# The Asset Durability Premium

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**Abstract**

Our paper examines the asset pricing implications of asset durability for equity risk. We develop a quantitative model with aggregate uncertainty where firms optimize asset durability driven by occasionally binding borrowing constraints. Our model reveals a novel risk premium channel emerging in general equilibrium, with durable capital harder to finance due to greater price risk sensitivities to financial frictions. Firms holding less durable capital provide hedging benefits against aggregate risk, as capital prices of varying durabilities affect firm risk. This model helps rationalize the asset durability premium documented in the cross-section of stock returns among financially constrained firms.

**JEL Codes:** E2, E3, G12

**Keywords:** asset durability; financial constraints; collateral; cross-section of stock returns

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# 1 Introduction

In this paper, we examine the asset pricing implications of the fact that firms endogenously react to financial constraints by optimizing their capital composition of different asset durabilities, with respect to how much they hold durable vs. non-durable capital, whereby the asset durability has been shown an essential feature of capital (Rampini, 2019).[1] We show compositional changes in asset durability not only determine the price cyclicalities of different assets, but significantly shift the firm risk in the cross-section.

The canonical macro-finance model featuring financial frictions predicts that economic downturns lead to deteriorated firms' financial conditions (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). Rampini (2019) shows that tightened financial constraints determine firms' choices over different types of capital characterized by asset durability. Our paper specifically examines the impacts of firms' asset compositional changes driven by financial frictions on the risk sensitivities of different capital prices in general equilibrium. Most importantly, we demonstrate the novelty of our results by showing both empirically and theoretically that firms' asset composition of different asset durabilities, by loading in different risk exposures to financial frictions, affects the firm risk and determines the cross-section of stock returns.

In particular, our general equilibrium model demonstrates that firms' substitution over asset durability delivers an important but under-explored *risk-premium* channel, which finds that the equilibrium price of durable capital is more procyclical, and durable capital is therefore riskier than non-durable capital. Our model insights are new in the sense that durable capital is harder to finance not only because it has a greater down payment, as in Rampini (2019), but also because it exhibits extra risk sensitivities to financial frictions relative to non-durable capital. Hence, as firms' holding less durable capital provides hedging against aggregate risk, properties of capital prices of different asset durabilities significantly shift the firm risk.

We first examine the empirical relationship between asset durability and the expected stock returns. Our paper contributes to the literature by providing the first empirical measure of asset durability at the firm level. Specifically, the novelty of our measure lies in the aggregation of the differed durability across refined asset categories of a firm's portfolio, which is derived from the depreciation data in the U.S. Bureau of Economic Analysis (BEA) fixed asset table. The BEA table provides detailed estimates of depreciation rates and net capital stocks at fixed costs, covering a broad array of assets. For each year, we construct the asset-level durability across assets listed in the BEA table, calculating the industry-level asset durability. We then obtain a firm-level measure of asset durability by calculating the value-weighted average of industry-level asset durability indices across the business segments in which the firm operates.

[1] Asset durability is defined as the inverse of the geometric depreciation rate of a given asset type as in Rampini (2019) and the degree of durability varies across asset types. For example, the depreciation rate can be as low as 1 percent for new residential structures, which are identified as more durable capital, whereas the depreciation rate can be as high as 31 percent for computing equipment, as labeled as less durable capital.

Next, we uncover significant heterogeneity in asset durability across firms and show that firms' portfolios shift toward less durable capital if they face tightened financial constraints. Particularly, we document that stock returns in the cross-section are strongly associated with firms' asset durability measures, especially when firms are financially constrained. In particular, we construct five portfolios that are univariate-sorted based on firms' asset durability relative to their industry peers and then examine the return differences across different portfolios. We show that there is a statistically significant asset durability return spread among financially constrained firms. The levered return spread between the highest durability quintile portfolio and the lowest durability quintile portfolio averages approximately 3.56% to 6.93% per year, depending on the specific measure that we use to sample financially constrained firms. Considering the fact that constrained firms with larger asset durability are on average more leveraged in the data, we find that the return spreads between the highest and lowest durability quintiles even after controlling for the leverage effect, known as the unlevered return spreads, are still sizeable ranging from 2.34% to 4.75%. Hence, termed as the "Asset Durability Premium", our documented return spread captures differences in average portfolio returns between the highest and lowest portfolios sorted by the asset durability measure, regardless of the leverage ratio differences across portfolios. We show that implementing a high-minus-low strategy based on asset durability spread results in an annualized Sharpe ratio of 0.59 and 0.49 for levered and unlevered returns respectively, comparable to that of the market portfolio. By contrast, we find no asset durability spread among unconstrained firms.

Our model then builds upon our documented empirical regularities in order to rationalize the asset durability premium particularly associated with financial frictions. In specific, we develop a quantitative general equilibrium model with aggregate uncertainty that allows firms grappling with tightened financial constraints during recessions to adjust the composition of durable and non-durable capital through financing. In our model, firms are ex-ante homogeneous and ex-post heterogeneous in idiosyncratic productivity realizations but subject to occasionally binding borrowing constraints. At the aggregate level, firms' profits are affected by aggregate productivity shocks along with financial shocks that unexpectedly liquidate firms' net worth. Importantly, firms pose capital as collateral to incur external debt financing (e.g., Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010)), which reflects the presence of financial frictions by which lending contracts cannot be fully enforced. In line with Rampini (2019), our model distinguishes between durable and non-durable capital with respect to their geometric depreciation rates while both capital types are collateralizeable for financing. However, differed from Rampini (2019) that studies the trade-off between choosing durable and non-durable capital under financial frictions in partial equilibrium, our model focuses on the asset pricing implications of such trade-off in a dynamic and stochastic framework featuring the general equilibrium. We show the general equilibrium delivers the novel and underexplored risk-premium channel, such that firms' choices of asset durability affect both the price riskiness of capital over business cycles and the stock returns across firms in the cross-section.

At the aggregate level, our model suggests that firms become more financially constrained under adverse aggregate productivity shocks and financial shocks, and shift asset holdings toward cheaper and less durable capital. In equilibrium, the price of more durable capital is more procyclical, thus riskier than less durable capital. Consequently, more durable capital commands higher risk compensation as expected returns. Importantly, we stress that this result partly follows from Rampini (2019), as acquiring more durable capital incurs a larger down payment despite of a lower frictionless user cost, which makes it harder to finance. In addition, our general equilibrium model insight is new in that durable capital is more costly also because of its larger price risk sensitivities to financial frictions. Most importantly, while firms with low net worth but high financing needs endogenously acquire less durable assets, heterogeneities in productivity and net worth then translate into endogenous cross-firm heterogeneities in capital composition of different asset durabilities. In the cross-section, firms with larger asset durability therefore earn higher risk premia. We demonstrate that such cross-sectional variations in firms' portfolio holdings are critically important for understanding the riskiness of financially constrained firms.

Quantitatively, our model, once calibrated to match both standard U.S. business cycle moments and different depreciation rates of more and less durable capital in the data, generates the substitution between capital types and the relative cyclicality of capital prices. Our model finds that when firms are financially constrained, they hold 3.05% less on durable capital and invest 2.68% less through acquiring durable capital relative to non-durable capital on average. In addition, we find that the price of durable capital is about three to four times more volatile than that of non-durable capital over business cycles as measured by unconditional price volatility or by the covariance between capital prices and the stochastic discount factor. Finally, we show that the riskiness of capital prices determines the firm risk given firms' portfolio heterogeneity in asset durability. Through cross-sectional simulation, our model exhibits extra riskiness for firms' holding durable capital and generates a levered (unlevered) return spread between the highest durability quintile portfolio and the lowest durability quintile portfolio at 4.34% (1.32%) annually, which explains at least about 80% (30%) of observed spreads in our data.

In particular, we emphasize that the asset durability is not a standalone "risk factor" that generates the cross-section pattern of stock returns in our model. Rather, our model predicts that the price of durable capital is much riskier than that of non-durable capital in terms of its risk exposure to financial shocks, which is the key source of risk that is priced in general equilibrium. The cross-section of stock returns is an outcome of heterogeneous firms holding different asset compositions of durable and non-durable capital on balance sheet, which then determines the distribution of equity risk across firms.

Importantly, our model nests and differentiates two offsetting channels that affect firms' substitution over asset durability and the cross-section of stock returns. First, there is the new risk premium channel highlighted in this paper, where durable capital is riskier due to its more procyclical equilibrium price. Second, the asset collateralizability channel, as discussed by Ai, Li, Li, and Schlag (2020a), indicates that financing non-durable capital is riskier because it lacks the collateralizability value. Empirically, we show in Subsections 2.3 and 2.4, by distinguishing between levered and unlevered returns, that firms holding more durable capital are riskier, even after controlling for the asset collateralizability channel that these are also more leveraged firms. In Section I.2.3 of Internet Appendix, we document additional firm-level evidence that controlling for both leverage and the collateralizability measure, i.e., the relative weight of the present value of the Lagrangian multipliers in the total value of the firm's assets as in Ai, Li, Li, and Schlag (2020a), firms with greater asset durability are significantly risker. Theoretically, we derive a clear decomposition from our model in Subsection 4.2 to illustrate these offsetting channels. We also discuss why the asset durability premium are pronounced among financially constrained firms with greater leverage in Subsection 4.3. Our quantitative exercises find that the newly documented risk premium channel dominates, which delivers a more pronounced general equilibrium price effect that increases the relative riskiness of more durable capital.

In addition, we demonstrate using our general equilibrium model that the asset durability premium observed in both the data and the model is consistent with the downward-sloping term structure of equity returns (e.g., Lettau and Wachter (2007), Weber (2018), Gonçalves (2021), and Gormsen and Lazarus (2023).) In the data, median cash flow duration for financially constrained firms declines from 20 years to 19 years as we move from the lowest to highest asset-durability-sorted portfolios. Our model closely replicates this pattern, generating a decline from 20 to 18 years when constraints bind, and demonstrates that the negative relationship persists even when financial constraints are not binding, though average duration increases as unconstrained firms are better able to smooth shocks intertemporally. The key mechanism driving this result is again that occasionally binding financial constraints generate greater price procyclicality for durable assets relative to non-durable assets due to substitution over asset durability. This causes the non-depreciated capital value to load more weight on contemporaneous cash flows and truncates the far-horizon tail of cash flow streams for firms with more durable assets thereby shortening measured duration.

We highlight three important features of our model. First, our model specifically focuses on the margin of firms' acquiring capital via asset purchase in response to financial frictions, leading to changes of capital composition. This differs from another margin by which firms can take leased capital when they are financially constrained (Rampini and Viswanathan, 2013b; Li and Tsou, 2022). Intentionally shutting down the leasing channel, our model is able to disentangle a risk-premium channel that involves firms adjusting the capital mix with purchased capital. Empirically, we provide the evidence showing that firms' adjusting capital mix with purchased capital significantly affect the asset durability and capital prices, even controlling for the leasing channel. Second, we stress that in line with Rampini (2019), the durability differences across asset types are fully captured by the varied depreciation rates. Non-durable capital is not associated with those of low quality or low productivity. For example, the differences between the multi-year licensed software and one-year licensed software suggest that the capital goods can be equally productive but operate in varied durability. Importantly, this exactly motivates our model choices of treating durable and non-durable capital as perfect substitutes in firms' production. Third, we solve our model with a global solution. This enables us to directly compare and contrast the firm riskiness with and without financial constraints. We also emphasize that by solving our model using a global solution, the risk-premium channel dominates the collateralizability channel in general equilibrium, whereby a local solution to the model when financial constraints are always binding mostly features the asset collateralizability channel as in Ai, Li, Li, and Schlag (2020a). The global solution is therefore critical, by generating greater variability of the stochastic discount factor and the extra riskiness of more durable capital, which better connects our model with the empirical facts.

We then continue with our empirical investigation that provides direct support for our model mechanism in general equilibrium. First, we construct the model-implied financial shocks from the data and examine the effects of financial frictions on relative changes in both quantities and prices between durable and non-durable capital. Second, we show that financial shocks are negatively priced in asset-durability-sorted portfolios. Finally, we perform various tests to rule out other explanations for the return predictability using asset durability.

To best connect our quantitative model predictions with the data, we follow the approach of Eisfeldt and Muir (2016) and Belo, Lin, and Yang (2018) by constructing a model-implied proxy for the unobserved financial shocks. Specifically, we first identify two key model-implied moments: the change in firms' aggregate debt-to-net-worth ratio and the spread between the shadow interest rate and the risk-free rate. Intuitively, as predicted by our model, adverse financial shocks unexpectedly force firms into liquidation without engaging in production, therefore well captured by firms' loss in net worth and increases in the debt-to-net-worth ratio. Similarly, an increase in the interest spread indicates tightened borrowing constraint, directly raising the cost of borrowing. Within our model, a linear combination of these two moments explains most of the variation in financial shocks, achieving an $R^2$ of 78% in a regression setting. Using the estimated regression coefficients, we then construct an empirical time series of model-implied financial shocks using their empirical counterparts of aggregate debt-to-net-worth ratio and the yield spread between Moody's Baa and Aaa corporate bond yields.

Next, using our constructed financial shock series, we examine the effects of financial frictions on relative changes in the quantities and prices of durable vs. non-durable capital, which is the key model mechanism that arises in general equilibrium. First, our model finds that holding durable capital is more expensive than non-durable capital among financially constrained firms. To empirically test this prediction, we regress the average durability of constrained and unconstrained firms on the model-implied financial shocks, respectively. We find a negative and statistically significant coefficient associated with the sample of financially constrained firms, which confirms the capital substitution effects driven by changing financial frictions. This indicates that financially constrained firms shift toward cheaper and less durable assets when borrowing constraints are tightened. In contrast, we find a slightly positive but insignificant coefficient for the sample of unconstrained firms. Such evidence directly suggests the presence of a "reallocation effect" such that unconstrained firms are not unloading durable capital as much as those constrained firms, reflecting the substitution effect in response to adverse financial shocks. Though the coefficient is insignificant, it is consistent with the additional wealth effect operating in the data that offsets the capital substitution effect. That is, adverse financial shocks are erasing firms' net worth leading to reduction in capital financing and holdings of both durable and non-durable asset for all firms. Further, as predicted by our model, the capital substitution also implies a difference in price risk sensitivities to financial frictions between durable and non-durable capital, with durable capital prices declining more significantly following negative financial shocks. We then regress price changes for high- and low-durability asset groups on the market factor along with the financial shocks and find a significantly negative coefficient on financial shocks for the high-durability group. In contrast, the low-durability group exhibits a slightly positive but statistically insignificant coefficient. This evidence again confirms the risk-premium channel highlighted in our general equilibrium model, indicating that durable capital exhibits greater price cyclicality.

In addition, we empirically assess the riskiness of asset-durability-sorted portfolios by estimating a two-factor asset pricing model that includes both a aggregate market factor and our model-implied financial shocks. We implement a generalized method of moments (GMM) estimation of Cochrane (2005) to test the price of financial shocks and the exposure to such risk of portfolios sorted on asset durability. Our two-factor model captures reasonably well the variation in the average returns of the asset-durability-sorted portfolios, and we find that the price of risk with respect to financial shocks is significantly negative, consistent with our model prediction. Moreover, GMM-implied alphas in the high-minus-low spread portfolio sorted on asset durability are not statistically significant. Finally, the goodness of fit of our two-factor model is driven by the increasing negative exposure of the high-durability portfolios to financial shocks. We then conduct similar tests using simulated data from our quantitative model and show the price of risk with respect to financial shocks is negative and statistically significant, more importantly, much well aligned with the empirical estimates. Taken together, high-asset-durability firms exhibit higher expected stock returns because they have negative betas on financial shocks that are negatively priced. This is exactly consistent with our model prediction that firms' asset composition, given different risk exposures due to differed asset durability, affects the firm risk and the cross-section of stock returns.

Finally, we demonstrate the robustness of our results using alternative measures and rule out competing explanations for the asset durability premium. First, instead of using our model-implied financial shocks, we re-estimate the two-factor model and the associated risk exposures using alternative measures: the yield spread between Moody's Baa and Aaa corporate bond yields and the GZ spread (Gilchrist and Zakrajšek, 2012). As shown in Table IA.1 of the Internet Appendix, these alternative measures confirm our main findings that financial shocks are negatively priced and that high-asset-durability portfolios exhibit greater negative exposure to financial shocks. Next, we rule out alternative explanations for the cross-sectional variation in portfolio returns sorted by asset durability. Conducting asset pricing factor tests, we find that alphas remain significant even after we account for Fama and French (2015) five factors or Hou, Xue, and Zhang (2015) (HXZ hereafter) q-factors. This implies that the positive asset-durability-return relationship cannot be explained by established firm characteristics like size, value, profitability, and investment. Moreover, we show the asset durability premium cannot be explained by the leasing channel (Rampini and Viswanathan, 2013b; Li and Tsou, 2022). Additionally, we employ monthly Fama and MacBeth (1973) regressions to assess the ability of using firm-level durability to predict cross-sectional stock returns. This approach allows us to control for an extensive list of firm characteristics that typically predict stock returns. The slope coefficient associated with a firm's lagged durability is both economically and statistically significant. For instance, controlling for firms' financial leverage, a one-standard-deviation increase in a firm's durability corresponds to a 2.13% increase in a firms' expected stock return. We then confirm that the positive durability-return relation is not driven by other known predictors correlated with the durability measure. Specifically, we consider potential confounding effects associated with capital collateralizability, operating leverage and adjustment costs, output durability, and financial distress.[2] Our estimation results all suggest that the asset-durability-return relation persists even when we control for firm characteristics associated with these channels.

**Related Literature.** First, our paper builds on the corporate finance literature that emphasizes the importance of collateral for firms' capital structure decisions. Albuquerque and Hopenhayn (2004) study dynamic financing with limited commitment. Rampini and Viswanathan (2010, 2013a) develop a joint theory of capital structure and risk management based on firms' asset collateralizability. Schmid (2008) considers the quantitative implications of dynamic financing with collateral constraints. Nikolov, Schmid, and Steri (2021) meanwhile examine the quantitative implications of various sources of financial frictions on firms' financing decisions, including the collateral constraint. Falato, Kadyrzhanova, Sim, and Steri (2022) provide empirical evidence for the link between asset collateralizability and leverage in aggregate time series and in the cross-section. Our paper departs from these papers in that we explicitly study firms' optimal asset acquisition decisions among assets with different durabilities under the context of an occasionally binding collateral constraint, as in Rampini (2019). However, different from Rampini (2019), we bring the asset durability decision into a general equilibrium framework, take aggregate shocks into accounts, and highlight important asset pricing implications of an underexplored channel for determining the riskiness of equilibrium capital prices and the cross-sectional stock returns.

Second, a rich literature that starts with Eisfeldt and Rampini (2006) has examined how durable assets are reallocated across firms. In this body of research, a consistent empirical observation is that financially constrained agents often engage in the acquisition of assets within secondary markets.

[2] Existing systematic risks that may explain the documented asset durability premium include collateralizability (e.g., Ai, Li, Li, and Schlag (2020a)), operating leverage and adjustment costs (e.g., Zhang (2005), Gu, Hackbarth, and Johnson (2018), and Kim and Kung (2017)), output durability (e.g., Gomes, Kogan, and Yogo (2009)), and financial distress (e.g., Griffin and Lemmon (2002), Bharath and Shumway (2008), and Campbell, Hilscher, and Szilagyi (2008)).

Specifically, Eisfeldt and Rampini (2007) examine investment decisions in new and used capital within the context of financial frictions, demonstrating that financially constrained firms tend to prefer older investment goods. Gavazza, Lizzeri, and Roketskiy (2014) explore welfare gains from secondary markets for durable goods, especially with respect to consumer heterogeneity. Lanteri (2018) looks into the market for used investment goods using a quantitative business-cycle model with firm heterogeneity subject to idiosyncratic productivity shocks. Rampini (2019) examines the effects of asset durability on investment financing in the presence of collateral constraints. Building upon these insights, Hu, Li, and Xu (2020) adjust firms' marginal product of capital (MPK) by considering leased capital, highlighting leasing as an additional channel for capital reallocation that alters patterns of capital misallocation. Gavazza and Lanteri (2021) emphasize the role of secondary markets in reallocating used consumer durable goods from wealthier to poorer households, proposing that this mechanism contributes to the transmission of credit shocks. Meanwhile, Ma, Murfin, and Pratt (2022) utilize a large dataset on equipment transactions and document a negative correlation between firm age and capital age. Lanteri and Rampini (2023) comprehensively evaluate the welfare cost of two types of pecuniary externalities involved in capital reallocation via the resale of old capital. Our paper extends the existing literature by further exploring asset pricing implications of firms' capital choices. In particular, we develop a quantitative general equilibrium model that features firms' endogenous choices over asset durabilities and our findings largely enrich the views that mainly focus on new vs. used capital. For example, two brand new models of machines could differ in their depreciation rates while older capital may have either a longer or shorter duration of remaining service years as compared to newer capital.

Third, our paper also builds on the large macroeconomics literature that studies the role of credit market frictions in generating fluctuations over business cycles (see Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012) for extensive reviews). The papers most related to ours emphasize the importance of borrowing constraints and contract enforcements, such as Kiyotaki and Moore (1997, 2012), Gertler and Kiyotaki (2010), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Elenev, Landvoigt, and Van Nieuwerburgh (2021). In addition, Gomes, Yamarthy, and Yaron (2015) study asset pricing implications of credit market frictions in a production economy. Our model examines the impacts of financial shocks on constraining firms' balance sheets occasionally over business cycles, which causes firms to optimally adjust their asset durability. We show that dynamic substitutabilities between durable and non-durable capital not only matter for the riskiness of capital prices in equilibrium, but also shift the cross-section of stock returns.

In addition, our paper contributes to the literature on production-based asset pricing, for which Kogan and Papanikolaou (2012) provide an excellent survey. From a methodological point of view, our general equilibrium model allows for a cross-section of firms with heterogeneous productivities and is related to previous papers including Gomes, Kogan, and Zhang (2003), Gârleanu, Kogan, and Panageas (2012), Ai and Kiku (2013), and Kogan, Papanikolaou, and Stoffman (2017). Compared to these papers, we incorporate financial frictions in our model and study asset pricing implications of firms' occasionally binding collateral constraints. In this regard, our paper is closely related to Ai, Li, Li, and Schlag (2020a), which study cross-sectional stock returns by focusing on the value of asset collateralizeability. They show that more collateralizable assets provide insurance against aggregate shocks by relaxing collateral constraints, especially in recessions when financial constraints are binding. Our paper differs from Ai, Li, Li, and Schlag (2020a) in two important dimensions. First, our model examines the importance of a novel and underexplored risk-premium channel arising in general equilibrium with firms subject to occasionally binding constraints. By providing a clean decomposition in Section 4.3, our model compares and contrasts the risk-premium channel and the collateralizeability channel that both affect the relative riskiness of durable and non-durable capital. Our model is therefore well equipped to uncover the net effect that makes durable capital riskier compared to less durable capital, despite durable capital is allowed to be more collateralizeable. Second, rather than imposing an always binding constraint, our model is solved globally such that firms' capital financing is constrained occasionally. We highlight that the global solution is critical, which ultimately generates volatile stochastic discount factor driven by changing in financial frictions, leading to greater riskiness of firms holding more durable capital in general equilibrium. By contrast, a local solution when constraint is always binding mostly features the collateralizeability channel. Our quantitative model solution then gives us the exact flexibility to examine the total effects of firms' changing asset durability over business cycles for asset pricing implications.

Lastly, our paper is also connected to a broader literature linking investment to the cross-section of expected returns. Zhang (2005) provides an investment-based explanation for the value premium. Tuzel (2010) documents a positive relationship between firms' real estate holding and expected returns and proposes an adjustment cost explanation. Li (2011) and Lin (2012) focus on the relationship between R&D investment and expected stock returns. Eisfeldt and Papanikolaou (2013) develop a model of organizational capital and expected returns. Also, Belo, Lin, and Yang (2018) meanwhile study implications of equity financing frictions on the cross-section of stock returns. Importantly, as non-residential real estate may be considered a particular type of durable capital, our paper still complements and differs from Tuzel (2010) in the following regards. First, our key model mechanism operates because firms are subject to occasionally binding borrowing constraints driven by financial shocks, which is the key source of aggregate risk for asset pricing in the cross-section. Our theory and empirics regarding the asset durability premium particularly focus on the financially constrained firms. This differs from the adjustment cost channel in Tuzel (2010) that capital adjustment is risky regardless of the financial frictions, as long as firms can be affected by adverse aggregate productivity shocks. Second, rather than building in the exogenous capital adjustment cost, our model is new by featuring an *endogenous* cost associated with asset compositional changes that necessarily arise from the general equilibrium, i.e., firms optimally shift away from holding durable capital whenever financial constraints are tightened. That is, financing durable capital incurs a larger cost for its higher down-payment and its greater risk exposure to financial frictions. Our model therefore examines a new and different channel considering firms' adjustment of the capital mix so as to rationalize the documented asset durability premium.

The rest of our paper is organized as follows. We summarize our empirical results on the relationship between asset durability and expected returns in Section 2. We introduce a general equilibrium model with occasionally binding collateral constraints in Section 3, and study the asset pricing implications in Section 4. In Section 5, we provide a quantitative analysis of our model and discuss our model results. In Section 6, we provide the supporting empirical evidence for our model mechanism, and then conclude in Section 7. Details on data construction are relegated to Section IV of the Internet Appendix. In Section V of the Internet Appendix, we provide details on our model solution and present additional empirical evidence to establish the robustness of our results.

# 2 Empirical Facts

In this section, we present empirical evidence demonstrating how financial constraints correlate with firms' decisions related to asset durability and capital structure. We highlight key findings that underscore the significance of asset durability in shaping the cross-section of stock returns, suggesting the existence of *asset durability premium* among financially constrained firms.

## 2.1 Measuring Asset Durability

To empirically examine the connection between asset durability and expected returns of stocks, we first develop the measures of asset durability concerning a wide range of assets. In the spirit of Rampini (2019), we measure an asset's durability based on its service life, calculated as the reciprocal of the asset's depreciation rate. Specifically, we construct the measures of asset durability from the Bureau of Economic Analysis (BEA) fixed asset table, which provides detailed estimates for implied depreciation rates and net capital stocks at a fixed cost across non-residential asset categories.[3] In particular, the depreciation rates of different assets are presented in the table by their uses across industries.[4] Our measures are constructed based on a data sample that spans from 1978 to 2016.[5]

[3]The Bureau of Economic Analysis (BEA) fixed asset table presents the break-down of the implied depreciation rates and net capital stocks by asset categories, encompassing a wide array of industries.

[4]The BEA employs the 1997 North American Industry Classification System (NAICS) for industry classification. For our empirical analysis, we align the 63 BEA industries with Compustat firms using the NAICS codes.

[5]Our empirical analysis utilizes fixed asset data from the BEA, specifically the asset- and industry-level depreciation rates calculated using the methodology developed by Hulten and Wykoff (1981). BEA's detailed fixed asset data typically becomes available with a lag exceeding one year. Our sample concludes in 2016, representing the most recent finalized estimates consistent with the original methodology that were released in a timely manner. More recent data from 2017 onward was delayed and released after the COVID-19

**Constructing Industry- and Firm-level Asset Durability Measures**

Our measures of asset durability capture the service life of both tangible and intangible assets utilized by firms operating across industries. Specifically, we define the asset durability of tangible and intangible assets $k$ used by industry $j$ in year $t$ as the reciprocal of the implied depreciation rate at the asset level provided in the BEA table, referred to as the Asset Durability Score. We then calculate an industry-level asset durability index by value-weighting the durability of individual assets:

$$\text{Asset Durability}_{j,t}^{m} = \sum_{k \in m} \bar{w}_{k,j,t} \times \text{Asset Durability Score}_{k,j,t}^{m},\ m \in \{\text{TAN},\ \text{INTAN}\}, \tag{1}$$

Here, Asset Durability$_{j,t}^{m}$ represents the asset durability for industry $j$ in year $t$. Specifically, Asset Durability$_{j,t}^{\text{TAN}}$ and Asset Durability$_{j,t}^{\text{INTAN}}$ differentiate the measured asset durability of tangible or intangible capital employed in industry $j$, respectively. $\bar{w}_{k,j,t}$ denotes the proportion of industry $j$'s capital stock attributed to asset $k$ relative to the total capital stock in year $t$.

