A Game For Teaching Cutoff Equilibria

Naomi M. Utgoff (utgoff@usna.edu) [1]   Oskar Zorrilla (zorrilla@usna.edu) [1]

[1]United States Naval Academy

Motivation

Cutoff equilibria are an important category of Bayesian Nash equilibria, the simplest of which study a binary decision made under incomplete information. We present an in-class vaccination game from Utgoff's game theory class, which she uses as a prelude to formal equilibrium calculation. Dramatizing the decision process develops and strengthens student intuition, which supports subsequent formal mathematical analysis of this game and others featuring cutoff equilibria.

Active Learning

The standard pedagogical method of teaching these equilibria begins by first defining the equilibrium concept and then considering examples. The active learning approach we describe here switches the order. Students first encounter an example of their own, and only *after* they have developed some intuition do they proceed to formally define the equilibrium concept.

We highlight four important learning strategies that this exercise covers.

- **Incentive Compatibility** By incentivizing students with chocolate, Utgoff ensures that they have "skin in the game" and think through the best strategy.
- **Generation** Approaching a problem before formal analysis with incentives (chocolate) encourages students to think about how to best to play the game.
- **Elaboration** Solving analogous problems (or coming up with analogous examples). By solving the problem first, the lesson becomes about explaining the example students have just participated in, rather than finding examples that fit the model. Having participated in the exercise allows students to develop intuition for the model before they actually study it formally.
- **Reflection** Thinking about thinking: how the problem was solved. Metacognition is an important step in learning and is facilitated by the discussion that students have after the game to "debrief" on why they chose the strategies they did. This allows students to think about their own thought processes and whether those led them to find optimal strategies.

Solution Concept and Regret

A cutoff equilibrium is a Bayesian Nash equilibrium characterized by one or more cutoff types which partition a player's type space into discrete pieces, each piece corresponding to an action.

There are two sources of equilibrium regret in this game:

1. Both players vaccinate
2. Both players do not vaccinate and a player gets sick

Vaccination Game

Preparation: We draw student types iid Uniform$[0, 1]$ distribution using Excel's RAND() function.

Setting Up The Experiment: We give each student

1. their induced type and a description of the game:
2. verbal instructions to keep their type private
3. a participation fee of four Hershey's kisses with instructions not to eat any until the end of the game.

Play Of The Game:

1. Each student writes Vaccinate (V) or Don't vaccinate (D) on their piece of paper.
2. We collect the papers and draw pairs at random to play.

Payoffs:

1. We take away one Hershey's kiss from any member of the pair who chose V.
2. If one of the pair chose D and the other chose V, the student who chose D keeps all four Hershey's kisses.
3. If both players chose D, we draw publicly two random numbers (one for each member of the pair) using Excel's RAND() function. If the number drawn is less than or equal to a player's type, we take away all four Hershey's kisses. If the number drawn is greater than a player's type, the student who selected D keeps all four Hershey's kisses.

Class Observation

Average play is a cutoff strategy: Immediately after all students have played we compute and observe

