibrium, supposing that each rancher owns an infinitesimally small amount of land. We can also verify this directly. The solution to the planner's problem implies an equilibrium path of prices that are exogenous from an individual rancher's perspective. We can then solve the rancher's optimal problem given this path of prices, and confirm that the individual rancher's cattle management and deforestation decisions correspond to the planner's aggregate decisions.

Figure 8 shows that Pigouvian deforestation taxes equal would completely halt deforestation, since that tax would far exceed the NPV that ranching generates per hectare. As noted above, the social value of carbon released by a hectare of deforestation in Amazonia is much higher than the values of pasture land observed in the data.

While deforestation taxes have been viewed by many as politically infeasible due to the costs on voters, Figure 8 suggests that ranchers may actually benefit from these policies: restricting supply leads to higher beef prices, which leads to higher profits per hectare. This means incumbent ranchers (particularly those who are not actively deforesting) stand to gain from increased enforcement of anti-deforestation measures. We note that policies restricting agricultural production to increase prices for farmers are common around the world.

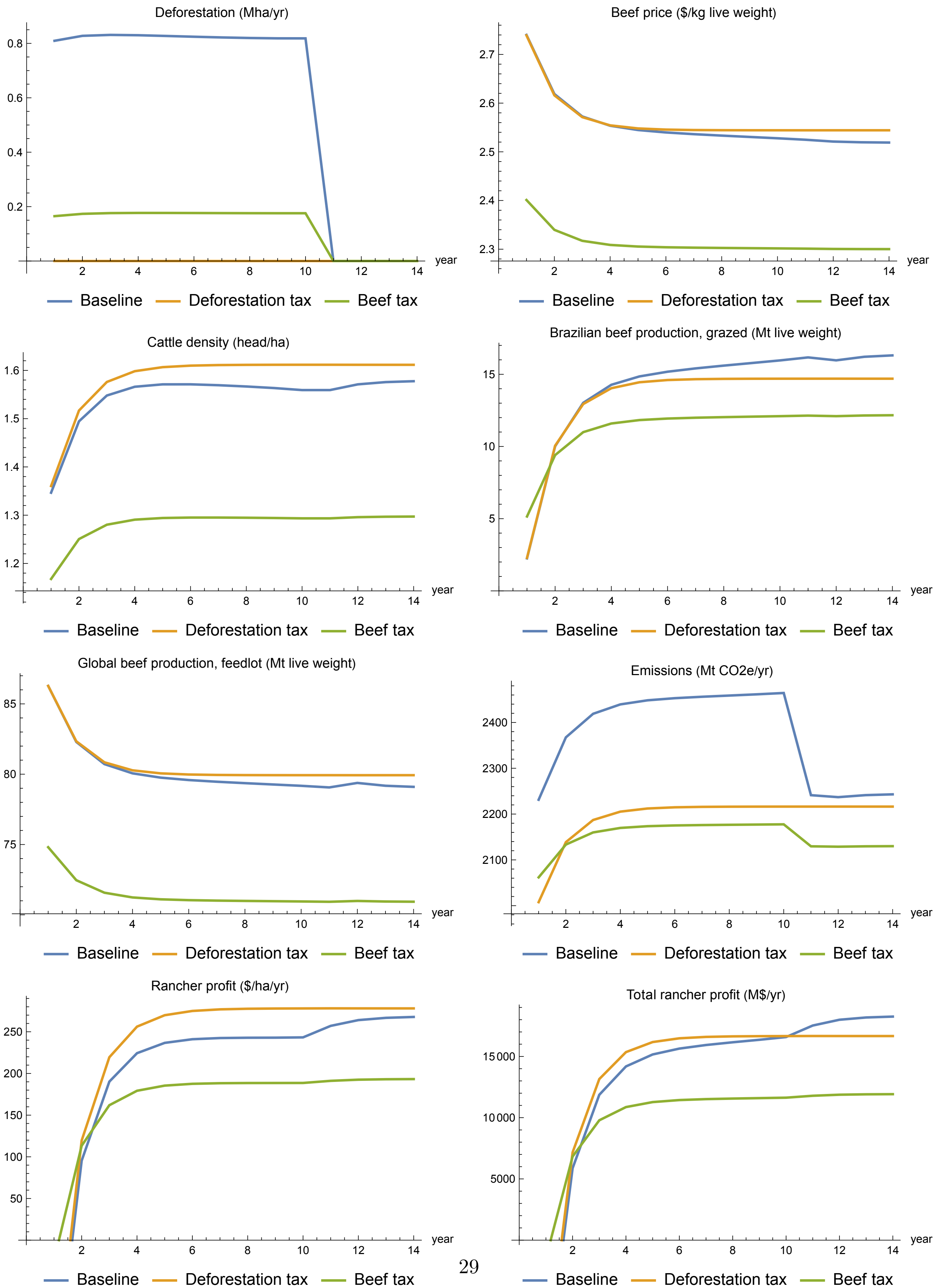
While deforestation taxes would increase profits per acre, pasture land would decrease, and Figure 8 shows that total profits for Brazilian ranchers would decrease slightly with deforestation taxes. The estimated loss of ranching income to Brazil would be \$1331.6 M/yr. Given that forest preservation programs such as REDD+ already receive hundreds of millions of dollars in funding per year, it may be feasible for other countries to compensate Brazil for this loss of income.

