A Risk-Based Liquidity Theory of International Currency*

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Abstract

I develop an open-economy monetary model with multiple currencies, where the ranking of currencies as media of exchange in international trade is endogenously determined in decentralized bilateral meetings. The ranking is driven by the currencies' ability to hedge against counterparties' uninsured income shocks, maximizing trade gains. Supporting the theory, countries with procyclical local currency valuations settle more trades in U.S. dollars, as observed in the data. The calibrated model explains the global role of the dollar as a vehicle currency, facilitating trades not directly involving the U.S., and the euro's role as a local medium of exchange in the Eurozone. Flight-to-quality, driven by risk aversion, increases demand for the USD as the global medium of exchange, highlighting the welfare consequences of its dominance.

Keywords: International currencies, medium of exchange, monetary search, liquidity, risk
JEL Codes: D51, D83, F4, E41

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1 Introduction

The U.S. dollar dominates international trade invoicing. This paper documents this tendency is particularly pronounced in countries where the USD valuation is more countercyclical relative to their national incomes, providing superior hedging against aggregate income fluctuations. I present a novel open-economy model with multiple currencies, where international trade is decentralized, with currencies serving as media of exchange for goods and agents facing uninsured future income fluctuations. I show that the hedging properties of currency returns against these fluctuations endogenously determine their ranking as medium of exchange, novelly formalizing the connection between hedging properties and trade invoicing observed in the data. By calibrating these hedging properties using foreign exchange rate data, I demonstrate that the model accounts for the global dominance of the dollar as the international medium of exchange facilitating global trades and the local role of the euro as medium of exchange within the Eurozone, despite their similar average real holding costs in terms of inflation. The demand for the dollar in trades becomes particularly pronounced under conditions of heightened risk aversion, illustrating the flight-to-quality effect in trade invoicing, as observed after the Global Financial Crisis and the European Debt Crisis, and resulting in sizable welfare gains from dollar trade settlements with the superior hedging property.

I begin by developing a tractable two-country model (the U.S. and Latin America) with two currencies (dollar and peso), in the spirit of Lagos and Wright (2005), where each period is divided into two subperiods. There are two types of agents in each country: buyers and sellers. In a subperiod, buyers can consume consumption goods and rebalance currencies in a Walrasian foreign exchange centralized market by using their labor income. On the other hand, sellers consume from the stochastic returns from their retained currency holdings and stochastic income, which are uninsurable in the foreign exchange market, as in Jacquet and Tan (2012) and Drenik, Kirpalani, and Perez (2021). The government in each country stochastically supplies currencies and determines the stochastic inflation rates.

In the decentralized international trade market in the subsequent subperiod, a buyer is randomly matched with a seller, potentially from a different country. Buyers wish to consume goods, but cannot produce them, while sellers do not wish to consume but are able to produce these goods. In the bilateral meeting, the buyer makes a take-it-or-leave-it offer to the seller, determining the terms of trade, which involves transferring a currency portfolio in exchange for the DM goods. Sellers do not impose any restrictions on the choice of currency as the medium of exchange.

My theory endogenously determines the ranking of the medium of exchange, based on the relative quality of currency returns as a hedge against the seller's income risk that will materialize in the following foreign exchange market. The buyer chooses to transfer a currency that provides better hedging against the seller's shock exposure, thereby securing better terms of trade. The endogenous, superior medium of exchange in the international trade generates the differential utility benefits of buying and holding alternative currencies in advance, captured by a higher liquidity premium. This mechanism determines a unique demand for each individual currency, and hence, nominal exchange rates in equilibrium, breaking the indeterminacy of Kareken and Wallace (1981).

I then provide novel empirical evidence for these theoretical predictions regarding the hedging properties, using data on the cross-country dispersion of the dollar's share in import invoicing, recently collected by Boz, Casas, Georgiadis, Gopinath, Mezo, Mehl, and Nguyen (2020), and data on local deposit currency holdings, as collected by Levy-Yeyati (2006). In the regressions, the hedging property of the dollar, proxied by the correlation between dollar returns and local GDP growth, significantly predicts the dollar share in both datasets, even when controlling for other predictors suggested by the theory, such as the shares of trade volumes with the U.S. and average local inflation rates. Innovatively, the significance of the hedging property in trade invoicing strengthened following the Global Financial Crisis and the European Debt Crisis, highlighting the flight-to-quality demand for dollar invoicing.

Supported by the empirical evidence from various countries, I extend the theory to a quantitative model with three trading regions: the U.S., the Eurozone, and Latin America. The selection of these regions is guided by previous empirical findings in the literature. As documented by Boz et al. (2020), the dollar has been the dominant international medium of exchange for several decades, while the euro has sought to compete but remains the locally dominant medium within the Eurozone. The Latin American region serves as a prototypical example, having adopted the dollar as both the medium in domestic and international trade, as well as the local deposit currency, as stated by Zhang (2014). Most importantly, I calibrate the key parameters of the model to data counterparts, without directly targeting currency shares in trade invoicing and local deposit holdings. Key moments, the hedging properties of currencies are calibrated to the covariances between currency returns and local real GDP growth, while the meeting probabilities with sellers from other countries in the DM are calibrated to trading volume shares.

I demonstrate that the hedging properties enable the model to successfully match the observed currency shares in trade invoicing and local deposit holdings. The average holding costs of the dollar and euro are comparable when only accounting for average inflation. Without hedging motives, this would lead to the indeterminacy of currency choice and nominal exchange rates, as shown in Kareken and Wallace (1981). However, with hedging motives, sellers in the U.S. and Latin America endogenously prefer to receive the dollar as the medium of exchange in international trades, due to its superior hedging properties. This happens in equilibrium since the dollar appreciates precisely when these regions experience negative income shocks, as observed in the data. As a result, buyers in both the U.S. and Latin America exclusively hold the dollar to secure the best terms of trade, as they are most likely to trade with sellers from these regions given the largest trade volume. Consequently, Eurozone sellers receive the dollar from Latin American buyers, even though U.S. agents are not directly involved in such trades, and despite the non-negligible trade volume between the Latin America region and the Eurozone. This highlights the dollar's role as a vehicle currency. The hedging motives and search frictions novelly rationalize the globally dominant role of the dollar as the international medium of exchange as an equilibrium outcome.

On the other hand, the Eurozone sellers endogenously prefer to receive the euro the most, as it provides the best hedge against their future income shocks. Consequently, Eurozone buyers choose to exclusively hold the euro to optimize the terms of trade, as they are most likely to trade with sellers from the Eurozone given the largest trade volume. Therefore, the model also rationalizes the local dominance of the euro within the Eurozone in equilibrium.

I then evaluate the welfare implications of counterfactual policies that prohibit the use of a part or all of foreign currencies in international trades and their holdings in deposits in the Latin American region. Specifically, prohibiting the dollar payment in trades could deteriorate the terms of trade that Latin American buyers face, as they would no longer be able to use the best medium with the superior hedging property. This welfare cost becomes particularly pronounced during times of flight-to-quality episodes, represented by heightened risk aversion, which further increases the hedging demand for the dollar in trades.

Finally, I analyze the optimal Ramsey problems for the global planner and national governments, who choose their state-dependent currency supplies and, consequently, inflation rates. The global planner maximizes the weighted average of countries' welfare by determining the supply of all currencies in the first stage, while private agents make their decisions in the second stage based on the planner's choices. I then study a simple policy game played between the national governments and private agents, where governments simultaneously choose their own currency supplies in the first stage, and private agents' decisions follow in the second stage.

I show that the global planner adopts the Friedman rule on average, such that the average growth of currency supplies equals the negative of the real rate for an illiquid bond. The planner chooses to maximize the hedging properties of currency returns to optimize gains from trade. Surprisingly, national governments also aim to maximize hedging properties, as they seek to improve the terms of trade for their citizens and increase the demand for their currencies to maximize seigniorage revenue. These results help rationalize the current superior hedging properties of the dollar by both the global planner's and local governments' objectives.

Related literature: This paper contributes to the empirical and theoretical literature on currency choice in trade invoicing and settlements. Recent work by Boz et al. (2020) constructs a comprehensive panel dataset of currency shares in trade invoicing and settlements, corroborating earlier findings of the persistent global dominance of the dollar and the limited dominance of the euro (Goldberg and Tille (2008) and Goldberg (2011)). Complementing this body of literature, I empirically show that the hedging property is the key determinant of currency choice in international trades and rationalize in a novel general equilibrium model that this property can endogenously explain the observed trade currency choice.

My theory of the international medium of exchange with uninsured income shocks is also related to a body of literature that develops money-search models in open economies pioneered by Matsuyama, Kiyotaki, and Matsui (1993), aiming to replicate currency shares in international trade and currency holdings by directly targeting them in calibrations (Zhang (2014), Gomis-Porqueras, Kam, and Waller (2017), and references therein). In contrast to these papers, I calibrate the key determinants of currency choice, such as average currency holding costs and hedging properties, to the data and demonstrate that the model can quantitatively account for trade invoicing and currency holding patterns without directly targeting them. Furthermore, the observed rise in dollar invoicing following the Global and European Debt Crises can be rationalized in my model by the increased demand for the dollar with heightened risk aversions. This result novelly extends the insights by Maggiori, Neiman, and Schreger (2019) of the heighened flight-to-quality demand for the USD to trade invoicing contexts.

The study of optimal policies in this paper builds on the framework from earlier literature that derives optimal deterministic currency supply rules (Li and Matsui (2009), Zhang (2014)). I focus on state-dependent currency supply rules in the presence of hedging motives. This introduces incentives for both the global planner and national governments to endogenously maximize the hedging properties of currencies, thereby improving the terms of trade. The optimal policies presented in this paper provide a rationale for the dollar's current superior hedging properties and the non-negligible seigniorage revenue reported by Portes and Rey (2002) and Goldberg (2011).

Finally, a distinct strand of literature has examined the role of currencies' hedging properties in currency choice, particularly in debt contracts within emerging countries (Bocola and Lorenzoni (2020), Drenik, Kirpalani, and Perez (2021), Oskolkov and Sorá (2023), Dalgic (2024)) and across countries (Obstfeld, Shambaugh, and Taylor (2010), Gourinchas, Rey, and Govillot (2017), Maggiori (2017)). These studies argue that the use of foreign currency denominations in domestic contracts arises from efficient risk-sharing between borrowers and lenders in Walrasian markets. Moreover, policies that eliminate foreign currencies from the choice of denomination can reduce welfare. In contrast, my theory emphasizes the hedging opportunities available in decentralized markets, which protect against shocks that remain uninsured in Walrasian markets. Relatedly, Lustig and Verdelhan (2007) documents the significant predictability of the U.S. aggregate consumption growth risk for cross-sectional dispersions of currency risk premia, underscoring the substantial hedging demand. This paper shows that the new channel in my model can amplify the negative welfare effects of policies aimed at eliminating foreign currencies by hindering the risk sharing in trades.

The structure of this paper: Sections 2 and 3,describe the baseline two-country model with multiple currencies, and Section 4 defines and characterizes the equilbria. Section 5 provides empirical support for theoretical predictions. Section 6 develops a three-region model with multiple currencies calibrated to data, compares the key model predictions with the data, and evaluate the welfare consequences of the policy that eliminates the foreign currencies from the currency choice in trades. Section 7 studies the optimal policy problem of the global planner and a simple policy game of national governments. Finally, Section 8 concludes.

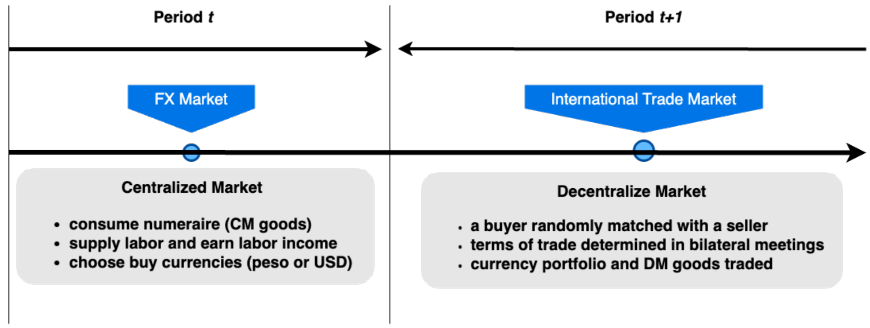

Figure 1: Timing of events 1

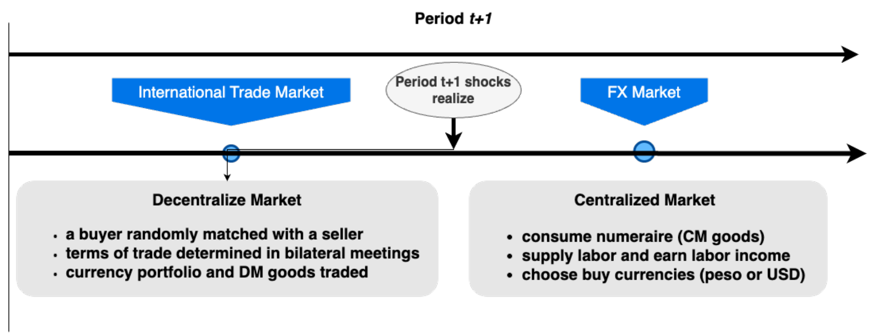

Figure 2: Timing of events 2

2 Environment

In this section, I develop a tractable two-country model to illustrate the main model mechanics studied throughout this paper in Section 3, and 4, and bring the model prediction to the data in Section 5. In Section 6, I extend the model to a quantitative version of a three-country model calibrated to the data.

Time is discrete and continues indefinitely. There are two countries, the U.S. (US) and the Latin America region (LM). Each period consists of two sub-periods: the first is for decentralized trade in local and foreign goods (DM) involving international trades, and the second is for settlement and currency exchange (CM). In each period, the state of the world is stochastic and independent and identically distributed (iid) over time. The state (shock) for a given period is revealed in the CM before the markets open.

Each country is populated by infinitely-lived agents. In US (LM), there are agents with mass 2 ($2n$), where n is the relative country size of country LM to US.

There are two types of agents in each country, which I will refer to as buyers and sellers, based on the roles they play in the DM. The agents are evenly divided between these two roles: in each period t, sellers from country $j \in \{US, LM\}$ can produce output $c_t^{s,j}$, but do no want to consume, while buyers want to consume but cannot produce. Sellers have immobile factors of production and are unable to produce the goods of the other country.

In the second sub-period, all trade occurs in a frictionless competitive market (CM). The type of an agent is denoted by $(x, j) \in \{b, s\} \times \{US, LM\}$, where the first element is either buyer or seller (b or s) and the second element is the origin of country.

In each period t, agents can consume a numeraire good, $C_t^{x,j}$, which is produced according to a linear production function in labor, and supply hours of work in the CM, $N_t^{x,j}$. On the other hand, sellers are not allowed to hold a positive amount of currencies at the end of each CM[1]. Figure 1 and 2 summarizes the timing of events and market structure, respectively.

Importantly, sellers are exposed to an income shock $1/\theta_{t+1}^{s,j}$ in each CM. They represent any co-movements with aggregate national economic fluctuations. On the other hand, buyers are assumed not to be exposed to these shocks.

Discussion of income shock $\theta_{t+1}^{s,j}$: *The shocks can be viewed as aggregate consumption risk, similar to the framework in standard consumption-based asset pricing literature (Lustig and Verdelhan (2007) and Chien, Lustig, and Naknoi (2020)). In emerging market economies, these aggregate shocks are often associated with economic crises, such as banking or currency crises. Following the approach of Chirstiano, Dalgic, and Nurbekyan (2022), this paper remains agnostic regarding the specific origins of these shocks, focusing instead on their implications for hedging motives and the choice of medium of exchange. Chirstiano, Dalgic, and Nurbekyan (2022) empirically documents that, unlike external financial flows, domestic deposit dollarization does not cause currency or banking crises in emerging markets. This finding aligns with the treatment of income shocks in this paper, where they are considered exogenous with respect to endogenous decisions, such as*

[1] Adams and Verdelhan (2025) empirically documents the limited hedging position of exporting and importing public firms against foreign exchange risks in Japan, Korea, Taiwan, and India. Alternatively, I could restrict the underlying parameter values such as lowering discount factor such that sellers do not want to hold any currencies in the CM.

currency portfolio choice.