We then construct a firm-level metric for asset durability of tangible and intangible capital by calculating the value-weighted average of industry-level asset durability indices across the various business segments in which a firm operates. The asset durability for a firm's tangible (TAN) or intangible (INTAN) capital stock is derived by aggregating the durability of the industry segments in which the firm operates:

$$\text{Asset Durability}_{i,t}^{m} = \sum_{j=1}^{n_{i,t}} \widetilde{w}_{i,j,t} \times \text{Asset Durability}_{j,t}^{m},\ m \in \{\text{TAN},\ \text{INTAN}\}, \tag{2}$$

where $n_{i,t}$ denotes the number of industry segments the firm operates in during year $t$, and $\widetilde{w}_{i,j,t}$ represents the proportion of the firm's sales attributed to industry segment $j$ relative to its total sales in year $t$. Asset Durability$_{j,t}^{m}$ is the durability of industry segment $j$ in year $t$, calculated using equation (1) for tangible or intangible assets.

Finally, the firm-level asset durability is derived as follows:

$$\text{Asset Durability}_{i,t} = w_{i,t}^{\text{TAN}} \times \text{Asset Durability}_{i,t}^{\text{TAN}} + (1 - w_{i,t}^{\text{TAN}}) \times \text{Asset Durability}_{i,t}^{\text{INTAN}}, \tag{3}$$

where $w_{i,t}^{\text{TAN}}$ represents the share of tangible capital for firm $i$ in year $t$ over its capital stock, which is measured by the summation of tangible and intangible capital. Specifically, firm's tangible capital is denoted by the PPEGT item in the Compustat and the intangible capital is measured in line with Peters and Taylor (2017).[6]

pandemic, and subsequent BEA releases since 2019 have adopted updated estimation procedures, making them less comparable with earlier data.

[6]Following Ai, Li, Li, and Schlag (2020a), we capitalize R&D and SGA expenditures using the perpetual inventory method.

## 2.2 Asset Durability and Financial Constraints

We then explore the relationship between a firm's financial constraints and asset durability. In line with Rampini (2019), we confirm that tightened financial constraints decrease firms' asset durability. Specifically, we use three metrics to gauge the extent of a firm's financial constraints: the dividend payment dummy (Farre-Mensa and Ljungqvist (2016), referred to as DIV), the Size-Age index (Hadlock and Pierce (2010), referred to as SA index), and the Whited-Wu index (Whited and Wu (2006), Hennessy and Whited (2007), referred to as WW index).[7]

We perform regressions of the firm-level asset durability measure on a set of firm characteristics, incorporating different measures of financial constraints across specifications, while controlling for firm and year fixed effects. Columns (1) to (4) in Table 1 report the estimated coefficients on financial constraints without other firm-level controls. Columns (5) to (7) present results associated with specifications that further include a set of firm characteristics as additional controls. Results across specifications consistently show a negative correlation between financial constraints and firms' asset durability. Specifically, the coefficients associated with the non-dividend dummy, as well as other constraint indicators such as the SA and WW indices, are significantly negative. Our estimation results strongly suggest that firms' capital composition of durable and non-durable capital is significantly correlated with the severity of their financial conditions. That is, firms with lower (higher) asset durability are more (less) financially constrained.

## 2.3 Asset Durability and Leverage

Considering that durable capital serving as collateral for financing is more collateralizeable (Ai, Li, Li, and Schlag, 2020a), firms with more durable capital may have greater leverage ratios. We present the evidence to confirm that more leveraged firms are holding more durable capital. Nonetheless, we will be explicitly clear in the following sections that the asset return differences associated with asset durability cannot be explained by the leverage differences. Though, we highlight that our model to be presented later helps rationalize the empirical regularities on both return differentials and the leverage ratios.

In Table 2, we report the summary statistics of the firm-level asset durability measure and the leverage for firms in Compustat. As we have shown that firms of lower asset durability are more financially constrained, we therefore compare and contrast the leverage ratios between financially constrained and unconstrained firms first, and then delve into the differences in leverage among financially constrained firms only.

Panel A of Table 2 displays the asset durability measures, the capital depreciation rates, and the book leverage ratios for the financially constrained and unconstrained firm groups.[8] Two key

[7] In contrast to the dividend payment dummy (DIV), the non-dividend payment dummy (Non-Div) denotes whether a firm does not pay dividends.

[8] Financially constrained firms are identified if they do not pay dividend by the end of June of a year, and unconstrained otherwise (i.e., the dividend payout dummy (Farre-Mensa and Ljungqvist, 2016).) We can

observations emerge. First, the average asset durability among financially constrained firms is much lower at 12.66 (larger capital depreciation rate of 0.17) compared to that of unconstrained firms at 16.54 (lower capital depreciation rate of 0.13). This again confirms the fact that financial constraints are associated with lower asset durability. Second, the average book leverage of constrained firms is 0.24, which is lower than that of the unconstrained firms at 0.33. This implies that financially constrained firms subject to external financing frictions are borrowing less, which is consistent with the findings in Ai, Li, Li, and Schlag (2020a) that firms with less collateralizeable capital is under-leveraged. In Panel B of Table 2, we sort the financially constrained firms into five quintiles based on their asset durability relative to peers within the same NAICS 3-digit industry and tabulate the statistics. Notably, we observe significant variability in average asset durability (depreciation), ranging from 7.69 (0.19) in the lowest quintile (Quintile L) to 18.00 (0.11) in the highest quintile (Quintile H). Furthermore, the book leverage exhibits an upward trend as we move from the lowest to the highest asset durability quintile. In summary, we demonstrate that firms' choices over asset durability are closely connected to firms' financing conditions and affect firms' capital structure.

## 2.4 Asset Durability and Stock Returns

Next, we document the stock return differences in the cross-section associated with asset durability. We show that firms' financial constraints are crucial to disentangle the connection between the asset durability and the expected returns.

Specifically, we examine the average excess returns of five annually sorted portfolios by firms' asset durability, controlling for industry differences.[9] In addition, we look into the return performance of a "high-minus-low" portfolio, which involves adopting a long position in the highest durability portfolio and a short position in the lowest asset durability portfolio. Importantly, we examine both levered and unlevered returns to control for the leverage effects that higher leverage ratio under financial constraints gives higher returns.[10]

In Table 3, we report levered returns in the left panel and unlevered returns in the right panel. Panel A presents results of the portfolio sorting of stocks of financially constrained firms identified by our three different criteria highlighted in Section 2.2, and Panel B shows results for our whole

show the results are very robust if using alternative financial constraint measures, including the SA index and WW index.

[9]Specifically, by the end of June in each year from 1978 to 2017, we sort firms according to their asset durability relative to their peers within the corresponding NAICS 3-digit industries. This classification generates industry-specific breaking points for quintile portfolios for each June. Therefore, the low (high) portfolio encompasses firms with the lowest (highest) asset durability within each industry. Particularly, we eliminate firms with asset values or sales lower than 1 million from our sample, so we minimize the influence of small firms on our findings.

[10]The unlevered return of a firm is defined as its levered return multiplied by one minus its leverage ratio. See further details per equation (46) in Section 4.3. Regarding the return calculations at the portfolio level, monthly returns of a stock are first averaged across the next twelve months (from July in year $t$ to June in year $t + 1$) and then value-weighted taking a firm's market capitalization at the time of a portfolio creation for aggregations up to the portfolio.

firm sample. Each section of the table displays *annualized* average excess stock returns (E[R]-$R_f$, relative to the risk-free rate), t-statistics, standard deviations, and Sharpe ratios for different portfolios sorted on asset durability.

In the first section of Panel A, among those financially constrained firms identified by the dividend payout dummy (DIV), the annualized average excess return for firms with high asset durability (Portfolio H) exceeds that of firms with low asset durability (Portfolio L) for both levered and unlevered returns. This divergence in returns is both economically substantial and statistically significant. First, regarding the results on the levered returns, we see a positive correlation between asset durability and the stock returns and the return is statistically significant for the long-short portfolio. Specifically, the high-minus-low portfolio exhibits a statistically significant average excess return of 6.93% (t-value of 2.86) and a Sharpe ratio of 0.59. Moving to the other three lower sections of Panel A, we highlight that this premium remains robust under alternative measures of financial constraint. Second, regarding the unlevered returns in the right panel, despite a less steep pattern, the expected excess returns again increase with asset durability across portfolios regardless of the financial constraint measures. Importantly, the unlevered returns of long-short portfolio remain statistically significant though the magnitudes are somewhat smaller reflecting the leverage effects. However, taking the whole sample including both constrained and unconstrained firms, our results in Panel B find that both the levered and unlevered returns for the long-short portfolio are quite small and lack statistical significance. In addition, the pattern that average excess returns increase with asset durability is much weakened.

In summary, we present compelling evidence that average excess returns increase with asset durability. Our results suggest that the return differences associated with asset durability are particularly pronounced among financially constrained firms, even after controlling for variations in leverage across portfolios. We refer to this return spread, driven by asset durability and observed in the long-short high-minus-low (Portfolio H-L) strategy, as the "asset durability premium."

In the following section, we will construct and present a general equilibrium model featuring heterogeneous firms and financial frictions. This model will allow us to quantitatively account for all documented empirical facts, most notably the positive asset durability premium.

# 3 A General Equilibrium Model

In this section, we describe the model we use for rationalizing the asset durability spread. The aggregate aspect of the model is intended to follow standard macro models with collateral constraints such as in Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010). Our model further allows for idiosyncratic productivity shocks as well as firms' entry and exit margin, which results in the heterogeneous durability of assets in the cross-section. These model features help deliver quantitatively plausible firm dynamics in order to study the implications of asset durability for the cross-section of equity returns.

## 3.1 Households

In our model, the representative household with infinite horizon consists of a continuum of workers and entrepreneurs. Workers (entrepreneurs) receive their labor (capital) incomes every period and submit them to the planner of the household, who makes decisions for consumption for all members of the household. Entrepreneurs and workers make their financial decisions separately.[11]

The household ranks the utility of consumption plans according to the following recursive preference as in Epstein and Zin (1989):

$$U_t = \left\{ (1-\beta) C_t^{1-\frac{1}{\psi}} + \beta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

in which $\beta$ is the time discount rate, $\psi$ is the intertemporal elasticity of substitution, and $\gamma$ denotes the degree of the relative risk aversion. As we show later in this paper, together with the endogenous growth and long-run risk, the recursive preference in our model generates a volatile pricing kernel and a sizable equity premium as in Bansal and Yaron (2004).

In every period $t$, the household consumes $C_t$ and purchases $B_{i,t}$ of risk-free bonds from entrepreneur $i$, from which she will receive $B_{i,t} R_{f,t+1}$ in the next period, in which $R_{f,t+1}$ denotes the risk-free interest rate from period $t$ to $t+1$. In addition, the household receives capital income $\Pi_{i,t}$ from entrepreneur $i$. We assume that the labor market is frictionless, and therefore the labor income from worker members is $W_t L_t$. The household budget constraint at time $t$ can therefore be written as:

$$C_t + \int B_{i,t} di = W_t L_t + R_{f,t} \int B_{i,t-1} di + \int \Pi_{i,t} di.$$

We let $M_{t+1}$ denote the stochastic discount factor of period $t$ as implied by household optimization. With recursive preference, the stochastic discount factor is denoted as:

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma},$$

and the optimality of the intertemporal saving decisions implies that the risk-free interest rate must satisfy

$$E_t[M_{t+1}] R_{f,t+1} = 1.$$

## 3.2 Entrepreneurs

There is a continuum of entrepreneurs indexed by $i$ who pursue productive ideas. An entrepreneur who starts at period 0 draws an idea with initial productivity $\bar{z}_0$ and start to operate

[11] Following Gertler and Kiyotaki (2010), we assume that household members make joint decisions on their consumption to avoid keeping the distribution of entrepreneur income as an extra state variable.

with an initial net worth $N_0$. Under our convention, $N_0$ is also the total net worth of all entrepreneurs at time 0 as the total measure of all entrepreneurs is normalized to one.

We let $N_{i,t}$ denote entrepreneur $i$'s net worth at time $t$, and let $B_{i,t}$ denote the total amount of risk-free bond the entrepreneur issues to the household at time $t$. Thus, the time-$t$ budget constraint for the entrepreneur is given as:

$$q_{d,t}K_{i,t+1}^{d} + q_{nd,t}K_{i,t+1}^{nd} = N_{i,t} + B_{i,t}. \tag{4}$$

In equation (4), we assume that two types of capital, type-$d$ and type-$nd$, differ in their asset durability; that is, the former capital is more durable, while the latter capital is less durable. For brevity's sake of model description, we simply call the latter the non-durable capital. We denote these two types of capital with a superscript $d$ for durable and $nd$ for non-durable, respectively. These two types of capital depreciate at geometric depreciation rates $\delta_d < \delta_{nd}$ each period, with $\delta_h \in (0,1)$, for $h \in \{d, nd\}$. We use $q_{d,t}$ and $q_{nd,t}$ to denote their prices at time $t$, respectively. $K_{i,t+1}^{d}$ and $K_{i,t+1}^{nd}$ are the amount of capital that entrepreneur $i$ purchases at time $t$, which can be used for production over the period from $t$ to $t+1$. We assume that the entrepreneur only has access to risk-free borrowing contracts (i.e., we do not allow for state-contingent debt). At time $t$, the entrepreneur is assumed to have an opportunity to default on his contract and abscond with $1-\theta$ of both types of capital. Because lenders can retrieve a $\theta$ fraction of the type-$h$ capital upon default, we assume entrepreneur's borrowing is subject to an occasionally binding constraint such that:

$$B_{i,t} \leq \theta \sum_{h\in\{d,nd\}} (1-\delta_h)\, q_{h,t}K_{i,t+1}^{h} \tag{5}$$

Following Rampini (2019), we assume that asset durability could well affect the degree of capital collateralizability as captured by $\theta(1-\delta_h)$ as in equation (5).[12] This implies that more durable capital (i.e. lower $\delta_h$) is more collateralizable. In our paper, we highlight that a clear distinction exists between the durability and collateralizability of an asset. According to Ai, Li, Li, and Schlag (2020a), an asset with higher colllateralizability lowers the riskiness of assets as insurance against aggregate shocks by relaxing the financing constraint. However, unlike that of the asset collateralizability, we show that asset durability, which is the key focus of our paper, determines not only the upper bound of capital financing, but also the price of collateralizeable assets. We show that the net effect of asset durability against the collateralizability of an asset means that the price of more durable assets exhibits greater risk sensitivities to aggregate shocks. Therefore in equilibrium, assets with longer durability embody higher riskiness than those with shorter durability.

Let $z_{i,t+1}$ denote entrepreneur $i$'s idiosyncratic productivity. From time $t$ to $t+1$, the produc-

[12] That is, the effective degree of collateralizability for a given type of capital, $\frac{B_{i,t}}{q_{h,t}K_{i,t+1}^{h}}$, when the borrowing constraint is binding is given by $\theta(1-\delta_h)$.

tivity of entrepreneur $i$ evolves according to the law of motion:

$$z_{i,t+1} = z_{i,t}e^{\varepsilon_{i,t+1}}, \tag{6}$$

in which $\varepsilon_{i,t+1}$ is a Gaussian shock with mean $\mu_\varepsilon$ and variance $\sigma_\varepsilon^2$, assumed to be i.i.d. across agents $i$ and over time. We use $\Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K^d_{i,t+1}, K^{nd}_{i,t+1}\right)$ to denote entrepreneur $i$'s equilibrium profit at time $t+1$ that arises from running a firm for production, in which $\bar{A}_{t+1}$ is the aggregate productivity realized in period $t+1$.[13] We provide the specification of the aggregate productivity process later in Section 5.1.

In period $t+1$, after production, the entrepreneur experiences a financial shock with probability $\lambda_{t+1}$, upon which that entrepreneur loses his idea and must liquidate all his net worth $N_{i,t+1}$ and thus cannot continue to the next period.[14] Specifically, if such a liquidation shock hits, then the entrepreneur restarts with a new idea with initial productivity $\bar{z}_{t+1}$ and an initial net worth $\chi S_{t+1}$, as a fraction $\chi \in (0,1)$ of the total asset of the economy in period $t+1$, $S_{t+1}$. The total asset value of the economy is then given by:

$$S_{t+1} = \Pi\left(\bar{A}_{t+1}, K^d_{t+1}, K^{nd}_{t+1}\right) + (1-\delta_d)\, q_{d,t+1}K^d_{t+1} + (1-\delta_{nd})\, q_{nd,t+1}K^{nd}_{t+1} \tag{7}$$

in which $\Pi\left(\bar{A}_{t+1}, K^d_{t+1}, K^{nd}_{t+1}\right)$ denotes the aggregate profit of all entrepreneurs who run firm productions as of period $t+1$. Besides the flow value of total profits, the stock value of both capital types after depreciation also determines the total asset value of the economy.

Conditional on no liquidation shock realized in period $t+1$, the net worth $N_{i,t+1}$ of entrepreneur $i$ at time $t+1$ is determined as:

$$\begin{aligned} N_{i,t+1} &= \Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K^d_{i,t+1}, K^{nd}_{i,t+1}\right) + (1-\delta_d)\, q_{d,t+1}K^d_{i,t+1} \\ &\quad + (1-\delta_{nd})\, q_{nd,t+1}K^{nd}_{i,t+1} - R_{f,t+1}B_{i,t}. \end{aligned} \tag{8}$$

The entrepreneur's net worth is the sum of profit that it receives from firm production and the non-depreciated capital of two types accounting for different depreciation rates $\delta_h$ after he pays back the debt borrowed from the last period plus interest. The aggregate net worth through integration over all entrepreneurs therefore satisfies:

$$N_{t+1} = (1-\lambda_{t+1})(S_{t+1} - R_{f,t+1}B_t) + \lambda_{t+1}\chi S_{t+1} \tag{9}$$

Whenever a liquidity shock hits, entrepreneurs submit their net worth to the household who choose consumption collectively for all members, and entrepreneurs then value their net worth using the same pricing kernel as the household. We let $V^i_t$ denote the value function of entrepreneur $i$.

[13] Therefore, we use firm and entrepreneur interchangeably depending on the context.

[14] This assumption effectively makes entrepreneurs less patient than the household and prevents them from saving their way out of the financial constraint.

It must satisfy the following Bellman equation:

$$V_t^i = \max_{\{K_{i,t+1}^d, K_{i,t+1}^{nd}, N_{i,t+1}, B_{i,t}\}} E_t\left[M_{t+1}\{\lambda_{t+1}N_{i,t+1} + (1-\lambda_{t+1})\, V_{t+1}^i\}\right], \tag{10}$$

subject to the budget constraint in equation (4), the collateral constraint in equation (5), and the law of motion of $N_{i,t+1}$ given by equation (8).

## 3.3 Production

**Final Output** As $z_{i,t}$ denotes the idiosyncratic productivity for entrepreneur $i$ running a firm production at time $t$, output $y_{i,t}$ of firm $i$ at time $t$ is assumed to be generated through the following production technology:

$$y_{i,t} = \bar{A}_t\left[z_{i,t}^{1-\nu}\left(K_{i,t}^d + K_{i,t}^{nd}\right)^\nu\right]^\alpha L_{i,t}^{1-\alpha} \tag{11}$$

In our formulation, $\alpha$ is the capital share, and $\nu$ is the span of control parameter as in Atkeson and Kehoe (2005). Importantly, in the spirit of Rampini (2019) by which asset durability differences across asset types are specifically associated with the depreciation rate differentials rather than the asset productivity or quality differences, our model treats durable and non-durable capital as perfect substitutes in production without introducing additional margins of differences. We will show that in spite of this simple model setup, our general equilibrium model is able to generate different riskiness of capital prices with respect to aggregate uncertainty.

Entrepreneur $i$'s profit from running this firm at time $t$, $\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd}\right)$ is given as:

$$\begin{aligned}\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd}\right) &= \max_{L_{i,t}} y_{i,t} - W_t L_{i,t}, \\ &= \max_{L_{i,t}} \bar{A}_t\left[z_{i,t}^{1-\nu}\left(K_{i,t}^d + K_{i,t}^{nd}\right)^\nu\right]^\alpha L_{i,t}^{1-\alpha} - W_t L_{i,t},\end{aligned} \tag{12}$$

in which $W_t$ is the equilibrium wage rate, and $L_{i,t}$ is the amount of labor hired by entrepreneur $i$ at time $t$.

It is convenient to write the profit function explicitly by maximizing labor in equation (12) and using the labor market-clearing condition $\int L_{i,t}di = 1$ to get:

$$L_{i,t} = \frac{z_{i,t}^{1-\nu}\left(K_{i,t}^d + K_{i,t}^{nd}\right)^\nu}{\int z_{i,t}^{1-\nu}\left(K_{i,t}^d + K_{i,t}^{nd}\right)^\nu di}, \tag{13}$$

so that entrepreneur $i$'s profit function becomes:

$$\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd}\right) = \alpha \bar{A}_t z_{i,t}^{1-\nu}\left(K_{i,t}^d + K_{i,t}^{nd}\right)^\nu \left[\int z_{i,t}^{1-\nu}\left(K_{i,t}^d + K_{i,t}^{nd}\right)^\nu di\right]^{\alpha-1}. \tag{14}$$

Given the output of entrepreneur $i$, $y_{i,t}$, from equation (11), the total output of the economy is given as:

$$
\begin{aligned}
Y_t &= \int y_{i,t} di, \\
&= \bar{A}_t \left[ \int z_{i,t}^{1-\nu} \left( K_{i,t}^d + K_{i,t}^{nd} \right)^\nu di \right]^\alpha . \qquad (15)
\end{aligned}
$$

**Capital Goods** We assume that capital goods are produced from a constant-return-to-scale technology subject to a convex adjustment cost function. That is, capital production, also known as investment, $I_t$, costs $G\left(I_t, K_t^d + K_t^{nd}\right)$ units of consumption goods. Therefore, the aggregate resource constraint is:

$$
C_t + I_t + G\left(I_t, K_t^d + K_t^{nd}\right) = Y_t. \qquad (16)
$$

We then take the standard assumption that the investment cost function is convex in investment capital ratio $\frac{I_t}{K_t}$ in which total capital stock as of time $t$, $K_t = K_t^d + K_t^{nd}$. Specifically:

$$
G(I_t, K_t^d + K_t^{nd}) = \frac{\tau}{2} \left( \frac{I_t}{K_t^d + K_t^{nd}} - \bar{ik} \right)^2 (K_t^d + K_t^{nd}). \qquad (17)
$$

$\tau > 0$ is a parameter that indexes the marginal adjustment cost on a capital investment relative to the long-run mean investment capital ratio, $\bar{ik}$.

For tractability of the model, we assume a normalization scheme such that, at the aggregate level, the proportion of two types of capital is fixed, such that $\frac{K_t^d}{K_t} = \zeta$, and $\frac{K_t^{nd}}{K_t} = 1 - \zeta$ for which $\zeta > 0$ leads to a constant ratio of type-$d$ to type-$nd$ capital, $\zeta / (1 - \zeta)$. This assumption helps such that the state of our model economy can be well summarized by a single state variable.[15] In addition to the technical merit, we further discuss the theoretical and empirical relevance of this assumption. First, this assumption helps achieve a nice aggregation across firms without discounting our model insights on the cross-section of stock returns, which is ultimately our key focus. Our model will deliver the simplicity that the equilibrium quantities and prices only depend on the aggregate states rather than the firm distribution. However, our model still maintains a rich cross-section of heterogeneous firms' capital ratios given different idiosyncratic productivities, capital, debt, and net worth positions across firms. The constant capital ratio assumption therefore gives us the leverage to focus on the asset pricing implications of the cross-sectional heterogeneities of firms' asset holdings of different asset durabilities without over-complicating our model dynamics. Second, in Section 6 of additional empirical analysis, we find that financially constrained firms reduce their asset durability given adverse financial shocks whereas unconstrained firms increase their asset durability through capital reallocation. Such evidence squares well with our model assumption

[15] Without this assumption, we must keep track of the ratio of two types of capital as an additional aggregate state variable; thus, we will not be able to achieve the recursion construction of the Markov equilibrium and the aggregation results as shown in Proposition 1.

that the total quantity ratios of durable and nondurable capital at the aggregate level are relatively stable at the business cycle frequency. Finally, given this assumption, we can characterize the fraction of the new investment goods in producing type-$d$ and type-$nd$ capital, $\phi_t$ and $1-\phi_t$, respectively, such that $\phi_t = (\delta_d - \delta_{nd})\,\zeta\,(1-\zeta)\,\frac{K_t}{I_t} + \zeta$. The aggregate stocks of type-$d$ and type-$nd$ capital are therefore:

$$K_{t+1}^{d} = (1-\delta_d)\,K_t^d + \phi_t I_t \tag{18}$$

$$K_{t+1}^{nd} = (1-\delta_{nd})\,K_t^{nd} + (1-\phi_t)\,I_t. \tag{19}$$

# 4 Equilibrium Asset Pricing

## 4.1 Aggregation

Our economy is one with both aggregate and idiosyncratic productivity shocks. In general, we use the joint distribution of capital and net worth as an infinite-dimensional state variable in order to characterize equilibrium dynamics recursively. In this section, we follow Ai, Li, Li, and Schlag (2020a) and maintain that the aggregate quantities and prices of our model can be characterized without any reference to firm distribution. Given aggregate quantities and prices, quantities and shadow prices at the individual firm level can be directly computed using equilibrium conditions.

**Distribution of Idiosyncratic Productivity** At the aggregate level, the heterogeneity of idiosyncratic productivity can be conveniently summarized by a simple statistic: $Z_t = \int z_{i,t} di$. Given the law of motion of $z_{i,t}$ from equation (6) and the fact that entrepreneurs receive a liquidation shock with probability $\lambda_t$, we have:

$$Z_{t+1} = (1-\lambda_t)\int z_{i,t} e^{\varepsilon_{i,t+1}} di + \lambda_t \bar{z}_{t+1}.$$

Only a fraction $1-\lambda_t$ of entrepreneurs will survive until the next period, while the rest will restart with productivity of $\bar{z}_{t+1}$ in period $t+1$. As the law of motion of firms' idiosyncratic productivity shocks is time-invariant and that of liquidation shocks are specified as stationary processes, the cross-sectional distribution of $z_{i,t}$ converges to a stationary distribution.[16] We assume that $\varepsilon_{i,t+1}$

[16] In fact, the stationary distribution of $z_{i,t}$ is a double-sided Pareto distribution. Our model is therefore consistent with the empirical evidence regarding the power law distribution of firm size.

is independent of $z_{i,t}$ and can integrate out $\varepsilon_{i,t+1}$ and rewrite the above equation as:[17]

$$\begin{aligned} Z_{t+1} &= (1-\lambda_t) \int z_{i,t} E\left[e^{\varepsilon_{i,t+1}}\right] di \ + \lambda_t \bar{z}_{t+1} \\ &= (1-\lambda_t) Z_t e^{\mu_\varepsilon + \frac{1}{2}\sigma_\varepsilon^2} + \lambda_t \bar{z}_{t+1}, \end{aligned} \tag{20}$$

in which the last equality follows from the fact that $\varepsilon_{i,t+1}$ is normally distributed. Clearly, if we choose the normalization $\bar{z}_{t+1} = \frac{1}{\lambda_t}\left[1-(1-\lambda_t) e^{\mu_\varepsilon + \frac{1}{2}\sigma_\varepsilon^2}\right]$ and initialize the economy by setting $Z_0 = 1$, then $Z_t = 1$ for all $t$. We assume as much for the rest of our paper.