7.1 Implications for ILUC and Leakage

Going beyond the explicit policy simulations above, we comment here on the broader conceptual implications of high elasticities for indirect land use change and leakage.

While our dynamic model does not have a single long-run elasticity of deforestation—long-run elaticities can be defined in many different ways—our results clearly indicate that deforestation is highly elastic in the long run. We can see this both in the policy functions and simulations results. As discussed above, we can see a long-run elasticity of approximately 3 in the policy functions displayed in Figure 7. Turning to Figure 8, we can see that the beef tax lowers the price received by ranchers by about 18% in the steady

Figure 8: Counterfactual Simulations



state; meanwhile, during the period when deforestation is allowed, deforestation rates are about 77% lower than in the baseline. This implies a long-run elasticity of about 4.3. Either way, we have a large long-run elasticity of deforestation, and this has important implications for policy concerns like indirect land use change and leakage.

Indirect land use change refers to the phenomenon that, when a policy causes the prices of agricultural commodities to rise, it can induce new agricultural production around the world, potentially far away from the places where the policy is implemented. For a given price change, the amount of indirect land use change in a given location is proportional to the price elasticity of land use in that location. Therefore, our result that deforestation is highly elastic in the long run indicates that policies that increase agricultural commodity prices (and particularly the price of beef) are likely to have large indirect land use change impacts in the Brazilian Amazon.

Highly price elastic deforestation also has important implications for leakage in the context of forest protection programs, such as the UN's REDD+ program. Consider a market clearing equation,

$$Q_D(P) = \lambda Q_S(P),$$

where $Q_D(P)$ is demand for some commodity, $Q_S(P)$ is supply, with $1-\lambda$ being the fraction of land protected. To keep this illustration simple, we ignore yield heterogeneity and targeted land protection, so $1-\lambda$ can represent both the reduction in production and the proportion of land protected. The implicit function theorem implies

$$\frac{dP^*}{d\lambda} = \frac{-Q_S(P)}{\frac{dQ_S(P)}{dP} - \frac{dQ_D(P)}{dP}}.$$

By the chain rule,

$$\frac{dQ_S(P^*)}{d\Delta} = \frac{dQ_S(P^*)}{dP}\frac{dP^*}{d\Delta} = Q_S(P)\frac{\frac{dQ_S(P)}{dP}}{\frac{dQ_S(P)}{dP} - \frac{dQ_D(P)}{dP}}.$$

Putting these expressions together,

$$\frac{d\left(\lambda Q_S(P^*)\right)}{d\lambda} = Q_S\left(P^*\right)\left(1 - \frac{\frac{dQ_S(P)}{dP}}{\frac{dQ_S(P)}{dP} - \frac{dQ_D(P)}{dP}}\right) = Q_S\left(P^*\right)\left(1 - \frac{\mathcal{E_S}}{\mathcal{E_S} - \mathcal{E_D}}\right),$$

where $\frac{\mathcal{E_S}}{\mathcal{E_S}-\mathcal{E_D}}$ is the rate of leakage.

With supply highly elastic (relative to demand), leakage can be close to 100%. While elasticities are not quite so extreme in the context of deforestation, it does seem to be the case that supply elasticities are substantially above demand elasticities. We had back-of-the-envelope estimates of the elasticity of deforestation of 3 and 4.3 above. Meanwhile, our borrowed elasticity of beef demand from the literature was $-.81$. Using these numbers, we have a back-of-the-envelope leakage estimates of 79% and 84%, implying that a large majority of forest protection is undermined by equilibrium effects.

8 Conclusion

This paper models dynamic deforestation and cattle management jointly. In the short run, deforestation may appear highly inelastic or even negatively correlated with prices, which helps explain previous empirical findings in reduced-form regressions. However, our model shows that, in the long run, deforestation becomes highly elastic—a result not captured in existing reduced-form analyses.

These findings have profound policy implications. A simplistic analysis, extrapolating from the reduced-form estimates, might suggest that deforestation is largely unresponsive to economic incentives, leading to the conclusion that policies affecting beef prices would have minimal impact on deforestation, and that forest protection policies would suffer from minimal leakage. Our results indicate otherwise.

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Online Appendix

A Model Details

A.1 Deriving the Euler Equations

A.1.1 Cattle Euler Equation

The first-order condition for the choice of culling B is

$$P_{mt}^{B}Y - \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C_{mt}^{c}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] + \beta E_{t}\left[\frac{d}{dB_{mt}}V\left(\mathbf{K}m, t+1, A_{m,t+1}\right)\right] = 0,$$

which implies

$$\begin{aligned}
&Y \times P_{mt}^{B} - \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C_{mt}^{c}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] \\
&+\beta E_{t}\left[-(1-\delta)\frac{d}{dK_{3,m,t+1}}V\left(K_{1,m,t+1}, K_{2,m,t+1}, K_{3,m,t+1}, A_{m,t+1}\right)\right. \\
&\left. - g\frac{d}{dK_{1,m,t+1}}V\left(K_{1,m,t+1}, K_{2,m,t+1}, K_{3,m,t+1}, A_{r,m,t+1}\right)\right] \\
&= 0,
\end{aligned} \tag{A1}$$

as each cow culled decreases next period's breeding stock by $(1-\delta)$ units and decreases the number of calves next period by g units.