Discussion on heterogeneity: *In the model, sellers are assumed to be more exposed to shocks than buyers, creating a key source of heterogeneity that drives the hedging motive for sellers. This type of heterogeneity between buyers and sellers has also been employed in the study of currency choice in financial contracts between borrowers and lenders, as explored by Drenik, Kirpalani, and Perez (2021). Similarly, in a search-theoretic framework, this assumption has been used by Jacquet and Tan (2012) to investigate the monetary policy transmission mechanism to asset prices via the liquidity channel. They argue that the heterogeneous exposure to aggregate shocks aligns with the risks faced by entrepreneurs, who encounter undiversifiable income shocks, compared to buyers, who represent households. Entrepreneurs and rich households, being more vulnerable to aggregate shocks, embody the spirit of Heaton and Lucas (2000), Moskowitz and Vissing-Jorgensen (2002), and Parker and Vissing-Jorgensen (2009), which applied similar assumptions to address the equity premium and risk-free rate puzzles*[2].

Chien, Lustig, and Naknoi (2020) also emphasizes the role of imperfect risk-sharing both within and across countries, due to limited participation and infrequent portfolio adjustments in financial markets. Similar to my current framework, their model concentrates world aggregate and country-specific risk on a subset of households. It successfully reconciles the observed smooth exchange rates with the high Sharpe ratios, despite the low international correlations of macroeconomic fundamentals.

In the recent international macroeconomics literature, this assumption has been utilized to analyze various phenomena, as seen in the works of Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), and Dalgic (2024). These studies highlight the relevance of such heterogeneity in explaining the observed dollarization of the domestic financial flows and in analyzing the welfare consequences of financial de-dollarization policies in emerging market countries.

For tractability, utilities for both buyers and sellers are additively separable and quasi-linear in

[2]Alternatively, building on the seminal work by Constantinides and Duffie (1996), Heaton and Lucas (1996) and Heaton and Lucas (1997) found the crucial roles of nontradable labor income risk for pricing risky financial assets.

hours[3]. For buyers from country j,

$$U_0^{b,j} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[u(c_t) + U(C_t) - N_t\big] \tag{1}$$

where c_t is the DM consumption, C_t is the CM consumption, N_t is the labor supply in the CM, $\beta \in (0,1)$ is the discount factor. Similarly, for sellers,

$$U_0^{s,i} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[-c_t + U(C_t) - N_t\big], \tag{2}$$

where c_t is the production cost in the CM.

I assume that u and U are twice continuously differentiable with $U' > 0$, $u' > 0$, $U'' < 0$, and $u'' < 0$. Furthermore, I assume that U and u satisfy the Inada conditions. Finally, there exists $c^{**} \in (0,\infty)$ such that $u'(c^{**}) = 1$ and $C^{**}(\theta) \in (0,\infty)$ such that $U'(C^{**}(\theta)) = \theta$ for all θ. In the following, I assume the CRRA utility function in both CM and DM: $u(c) = c^{1-\gamma}/(1-\gamma)$ and $U(C) = C^{1-\gamma}/(1-\gamma)$.

Discussion on the roles of buyers and sellers: *In this model, buyers can be interpreted as a composite entity that includes households, who save their incomes in the CM, and firms, which borrow funds from households to finance international trade expenses for imports in the DM*[4] *On the other hand, sellers can be viewed as households that own the exporting firms, either directly or indirectly through stock holdings. The income shock $\theta_{t+1}^{s,i}$ can then be attributed to the aggregate shock concentrating on these shareholders, as in Chien, Lustig, and Naknoi (2020), generating hedging motives for firms against these risks, as in the standard consumption-based asset pricing literature, as in Lustig and Verdelhan (2007).*

US and LM issue their own fiat currency (USD and peso), respectively, which are both perfectly divisible and storable. Currency $M_{j,t} \in \mathbb{R}_+$ issued in country $j \in \{US, LM\}$ is valued at $\phi_{j,t}$, the price of money in terms of the numeraire. The nominal exchange rate is defined as the price of

[3] Mukhin (2022) also adopts the quasi-linear utility to simplify the equilibrium consumption and labor supply schedule and focus on the equilibrium analysis of the invoicing currency choice and pricing decisions by firms in an open-economy New Keynesian model

[4] Chahrour and Valchev (2022) explicitly models this type of environment, emphasizing the complementarity between households' portfolio choices and firms' selection of collateral assets for borrowing. In their framework, this interaction drives the emergence of a dominant store of value and medium of exchange in international trade.

peso in terms of USD: $e_t \equiv \phi_{LM,t}/\phi_{US,t}$. Since market clearing in the CM implies that the law of one price holds, agents can trade currencies at the market-clearing exchange rate. Hence, the CM also functions as a foreign exchange market.

Money supplies, $M_{j,t}$, grow or shrink each period, independent and identically distributed (iid) over time, by a factor of $\gamma_{j,t+1}$, where $\gamma_{j,t+1} \equiv M_{j,t+1}/M_{j,t}$. Changes in the money supply are implemented through lump-sum monetary transfers or taxes in domestic currency within the CM to each country's young buyers, denoted by $T_{j,t}$. The government budget constraint for each country $j \in \{US, LM\}$ is given by:

$$\phi_{j,t}(M_{j,t} - M_{j,t-1}) = T_{j,t}. \tag{3}$$

I seek a stationary equilibrium where the stock of real balances remains constant over time and across different states. Specifically, I look for a stationary equilibrium where $\phi_{j,t}M_t = \phi_{j,t+1}M_{j,t+1}$ for all t and all states, implying that $\gamma_{j,t+1}^{-1} = \phi_{j,t+1}/\phi_{j,t}$[5]. Consequently, the nominal exchange rate growth and real currency return are solely driven by the stochastic money supply rule $\gamma_{j,t+1}$.

Discussion on market structure: *Adopting the trading structure of Lagos and Wright (2005) allows for a clear distinction between the two roles a currency can serve: as a medium of exchange and as a store of value. Unlike existing studies in international macroeconomics and finance, which primarily focus on the optimal saving choices of alternative currencies with different hedging properties—such as Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), and Dalgic (2024)—or on currencies as units of account Drenik, Kirpalani, and Perez (2021), this paper centers on the hedging properties of currencies that determine their selection as a medium of exchange, rather than as a store of value.*

3 Model

This section describes the characterizations of currency choice in trades and holdings. I solve the agents' individual problems using a backward induction approach. I start from characterizing the valuations of currencies for each agent in period-$t+1$ DM and then I show how they determine the currency choice in trades in DMs and also currency holdings in the CM in period t.

[5] Real balance is constant over time since the underlying exogenous state is iid.

3.1 Value functions in CM

In the CM in period $t+1$, let real balance holdings of monies for an agent with type (x, j) holding be $\mathbf{q}^j_{t+1} \equiv (q^j_{US,t+1}, q^j_{LM,t+1}) \equiv (\phi_{US,t+1} m^j_{US,t+1}, \phi_{LM,t+1} m^j_{LM,t+1}) \in \mathbb{R}^2_+$, where m^j_c is the nominal amount of holdings in currency issued in country c, and let $W^{x,j}_{t+1}$ and $V^{x,j}_{t+1}$ denote value functions for this agent in the CM and DM in period $t+1$, respectively.

At the start of the CM, the agent with type (x, j) faces the following maximization problem:

$$W^{x,j}_{t+1}(\mathbf{q}^j_{t+1}) = \max_{C_{t+1}, N_{t+1}, \mathbf{q}_{t+2}} \{U(C_{t+1}) - N_{t+1} + \beta V^{x,j}_{t+2}(\mathbf{q}_{t+2})\} \tag{4}$$

subject to

$$C_{t+1} + \sum_c q_{c,t+2} = \frac{1}{\theta^{x,j}_{t+1}} \times N_{t+1} + \sum_c \gamma^{-1}_{c,t+1} q_{c,t+1} + T_{j,t+1} \times \chi_{\{x=b\}}, \tag{5}$$

and if the agent is of seller type $(x = s)$, the real balance holding at the end of this CM should be zero, $\mathbf{q}_{t+2} = 0$. Then the optimality condition implies the labor supply:

$$N^{x,j}_{t+1} = C^{**}(\theta^{x,j}_{t+1}) - \sum_c \gamma^{-1}_{c,t+1} q_{c,t+1} + T_{j,t+1} \times \chi_{\{x=b\}} + \sum_c q_{c,t+2}, \tag{6}$$

where $U'(C^{**}(\theta)) = \theta$. In the case of buyers, $\theta^{b,j}_{t+1} = \theta^b = 1$. The expected value function, conditional on the current currency portfolio without knowing the state realization in period $t+1$, is given by

$$\mathbb{E}_{t+1-} W^{x,j}_{t+1}(\mathbf{q}) = \mathbb{E}_{t+1-}[W^{x,j}_{t+1}(0)] + v^{x,j}_{US} q^j_{US,t+1} + v^{x,j}_{LM} q_{LM,t+1}, \tag{7}$$

where $\mathbb{E}_{t+1-}$ is the conditional expectations on not knowing the state realization and the state-independent valuation of a currency issued in c is written as

$$v^{x,j}_c \doteq \underbrace{\mathbb{E}_{t+1-}\gamma^{-1}_{c,t+1}}_{\textit{Inflation Cost}} \times \mathbb{E}_{t+1-}\theta^{x,j,t}_{t+1} + \underbrace{Cov_{t+1-}(\gamma^{-1}_{c,t+1}, \theta^{x,j}_{t+1})}_{\textit{Insurance}}, \tag{8}$$

where Cov_{t+1-} is the covariance operator conditional on not knowing the state realization in period $t+1$. If an agent is a buyer, his valuation of a currency is determined solely by the expected holding cost of the currency, expressed in terms of the inverse of inflation, as captured in the first term. In contrast, if an agent is a seller, his valuation of a currency is also influenced by its hedging properties against future income risk, as represented in the second term. Specifically, he values a currency more if it appreciates when his income deteriorates in the future, aligning with the modelings of Jacquet and Tan (2012) and Drenik, Kirpalani, and Perez (2021). As discussed earlier, this model offers a parsimonious approach to incorporating the seller's exposure to aggregate shocks affecting the country.

3.2 Terms of trade in DM

This section characterizes the terms of trade (ToT) in the DM, assuming agents are restricted to holding non-negative quantities of each currency and taking as given their currency portfolios, which are the choices in the CM beforehand that I will describe later.

Suppose that a buyer from country j meets a seller from country i. Let $\mathbf{q}_{t+1}^{b,j}$ denote the buyer's currency portfolio. The value of such a buyer, conditional on meeting this seller, is given by

$$u(c_{t+1}^{j,i}) + \mathbb{E}_{t+1-} W_{t+1}^{b,j}(\mathbf{q}_{t+1}^{b,j} - \mathbf{d}_{t+1}^{j,i}), \tag{9}$$

where $\mathbf{d}_{t}^{j,i}$ is the currency transfer from the buyer to the seller in the meeting. The seller's value function, conditional on having this meeting in DM, is given by

$$-c_{t+1}^{j,i} + \mathbb{E}_{t+1-} W_{t+1}^{s,i}(\mathbf{d}_{t+1}^{j,i}). \tag{10}$$

Note that the seller does not have any currency since they are not allowed to hold currencies at the end of the previous CM. Combined with the linearity of the value function in the next DM, the seller will participate in a trade if the terms of trade satisfy $v_{US}^{s,i} d_{US,t+1}^{j,i} + v_{LM}^{s,i} d_{LM,t+1}^{j,i} \geq c_{t+1}^{j,i}$.

The buyer makes a take-it-or-leave-it offer (TIOLI) to the seller. The TIOLI offer maximizes the buyer's surplus by choosing the DM consumption and currency portfolio transfer, subject to the seller's incentive compatibility (IC) and feasibility constraint of the currency transfer:

$$\max_{c_{t+1}^{j,i}, d_{US,t+1}^{j,i}, d_{LM,t+1}^{j,i}} V_{t+1}^{b,j}(\mathbf{q}_{t+1}^{b,j}) - \mathbb{E}_t W_{t+1}^{b,j}(\mathbf{q}_{t+1}^{b,j}) = \max_{c_{t+1}^{j,i}, d_{US,t+1}^{j,i}, d_{LM,t+1}^{j,i}} u(c_{t+1}^{j,i}) - (v_{US}^{b,j} d_{US,t+1}^{j,i} + v_{LM}^{b,j} d_{LM,t+1}^{j,i}) \tag{11}$$

subject to:

$$v_{US}^{s,i} d_{US,t+1}^{j,i} + v_{LM}^{s,i} d_{LM,t+1}^{j,i} \geq c_{t+1}^{j,i}, \qquad \textit{(seller's IC)} \tag{12}$$

and for $c \in \{US, LM\}$:

$$0 \leq d_{c,t+1}^{j,i} \leq q_{c,t+1}^{b,j} \qquad \textit{(feasibility).} \tag{13}$$

Note that the seller's IC constraint always binds at the optimum. The marginal gains from trading the DM goods are infinitely large at $c_{t+1}^{j,i} = 0$, which implies that trade will occur provided that the buyer holds a non-empty portfolio. The first-order condition (FOC) for the real balance transfer of currency $c \in \{US, LM\}$ implies

$$\underbrace{v_c^{b,j}}_{\textit{Marginal Cost of Real Balance Transfer}} \geq \underbrace{v_c^{s,i} u'(c_{t+1}^{j,i}) - v_c^{s,i} \lambda_c^{j,i}}_{\textit{Marginal Benefit of Real Balance Transfer}}, \tag{14}$$

with equality if $d_{c,t+1}^{j,i} > 0$, where $v_c^{s,i} \lambda_c^{j,i} \geq 0$ is Lagrange multiplier on the upper bound on the feasibility constraint. Equivalently,

$$\epsilon_c^{j,i} \geq u'(c_{t+1}^{j,i}) - \lambda_c^{j,i}, \tag{15}$$

where $\epsilon_c^{j,i} \doteq v_c^{b,j} / v_c^{s,i}$ is the relative marginal benefit of holding real balance of currency c for the buyer to the seller, or the marginal cost of transferring that currency. Therefore, the buyer does not transfer currency c, even with a positive amount, if the relative marginal benefit of holding it (on the left hand side) exceeds the marginal benefit of consuming the DM goods (on the right hand side).

To illustrate the terms of trade, consider a domestic meeting between a buyer and a seller both from LM, where the Latin American peso (issued in LM) serves as the national currency and the U.S. dollar (USD, issued in the U.S.) is the foreign currency. The country superscripts are omitted to simplify the notation.