**Firm Profits** We assume that $\varepsilon_{i,t+1}$ is observed at the end of period $t$ when entrepreneurs plan the next period's capital. As we show in Section III of the Internet Appendix, this implies that entrepreneur $i$ will choose $K_{i,t+1}^d + K_{i,t+1}^{nd}$ to be proportional to $z_{i,t+1}$ in equilibrium. Additionally, because $\int z_{i,t+1} di = 1$, we must have:

$$K_{i,t+1}^d + K_{i,t+1}^{nd} = z_{i,t+1}\left(K_{t+1}^d + K_{t+1}^{nd}\right), \tag{21}$$

in which $K_{t+1}^d$ and $K_{t+1}^{nd}$ are the aggregate quantities of type-$d$ and type-$nd$ capital, respectively.

The assumption that capital is chosen after $z_{i,t+1}$ is observed rules out capital misallocation and implies that total output does not depend on the joint distribution of idiosyncratic productivity and capital. This is because given idiosyncratic shocks, all entrepreneurs choose the optimal level of capital such that the marginal productivity of capital is the same for all entrepreneurs. Thus, $Y_t = \bar{A}_t \left(K_t^d + K_t^{nd}\right)^{\alpha\nu} \int z_{i,t} di = \bar{A}_t \left(K_t^d + K_t^{nd}\right)^{\alpha\nu}$. It also implies that the profit at the firm level is proportional to aggregate productivity such that:

$$\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd}\right) = \alpha \bar{A}_t z_{i,t}\left(K_t^d + K_t^{nd}\right)^{\alpha\nu},$$

and the marginal products of capital are equalized across firms for the two types of capital:

$$\frac{\partial}{\partial K_{i,t}^d}\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd}\right) = \frac{\partial}{\partial K_{i,t}^{nd}}\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd}\right) = \alpha\nu\bar{A}_t\left(K_t^d + K_t^{nd}\right)^{\alpha\nu-1}. \tag{22}$$

To derive equation (22), we take derivatives of firm $i$'s output function in equation (11) with respect to $K_{i,t}^d$ and $K_{i,t}^{nd}$, and then impose optimality conditions in equations (13) and (21).

**Intertemporal Optimality** Having simplified profit functions, we can derive the optimality conditions for the entrepreneur's maximization problem in equation (10). We denote the

[17]The first line requires us to define the set of firms and the notion of integration in a mathematically careful way. Rather than reviewing the technical details, we refer readers to Feldman and Gilles (1985) and Judd (1985). Constantinides and Duffie (1996) use a similar construction in the context of heterogeneous consumers. See footnote 5 in Constantinides and Duffie (1996) for a more careful discussion on possible constructions of an appropriate measurable space under which the integration is valid.

marginal value of net worth for entrepreneur $i$ using $\mu_t^i$ and let $\eta_t^i$ be the Lagrangian multiplier associated with the collateral constraint in equation (5). The first-order condition with respect to $B_{i,t}$ implies:

$$\mu_t^i = E_t\left[\widetilde{M}_{t+1}^i\right] R_{f,t+1} + \eta_t^i, \tag{23}$$

where:

$$\widetilde{M}_{t+1}^i \equiv M_{t+1}[(1-\lambda_{t+1})\,\mu_{t+1}^i + \lambda_{t+1}]. \tag{24}$$

We find that one unit of net worth allows an entrepreneur to reduce one unit of borrowing, the present value of which is $E_t\left[\widetilde{M}_{t+1}^i\right] R_{f,t+1}$, and relaxes the collateral constraint, the benefit of which is measured by $\eta_t^i$.

Similarly, the first-order condition for $K_{i,t+1}^d$ is:

$$\mu_t^i = E_t\left[\widetilde{M}_{t+1}^i \frac{\frac{\partial}{\partial K_{i,t+1}^d}\Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right) + (1-\delta_d)\,q_{d,t+1}}{q_{d,t}}\right] + \theta(1-\delta_d)\eta_t^i. \tag{25}$$

It implies that an additional unit of net worth allows an entrepreneur to purchase $\frac{1}{q_{d,t}}$ units of capital, which pays a profit of $\frac{\partial}{\partial K_{i,t+1}^d}\Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right)$ over the next period before it depreciates at rate $\delta_d$. In addition, a fraction $\theta$ of type-$d$ capital can be used as collateral to relax the borrowing constraint adjusted for its collateralizability. Similarly, the optimality with respect to the choice of type-$nd$ capital follows:

$$\mu_t^i = E_t\left[\widetilde{M}_{t+1}^i \frac{\frac{\partial}{\partial K_{i,t+1}^{nd}}\Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right) + (1-\delta_{nd})\,q_{nd,t+1}}{q_{nd,t}}\right] + \theta(1-\delta_{nd})\eta_t^i. \tag{26}$$

**Recursive Construction of the Equilibrium** In our model, entrepreneurs have different levels of net worth. First, net worth depends on the entire history of idiosyncratic productivity shocks, as can be seen from equation (6) by which $z_{i,t+1}$ depends on $z_{i,t}$, which in turn depends on $z_{i,t-1}$ and so forth. Furthermore, net worth also depends on the need for capital, which relies on the realization of the next period's productivity shock. Therefore, the marginal benefit of net worth, $\mu_t^i$, and the tightness of the collateral constraint, $\eta_t^i$, generally depend on an individual firm's entire history. We next show that despite the heterogeneity in net worth and capital holdings across firms, our model permits an equilibrium in which $\mu_t^i$ and $\eta_t^i$ are equalized across firms, and that aggregate quantities can be determined independently of the distribution of net worth and capital.

In addition, assumptions that type-$d$ and type-$nd$ capital are perfect substitutes in production and that the idiosyncratic shock $z_{i,t+1}$ is observed before the decisions on $K_{i,t+1}^d$ and $K_{i,t+1}^{nd}$ both greatly simplify our model equilibrium. As a result, the marginal product of both types of capital are equalized within and across firms as shown in equation (22), and $\mu_t^i$ and $\eta_t^i$ are no longer firm-specific according to equations (23) to (26). Intuitively, as the marginal product of capital depends only on the sum of $K_{i,t+1}^{d}$ and $K_{i,t+1}^{nd}$, entrepreneurs only choose the total amount of capital that equalize marginal product across firms. Depending on the specific borrowing need when $z_{i,t+1}$ is observed before $t+1$, an entrepreneur then determines $K_{i,t+1}^{d}$ and $K_{i,t+1}^{nd}$ with realized $z_{i,t+1}$ consistent with the firm-specific collateral constraint.

We formalize this observation by constructing a recursive equilibrium in two steps. First, we show that aggregate quantities and prices can be characterized by a set of equilibrium functionals. Second, we further construct an individual firm's quantities from aggregate quantities and prices. We make one final assumption: that aggregate productivity is given by $\bar{A}_t = A_t(K_t^d + K_t^{nd})^{1-\nu\alpha}$, in which $\{A_t\}_{t=0}^{\infty}$ is an exogenous Markov productivity process. On the one hand, this assumption follows Frankel (1962) and Romer (1986) and is a parsimonious way to generate endogenous growth. On the other hand, this assumption, when combined with recursive preferences, increases the volatility of the pricing kernel, as in the literature on long-run risk models (see, e.g., Bansal and Yaron (2004) and Kung and Schmid (2015)). From a technical point of view, this assumption means that equilibrium quantities are homogenous of degree one in the total capital stock, $K_t = K_t^d + K_t^{nd}$, and equilibrium prices do not depend on $K_t$. It is therefore convenient to work with normalized quantities.

To begin, we denote a generic variable in current period as $X$ and in future period as $X'$ and then let the lowercase variables denote aggregate quantities normalized by the current total capital stock; for instance, the current period aggregate net worth $n$ denotes aggregate net worth $N$ normalized by the total capital stock $K$. Abstracting from the time indexation, the equilibrium objects of our model include the normalized consumption, $c(A,\lambda,n)$, investment, $i(A,\lambda,n)$, the marginal value of net worth, $\mu(A,\lambda,n)$, the Lagrangian multiplier on the collateral constraint, $\eta(A,\lambda,n)$, the price of type-$d$ capital, $q_d(A,\lambda,n)$, the price of type-$nd$ capital, $q_{nd}(A,\lambda,n)$, and the risk-free interest rate, $R_f(A,\lambda,n)$ as functions of the realized exogenous state variables $A$ and $\lambda$, as well as the endogenous state of normalized aggregate net worth, $n$.

We can define the growth rate of total capital stock as:

$$\Gamma(A,\lambda,n) \equiv \frac{K'^d + K'^{nd}}{K^d + K^{nd}} = (1-\delta_{nd}) + (\delta_{nd} - \delta_d)\zeta + i(A,\lambda,n)$$

Then the law of motion of the endogenous state variable $n$ follows from equation (9):[18]

$$\begin{aligned} n' \;=\;& (1-\lambda'+\lambda'\chi)\Big[\;\alpha A' + \zeta(1-\delta_d)\,q_d(A',\lambda',n') + (1-\zeta)(1-\delta_{nd})\,q_{nd}(A',\lambda',n')\;\Big] \\ & -(1-\lambda')\frac{b(A,\lambda,n)R_f(A,\lambda,n)}{\Gamma(A,\lambda,n)}. \end{aligned} \tag{27}$$

Given optimal consumption and capital growth rates, we obtain the normalized utility of the

[18] We make use of the property that the ratio of $K_t^d$ over $K_t^{nd}$ is always equal to $\zeta/(1-\zeta)$, as implied by the laws of motion of the capital stock for both types.

household as the functional fixed point of:

$$u(A,\lambda,n)=\left\{(1-\beta)c(A,\lambda,n)^{1-\frac{1}{\psi}}+\beta\Gamma(A,\lambda,n)^{1-\frac{1}{\psi}}\left(E[u(A',\lambda',n')^{1-\gamma}]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The stochastic discount factors can be rewritten as:

$$M' = \beta\left[\frac{c(A',\lambda',n')\Gamma(A,\lambda,n)}{c(A,\lambda,n)}\right]^{-\frac{1}{\psi}}\left[\frac{u(A',\lambda',n')}{E\left[u(A',\lambda',n')^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}, \quad (28)$$

$$\widetilde{M}' = M'[(1-\lambda')\mu(A',\lambda',n')+\lambda']. \quad (29)$$

We next construct a Markov equilibrium for which all prices and quantities at the aggregate level are functions of the state variables $(A,\lambda,n)$. For simplicity's sake, we assume that the initial idiosyncratic productivity across all firms satisfies $\int z_{i,1}di=1$, the initial aggregate net worth is $N_0$, aggregate capital holdings start with $\frac{K_1^d}{K_1^{nd}}=\frac{\zeta}{1-\zeta}$, and a firm's initial net worth satisfies $n_{i,0}=z_{i,1}N_0$ for all $i$. The full equilibrium of our model then can be characterized as a set of aggregate quantities, $\{C_t,B_t,\Pi_t,K_t^d,K_t^{nd},I_t,N_t\}$, individual entrepreneur choices, $\left\{K_{i,t}^d,K_{i,t}^{nd},L_{i,t},B_{i,t},N_{i,t}\right\}$, and prices $\left\{M_t,\widetilde{M}_t,W_t,q_{d,t},q_{nd,t},\mu_t,\eta_t,R_{f,t}\right\}$ such that, given prices, quantities satisfy the household's and the entrepreneurs' optimality conditions, the market-clearing conditions, and relevant resource constraints. The following proposition provides details regarding the recursive stochastic equilibrium of our model.

**Proposition 1.** *(Markov Equilibrium)*

*Suppose there exists a set of equilibrium functionals* $\{c(A,\lambda,n),u(A,\lambda,n),,b(A,\lambda,n),i(A,\lambda,n),\mu(A,\lambda,n),\eta(A,\lambda,n),q_d(A,\lambda,n),q_{nd}(A,\lambda,n),R_f(A,\lambda,n),\phi(A,\lambda,n)\}$ *satisfying the following set of functional equations:*

$$E\left[M'\middle|A,\lambda,n\right]R_f(A,\lambda,n)=1, \quad (30)$$

$$\mu(A,\lambda,n)=E\left[\widetilde{M}'\middle|A,\lambda,n\right]R_f(A,\lambda,n)+\eta(A,\lambda,n), \quad (31)$$

$$\mu(A,\lambda,n)=E\left[\widetilde{M}'\frac{\alpha\nu A'+(1-\delta_d)q_d(A',\lambda',n')}{q_d(A,n)}\middle|A,\lambda,n\right]+\theta(1-\delta_d)\eta(A,\lambda,n), \quad (32)$$

$$\mu(A,\lambda,n)=E\left[\widetilde{M}'\frac{\alpha\nu A'+(1-\delta_{nd})q_{nd}(A',\lambda',n')}{q_{nd}(A,n)}\middle|A,\lambda,n\right]+\theta(1-\delta_{nd})\eta(A,\lambda,n), \quad (33)$$

$$\frac{n+b(A,\lambda,n)}{\Gamma(A,\lambda,n)}=\zeta q_d(A,\lambda,n)+(1-\zeta)q_{nd}(A,\lambda,n), \quad (34)$$

$$\eta(A,\lambda,n)\{b(A,\lambda,n)-\theta[\zeta(1-\delta_d)q_d(A,\lambda,n)+(1-\zeta)(1-\delta_{nd})q_{nd}(A,\lambda,n)]\Gamma(A,\lambda,n)\}=0, \quad (35)$$

$$G'(i(A,\lambda,n))=\phi(A,\lambda,n)q_d(A,\lambda,n)+(1-\phi(A,\lambda,n))q_{nd}(A,\lambda,n), \quad (36)$$

$$c(A,\lambda,n)+i(A,\lambda,n)+g(i(A,\lambda,n))=A, \quad (37)$$

$$\phi(A,\lambda,n) = \frac{(\delta_d - \delta_{nd})(1-\zeta)\zeta}{i(A,\lambda,n)} + \zeta, \tag{38}$$

*where the law of motion of $n$ is given by equation (27), and the stochastic discount factors $M'$ and $\widetilde{M}'$ are defined in equations (28) and (28). Then, equilibrium prices and quantities can be constructed as follows, thereby constituting a Markov equilibrium:*

1. *Given the sequence of exogenous shocks $\{A_t, \lambda_t\}$, the sequence of $n_t$ can be constructed using the law of motion in equation (27), and the normalized policy functions are constructed as:*

$$x_t = x(A_t, \lambda_t, n_t), \ \ for\ x = c, u, b, i, \mu, \eta, q_d, q_{nd}, R_f, \phi,$$

   *and are jointly determined by equations (30)-(38).*

2. *Given the sequence of normalized quantities, aggregate quantities are constructed as:*

$$K^d_{t+1} = K^d_t(1-\delta_d) + \phi_t I_t, K^{nd}_{t+1} = K^{nd}_t(1-\delta_{nd}) + (1-\phi_t) I_t,$$
$$X_t = x_t \left[K^d_t + K^{nd}_t\right],$$

   *for $x = c, i, b, n$, $X = C, I, B, N$, and all $t$.*

3. *Given the aggregate quantities, individual entrepreneurs' net worth follows from equation (8). Given the sequences $\{N_{i,t}\}$, the quantities $B_{i,t}, K^d_{i,t}$ and $K^{nd}_{i,t}$ are jointly determined by equations (4), (5), and (21). Finally, $L_{i,t} = z_{i,t}$ for all $i, t$.*

We first provide the interpretations on our equilibrium conditions. Equation (30) is the household's intertemporal Euler equation with respect to the choice of risk-free asset. Equation (31) is the firm's optimality condition for the choice of debt. Equations (32) and (33) are the firm's first-order conditions with respect to the choice of type-$d$ and type-$nd$ capital. Equation (34) is the budget constraint of firms. Equation (35) governs the condition of complementary slackness, which gives the endogenous upper limit of borrowing for each period. Equation (36) is the optimality condition for capital goods production, equation (37) is the aggregate resource constraint, and equation (38) separates the allocation of new investment into two types of capital.

Proposition 1 implies that conditions in equations (30)-(38) are not only necessary but also sufficient for the construction of equilibrium quantities and prices. This proposition implies that we can solve for aggregate quantities first and then use the firm-level budget constraint and the law of motion of idiosyncratic productivity to construct the cross-section of net worth and capital holdings. Our construction of the equilibrium allows $\eta(A,\lambda,n) > 0$ for some values of $(A,\lambda,n)$; that is, our general setup allows for occasionally binding constraints. Numerically, we resort to the parameterized expectation algorithm as outlined in Christiano and Fisher (2000) and solve the aggregate quantities and prices globally over the permissible domain of state variables.

Importantly, type-$d$ capital can perfectly substitute for type-$nd$ capital in production and both types of capital are freely traded on the market; thus, the marginal product of capital must be equalized within and across firms. The trading of capital therefore equalizes the Lagrangian multiplier of financial constraints across firms. This is the key feature of our model that allows us to construct a Markov equilibrium without including the distribution of capital as a state variable.[19]

## 4.2 User Cost, Down Payment, and Risk Sensitivity

Following Proposition 1, aggregate quantities and prices do not depend on the joint distribution of individual entrepreneur-level capital and net worth. In this section, we define the user costs of type-$d$ and type-$nd$ capital in the presence of collateral constraint and aggregate risks by extending the definition in Jorgenson (1963). The optimal decision to choose between type-$d$ and type-$nd$ capital is achieved when user costs of two types of capital are equalized. The definitions in this section clarify a novel *risk-premium channel* in equilibrium that affects the relative attractiveness between two types of capital, which has not been emphasized in the literature.

First, we provide the intuition about the trade-off underlying type-$d$ versus type-$nd$ decisions by comparing their user costs. The user cost of capital, $\tau_{h,t}$, $h \in \{d, nd\}$, is:

$$\tau_{h,t} \quad = \quad \vartheta_{h,t} - E_t \left[ \frac{\widetilde{M}_{t+1}}{\mu_t} \left\{ q_{h,t+1} \left(1 - \delta_h\right) - R_{f,t+1} \frac{B_{h,t}}{K_{h,t+1}} \right\} \right], \tag{39}$$

We denote $B_{h,t}$ as the act of borrowing for financing type-$h$ capital of amount $K_{h,t+1}$. User costs can be measured by the difference between the minimum down payment per unit of capital paid upfront, $\vartheta_{h,t} = \frac{q_{h,t}K_{h,t+1} - B_{h,t}}{K_{h,t+1}}$, which is the first term in equation (39), and the present value of the fractional capital resale value next period that cannot be pledged, which is the second term in the equation.

For simplicity, we first define a shadow interest rate for borrowing among entrepreneurs, $R_{I,t}$, which is given by:

$$1 = E_t \left( \frac{\widetilde{M}_{t+1}}{\mu_t} \right) R_{I,t+1}. \tag{40}$$

Based on equation (23) and the definition in equation (40), we can obtain an interest rate spread, $\Delta_{f,t+1}$, between two interest rates:

$$\Delta_{f,t+1} = R_{I,t+1} - R_{f,t+1} = \frac{\eta_t}{\mu_t} R_{I,t+1}.$$

Given the occasionally binding constraint as in equation (5), we obtain a measure of aggregate slackness of the credit constraint of each period $\Delta_t$ such that:

$$\Delta_t = \theta - \frac{B_t}{[(1-\delta_d)q_{d,t}\zeta + (1-\delta_{nd})q_{nd,t}(1-\zeta)]K_{t+1}} \geq 0. \tag{41}$$

[19] Because of these simplifying assumptions, our model is not generating a cross-section of firms among which some firms are financially constrained while others are not.

Thus, when all firms are financially constrained in a period, $\Delta_t = 0$.

As a result, we can simplify the user cost of financing for a unit of type-$h$ capital as:

$$\begin{aligned}\tau_{h,t} &= q_{h,t}\left[1-(\theta-\Delta_t)(1-\delta_h)\right] \\ &\quad -(1-\delta_h)\,Cov_t\left(\frac{\widetilde{M}_{t+1}}{\mu_t}, q_{h,t+1}\right) - \frac{1}{R_{f,t+1}+\Delta_{f,t+1}} E_t\varphi_{h,t+1}. \end{aligned} \tag{42}$$

We obtain the second line while the second term in equation (39) is expanded using a covariance term. Factoring out the discount factor $\frac{1}{R_{f,t+1}+\Delta_{f,t+1}}$, the capital resale value for the next period can be summarized as $\varphi_{h,t+1}$ such that:

$$\varphi_{h,t+1} = (1-\delta_h)\left[q_{h,t+1} - R_{f,t+1}(\theta-\Delta_t)\right]. \tag{43}$$

Next, we derive the difference in the user costs of the two types of capital and show that three important wedges appear to drive our main mechanism that determines firms' trade-off between holding type-$d$ and type-$nd$ capital.

$$\tau_{d,t} - \tau_{nd,t} = (\vartheta_{d,t} - \vartheta_{nd,t}) + \Delta_{rp,t} - \frac{1}{R_{f,t+1}+\Delta_{f,t+1}} E_t\left[\varphi_{d,t+1} - \varphi_{nd,t+1}\cdot\right] \tag{44}$$

The first component in equation (44) denotes the down-payment differences highlighted in Rampini (2019), which appears to be positive in the sense that durable capital is more "expensive" for financing a higher down-payment. It directly affects the trade-off for substitution between durable and less durable capital.

Our model, in particular, highlights an *additional* risk-premium wedge as captured by the second term, $\Delta_{rp,t}$. This wedge denotes the difference in the risk premium evaluated by entrepreneurs' stochastic discount factors for type-$d$ versus type-$nd$ capital because of different covariances between capital prices and the discount factor. In particular, this wedge follows that:

$$\Delta_{rp,t} = -(1-\delta_d)Cov_t\left(\frac{\widetilde{M}_{t+1}}{\mu_t}, q_{d,t+1}\right) + (1-\delta_{nd})Cov_t\left(\frac{\widetilde{M}_{t+1}}{\mu_t}, q_{nd,t+1}\right).$$

While substitution across asset durability affects capital prices, such price effects reflect the relative riskiness in general equilibrium, and effectively introduces additional variations to user cost differences across capital types and over time. As a result, this wedge delivers the co-movement of capital prices with stochastic discount factors, which generates risk exposures in stock returns. We show in equilibrium that adverse aggregate productivity and financial shocks tend to trigger severe financial frictions on all firms, and that firms will acquire less expensive and less durable capital. Hence, durable capital not only exhibits greater price cyclicality but its prices are also more sensitive to aggregate shocks. On average, $\Delta_{rp} > 0$ helps explain why durable capital can be considered increasingly more expensive relative to financing for less durable capital (i.e., its incremental risk exposure to aggregate shocks).

Conditional on the same discount factor, the third term in the expectation in equation (44) gives that:

$$E_t\left[\varphi_{d,t+1}-\varphi_{nd,t+1}\right]=(1-\delta_d)\,E_t\left[q_{d,t+1}-R_{f,t+1}(\theta-\Delta_t)\right]-(1-\delta_{nd})\,E_t\left[q_{nd,t+1}-R_{f,t+1}(\theta-\Delta_t)\right],$$

which denotes the difference in the expected capital resale value for the next period. For $\delta_d < \delta_{nd}$ and with higher durable capital price in equilibrium $q_{d,t} > q_{nd,t}$ on average, it can be easily shown that $E_t\left[\varphi_{d,t+1}-\varphi_{nd,t+1}\right] > 0$. This term thus reflects the marginal benefit of acquiring durable capital relative to non-durable capital. This relative benefit term partly offsets the higher down payment and greater price riskiness of durable capital to determine total relative user costs between durable and non-durable capital.

In summary, our decomposition exercises suggest that it is costly for a firm to buy durable capital for two reasons. First, acquiring durable capital may be relatively more costly because it requires a larger down payment. Second, given that aggregate shocks will trigger a firm's substitutabilities over asset durability, the greater risk sensitivities of more durable capital relative to that of less durable capital commands a positive risk premium wedge that makes durable capital more expensive in equilibrium. User cost differences that are driven by different down payments have been emphasized in Rampini (2019), while the additional wedge delivered by a risk premium component is a key novel channel that we highlight in our paper.

We then consider a special case that can much highlight our model contribution. Suppose the capital prices of both types are fixed over time; this can be achieved, for instance, if there is no adjustment cost for producing capital goods in our model. It then implies that:

$$\tau_{d,t}-\tau_{nd,t}=(\vartheta_{d,t}-\vartheta_{nd,t})-\frac{1}{R_{f,t+1}+\Delta_{f,t+1}}\left[\varphi_{d,t+1}-\varphi_{nd,t+1}\right].$$

Importantly, in such a case, capital prices do not fluctuate, and risk sensitivities of capital prices do not affect user cost differentials; thus, the risk premium wedge disappears. The asset durability trade-off can be traced back to Rampini (2019), thereby shutting off the risk-premium channel after fixing the stochastic discount factor.

Therefore, we emphasize that our highlighted risk premium channel for affecting choices over asset durability naturally arises in the general equilibrium over business cycles. This premium channel operates as long as more durable capital exhibits greater risk sensitivities to aggregate shocks regardless of whether or not entrepreneurs' financial constraints are binding. It can be shown that down payment differences and relative benefits of resale values of durable capital are both greatly weakened when entrepreneurs' borrowings are constrained. Specifically, according to equation (42), for any given $q_{d,t}$ and $q_{nd,t}$, the binding constraint for $\Delta_t = 0$ reduces down payments of $\vartheta_{d,t}$ and $\vartheta_{nd,t}$ on both capital types. The relative marginal benefit of acquiring durable capital is less important as $\frac{1}{R_{f,t+1}+\Delta_{f,t+1}}$ is smaller when the constraint is binding for $\eta_t > 0$ and entrepreneurs borrow at a rate with a positive spread $\Delta_{f,t+1} > 0$ over the risk-free rate. With $\Delta_t = 0$, the resale value of both capital goods, $\varphi_{d,t+1}$ and $\varphi_{nd,t+1}$, will be smaller as well. The relative benefit will become less important in determining the substitutabilities of asset durability when adverse financial shocks hit. Hence, on relative terms, the risk-premium channel predominantly affects capital substitutabilities particularly when financial frictions are more severe.

In sum, we use this paper to highlight an additional risk premium channel by building a dynamic choice of asset durability into a general equilibrium model with financial frictions and aggregate risks. We show that because of the different risk sensitivities of durable and non-durable capital over business cycles driven by aggregate shocks, firms' decisions over durable vs. non-durable capital goods are additionally affected by a risk premium channel. More importantly, when financial frictions are more severe than those that bind entrepreneurs' borrowing, this channel is comparatively much stronger and determines firms' choices over asset durability.

## 4.3 Asset Pricing Implications

In this section, we study the asset pricing implications of our model both at the aggregate and the firm level. It can be shown that the relative price riskiness of durable and non-durable capital naturally translates into a cross-section of stock returns characterized by the firm heterogeneity in asset durability.