Next, we consider the derivatives of the value function with respect to the cattle stock state variables, using the Envelope Theorem. Recall that $K_{mt} = \gamma_1 K_{1mt} + \gamma_2 K_{2mt} + K_{3mt}$. First, consider the derivative with respect to K_{3mt}. To reduce clutter, we write V_{mt} without its arguments:

$$\begin{aligned}
\tfrac{d}{dK_{3mt}}V_{mt} &= \tfrac{d}{dK_{3mt}}\left[-\left(K_{mt}-B_{mt}\right)C_{mt}^{c}\left(\tfrac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] + \beta E_{t}\left[\tfrac{d}{dK_{3mt}}V_{m,t+1}\right] \\
&= \tfrac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C_{mt}^{c}\left(\tfrac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] \\
&\quad +\beta E_{t}\left[(1-\delta)\tfrac{d}{dK_{3,m,t+1}}V_{m,t+1} + g\tfrac{d}{dK_{1,m,t+1}}V_{m,t+1}\right] \\
&= Y \times P_{mt}^{B},
\end{aligned} \tag{A2}$$

where the second equality uses the fact that

$$\frac{d}{dK_{mt}}\left[-\left(K_{mt}-B\right)C_{mt}^{c}\left(\frac{K_{mt}-B}{A_{rmt}}\right)\right] = \frac{d}{dB}\left[\left(K_{mt}-B\right)C_{mt}^{c}\left(\frac{K_{mt}-B}{A_{rmt}}\right)\right];$$

and the last line follows from (A1).

Next, we consider the value function's derivative with respect to the yearling stock K_{2mt}:

$$\begin{aligned}
\frac{d}{dK_{2mt}}V_{mt} &= \gamma_2 \frac{d}{dK_{mt}}\left[-\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] + \beta E_t\left[\frac{d}{dK_{2mt}}V_{m,t+1}\right] \\
&= \gamma_2 \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] + \beta E_t\left[\frac{d}{dK_{3,m,t+1}}V_{m,t+1}\right] \\
&= \gamma_2 \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] + \beta E_t\left[Y \times P^B_{m,t+1}\right].
\end{aligned} \tag{A3}$$

Now, we take the value function's derivative with respect to the calf stock K_{1mt}:

$$\begin{aligned}
\frac{d}{dK_{1mt}}V_{mt} &= \gamma_1 \frac{d}{dK_{mt}}\left[-\left(K_{mt}-B\right)C^c_{mt}\left(\frac{K_{mt}-B}{A_{rmt}}\right)\right] + \beta E_t\left[\frac{d}{dK_{1mt}}V_{m,t+1}\right] \\
&= \gamma_1 \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] + \beta E_t\left[\frac{d}{dK_{2,m,t+1}}V_{m,t+1}\right], \\
&= \gamma_1 \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] \\
&\quad +\beta E_t\left[\gamma_2 \frac{d}{dB_{m,t+1}}\left[\left(K_{m,t+1}-B_{m,t+1}\right)C^c_{m,t+1}\left(\frac{K_{m,t+1}-B_{m,t+1}}{A_{r,m,t+1}}\right)\right] + \beta E_{t+1}\left[Y \times P^B_{m,t+2}\right]\right]
\end{aligned} \tag{A4}$$

where we have substituted for $\frac{d}{dK_{2,m,t+1}}V_{t+1}$.

We then substitute for the value functions in A1 and get

$$\begin{aligned}
Y \times P^B_{mt} &= \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] \\
&\quad +\beta E_t\left[(1-\delta)\frac{d}{dK_{3,m,t+1}}V_{m,t+1}\right] + \beta E_t\left[g\frac{d}{dK_{1,m,t+1}}V_{m,t+1}\right] \\
&= \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] \\
&\quad +\beta(1-\delta)E_t\left[Y \times P^B_{m,t+1}\right] \\
&\quad +\beta g E_t\left[\gamma_1 \frac{d}{dB_{m,t+1}}\left[\left(K_{m,t+1}-B_{m,t+1}\right)C^c_{m,t+1}\left(\frac{K_{m,t+1}-B_{m,t+1}}{A_{r,m,t+1}}\right)\right]\right] \\
&\quad +\beta g E_t\left[\beta E_{t+1}\left[\gamma_2 \frac{d}{dB_{m,t+2}}\left[\left(K_{m,t+2}-B_{m,t+2}\right)C^c_{m,t+2}\left(\frac{K_{m,t+2}-B_{m,t+2}}{A_{r,m,t+2}}\right)\right]\right]\right] \\
&\quad +\beta g E_t\left[\beta E_{t+1}\left[\beta E_{t+2}\left[Y \times P^B_{m,t+3}\right]\right]\right]
\end{aligned} \tag{A5}$$

Rearranging:

$$\begin{aligned}
Y \times P^B_{mt} &= \frac{d}{dB_{mt}}\left[\left(K_{mt}-B_{mt}\right)C^c_{mt}\left(\frac{K_{mt}-B_{mt}}{A_{rmt}}\right)\right] \\
&\quad +\beta(1-\delta)E_t\left[Y \times P^B_{m,t+1}\right] + g\beta^3 E_t\left[Y \times P^B_{m,t+3}\right] \\
&\quad +g\beta E_t\left[\gamma_1 \frac{d}{dB_{m,t+1}}\left[\left(K_{m,t+1}-B_{m,t+1}\right)C^c_{m,t+1}\left(\frac{K_{m,t+1}-B_{m,t+1}}{A_{r,m,t+1}}\right)\right]\right] \\
&\quad +g\beta^2 E_t\left[\gamma_2 \frac{d}{dB_{m,t+2}}\left[\left(K_{m,t+2}-B_{m,t+2}\right)C^c_{m,t+2}\left(\frac{K_{m,t+2}-B_{m,t+2}}{A_{r,m,t+2}}\right)\right]\right]
\end{aligned} \tag{A6}$$

where nested expectations have been suppressed following the law of iterated expectations.