3.2.1 Indeterminacy with common relative currency valuations

This section characterizes the terms of trade when two currencies have the same relative valuation, i.e., $\epsilon_{US} = \epsilon_{LM} = \epsilon$, following the spirit of Jacquet and Tan (2012), but in the context of mul-

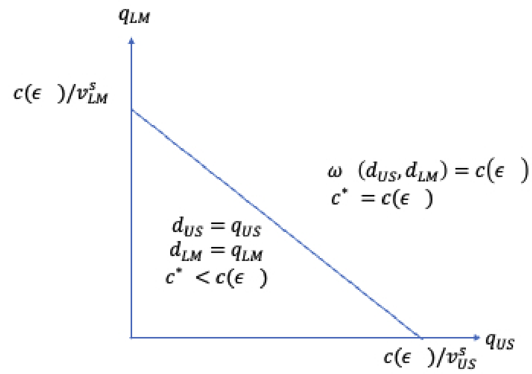

Figure 3: Terms of trade when symmetric currencies

tiple currencies rather than a single currency and Lucas tree. This situation arises, for example, when the expected currency holding costs are equalized, i.e., $\mathbb{E}_{t+1-}(\gamma^{-1}_{US,t+1}) = \mathbb{E}_{t+1-}(\gamma^{-1}_{LM,t+1})$, across currencies, and the hedging properties are also identical, i.e., $Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{US,t+1}) = Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{LM,t+1})$. In this special case, the relative cost of transferring these currencies is exactly equalized from the buyer's perspective. As a result, the composition of the optimal currency portfolio transfer is indeterminate, as shown in the following proposition:

Proposition 1. *Let $\omega_{LM}(d_{US}, d_{LM})$ be the expected value of a transfer (d_{US}, d_{LM}) for a LM seller, $v^s_{US}d_{US} + v^s_{LM}d_{LM}$ and $c(\epsilon)$ be the DM consumption of a buyer such that $u'(c(\epsilon)) = \epsilon$. Then the terms of trade in the DM (c^*, d_{US}, d_{LM}) satisfy $c^* = \omega(d_{US}, d_{LM}) = \min\{\omega_{LM}(q_{US}, q_{LM}), c(\epsilon)\}$.*

Proof. See section A.1 in the Appendix A. □

Figure 3 depicts the terms of trade graphically. If the buyer holds sufficient real balances such that $\omega(q_{US}, q_{LM}) \geq c(\epsilon)$, he purchases $c(\epsilon)$ amounts of goods from the seller by transferring the real balance worth $c(\epsilon)$ to the seller[6]. In particular, when unconstrained, the buyer is indifferent as to which currency to use as a medium of exchange. To obtain an additional unit of goods, the buyer incurs a cost of ϵ in utility, which is independent of the choice of payment currency. If the buyer does not hold real balances to purchase this unconstrained optimal amounts, then he transfers his entire portfolio holdings to the seller to obtain the maximum level of consumption within his budget.

[6] Note that sellers overproduce DM goods relative to the case of no hedging motive or market completeness $\theta^s \equiv 1$, when $\epsilon > 1$.

3.2.2 Terms of trade with different relative valuations

Now, I turn to the more empirically relevant case where two alternative currencies have different relative valuations. This situation occurs, for example, when the USD appreciates more than the LM peso when the incomes of LM sellers decrease, leading to differing hedging properties of the currencies: $Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{US,t+1}) > Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{LM,t+1})$. In this case, the buyer has a strict preference for using one of the two currencies as a medium of exchange, as shown in the following proposition.

Proposition 2. *Suppose $\epsilon_{LM} > \epsilon_{US}$ and buyers' currency portfolio is given by (q_{US}, q_{LM}). Then there exist $q_{US}(\epsilon_{LM})$ and $q_{US}(\epsilon_{LM})$ with $q_{US}(\epsilon_{US}) > q_{US}(\epsilon_{LM})$ such that*

$$(c^*(q_{US}, q_{LM}), d_{US}, d_{LM}) = \begin{cases} (c(\epsilon_{US}), q_{US}(\epsilon_{US}), 0), & \textit{if} \quad q_{US} > q_{US}(\epsilon_{US}) \\ (v^s_{US} q_{US}, q_{US}, 0), & \textit{if} \quad q_{US} \in [q_{US}(\epsilon_{LM}), q_{US}(\epsilon_{US})] \\ (c(\epsilon_{LM}), q_{US}, (c(\epsilon_{LM}) - v^s_{US} q_{US})/v^s_{LM}), & \\ \quad \textit{if } q_{US} < q_{US}(\epsilon_{LM}) \textit{ and } \omega(q_{US}, q_{US}) \geq c(\epsilon_{LM}) & \\ (v^s_{US} q_{US} + v^s_{LM} q_{LM}, q_{US}, q_{LM}), & \textit{otherwise.} \end{cases} \tag{16}$$

Proof. See section A.2 in Appendix A. □

The result of this proposition is graphically described in Figure 4. When the buyer holds a sufficient amount of the real balance of the seller's preferred currency (USD) such that $q_{US} > q_{US}(\epsilon_{US})$, he purchases the unconstrained optimal amount of goods $c(\epsilon_{US})$. If the real balance falls short of this level, consumption is constrained, and the buyer transfers the entire real balance of USD but does not use the less preferred currency (peso) as long as consumption remains above $c(\epsilon_{LM})$. This occurs because, when the buyer uses LM peso as a medium of exchange, the marginal cost of obtaining additional units of obtaining additional units of consumption, ϵ_{LM}, is still higher than the marginal utility of consumption. As the consumption level falls below $c(\epsilon_{LM})$, the buyer begins to transfer LM peso to the seller.

The result illustrates the novel endogenous ranking of currencies as medium of exchange in decentralized trades. This ranking is determined by the hedging properties against the future

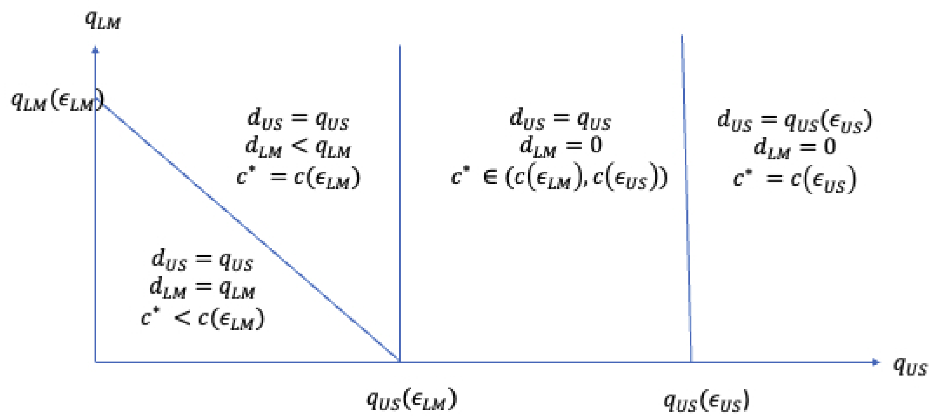

Figure 4: Terms of trade when asymmetric currencies

aggregate income shock on sellers that materialize in the subsequent CM[7].

3.2.3 Relative valuations and liquidity

In this model, the notion of liquidity capture the idea that, beyond physical or legal frictions limiting the trading of an asset, whether an asset is actively traded also reflects the choices made by agents, in the spirit of Jacquet and Tan (2012). Since I do not assume any type of trading restrictions or transactions costs in either sub-period, both currencies are equally liquid in the CM. Therefore, I focus on the liquidity properties of assets in the DM. Let $c^*(q_{US}.q_{LM})$ denote the quantity consumed in the DM by a buyer with portfolio (q_{US}, q_{LM}) as given in propositions 1 and 2, I define *the degree of liquidity* for each currency issued in $c \in \{US, LM\}$,

$$\mathcal{L}^c(q_{US}, q_{LM}) = \frac{u'(c^*(q_{US}, q_{LM}))\frac{\partial c^*(q_{US}, q_{LM})}{\partial d_c}}{v_c^b} - 1. \tag{17}$$

Definition 1.

1. *If $\mathcal{L}^c \geq 0$, then currency c is said to provide liquidity to buyers, while if $\mathcal{L}^i < 0$, then currency*

[7] Mukhin (2022) also examines firms' choice of trade invoicing in a sticky-price model with strategic complementarities in currency choice. In his framework, a firm's currency decision is influenced by the choices made by intermediary goods producers and final goods producers. Firms then select the currency that minimizes the variance of the optimal prices denominated in that currency. However, his framework abstracts from the state-dependent valuations of firms' cash flows by approximating the stochastic discount factor as constant. As a result, the model does not incorporate the role of hedging motives in currency choice, particularly against stockholders' future income risk exposures.

c is said to not provide liquidity to buyers.

2. *If $\mathcal{L}^c > \mathcal{L}^{c'} > 0$, then currency c is said to provide more liquidity than currency c'.*

If $\mathcal{L}^c \geq 0$, it implies that $u'(c^*(q_{US}, q_{LM}))\frac{\partial c^*(q_{US},q_{LM})}{\partial d_c} > v_c^b$, meaning that if buyers were given an extra unit of currency c, they would spend it in the DM meeting. Conversely, if $\mathcal{L}^c < 0$ on the contrary, agents strictly prefer not to sell currency c for consumption in the DM meeting, meaning that the currency does not provide liquidity to agents.

Lemma 1. *For each currency issued in $c \in \{US, LM\}$, $\mathcal{L}^c = \frac{u'(c^*)}{\epsilon_c} - 1$.*

Proof. The proof can be immediately completed by using the expression for the DM consumption in proposition 1 or 2 to compute the partial derivative $\frac{\partial c^*(q_{US},q_{LM})}{\partial d_c}$. □

When buyers enter the DM with very little in their portfolio $\omega(q_{US}, q_{LM}) < c(\epsilon_{LM})$, the marginal utility of using each currency in the DM exceeds its discounted expected marginal value in the next CM, even if buyers spend all of their portfolio in the DM. As the value of buyers' portfolio increases, the liquidity provided by each currency falls because buyers can purchase more DM consumption. If $\epsilon_{US} < \epsilon_{LM}$, when the value of buyers' currency portfolio exceeds $q_{US}(\epsilon_{LM})$, LM peso stops prividing liquidity, but USD still provides liquidity. In this case $\epsilon_{US} < \epsilon_{LM}$, in equilibrium, USD always provide liquidity, whereas it is possible for LM peso not to provide liquidity.

Proposition 3.

1. *If $\epsilon_{US} = \epsilon_{LM} = \epsilon$, then $\mathcal{L}^{US} = \mathcal{L}^{LM} > 0$ if and only if $\omega(q_{US}, q_{LM}) < c(\epsilon)$, and $\mathcal{L}^{US} = \mathcal{L}^{LM} = 0$, otherwise.*

2. *If $\epsilon_{US} < \epsilon_{LM}$, then:*

   (a) *for USD, $\mathcal{L}^{US} > 0$ if and only if $q_{US} < q_{US}(\epsilon_{US})$ and $\mathcal{L}^{US} = 0$, otherwise. For LM peso, $\mathcal{L}^{LM} > 0$ if and only if $\omega(q_{US}, q_{LM}) < c(\epsilon_{LM})$; $\mathcal{L}^{LM} = 0$, if and only if $q_{US} \leq q_{US}(\epsilon_{LM})$ and $\omega(q_{US}, q_{LM}) \geq c(\epsilon_{LM})$; and $\mathcal{L}^{LM} < 0$, otherwise.*

   (b) *$\mathcal{L}^{US} > \mathcal{L}^{LM}$ for all (q_{US}, q_{LM}).*

Proof. The proof relies on the result from lemma 1 and the first-order conditions (15). □

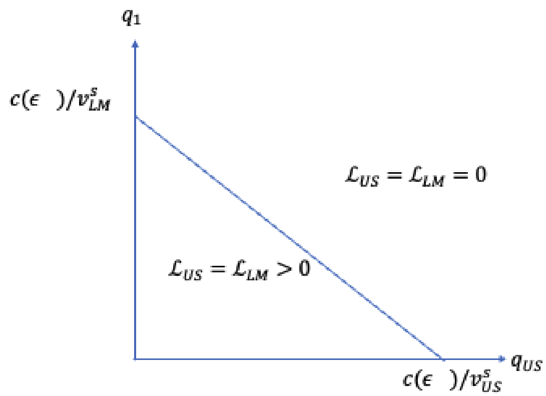

Figure 5: Liquidity in symmetric case

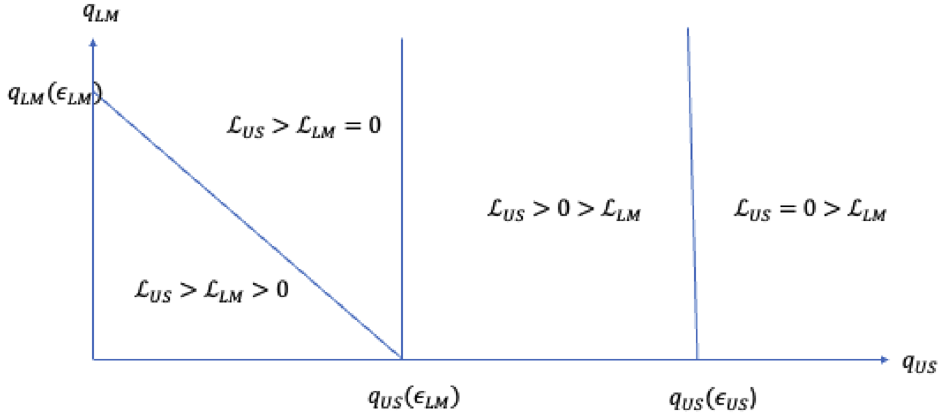

Figure 6: Liquidity in asymmetric case

In case 1, Figure 5 provides a graphical summary of part (a) in proposition 3, where both currencies provide the same degree of liquidity, but both will strictly provide liquidity if and only if the value of buyer's portfolio is small. On the other hand, as seen in Figure 6, for case (b), USD always provides more liquidity than LM peso, when there is a difference in relative valuation in favor of USD. Since the liquidity service provided by a currency depends on agents' optimal portfolio decisions as well as the terms of trade for the DM, the liquidity provided by each currency is endogenous, as are the liquidity differences across currencies. Importantly, this liquidity difference across currencies is partly driven by the different hedging properties.

3.3 Value functions at the beginning of DM

Now I describe the value functions of buyers and sellers at the beginning of the DM before knowing whom they meet, given the currency portfolio chosen in the previous CM. Let $\lambda^b_{j,i}$ ($\lambda^s_{j,i}$) denote the probability of a buyer (seller) from country j meeting a seller (buyer) from country i in the DM.

The value function of a buyer from country j born in period t is given by

$$\begin{aligned} V^{b,j}_{t+1}(\mathbf{q}_{t+1}) &= \sum_i \lambda_{ji}\big\{u[c^{j,i}_{t+1}(\mathbf{q}_{t+1})] + \mathbb{E}_{t+1-}[W^{b,j}_{t+1}(\mathbf{q}_{t+1} - \mathbf{d}^{j,i}_{t+1}(\mathbf{q}_{t+1})]\big\} \\ &\quad + \left(1 - \sum_i \lambda_{ji}\right)\mathbb{E}_{t+1-}\big[W^{b,j}_{t+1}(\mathbf{q}_{t+1})\big] \\ &= \sum_i \lambda_{ji}\big\{u[c^{j,i}_{t+1}(\mathbf{q}_{t+1})] - v^{b,j}\cdot \mathbf{d}^{j,i}_{t+1}(\mathbf{q}_{t+1})\big\} + \mathbb{E}_{t+1-}\big[W^{b,j}_{t+1-}(\mathbf{q}_{t+1})\big], \end{aligned} \tag{18}$$

where λ_{ji} is a probability of a buyer from country j meeting a seller from i and $v^{b,j}$ is the vector of buyer j's currency valuations. Similarly, the value function of a seller from country i born in period t is given by

$$V^{s,i}_{t+1} = \sum_j \lambda^s_{ij}\big\{-c^{j,i}_{t+1}(\mathbf{q}_{t+1}) + v^{s,i}\cdot \mathbf{d}^{j,i}_{t+1}(\mathbf{q}_{t+1})\big\} + \mathbb{E}_{t+1-}\big[W^{s,i}_{t+1}(0)\big], \tag{19}$$

where $\lambda^s_{i,j}$ is a probability of a seller from country i meeting a buyer from j and $v^{s,i}$ is the vector of seller i's currency valuations.

3.4 Currency portfolio choice for buyers in CM

In this section, I characterize the currency portfolio choice for buyers in period t, who choose the medium of exchange used in the subsequent DM, and I define the stationary monetary equilibrium.

The problem of a buyer born in period t in the CM is to choose consumption $C^{b,j}_t$, labor supply $N^{b,j}_t$, and currency real balances $\mathbf{q}^{b,j}_{t+1}$ to bring into the DM, aiming to maximize his lifetime expected discounted utility. A buyer's problem in the CM is identical every period since the shock is i.i.d., and I focus on monetary policy rules where the inflation rate between t and $t+1$ depends only on the state of the world in period $t+1$. Consequently, the demands for real balances are the same every period in all states of nature, in terms of the numeraire CM goods.