**Asset Durability Spread at the Aggregate Level** We first discuss the importance of differentiating between *levered* and the *unlevered* returns on durable and non-durable capital. Given that one unit of type-$h$ capital costs $q_{h,t}$ in period $t$ and pays off $\Pi_{h,t+1} + (1-\delta_h)\, q_{h,t+1}$ in the next period, for $h \in \{d, nd\}$, unlevered returns therefore follow such that:

$$R_{h,t+1} = \frac{\alpha\nu A_{t+1} + (1-\delta_h)\, q_{h,t+1}}{q_{h,t}} \quad (h = d, nd). \tag{45}$$

The levered return on type-$d$ (type-$nd$) capital is similarly defined by adjusting for the leverage ratio and net worth:

$$\begin{aligned} R^{Lev}_{h,t+1} &= \frac{\alpha\nu A_{t+1} + (1-\delta_h)\, q_{h,t+1} - R_{f,t+1}(q_{h,t} - n_t/\Gamma_t)}{n_t/\Gamma_t} \\ &= \frac{1}{1-\psi_{h,t}} \left(R_{h,t+1} - R_{f,t+1}\right) + R_{f,t+1}, \end{aligned} \tag{46}$$

in which $n_t/\Gamma_t$ denotes the amount of internal net worth used to buy one unit of capital of a given type for the period $t+1$, thereby serving as the down payment. The financial leverage ratio specific to that capital type is thus defined as $\psi_{h,t} = \frac{B_{h,t}}{q_{h,t}K_{h,t+1}} = 1 - \frac{n_t}{q_{h,t}\Gamma_t}$. Regarding the first line in equation (46), the numerator captures the next period's return to the type of capital after subtracting the debt financing repayment for buying that one unit of capital. Finally, we see that excess returns derived from levered returns and those of un-levered returns are governed by the following relation:

$$R_{h,t+1}^{Lev} - R_{f,t+1} = \frac{1}{1-\psi_{h,t}}(R_{h,t+1} - R_{f,t+1}). \tag{47}$$

Importantly, when firms' credit constraints are binding, we see that borrowing for acquiring durable capital incurs a greater leverage $\psi_{d,t} = \theta(1-\delta_d) > \psi_{nd,t} = \theta(1-\delta_{nd})$. This generically increases levered returns on financing for durable capital $R_{d,t+1}^{Lev}$ according to equation (46) and follows Ai, Li, Li, and Schlag (2020a) in that more durable capital is more collateralizeable. We therefore report both the levered and the unlevered returns along with their return spreads for asset pricing implications in the following section. Specifically, we show that durable capital indeed has more collateralizeablility value, but is also riskier in equilibrium.

Next, we derive and focus on the spread of expected unlevered returns on durable and non-durable capital investment. Combining the two Euler equations as of equations (32) and (33), we have:

$$E_t\left[\widetilde{M}_{t+1}R_{h,t+1}\right] = \mu_t - \theta(1-\delta_h)\eta_t.$$

and the return spread follows:

$$\begin{aligned} & E_t\left(R_{d,t+1} - R_{nd,t+1}\right) \\ = & -\frac{1}{E_t\left(\widetilde{M}_{t+1}\right)}\left(Cov_t\left[\widetilde{M}_{t+1}, R_{d,t+1}\right] - Cov_t\left[\widetilde{M}_{t+1}, R_{nd,t+1}\right]\right) - \Omega_t \end{aligned} \tag{48}$$

in which $\Omega_t = \frac{\theta(\delta_{nd}-\delta_d)}{E_t\left(\widetilde{M}_{t+1}\right)}\eta_t$.

According to equation (48), it shows that the return spread between investing in durable and non-durable capital at the aggregate level is driven by two components: the first term captures the risk premium differences in the covariance of the stochastic discount factor and the asset payoff; the second term gives the return differences due to different marginal gains from financing the durable capital relative to less durable capital for different collateralizablility value of each capital type $\delta_d \neq \delta_{nd}$ when borrowing constraint matters $\eta_t > 0$. Therefore, we label the first component as the risk premium channel, our key focus in this paper, and the second component the collateralizability channel (Ai, Li, Li, and Schlag, 2020a), under both of which more durable and less durable capital would differ in the relative riskiness of prices and the expected returns in equilibrium.

It is critically important to emphasize that equation (48) gives us exactly the very decomposition to compare and contrast the risk-premium and the collateralizability channel, which appear to be offsetting each other. In particular, for the first component, according to equation (45), the main driving force of return spread differences between durable and non-durable capital comes from the resale price $(1-\delta_h)\,q_{h,t+1}$ rather than from the marginal product of capital $\alpha\nu A_{t+1}$, which is common for both capital types. If the price of type-$d$ capital exhibits higher cyclicality, then it is more covaried with the stochastic discount factor and is thus more sensitive to aggregate shocks. As we have discussed previously that this risk-premium channel affects the incentives for firms' optimization over asset durability, $R_{d,t+1}$ is more riskier than its counterpart $R_{nd,t+1}$ over business cycles, and the first term is positive. As for the second term $\Omega_t$, since $\delta_{nd} > \delta_d$, the marginal gain from the collateralizability value of durable capital is positive $\frac{\theta(\delta_{nd}-\delta_d)}{E_t\left(\widetilde{M}_{t+1}\right)} > 0$ as long as the borrowing constraint is binding for $\eta_t > 0$, and the return spread is therefore offset by $\Omega_t > 0$.

By solving our model using a global solution, we can numerically evaluate the relative magnitudes of these offsetting channels given firms' borrowing constraints are occasionally binding over business cycles. Before proceeding to quantitative exercises, several key observations warrant mention. First, the global solution specifically allows constraints to bind occasionally, generating additional variations in the shadow value of financial frictions $\eta_t$ that are more volatile and exposed to aggregate shocks. This highlights the importance of the risk-premium channel in making durable capital riskier. Second, according to equation (48), the asset collateralizability channel becomes more pronounced when constraints are binding. This is shown in Ai, Li, Li, and Schlag (2020a), where their model is solved using a local solution that imposes an always binding constraint. Our different solution approach thus provides a more comprehensive understanding of how financial frictions affect the relative riskiness of durable capital. In subsequent sections, we demonstrate that the risk-premium channel is sufficiently strong to dominate the collateralizability channel when entrepreneurs borrow up to their financial constraints. Our findings are therefore novel and important, by showing that durable capital is riskier on average in equilibrium even though it has greater collateralizeability value than that of non-durable capital even when the credit constraints do not always bind.

**Asset Durability Spread at the Firm Level** We then derive model implications on firm risk. We define the equity return on an entrepreneur's net worth to be approximately $\frac{N_{i,t+1}}{N_{i,t}}$.[20] We can use equations (4) and (8) and write out the return as below:

$$
\begin{aligned}
R_{i,t+1} &= \frac{\alpha\nu A_{t+1}\left(K^d_{i,t+1}+K^{nd}_{i,t+1}\right)+(1-\delta_d)\,q_{d,t+1}K^d_{i,t+1}+(1-\delta_{nd})\,q_{nd,t+1}K^{nd}_{i,t+1}-R_{f,t+1}B_{i,t}}{N_{i,t}} \\
&= \frac{\vartheta^i_{d,t}}{N_{i,t}}R^{Lev}_{d,t+1}+\frac{\vartheta^i_{nd,t}}{N_{i,t}}R^{Lev}_{nd,t+1}.
\end{aligned}
$$

[20]In Section III of the Internet Appendix, we recast the firm value in the form of $V^i\left(N_{i,t},z_{i,t+1}\right) = \mu\left(A_t,\lambda_t,n_t\right)N_{i,t}+\Theta\left(A_t,\lambda_t,n_t\right)\left(K^d_t+K^{nd}_t\right)z_{i,t+1}$. We show $\Theta_t = 0$ when $\nu = 1$ in equation (III.25). As in our calibration, $\nu$ is large and close to one, and we ignore the second part in firms' values for illustrative purposes here. In our quantitative evaluations in Section 5, we examine precisely defined returns on firms' equity.

This expression has an intuitive interpretation: the firm's equity return is a weighted average of the levered returns on type-$d$ capital, $R^{Lev}_{d,t+1}$, and the return on type-$nd$ capital, $R^{Lev}_{nd,t+1}$. The weights $\frac{\vartheta^i_{d,t}}{N_{i,t}}$ and $\frac{\vartheta^i_{nd,t}}{N_{i,t}}$ are the fractions of the down payment for purchasing some amounts of durable capital and non-durable capital, respectively, in entrepreneur $i$'s net worth such that $\frac{\vartheta^i_{d,t}}{N_{i,t}} + \frac{\vartheta^i_{nd,t}}{N_{i,t}} = 1$. Given unlevered returns, it follows that the excess stock returns of firm $i$ can be rewritten as follows:

$$R_{i,t+1} - R_{f,t+1} = \frac{\vartheta^i_{d,t}}{N_{i,t}} \frac{1}{1-\psi_{d,t}} (R_{d,t+1} - R_{f,t+1}) + \frac{\vartheta^i_{nd,t}}{N_{i,t}} \frac{1}{1-\psi_{nd,t}} (R_{nd,t+1} - R_{f,t+1}).$$

Accordingly, as returns $R_{h,t+1}$ and leverages $\psi_{h,t}$ are common across all firms in our model, expected returns differ across firms only because firms' composition of nominal expenditure on type-$d$ versus the type-$nd$ capital are different.

Importantly, we demonstrate the very novelty of our model in the sense that firms' asset composition of different asset durabilities, by loading in different risk exposures to aggregate risk, could well affect the firm risk and determine the cross-section of stock returns. Especially, the composition of nominal expenditure on different types of capital can be effectively summarized by the measure of asset durability of a firm in our data. Such parallel between our model and our empirical results allows our model to quantitatively reproduce the asset durability spread that we observe in our data.

# 5 Quantitative Model Predictions

In this section, we first discuss the calibration of our model and evaluate its ability to replicate key aggregate moments of both macroeconomic quantities and asset prices. We then investigate the model's performance in quantitatively accounting for key features of firm characteristics and generating the asset durability premium in the cross-section. In particular, we highlight that firms optimally adjust the composition of asset durability on their balance sheets by substituting between durable and non-durable capital over the business cycle, which results in different risk sensitivities and price cyclicalities for the two capital types in equilibrium. This risk-premium channel is the key mechanism that drives return spreads across portfolios sorted on asset durability. Importantly, our model finds that although durable capital has greater collateral value than non-durable capital, it is substantially riskier in equilibrium. We show that our model not only helps explain the asset durability premium among financially constrained firms but also generates a negative relationship between cash flow duration and asset durability across duration-sorted portfolios regardless of whether firms' financial constraints are binding.

## 5.1 Specification of Aggregate Shocks

We formalize the specification of the exogenous processes of aggregate shocks for our model economy. First, the aggregate productivity in natural logarithm $a \equiv \log(A)$ is:

$$a_t = a_{ss}(1-\rho_A) + \rho_A a_{t-1} + \sigma_A \varepsilon_{A,t}, \tag{49}$$

in which $a_{ss}$ denotes the steady-state value of $a$. In addition, following Ai, Li, and Yang (2020b), we introduce a second type of aggregate shocks to the probability of entrepreneurs' net worth being liquidated, $\lambda_t$. This shock originates directly from the financial sector, following Jermann and Quadrini (2012). We incorporate both types of shocks mainly to improve the quantitative performance of our model. As in all standard real business cycle models, it is hard to generate large enough variations in asset prices with aggregate productivity shocks only.

Specifically, the shocks to entrepreneurs' liquidation probability directly affect entrepreneurs' discount rate, as can be seen from equation (24), which allows for studying asset pricing implications.[21] We also note that technically $\lambda_t \in (0,1)$. For brevity's sake, we set:

$$\lambda_t = \frac{\exp(x_t)}{\exp(x_t) + \exp(-x_t)},$$

and $x_t$ itself follows an autocorrelated process:

$$x_t = x_{ss}(1-\rho_x) + \rho_x x_{t-1} + \sigma_x \varepsilon_{x,t}.$$

We assume innovations to the two exogenous processes governed by:

$$\begin{bmatrix} \varepsilon_{A,t+1} \\ \varepsilon_{x,t+1} \end{bmatrix} \sim Normal\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{A,x} \\ \rho_{A,x} & 1 \end{bmatrix} \right),$$

in which parameter $\rho_{A,x}$ captures the correlation between these two shocks. Following Quadrini (2011) and Bigio and Schneider (2017), we assume a negative correlation $\rho_{A,x}$ in our calibration, which indicates that a negative productivity shock is associated with a positive discount rate shock. This is partly motivated from structural VAR estimations. Negative correlations of productivity shocks and liquidity shocks are needed in our model framework to quantitatively generate a positive correlation between consumption and investment growth that is consistent with the data.[22]

[21] Macro models with financial frictions, as portrayed in Gertler and Kiyotaki (2010) and Elenev et al. (2021), use a similar device for the same reason.

[22] This is a classic problem shared by many neoclassical macroeconomic models with flexible prices. See discussions in Kiyotaki and Moore (2019).

## 5.2 Calibration

We calibrate our model to target data moments of annual frequency. To compute these data moments, we use macroeconomic data on a per capita basis from a long sample that ranges from 1930 to 2017. Our consumption, output, and physical investment data are from the Bureau of Economic Analysis (BEA). To complete cross-sectional analyses, we use several data sources at the micro-level that help us evaluate our model predictions, which we summarize in Section IV of the Internet Appendix.

Table 4 reports the list of parameters and the corresponding macroeconomic moments in our calibration procedure. We group our parameters into four blocks. In the first block, we list the parameters that we borrow directly from the literature. In particular, we set the relative risk aversion $\gamma$ to 10 and the intertemporal elasticity of substitution $\psi$ to 2. These are parameter values in line with the long-run risks literature (e.g., Bansal and Yaron (2004)). The capital share parameter, $\alpha$, is set to 0.32, close to the number used in the standard RBC literature (e.g., Kydland and Prescott (1982)). The span of control parameter $\nu$ is set to 0.85, consistent with Atkeson and Kehoe (2005). We also set the discount factor $\beta = 0.984$ and the average annual entrepreneurs' liquidation probability $E(\lambda) = 0.12$ to jointly match the level of risk-free interest rate for household loans to about 1.2% in the data and set an average firm's life span to 10 years in Compustat. The elasticity parameter of the investment adjustment cost functions is set at $\tau = 7$, which is standard in the RBC literature and allows our model to achieve a reasonably large volatility of investment in line with the data.

We determine the parameters in the second block by matching a set of first moments of quantities and prices to their empirical counterparts. We first set the depreciation rates for durable and non-durable capital to be 0.05 and 0.19, respectively, which correspond to empirical estimates of a lower and upper bound across the refined capital categories that are based on our calculations of the BEA data. We then pick $\zeta = 0.645$, which delivers a total annual depreciation rate of weighted averages of approximately 10%. Given that the average consumption-to-investment ratio $E(C/I)$ is 4, we back out the average economy-wide productivity growth rate $E(A_{ss})$ to match a mean growth rate of the U.S. economy of 2% per year conditional on the depreciation rates of capital. We calibrate the remaining parameters related to financial frictions, namely, the collateralizability parameter, $\theta$, and the transfer to entering entrepreneurs, $\chi$, by jointly matching two moments: the median leverage ratio of 0.31 among U.S. non-financial firms in Compustat and the equity over total asset ratios of approximately 0.48 among younger and newer U.S. private firms aged less than 10 years (Dinlersoz, Kalemli-Ozcan, Hyatt, and Penciakova, 2018).

The parameters in the third block are based on the conversion of standard parameter values that we estimate using quarterly data. Based on quarterly estimates from Bayesian estimations of a structural model with both macroeconomic and financial blocks (Guerron-Quintana and Jinnai, 2019), we convert quarterly values to their annual counterparts associated with the exogenous processes.[23] The shock correlation is set to $\rho_{A,x} = -0.85$, which lies between the number of $-0.75$ as derived from the positive correlation between the abundance of credit supply and the aggregate productivity in Bigio and Schneider (2017) and $-1$ as assumed in Ai, Li, Li, and Schlag (2020a).

The last block contains parameters related to idiosyncratic productivity shocks. We calibrate them to match the mean and standard deviation of the idiosyncratic productivity growth of financially constrained firms in our U.S. Compustat database.

## 5.3 Numerical Solution and Simulation

We briefly summarize our model's numerical solution in this subsection. In particular, we solve our model globally for aggregate quantities and prices by allowing the credit constraint to be binding only occasionally over time. Our numerical analysis involves two major steps. First, we solve the model featuring the aggregate dynamics of quantities and prices. Second, we take the firm's policy functions and simulate a large panel of firms subject to idiosyncratic shocks, so we may compute corporate behaviors and their return profiles across sorted portfolios.

Specifically, we follow Christiano and Fisher (2000) and apply the modified Parameterized Expectation Algorithm (PEA) to directly approximate all expectation terms on the Euler Equations using Chebyshev Polynomials. Conditional on states, the approximated functionals related to policy functions can easily back out the functional values of $\eta_t$, which indicate if the credit constraint is binding occasionally. It is important to note that abstracting away from a time-varying firm distribution, our model solution features results that all firms are either constrained at a time or unconstrained at another time along the simulation path. This saves the computational burden if the distribution of firms is a state variable but without sacrificing our model predictability on cross-sectional returns. For a given calibration and our predefined dimension of functional approximation exercises, our model can be solved very quickly and efficiently. We relegate Section II of the Internet Appendix for additional details on our algorithm and implementation.

Once functional approximations are obtained for aggregate quantities and prices, we move to the simulation stage. For each simulation, we simulate the model for 600 periods, and drop the first 100 periods of simulated data. We then run 100 separate simulations and compute the averages of data moments aggregated across 10,000 firms conditional on sorted portfolios or over time for aggregate results. Finally, we report the aggregate moments, the return spreads, and corporate ratios across portfolios from our model and compare them with our data.

[23] The persistence parameters are pinned down by having $\rho_A = 0.9543^4 = 0.8294$ and $\rho_x = 0.9870^4 = 0.949$, respectively. The standard deviation of the liquidation shocks and that of the productivity shocks can be obtained such that $\sigma_x = 0.0949 \cdot \sqrt{\sum_{j=0}^{q=3} 0.9870^{2j}} = 0.1862$ and $\sigma_A = 0.0144 \cdot \sqrt{\sum_{j=0}^{q=3} 0.9543^{2j}} = 0.0269$.

## 5.4 Aggregate Moments

We first examine the quantitative performance of our model at the aggregate level and document our model's success in matching a wide set of conventional moments in macroeconomic quantities and asset prices. Most importantly, our model delivers a sizable asset durability spread at the aggregate level.

Table 5 reports the key moments of macroeconomic quantities (top panel) and those of asset returns (bottom panel), respectively, and compares them to their counterparts in the data when available. The top panel shows that the model simulated data are broadly consistent with the basic features of the aggregate macro-economy in terms of volatilities, correlations, and persistence of output, consumption, and investment. In sum, our model is as successful as neoclassical growth models in accounting for the dynamics of macroeconomic quantities.

With respect to the asset pricing moments (bottom panel), we make two observations. First, our model is reasonably successful in generating moments related to financial frictions and asset pricing at the aggregate level. In particular, it replicates a low and smooth risk-free rate, with a mean of 1.2% and a volatility of 0.56%. The equity premium and leverage ratio in this economy are 6.98% and 0.44, respectively, and broadly consistent with the empirical target of 5.71% and 0.31 in the data. Our model also delivers large levered and unlevered returns on acquiring durable capital (i.e., 10.88% and 6.18%.) Second, our model confirms that return differences between durable and non-durable capital investment are more pronounced when credit constraint is binding. This holds regardless of whether the durable capital spread is measured using levered returns or unlevered returns. Specifically, expected return differences between durable and non-durable capital investment are 6.94% (levered returns) and 3.07% (unlevered returns), respectively, when the collateral constraint is binding. Though we are unable to directly uncover empirical moments on returns at the asset level from the BEA table, our model later evaluates the asset durability spread against data as observed in the cross-section of stock portfolios.

## 5.5 Model Mechanisms

In this subsection, we numerically evaluate the performance of our quantitative model and further explore the model mechanisms that give rise to the asset durability premium.

We first examine firms' efforts to replace durable capital using non-durable capital when constraints are binding (i.e., substitution of asset durability driven by financial frictions). Our first measure is the capital expense ratio on durable capital goods as a fraction of total capitalization, $Expense_t = \frac{q_{d,t}K_{d,t}}{q_{d,t}K_{d,t}+q_{nd,t}K_{nd,t}}$ with and without a binding constraint. Results from the top panel of Table 6 clearly suggest that firms' balance sheets shift toward more non-durable capital when constraints are binding, which amounts to a reduction of 3.05% of capital expense on durable capital moving from an unconstrained to a constrained situation. Our model solution accommodates a constant share of durable over non-durable ratio in quantities for $K_{d,t} = \zeta K_t$ and $K_{nd,t} = (1-\zeta)K_t$.

Such drops in durable capital expense at the aggregate level are largely reflective of the relative price changes in equilibrium. Therefore, our model clearly shows that the asset durability substitution between durable and non-durable capital is consistent with the model prediction of greater price cyclicality of more durable capital, i.e., our highlighted risk-premium channel. Next, we examine the difference as measured by "excess investment" in durable capital relative to non-durable capital, i.e. $\frac{K_{d,t+1}-K_{nd,t+1}}{K_t} = \zeta\Gamma_t - (1-\zeta)\Gamma_t$. This measure factors out the impact of relative price changes and focuses on quantity substitution. Results from the bottom panel of Table 6 suggest that less capital investment goes to durable capital accumulation if firms are more financially constrained. This loss of investment in durable capital because of tightened constraints amounts to a 2.68% decrease from that under the unconstrained scenario. Therefore, we demonstrate that regardless of how we measure the asset durability substitution, either in nominal terms or real terms, aggregate shocks that bind credit constraints trigger firms' asset holdings toward less expensive and less durable capital.

Second, we specifically examine the risk-premium channel (i.e., equilibrium asset prices for more durable capital goods are more volatile over business cycles), which commands a larger risk premium for holding more durable capital. Table 7 summarizes statistics that indicate that prices of more durable capital goods in our model economy are riskier. Specifically, we first simulate the aggregate time series and simply compute the average standard deviations of log capital prices of both types across model simulations. Our model generates more cyclical durable capital prices (with the standard deviation of 0.169), which are unconditionally more volatile compared to that of non-durable capital (with the standard deviation of 0.075). Next, we examine the source of this large price variability of durable capital by computing the covariance between the weighted stochastic discount factor $\widetilde{M}_{t+1}$ as in equation (28) and next period capital prices $q_{d,t+1}$ and $q_{nd,t+1}$ conditional on time $t$'s information using simulated aggregate data. Our results in the third and fourth rows of the table suggest that durable capital prices are more negatively correlated with the stochastic discount factor (with the covariance of -0.059) as compared to that of the non-durable capital prices (with the covariance of -0.013). Since durable capital exhibits greater risk sensitivities to business cycles, a larger risk premium is associated with holding the durable capital. Finally, we compute the *elasticity* of log differences in capital prices with respect to changes in the liquidation probabilities in logs, by which financial shocks are the primary triggers for credit constraints to be binding. Our model results suggest that, on average, durable capital prices are more responsive to financial shocks (with the correlation of -0.925) compared to those of non-durable capital goods (with the correlation of -0.85). Therefore, across different measures, our model predicts that the price of durable capital can be as much as about three to four times more volatile than that of non-durable capital over business cycles, as measured by unconditional price volatility or by risk sensitivities driven by aggregate shocks. Hence, our quantitative model confirms the importance of our highlighted risk-premium channel in showing that durable capital is riskier in equilibrium.

Next, we evaluate how important our risk-premium channel is to the point that durable capital investment commands a higher expected return in our model. Table 8 presents our model results when we fix capital prices to be constant over time. As we have shown in Subsection 4.2, the risk-premium channel is shut off if we do not allow for variation of capital prices over business cycles. In particular, we fix the capital prices at their respective steady-state values as they are in our baseline model by which $q_d^{ss} > q_{nd}^{ss}$. Hence, relative to our baseline model case, if acquiring durable capital now is not compensated for additional risk premium while durable capital is still more expensive for its larger down payment, then investing in durable capital investment is not an attractive option, for it fails to yield a higher expected return for entrepreneurs. Our counterfactual analysis suggests that expected return spreads between investing in durable relative to non-durable shrink and even turn negative on average when firms are unconstrained. When firms are more constrained, we show from equation (42) in Subsection 4.2 that the impacts of down payments and relative benefits between durable and non-durable capital are less pronounced; therefore, the expected return spread is somewhat less negative. In sum, the risk-premium channel is critically important for a general equilibrium analysis, both qualitatively and quantitatively, as it generates a large expected return spread for investing in durable capital relative to non-durable capital.

Finally, we compute the mean return spread reduction when financial constraint is binding, $\Omega_t = E[\theta\eta_t(\delta_{nd} - \delta_d)/E_t(\tilde{M})]$ as in equation (48), which measures the size of the capital collateralizeability effect that durable capital can be less risky as it provides extra collateral value. The collateralizability differences between type-$d$ and type-$nd$ capital goods, therefore, partially offset the model return spread commanded by acquiring more durable capital. The reduction of return spread matters only when constraints are binding $\eta_t > 0$. When we load our baseline calibration, as shown in Table 5, we find that this effect is small and reduces the return spread by about 23 basis points, which accounts for a tiny share of 3.3% (7.5%) of our total durable spread of 6.94% (3.07%) as measured in levered (unlevered) returns on financing durable capital investment when constraints are binding. Therefore, our quantitative model results suggest that the risk-premium channel dominates the offsetting collateralizeability channel, regardless of the effects driven by leverage ratios.

## 5.6 Impulse Response Functions

We further show that the impacts of our model mechanism on asset pricing can be best illustrated by looking into the model-implied impulse response functions of quantities and prices in response to exogenous aggregate shocks.

In Figure 1, we plot the percentage deviations of quantities and prices from the steady state in response to one-standard-deviation of aggregate productivity shocks for shock period 1 (i.e., the shock to $a$.) over a 20-period horizon. In particular, since our model allows for collateral constraints to be binding only occasionally, the steady state of the shadow value of relaxing the borrowing constraint $\eta_t$ has factored in both binding and non-binding periods. Three observations are summarized as follows. First, a positive shock to $a$ (top panel in the left column) works as a negative discount rate shock to entrepreneurs, and the shock leads to a relaxation of the collateral constraint as reflected by a drop in the Lagrangian multiplier, $\eta$ (top panel in the right column).

Second, relaxed collateral constraints translate into positive growth in the aggregate investment (second panel in the left column). Upon a positive productivity shock, not only does an entrepreneur's net worth jump sharply (third panel in the left column), but the price of type-$d$ capital also increases sharply (second panel in the right column). However, the price of type-$nd$ capital rises with a smaller magnitude, in contrast to the price of type-$d$ capital. This observation suggests that the price of durable capital presents higher risk sensitivities and greater price fluctuations driven by aggregate productivity shocks. The different risk profiles are also reflected in the different responses of the unlevered return on durable capital, $r_d$, and that on non-durable capital, $r_{nd}$ when we factor impacts of leverage changes. The return of type-$d$ capital responds much more to productivity shocks than that of type-$nd$ capital (third panel in the right column). All these findings are consistent with our key model mechanism on asset durability substitution driven by a risk-premium channel.

Lastly and most importantly, we confirm the operation of asset durability substitution in economic expansions, when firms are collectively less constrained, they will prefer "more expensive" durable capital. We show the impulse response of durable capital expenditure as a fraction of total asset (i.e., Expense Ratio of $K_{d,t+1}$), reacting to a positive productivity shock (bottom panel in the left column). It shows that the aggregate acquisition of durable capital across all firms increases relative to share of non-durable capital expenditure. In addition, in terms of the quantities of investment in durable capital in excess of non-durable capital (bottom panel in the right column), we see more investment to support accumulating durable capital relative to non-durable capital when the economy sees positive aggregate productivity shocks. All these impulse responses reflect the key channel on asset durability substitution, which explains why the price and returns on type-$d$ capital increase even more significantly, as shown in the second and third panel in the right column.

Next, we introduce one standard deviation positive shocks to raise the liquidation probability $\lambda_t$. We then present the impulse responses of these key variables of interest to such adverse financial shocks in Figure 2.