A.1.2 Deforestation Euler Equation

For the choice of deforestation D, we first recall that $D_{mt} = A_{m,t+1} - A_{mt}$. The rancher's first-order condition with respect to $A_{m,t+1}$ is:

$$\frac{d}{dA_{m,t+1}} C^d_{m,t} \left(A_{m,t+1} - A_{mt}\right) + \beta E_t \left[\frac{d}{dA_{m,t+1}} V_{m,t+1}\right] = 0, \tag{A7}$$

or equivalently,

$$\frac{d}{dD_{mt}} C^d_{mt} \left(D_{mt}\right) + \beta E_t \left[\frac{d}{dA_{m,t+1}} V_{m,t+1}\right] = 0, \tag{A8}$$

Next, let's consider the derivative of the value function with respect to state variable A_{mt}, using the Envelope Theorem:

$$\frac{d}{dA_{mt}} V_{mt} = \frac{d}{dA_{mt}} \pi_{m,t} \left(B_{mt}, \mathbf{K}_{mt}, A_{mt}\right) + \frac{d}{dA_{mt}} C^d_{mt} \left(A_{m,t+1} - A_{mt}\right). \tag{A9}$$

We can express the derivative of agricultural payoffs in terms of the marginal returns to land in either pasture or cropland:

$$\begin{aligned} \tfrac{d}{dA_{mt}} \pi_{m,t} \left(B_{mt}, \mathbf{K}_{m,t}, A_{mt}\right) &= \tfrac{d}{dA_{mt}} \left(\pi_{rmt} \left(B_{mt}, \mathbf{K}_{mt}, A^*_{rmt}\right) + \pi_{cmt} \left(\tfrac{A^*_{cmt}}{A_{mt}}, A_{mt}\right)\right) \\ &= \tfrac{dA^*_{rmt}}{dA_{mt}} \tfrac{d}{dA^*_{rmt}} \pi_{rmt} \left(B_{mt}, \mathbf{K}_{mt}, A^*_{rmt}\right) \\ &\quad + \tfrac{dA^*_{cmt}}{dA_{mt}} \tfrac{d}{dA^*_{cmt}} \pi_{cmt} \left(\tfrac{A^*_{cmt}}{A_{mt}}, A_{mt}\right) \\ &= \tfrac{d}{dA_{cmt}} \pi_{cmt} \left(\tfrac{A_{cmt}}{A_{mt}}, A_{mt}\right). \\ &= \tfrac{d}{dA_{rmt}} \pi_{rmt} \left(B_{mt}, \mathbf{K}_{mt}, A_{rmt}\right) \end{aligned} \tag{A10}$$

The second equality comes from the chain rule. The third and fourth equalities rely on the fact that agricultural land must be divided between pasture and cropland, so that $\frac{dA^*_{rmt}}{dA_{mt}} + \frac{dA^*_{cmt}}{dA_{mt}} = 1$, and from the fact that the marginal returns to land is equalized across crops and pasture (equation **??**).

Next, we can combine equations A8, A9, and (A10):

$$\frac{d}{dD_{mt}} C^d_{mt} \left(D_{mt}\right) = \beta E_t \left[-\frac{d}{dA_{r,m,t+1}} \pi_{rmt} \left(B_{m,t+1}, \mathbf{K}_{m,t+1}, A_{r,m,t+1}\right) + \frac{d}{dD_{m,t+1}} C^d_{m,t+1} \left(D_{m,t+1}\right)\right], \tag{A11}$$

and since pasture land affects payoffs through the holding costs:

$$\frac{d}{dD_{mt}} C^d_{mt} \left(D_{mt}\right) = \beta E_t \left[\frac{d}{dA_{r,m,t+1}} \left(K_{m,t+1} C^r_{m,t+1} \left(\frac{K_{m,t+1}}{A_{r,m,t+1}}\right)\right) + \frac{d}{dD_{m,t+1}} C^d_{m,t+1} \left(D_{m,t+1}\right)\right], \tag{A12}$$

Given the equivalence of the marginal returns to land across crops and pasture (equation **??**), we can also express this deforestation Euler equation in terms of cropland:

$$\frac{d}{dD_{mt}} C^d_{mt}\left(D_{mt}\right) = \beta E_t \left[\frac{d}{dA_{c,m,t+1}} \pi_{c,m,t+1} \left(\frac{A_{c,m,t+1}}{A_{m,t+1}}, A_{m,t+1} \right) + \frac{d}{dD_{m,t+1}} C^d_{m,t+1}\left(D_{m,t+1}\right) \right]. \quad \text{(A13)}$$

A.2 The Representative Rancher

Case 1: no deforestation

Suppose there are potentially many ranchers indexed by r, each of whom owns a pasture area a_r. Let $A \equiv \sum_r a_r$. We're going to compare the situation with many ranchers in a competitive equilibrium to the situation with a single rancher operating the full pasture area A. In either case, the path of beef prices P_t is exogenous

The representative rancher maximizes

$$\max_{\{B_t \geq 0\}} E\left[\sum_t \beta^t A\pi\left(B_t, K_t/A, P_t\right) \right].$$

subject to

$$K_{t+1} = (1 + g - \delta)\left(K_t - B_t\right).$$

Rearranging,

$$B_t = K_t - \frac{K_{t+1}}{1+g-\delta}.$$

Then we can rewrite the problem

$$\max_{\{K_t\}} \sum_t \beta^t A\pi \left(K_t - \frac{K_{t+1}}{1+g-\delta}, K_t/A, P_t \right).$$

The constraint $B_t \geq 0$ in the original problem will translate to the constraint

$$K_t - \frac{K_{t+1}}{1+g-\delta} \geq 0$$

in the modified problem.