The maximization problem of a buyer with type (b, j) is formulated as

$$W_t^{b,j}(\mathbf{q}_t) = \max_{C_t, N_t, \mathbf{q}_{t+1}} U(C_t) - N_t + \beta V_{t+1}^{b,j}(\mathbf{q}_{t+1}) \tag{20}$$

subject to

$$C_t + \sum_c q_{c,t+1} = N_t + \sum_c \gamma_{c,t+1}^{-1} q_{c,t} + T_{j,t}. \tag{21}$$

From the quasi-utility function, it follows that the optimal labor supply is $N_t^{b,j,t} = C^{**}(1) + \sum_c q_{c,t+1} - \sum_c \gamma_{c,t+1}^{-1} q_{c,t}^{b,j,t} - T_{j,t}$. Then the currency portfolio choice problem is given by

$$\max_{\mathbf{q}_{t+1}} - \sum_c q_{c,t+1} + \beta V_{t+1}^{b,j}(\mathbf{q}_{t+1}). \tag{22}$$

The first-order condition for currency c real balance in the CM is given by

$$1 \geq \beta \partial_{q_{c,t}} V_{t+1}^{b,j}(\mathbf{q}_{t+1}) \tag{23}$$

with equality if $q_{c,t+1} > 0$. The following proposition characterizes this optimality condition in terms of the liquidity:

Proposition 4. *Let $l_{j,i}^c = \max\{0, \mathcal{L}_{j,i}^c\}$ be the liquidity premium currency c carries, where $\mathcal{L}_{j,i}^c$ is the liquidity of currency c in the meeting between a buyer from country j and a seller from country i. The FOC in equation (23) can be written as*

$$1 \geq \beta v_c^{b,j} \times [1 + \sum_i \lambda_{ji} \times l_{j,i}^c(\boldsymbol{q}_{t+1}^{b,j})]. \tag{24}$$

Proof. See section A.3 in the Appendix A. □

Note that the optimal currency choice depends on the meeting probability with sellers and liquidity premia in those meetings. The liquidity $\mathcal{L}_{j,i}^c$ appears in the first-order conditions for the currency choice of buyers if and only if the currency strictly provides liquidity to the buyer in one of the DM meetings. If a currency is illiquid in a particular meeting, it does not appear in the first-order conditions because buyers do not wish to use any of it for purchasing consumption in that meeting−they prefer to hold onto it until the next CM.

Parameter	Description	Value
γ	Relative risk aversion	2
β	Discount factor	0.9
$v_1^{b,LM}$	Buyer's valuation of LM peso	0.9
$v_2^{b,LM}$	Buyer's valuation of USD	0.9
$\lambda_{LM,LM}$	Probability of meeting LM seller	0.5
$\lambda_{LM,US}$	Probability of meeting US seller	0.5
$\epsilon_{LM}^{LM,US}$	Relative transfer cost of LM peso to US seller	2
$\epsilon_{US}^{LM,LM}$	Relative transfer cost of USD to LM seller	2

Table 1: Parameter values in numerical experiment

4 Equilibrium

This section provides the definition of the stationary equilibrium that is of my interest and its characterizations of the patterns of currency portfolio holding and means of payment across different meetings under alternative parameter configurations.

Definition 2 (Stationary Monetary Equilibrium)**.** *A stationary monetary equilibrium is a list of quantities traded* $\{C_t^{b,j}, c_t^{j,i}, \boldsymbol{d}_t^{j,i}\}$*, labor supply* $(\{N_t^{b,j}\}, \{N_t^{s,j}\})$*, and currency portfolios* $\{\boldsymbol{q}_{t+1}^{b,j,t}\}$ *for all* t *and* $j \in \{US, LM\}$ *s.t.*

1. *The first-order conditions for currency choices, equation (24), are satisfied;*
2. *The terms of trade* $(c_t^{j,i}, \boldsymbol{d}_t^{j,i})$ *satisfy the optimal TIOLI offers in propositions 1 or 2;*
3. *The choice in the CMs is consistent with optimality conditions.*

4.1 Equilibrium characterizations

I illustrate various equilibrium patterns of the medium of exchange under the parameter values reported in Table 1, where the two currencies are identical with respect to the expected valuations for LM buyers, and they meet sellers from different countries with equal probability (0.5). I vary the currency c's hedging properties in the meeting with sellers from country c, whose government issues this currency, $Cov_{t+1-}(\gamma_{c,t+1}^{-1}, \theta_{t+1}^{s,c})$, while assuming that it is better to accept the local currency than a foreign currency in such a meeting. Specifically, the LM peso is more valued than the USD in the meeting with LM sellers ($\epsilon_{LM}^{LM,LM} < \epsilon_{US}^{LM,LM}$), i.e., in local meetings, and vice versa in the meeting with US sellers ($\epsilon_{US}^{LM,US} < \epsilon_{LM}^{LM,US}$).

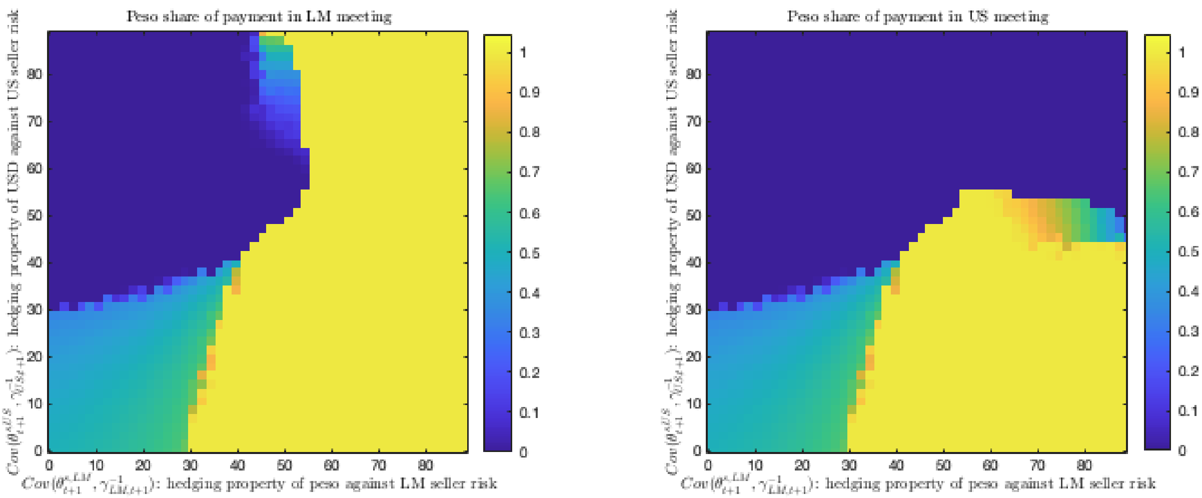

Figure 7: Currency 1 shares in DM transfers

The choice of these parameters is not intended to replicate empirical patterns but rather to demonstrate the model's ability to generate diverse equilibrium patterns of currency choices based on hedging properties. In the subsequent section 6, I extend the analysis to a three-country model, where parameter values calibrated from data are used to evaluate the model's ability to reproduce empirical patterns of currency choices.

Figure 7 visualizes the share of real balance transfers of the LM peso in both domestic and foreign meetings, quantifying the degree of the dollar share in terms of means of payment. The model generates all possible payment patterns in the DM, depending on the hedging properties of the two currencies. When the LM peso serves as a superior hedge in domestic meetings (higher $Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{LM,t+1})$) and the USD is a poor hedge against US sellers' income risk (low $Cov_{t+1-}(\theta^{s,US}_{t+1}, \gamma^{-1}_{US,t+1})$), the buyer always uses the LM peso (local currency) as the means of payment, regardless of the sellers' origins (yellow regions in both panels). This pattern is referred to as the local currency regime, as all DM transactions are processed using the buyer's national currency.

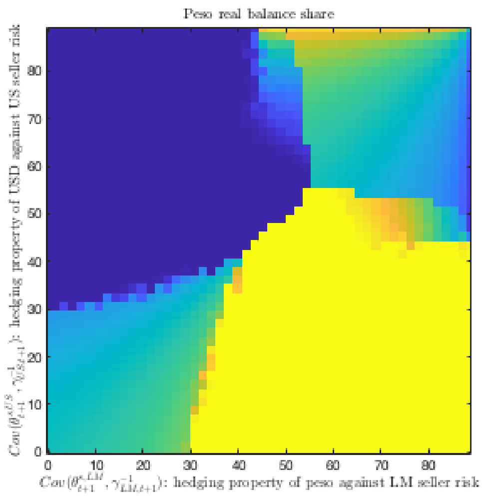

Figure 8: Currency 1 share in CM real balance

Conversely, when the national currency (LM peso) is a poor hedge against LM sellers' income shocks (low $Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{LM,t+1})$) and the foreign currency (USD) is a good hedge against US sellers' income shocks (high $Cov_{t+1-}(\theta^{s,US}_{t+1}, \gamma^{-1}_{US,t+1})$), the buyer always uses the foreign currency (USD) as the means of payment. This pattern represents the dollar dominance in means of payment, as all DM transactions are conducted using the USD (upper left areas in blue regions in both panels).

When LM peso is a good hedge against LM sellers' income shocks (high $Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{LM,t+1})$) and USD is also a good hedge against US sellers' shocks (high $Cov_{t+1-}(\theta^{s,US}_{t+1}, \gamma^{-1}_{US,t+1})$), buyers always use the sellers' national currencies (producer currencies). This regime, which is shown in the right upper corner in blue in both panels, represents the producer currency regime. In this scenario, sellers prefer to receive their national currencies, offering better terms of trade in exchange. Therefore, the means of payment depend on the nationality of the sellers (producers), distinguishing this regime from the local currency regime.

Finally, when the LM peso is a poor hedge against LM sellers' income shocks, meaning low $Cov_{t+1-}(\theta^{s,LM}_{t+1}, \gamma^{-1}_{LM,t+1})$ and USD is also a bad hedge against US sellers' shocks, implying low $Cov_{t+1-}(\theta^{s,US}_{t+1}, \gamma^{-1}_{US,t+1})$, buyers will use both currencies, as they hold little of either currency and are constrained to buy DM goods in both meetings. This coexistence regime appears in the left bottom areas, marked in sky blue, in both panels.

Figure 8 illustrates the share of real balance holdings of LM peso relative to USD, quantifying the degree of dollarization as a store of value. The values in this figure are approximately a linear combination of the two panels in Figure 7, reflecting the fact that buyers primarily save with the intention of spending their currency portfolio in the subsequent DM meetings, rather than saving for the next CM.

The figure delineates four distinct regions, each representing a possible currency holding pattern. In the yellow region, buyers hold only the national currency (LM peso) because it will be used in both domestic and foreign meetings in the DM, while the foreign currency (USD) is not used due to its relatively weaker hedging properties. This equilibrium regime is referred to as the national currency regime. Conversely, in the blue region, buyers exclusively hold the foreign currency (USD), as it will be used in both local and foreign meetings due to its superior hedging properties. This region corresponds to dollarization of stores of value.

Finally, when both USD and LM peso are effective hedges for sellers from the U.S. and LM, respectively, buyers diversify their holdings, obtaining both currencies in the CM. This pattern, seen in the sky-blue regions, leads to the coexistence regime of multiple currencies as stores of value. The model determines the demand function for each currency, which in turn determines the nominal exchange rate, resolving the nominal exchange rate indeterminacy discussed in Kareken and Wallace (1981)[8].

5 Empirical Analysis

In this section, I provide empirical support for the hedging motive behind currency choice in trades and real balance holdings. As shown in Figure 7, buyers use a currency more when it offers their sellers better hedging against future income shocks. Consequently, buyers hold such a currency more in the CM beforehand, as its liquidity premia—captured in the first-order condition, eq. (24)—are higher, as illustrated in Figure 8.

[8]In the current setup, the currency valuations are endogenously determined, but their time variations are solely driven by the state-dependent money supply rules. Alternatively, the model would be extended by incorporating currency demand shocks in the form of discount rate shocks on β, so that the currency valuations are also driven by the time-varying currency demand. If this shock is also countercyclical, the equilibrium currency valuations would become countercyclical, providing a similar qualitative implications to the setup in the main text. Nevertheless, I abstract from such an extension since after all, I will calibrate the cyclical properties of currency valuations directly to the empirical moments. Consequently, the sources of shocks do not matter significantly for the implications for currency choice in trades, which is the main focus in this paper.

Moreover, the same optimality condition suggests that the holding of a particular currency depends positively on: (i) the inverse of expected holding costs (i.e., the inverse inflation rate) $v_c^{b,j}$, and (ii) meeting probabilities $\lambda_{j,i}$, which interact with the liquidity premia $l^c_{j,i}$. This implies that buyers will hold a currency more when its holding cost (inflation rate) is lower.

In the remainder of this section, I test these theoretical implications of currency hedging benefits in relation to inflation rates and meeting probabilities. Specifically, I apply these concepts to the currency choices in international trade settlement/invoicing and deposits. While the hedging implications for the choice of trade settlement currency in the cross-section of countries are novel, the implications for deposit dollarization have been previously explored in the literature (Levy-Yeyati (2006), Bocola and Lorenzoni (2020), Chirstiano, Dalgic, and Nurbekyan (2022), Dalgic (2024)). Moreover, I provide a novel time-series evidence that the dollar invoicing becomes more pronounced after global financial crises, suggesting that flight-to-quality demand also arises in trade invoicing.

5.1 Measurement and data

I measure the sellers' labor income shocks, $\theta^{s,i}_{t+1}$, using real GDP and aggregate income data from each country. This approach is motivated by the theory in this paper and the findings in the literature that agents have a significant hedging motive against adverse aggregate shocks. Such hedging has been studied in countries experiencing deposit and credit dollarization, particularly in emerging economies, as discussed by Chirstiano, Dalgic, and Nurbekyan (2022), Drenik, Kirpalani, and Perez (2021), and Dalgic (2024)[9]. These studies argue that depositors in these countries hold foreign currency (typically the U.S. dollar) as hedging against income risk during recessions, when their local currencies tend to devalue against the U.S. dollar.

Moreover, a large body of literature, including works by Gourinchas, Rey, and Govillot (2017), Obstfeld, Shambaugh, and Taylor (2010), and Maggiori (2017), emphasizes the hedging role of currencies, particularly the U.S. dollar, in response to both global and national aggregate shocks. This literature helps explain the large demand for the U.S. dollar as a store of values globally.