First, a positive shock to $x_t$ raises the likelihood of a firm being liquidated $\lambda$ as of the shock period 1 shown in the figure (top panel in the left column). It then works as a positive discount rate shock to entrepreneurs, which leads to a tightening of the collateral constraint, and results in an increase in the Lagrangian multiplier, $\eta$ (top panel in the right column.)

Second, tightened collateral constraints result in slumps both in investment (second panel in the left column) and in entrepreneurs' net worth (third panel in the left column). In addition, the price of type-$d$ capital drops dramatically (second panel in the right column), although the price of type-$nd$ capital tumbles only slightly. We also see drops in both the unlevered return on type-$d$ capital, $r_d$, and that on type-$nd$ capital, $r_{nd}$, although the former has relatively larger decreases (third panel in the right column). Overall, we see durable capital as a riskier asset than non-durable capital by exhibiting larger risk sensitivities in case of a bad financial shock.

Finally, impulse responses of the relative capital expenditure on durable capital and excess investments in durable capital accumulation again confirm capital substitutabilities effects with negative financial shocks. Intuitively, when firms are more constrained in recessions after a bad liquidation shock, they prefer acquiring "cheaper" non-durable capital. The economy starts spending more on non-durable capital relative to a share of the total capital (bottom panel in the left column). The capital accumulation using durable capital in excess of non-durable capital through investment also shrinks (bottom panel in the right column.) All these substitutions also rationalize the different risk profiles of durable and non-durable capital to financial shocks in addition to productivity shocks (the second and third panel in the right column).

In summary, our model-implied impulse response functions of key variables to both aggregate shocks all suggest that returns on type-$d$ capital, $r_d$ respond much stronger than that on type-$nd$ capital, $r_{nd}$, to aggregate shocks by exhibiting larger risk sensitivities. Hence, durable capital is indeed much riskier than non-durable capital over business cycles driven by both types of shocks; therefore, holding durable capital necessarily commands for a greater expected return spread.

## 5.7 Asset Durability Spread

We now examine the implications of our model for the cross-section of asset durability-sorted portfolios. We simulate firms using the model, measure their asset durability, and apply the same portfolio-sorting procedure as in our empirical analysis.[24] Table 9 reports the average returns of the sorted portfolios, along with the median values of several key characteristics from the actual data and the mean values from the simulated data.

Table 9 first reports several other characteristics of the asset durability-sorted portfolios that inform the economic mechanism we emphasize in our model. First, not surprisingly, the asset durability measure is monotonically increasing across asset durability-sorted portfolios.[25] In addition, our model-based portfolios with largest durability and lowest durability exhibit similar depreciation rates, 0.08 and 0.18, respectively, as correspondence to the depreciation rates of portfolios constructed in the data, 0.11 and 0.19, respectively. For each portfolio in between, the model-based depreciation rates are close enough to the data counterparts. This provides important validity of

[24]In the simulation, extremely financially constrained firms may attempt to acquire negative type-$d$ capital by selling expensive capital and purchasing less expensive type-$nd$ capital instead. This scenario could lead to negative accumulated net worth. To maintain consistency with our empirical analysis, we impose the restriction that type-$d$ capital, type-$nd$ capital, and net worth must be strictly positive for all simulated firms. We then perform the univariate portfolio sorting exercise following our empirical approach.

[25]Following the construction of the asset durability measure in Section 2, we define the asset durability in our simulation as the weighted average of the reciprocal of the depreciation rate with respect to durable and non-durable capital:

$$\text{Asset Aurability} = \frac{K_d}{K_d + K_{nd}} \times \delta_d^{-1} + \frac{K_{nd}}{K_d + K_{nd}} \times \delta_{nd}^{-1}. \tag{50}$$

our model for studying the return spread across portfolios sorted by asset durability, even if we are not calibrating our model to target the degree of asset durability for each portfolio. Second, as in the data, leverage is increasing in asset durability. This implication of our model is consistent with the data and the broader corporate finance literature (e.g. Ai, Li, Li, and Schlag (2020a)). However, the dispersion in leverage ratios in our model is slightly larger than in the data. Third, as we show in equation (III.23) in the Internet Appendix, higher idiosyncratic productivity tends to boost firms' production triggering investment demand, therefore tightening their financial constraints. Lacking sufficient net worth to obtain durable assets leads them to prefer less costly, non-durable assets, which reduces their asset durability. We report this firm characteristic in the fourth rows on both panels.[26] Our model again captures well of this empirical fact that firms with larger asset durability are less productive firms.

We next examine the asset durability premium in our model. As in the data, our simulated firms with high asset durability have a significantly higher average return than those with low asset durability. Quantitatively, our model produces a levered and unlevered asset durability spread of 4.36% and 1.32%, respectively. We see that our model rationalizes about 63% (30%) of the return spread differences in the data of 6.93% in levered returns (4.75% in unlevered returns). Taking the average asset durability premium from the data, 5.01%, across the scenarios of using four different financial constraint measures, according to Panels A to D in Table 3. Our model thus accounts for more than 80% of the levered-return based spread in the data. Hence, regardless of measures, our model predicts a sizable and positive asset durability premium.

Given our highlighted model mechanism of asset durability substitution driven by aggregate shocks that tighten credit constraints over business cycles, our model produces sizeable asset durability premium quite well. The size of the premium is determined by the difference in the risk covariance as well as by cyclical properties of prices of durable and non-durable capital. In the cross-section, firms holding more durable capital are necessarily riskier for extra risk exposure and sensitivities; therefore, equity returns on these firms require extra risk compensations.

In addition, we emphasize that our model results shed light on the relationship between the tightness of firms' financial constraints and the expected stock returns. Our model mechanism features the fact that financially constrained firms may choose to acquire more non-durable capital on balance sheet as holding non-durable capital helps hedging against the aggregate uncertainty. These firms therefore appear to be less risky in equilibrium and command lower expected returns. Specifically, these constrained firms are not only less risky than those constrained firms who hold more durable capital, but also less risky as compared to those unconstrained firms with larger durable over non-durable capital ratio. Hence, our model insights may help ease the overly strong prediction of a positive relationship between financial constraints and the exposure to aggregate shocks as predicted by standard theories (e.g. Whited and Wu (2006), Buehlmaier and Whited (2018), and more recently Nikolov, Schmid, and Steri (2021)).

[26] Following Ai, Croce, and Li (2012), we estimate firm-level productivity from Compustat.

## 5.8 Cash Flow Duration

Next, we explore the relationship between cash flow duration and asset durability both in the data and in our quantitative model. Importantly, Weber (2018), Gonçalves (2021), and Gormsen and Lazarus (2023) show that firms with longer cash flow duration earn lower average returns than those with shorter duration. We demonstrate that our quantitative model not only helps explain the asset duration premium among financially constrained firms but also generates a negative relationship between cash flow duration and asset durability across duration-sorted portfolios – even when financial constraints are not binding.

Table 10 reports average cash flow duration across asset durability-sorted portfolios for both actual data (Panel A) and model-simulated data (Panel B). In Panel A, we calculate cash flow duration following Dechow, Sloan, and Soliman (2004) and Weber (2018) for financially constrained firms in the data. We find that firms with more durable capital exhibit shorter cash flow duration. Specifically, moving from the portfolio with the least durable assets (Low) to the most durable (High), median cash flow duration declines from 20 years to 19 years. This negative relationship between asset durability and cash flow duration is consistent with the downward-sloping term structure of equity returns documented for financially constrained firms.

In Panel B, we compute cash flow duration using our model's simulated data. For each firm, cash flow in period $t$ is defined as the liquidated net worth net of the average payout to replacement firms, as specified in equations (7), (8), and (9):

$$\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}^d, K_{i,t}^{nd}\right) + (1-\delta_d)\, q_{d,t} K_{i,t}^d + (1-\delta_{nd})\, q_{nd,t} K_{i,t}^{nd} - \chi S_t. \tag{51}$$

Our baseline measure uses a constant discount rate of 8% to match the methodology in Dechow et al. (2004).[27] When financial constraints are binding, the model replicates the negative relationship observed in the data with remarkably close magnitudes: cash flow duration declines from approximately 20 years for firms with the least durable assets to 18 years for those with the most durable assets, closely matching the 20-to-19-year decline in Panel A. When constraints are not binding, cash flows become more persistent across all portfolios, increasing duration levels, as firms in unconstrained periods are better able to smooth shocks intertemporally, shifting cash flows to the distant future. Nonetheless, the downward-sloping pattern across asset durability portfolios remains.

One may reason that more durable assets could sustain more persistent cash flows, which would push cash flows to the far end of the equity term structure. However, both our calculations using actual data and simulated data suggest the opposite. According to our model, the key reason hinges on the role that non-depreciated capital value plays in the cash flow duration measure (i.e., $(1-\delta_d)\, q_{d,t} K_{i,t}^d + (1-\delta_{nd})\, q_{nd,t} K_{i,t}^{nd}$.) Specifically, occasionally binding constraints driven by

[27] Given our model's average risk premium of roughly 6% and aggregate growth rate of 2%, we choose 8% as a constant cost of capital to best correspond to the implied equity premium of 6%, given a 12% cost of capital and 6% growth rate as in Dechow et al. (2004).

financial frictions in our model generate greater price procyclicality of durable assets relative to non-durable assets in equilibrium. In good states with more positive aggregate productivity shocks and favorable financial shocks, $q_{d,t}$ surges more relative to $q_{nd,t}$ because firms invest in durable assets more sharply thanks to the asset substitution effect. Non-depreciated capital value $(1-\delta_d)\, q_{d,t} K^d_{i,t}$ therefore loads more weight on contemporaneous cash flows for $\delta_d < \delta_{nd}$ among firms with more durable assets (larger $K^d_{i,t}$ relative to $K^{nd}_{i,t}$). In bad states with negative productivity shocks and adverse financial shocks, $q_{d,t}$ drops more relative to $q_{nd,t}$, leading to a greater shortage of non-depreciated capital value, which again loads more weight on contemporaneous cash flows. All these effects together mute the propagation of shocks, either good or bad, into future durable capital asset accumulation, thus truncating the far-horizon tail of cash flow streams for firms with more durable assets. Especially under collateral constraints, the bad states prevent $K^d_{i,t}$ from scaling up the far-horizon leg, which further reinforces the larger near-term effects for cash flow duration. To see this point more clearly, we further recompute cash flow duration using only the flow profits (i.e., $\Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K^d_{i,t+1}, K^{nd}_{i,t+1}\right)$), without considering the non-depreciated capital value and the reinvestment to new firms, which are independent of the asset durability concern. Results in Panel B suggest that the model-implied cash flow duration now becomes positively sloped with asset durability across portfolios, contrasting with the data and our benchmark results.

To further establish the robustness of our results, we compute the exact cash flow duration as implied by our model. Instead of using constant discounting as in the empirical literature, we discount the cash flow stream using our endogenous SDF and report the results in Panel B. Particularly in our model, cash flows are low during bad times when the marginal utility of consumption and thus the SDF is high, generating a negative covariance between the SDF and cash flows. Our exact cash flow duration measures preserve the main finding: firms with more durable assets exhibit shorter cash flow duration. Firms with the lowest asset durability are associated with an average cash flow duration of about 11 years, whereas those with the highest asset durability exhibit a duration of about 10 years. Regarding the reduction in absolute magnitudes relative to the constant discount rate case, we provide additional analytical results in Section I.1.2 of the Internet Appendix showing that cash flow duration is shortened when the model-implied negative covariance between the SDF and cash flows is properly accounted for in the calculation.

# 6 Empirical Analysis

In this section, we present direct evidence supporting our model's mechanisms and predictions. First, we construct a model-implied financial shock series and examine its impacts on the relative changes in quantities and prices between durable and non-durable capital, thereby identifying the risk-premium channel predicted by our general equilibrium model. Second, we employ a GMM test to demonstrate that financial shocks are negatively priced in the cross-section of test asset returns. When combined with the decreasing exposure of asset-durability-sorted portfolios to this risk factor, these results clearly confirm the underlying mechanism of the risk premium channel. Finally, we demonstrate that the positive relationship between asset durability and returns cannot be explained by various alternative explanations proposed in the existing literature.

## 6.1 Estimation of the Financial Shocks

Our model suggests that changes in the financial friction trigger compositional changes in firms' asset durability over business cycles, leading to the price cyclicality differences between durable and non-durable capital. Our empirical analysis aims to uncover from the data such mechanism by which shocks affecting the financial frictions are shifting the prices and quantities of asset durability changes. In our model, we study the shocks that affect the probability of liquidation of entrepreneurs by affecting their discount rates and their financing and capital decisions (i.e., financial shocks.) Because the direct empirical correspondence of such liquidation shocks is not readily available in the data, our empirical strategy is to exploit the structure of our model to identify the financial shock proxy from the data.

Following Eisfeldt and Muir (2016) and Belo, Lin, and Yang (2018), we project the structural financial shocks in our model onto a set of moments-which capture most of the variations driven by financial shocks and are also measurable in the data—using a multivariate linear regression. We then use the estimated regression coefficients along with the corresponding empirical moments at each date, to construct the fitted values of the projection and obtain our identified financial shocks, denoted as $\varepsilon_{x,t}^{\text{Data}}$.[28]

We take two moments for the projection exercises – the changes in the aggregate debt-to-net-worth ratio, denoted by $\Delta\text{BN}$, and the spread between the shadow interest rate and the risk-free rate, denoted by $\Delta_{f,t}$ – and estimate the following regression using simulated data generated by our model

$$\varepsilon_{x,t} = \beta_1 \times \widetilde{\Delta\text{BN}}_t + \beta_2 \times \widetilde{\Delta}_{f,t} + u_t, \tag{52}$$

where $\widetilde{\Delta\text{BN}}_t$ and $\widetilde{\Delta}_{f,t}$ denote the standardized moments, each with a mean of zero and a standard deviation of one. Our regression results suggest that these two moments account for most of the variation in the financial shocks within the model as determined by an $R^2$ around 78%.

Specifically, we estimate equation (52) using simulated data for 100 times with each simulation path at a time. We then take the averaged estimated coefficients $\beta_1$ and $\beta_2$ across all simulation, obtaining the estimated slope coefficients $\beta_1 = 0.98$ and $\beta_2 = 0.09$. Intuitively, the positive slope coefficient on the change in the debt-to-net-worth ratio reflects that large liquidation shocks $\varepsilon_{x,t}$ are correlated with increased aggregate leverage. That is, an increase in the liquidation probability leads to a decline in aggregate net worth, as illustrated in equation (9), significantly tightening borrowing constraints. This tightening occurs because some firms are unexpectedly liquidated

[28]The structural innovation to financial shocks, denoted as $\varepsilon_x$, is specified in Section 5.1. By the regression specification per equation (52), we obtain the fitted value of financial shocks as $\varepsilon_{x,t}^{\text{Data}}$, which serves as the model-implied financial shocks.

without engaging in production, while those firms without liquidation still reduce their production, given the negative correlation between financial shocks and productivity shocks. Consequently, aggregate net worth declines, while the debt-to-net-worth ratio rises. In addition, the positively estimated coefficient on $\widetilde{\Delta}_{f,t}$ suggests that tightened borrowing constraints also have a price impact on the cost of borrowing.

Taking the two corresponding empirical moments related to the U.S. economy, we finally obtain a time series of annual frequency to proxy for the financial shocks covering years from 1960 to 2016. The empirical moment $\Delta \text{BN}_t$ is constructed using the aggregate debt-to-equity ratio, while the spread $\Delta_{f,t}$ is measured by the yield spread between Moody's Baa and Aaa corporate bond yields, obtained from the Federal Reserve Economic Data (FRED).

We then plot the constructed model-implied financial shocks and the time series of the two empirical moments in the two panels of Figure 3. Across the panels, we see in the plots that our constructed financial shocks spiked while the debt-to-net-worth ratio and the yield spread also jumped during recessions. In addition, it is clear that financial innovations do not have to be always coupled with economic downturns as financial shocks can well increase outside the years of economic recession. For instance, sharp increases in $\varepsilon_{x,t}^{\text{Data}}$ occur in 1978, 1985, and 1988, despite these years not being classified as recessions. Taking together, we find our model-implied financial shocks are generally counter-cyclical. Importantly, we note firms' financial conditions are not perfectly aligned with the business cycles, supporting our modeling approach in which aggregate economic conditions in our model are driven by both financial and productivity shocks and they are not perfectly correlated.

## 6.2 Financial Frictions, Price Cyclicality, and Asset Durability

In this subsection, we use the model-implied financial shocks constructed in Section 6.1 to examine the effects of changes in financial frictions on relative changes in quantities and prices between durable and non-durable capital. We show our evidence is consistent with the risk-premium channel that drives the capital substitutability as our key model mechanism in general equilibrium as highlighted in Section 5.5.

Our model predicts that firms, especially those more financially constrained, prefer cheaper and less durable assets during recessions. To validate this prediction, we first calculate the cross-sectional averages of asset durability for the constrained and unconstrained firms each year, according to firms' dividend payment dummy (DIV). We then examine the reactions of asset durability to our model-implied financial shocks, by lagging the shock series by one year and controlling for other firm characteristics, per the following regression:

$$\text{Durability}_{h,t+1} = a + b \times \varepsilon_{x,t}^{\text{Data}} + c \times \text{Controls}_{h,t} + u_{h,t},\ h \in \{\text{Const.}, \text{Unconst.}\} \tag{53}$$

As shown in Table 11, we present the estimation results of equation (53) for groups of financially constrained and unconstrained firms, respectively. The estimated coefficient $b$ is $-0.16$ for the constrained group in Column 1. This finding directly supports our primary model mechanism: the durability of financially constrained firms decreases with realized adverse financial shock, indicating a preference for cheaper, less durable assets when borrowing constraints are binding. This result also aligns with our quantitative prediction in Table 6 of Section 5.5.

In Column 2, the slightly positive but insignificant coefficient in the unconstrained group reflects the interplay between the capital substitution effect and the wealth effect in the data. On the one hand, the substitution effect suggests the presence of capital reallocation such that unconstrained firms would increase their holdings of durable capital by acquiring it from their financially constrained firms. On the other hand, the wealth effect also operates and implies that the realization of bad financial shocks pushes the economy into a recession, leading both constrained and unconstrained firms to reduce their holdings of durable capital. The coefficient in Column 3 indicates a significantly negative change in durability measured by the between-group differences in response to financial shocks. Taken together, these opposing forces (e.g., the substitution and wealth effects) offset each other, which explains the small and statistically insignificant coefficient observed for the unconstrained group in the second column of Table 11.

We then take the log price differences for each asset in the Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA) tables while classify the detailed assets into high- and low-durability groups according to their durability scores, as in Table 1 in Section 2.1. We finally construct the time series of capital price changes by averaging the price changes within each group.[29] We run the following regression for each asset durability group to estimate the sensitivity of its price changes with respect to the model-implied financial shock series in addition to the market factor:

$$\Delta \text{Log } q_{h,t+1} = a_q + b_1 \times \widetilde{\text{MKT}}_t + b_2 \times \varepsilon_{x,t}^{\text{Data}} + u_{h,t}, \ h \in \{H, L\}, \tag{54}$$

where $\Delta \text{Log } q_{h,t+1}$ denotes the log price changes in the high or low asset durability group. $\widetilde{\text{MKT}}_t$ represents the market factor that is orthogonal to the financial shocks, and $\varepsilon_{x,t}^{\text{Data}}$ stands for our model-implied financial shocks.[30]

We present the estimation results in Table 12. In Column H, we find a negative and statistically significant coefficient associated with the financial shocks term when price changes are associated with assets in the high-asset-durability group. In contrast, Column L shows an insignificant and

[29]For further insights into price indexes related to structures, equipment, and intellectual property products, we refer to NIPA Table 5.4.4, 5.5.4, and 5.6.4 (https://apps.bea.gov/iTable/iTable.cfm?reqid=19&step=2).

[30]According to our model, the total market factor (i.e., the market risk premium) is confounded by variations driven by both the aggregate productivity and financial shocks. To best connect the data and our model, we isolate the component from the market factor associated with the aggregate productivity shocks only, by projecting the market factor onto the model-implied financial shocks $\varepsilon_{x,t}^{\text{Data}}$. The residual denoted by $\widetilde{\text{MKT}}_t$ therefore captures the variation orthogonal to financial shocks.

slightly positive coefficient for price changes in the low-asset-durability group. These results provide direct empirical support for our model's prediction: when facing financial shocks, financially constrained firms reallocate their asset holdings toward cheaper, less durable capital. Capital reallocation subject to financial frictions then leads to a larger price decline for durable capital relative to less durable capital in equilibrium. In contrast, the preferred less durable capital exhibits smaller price fluctuations, making it less sensitive to financial shocks. Moreover, our empirical findings are consistent with our quantitative prediction regarding the price cyclicality and the underlying risk premium channel, as demonstrated in Table 7 of Section 5.5, which shows that the price of durable capital is more correlated with the stochastic discount factor and exhibits greater price cyclicality.

## 6.3 Market Price and Exposure of Macroeconomic Shocks

In this subsection, we further explore a series of testable implications derived from our model. First, we apply the generalized method of moments (GMM) test to demonstrate that the model-implied financial shocks manifest a negative price of risk within the cross-section of asset-durability-sorted portfolios. This result is well aligned with the model prediction presented in Section 5.6. Second, we employ additional asset pricing tests using real and the simulated data from our model, and show that both the price of risk and the risk exposure of asset-durability-sorted portfolios exhibit a pattern consistent between data and our model.

Specifically, we examine the risk exposure of asset-durability-sorted portfolios to model-implied financial shocks using a two-factor asset pricing framework motivated by our general equilibrium model, in which financial shocks are a key source of systematic risk. Following Cochrane (2005), we specify a stochastic discount factor (SDF) to evaluate the pricing of financial shocks by identifying the sources of risk that affect investors' marginal utility. To this end, we consider two versions of the SDF: one based on the data and the other derived from our model.

**SDF (Data):**

$$\mathrm{SDF}_t = 1 - b_M \times \widetilde{\mathrm{MKT}}_t - b_x \times \varepsilon_{x,t}^{\mathrm{Data}}, \tag{55}$$

where $\widetilde{\mathrm{MKT}}_t$ represents the market factor that is orthogonal to financial shocks and $\varepsilon_{x,t}^{\mathrm{Data}}$ denotes our model-implied financial shocks.

**SDF (Model):**

$$\mathrm{SDF}_t = 1 - b_A \times \widetilde{\varepsilon}_{A,t} - b_x \times \widetilde{\varepsilon}_{x,t}, \tag{56}$$

where $\widetilde{\varepsilon}_{A,t}$ and $\widetilde{\varepsilon}_{x,t}$ are the productivity and financial shocks in the model, respectively, which have been orthogonalized through a structural decomposition.[31]

In both settings, we focus on estimating the price of risk associated with financial shocks,

[31] As in Section 5.1, structural shocks to aggregate productivity and the probability of liquidation ($\varepsilon_{A,t}$ and $\varepsilon_{x,t}$) are correlated and their correlation is captured by $\rho_{A,x}$. To align the estimation of using model simulation with that of using the actual data, we therefore recover the productivity shocks that are orthogonal to financial shocks, $\widetilde{\varepsilon}_{A,t}$, which is to best map our measure to the market factor that is orthogonal to the

captured by the coefficient $b_x$. A significantly negative estimate of $b_x$ would support the hypothesis that the model-implied financial shock is indeed a well-priced risk factor in the cross-section of asset returns.

To assess $b_x$, we employ two sets of test assets in the data and the simulation. In the data, we use six asset-durability-sorted portfolios (as presented in Table 3), six size-momentum portfolios, and five industry portfolios. In the model simulations, we use six asset-durability-sorted portfolios (as presented in Table 9), five book-to-market-sorted portfolios, and five leverage-sorted portfolios.[32] Subsequently, we estimate the generalized method of moments (GMM) using the following set of moment conditions:[33]

$$\mathrm{E}[R_i^e] = -\mathrm{Cov}(\mathrm{SDF}_t, R_i^e), \tag{57}$$

which is the empirical equivalent to the Euler equation of our model, but with the conditional moments replaced by their unconditional counterparts. We essentially assess the ability of the financial shocks (i.e., $\varepsilon_{x,t}^{\mathrm{Data}}$ in the data and $\widetilde{\varepsilon}_{x,t}$ in the model) to price test assets based on residuals of the Euler equation.

Moreover, we adhere to practices in the literature, such as those outlined in Papanikolaou (2011), Eisfeldt and Papanikolaou (2013), and Kogan and Papanikolaou (2014), to compute two statistics that facilitate cross-sectional fitting. These statistics encompass the sum of squared errors (SSQE) and the mean absolute percent errors (MAPE). Additionally, we calculate the $J$-statistic for the overidentifying restrictions of our model. An insignificantly low $J$-statistic implies the non-rejection of the null hypothesis of zero pricing errors.

In Panel A of Table 13, we tabulate the estimation results of the fitted CAPM (Specification 1) and our two-factor SDF model (Specification 2). In Specification 1, we isolate the price of risk of the market factor, which is notably significant. After we incorporate the market factor with the financial shocks in Specification 2 as our reference, we observe that the price of the financial shocks is negative -0.53 and statistically significant at the 1% level.

To assess the asset pricing errors, the CAPM results exhibit SSQE and MAPE values of 5.70% and 5.13%, respectively. Upon the introduction of the financial shocks as the second source of risk to our model in Specification 2, these figures decrease to 5.64% and 5.11%, respectively. Despite the statistically insignificant outcome of the $J$-test in the CAPM model, we find that including the financial shocks effectively enhances model fitness by reducing the pricing errors. Notably, the $J$T

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model-implied financial shocks, $\widetilde{\mathrm{MKT}}_t$. We do the following decomposition:

$$\begin{aligned} \widetilde{\varepsilon}_{x,t} &= \varepsilon_{x,t} \\ \widetilde{\varepsilon}_{A,t} &= \frac{\varepsilon_{A,t} - \rho_{A,x}\widetilde{\varepsilon}_{x,t}}{\sqrt{1-\rho_{A,x}^2}} \end{aligned}$$

[32]This selection of test assets is in line with Belo, Li, Lin, and Zhao (2017) and Lin, Palazzo, and Yang (2020).

[33]For detailed insights into moment conditions, please refer to Table 13.

difference test reveals statistical significance between the CAPM model and our two-factor model in Specifications 2.

Based on the estimation results using simulated data as shown in Specification 3 of Panel A, the productivity shock carries a positive price of risk, while the financial shock exhibits a negative price of risk. Both estimates are statistically significant at the 5% significance level or better. The evidence on the price of risk is further validated by our model simulations.

In addition to the price of risk, the upper panel of Panel B in Table 13 presents the estimated risk exposures (GMM-implied betas) of asset-durability-sorted portfolios to the two factors, along with the corresponding GMM-implied alphas in the data. We find that the exposure to the market factor ($\beta^i_{\text{MKT}}$) displays a relatively flat pattern across the portfolios.[34] More importantly, we observe that the risk exposure to model-implied financial shocks ($\beta^i_{x,\text{Data}}$) exhibits a clear and significant declining trend from the low- to high-asset-durability portfolios. This pattern highlights an increasing covariance with the negatively priced financial shocks as asset durability rises.

We observe a similar pattern in the simulation results, shown in the lower panel of Panel B. Specifically, the exposure to productivity shocks ($\beta^i_A$) increases, while the exposure to financial shocks ($\beta^i_x$) decreases from the low- to high-asset-durability portfolios. These trends are consistent with our empirical findings and demonstrate that, both in the data and in the model, the variation in expected returns is driven by an increasing covariance with negatively priced financial shocks as asset durability rises.

## 6.4 Additional Empirical Asset Pricing Tests

Finally, we provide additional evidence to establish the robustness of our results using alternative measures and rule out other explanations that may potentially drive the results on asset pricing related to the asset durability.