First-order condition, assuming interior solution for all t:

$$\partial K_t: \qquad E\left[\frac{d\pi\left(K_{t-1} - \frac{K_t}{1+g-\delta}, K_{t-1}/A, P_{t-1}\right)}{dK_t} + \beta \frac{d\pi\left(K_t - \frac{K_{t+1}}{1+g-\delta}, K_t/A, P_t\right)}{dK_t} \right] = 0 \ ,$$

which we can rewrite

$$\partial K_t: \qquad E\left[\tfrac{d\pi\left(B_{t-1},K_{t-1}/A,P_{t-1}\right)}{dB_{t-1}}+\beta\tfrac{d\pi\left(B_t,K_t/A,P_t\right)}{dK_t}\right] \quad = \quad 0.$$

Notice that these first order conditions depend only on K_t/A for various t—cattle densities. Therefore, if $\{k_t\}$, where $k_t = K_t/A_t$ satisfy the first order conditions for the representative rancher, then the same densities of $\{k_t\}$ solve the first order conditions for a rancher of any size. Thus, the representative rancher will choose the same cattle density that smaller ranchers would.

Case 2: with deforestation

Now we add the decision of how much to deforest, D_t. This cost function is thought to be concex, i.e., $C(D) = \gamma_D D^2$. Deforestation increases the pasture area: $A_{t+1} - A_t = D_t$ The representative rancher's problem is now

$$\max_{\{K_t,A_t\}}\sum_t \beta^t\left[A_t\pi\left(K_t-\frac{K_{t+1}}{1+g-\delta},K_t/A_t,P_t\right)-C\left(A_{t+1}-A_t\right)\right].$$

With the representative rancher, we now have these first-order conditions:

$$\begin{aligned}
\partial K_t: &\qquad E\left[A_{t-1}\tfrac{d\pi\left(B_{t-1},K_{t-1}/A_{t-1},P_{t-1}\right)}{dB_{t-1}}+\beta A_t\tfrac{d\pi\left(B_t,K_t/A_t,P_t\right)}{dK_t}\right] &= \quad 0\\
\partial A_t: &\qquad E\left[-C'\left(A_t-A_{t-1}\right)+\beta\left(A_t\tfrac{d\pi\left(B_t,K_t/A_t,P_t\right)}{dA_t}+\pi\left(B_t,K_t/A_t,P_t\right)+C'\left(A_{t+1}-A_t\right)\right)\right] &= \quad 0.
\end{aligned}$$

For the competitive market, we imagine that there is a continuum of forest landowner engaging in deforestation, each of whom owns some infinitesimal amount of land. Landowner i's decision variable is D_i. For each of them, the marginal cost of deforestation is given by $2\gamma_D D$, where $D = \int D_i di$. The idea here is that the convexity of deforestation cost does not come from convexity in a firm-specific deforestation cost function, but from some underlying scarcity—e.g., we can suppose that labor is the main input into deforestation, and the local labor supply curve is upward sloping. Then, the wages that clear the market are increasing with respect to the level of deforestation.

Landowners sell land to ranchers in a frictionless land market. Let P_t^A be the price of pasture land per hectare in time t.

A rancher's objective function is

$$\max_{\{K_t,a_t\}}\sum_t \beta^t a_t\pi\left(K_t-\frac{K_{t+1}}{1+g-\delta},K_t/a_t,P_t\right)-P_t^A\left(a_t-a_{t-1}\right).$$

A rancher's first order conditions will be

$$\begin{aligned}\partial K_t: \qquad E\left[a_{t-1}\frac{d\pi\left(B_{t-1},K_{t-1}/a_{t-1},P_{t-1}\right)}{dB_{t-1}}+\beta a_t\frac{d\pi\left(B_t,K_t/a_t,P_t\right)}{dK_t}\right] &= 0\\ \partial a_t: \quad E\left[\left(a_t\frac{d\pi\left(B_t,K_t/a_t,P_t\right)}{da_t}+\pi\left(B_t,K_t/a_t,P_t\right)-P_t^A\right)+\beta P_{t+1}^A\right] &= 0.\end{aligned}$$

A forest landowner's objective function, assuming they always sell the land they deforest immediately

$$\max_{\left\{A_{it}^F\right\}}\sum_t \beta^t\left[P_t^A\left(A_{it-1}^F-A_{it}^F\right)-\left(A_{it-1}^F-A_{it}^F\right)C'\left(D_t\right)\right],$$

noting that the aggregate deforestation rate D_t is exogenous from the perspective of an infinitesimally small landowner.

$$\partial A_t^F : E\left[-P_t^A+C'\left(D_t\right)+\beta\left(P_{t+1}^A-C'\left(D_{t+1}\right)\right)\right]=0.$$

Substituting the deforester's Euler equation into the rancher's a_t Euler equation,

$$\partial a_t: \quad E\left[\left(a_t\frac{d\pi\left(B_t,K_t/a_t,P_t\right)}{da_t}+\pi\left(B_t,K_t/a_t,P_t\right)-C'\left(D_t\right)\right)+\beta C'\left(D_{t+1}\right)\right] = 0.$$

Now, if we consider a solution to the represenative rancher's problem and let $k_t = K_t/A_t$, that same cattle density and $a_t/a_{t-1} = A_t/A_{t-1}$ will solve a small rancher's decision problem.