To measure sellers' hedging motives, I use their connections to a country's (LM in this example)

[9]For advanced economies, see Lustig and Verdelhan (2007), which shows the significant predictive power of such hedging motives for currency risk premia.

nominal GDP in the model, evaluated with its currency (LM peso) as the numeraire[10]:

$$\begin{aligned}
\textit{Nominal GDP}_{LM} = \quad & \underbrace{\lambda^{s}_{LM,LM}\left(\frac{d^{LM,LM}_{LM}}{\phi_{LM}} + \frac{d^{LM,LM}_{US}}{\phi_{US}}\right)}_{\textit{Value added in the DM meeting with domestic buyers}} \\
+ \quad & \underbrace{\lambda^{s}_{LM,US}\left(\frac{d^{US,LM}_{LM}}{\phi_{LM}} + \frac{d^{US,LM}_{US}}{\phi_{US}}\right)}_{\textit{Value added in the DM meeting with foreign buyers}} \qquad (25) \\
+ \quad & \underbrace{\frac{C^{b,LM}}{\phi_{LM}}}_{\textit{Buyer's CM consumption}} + \underbrace{\frac{C^{s,LM}(\theta^{s,LM})}{\phi_{LM}}}_{\textit{Sellers' CM consumption}} .
\end{aligned}$$

Note that only the last term involving sellers' CM consumption is stochastic in this economy. The real GDP is defined as $\textit{Real GDP} \equiv \textit{Nominal GDP} \times \phi_i$ and the real GDP growth is given by

$$\begin{aligned}
\Delta \log RGDP_{LM,t+1} \equiv \log RGDP_{LM,t+1} - \log RGDP_{LM,t} &\approx \frac{RGDP_{LM,t+1} - RGDP_{LM,t}}{RGDP_{LM,t}} \\
&= \frac{C^{s,LM}(\theta^{s,LM}_{t+1}) - C^{s,LM}(\theta^{s,LM}_{t})}{C^{s,LM}(\theta^{s,LM}_{t})} \approx \log C^{s,LM}(\theta^{s,LM}_{t+1}) - \log C^{s,LM}(\theta^{s,LM}_{t}) \\
&= -\frac{1}{\gamma}(\log \theta^{s,LM}_{t+1} - \log \theta^{s,LM}_{t}),
\end{aligned} \qquad (26)$$

where the last equality follows from the assumption that the utility function is CRRA. Then the covariance between the income shock and the currency return can be empirically observed as

$$Cov(\log RGDP_{LM,t+1}, \Delta \log e_{US,LM,t+1}) = \frac{1}{\gamma} Cov\left(\log \theta^{s,LM}_{t+1}, \log \gamma_{US,t+1} - \log \gamma_{LM,t+1} \right), \qquad (27)$$

where $\log \theta^{s,LM}_{t}$ drops out since it is orthogonal to the future inflation rates $\log \gamma_{c,t+1}$. This equation shows that the measurable covariance between real GDP growth and the currency appreciation rate serves as a good proxy for the relative hedging property of a currency against the sellers' income shock. A positive covariance implies that the local currency provides better hedging against income shocks, which translates to a better terms of trade in trades provided by sellers in the DM.

I use the export/import settlement and invoicing currency share dataset from Boz et al. (2020)

[10]In the rest of this paper, the time subscript will not be explicitly indicated unless they are noted.

as proxies for the medium of exchange in international trade in the model. While it would be ideal to separate the settlement and invoicing currencies in the data, Goldberg and Tille (2008) document that the currency used in trade invoicing is typically the same as the one used for actual payments. Therefore, this approach remains consistent with the model.

As a proxy for the currency portfolio shares of buyers, I employ the extended dataset of deposit dollarization from Levy-Yeyati (2006), where the deposit dollarization for country i and year t, as

$$\textit{deposit dollarization}_{i,t} \equiv \frac{\textit{value of foreign currency deposits held by domestic residents in domestic banks}}{\textit{total deposits held by domestic residents in domestic banks}},$$

where both the numerator and denominator are expressed in local currency units. I follow the literature (see Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022)) in referring to 'foreign currency' as the dollar.

I proxy the USD return by the appreciation rate against the local currency, adjusted by the local inflation rate γ_{LM}: $\Delta \log e_{US,LM,t+1} - \log \gamma_{LM,t+1} \equiv \log e_{US,LM,t+1} - \log e_{US,LM,t} - \log \gamma_{LM,t+1} = -\log \gamma_{US,t+1}$, where the last equality comes from the definition of a nominal exchange rate in the model: $\Delta e_{US,LM,t+1} \equiv \log e_{US,LM,t+1} - \log e_{US,LM,t} = \log \gamma_{LM,t+1} - \log \gamma_{US,t+1}$. The local inflation rates are measured from CPIs [11]. Lustig and Verdelhan (2007) similarly constructs unexpected currency returns that are relevant for hedging properties against aggregate risk.

Finally, the trade share data from the OECD Inter-Country Input-Output (ICIO) table are used as proxies for the meeting probabilities with US sellers in the DM (Decentralized Market) in the model.

The baseline merged dataset spans annual data from 1995 to 2009, with the number of countries varying based on the availability of each data series at any given time. The trade invoicing data, however, spans from 1995 to 2019. To construct the cross-country dataset for dollar shares, I compute the time-series average for each variable, following the methodology outlined in the literature Levy-Yeyati (2006), Chirstiano, Dalgic, and Nurbekyan (2022), and Dalgic (2024). The average local inflation and trade flows with the U.S. are computed as time-series averages of CPI inflation rates and of exports to and imports from the U.S. The correlations between real GDP growth rates and USD appreciation rates are calculated using the entire annual time series from 1995 to 2009.

[11] Mukhin (2022) also proxies the inflation rates in the model by the average of the CPI inflation rates and local currency depreciation rates.

Variable	deposit dollarization	import USD share	export USD share
Intercept	0.025	0.318***	0.217**
$corr(\Delta \log GDP_i, -\Delta \log \gamma_{US})$	-0.400***	-0.259**	-0.386**
$corr(\Delta \log GDP_i, -\Delta \log \gamma_i)$	0.175	0.084	0.074
Average inflation rate γ_i	2.00***	0.853	1.797**
$US\ import\ share_i$	-1.049	1.459***	-
$US\ export\ share_i$	1.2628*	-	1.634***
R-squared	0.4434	0.4822	0.4818
Adj. R-squared	0.3321	0.3922	0.3875

Table 2: Empirical results

Notes: * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$. The definitions and measurements of variables are described in section 5.1. The standard errors are computed by the standard Newey-West estimators.

5.2 Econometric specification and results

The econometric specification in this paper follows Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022):

$$\begin{aligned} Dollarization\ index_i = \alpha + \beta_1 \cdot corr(\Delta \log Real\ GDP_i, &\overbrace{-\log \gamma_{US}}^{USD\ appreciation\ rate})+ \\ + \beta_2 \cdot corr(\Delta \log Real\ GDP_i, &\overbrace{-\log \gamma_i}^{Local\ currency\ appreciation\ rate})+ \\ + \beta_3 \cdot \underbrace{\gamma_i}_{Average\ local\ inflation\ rate} &+ \beta_4 \cdot US\ import\ share_i + \beta_5 \cdot US\ export\ share_i + u_i, \end{aligned}$$

where u_i represents the regression residual.

Table 2 presents the regression results, which align with the theoretical predictions. Consistent with the empirical findings of Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022), the second column demonstrates that more countercyclicality of the USD valuations are significantly associated with higher degrees of deposit dollarization ($\beta_1 < 0$). The predictability of the hedging properties by the local currencies β_2 is statistically insignificant. The average local inflation rate also predicts higher dollarization ($\beta_3 > 0$), which is consistent with the theory.

The third and last columns reveal novel empirical patterns for international settlement currencies, consistent with the theory. More countercyclical valuations of the USD predict higher shares of U.S. dollar invoicing/settlement in international trade ($\beta_1 < 0$), although the hedging property

Variable	import USD share	export USD share
Intercept		
- Before 2013	0.295***	0.270**
- After 2013	0.318***	0.217**
$corr(\Delta \log GDP_i, -\Delta \log \gamma_{US})$		
- Before 2013	-0.191	-0.211
- After 2013	-0.2596**	-0.3869**
$corr(\Delta \log GDP_i, -\Delta \log \gamma_i)$		
- Before 2013	0.015	-0.033
- After 2013	0.084	0.074
Average inflation rate γ_i		
- Before 2013	0.823	1.437
- After 2013	0.853	1.797**
$US\ import\ share_i$		
- Before 2013	1.579***	-
- After 2013	1.459***	-
$US\ export\ share_i$		
- Before 2013	-	1.907***
- After 2013	-	1.634***
R-squared		
- Before 2013	0.5058	0.3572
- After 2013	0.4822	0.4818
Adj. R-squared		
- Before 2013	0.3960	0.2143
- After 2013	0.3922	0.3875

Table 3: Empirical results in subsamples before and after 2013

Notes: * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$. The table reports the regression results in the subsample before 2013 and after 2014. The definitions and measurements of variables are described in section 5.1. The standard errors are computed by the standard Newey-West estimators.

of local currencies play statistically insignificant roles (β_2). Additionally, larger trade flows with the U.S. contribute to higher U.S. dollar shares, aligning with the theoretical prediction that if U.S. agents prefer to use the U.S. dollar as a medium of exchange, as they do in reality, increased trade with U.S. buyers and sellers will raise the aggregate U.S. dollar share ($\beta_4 > 0$, $\beta_5 0$).

Moreover, the significance of β_1 even after controlling for trade volume with the U.S. highlights the distinct role of hedging motives in driving currency choice in trades. Specifically, the significant β_1 in import invoicing suggests the hedging motive of local sellers, in line with the theory. On the other hand, higher local average inflation does not significantly increase the U.S. dollar share in international trade settlement.

These empirical findings for both deposit and international trade settlement patterns suggest that the hedging properties of currencies can also drive the choice of medium of exchange, aligning with the theory presented in this paper. This contrasts with the existing literature (e.g., Levy-Yeyati (2006), Chirstiano, Dalgic, and Nurbekyan (2022), and Dalgic (2024)), which focuses primarily on currencies serving as stores of value. These studies explore the positive and normative implications of deposit dollarization in emerging countries, where it acts as a store of value and an efficient redistributive mechanism during economic crises. They emphasize the welfare cost of deposit de-dollarization, which removes effective insurance mechanisms within countries, and empirically argue that although the high riskiness of local currencies is driven by economic crises, intra-national deposit dollarization does not systematically cause such crises.

This paper complements their analysis by highlighting the efficiency of deposit dollarization through improved terms of trade, due to the superior medium of exchange. Specifically, foreign currency holdings in deposits facilitate the use of a more effective medium of exchange with better hedging properties. Indeed, Section 7 demonstrates that welfare-maximizing monetary policy should focus on enhancing the hedging properties of national currencies against such shocks, rather than entirely restricting deposit dollarization.

Table 3 presents the empirical results from two subsample regressions using trade invoicing data before and after the European Debt Crisis in 2013. The results indicate that the coefficient on the USD hedging term becomes larger and statistically more significant in the second subsample. The same quantitative pattern is observed for the subsamples before and after the Global Financial Crisis in 2009. These findings suggest that the hedging demand for the USD strengthened significantly after both crises. This post-crisis flight-to-quality demand for the USD in trade invoicing aligns with the narratives provided by Ito and Chinn (2015) and Maggiori, Neiman, and Schreger (2019) that heightened risk aversion after crises increased the hedging dollar demand in the form of flight-to-safety.

In Section 6, I quantify the welfare implications of the hedging-motive channels in trade invoicing, using a calibrated three-region model that accounts for the empirical patterns of trade invoicing in those regions. I show that the large flight-to-quality demand, driven by heightened risk aversion, exacerbates the welfare cost of a policy prohibiting USD holdings and invoicing outside of the U.S., as it worsens the terms of trade in exchange for a second-best medium of exchange.

6 Quantitative Analysis

In this section, I extend the baseline model to a three-country open economy framework to quantitatively assess its ability to replicate the trade settlement and currency holding patterns observed in the data. Specifically, I calibrate the model to match aggregate moments and empirical currency return properties of three regions: the U.S., the Eurozone, and Latin America. I then evaluate how well the calibrated model replicates observed currency choices in international trade. Furthermore, I provide an interpretation of why the U.S. dollar is widely used in international trade, even in transactions where the U.S. is not directly involved. This dollar dominance has been regarded as a puzzle, given that the euro was expected to compete with the dollar as an alternative international currency as its economic size and trade volume with the rest of the world grew (Boz et al. (2020)).

Building on this quantitative model, I quantify the welfare gains for the global economy resulting from the widespread use of the U.S. dollar, owing to its superior hedging properties. This is done by comparing welfare outcomes with two counterfactual scenario in which the Latin American region implements policies banning currency deposits and trade invoicing in the USD or all of the foreign currencies. The welfare differences are expressed in terms of annual consumption-equivalent terms. Notably, the welfare effects are magnified when agents are more risk-averse, as exemplified by flight-to-quality episodes.

6.1 Calibration

To calibrate the model, the global economy is divided into three trading blocs: the United States, the Eurozone, and Latin America. After discussing the parameters that are conventionally calibrated in the existing literature, I outline the calibration procedure for the hedging properties of those three currencies, which are directly estimated from the data. I then evaluate how well the model replicates the currency usage patterns observed in the data. All data used are on an annual basis, covering the period from 2000 to 2009, unless otherwise specified. This time frame is chosen based on the availability of deposit dollarization data and euro exchange rate data.

The functional forms for the utility function is the standard CRRA utility function $U(c) =$

Parameter	Value	Source	Description
β	0.966	Standard	discount factor
θ_b	1	Standard	buyer and seller's mean utility cost of labor supply
n_{US}	0.393	US GDP share	US country size
n_{EU}	0.408	Euro area GDP share	Euro area size
n_{LM}	0.198	LM GDP share	LM region size
$\lambda_{US,EU}$	0.143	Import share of US from EU	US buyer meeting prob. w. EU seller
$\lambda_{US,LM}$	0.173	Import share of US from LM	US buyer meeting prob. w. LM seller
$\lambda_{EU,US}$	0.092	Import share of EU from US	EU buyer meeting prob. w. US seller
$\lambda_{EU,LM}$	0.023	Import share of EU from LM	EU buyer meeting prob. w. LM seller
$\lambda_{LM,US}$	0.377	Import share of LM from US	LM buyer meeting prob. w. US seller
$\lambda_{LM,EU}$	0.136	Import share of LM from EU	LM buyer meeting prob. w. EU seller
$\mathbb{E}(\gamma_{dollar}^{-1})$	0.9750	Average reciprocal of inflation in US	USD annual return
$\mathbb{E}(\gamma_{euro}^{-1})$	0.9754	Average reciprocal of inflation in EU	Euro annual return
$\mathbb{E}(\gamma_{peso}^{-1})$	0.9397	Average reciprocal of inflation in LM	LM annual return

Table 4: Calibrated parameters

Parameters	Value	Source	Description
γ	2	Standard	Relative risk aversion parameter
$\epsilon_{dollar}^{\cdot,US}$	0.9992	$\gamma \cdot Cov(\Delta \log RGDP_{US,t}, \log \gamma_{dollar,t}^{-1})$	Transfer cost of USD to US seller
$\epsilon_{euro}^{\cdot,US}$	1.0063	$\gamma \cdot Cov(\Delta \log RGDP_{US,t}, \log \gamma_{euro,t}^{-1})$	Transfer cost of euro to US seller
$\epsilon_{peso}^{\cdot,US}$	1	$\gamma \cdot Cov(\Delta \log RGDP_{US,t}, \log \gamma_{peso,t}^{-1})$	Transfer cost of peso to US seller
$\epsilon_{dollar}^{\cdot,EU}$	1.0027	$\gamma \cdot Cov(\Delta \log RGDP_{EU,t}, \log \gamma_{dollar,t}^{-1})$	Transfer cost of USD to EU seller
$\epsilon_{euro}^{\cdot,EU}$	0.9992	$\gamma \cdot Cov(\Delta \log RGDP_{EU,t}, \log \gamma_{euro,t}^{-1})$	Transfer cost of euro to EU seller
$\epsilon_{peso}^{\cdot,EU}$	1.0022	$\gamma \cdot Cov(\Delta \log RGDP_{EU,t}, \log \gamma_{peso,t}^{-1})$	Transfer cost of peso to EU seller
$\epsilon_{dollar}^{\cdot,LM}$	0.9768	$\gamma \cdot Cov(\Delta \log RGDP_{LM,t}, \log \gamma_{dollar,t}^{-1})$	Transfer cost of USD to LM seller
$\epsilon_{euro}^{\cdot,LM}$	0.9836	$\gamma \cdot Cov(\Delta \log RGDP_{LM,t}, \log \gamma_{euro,t}^{-1})$	Transfer cost of euro to LM seller
$\epsilon_{peso}^{\cdot,LM}$	09971	$\gamma \cdot Cov(\Delta \log RGDP_{LM,t}, \log \gamma_{peso,t}^{-1})$	Transfer cost of peso to LM seller

Table 5: Calibrated transfer costs of currencies in DM

$u(c) = c^{1-\gamma}/(1-\gamma)$. where the constant relative risk aversion (CRRA) parameter is set to a standard value of 2. The discount factor is set to $\beta = 0.966$, consistent with an annual real interest rate of 3.5%. The marginal labor utility cost for buyers is unity following the literature, Lagos and Wright (2005).