First, instead of using our model-implied financial shocks for the asset pricing tests, we take alternative measures of financial shocks for estimating the two-factor model using GMM and for estimating the risk-exposure (beta) to the two sources of aggregate risk along the asset durability-sorted portfolios. In Table IA.1 of Internet Appendix, we summarize the estimation results using the the yield spread between Moody's Baa and Aaa corporate bond yields and the GZ spread (Gilchrist and Zakrajšek, 2012) to denote the financial shocks individually. We again confirm the robustness of our results that financial shocks are negatively priced and the high-durability portfolio is more negatively exposed to the financial shocks.

Second, we examine the degree to which the variability in asset durability for return predictability can be accounted for by conventional risk factors or certain firm characteristics. We execute an array of asset pricing factor assessments and report the results in Table IA.2 in our Internet Appendix. We find that the observed positive relationship between asset durability and returns

[34] We modify the code of Kan, Robotti, and Shanken (2013) to compute the alphas and $t$-statistics for the test assets based on Chapter 12 of Cochrane (2005).

remains largely unaltered even if we include other systematic risks and firm characteristics. Our findings underscore that the dispersion of returns across portfolios sorted on asset durability cannot be explained by risk exposures to other risk factors. Notably, the alphas in the long-short portfolio retain their statistical significance.

Third, to further explore the asset-durability-return relationship, we employ Fama and MacBeth (1973) regressions as outlined in Section I.2.3 of our Internet Appendix. We do so to rule out potential alternative explanations. The outcomes of these Fama-Macbeth regressions closely mirror our previous findings, particularly when we arrange portfolios based on asset durability. As shown in Table IA.3, the asset durability continues to significantly and positively predict future stock returns. Most notably, this predictability remains robust even in the presence of established predictors for stock returns that are found in the literature, e.g., firms' leverage and the collateralizeability value of assets.

# 7 Conclusion

Durable capital is more expensive to finance not only for its greater down payment but also for its larger price risk sensitivities to financial frictions. We highlight a general equilibrium price effect that has critically important asset pricing implications for understanding firms' equity risk due to asset durability, especially when financial constraints matter. With a novel metric to gauge asset durability based on firms' assets, we document a substantial return differential of 5% annually between firms with high asset durability and those with low asset durability when firms are financially constrained. Considering firms' dynamic capital choices between choosing durable and non-durable capital, we develop a general equilibrium asset pricing model incorporating heterogeneous firms and occasionally binding collateral constraints. Tightened financial constraints trigger firms to opt for reduced position in holding durable capital, which helps alleviate collateral constraints under adverse aggregate shocks. When the model is solved globally by allowing for occasionally binding constraints, our model uncovers an quantitatively important risk-premium channel in general equilibrium such that durable asset prices exhibit greater cyclicality and larger risk exposure to aggregate risk. In the cross-section, firms with larger asset durability are therefore more exposed to financial frictions driven by aggregate shocks and earn higher expected returns. Our model generates substantial variation in risk exposure across asset durability-sorted portfolios, which helps explain the asset durability premium observed among financially constrained firms. We further demonstrate that our model produces a negative relationship between cash flow duration and asset durability, consistent with the empirical evidence of a downward-sloping term structure of equity returns.

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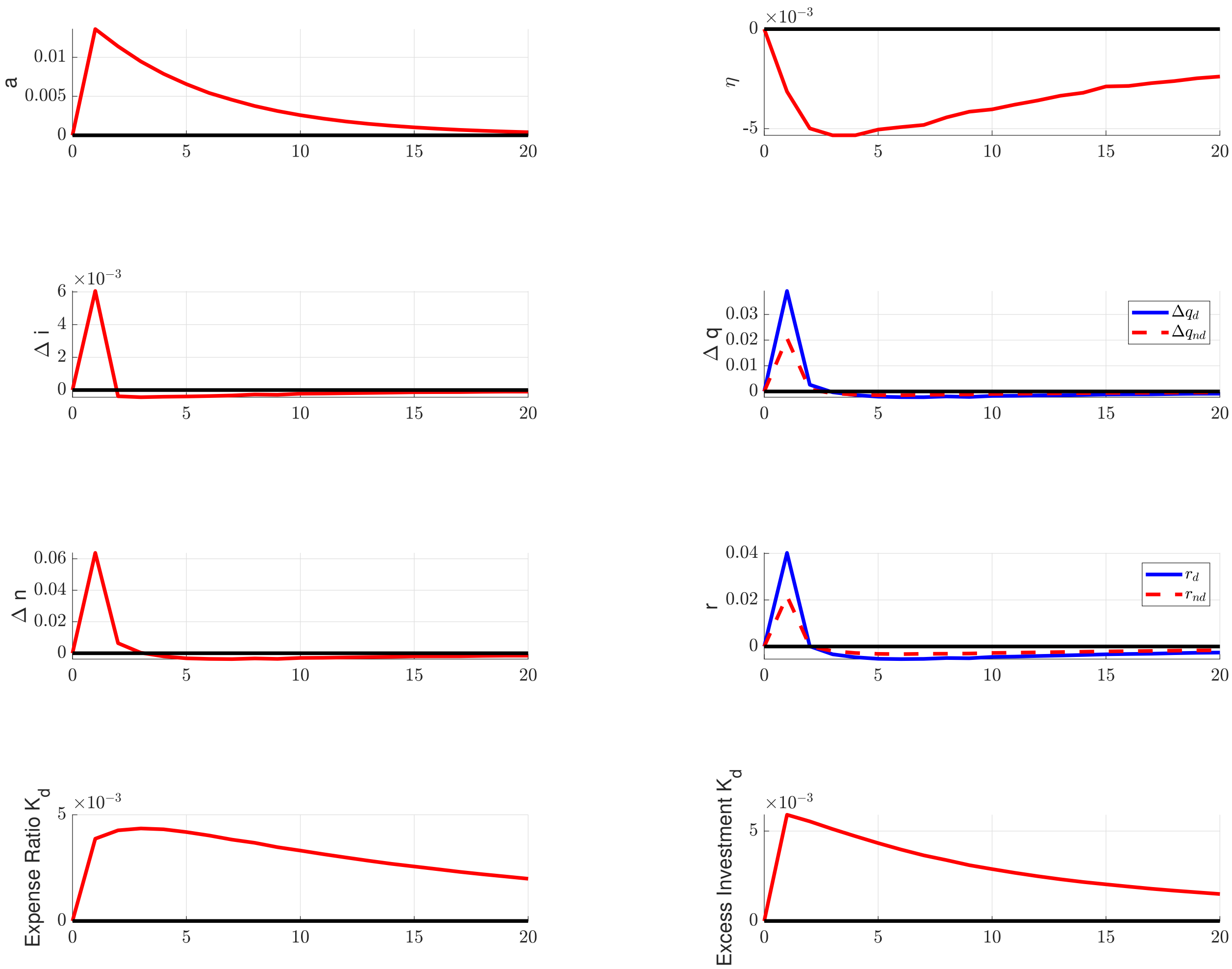
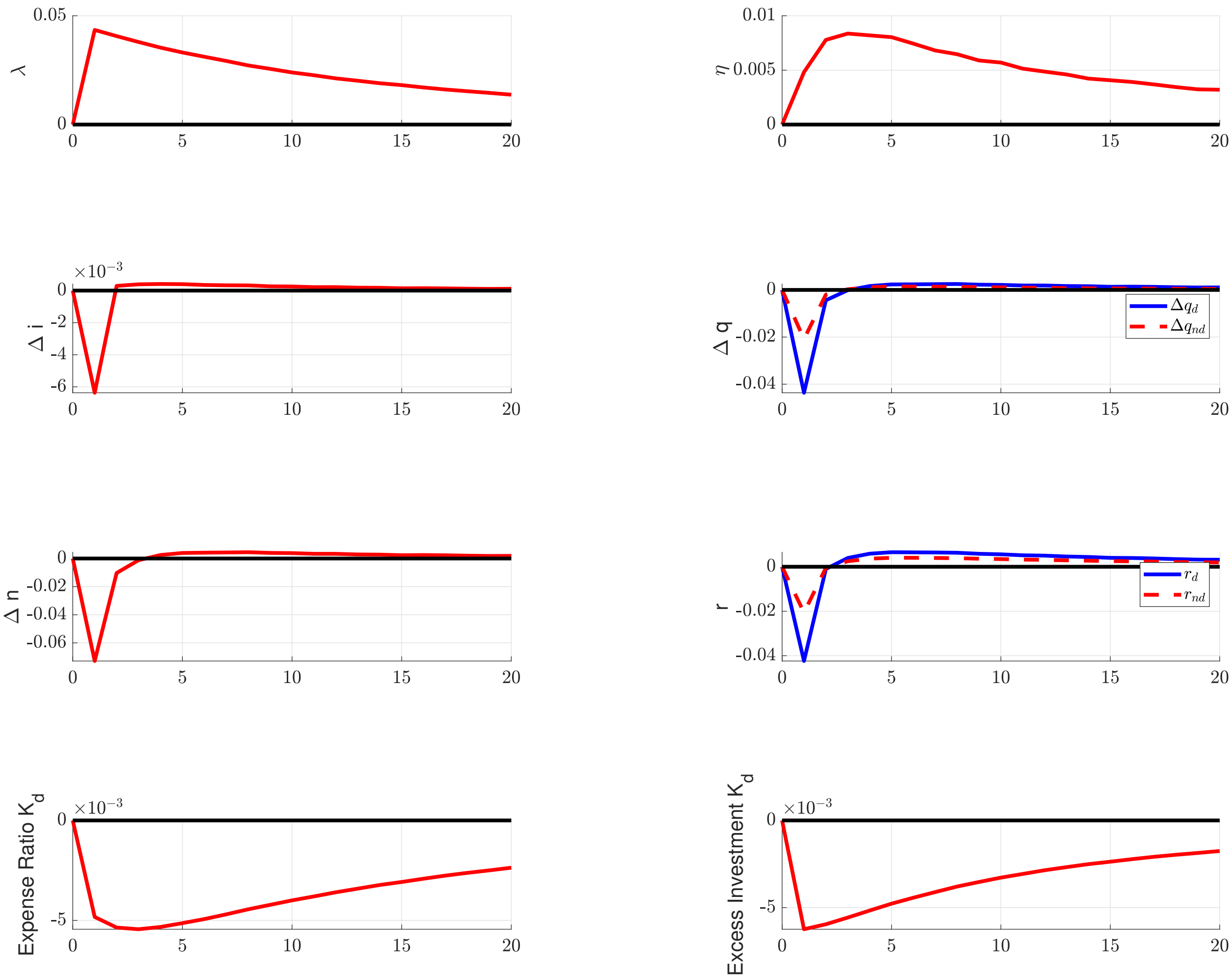
## Figure 1. Impulse Responses to 1 S.D. Productivity Shock

This figure plots the log-deviations from the steady state for quantities and prices associated with a one-standard-deviation positive shock to $a_t$ in period 1. One period is a year. All parameters are calibrated as in Table 4.



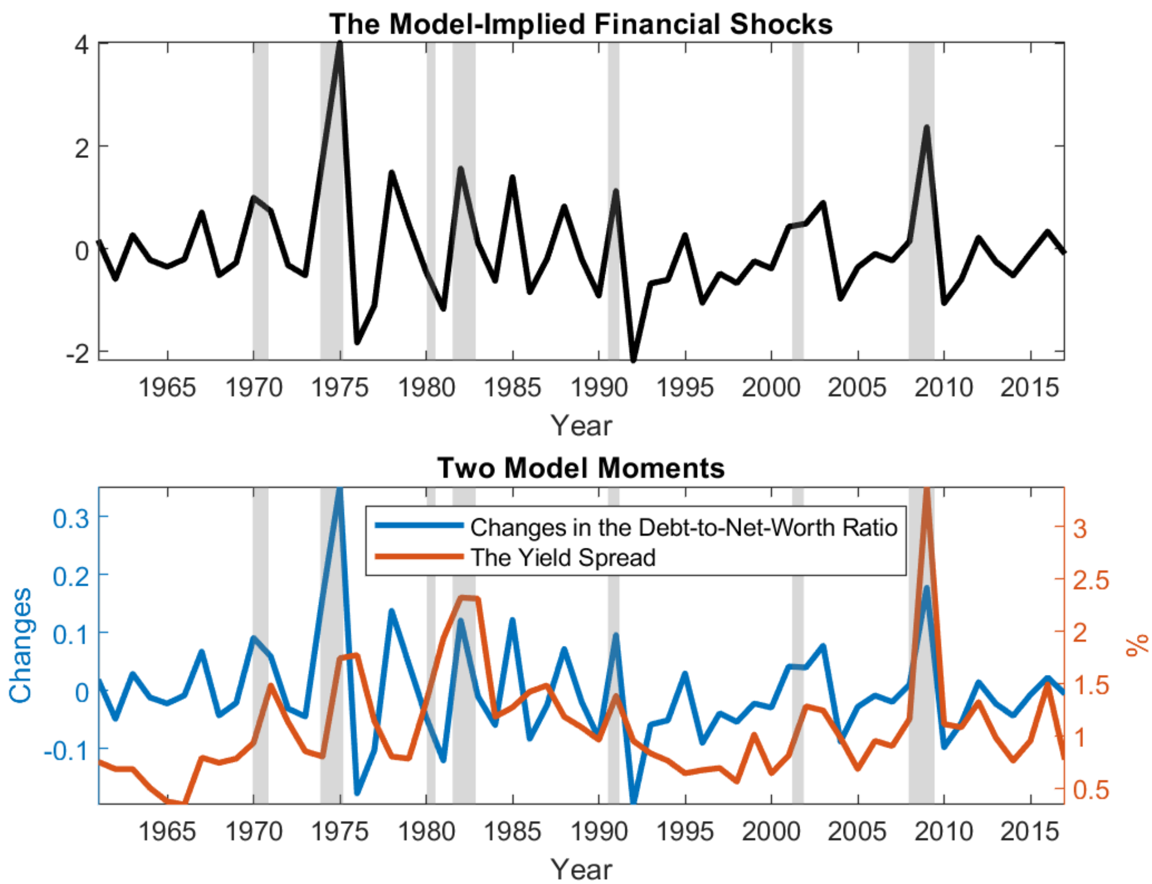
## Figure 2. Impulse Responses to 1 S.D. Financial Shock

This figure plots the log-deviations from the steady state for quantities and prices associated with a one-standard-deviation positive shock to $x_t$ in period 1. One period is a year. All parameters are calibrated as in Table 4.



## Figure 3. Model-implied Financial Shocks

This figure reports the time series of the model-implied financial shocks (top panel) and the time series of the change in the debt-to-net-worth ratio and the yield spread between Moody's Baa and Aaa corporate bond yields (bottom panel). Shaded bars represent NBER recession years. Data are annual from 1960 to 2016.



## Table 1: Durability and Financial Constraints

This table presents the regression coefficients of asset durability on different financial constraints along with coefficients associated with other controls if any:

$$\text{Asset Durability}_{i,t} = a + b \times \text{Financial Constraint}_{i,t} + c \times \text{Controls}_{i,t} + u_{it}.$$

We provide the definitions of variables in Table IA.6 of the Internet Appendix. All independent variables possess a mean of zero and a standard deviation of one, following winsorization at the 1st and 99th percentiles of their empirical distribution. Our reported $t$-statistics in parentheses are based on standard errors clustered at the firm level. Our sample omits firms of utility, financial, public administrative, and public administrative industries, and covers the period from 1977 to 2016.

| **Variables** | **(1)** | **(2)** | **(3)** | **(4)** | **(5)** | **(6)** | **(7)** |
|---|---|---|---|---|---|---|---|
| Non-DIV | -0.10 | | | | -0.10 | | |
| [t] | -1.92 | | | | -1.91 | | |
| SA | | -1.41 | | | | -1.34 | |
| [t] | | -14.28 | | | | -13.20 | |
| WW | | | -0.32 | | | | -0.38 |
| [t] | | | -4.23 | | | | -5.52 |
| ROA | | | | 0.14 | 0.31 | 0.24 | 0.30 |
| [t] | | | | 6.57 | 14.14 | 10.81 | 12.92 |
| Log ME | | | | | -0.02 | -0.52 | -0.31 |
| [t] | | | | | -0.34 | -8.15 | -5.08 |
| Log B/M | | | | | 0.14 | -0.07 | -0.01 |
| [t] | | | | | 5.36 | -2.48 | -0.59 |
| I/K | | | | | -0.22 | -0.23 | -0.17 |
| [t] | | | | | -12.28 | -12.58 | -9.05 |
| Book Lev. | | | | | 0.31 | -0.39 | -0.14 |
| [t] | | | | | 1.88 | -2.40 | -0.84 |
| Cash/AT | | | | | 0.43 | 0.44 | 0.45 |
| [t] | | | | | 13.98 | 14.12 | 14.29 |
| Redp | | | | | 0.04 | 0.02 | 0.02 |
| [t] | | | | | 0.24 | 0.17 | 0.13 |
| TANT | | | | | 3.01 | 3.00 | 2.93 |
| [t] | | | | | 38.07 | 38.38 | 36.06 |
| Observations | 133,830 | 133,830 | 123,654 | 133,685 | 99,292 | 99,292 | 94,299 |
| R-squared | 0.87 | 0.88 | 0.88 | 0.87 | 0.91 | 0.91 | 0.92 |
| Controls | No | No | No | No | Yes | Yes | Yes |
| Time FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Firm FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Cluster SE | Yes | Yes | Yes | Yes | Yes | Yes | Yes |

## Table 2: Summary Statistics

This table provides a comprehensive overview of the summary statistics pertaining to both the main outcome variables and control variables within our sample. The precise definitions of asset durability and depreciation measures are outlined in Section 2.1. Panel A presents the summary of two sub-samples of firms, i.e., financially constrained and unconstrained ones based on a dividend payment dummy (DIV), as classified by Farre-Mensa and Ljungqvist (2016), at the end of each June. We show the pooled means of these variables, weighted by firm market capitalization at fiscal year-end. Panel B showcases time-series averages representing the cross-sectional median of firm characteristics within constrained firms. These firms are segmented into five portfolios based on their asset durability relative to industry peers. We use NAICS 3-digit industry classifications to carry out our categorizations. Further definitions of our variables can be found in Table IA.6 of the Internet Appendix. Our sample spans the period from 1977 to 2016, and excludes firms of financial, utility, and public administrative sectors.

| | **Panel A: Pooled Statistics** | | **Panel B: Firm Characteristics** | | | | |
|---|---|---|---|---|---|---|---|
| | **Const.** | **Unconst.** | **Portfolios** | | | | |
| **Variables** | **Mean** | | **L** | **2** | **3** | **4** | **H** |
| Durability | 12.66 | 16.54 | 7.69 | 9.99 | 11.45 | 14.24 | 18.00 |
| Depreciation | 0.17 | 0.13 | 0.19 | 0.16 | 0.15 | 0.13 | 0.11 |
| Book Lev. | 0.24 | 0.33 | 0.13 | 0.19 | 0.21 | 0.28 | 0.32 |

## Table 3: Portfolios Sorted on Asset Durability

This table shows average excess returns for five portfolios sorted on asset durability across firms relative to their industry peers. To obtain these results, we use NAICS 3-digit industry classifications and rebalance portfolios at the end of every June. Our results reflect monthly data from July 1978 to December 2017 and exclude firms of utility, financial, public administrative, and public administrative industries. We split the whole sample into financially constrained and unconstrained subsamples at the end of every June, as classified by the dividend payment dummy, the SA index, and the WW index. We report average levered and unlevered excess returns over the risk-free rate E[R]-$R_f$, standard deviations Std, as well as the Sharpe ratios SR across five portfolios in constrained subsamples (Panel A) and in the whole sample (Panel B). The unlevered return of a firm is defined as its levered return multiplied by one minus its leverage ratio. See further details per equation (46) in Section 4.3. We estimate standard errors by using the Newey-West correction. We also include t-statistics in parentheses and annualize portfolio returns by multiplying by 12. All returns, standard deviations, and Sharpe ratios have been annualized.

| | **Levered Returns** | | | | | | **Unlevered Returns** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Panel A: Constrained Subsample** | | | | | | | | | | | |
| | **L** | **2** | **3** | **4** | **H** | **H-L** | **L** | **2** | **3** | **4** | **H** | **H-L** |
| | **DIV** | | | | | | | | | | | |
| E[R]-$R_f$ (%) | 5.39 | 9.57 | 9.34 | 9.03 | 12.32 | 6.93 | 3.73 | 6.91 | 6.84 | 6.90 | 8.93 | 5.20 |
| [t] | 1.48 | 2.81 | 2.81 | 2.92 | 3.62 | 2.86 | 1.32 | 2.52 | 2.77 | 2.86 | 3.57 | 3.17 |
| Std (%) | 26.79 | 25.32 | 24.81 | 24.05 | 24.09 | 11.8 | 20.25 | 19.84 | 18.79 | 18.60 | 17.34 | 9.22 |
| SR | 0.20 | 0.38 | 0.38 | 0.38 | 0.51 | 0.59 | 0.18 | 0.35 | 0.36 | 0.37 | 0.51 | 0.56 |
| | **SA** | | | | | | | | | | | |
| E[R]-$R_f$ (%) | 4.53 | 7.59 | 7.97 | 8.39 | 9.63 | 5.10 | 2.94 | 5.81 | 5.40 | 5.60 | 6.62 | 3.68 |
| [t] | 1.12 | 1.89 | 1.98 | 2.35 | 2.77 | 2.54 | 1.07 | 2.75 | 2.54 | 3.19 | 4.05 | 2.13 |
| Std (%) | 24.45 | 23.55 | 24.34 | 21.09 | 20.70 | 11.58 | 18.77 | 18.07 | 18.57 | 14.7 | 14.33 | 9.97 |
| SR | 0.19 | 0.32 | 0.33 | 0.40 | 0.47 | 0.44 | 0.16 | 0.32 | 0.29 | 0.38 | 0.46 | 0.37 |
| | **WW** | | | | | | | | | | | |
| E[R]-$R_f$ (%) | 6.09 | 8.24 | 9.13 | 9.59 | 9.65 | 3.56 | 4.42 | 6.55 | 7.01 | 7.00 | 6.85 | 2.42 |
| [t] | 2.13 | 2.78 | 3.68 | 3.78 | 3.85 | 2.23 | 1.99 | 2.78 | 3.40 | 3.74 | 3.67 | 1.76 |
| Std (%) | 25.70 | 24.18 | 23.67 | 21.10 | 20.85 | 11.04 | 20.07 | 18.96 | 18.71 | 15.23 | 14.92 | 9.66 |
| SR | 0.24 | 0.34 | 0.39 | 0.45 | 0.46 | 0.32 | 0.22 | 0.35 | 0.37 | 0.46 | 0.46 | 0.25 |
| | **Panel B: Whole Sample** | | | | | | | | | | | |
| E[R]-$R_f$ (%) | 7.36 | 8.10 | 8.12 | 8.65 | 8.79 | 1.44 | 4.85 | 5.30 | 5.82 | 5.60 | 5.75 | 0.90 |
| [t] | 2.70 | 3.49 | 3.26 | 4.17 | 3.55 | 1.03 | 2.6 | 3.29 | 3.58 | 3.65 | 3.62 | 0.98 |
| Std (%) | 19.25 | 16.75 | 15.14 | 15.15 | 17.37 | 8.72 | 12.96 | 11.4 | 10.53 | 10.77 | 11.4 | 5.94 |
| SR | 0.38 | 0.48 | 0.54 | 0.57 | 0.51 | 0.17 | 0.37 | 0.46 | 0.55 | 0.52 | 0.50 | 0.15 |

## Table 4: Calibration

This table reports the parameter values of our quantitative model. Parameter values are based on calibration to the U.S. data of annual frequency.

| **Parameter** | **Symbol** | **Value** |
|---|---|---|
| Relative risk aversion | $\gamma$ | 10 |
| IES | $\psi$ | 2 |
| Capital share | $\alpha$ | 0.32 |
| Span of control parameter | $\nu$ | 0.85 |
| Time discount factor | $\beta$ | 0.984 |
| Death rate of entrepreneurs | $E(\lambda)$ | 0.12 |
| Inv. adj. cost parameter | $\tau$ | 7 |
| Mean productivity growth rate | $E(\tilde{A})$ | 0.599 |
| Durable capital dep. rate | $\delta_d$ | 0.05 |
| Non-durable capital dep. rate | $\delta_{nd}$ | 0.19 |
| Mean fraction of durable capital over total asset | $\zeta$ | 0.645 |
| Collateralizability parameter | $\theta$ | 0.511 |
| Entering entrepreneurs' net worth over capital ratio via transfers | $\chi$ | 0.35 |
| Persistence of aggregate productivity shocks | $\rho_A$ | 0.83 |
| Persistence of liquidation shocks | $\rho_x$ | 0.95 |
| S.D. of aggregate productivity shocks | $\sigma_A$ | 0.027 |
| S.D. of liquidation shocks | $\sigma_x$ | 0.186 |
| Shock correlation coefficient | $\rho_{A,x}$ | -0.85 |
| Mean idio. productivity growth | $\mu_\epsilon$ | 0.005 |
| S.D. of idio. productivity growth | $\sigma_\epsilon$ | 0.14 |

## Table 5: Model Simulations and Aggregate Moments

This table presents annualized moments from our model simulations and the data whenever available. The model moments are calculated based on repetitions of sample simulations. We carry out our simulation at annual frequency and we report annualized moments. The market return ($R_M$) reflects the return on entrepreneurs' net worth, incorporating endogenous financial leverage. $R_h^{Lev}$ and $R_h$ represent the returns on maximally levered and non-levered capital for capital type $h \in \{d, nd\}$, respectively, which we compute based on average financial leverage in the economy. Volatility, correlations, and first-order autocorrelation are denoted as $\sigma(.)$, $corr(.,.)$ and $AC1(.)$, respectively. The average reduction in the capital durability return spread driven by the asset collateralizeability channel is denoted by $\Omega_t = E[\theta\eta_t(\delta_{nd} - \delta_d)/E_t(\tilde{M})]$ defined in equation (48). "Constrained" and "Unconstrained" refer to model moments computed using the subsamples of our simulation when the shadow value of borrowing constraint $\eta_t > 0$ and $\eta_t = 0$, respectively. Returns and return spreads are all expressed in percent (%).

| Moments | Data | Model |
|---|---|---|
| $\sigma(\Delta y)$ | 3.05 | 3.08 |
| $\sigma(\Delta c)$ | 2.53 | 2.62 |
| $\sigma(\Delta i)$ | 10.30 | 6.58 |
| $corr(\Delta c, \Delta i)$ | 0.39 | 0.58 |
| $AC1(\Delta c)$ | 0.49 | 0.34 |
| Leverage ratio | 0.31 | 0.44 |
| $E[R_M - R_f]$ | 5.71 | 6.98 |
| $\sigma(R_M - R_f)$ | 20.89 | 8.73 |
| $E[R_f]$ | 1.2 | 1.2 |
| $\sigma(R_f)$ | 0.97 | 0.56 |
| $E[R_d^{Lev}]$ | | 10.88 |
| $E[R_d^{Lev} - R_{nd}^{Lev}]$ | | 5.27 |
| $E[R_d^{Lev} - R_{nd}^{Lev}]$ (Constrained) | | 6.94 |
| $E[R_d^{Lev} - R_{nd}^{Lev}]$ (Unconstrained) | | 2.9 |
| $E[R_d]$ | | 6.18 |
| $E[R_d - R_{nd}]$ | | 2.39 |
| $E[R_d - R_{nd}]$ (Constrained) | | 3.07 |
| $E[R_d - R_{nd}]$ (Unconstrained) | | 1.43 |
| $\Omega_t$ | | 0.23 |
| $\frac{\Omega_t}{E[R_d^{Lev} - R_{nd}^{Lev}]}$ (Constrained) | | 0.075 |
| $\frac{\Omega_t}{E[R_d - R_{nd}]}$ (Constrained) | | 0.033 |