B Data Appendix

B.1 Transportation Costs

Our strategy for measuring transportation costs to the port is similar to that in Souza-Rodrigues (2019).

We begin by defining the locations of thirteen ports, one location for each Brazilian state that has access to the ocean. Locations were selected by hand using the Brazilian government's shapefiles describing port locations.[8] The list of points is

- Santos, São Paulo: latitude -23.961°, longitude -46.294°
- Paranaguá, Paraná: latitude -25.502°, longitude -48.506°
- Rio Grande, Rio Grande do Sul: latitude -32.043°, longitude -52.076°
- Vitória, Espírito Santo: latitude -20.277°, longitude -40.236°
- São Francisco do Sul, Santa Catarina: latitude -26.239°, longitude -48.635°
- São Luís, Maranhão: latitude -2.569°, longitude -44.370°

[8] Obtained from https://www.gov.br/transportes/pt-br/assuntos/dados-de-transportes/bit/bit-mapas.

- Salvador, Bahia: latitude -12.786°, longitude -38.477°
- Recife, Pernambuco: latitude -8.054°, longitude -34.868°
- Maceió, Alagoas: latitude -9.683°, longitude -35.726°
- Pecém, Ceará: latitude -3.527°, longitude -38.796°
- Rio de Janeiro, Rio de Janeiro: latitude -22.892°, longitude -43.195°
- Barcarena, Pará: latitude -1.492°, longitude -48.591°
- Aracaju, Sergipe: latitude -10.906°, longitude -37.048°

The next step is to construct a transportation network. Because beef is primarily transported by road, we ignore railways and rivers for this exercise, but we distinguish between paved and unpaved roads. Brazil's National Bureau of Infrastructure provides shapefiles of federal highways and waterways (`https://www.gov.br/dnit/pt-br/assuntos/atlas-e-mapas/pnv-e-snv`). Brazil's Ministry of Transport provides shapefiles for the federal road network, railway network, waterways, and ports (`https://www.gov.br/transportes/pt-br/assuntos/dados-de-transportes/bit/bit-mapas`). Finally, a shapefile of state highways was obtained from the World Bank (`https://datacatalog.worldbank.org/search/dataset/0038536`).

We use proprietary data from ESALQ the per-kilometer cost of transporting beef by road. We directly use freight costs from this dataset as our measure of the cost of transportation by paved road. We then assume that unpaved roads have a 30% higher cost per kilometer than paved roads, following Souza-Rodrigues (2019).

Then, for each municipality-port pair, we compute lowest possible cost of transporting beef from the centroid of the municipality to the port, taking into account the modes of transportation along each potential routes. To do this, we use a shortest path algorithm, noting that we search for the path that minimizes transportation cost, not distance. Then, for each municipality, we choose the port with the lowest transportation cost. The cost-minimizing path does not vary across years because each potential route's cost is linearly proportional to the per-kilometer cost of transporting beef.

The ESALQ data only covers 1997-2012, so we use this procedure to construct transportation costs c_{mt} for each municipality m and year t through 2012. For subsequent years, we project transportation costs using diesel prices obtained from the Brazilian National Agency for Petroleum, Natural Gas and Biofuels. Specifically, we use OLS to estimate the equation

$$c_{mt} = \alpha_{0m} + \alpha_P P_{diesel,t} \cdot \text{dist}_m + \epsilon_{mt},$$

where dist_m is the distance of the selected path for municipality m. For 2013 onward, we use fitted values from this regression to impute transportation costs.

B.2 Land Prices

We run some descriptive regressions with our land price data to illustrate that it has sensible correlations with observable municipality-level characteristics. As anticipated, higher land productivity, proxied by grass yields, is associated with increased land prices, while higher transportation costs to the port lower land values. Moreover, larger cattle stocks and a greater share of cropland within a municipality indicate higher land demand, which pushes prices upward.

Table B1: Land prices and covariates

	(1) ln(Land price)	(2) ln(Land price)	(3) ln(Land price)	(4) ln(Land price)
ln(Grass yield)	5.19***	4.70***	4.20***	4.20***
	(0.54)	(0.55)	(0.52)	(1.56)
ln(Transportation cost)	-1.62***	-1.45***	-1.26***	-1.26***
	(0.13)	(0.13)	(0.14)	(0.43)
Herd density		0.33***	0.32***	0.32**
		(0.046)	(0.047)	(0.14)
Cropland share			1.12***	1.12**
			(0.16)	(0.49)
Observations	902	902	902	902
R-squared	0.614	0.632	0.645	0.645
Year FE	✓	✓	✓	✓
State FE	✓	✓	✓	✓
Clustered by municipality				✓

Observations are by municipality and year. Robust standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

Figure B1 Displays a histogram of land prices. Note that even the highest land prices are well below $10,000 per hectare, which would be a conservative estimate of the value of carbon released by a hectare of slash and burn deforestation in the Amazon.