Since the model assumes that gross money growth rates are equal to gross inflation rates in a stationary equilibrium, the expected inflation rates $\mathbb{E}(\gamma_{US}^{-1})$, $\mathbb{E}(\gamma_{EU}^{-1})$, and $\mathbb{E}(\gamma_{LM}^{-1})$ are set to the inverse of the average annual inflation rates for the period 1999 to 2009. The inflation rates are approximately 0.9750 for the U.S., 0.9754 for the Eurozone, and 0.9397 for the Latin American region, using data from the World Bank.[12]

The parameters governing the Eurozone and Latin American regions are computed as real GDP-weighted averages of member countries' values. The Eurozone and U.S. have the lowest average inflation rates and hence, the euro and dollar have the lowest holding costs, followed by the Latin American peso by a wide margin. This ranking implies that, in the absence of insurance motives, both the euro and dollar would be the equally most preferable store of value, and thus the preferred medium of exchange. Consequently, the nominal exchange rate would be indeterminate without hedging motives as shown in Kareken and Wallace (1981).

The next set of parameters concerns the model's meeting parameters for each country pair, denoted as $\lambda_{j,i}$. These six international meeting parameters, $\lambda_{j,i}$, are calibrated using bilateral trade data from the OECD Inter-Country Input-Output (ICIO) table. The share of each region's population is determined by its share of Real GDP. The size of each region n_j is the real GDP share in the three regions in this economy.

The final set of parameters involves the costs associated with transferring a currency in the decentralized market (DM), denoted as $\epsilon_c^{j,i}$[13]. This requires the measurement of the covariance between labor earning risk and currency returns, $Cov(\theta_{t+1}^{s,i}, -\log \gamma_{c,t+1})$. They are calibrated using data on foreign exchange rate growth, inflation rates and real GDPs. The measurement of $\log \theta_{t+1}^{s,i}$ from the real GDP data follows the procedure outlined in Section 5.1. Both $\theta_{i,t+1}^{s,i}$ and $\gamma_{c,t+1}$ are assumed to be jointly log-normally distributed.

To measure the real returns for currency j, $\gamma_{j,t+1}^{-1}$ for sellers in country i, I use the following approach as in Section 5: when $j \neq i$, the real return is given by $\gamma_{j,t+1}^{-1} = \exp(\log \Delta e_{ji,t+1} - \log \gamma_{i,t+1})$, where $\gamma_{i,t+1}$ represents the inflation rate of country i. For the domestic currency, the return is simply the inverse of the inflation rate. The iid assumption of currency returns approximates the random walk property of the nominal exchange rates in the data as documented by Meese and Rogoff (1983), since the change in the nominal exchange rates is given by the currency return differentials in terms of numeraire: $\Delta \log e_{ji,t+1} = -\log \gamma_{j,t+1} + \log \gamma_{i,t+1}$.

Table 4 and 5 summarize the baseline calibration results for the three-region model. This cali-

[12]Note that the U.S. inflation rate is more conservative in this paper, as it spans data up to 2009, whereas Zhang (2014) includes data only up to 2005.

[13]It is important to note that, in the model, buyers are assumed to be identical, regardless of their country of origin. Consequently, there remain only nine $\epsilon_c^{j,i}$'s to calibrate.

	Description	Model	Data
US	USD deposit share	1	-
	USD share in imports	1	0.94
	USD share of EU import	1	-
	USD share of LM import	1	-
	Seigniorage in RGDP	0.11 %	0.1-0.2%
EU	USD deposit share	0	0.036
	Euro deposit share	1	-
	Euro share in imports	1	0.7
	Euro share of US import	1	-
	Euro share of LM import	1	-
LM	USD deposit share	1	0.3
	USD share in LM transactions	1	-
	USD share in imports	1	0.97
	USD share of US import	1	-
	USD share of EU import	1	-

Table 6: Quantitative results

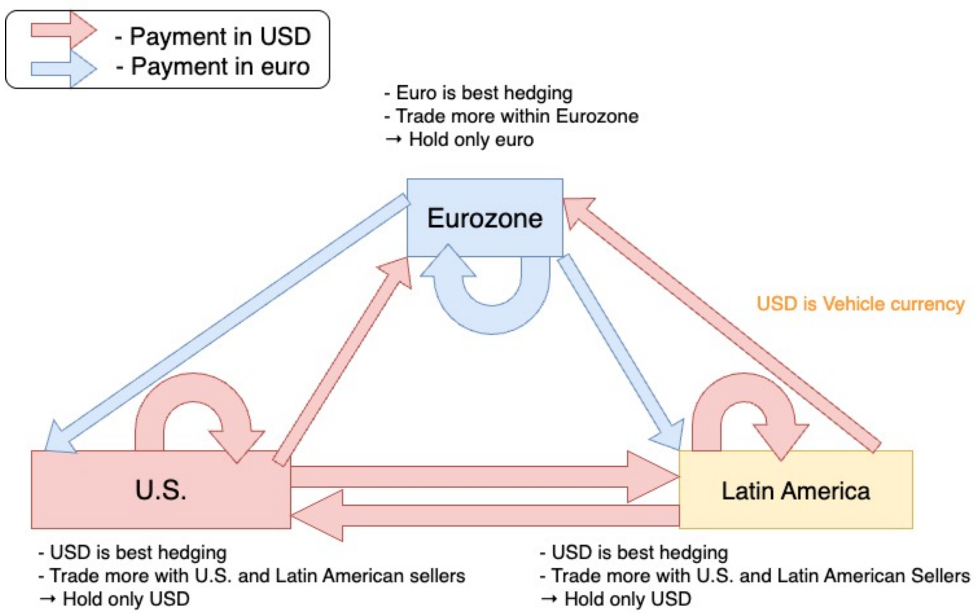

Figure 9: Payment patterns in the baseline calibration

bration indicates that the US dollar is the most effective as a hedging device. This superior hedging property distinguishes the U.S. dollar from the euro as the international medium of exchange despite their similarity in expected currency holding costs.

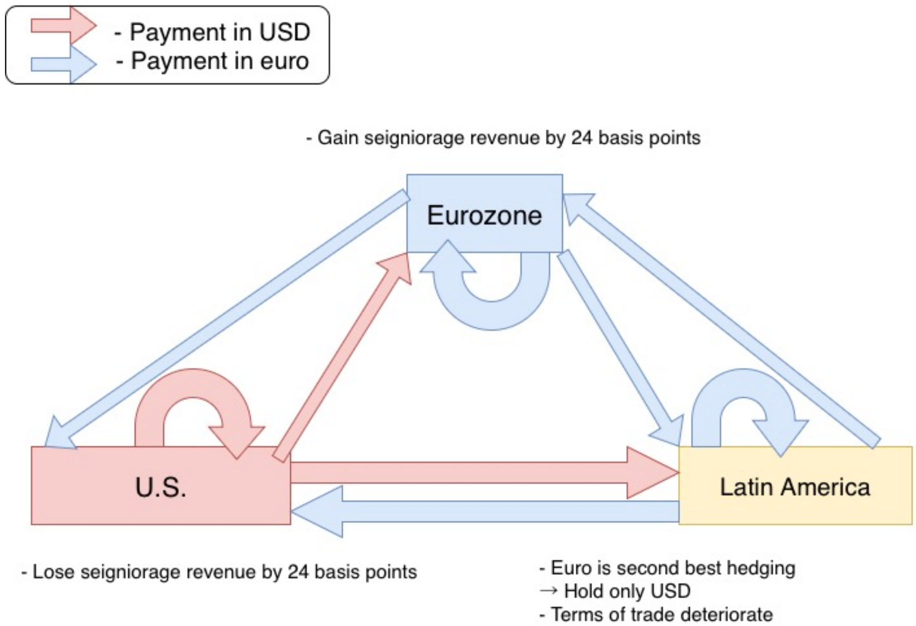

Figure 10: Payment patterns under the counterfactual where the dollar is prohibited in the Latin America region

6.2 Quantitative results and welfare benefits of superior hedging properties of the dollar

Table 6 presents the quantitative implications of the extended model, compared with the data. Figure 9 graphically summarizes the qualitative results. The red arrows represent the payment in dollar, while the blue ones show the payment in euro. The widths of those arrows represent the trade volumes.

The U.S. dollar is used as the medium of exchange and is held by agents in both the U.S. and Latin America, capturing the phenomenon of dollarization observed in many Latin American and Eastern European economies. In contrast, the euro functions as the local currency, used exclusively within the Eurozone, despite the fact that its average holding cost (inflation rate) is similar to that of the dollar. However, the dominance of the dollar stems from its superior hedging property, consistent with the argument by Ito and Chinn (2015) that the hedging motive is the key driver of the surge in the U.S. dollar as the international medium of exchange. This rationalizes the observations made by Gourinchas, Rey, and Sauzet (2019) and Boz et al. (2020), with the model's implications aligning with key empirical patterns in the current international monetary system, where the U.S. dollar remains the dominant global currency, while the euro continues to serve as the regional currency mainly used in transactions involving Eurozone agents.

Furthermore, the sixth row in Table 6 indicates that the model predicts seigniorage revenue of 0.11% of U.S. real GDP, consistent with empirical estimates ranging from 0.1% to 0.2% by Portes and Rey (2002) and Goldberg (2011).

The third row in Table 7 provides more detailed patterns of currency usage in the baseline calibration. Notably, the U.S. dollar is used as the settlement currency in international trade between the Eurozone and Latin America, despite the U.S. not being directly involved in this trade. In this sense, the U.S. dollar functions as a vehicle currency, dominating the euro in the baseline economy, consistent with the empirical findings of Gourinchas, Rey, and Sauzet (2019) and Boz et al. (2020).

This result is driven by search frictions in decentralized international trade. Latin American buyers expect a higher likelihood of meeting with U.S. and Latin American sellers in the decentralized market (DM), who prefer to receive the U.S. dollar and offer better terms of trade in exchange for it. As a result, Latin American buyers hold only U.S. dollars in the centralized market (CM) beforehand and transfer them even to Eurozone sellers. Although their probability of meeting Eurozone sellers is nonnegligible (13.6%) and the euro is the best medium of exchange for transferring to them, the benefit of holding the euro is relatively low compared to holding the U.S. dollar, as the euro is not the best medium of exchange for the more probable meetings with U.S. and Latin American sellers.

These results highlight the dominant role of the U.S. dollar as the primary international medium of exchange. Importantly, it is the superior hedging properties of the U.S. dollar that drive this status, rather than its average holding cost relative to other currencies or the coincidence of multiple equilibria through coordination. The model suggests that the observed hedging properties may be an overlooked in the literature, yet crucial driver of the U.S. dollar's dominance as the international medium of exchange in the global monetary system.

Finally, I conduct two counterfactual policy analyses to quantify the welfare implications of USD trade invoicing with its superior hedging properties. The first policy prohibits the use of USD in trades and currency holdings by LM buyers, while allowing the use of the euro. Figure 10

Country	Description	Baseline	Counterfactual 1	Counterfactual 2
US	USD deposit share	1	1	1
	USD share in US transactions	1	1	1
	USD share of EU import	1	1	1
	USD share of LM import	1	1	1
EU	Euro deposit share	1	1	1
	Euro share in EU transactions	1	1	1
	Euro share of US import	1	1	1
	Euro share of LM import	1	1	1
LM	USD deposit share	1	0	0
	USD share in LM transactions	1	0	0
	USD share of US import	1	0	0
	USD share of EU import	1	0	0
	Euro deposit share	0	1	0
	Euro share in LM transactions	0	1	0
	Euro share of US import	0	1	0
	Euro share of EU import	0	1	0
	Peso deposit share	0	0	1
	Peso share in LM transactions	0	0	1
	Peso share of US import	0	0	1
	Peso share of EU import	0	0	1

Table 7: Currency choice under alternative scenarios

Notes: Counterfactual 1 corresponds to the results under the policy which prohibits the U.S. dollar holdings and import payment in the LM region. Counterfactual 2 corresponds to the results under the policy which prohibits the holdings and import payment by all the foreign currencies including both the U.S. dollar and the euro in the LM region.

graphically illustrates the payment patterns under this scenario, simliar to Figure 9.

The fourth and fifth columns in Table 7 show the currency use patterns under these two counterfactual policies. In the first counterfactual listed in the fourth column, LM buyers substitute the USD with the euro, as the euro has a comparable expected currency holding cost to the USD and offers better hedging properties than the LM peso. Consequently, the welfare difference between the baseline and this counterfactual captures the welfare benefit arising from the superior hedging properties of the USD[14].

The second counterfactual policy prohibits the use of foreign currencies, including both the USD and the euro, by LM buyers, quantifying the welfare consequences of using the LM peso, which

[14]The model does not have any transition dynamics since there is no endogenous state variable. Consequently, the welfare comparison between stochastic steady states is the right exercise to quantify the welfare consequences of different counterfactuals.

Country	Description	BL: $\gamma = 2$ ($\gamma = 10$)	CF1: $\gamma = 2$ ($\gamma = 10$)	C.E.: $\gamma = 2$ ($\gamma = 10$)
Global	Total	-2.0001	-2.0006	
US	Total	-0.7821	-0.7870	-0.24 % (-0.25%)
	Gains from Trade	-0.7870	-0.7870	
	Foreign currency holding cost	0	0	
	Seigniorage revenue	0.0048 (0.0049)	0	
EU	Total	-0.8173	-0.8124	0.237% (0.24%)
	Gains from Trade	-0.8173	-0.8173	
	Foreign currency holding cost	0	0	
	Seigniorage revenue	0	0.0049 (0.0050)	
LM	Total	-0.4005	-0.4012	-0.06 % (-0.297%)
	Gains from Trade	-0.3957 (-0.2178)	-0.3963 (-0.2206)	
	Foreign currency holding cost	-0.0048 (-0.0049)	-0.0049 (-0.0050)	
	Seigniorage revenue	0	0	

Table 8: Welfare quantification when banning the U.S. dollar in LM

Notes: BL (CF1) in the first row represent the baseline (counterfactual 1) results. The counterfactual policy prohibits the use of the U.S. dollar in the currency holdings and import invoicing by LM buyers. The numbers in parentheses are results from the case with a higher risk aversion $\gamma = 10$. C.E. in the first row means the annual consumption equivalent compensation for the transition from the baseline equilibrium to the counterfactual equilibrium.

has the highest currency holding cost and the worst hedging properties. By the construction of this policy, the fifth column in Table 7 shows that the LM buyers only hold their local currencies. Therefore, the welfare difference between the baseline and this counterfactual implies the cost of incurring high currency holding costs and using the local currency with a poor hedging property.