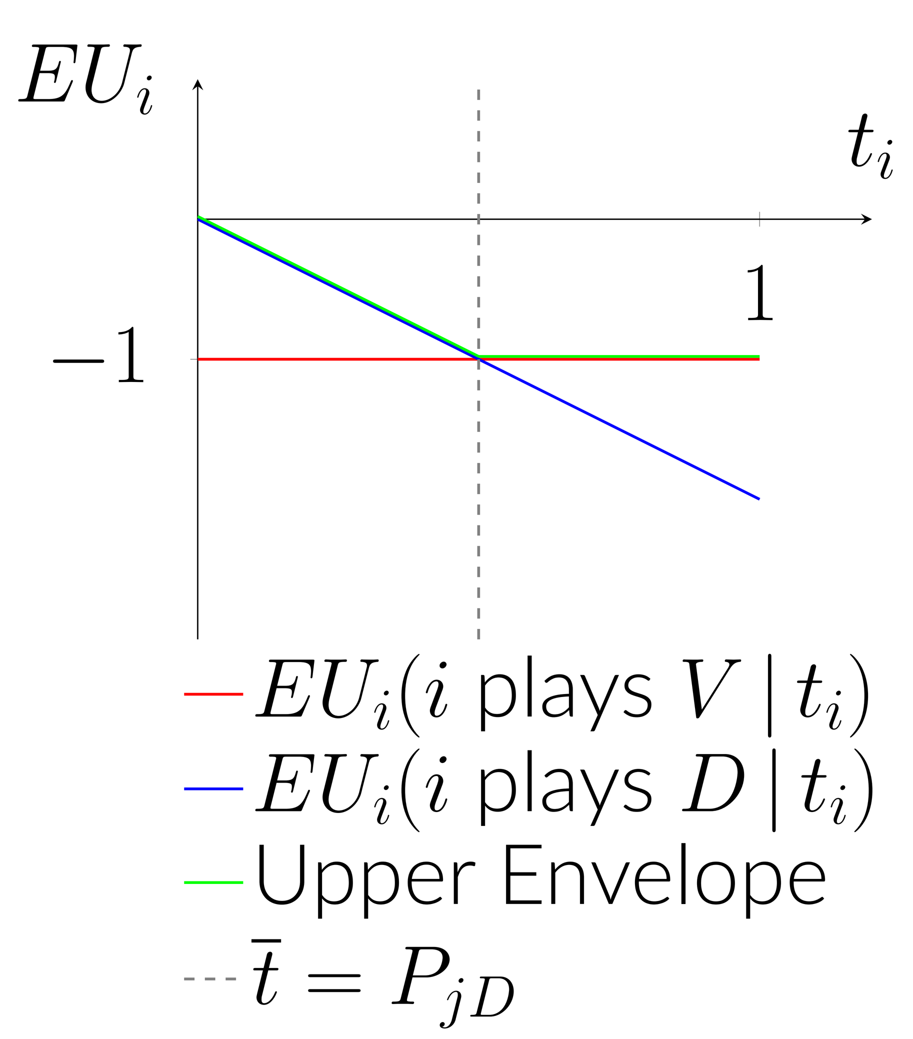
$$E[t \mid V] > E[t \mid D].$$

Students discuss the reason behind their actions, and their experience of regret (or not).

Over 81 observations since 2024, we observe

$$E[t \mid V] = 0.54 > E[t \mid D] = 0.36.$$

Graphical Solution



Analytic Solution

If $t_i = 0$, D weakly dominates V. The Bayesian Nash equilibrium strategy must take the form

$$\begin{array}{l} D \text{ if } t_i < \bar{t} \\ V \text{ if } t_i \geq \bar{t} \end{array}$$

for some $\bar{t}$.

In Bayesian Nash equilibrium $\bar{t}$ satisfies

$$\begin{aligned} EU_i[V \mid t_i = \bar{t}] &= EU_i[D \mid t_i = \bar{t}] \\ -1 &= P_{jD}(-4\bar{t}) + (1 - P_{jD})(0) \\ -1 &= -4\bar{t}^2 \\ \bar{t} &= 1/2. \end{aligned}$$

The Bayesian Nash equilibrium is a cutoff equilibrium given by

$$\begin{array}{l} D \text{ if } t_i < 1/2 \\ V \text{ if } t_i \geq 1/2. \end{array}$$

References

[1] Peter C. Brown, Henry L. Roediger III, and Mark A. MacDaniel. *Make it Stick: The Science of Succesful Learning*. Harvard University Press, Cambridge, MA, 2014.

[2] Naomi Utgoff. Game theory 39: Vaccination with incomplete information. https://www.youtube.com/watch?vč8H4DDTeTV4, 2020.

[3] Oskar Zorrilla. *Macroeconomics: An Active Learning Approach*. MIT Press, Cambridge, MA, 2025.