## Table 6: Substitution of Durable vs. Non-durable Capital

This table presents model-implied moments measuring the degree of firms' capital substitution between asset durability concerning the tightness of borrowing constraints. The model moments are calculated based on repetitions of sample simulations of annual frequency. $E(\frac{q_{d,t}K_{d,t}}{q_{d,t}K_{d,t}+q_{nd,t}K_{nd,t}})$ denotes the average "expense ratio" which measures firm's relative capital expense on durable capital over total asset. $E(\frac{K_{d,t+1}-K_{nd,t+1}}{K_t})$ denotes the average "excess investment" in capital accumulation of durable relative to non-durable capital. "Constrained" and "Unconstrained" refer to model moments computed using the subsamples of our simulation when the shadow value of borrowing constraint $\eta_t > 0$ and $\eta_t = 0$, respectively. Relative change refers to the average drop of "expense ratio" and "excess investment" in durable capital when firms are constrained as compared to those under unconstrained time in relative terms and in percent.

| **Moments** | **Model** |
|---|---|
| $E(\frac{q_{d,t}K_{d,t}}{q_{d,t}K_{d,t}+q_{nd,t}K_{nd,t}})$ | 0.79 |
| $E(\frac{q_{d,t}K_{d,t}}{q_{d,t}K_{d,t}+q_{nd,t}K_{nd,t}})$ $(Constrained)$ | 0.78 |
| $E(\frac{q_{d,t}K_{d,t}}{q_{d,t}K_{d,t}+q_{nd,t}K_{nd,t}})$ $(Unconstrained)$ | 0.805 |
| *Relative Change* | -3.05% |
| $E(\frac{K_{d,t+1}-K_{nd,t+1}}{K_t})$ | 0.297 |
| $E(\frac{K_{d,t+1}-K_{nd,t+1}}{K_t})$ $(Constrained)$ | 0.294 |
| $E(\frac{K_{d,t+1}-K_{nd,t+1}}{K_t})$ $(Unconstrained)$ | 0.302 |
| *Relative Change* | -2.68% |

## Table 7: Cyclicality of Equilibrium Capital Prices

This table presents model-implied moments measuring the unconditional and conditional variability of capital prices of both capital types. The model moments are calculated based on repetitions of sample simulations of annual frequency. Volatility, covariance, and correlation coefficient are denoted as $\sigma(.)$, $cov(.,.)$ and $corr(.,.)$, respectively. For each type of capital $h \in \{d, nd\}$, $log[q_{h,t}]$ denotes capital prices in natural logarithm. $\widetilde{M}_{t+1}$ is the augmented stochastic discount factor as defined in equation (28). $\lambda_t$ denotes the probability of a firm being liquidated in period $t$.

| **Moments** | **Model** |
|---|---|
| $\sigma(\log[q_{d,t}])$ | 0.169 |
| $\sigma(\log[q_{nd,t}])$ | 0.075 |
| $cov(q_{d,t+1}, \widetilde{M}_{t+1})$ | -0.059 |
| $cov(q_{nd,t+1}, \widetilde{M}_{t+1})$ | -0.013 |
| $corr(\Delta\log[\lambda_t], \Delta\log[q_{d,t}])$ | -0.925 |
| $corr(\Delta\log[\lambda_t], \Delta\log[q_{nd,t}])$ | -0.85 |

## Table 8: Additional Model Results: Fixed Capital Prices

This table presents annualized moments from the model simulations for returns on capital investment. The model moments are calculated based on repetitions of sample simulations. We carry out our simulation at an annual frequency. We fix capital prices to be constant over time and at their respective steady state values as they are in our baseline model by which $q_d^{ss} > q_{nd}^{ss}$ when financial constraints are imposed. $R_h^{Lev}$ and $R_h$ represent the returns on maximally levered and non-levered capital for capital type $h \in \{d, nd\}$, respectively. "Constrained" and "Unconstrained" refer to model moments computed using the subsamples of our simulation when the shadow value of borrowing constraint $\eta_t > 0$ and $\eta_t = 0$, respectively.

| **Moments** | **Model** |
|---|---|
| $E[R_d^{Lev}]$ | 8 |
| $E[R_d^{Lev} - R_{nd}^{Lev}]$ | -0.3 |
| $E[R_d^{Lev} - R_{nd}^{Lev}]$ (Constrained) | -0.28 |
| $E[R_d^{Lev} - R_{nd}^{Lev}]$ (Unconstrained) | -0.48 |
| $E[R_d]$ | 5.38 |
| $E[R_d - R_{nd}]$ | -0.56 |
| $E[R_d - R_{nd}]$ (Constrained) | -0.54 |
| $E[R_d - R_{nd}]$ (Unconstrained) | -0.76 |

## Table 9: Asset Durability Spread, Data, and Model Comparison

This table provides a comparison of moments based on samples between using actual data (Panel A) and using our model simulated data (Panel B) at the portfolio level. Panel A presents statistics computed from the subset of financially constrained firms in the data, categorized by the dividend payment dummy (DIV). In Panel B, we conduct a model simulation and replicate the same portfolio sorting that we conducted using actual real data. Both Panel A and Panel B present the time-series average of cross-sectional median firm characteristics, utilizing year-end values. These characteristics include asset durability, depreciation rate, book leverage, idiosyncratic productivity, and levered and unlevered return on equity. Additionally, we report excess returns E[R]-$R_f$(%) (annualized by multiplying by 12, in percentage terms) for quintile portfolios sorted based on asset durability.

| | **L** | **2** | **3** | **4** | **H** | **H-L** |
|---|---|---|---|---|---|---|
| | | | **Panel A: Data** | | | |
| Asset Durability | 7.69 | 9.99 | 11.45 | 14.24 | 18.00 | |
| Depreciation | 0.19 | 0.16 | 0.15 | 0.13 | 0.11 | |
| Book Lev. | 0.13 | 0.19 | 0.21 | 0.28 | 0.32 | |
| Idio. Productivity | 1.04 | 1.02 | 1.00 | 0.96 | 0.85 | |
| E[R]-$R_f$ (%) | 5.39 | 9.57 | 9.34 | 9.03 | 12.32 | 6.93 |
| E[R]-$R_f$ Unlevered (%) | 4.09 | 7.43 | 6.77 | 7.29 | 8.84 | 4.75 |
| | | | **Panel B: Model** | | | |
| Asset Durability | 6.14 | 7.57 | 9.32 | 12.02 | 16.72 | |
| Depreciation | 0.18 | 0.17 | 0.15 | 0.13 | 0.08 | |
| Book Lev. | 0.22 | 0.25 | 0.28 | 0.33 | 0.41 | |
| Idio. Productivity | 1.03 | 1.02 | 1.01 | 0.99 | 0.97 | |
| E[R]-$R_f$ (%) | 7.22 | 7.99 | 8.77 | 9.88 | 11.57 | 4.36 |
| E[R]-$R_f$ Unlevered (%) | 5.42 | 5.83 | 6.17 | 6.53 | 6.74 | 1.32 |

## Table 10: Cash Flow Duration, Data, and Model Comparison

This table compares cash flow duration measurements between actual data (Panel A) and model-simulated data (Panel B) at the portfolio level. Portfolios are sorted on asset durability relative to industry peers, as shown in Table 9. Panel A presents portfolio-level median cash flow duration measures for financially constrained firms in the data, categorized by dividend payment status (DIV). Panel B replicates the same portfolio sorting methodology using simulated data from our model. We report time-series averages of cross-sectional mean cash flow duration measures for each portfolio. In Panel A, the reported cash flow duration measurements are based on estimations using the methodology developed by Dechow et al. (2004).

$$\text{CF Duration}_{\text{t}} = \frac{\sum_{s=1}^{T} s \times CF_{t+s}(1+r)^{-s}}{P_t} + \left(T + \frac{1+r}{r}\right)\frac{P_t - \sum_{s=1}^{T} CF_{t+s}(1+r)^{-s}}{P_t}.$$

$P_t$ denotes current market equity value of the firm. Cash flows $CF_t$ are measured assuming clean surplus accounting $CF_t = E_t - (BV_t - BV_{t-1}) = BV_{t-1} \cdot (ROE_t - g_t^{BV})$ and we obtain forecasts following Nissim and Penman (2001). In line with Dechow et al. (2004) and Nissim and Penman (2001), we forecast the return on equity (ROE) based on a first-order autoregressive process with an autocorrelation coefficient of 0.41, and a long-run mean equal to the cost of equity of $r = 12\%$. In addition, we assume growth in book equity (BV) $g_t^{BV}$ similarly follows a first-order autoregressive process with an autocorrelation coefficient of 0.24, equal to the long-run average rate of mean reversion in sales growth, and a mean equal to the long-run gross domestic product (GDP) growth rate of 6%. We set the terminal period $T$ to 15 years for the perpetuity assumption. In Panel B, we calculate our baseline cash flow duration measure using model-implied cash flow payouts up to 100 periods as follows:

$$\text{CF Duration}_{\text{i,t}} = \frac{\sum_{s=1}^{100} s \times CF_{i,t+s}(1+r)^{-s}}{P_{i,t}}, \quad P_{i,t} = \sum_{s=1}^{100} CF_{i,t+s}(1+r)^{-s}$$

where we use $r = 8\%$ for constant discounting, reflecting a model-implied total output growth rate of 2% and an equity risk premium of approximately 6%. For firm $i$, we compute cash flow as:

$$\text{CF}_{\text{i,t}} = \alpha A_t(K_{i,t}^{d} + K_{i,t}^{nd}) + (1-\delta_d)q_{d,t}K_{i,t}^{d} + (1-\delta_{nd})q_{nd,t}K_{i,t}^{nd} - \chi\big[(\alpha A_t(K_t^{d} + K_t^{nd}) + (1-\delta_d)q_{d,t}K_t^{d} + (1-\delta_{nd})q_{nd,t}K_t^{nd}\big].$$

Additionally, we present a "truncated" cash flow duration measure that considers flow profits only:

$$\text{CF}_{\text{i,t}} = \alpha A_t(K_{i,t}^{d} + K_{i,t}^{nd}).$$

Lastly, we report the exact cash flow duration implied by our model, incorporating the endogenously determined time-varying stochastic discount factor sequence to discount our baseline cash flows:

$$\text{CF Duration}_{\text{i,t}} = \frac{\sum_{s=1}^{100} s \times M_{t+s} \cdot CF_{i,t+s}}{P_{i,t}}, \quad P_{i,t} = \sum_{s=1}^{100} M_{t+s} \cdot CF_{i,t+s}.$$

"Constrained" and "Unconstrained" refer to model moments computed using the subsamples of our simulation when the shadow value of borrowing constraint $\eta_t > 0$ and $\eta_t = 0$, respectively.

| | **L** | **2** | **3** | **4** | **H** |
|---|---|---|---|---|---|
| | | | **Panel A: Data** | | |
| Measured CF Duration | 20.43 | 20.15 | 19.99 | 19.47 | 18.96 |
| | | | **Panel B: Model** | | |
| Measured CF Duration (Constrained) | 19.93 | 19.63 | 19.19 | 18.58 | 17.75 |
| Measured CF Duration (Unconstrained) | 23.62 | 23.79 | 23.13 | 22.35 | 21.20 |
| Measured CF Duration (Constrained) - Profits Only | 17.40 | 17.54 | 17.62 | 17.68 | 17.78 |
| Measured CF Duration (Unconstrained) - Profits Only | 19.91 | 20.57 | 20.85 | 21.15 | 21.41 |
| Exact CF Duration (Constrained) | 11.35 | 11.21 | 10.97 | 10.64 | 10.20 |
| Exact CF Duration (Unconstrained) | 11.56 | 11.48 | 11.25 | 10.96 | 10.54 |

## Table 11: The Effects of Financial Shocks on Quantity Dynamics

This table presents the sensitivity of asset durability in year $t+1$ to the model-implied financial shocks $\varepsilon_x^{\text{Data}}$. We first calculate the cross-sectional average asset durability for both constrained and unconstrained firms each year, using the dividend payment dummy (DIV). We then examine the sensitivity of asset durability to the model-implied financial shocks, along with other firm characteristics, by reporting the estimated coefficients on asset durability. All estimates are based on the following time-series regressions:

$$\text{Durability}_{h,t+1} = a + b \times \varepsilon_{x,t}^{\text{Data}} + c \times \text{Controls}_{h,t} + u_{h,t},\ h \in \{\text{Const.}, \text{Unconst.}\}.$$

Standard errors are calculated using the Newey-West correction. All independent variables are standardized to have a mean of zero and a standard deviation of one.

| | **Const.** | **Unconst.** | **Const.-Unconst.** |
|---|---|---|---|
| $\varepsilon_x^{\text{Data}}$ | -0.16 | 0.00 | -0.06 |
| [t] | -2.11 | 0.24 | -2.09 |
| Log ME | -1.38 | -1.75 | -0.04 |
| [t] | -13.75 | -14.60 | -0.55 |
| Log B/M | 0.18 | -0.23 | 0.26 |
| [t] | 2.49 | -2.24 | 5.54 |
| I/K | -0.08 | -0.04 | 0.02 |
| [t] | -1.18 | -0.58 | 1.55 |
| ROA | 0.00 | -0.15 | 0.11 |
| [t] | 0.01 | -1.70 | 2.07 |
| | | | |
| Observations | 40 | 40 | 40 |
| R-square | 0.95 | 0.94 | 0.66 |

## Table 12: The Effects of Financial Shocks on Price Dynamics

This table shows the exposure of price dynamics to the model-implied financial shocks. We classify detailed assets into high- and low-durability groups based on their durability scores. Next, we compute the price changes for each asset and average them within each group across years to construct the time series of price changes. All estimates are based on the following time-series regressions:

$$\Delta \text{Log } q_{h,t+1} = a_q + b_1 \times \widetilde{\text{MKT}}_t + b_2 \times \varepsilon_{x,t}^{\text{Data}} + \varepsilon_{h,t},\ h \in \{H, L\},$$

in which $\Delta \text{Log } q_{h,t+1}$ represents the log difference of price changes in the high H and low L asset durability groups. $\widetilde{\text{MKT}}_t$ represents the market factor that is orthogonal to financial shocks, and $\varepsilon_{x,t}^{\text{Data}}$ stands for the model-implied financial shocks. $\widetilde{\text{MKT}}$ refers to the market factor that is orthogonal to the model-implied financial shocks. Standard errors are computed using the Newey-West correction, and $t$-statistics are reported in parentheses.

| | **L** | **H** |
|---|---|---|
| MKT | -0.48 | -0.05 |
| [t] | -0.59 | -0.17 |
| $\varepsilon_x^{\text{Data}}$ | 0.12 | -0.74 |
| [t] | 0.21 | -2.68 |
| | | |
| Observations | 40 | 40 |
| R-square | 0.07 | 0.27 |

## Table 13: Estimating the Market Price of Risk

In Panel A, we present the GMM estimates of the parameters of the stochastic discount factor (SDF), given by $\text{SDF} = 1 - b_M \times \widetilde{\text{MKT}} - b_x \times \varepsilon_x^{\text{Data}}$ ($\text{SDF} = 1 - b_A \times \widetilde{\varepsilon}_A - b_x \times \widetilde{\varepsilon}_x$), using quintile portfolios sorted by asset durability. $\varepsilon_x^{\text{Data}}$ represents the model-implied financial shock in the data, while $\widetilde{\varepsilon}_a$ and $\widetilde{\varepsilon}_x$ correspond to orthogonalized productivity and financial shocks in the model. $\widetilde{\text{MKT}}$ refers to the market factor that is orthogonal to the model-implied financial shocks. $\widetilde{\varepsilon}_A$ denotes the aggregate productivity shocks that are orthogonal to the structural innovations to the liquidation probability, $\widetilde{\varepsilon}_x = \varepsilon_x$. We normalize the SDF such that $E[\text{SDF}] = 1$, following Cochrane (2005). We report $t$-statistics and standard errors computed using the Newey-West procedure with three lags. To assess model fit, we include the sum of squared errors (SSQE), mean absolute pricing errors (MAPE), and the $J$-statistic for the overidentifying restrictions. Given the Euler equation $\text{E}[\text{SDF} \times R_i^e] = 0$, SSQE and MAPE are computed based on each testing asset $i$'s moment error $u_i = \frac{1}{T}\sum_{t=1}^{T}\left[\widehat{\text{SDF}} \times R_{i,t}^e\right]$. SSQE and MAPE are defined as $\sum_{i=1}^{N} u_i \times u_i$ and $\frac{1}{N}\sum_{i=1}^{N}|u_i|$, in which $N$ denotes the number of testing assets. In Panel B, we present the GMM-implied risk exposure of the testing portfolios: ($\beta_{\text{MKT}}^i$ and $\beta_{x,\text{Data}}^i$) to the market factor and the model-implied financial shocks in the upper panel, as well as ($\beta_A^i$ and $\beta_x^i$) to productivity and financial shocks in the lower panel, together with GMM-implied pricing errors ($\alpha^i$) in percentage.

| **Panel A: Price of Risk** | | | | |
|---|---|---|---|---|
| | **Data** | | | **Model** |
| | **(1)** | **(2)** | | **(3)** |
| $\widetilde{\text{MKT}}$ | 0.69 | 0.69 | $\widetilde{\varepsilon}_A$ | 0.75 |
| [t] | 9.33 | 7.43 | | 2.01 |
| $\varepsilon_x^{\text{Data}}$ | | -0.53 | $\widetilde{\varepsilon}_x$ | -1.26 |
| [t] | | -3.16 | | -3.03 |
| SSEQ (%) | 5.70 | 5.64 | | 0.13 |
| MAPE (%) | 5.13 | 5.11 | | 0.67 |
| $J$-test | 9.85 | 9.38 | | 5.48 |
| p | 0.83 | 0.86 | | 0.38 |
| $J$T-Diff | | 17.39 | | |
| p | | 0.00 | | |

| **Panel B: Risk Exposure** | | | | | | |
|---|---|---|---|---|---|---|
| | **L** | **2** | **3** | **4** | **H** | **H-L** |
| **Data: SDF ($\widetilde{\text{MKT}} + \varepsilon_{\mathbf{x}}^{\textbf{Data}}$)** | | | | | | |
| $\beta_{\text{MKT}}^i$ | 25.13 | 23.27 | 20.95 | 22.46 | 22.25 | -2.88 |
| [t] | 14.11 | 10.18 | 13.37 | 7.71 | 11.94 | -1.54 |
| $\beta_{x,\text{Data}}^i$ | 3.84 | 0.80 | -0.54 | 1.34 | -0.99 | -4.83 |
| [t] | 1.48 | 0.32 | -0.29 | 0.48 | -0.38 | -2.62 |
| $\alpha^i$ | -3.98 | -0.12 | -1.94 | 0.04 | 2.21 | 6.19 |
| [t] | -1.58 | -0.05 | -0.83 | 0.02 | 0.96 | 0.49 |
| **Model: SDF ($\widetilde{\varepsilon}_{\mathbf{A}} + \widetilde{\varepsilon}_{\mathbf{x}}$)** | | | | | | |
| $\beta_A^i$ | 2.15 | 2.12 | 2.09 | 2.06 | 2.01 | -0.14 |
| [t] | 9.74 | 8.79 | 7.89 | 6.96 | 5.40 | -0.86 |
| $\beta_x^i$ | -5.61 | -6.37 | -7.14 | -8.16 | -9.60 | -4.00 |
| [t] | -26.12 | -27.93 | -28.91 | -29.56 | -26.93 | -25.43 |
| $\alpha^i$ | 0.06 | 0.01 | -0.06 | -0.06 | 0.06 | -0.02 |
| [t] | 0.36 | 0.02 | -0.28 | -0.24 | 0.23 | -0.13 |

# Internet Appendix for
# “The Asset Durability Premium” *

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## Contents



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# I Supplemental Materials on Empirical Analysis

This section outlines the construction of cash flow duration and shows why accounting for the endogenous and negative covariance between the SDF and cash flows reduces the magnitude of cash flow duration, in contrast to the constant discounting approach used in the empirical literature. We provide additional empirical results to support our model predictions.

## I.1 Cash Flow Duration

### I.1.1 Empirical Measurement

We construct one version of firm $i$'s cash flow duration, following the approach of Lettau and Wachter (2007), to reflect the timing of cash flows. Duration ($Dur$) is the equity-implied cash flow duration. Dechow, Sloan, and Soliman (2004) proposes the measure of cash flow duration and documents a negative relationship between cash flow durations and stock returns; in addition, Weber (2018) recently studies asset pricing implications, including exposure to existing risk factors, time variations in the slope, and the effect of short-sale constraints.

Duration ($Dur$) resembles the traditional Maculay duration for bonds, which reflects the weighted average time to maturity of cash flows. The ratio of discounted cash flows to price determines the weights:

$$Dur_{i,t} = \frac{\sum_{s=1}^{T} s \times CF_{i,t+s}/(1+r)^s}{P_{i,t}}, \tag{I.1}$$

in which we denote $Dur_{i,t}$ as firm $i$'s duration at the end of fiscal year $t$, $CF_{i,t+s}$ as the cash flow at time $t+s$, $P_{i,t}$ as the current stock price, and $r$ as the expected return on equity. Following Dechow, Sloan, and Soliman (2004), we assume the expected return on equity to be constant across both stocks and time. Relaxing such an assumption for firm-specific discount rates generates larger cross-sectional variations in the duration measure since firms with high cash flow duration tend to be growth firms with lower stock returns. For simplicity's sake, we focus on the measure of cash flow duration by using the constant expected return.[1]

In contrast to fixed income securities, such as bonds, stocks cannot have a well-defined finite maturity, $t+T$, and cash flows are not known in advance. Therefore, we split the duration formula into a finite detailed forecasting period and an infinite terminal value. We

[1] According to Weber (2018), the variation over time in the return on equity $r$ does not alter the cross-sectional ranking, which alleviates the concern for cross-sectional implications.

also assume that the latter component is paid out as the level perpetuity for simplicity's sake. Such an assumption allows us to rewrite equation (I.1) as follows:

$$Dur_{i,t} = \frac{\sum_{s=1}^{T} s \times CF_{i,t+s}/(1+r)^s}{P_{i,t}} + \left(T + \frac{1+r}{r}\right) \times \frac{P_{i,t} - \sum_{s=1}^{T} CF_{i,t+s}/(1+r)^s}{P_{i,t}}. \quad \text{(I.2)}$$

Moreover, we impose a clean surplus accounting, based on an accounting identity, and forecast cash flows via forecasting return on equity ($ROE$), $E_{i,t+s}/BV_{i,t+s-1}$, and growth in book equity, $(B_{i,t+s} - B_{i,t+s-1})/BV_{i,t+s-1}$:

$$\begin{aligned} CF_{i,t+s} &= E_{i,t+s} - (B_{i,t+s} - B_{i,t+s-1}) \\ &= B_{i,t+s-1} \times \left[\frac{E_{i,t+s}}{B_{i,t+s-1}} - \frac{B_{i,t+s} - B_{i,t+s-1}}{B_{i,t+s-1}}\right]. \end{aligned} \quad \text{(I.3)}$$

Following Dechow, Sloan, and Soliman (2004), we model returns on equity and growth in equity as an autoregressive process based on recent findings in the financial accounting literature. In Weber (2018), the author estimates autoregressive parameters by using the merged CRSP-Compustat universe and assumes the mean reversion of $ROE$ to the average cost of equity. We also follow the estimated autoregressive parameters in Weber (2018) by assuming that the growth in book equity is mean reverting to the long-run average growth rate in the economy with a coefficient of mean reversion equal to the average historical mean reversion in sales growth. The persistence of AR(1) model is 0.41 for $ROE$ and 0.24 for $BV$, respectively. We assign the discount rate $r$ to a value 0.12, which is equal to a steady-state average cost of equity of 0.12. Finally, we assign the average long-run nominal growth rate to a value of 0.06, and use a detailed forecasting period of 15 years.

### I.1.2 Accounting for the Negative SDF–Cash Flow Covariance

In our model, cash flows are low during bad times when the marginal utility of consumption – and thus the SDF $M_t$ – is high, generating a negative covariance between the SDF and cash flows. In the following, we show making $\text{Cov}(M_t, CF_t)$ more negative at long horizons compresses tail present values, shifting weight to earlier dates to keep $\sum_t w_t = 1$. Because duration is the first moment of this weight distribution, compressing the tail strictly reduces duration.

**Setup.** Let $\text{PV}_t = \mathbb{E}[M_t CF_t] = \mathbb{E}[M_t]\mathbb{E}[CF_t] + \text{Cov}(M_t, CF_t)$, price $P = \sum_t \text{PV}_t \in (0, \infty)$, weights $w_t = \text{PV}_t/P$, and duration $D = \sum_t t\, w_t$. Consider two environments $A$ and $B$ with the same means $\{\mathbb{E}[M_t], \mathbb{E}[CF_t]\}$, but possibly different covariances. Assume $\text{PV}_t \geq 0$ and prices are finite and positive in both environments.

**Assumption (Tail dominance).** There exists $T$ such that for all $t \geq T$,

$$\mathrm{Cov}_B(M_t, CF_t) \leq \mathrm{Cov}_A(M_t, CF_t),$$

with strict inequality for at least one $t \geq T$, and equality for $t < T$.

**Step 1: Tail PVs shrink in $B$.** For $t \geq T$,

$$\mathrm{PV}_t^B = \mathbb{E}[M_t]\mathbb{E}[CF_t] + \mathrm{Cov}_B(M_t, CF_t) \leq \mathbb{E}[M_t]\mathbb{E}[CF_t] + \mathrm{Cov}_A(M_t, CF_t) = \mathrm{PV}_t^A,$$

with strict inequality for at least one $t \geq T$. Summing, $\sum_{t \geq T} \mathrm{PV}_t^B < \sum_{t \geq T} \mathrm{PV}_t^A$. Since $\mathrm{PV}_t^B = \mathrm{PV}_t^A$ for $t < T$, we also have $P_B \leq P_A$.

**Step 2: Tail weights shrink in $B$.** Using $w_t^X = \mathrm{PV}_t^X / P_X$,

$$\sum_{t \geq T} w_t^B = \frac{\sum_{t \geq T} \mathrm{PV}_t^B}{P_B} \leq \frac{\sum_{t \geq T} \mathrm{PV}_t^B}{P_A} < \frac{\sum_{t \geq T} \mathrm{PV}_t^A}{P_A} = \sum_{t \geq T} w_t^A.$$

Thus the weight distribution in $B$ has strictly less mass on the tail and, by conservation of total mass, weakly more mass on earlier dates.

**Step 3: Less tail mass implies lower duration.** Write

$$D_X = \sum_{t < T} t\, w_t^X + \sum_{t \geq T} t\, w_t^X \geq (T-1) \sum_{t < T} w_t^X + T \sum_{t \geq T} w_t^X = (T-1) + \sum_{t \geq T} w_t^X,$$

which is strictly increasing in the tail mass $\sum_{t \geq T} w_t^X$. Since $\sum_{t \geq T} w_t^B < \sum_{t \geq T} w_t^A$, it follows that

$$D_B < D_A.$$

## I.2 Empirical Asset Pricing Tests

### I.2.1 Market Price and Exposure to Macroeconomic Shocks

In this subsection, we assess the robustness of our risk-based explanation for the asset durability premium. We employ the yield spread between Moody's Baa and Aaa corporate bond yields and GZ credit spread as proxies for the macroeconomic risk driven by financial shocks and conduct a generalized method of moments (GMM) test to evaluate their role in pricing stock returns.