I compare the welfare outcomes between the baseline and counterfactual scenarios, based on the decomposition of the welfare effects of inflation:

Proposition 5. *The monetary policies setting the state-dependent money supply at $t+1$, ($\gamma_{c,t+1}$ affect the expected welfare of country j's citizens through the following channels: gains from trade, foreign currency holding costs, and seigniorage revenues:*

$$\mathcal{W}_{j,t+1} = \underbrace{S_j(q^{b,j}_{t+1})}_{\textit{Gains from trade}} \underbrace{+\theta^b \sum_{c \neq j} (\mathbb{E}[\gamma^{-1}_{c,t+1}] - 1) q^{b,j}_{c,t+1} +}_{\textit{Foreign currency holding costs}} \underbrace{\theta^b \mathbb{E}[1 - \gamma^{-1}_{j,t+1}] \sum_{c \neq j} q^{b,c}_{j,t+1}}_{\textit{Seigniorage revenue from other countries}}, \quad (28)$$

where

$$S_j(q^{b,j,t}_t) = \sum_i \lambda_{j,i} [u(c^{j,i}_{t+1}) - \sum_c v^{b,j}_c d^{j,i}_{c,t}].$$

Country	Description	BL: $\gamma = 2$ ($\gamma = 10$)	CF2: $\gamma = 2$ ($\gamma = 10$)	C.E.: $\gamma = 2$ ($\gamma = 10$)
Global	Total	-2.0001	-2.0008	
US	Total	-0.7821	-0.7870	-0.24 % (-0.25%)
	Gains from Trade	-0.7870	-0.7870	
	Foreign currency holding cost	0	0	
	Seigniorage revenue	0.0048 (0.0049)	0	
EU	Total	-0.8173	-0.8173	0
	Gains from Trade	-0.8173	-0.8173	
	Foreign currency holding cost	0	0	
	Seigniorage revenue	0	0	
LM	Total	-0.4005	-0.3964	0.41 % (0.27%)
	Gains from Trade	-0.3957 (-0.2178)	-0.3964 (-0.2198)	
	Foreign currency holding cost	-0.0048 (-0.0049)	0	
	Seigniorage revenue	0	0	

Table 9: Welfare quantification when banning all of the foreign currencies in LM

Notes: BL (CF) in the first row represent the baseline (counterfactual) results. The counterfactual policy prohibits the use of all of the foreign currencies including both USD and euro in the currency holdings and import invoicing by LM buyers. The numbers in parentheses are results from the case with a higher risk aversion $\gamma = 10$. C.E. in the first row means the annual consumption equivalent compensation for the transition from the baseline equilibrium to the counterfactual equilibrium.

Therefore, the global welfare is the sum of those welfare criteria in all countries:

$$\mathcal{W}_g = \sum_j \mathcal{W}_j = \sum_j S_j(q_t^{b,j,t}). \tag{29}$$

Proof. See section A.4 in the Appendix A. □

According to Proposition 5, the superior hedging property of the U.S. dollar influences a country's welfare through seigniorage transfers between countries and the improved terms of trade resulting from liquidity provision in trades.

The third and fourth columns in Table 8 compare the baseline scenario with a counterfactual scenario in which agents in the Latin America region are restricted from holding or using any USD. In this counterfactual, the global welfare would decline. LM agents would hold and use the euro instead, transferring the inflation tax burden to the Eurozone, which would benefit from this policy. On the other hand, the U.S. could lose the seigniorage revenue originally collected from the LM buyers. The terms of trade for LM buyers would worsen significantly because they would exchange DM goods using the euro in the DM meetings with both LM and U.S. sellers, who prefer to receive the USD. Notably, when risk aversion increases from 2 to 10, as seen in the flight-to-quality episodes following the Global Financial Crisis and European Debt Crisis, this welfare cost is amplified, as shown in parentheses. The rise in foreign currency holding costs with higher risk aversion also implies a greater demand for USD invoicing (LM's foreign currency holding cost on the third column), consistent with the empirical finding in Section 5 that after global crises, the US dollar invoicing share significantly increased.

Table 9 reports the results from the second counterfactual policy that eliminates both USD and euro from LM buyer's currency portfolios. The global welfare would decline more significantly than the first policy. Unlike the previous counterfactual, Latin American agents would enjoy welfare gains due to the reduction of the inflation tax paid to foreign countries. The gains from the trade worsens slightly more for the LM than under the first counterfactual due to the higher currency holding cost of the LM peso. Again, the heightened risk aversion could erode the welfare gains from the policy due to the higher demand for the USD invoicing with the superior hedging property as reported in parenthesis.

Finally, I compute the annual equivalent consumption compensation for the transition from the baseline economy to the counterfactual economy, following the approach of Lucas (1982). Let $\mathcal{W}_j$ represent the annual utility of the buyers and sellers in region j in the baseline, and $\mathcal{W}_j^c(\Delta_j)$ represent the annual utility in the counterfactual economy, where consumption in both the CM and DM is multiplied by $\Delta_j > 0$. I report the value of $(\Delta_j - 1)\%$ in the last columns of Table 8 and 9, such that the baseline utility level becomes equal to that in the counterfactual economy with consumption compensation, i.e., $\mathcal{W}_j = \mathcal{W}_j^c(\Delta_j)$. If $\Delta_j - 1 > 0$, the region's welfare improves by transitioning from the baseline to the counterfactual economy. Conversely, if $\Delta_j - 1 < 0$, the region experiences a welfare loss due to the transition. [15]

The results show that the seigniorage revenue for the U.S. and the Eurozone could be significant, accounting for 0.24–0.25% of their annual consumption. Moreover, the welfare benefits from using the U.S. dollar are considerable, particularly due to its superior hedging properties, especially under conditions of heightened risk aversion, which reflect the aftermath of recent crises. The consumption equivalent compensation could reach approximately 0.3% in terms of annual consumption in the LM region with $\gamma = 10$.

[15] The derivation of the equivalent consumption compensation Δ_j partly relies on the assumption that $\theta_{t+1}^{s,i,t}$ is log-normally distributed.

These welfare changes are similar in magnitude to those observed for long-run inflation welfare costs and benefits in monetary models, such as Zhang (2014). However, the welfare effects in this paper arise from the second-moment properties of real returns on alternative currencies, rather than from the first-moment effects. This suggests that the hedging channel may play a significant role when considering the welfare implications of inflation in monetary models.

Existing studies on the welfare implications of deposit de-dollarization policies have primarily focused on limited risk-sharing and seigniorage arising from the role of currencies as stores of value, often overlooking the terms of trade channel (see, e.g., Bocola and Lorenzoni (2020), Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), Dalgic (2024)). This paper contributes to the discussion by highlighting the additional welfare costs affecting the availability of the prefered medium of exchange, both locally and globally, in the context of a similar policy.

7 Global planner's problem and a simple policy game

Finally, I discuss the state-dependent optimal monetary policies for the global planner who maximizes the global welfare and national governments who maximize the welfare of their national citizens in a simple policy game in the spirit of Zhang (2014).

Note from Proposition 5 that the variances of currency returns $\gamma_{c,t+1}^{-1}$ do not directly affect the welfare but rather influence and mitigate the restrictions on the covariances between the currency return and shocks $\theta_{t+1}^{s,i}$, which have a direct impact on welfare. This insight leads to the following proposition for the optimal policy of the global planner:

Proposition 6. *Suppose that $Var(\gamma_{c,t+1}) \leq \overline{M}_c$ for some $\overline{M}_c > 0$. The global planner chooses the maximum variances of the currency returns. To maximize gains from trade in DM, the global planner adopts the Friedman rule on average s.t. $\mathbb{E}[\gamma_{c,t+1}^{-1}] = 1/\beta$ and maximizes the covariances between the currency returns and shocks, $\{Cov(\gamma_{c,t+1}^{-1}, \theta_{t+1}^{s,i})\}_{c,i}$.*

Proof. See section A.5 in the Appendix A. □

Now I am studying the Nash equilibrium of a simple one-shot policy game in the spirit of Zhang (2014), where all national governments, acting as first movers, simultaneously choose their domestic state- and time-dependent money supply rules (i.e., inflation rates) in the first stage. Then, in the second stage, private agents, as second movers, choose their actions. The national governments are aware of how their money supply rules affect the subsequent actions of private agents through inflation rates when deciding on their policies.

The national governments will adopt a policy regarding the second moments of their national currency return (inverse inflation rate) from the global planner's policy. By doing so, they can maximize the demand for their currencies, and consequently, their seigniorage revenues. However, they might deviate from the Friedman rule on average due to the incentive to collect higher seigniorage from other countries. The following proposition summarize the results:

Proposition 7. *Suppose that* $Var(\gamma_{c,t+1}) \leq \overline{M}_c$ *for some* $\overline{M}_c > 0$*. National government c chooses the maximum variances of the currency returns. To maximize gains from trade in DM, the national government c maximizes the covariances between the currency returns and shocks,* $\{Cov(\gamma_{c,t+1}^{-1}, \theta_{t+1}^{s,i})\}_i$*. However, it might deviate from the Friedman rule on average:* $\mathbb{E}[\gamma_{c,t+1}^{-1}] < 1/\beta$*.*

Proof. See section A.6 in the Appendix A. □

These two propositions show that both the global planner and national governments align in terms of providing the maximum degree of hedging. Therefore, even national governments should have sufficient incentives to make their currencies a better hedging device as envisioned by the global planner. This result rationalizes the superior hedging role of the U.S. dollar observed in the data in terms of the global and national governments' optimal policies.

Moreover, the national government does not exclude the use of foreign currencies as media of exchange to eliminate the inflation taxes paid to foreign countries, if foreign currencies are more superior medium of exchange in some trades than the national currency. This also rationalizes the dominance of the U.S. dollar invoicing observed in the data in terms of the optimal policies of the national governments.

8 Conclusion

In this paper, I examined both the positive and normative implications of the hedging properties of currencies for their role as international mediums of exchange. Theoretically, currencies with stronger hedging properties are more likely to be preferred as mediums of exchange and dominate currency holdings, which aligns with novel empirical evidence from the trade settlement data. The quantitative version of the model, calibrated to exchange rate cyclicality, is able to account for observed empirical patterns in both the international use of currencies and currency holdings in deposits.

Furthermore, the model predicts that the hedging properties of currencies have a sizable welfare implications of policies often discussed in emerging market countries, which restrict the foreign currency use in trades and holdings. These welfare consequences can primarily arise from seigniorage changes and shifts in the terms of trade, as removing the preferred medium of exchange—such as the US dollar with the superior hedging property. The welfare cost from deteriorating terms of trade could be considerable, potentially offsetting the large welfare gains from a reduced inflation tax, particularly in cases where agents are highly risk-averse as exemplified during flight-to-quality episodes.

The framework developed here could be further extended by incorporating alternative financial assets, such as government bonds, as well as more general forms of preferences and uncertainty. Such extensions would allow for a joint analysis of risk and liquidity premia in both currency and bond markets, providing a framework for the analysis of various polices such as quantatitive easing and foreign exchange interventions. I leave these extensions for future work.

References

Adams, Patrick and Adrian Verdelhan. 2025. “Exchange rate risk in public firms.” *mimeo* .

Bocola, Luigi and Guido Lorenzoni. 2020. “Financial crises, dollarization, and lending of last resort in open economies.” *American Economic Review* 110 (8):2524–2557.

Boz, Emine, Camila Casas, Georgios Georgiadis, Gita Gopinath, Helena Le Mezo, Arnaud Mehl, and Tra Nguyen. 2020. “Patterns in invoicing currency in global trade.” *mimeo* .

Chahrour, Ryan and Rose Valchev. 2022. “Trade finance and the durability of the dollar.” *Review of Economic Studies* 89:1873–1910.

Chien, YiLi, Hanno Lustig, and Kanda Naknoi. 2020. “Why Are Exchange Rates So Smooth? A Household Finance Explanation.” *Journal of Monetary Economics* 112:129–144.

Chirstiano, Lawrence, Husnu Dalgic, and Armen Nurbekyan. 2022. “Financial dollarization: efficient intranational risk sharing or prescription for disaster?” *mimeo* .

Constantinides, George M. and Darrell Duffie. 1996. “Asset pricing with heterogeneous consumers.” *Journal of Political Economy* 104 (2):219–240.

Dalgic, Husnu C. 2024. “Financial dollarization in emerging markets: an insurance arrangement.” *International Economic Review* 65 (3):1189–1219.

Drenik, Andres, Rishabh Kirpalani, and Diego Perez. 2021. “Currency choice in contracts.” *Review of Economic Studies* 89 (5):2529–2558.

Goldberg, Linda. 2011. “The international role of the dollar: does it matter if this changes?” *Staff Reports* 522:Federal Reserve Bank of New York.

Goldberg, Linda and Cedric Tille. 2008. “Vehicle currency use in international trade.” *Journal of International Economics* 76 (2):177–192.

Gomis-Porqueras, Pedro, Timothy Kam, and Christopher Waller. 2017. “Nominal exchange rate determinacy under the threat of currency counterfeiting.” *American Economic Journal: Macroeconomics* 23 (2):365–388.

Gourinchas, Pierre-Olivier, Hélène Rey, and Nicholas Govillot. 2017. "Exorbitant previledge and exorbitant duty." *mimeo* .

Gourinchas, Pierre-Olivier, Hélène Rey, and Maxime Sauzet. 2019. "The international monetary and financial system." *Annual Review of Economics* 11:859–93.

Heaton, John and Deborah Lucas. 1996. "Evaluating the effects of incomplete markets on risk sharing and asset pricing." *Journal of Political Economy* 104 (3):443–487.

———. 2000. "Portfolio choice and asset prices: the imporrtance of entrepreneurial risk." *Journal of Finance* 55 (3):1163–1198.

Heaton, John C. and Deborah Lucas. 1997. "Market frictions, saving behavior and portfolio choice." *Journal of Political Economy* 1 (1):76–101.

Ito, Hiro and Menzie Chinn. 2015. "The rise of the redback: evaluating the prospects for renminbi use in invoicing." *Renminbi Internationalization: Achievements, Prospects, and Challenges* 111-58.

Jacquet, Nicolas and Serene Tan. 2012. "Money and asset prices with uninsurable risks." *Journal of Monetary Economics* 59:784–797.

Kareken, John and Neil Wallace. 1981. "On the indeterminacy fo equilbrium exchange rates." *Quaterly Journal of Economics* 96 (2):207–222.

Lagos, Ricardo and Randall Wright. 2005. "A unified framework for monetary theory and policy analysis." *Journal of Political Economy* 113 (3):464–484.

Levy-Yeyati, Eduardo. 2006. "Financial dollarization: evaluating the consequences." *Economic Policy* 21:62–118.

Li, Yiting and Akihiko Matsui. 2009. "A theory of international currency: competition and discipline." *Journal of Japanese and Interantional Economies* 23:407–426.

Lucas, Robert E. 1982. *Models of Business Cycles.* Basil Blackwell, Oxford.

Lustig, Hanno and Adrian Verdelhan. 2007. "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk." *American Economic Review* 97 (1):89–117.

Maggiori, Matteo. 2017. "Financial intermediation, international risk sharing, and reserve currencies." *American Economic Review* 107 (10):3038–71.

Maggiori, Matteo, Brent Neiman, and Jesse Schreger. 2019. "The rise of the dollar and fall of the euro as international currencies." *AEA Papers and Proceedings* 109:521–526.

Matsuyama, Kiminori, Nobuhiro Kiyotaki, and Akihiko Matsui. 1993. "Toward a theory of internationla currency." *Review of Economic Studies* 60:283–307.

Meese, Richard A. and Kenneth Rogoff. 1983. "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?" *Journal of International Economics* 14 (1):3–24.

Moskowitz, Tobias J. and Annette Vissing-Jorgensen. 2002. "The returns to entrepreneurial investment: a private equity premium puzzle?" *American Economic Review* 92 (4):745–778.

Mukhin, Dmitry. 2022. "An equilibrium model of international price system." *American Economic Review* 112 (2):650–688.

Obstfeld, Maurice, Jay C. Shambaugh, and Alan M. Taylor. 2010. "Financial stability, the trilemma, and international reserves." *American Economic Journal: Macroeconomics* 2 (2):57–94.

Oskolkov, Aleksei Oskolkov and Marcos Sorá. 2023. "Macroprudential policy for internal financial dollarization." *Journal of International Economics* 145:103838.

Parker, Jonnathan A. and Antte Vissing-Jorgensen. 2009. "Who Bears Aggregate Fluctuations and How?" *American Economic Review, Papers and Proceedings* 99 (2):399–405.

Portes, Richard and Helene Rey. 2002. "Euro vs. dollar: will the euro replace the dollar as the world currency?" *Economic Policy* 13:26.

Zhang, Cathy. 2014. "An information-based theory of international currency." *Journal of International Economics* 93:286–301.

Online Appendix Not For Publication

A Proofs

In the Appendix, I drop the time subscript of random variables since they are iid and the conditional information is not relevant. The currencies are denoted by numbers such as 1, 2, and 3 for more generic notations rather than US or LM.