B.3 Culling Rate Measurement

Our cattle stock data include only total headcounts, not headcounts by age. This limitation makes it challenging to precisely measure culling rates, as we cannot directly observe the age distribution of the herd. We let $K_{mt}^{count} = K_{1mt} + K_{2mt} + K_{3mt}$ denote the observed cattle stock in year t. Note that this raw count is different from the model's aggregate K_{mt}, in which the calves and yearlings are weighted down to produce an adult-equivalent measure.

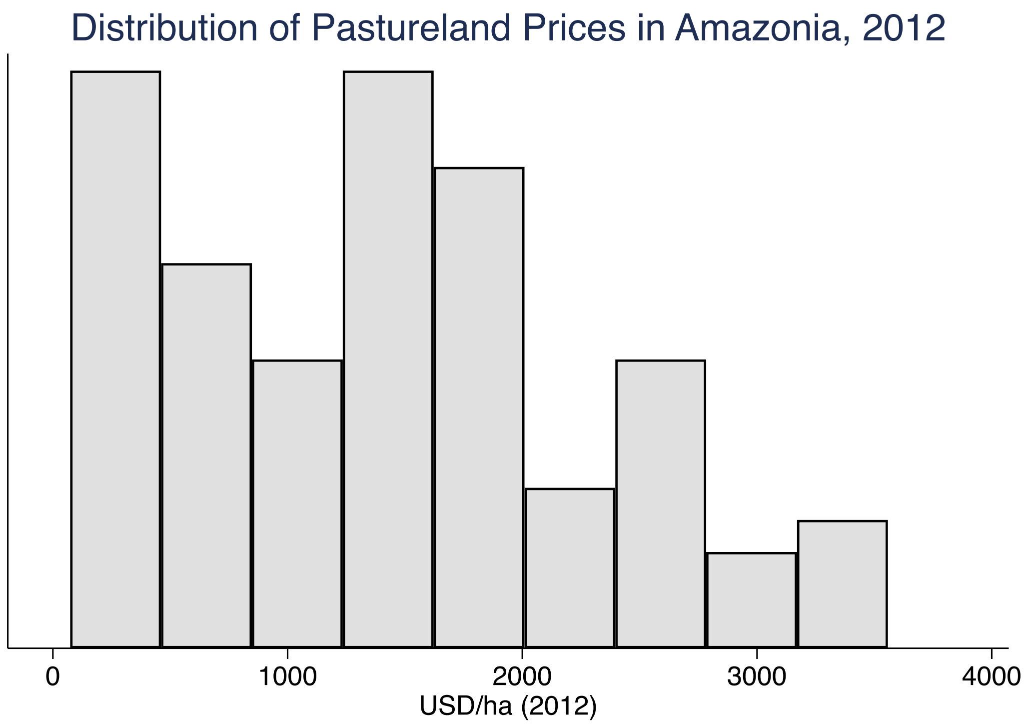
Given the herd dynamics in equations (2)-(4), we can express the time-$t+1$ cattle stock as follows:

$$K_{m,t+1}^{count} = (1 - \delta + g)(K_{3mt} - B_{mt}) + K_{1mt} + K_{2mt}.$$

Rearranging, we have

$$B_{mt} = K_{3mt} - \frac{K_{m,t+1}^{count} - K_{1mt} - K_{2mt}}{1 - \delta + g}. \tag{B14}$$

Figure B1: Histogram of log land prices (in $ per hectare)



If the initial cattle stocks by age were observed, equation (B17) would allow us to measure the culling rate directly. In the absence of such data, we pursue two strategies to estimate the culling rate.

The first is that we assume the initial period is in a steady state. However, there is a continuum of steady states with different ratios of young and old cattle. Observe that if we impose a steady state with constant young and adult stocks—that is, $K_y = K_{1mt} = K_{2mt}$ and and $K_3 = K_{3mt}$, but not necessarily $K_3 = K_y$—then the cattle dynamic equations imply

$$K_3 = (1 - \delta)(K_3 - B) + K_y \tag{B15}$$

$$K_y = g(K_3 - B). \tag{B16}$$

Here, we have two equations and three unknowns (K_3, K_y, and B), so we need to impose another condition to pin down a unique steady state. Aggregate statistics from IBGE show a ratio of adult cows to yearlings of about 2.8. We therefore impose $K_3/K_y = 2.8$ in the initial period. That is, in the initial period $t = 1$ we assume that $K_{3mt} = 2.8K_{1mt} = 2.8K_{2mt}$, which implies $K_{1mt} = K_{2mt} = K_{mt}^{count}/4.8$, and $K_{3mt} = 2.8K_{mt}^{count}/4.8$. We can then use (B17) to measure the culling rate in the initial period, and we can use the herd dynamic equations to infer the cattle stocks by age and culling rates for subsequent periods. In the text, we refer to this measure, which imposes a demographic assumption on the initial cattle stock, as our *primary measure* of the culling rate.