A.1 Proof of Proposition 1

Proof. The proof relies on the standard Kuhn-Tucker condition. Since the obtaining an additional unit of the DM consumption by using either currencies is identical, the first-order necessary and sufficient optimality condition is given by

$$\epsilon_c \equiv \epsilon \geq u'(c) - \lambda_c, \tag{A.1}$$

coupled with the nonnegative currency transfer condition $d_c \geq 0$, the feasibility condition $d_c \leq q_c$ for all $c \in \{1, 2\}$, and complementary slackness conditions. Define a consumption level $c(\epsilon)$ such that the marginal utility is equal to ϵ, i.e. $u'(c(\epsilon)) = \epsilon$. Consider two cases: (i) $c(\epsilon)$ is budget feasible and (ii) $c(\epsilon)$ is not budget feasible.

(i) $\underline{v_1^s q_1 + v_2^s q_2 \geq c(\epsilon)}$ The terms of trade involve the consumption $c = c(\epsilon)$ and the currency portfolio transfer $v_1^s d_1 + v_2^s d_2 \geq c(\epsilon)$. To see this, set the Lagrange multiplier $\lambda_c = 0$. The definition of $c(\epsilon)$ implies that the first-order condition in eq.(A.1) holds with equality. Moreover, the the currency portfolio transfer (d_1, d_2) satisfies the feasibility condition and can be structured to satisfy the nonnegative amount of each currency transfer.

(ii) $\underline{v_1^s q_1 + v_2^s q_2 < c(\epsilon)}$ The terms of trade in this case are given by the consumption $c = v_1^s d_1 + v_2^s d_2$ and $d_c = q_c$ for all $c \in \{1, 2\}$. In words, the buyer gives all the currency holdings to the seller to get the maximum level of consumption. To see this, set λ_c's in eq.(A.1) such that the first-order condition in eq.(A.1) holds with equality. Moreover, the currency portfolio transfer satisfies the nonnegativity and feasibility. □

A.2 Proof of Proposition 2

Proof. The proof utilizes relies on the standard Kuhn-Tucker condition. Anticipating the extension of the model in section 6, I provide the proof of three-currency version of the model.

Without loss of generality, assume that $\epsilon_1 < \epsilon_2 < \epsilon_3$ so that the currency 1 is the best medium of exchange, followed by currency 2 and then currency 3. This assumption implies that $c(\epsilon_3) < c(\epsilon_2) < c(\epsilon_1)$ since each ϵ represents the marginal cost of obtaining an additional unit of consumption by trasferring a currency. Define $q_1(\epsilon_1)$ be the real balance of currency 1 such that $v_1^s q_1(\epsilon_1) = c(\epsilon_1)$, i.e. the real balance holding of currency 1 enough to cover the consumption expense at the level of $c(\epsilon_1)$. Analogously, define the real balance holding of currency 1 enough to expense $c(\epsilon_2)$, i.e. $v_1^s q_1(\epsilon_2) = c(\epsilon_2)$. Obviously, $q_1(\epsilon_2) < q_1(\epsilon_2)$ since $c(\epsilon_2) < c(\epsilon_1)$.

There are three different first-order conditions: for $c \in \{1, 2, 3\}$,

$$\epsilon_c \geq u'(c) - \lambda_c. \tag{A.2}$$

I consider three distinct cases depending on the buyer's currency holding.

(i) $\underline{q_1 \geq q_1(\epsilon_1)}$ This case describes the situation where the buyer holds an adequate amount of currency 1 to cover all the necessary consumption expenditure solely with that preferrd medium of exchange. The terms of trade are given by the consumption $c = c(\epsilon_1)$ and currency transfer $d_1 = c(\epsilon_1)/v_1^s$, $d_2 = d_3 = 0$, implying that the buyer only uses the most preferred medium of exchange, currency 1 to purchase the sufficient amount of consumption. By setting $\lambda_1 = 0$, the first-order condition (A.2) for currency 1 holds with equality. The first-order conditions for currency 2 and 3 hold with inequality by setting $\lambda_c = 0$ for $c \in \{2, 3\}$ since the marginal cost of transferring those currencies is strictly larger than the marginal utility of consumption. Moreover, the currency portfolio transfer is budget feasible and trivially nonnegative.

(ii) $\underline{q_1(\epsilon_2) < q_1 < q_1(\epsilon_1)}$ This subcase corresponds to the scenario where the buyer does not hold enough amount of currency 1 to buy $c(\epsilon_1)$ with that currency but still adequate to purchase $c(\epsilon)$ amount of consumption without using other currencies. In this case, the terms of trade involve the consumption level $c = v_1^s q_1$ and the currency portfolio transfer $d_1 = q_1$, $d_2 = d_3 = 0$. To see this, set $\lambda_1 = u'(c) - \epsilon_1 > 0$ and $\lambda_2 = \lambda_3 = 0$, implying that the currency 1 transfer is now constrained.

Then the first order condition (A.2) for currency 1 holds with equality while those for currency 2 and 3 hold with inequality. The currency transfers are budget feasible and nonnegative.

(iii) $q_1 < q_1(\epsilon_2)$ and $v_1^s q_1 + v_2 q_2 \geq c(\epsilon_2)$ This describes the case where the buyer holds enough amount of currency 1 and 2 to cover the consumption expense at the level of $c(\epsilon_2)$ without using currency 3, the worst medium of exchange. The solution is the consumption $c = c(\epsilon_2)$ and currency transfers $d_1 = q_1$, $d_2 = (c(\epsilon_2) - v_1^s q_1)/v_2^s$, and $d_3 = 0$. To confirm this is actually the solution, set $\lambda_1 = u'(c) - \epsilon_1 > 0$ and $\lambda_2 = \lambda_3 = 0$. Then the first-order conditions (A.2) hold with equality for currency 1 and 2, and with inequality for currency 3. The currency transfer is nonnegative and budget-feasible.

(iv) $c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 < c(\epsilon_2)$ This corresponds to the situtation where the currency 1 and 2 holdings are not enough to obtain $c(\epsilon_2)$ units of consumption but adequate to expense $c(\epsilon_3)$ units of consumption. The terms of trade in this case are characterized by the consumption $c = v_1^s q_1 + v_2^s q_2$ and the currency transfer $d_1 = q_1$, $d_2 = q_2$, and $d_3 = 0$. To check if this is actually the solution, set $\lambda_c = u'(c) - \epsilon_c$ for currency $c \in \{1, 2\}$ and $\lambda_3 = 0$. Then the first order conditions (A.2) hold for currency 1 and 2 with equality while it holds for currency 3 with inequality.

(v) $v_1^s q_1 + v_2^s q_2 < c(\epsilon_3)$ and $c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 + v_3 q_3$ This is the case where the holdings of currency 1 and 2 are not sufficient to obtain $c(\epsilon_3)$ but the currency 3 holding can cover the rest of the expense. The terms of trade are provided by $c = c(\epsilon_3)$ and the currency transfer $d_1 = q_1$, $d_2 = q_2$, and d_3 such that $c(\epsilon_3) = v_1^s q_1 + v_2^s q_2 + v_3 d_3$. This is nonnegative and budget-feasible. To check if this satisfies the first-order condition (A.2), set $\lambda_c = u'(c) - \epsilon_c$ for currency $c \in \{1, 2\}$ and $\lambda_3 = 0$. Then the first order conditions hold for currency 1 and 2 with equality while it holds for currency 3 with inequality.

(vi) $v_1^s q_1 + v_2^s q_2 + v_3 q_3 < c(\epsilon_3)$ The terms of trade in this case corresponds to $c = v_1^s q_1 + v_2^s q_2 + v_3 q_3$ and currency transfers $d_c = q_c$ for all $c \in \{1, 2\}$. The first-order conditions (A.2) hold with equality for all currencies by setting $\lambda_c = u'(c) - \epsilon_c$. □

A.3 Proof of Proposition 4

Proof. Following the proof of proposition 2 in appendix A.2, consider six different cases to compute $\partial_{q_{c,t}^{b,j,t}} V_{t+1}^{b,j,t}$. I denote $\partial_{q_{c,t+1}} V_{t+1}^{b,j} = \partial_c V^b$.

(i) $\underline{v_1^s q_1 + v_2^s q_2 \geq c(\epsilon)}$ In this case, the buyer never spend the additional unit of currency 1 in the DM and carry over to the next CM. Consequently, $\partial_1 V^b = v_1^b = (v_1^b/v_1^s)v_1^s = u'(c)v_1^s$, where the last equality comes from the first-order condition with respect to currency 1 (A.2). Moreover, $\partial_c V^b = v_c^b$, since those currencies will never be used in this current case and be carried over to the next CM.

(ii) $\underline{q_1(\epsilon_2) < q_1 < q_1(\epsilon_1)}$ The buyer uses the additional holding of currency 1 to obtain v_1^s units of consumption goods, yielding $\partial_1 V^b = u'(c)v_1^s$. Moreover, $\partial_c V^b = v_c^b$ for currency $c \in \{2, 3\}$ since those currencies will never be used in this current case and be carried over to the next CM.

(iii) $\underline{q_1 < q_1(\epsilon_2) \text{ and } v_1^s q_1 + v_2 q_2 \geq c(\epsilon_2)}$ In this case, the buyer uses the additional holding of currency 1 and reduce the transfer of currency 2 by v_1^s/v_2^s, which is the seller's exchange rate between currency 1 and 2, while maintaining the DM consumption at the same level. This yields $\partial_1 V^b = v_2^b(v_1^s/v_2^s) = u'(c)v_1^s$, where the last equality follows from the first-order condition with respect to currency 2 (A.2). Since the additional holding of currency 2 and 3 will never be used in this case, $\partial_c V^b = v_c^b$ for currency $c \in \{2, 3\}$.

(iv) $\underline{c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 < c(\epsilon_2)}$ The buyer spends the additional holding of currency 1 to obtain additional v_1^s units of DM goods, yielding $\partial_1 V^b = u'(c)v_1^s$. Moreover, the buyer uses the additional holding of currency 2 to obtain the additional v_2^s units of DM goods, yielding $\partial_2 V^b = u'(c)v_2^s$. On the other hand, he will never spend the additional holding of currency 3 so that $\partial_3 V^b = v_3^b$.

(v) $\underline{v_1^s q_1 + v_2^s q_2 < c(\epsilon_3) \text{ and } c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 + v_3 q_3}$ In this case, the buyer uses the additional holding of currency 1 or 2 and reduce the transfer of currency 3 by v_1^s/v_3^s or v_2^s/v_3^s, which is the seller's exchange rate between currency 1 (or 2) and 3, while maintaining the DM consumption at the same level. This yields $\partial_c V^b = v_3^b(v_1^s/v_3^s) = u'(c)v_c^s$ for currency $c \in \{1, 2\}$, where the last equality follows from the first order condition with respect to currency 3 in (A.2). Finally, the buyer will never spend the additional holding of currency 3 so that $\partial_3 V^b = v_3^b$.

(vi) $\underline{v_1^s q_1 + v_2^s q_2 + v_3 q_3 < c(\epsilon_3)}$ The buyer spends the additional holding of currency 1, 2, or 3 to obtain the additional v_c^s units of DM goods, yielding $\partial_c V^b = u'(c)v_c^s$, $c \in \{1, 2, 3\}$.

Using these results, I can rewrite the first-order condition with respect to currency $c \in \{1, 2, 3\}$

in (23) as

$$\theta^{b,j} \geq \beta\left[\partial_c V^b + \left(1 - \sum_i \lambda_{j,i}\right) v_c^b\right] = \beta v_1^b \sum_i \lambda_{j,i}\left[1 + \max\left\{0, \frac{1}{\epsilon_c^{j,i}} u'(c) - 1\right\}\right]$$
$$= \beta v_c^b\left[1 + \sum_i \lambda_{j,i} l_c^{j,i}(\mathbf{q})\right]. \tag{A.3}$$

□

A.4 Proof of Proposition 5

Proof. I start with expressing the government lump-sum transfer in the first line as

$$T_{j,t} = \phi_{j,t} M_{j,t+1}\left(1 - \frac{M_{j,t}}{M_{j.t+1}}\right) = (1 - \gamma_{j,t}^{-1}) \sum_c q_{j,t+1}^{b,c}, \tag{A.4}$$

where the last equality uses the market clearing condition in period $t-1$ for the real balance of currency c: $\phi_{j,t} M_{j,t+1} = \sum_c q_{j,t+1}^{b,c}$. Since the buyer makes TIOLI offer in the DM, the sellers' welfare does not change in the inflation rates.

The buyer's lifetime value can be decomposed as

$$U_0^{b,j} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_i \lambda_{ji} \{u[c_t^{j,i}(\mathbf{q}_t^{b,j})] - v^{b,j} \cdot \mathbf{d}_t(\mathbf{q}_t^{b,j})\}\right] + U[C^{**}(1)] - C^{**}(1)$$
$$+ T_{j,t} + \sum_c \gamma_{c,t}^{-1} q_{c,t}^{b,j} - \sum_c q_{c,t+1}^{b,j}$$
$$= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_i \lambda_{ji} \{u[c_t^{j,i}(\mathbf{q}_t^{b,j})] - v^{b,j} \cdot \mathbf{d}_t(\mathbf{q}_t^{b,j})\}\right] + U[C^{**}(1)] - C^{**}(1)$$
$$- \sum_{c \neq j} \mathbb{E}_0[1 - \gamma_{c,t}^{-1}] q_{c,t}^{b,j} + \sum_{c \neq j} \mathbb{E}_0[1 - \gamma_{j,t}^{-1}] q_{j,t}^{b,c} \tag{A.5}$$

where the last equality follows from the fact that $q_{c,t+1}^{b,i} = q_{c,t}^{b,j}$ for all c and i.

Collecting all these terms depending on the period-$t+1$ inflation rate $\gamma_{c,t+1}$, the relevant terms in total welfare of country j are given by

$$\mathcal{W}_{j,t+1} = S(\mathbf{q}_{t+1}^{b,j}) - \underbrace{\sum_{c \neq j} \mathbb{E}_{t+1-}[1 - \gamma_{c,t+1}^{-1}] q_{c,t+1}^{b,j}}_{\text{Inflation tax paid to foreign countries}} + \underbrace{\sum_{c \neq j} \mathbb{E}_{t+1-}[1 - \gamma_{j,t+1}^{-1}] q_{j,t+1}^{b,c}}_{\text{Seigniorage collected from foreign countries}} \tag{A.6}$$

where β^{gov} is the government's discount factor. Letting $\beta = \beta^{gov} \to 1$, I obtain the desired result in eq. (28). Moreover, the global welfare (29) can be obtained by summing up the local welfare (28).

□

A.5 Proof of Proposition 6

Proof. The global welfare does not directly depend on the variance of inflation rates but only through the restrictions on the values that the covariance terms can take $Cov(\theta^{s,i}_{t+1}, \gamma^{-1}_{j,t+1})$. Moreover, the global welfare is nondecreasing in those covariances through the hedging and hence, affecting the terms of trade in DM. Consequently, the global planner chooses the largest variances and covariances of the inflation rates with the labor earning risks $\theta^{s,i}_{t+1}$. Moreover, setting the average currency return equal to the inverse discount factor $\mathbb{E}[\gamma^{-1}_{j,t+1}] = 1/\beta$, the currency j holdings become satiated to approach to infinity for all agents, which weakly increases the global gains from trade and hence, the global welfare.

□

A.6 Proof of Proposition 7

Proof. Notice that the local welfare in (28) does not directly depend on the national inflation rate but only through the restrictions on the values that the covariance terms can take $Cov(\theta^{s,i}_{t+1}, \gamma^{-1}_{j,t+1})$. Consequently, it is optimal for the local planner to maximize the variance and covariances. The local planner might not choose the Friedman rule on average, $\mathbb{E}[\gamma^{-1}_{j,t+1}] < 1/\beta$ since the local welfare might be improved by raising the inflation tax and increasing the seignirage collected from foreign agents. □