An alternative measure imposes the herd demographic assumptions within each period. Specifically, we impose $K_{1mt} = K_{2mt} = K_{mt}^{count}/4.8$ and $K_{3mt} = 2.8K_{mt}^{count}/4.8$ within each time period, modifying equation (B17) to obtain

$$B_{mt} = 2.8K_{m,t}^{count}/4.8 - \frac{K_{m,t+1}^{count} - 2K_{m,t}^{count}/4.8}{1-\delta+g}. \tag{B17}$$

In other words, we do for every period what the primary measure does for the initial period. In the text, we refer to this measure, which imposes a demographic assumption on each period's cattle stock, as our *alternative measure* of the culling rate.

B.3.1 Simplified Model

Note that in the simplified version of the model without time-to-build, it is straightforward to infer the culling rate from observed changes in the cattle stock. In this version of the model, where there is only one measure of the cattle stock, the herd dynamics are characterized by

$$K_{t+1} = (1+g-\delta)(K_t - B_t).$$

We can rearrange this equation to express the culling rate as

$$B_t = K_t - \frac{K_{t+1}}{1-\delta+g}.$$

C Robustness

C.1 Dropping low deforestation rates

While the data do not include zero deforestation rates, this is likely due to measurement error. Some municipalities have very low deforestation rates, and misclassification can produce spurious non-zero rates. See Torchiana et al. (2025) for a discussion of measurement error in the context of measuring land use change.

To assess robustness to potential zeros, we re-estimate the model after dropping very low deforestation rates. Table (C2) demonstrates that our cost estimates are robust to dropping municipalities with very low deforestation rates.

C.2 Dropping low deforestation rates

One concern is that Amazonia is an expansive region with variation in agricultural practices across it. Table (C3) presents cost estimates for two sub-samples, in contrast to our main sample In column (2), we drop municipalities that always have forest cover above 95%. These tend to be very large municipalities in the Amazonas and Pará states. In column (3), we drop all municipalities within the states of Mato Grosso and Rondônia, which have relatively high stocking densities and are known to have a non-trivial

Table C2: Robustness to Zeros: Simplified Cattle Dynamics

	(1)	(2)	(3)
θ_{r0}	219.858	211.237	194.157
	(36.41,371.39)	(23.78,363.73)	(8.5227,348.66)
θ_{r1}	1403.39	1399.97	1380.36
	(1104.3,1841.8)	(1097.7,1845.7)	(1077.3,1814.4)
θ_d	34175.6	33820.3	31235.2
	(22233.,54371.)	(21871.,49778.)	(20534.,46019.)
Post-2006	720.299	743.919	772.51
	(327.92,943.23)	(365.01,978.71)	(343.88,1044.4)
Blacklist	470.119	447.35	403.305
	(31.549,1168.4)	(8.1036,1059.8)	(−24.071,1052.6)
Mean holding cost	427.965	424.559	416.721
	(298.15,545.14)	(291.84,542.89)	(280.17,536.46)
Mean DF cost	904.263	933.564	969.364
	(743.19,1061.7)	(759.56,1102.2)	(780.06,1167.3)
Mean marginal DF cost	1240.	1278.34	1332.57
	(1079.,1462.1)	(1110.7,1493.6)	(1154.2,1557.6)
Observations	12773	12274	10518

Notes: The table presents cost parameter estimates and derived statistics for specifications with simplified cattle dynamics (i.e., no calf or yearling stocks). Column (1) uses the full sample. Column (2) drops observations involving deforestation rates below .05%. Column (3) drops observations involving deforestation rates below .2%. Mean cost statistics average over municipalities and years in our sample. The Post-2006 and Blacklist dummy variables are included in the deforestation cost function. Standard errors, shown in parentheses, are clustered by year and calculated using the block bootstrap procedure. Observations are by municipality-year.

Table C3: Robustness in Subsamples: Simplified Cattle Dynamics

	(1)	(2)	(3)
θ_{r0}	219.858	208.144	363.641
	(36.41,371.39)	(18.845,360.33)	(−335.61,529.55)
θ_{r1}	1403.39	1427.88	1032.87
	(1104.3,1841.8)	(1120.2,1876.9)	(606.74,3807.)
θ_d	34175.6	33351.4	30888.1
	(22233.,54371.)	(21630.,53042.)	(15122.,2.23835×10^5)
Post-2006	720.299	757.683	111.954
	(327.92,943.23)	(345.63,988.31)	(−2277.,381.52)
Blacklist	470.119	474.503	255.3
	(31.549,1168.4)	(31.699,1198.1)	(−155.8,1744.5)
Mean holding cost	427.965	422.804	501.516
	(298.15,545.14)	(290.6,540.88)	(172.57,630.98)
Mean DF cost	904.263	939.992	410.104
	(743.19,1061.7)	(776.,1107.)	(225.78,676.63)
Mean marginal DF cost	1240.	1281.86	729.108
	(1079.,1462.1)	(1116.5,1505.8)	(564.29,2890.1)
Observations	12773	12178	9549

Notes: The table presents cost parameter estimates and derived statistics for specifications with simplified cattle dynamics (i.e., no calf or yearling stocks). Column (1) uses the full sample. Column (2) drops observations from municipalities that always have forest cover above 95%. Column (3) drops all municipalities from Mato Grosso and Rondônia. Mean cost statistics average over municipalities and years in our sample. The Post-2006 and Blacklist dummy variables are included in the deforestation cost function. Standard errors, shown in parentheses, are clustered by year and calculated using the block bootstrap procedure. Observations are by municipality-year.

presence of feedlots recently.

Results are relatively robust, with the only notable difference is that holding costs appear to be slightly higher when dropping Mato Grosso and Rondônia.