We estimate a two-factor stochastic discount factor (SDF) model, where the first factor is the market factor, and the second factor is related to financial shocks, represented by the yield spread or GZ credit spread:

$$\text{SDF}_t = 1 - b_M \times \widetilde{\text{MKT}}_t - b \times \text{Macro}_t, \tag{I.4}$$

in which $\text{Macro}_t$ denotes either the yield spread or the GZ credit spread. $\widetilde{\text{MKT}}_t$ represents the component of the market factor orthogonal to the yield spread or GZ credit spread. Specifically, we isolate the portion of the market factor driven solely by aggregate productivity shocks by projecting the market factor onto each spread separately and retaining the residuals, denoted as $\widetilde{\text{MKT}}$. We then examine the sensitivity of $b$, which captures the price of macroeconomic risk.

We conduct GMM estimation using a set of test assets, including six asset-durability-sorted portfolios (Table 3), six size-momentum portfolios, five industry portfolios.[2] The moment condition used is:

$$\mathbb{E}[R_i^e] = -\text{Cov}(\text{SDF}_t, R_i^e), \tag{I.5}$$

which is the empirical equivalent of our model's Euler equation. We evaluate the ability of macroeconomic shocks ($\text{Macro}_t$) to price test assets.

Panel A of Table IA.1 presents the results:

- **Specification 1:** The market factor alone exhibits a significant price of risk.
- **Specification 2:** Adding the yield spread reduces pricing errors.
- **Specification 3:** Incorporating the GZ credit spread improves model fit.

The yield spread and GZ credit spread have significant negative prices of risk, aligning with our model predictions. The inclusion of macroeconomic shocks improves the performance of the CAPM model in pricing stock returns. The $JT$ difference test between the CAPM and our two-factor model is statistically significant at the 5% level.

**[Place Table IA.1 about here]**

Panel B of Table IA.1 reports GMM-implied risk exposures (betas) and pricing errors (alphas):

- Market betas ($\beta^i_{\text{MKT}}$) are relatively uniform across portfolios.

[2] Following Belo, Li, Lin, and Zhao (2017) and Lin, Palazzo, and Yang (2020).

- Financial shock betas ($\beta^i_{\text{Yield}}$, $\beta^i_{\text{GZ}}$) decline from low- to high-asset-durability portfolios, reflecting higher exposure to negatively priced financial shocks.

These results further validate our risk-based explanation by demonstrating that financial shocks are a key driver of cross-sectional variation in stock returns associated with asset durability.

#### I.2.2 Asset Pricing Factor Regressions

In this subsection, we consider the extent to which the variability in the average returns of the durability-sorted portfolios in our analysis can be explained by exposure to standard risk factors that are proposed by the Fama and French (2015) five-factor model, the Hou, Xue, and Zhang (2015) q-factor model, or, notably, the collateralizability factor identified in Ai, Li, Li, and Schlag (2020).[3]

To test the standard risk factor models, we perform time-series regressions of asset durability-sorted portfolios' excess returns on the Fama and French (2015) five-factor model (the market factor-MKT, the size factor-SMB, the value factor-HML, the profitability factor-RMW, and the investment factor-CMA), and of the collateralizability factor-COL (i.e., the long-short portfolio sorted on collateralizability) in Panel A, as well as on the Hou, Xue, and Zhang (2015) q-factor model (the market factor-MKT, the size factor-SMB, the investment factor-I/A, and the profitability factor-ROE), and the long-short portfolio sorted on collateralizability (COL) in Panel B, respectively. We use these time-series regressions to estimate the betas (i.e., risk exposures) of each portfolio's excess return on various risk factors and also to estimate each portfolio's risk-adjusted return (i.e., alphas in %). We annualize the excess returns and alphas in Table IA.2.

**[Place Table IA.2 about here]**

As presented in Table IA.2, the risk-adjusted returns (intercepts) of the high-minus-low portfolio sorted by asset durability remain notably large and statistically significant. These intercepts range from 8.14% for the Fama and French (2015) five-factor model in Panel A to 8.54% for the Hou, Xue, and Zhang (2015) q-factor model in Panel B. These intercepts are all at least 3.38 standard errors above zero, indicating high statistical significance. Additionally, the alphas estimated by both the Fama-French five-factor model and the HXZ q-factor model remain comparable to the durability spread observed in the univariate sorting (Table 3).

[3] The Fama and French factors are sourced from Kenneth French's data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The HXZ factors are obtained from the q-factors data library (http://globalq.org/index.html).

Furthermore, the high-minus-low portfolio's returns exhibit significantly negative market betas in relation to both the Fama and French (2015) five-factor model and the Hou, Xue, and Zhang (2015) q-factor model. However, these returns show insignificantly negative betas with respect to both models. Lastly, the asset durability spread cannot be explained by the collateralizability factor (COL), despite the association between higher asset durability and asset collateralizability.

Overall, the outcomes from our asset pricing factor tests detailed in Table IA.2 indicate that the variation in cross-sectional returns among portfolios categorized by asset durability cannot be absorbed by the Fama and French (2015) five-factor model, the HXZ q-factor model (Hou, Xue, and Zhang (2015)), or the collateralizability premium. Consequently, the elevated returns linked to asset durability are not explained by common risk factors. In our next subsection, we reinforce the association between asset durability and returns by utilizing Fama-Macbeth regressions.

### I.2.3 Firm-level Return Predictability Regressions

We further investigate the predictive capacity of asset durability for cross-sectional stock returns using Fama-MacBeth cross-sectional regressions (Fama and MacBeth (1973)). This analytical method enables us to account for an extensive array of firm characteristics that predict stock returns. Moreover, it allows us to explore whether the positive relationship between asset durability and returns can be attributed to other established predictors at the firm level that are captured in the literature.[4]

We perform cross-sectional regressions for each month spanning from July of year $t$ to June of year $t+1$ as expressed in the following equation:

$$R_{i,t+1} - R_{f,t+1} = a + b \times \text{Asset Durability}_{i,t} + c \times \text{Controls}_{i,t} + u_{it}. \tag{I.6}$$

Within each month, we regress the monthly returns of individual stocks (annualized by multiplying by 12) against the asset durability of year $t-1$ (reported by the end of December of year $t-1$), diverse sets of control variables known by the end of June of year $t$, and industry fixed effects. Our control variables encompass the natural logarithm of market capitalization at the end of each June (Size), which is deflated by the CPI index, the natural logarithm of the book-to-market ratio (B/M), the investment rate (I/K), profitability (ROA),

[4]Using this approach is advantageous compared to using portfolio tests, as the latter not only necessitate predetermined breaking points for sorting firms into portfolios, but also involve the selection of the number of portfolios. Moreover, since incorporating multiple sorting variables with distinct information about future stock returns through a portfolio approach is intricate, Fama-MacBeth cross-sectional regressions offer a reliable cross-validation mechanism.

R&D intensity (R&D/AT), organization capital ratio (OC/AT), book leverage, and industry indicators based on NAICS 3-digit industry classifications. To mitigate the impact of outliers, all independent variables are normalized to possess a zero mean and one standard deviation, following winsorization at the 1st and 99th percentiles.

**[Place Table IA.3 about here]**

Table IA.3 displays the outcomes of cross-sectional predictability regressions conducted on a monthly basis. The presented coefficient represents the mean slope derived from monthly regressions, while the accompanying $t$-statistics are obtained by dividing the average slope by its standard error across the time series. These Fama-MacBeth regression results are aligned with the patterns that we observe in portfolios organized with respect to asset durability.

Our Fama-MacBeth regression results corroborate our findings from portfolios sorted with respect to asset durability. To address the potential influence of leveraged positions, we incorporate a control for firm-level book leverage in each specification. In Specification 1, the relationship between asset durability and future stock returns is statistically significant and positive, characterized by a slope coefficient of 2.13, which is 3.44 standard errors from zero. This outcome underscores that the asset durability-return relation is predominantly driven by the leverage channel. For Specification 2, we introduce firm-level collateralizability as outlined by Ai, Li, Li, and Schlag (2020). Notably, the slope coefficient associated with asset durability remains significant and even increases in magnitude, even after we explicitly account for firm-level collateralizability. Simultaneously, collateralizability exhibits a significant and negative prediction for stock returns, which aligns with findings in Ai, Li, Li, and Schlag (2020).

We next explore potential alternative explanations grounded in systematic risks proposed by previous studies. Specifically, we investigate four alternative channels that could account for variations in our asset-durability-sorted portfolios:

**Operating Leverage and Adjustment Costs:** High-asset-durability firms might experience elevated expected returns due to the presence of higher fixed or adjustment costs associated with the downsizing of capital stock, especially during periods of economic decline. This aligns with the literature (e.g., Zhang (2005), Gu, Hackbarth, and Johnson (2018), Kim and Kung (2017)) which posits that firms with durable assets face challenges and costs when downsizing their production capacity, thus contributing to our observed pattern of returns in our analysis.

**Output Durability:** Firms with high asset durability often generate durable goods as outputs, making their cash flows more sensitive to business cycle fluctuations. This could contribute to observed differences in returns. This concept corresponds to the theory proposed by Gomes, Kogan, and Yogo (2009).

**Financial Distress:** Lower asset durability might expose firms to a higher risk of financial distress, resulting in comparatively lower average returns. This possibility is consistent with research by Griffin and Lemmon (2002), Bharath and Shumway (2008), and Campbell, Hilscher, and Szilagyi (2008).

In sum, these alternative explanations suggest that the observed return differentials could be driven by factors beyond asset durability, such as operating dynamics, output characteristics, and financial vulnerabilities.

If operating leverage (Zhang (2005) and Gu et al. (2018)) or adjustment costs (Kim and Kung (2017)) prove to be the driving factors behind the asset durability premium, we would then anticipate that this premium would diminish when we account for operating leverage in Specifications 3 and 4, or for asset redeployability in Specification 5. However, the significantly persistent positive slope coefficients on asset durability at the 1% level in these specifications indicate that the observed return predictability is not attributed to systematic risk that stems from either operating leverage or adjustment costs.

We also explore the concept of output durability as proposed by Gomes, Kogan, and Yogo (2009) and examine its relationship with our asset durability measure.[5] Gomes, Kogan, and Yogo (2009) posit that producers of durable goods experience cash flow sensitivity to aggregate economic fluctuations due to the procyclicality of demand for their products. This elevated sensitivity renders their stocks riskier and yields higher average returns. In Specification 6, we observe that firm-level asset durability continues to predict stock returns, even after we account for the Durable Output dummy that reflects a firm operates in durable goods producing industries. This persistence in positive predictability suggests that our asset durability measure encapsulates distinct information compared to that of output durability. However, we recognize that variations in stock returns that we highlight in Gomes, Kogan, and Yogo (2009) primarily stem from differences between durable and service industries (across industries). On the contrary, our asset durability's predictability revolves around disparities in firms' asset durability in relation to their industry peers (within the industry). Consequently, the concepts of output durability and capital durability complement each other, although they stem from different economic mechanisms.

For Specifications 7 through 10, we introduce the firm-level O index, the Z index, default probability, and failure probability as measures of a firm's financial distress, as proposed by Griffin and Lemmon (2002), Bharath and Shumway (2008), and Campbell, Hilscher, and Szilagyi (2008). Notably, we observe that the coefficients on asset durability remain signifi-

[5]Detailed classifications for output durability are obtained from Motohiro Yogo's personal website (https://sites.google.com/site/motohiroyogo/).

cant and, if anything, are slightly more pronounced in magnitude when we explicitly account for these firm-level financial distress measures. Our findings in Specifications 7 to 10 have important implications. Firstly, they underscore that the positive asset durability premium stands apart from the negative relation between distress and expected returns, which is commonly documented in the literature. These specifications reaffirm that asset durability's predictability is independent of financial distress and that it encompasses information that goes beyond what is captured by financial distress. Secondly, our theoretical framework may provide insights into the financial distress puzzle, suggesting that financially distressed firms exhibit lower risk and consequently lower average returns since they tend to use more economical non-durable assets and experience less price cyclicality.

In our final specification, Specification 12, we find that the predictability of asset durability for stock returns remains intact even when we account for all the known predictors and control variables together in a comprehensive analysis. This horse racing test demonstrates that these variables do not undermine the predictive power of asset durability. In summary, our findings suggest that asset durability's ability to forecast stock returns is distinct and not overshadowed by these established predictors.

# II Computation Details on Model Solutions

To study asset pricing implications, we first solve the model regarding dynamics of aggregate prices and quantities only and then take policy functions to simulate a large panel of firms for computing return profiles. In this section, we describe our computational procedures for solving the model about aggregates.

Specifically, we use the modified Parameterized Expectation Algorithm (PEA) as in Christiano and Fisher (2000) to solve our model for the sequence of functional objects. We thus solve our model using a global method, which allows for occasionally binding constraints and distinguishes ours from the literature that imposes a binding constraint at the stochastic steady state. As we abstract away from a time-varying firm distribution, our model solution shows that all firms could be constrained or unconstrained in different times along the simulation path.

## II.1 Recast of the Law of Motion for Ease of Computation

Our numerical implementation reduces the computational burden by avoiding the iterative root-finding that is extremely time-consuming but routinely associated with a dynamic programming problem. That is, our computation can be very iteratively solved for the root of an equilibrium functional $n^{prime}(A, \lambda, n)$ that fits the path of the law of motion per equation (III.4).

Instead, we perform some change of variable that effectively reconstructs endogenous state variables by which the law of motion of net worth is no longer intertwined with equilibrium functionals. Specifically, we change the normalization of the total cost of borrowing $R_{f,t+1}B_t$ as of period $t+1$ using future capital stock of $K_{t+1}$, which gives $\tilde{b}_t = \frac{R_{f,t+1}b_t}{\Gamma_t}$. The redefined debt position $\tilde{b}_t$ thus enters the law of motion such that:

$$n_{t+1} = (1 - \lambda_{t+1})(s_{t+1} - \tilde{b}_t) + \lambda_{t+1}\chi s_{t+1} \tag{II.1}$$

in which $s_{t+1} = \alpha\nu A_{t+1} + (1 - \delta_d)\zeta q_{d,t+1} + (1 - \delta_{nd})(1 - \zeta)q_{nd,t+1}$. When we combine this with the balance sheet constraint $n_t + b_t = \Gamma_t(\zeta q_{d,t} + (1 - \zeta)q_{nd,t})$, we have the law of motion refined over this redefined debt position, which directly builds on the predefined grids of $\tilde{b}_t$ without solving for any root functional.

$$\begin{aligned}\tilde{b}_t = & \frac{R_{f,t+1}}{\Gamma_t}(1 - \lambda_t)\tilde{b}_{t-1} + R_{f,t+1}[\zeta q_{d,t} + (1 - \zeta)q_{nd,t}] \\ & - \frac{R_{f,t+1}}{\Gamma_t}[1 - \lambda_t(1 - \chi)]s_t\end{aligned} \tag{II.2}$$

In particular, the occasionally binding borrowing constraint based on the redefined debt position is formulated as:

$$\tilde{b}_t \leq R_{f,t+1}\theta[(1 - \delta_d)\zeta q_{d,t} + (1 - \delta_{nd})(1 - \zeta)q_{nd,t}] \tag{II.3}$$

## II.2 Recast of the Recursive Equilibrium

We then recast the model equilibrium conditions and solve a sequence of equilibrium functional $X(A_t, \lambda_t, \tilde{b}_{t-1})$ defined over a predetermined debt position $\tilde{b}_{t-1}$ and the aggregate states $A_t$ and $\lambda_t$ as of time $t$. We show that our recast model structure is not subject to time-consuming root-finding iterations. Similarly, we denote the generic variable in period $t$ as $X$ and $X'$ for period $t+1$ and $x$ and $X$ to characterize a generic normalized and non-normalized quantity, respectively. The model equilibrium can be similarly rewritten as a set of a set of equilibrium functional $\{c\left(A, \lambda, \tilde{b}\right), \tilde{b}'(A, \lambda, \tilde{b}), i\left(A, \lambda, \tilde{b}\right), \mu\left(A, \lambda, \tilde{b}\right), \eta\left(A, \lambda, \tilde{b}\right), q_d\left(A, \lambda, \tilde{b}\right),$ $q_{nd}\left(A, \lambda, \tilde{b}\right), R_f\left(A, \lambda, \tilde{b}\right), \phi\left(A, \lambda, \tilde{b}\right), M'\left(A, \lambda, \tilde{b}\right), \widetilde{M}'\left(A, \lambda, \tilde{b}\right), n\left(A, \lambda, \tilde{b}\right), \Gamma\left(A, \lambda, \tilde{b}\right)\}$ satisfying the following set of functional equations:

$$M' = \beta \left[ \frac{c\left(A', \lambda', \tilde{b}'\right) \Gamma\left(A, \lambda, \tilde{b}\right)}{c\left(A, \lambda, \tilde{b}\right)} \right]^{-\frac{1}{\psi}} \left[ \frac{u\left(A', \lambda', \tilde{b}'\right)}{E\left[u\left(A', \lambda', \tilde{b}'\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma}, \tag{II.4}$$

$$\widetilde{M}' = M'[(1-\lambda')\,\mu\left(A', \lambda', \tilde{b}'\right) + \lambda'], \tag{II.5}$$

$$E\left[ M' \middle| A, \lambda, \tilde{b} \right] R_f\left(A, \lambda, \tilde{b}\right) = 1, \tag{II.6}$$

$$\mu\left(A, \lambda, \tilde{b}\right) = E\left[ \widetilde{M}' \middle| A, \lambda, \tilde{b} \right] R_f\left(A, \lambda, \tilde{b}\right) + \eta\left(A, \lambda, \tilde{b}\right), \tag{II.7}$$

$$\mu\left(A, \lambda, \tilde{b}\right) = E\left[ \widetilde{M}' \frac{\alpha\nu A' + (1-\delta_d)\, q_d\left(A', \lambda', \tilde{b}'\right)}{q_d\left(A, \lambda, \tilde{b}\right)} \middle| A, \lambda, \tilde{b} \right] + \theta(1-\delta_d)\eta\left(A, \lambda, \tilde{b}\right), \tag{II.8}$$

$$\mu\left(A, \lambda, \tilde{b}\right) = E\left[ \widetilde{M}' \frac{\alpha\nu A' + (1-\delta_{nd})\, q_{nd}\left(A', \lambda', \tilde{b}'\right)}{q_{nd}\left(A, \lambda, \tilde{b}\right)} \middle| A, \lambda, \tilde{b} \right] + \theta(1-\delta_{nd})\eta\left(A, \lambda, \tilde{b}\right), \tag{II.9}$$

$$\begin{aligned} \tilde{b}'(A, \lambda, \tilde{b}) = {} & \frac{R_f(A, \lambda, \tilde{b})}{\Gamma(A, \lambda, \tilde{b}))} (1-\lambda)\, \tilde{b} + R_f(A, \lambda, \tilde{b})[\zeta q_d(A, \lambda, \tilde{b}) + (1-\zeta) q_{nd}(A, \lambda, \tilde{b})] \\ & - \frac{R_f(A, \lambda, \tilde{b})}{\Gamma(A, \lambda, \tilde{b})} [1-\lambda(1-\chi)](a\nu A + (1-\delta_d)\zeta q_d(A, \lambda, \tilde{b}) + (1-\delta_{nd})(1-\zeta) q_{nd}(A, \lambda, \tilde{b})), \end{aligned} \tag{II.10}$$

$$\frac{n(A, \lambda, \tilde{b}) R_f(A, \lambda, \tilde{b})}{\Gamma(A, \lambda, \tilde{b})} + \tilde{b}'(A, \lambda, \tilde{b}) = R_f(A, \lambda, \tilde{b})[\zeta q_d(A, \lambda, \tilde{b}) + (1-\zeta)\, q_{nd}(A, \lambda, \tilde{b})], \tag{II.11}$$

$$\eta\left(A, \lambda, \tilde{b}\right) \{\tilde{b}'(A, \lambda, \tilde{b}) - R_f\left(A, \lambda, \tilde{b}\right) \theta[\zeta(1-\delta_d) q_d\left(A, \lambda, \tilde{b}\right) + (1-\zeta)(1-\delta_{nd}) q_{nd}\left(A, \lambda, \tilde{b}\right)]\} = 0, \tag{II.12}$$

$$G'\left(i\left(A, \lambda, \tilde{b}\right)\right) = \phi\left(A, \lambda, \tilde{b}\right) q_d\left(A, \lambda, \tilde{b}\right) + \left(1 - \phi\left(A, \lambda, \tilde{b}\right)\right) q_{nd}\left(A, \lambda, \tilde{b}\right), \tag{II.13}$$

$$c\left(A, \lambda, \tilde{b}\right) + i\left(A, \lambda, \tilde{b}\right) + g\left(i\left(A, \lambda, \tilde{b}\right)\right) = A, \tag{II.14}$$

$$\phi\left(A, \lambda, \tilde{b}\right) = \frac{(\delta_d - \delta_{nd})\,(1-\zeta)\,\zeta}{i\left(A, \lambda, \tilde{b}\right)} + \zeta, \tag{II.15}$$

$$\Gamma\left(A, \lambda, \tilde{b}\right) = i\left(A, \lambda, \tilde{b}\right) + [1 - \zeta\delta_d - (1-\zeta)\delta_{nd}]. \tag{II.16}$$

## II.3 Functional Approximation

Following Christiano and Fisher (2000), we solve the sequence of functional objects in equilibrium by using functional approximations based on Chebyshev polynomials. To implement our numerical algorithm, we use Chebyshev polynomial basis functions $T_k(x)$ up to n-orders, i.e., from order-0 to order-(n-1) $k \in \{0, 1, ..., n-1\}$, so we can approximate six equilibrium functional objects (i.e., the asset prices of durable capital $q_d(A, \lambda, \tilde{b})$) of non-durable capital $q_{nd}(A, \lambda, \tilde{b})$, the equilibrium risk-free rate $R_f(A, \lambda, \tilde{b})$, the marginal product of capital investment $\mu(A, \lambda, \tilde{b})$, the utility function $u(A, \lambda, \tilde{b})$, and the policy function on optimal consumption $c(A, \lambda, \tilde{b})$. Carrying on basis coefficients of the Chebyshev functional approximates can sufficiently help us back out the rest of the equilibrium functional objects defined in the recursive equilibrium. The six Chebyshev approximated functionals are stated as:

$$q_d(x; \lambda_i, A_j) = \sum_{k=0}^{k=n-1} d_{q_d,k,i_\lambda,j_A}(x) T_k(x) \tag{II.17}$$

$$q_{nd}(x; \lambda_i, A_j) = \sum_{k=0}^{k=n-1} d_{q_{nd},k,i_\lambda,j_A}(x) T_k(x) \tag{II.18}$$

$$R_f(x; \lambda_i, A_j) = \sum_{k=0}^{k=n-1} d_{Rf,k,i_\lambda,j_A}(x) T_k(x) \tag{II.19}$$

$$\mu(x; \lambda_i, A_j) = \sum_{k=0}^{k=n-1} d_{\mu,k,i_\lambda,j_A}(x) T_k(x) \tag{II.20}$$

$$u(x; \lambda_i, A_j) = \sum_{k=0}^{k=n-1} d_{U,k,i_\lambda,j_A}(x) T_k(x) \tag{II.21}$$

$$c(x; \lambda_i, A_j) = \sum_{k=0}^{k=n-1} d_{c,k,i_\lambda,j_A}(x) T_k(x) \tag{II.22}$$

in which $x$ takes discrete values of $x_j = 2(\tilde{b}_j - \underline{b})/(\bar{b} - \underline{b}) - 1$ derived from the grid space of $\tilde{b}_{t-1} \in \{\tilde{b}_1, ..., \tilde{b}_{n_b}\}$. We note that such changes of variables accommodates the fact that Chebyshev polynomial basis functions $T_k(x)$ are defined over $x \in [-1, 1]$. $\bar{b}$ and $\underline{b}$ thus capture the upper and lower bounds of the predetermined redefined debt position. $A_t = A_j \in \mathbf{A}$ and $\lambda_t = \lambda_i \in \mathbf{\Lambda}$ take discrete values from some discretization on the TFP $A_t$ and the liquidity shock process $\chi_t$ of grid points of $n_A$ and $n_\chi$ considering their correlations. The basis coefficient vectors $d_{q_d,k,i_\lambda,j_A}(x)$, $d_{q_{nd},k,i_\lambda,j_A}(x)$, $d_{Rf,k,i_\lambda,j_A}(x)$, $d_{\mu,k,i_\lambda,j_A}(x)$, $d_{U,k,i_\lambda,j_A}(x)$, and $d_{c,k,i_\lambda,j_A}(x)$ are therefore specific to the discrete values of aggregate states.

We reach our model solution once the iterations over the basis coefficients on different orders of Chebyshev polynomials at selected nodes of state variables obtain numerical convergence. This would effectively pin down the equilibrium objects. In terms of implementation, our functional approximations are based on Chebyshev polynomial basis functions $T_k(x)$ up to 3-orders, and we confirm that our results are not sensitive to increasing the order of the Chebyshev polynomials. We also select three nodes that are the roots of the Chebyshev polynomials and effectively map them to the grids of the redefined debt positions. We set the $\bar{b} = 5$ and $\underline{b} = 0$ for reaching the model solutions. It can be shown that our model solution is robust to expanding or shrinking the width of the node grids.

## II.4 Outline of the Algorithm

Finally, we outline the exact algorithm of our numerical routines to obtain the basis coefficients of Chebyshev polynomials that best approximate the equilibrium functional objects. Following Galindev and Lkhagvasuren (2010), we first generate discretized nodes of TFP shocks and the liquidity shocks of dimension $N^A \times N^\lambda = 3^2 = 9$, so we may consider their shock correlations. With $N^{\tilde{b}} = 3$ nodes of debt grids, the procedure continues in the form of iterations as below:

1. Conditional on each predetermined debt position $\tilde{b}_{t-1} = b_j$ and the realizations of $A_{i_\lambda}$ and $A_{j_A}$, we compute the conjectured $q^0_{d,t}$, $q^0_{nd,t}$ using guess basis coefficients. Thereafter, we solve for the implied investment-capital ratio $ik^1_t$, the consumption-capital ratio $c^1_t$, and the investment share into durable capital goods $\phi^1_t$, given Equations (II.13)(II.14) and (II.15).
2. Following the law of motion per Equation (II.10), conjectured $R^0_{f,t+1}$, and the computed capital growth $\Gamma_t$ by Equation (II.16), we solve for the implied $\tilde{b}_t$ and the networth $n_t$. As a result, we can compute conjectured future consumption $c^0_{t+1}$ and utility values $u^0_{t+1}$ to pin down the implied risk-free rate $R^1_{f,t+1}$ and the implied utility values $u^1_t$. Using Equations (II.4) and (II.5), we compute the stochastic discount factors $\tilde{M}_{t+1}$ and $M_{t+1}$
3. We then compute the implied borrowing limit $\tilde{b}^1 = R_{f,t+1}\theta[(1-\delta_d)\zeta q^0_{d,t} + (1-\delta_{nd})(1-\zeta)q^0_{nd,t}]$, and proceed to check if the constraint is binding. If $\tilde{b}_t > \tilde{b}^1$, we set $\tilde{b}_t = b^l$ and recompute the implied expected marginal product of capital $\mu^1$ and updated $\eta^1_t$ using Equations (II.8) and (II.9). Otherwise, $\eta^1_t = 0$, and we leave the redefined debt unchanged and solve for $\mu^1_t$ per Equation (II.7)
4. Depending on updated values of $\mu^1_t$, we solve for implied market equilibrium asset prices $q^1_{d,t}$ and $q^1_{nd,t}$ from the non-arbitrage conditions of capital investment in durable capital and non-durable capital, respectively.

5. Finally, we solve the basis coefficient vector in a linear equation system evaluated at each node of $x_j$ at the implied values of $q^1_{d,t}, q^1_{nd,t}, R^1_{f,t+1}, \mu^1_t, u^1_t$ and $c^1_t$, and then update the basis coefficient vectors stacked in a long vector $d$ in the following routine $d^* = z \cdot d^* + (1-z) \cdot d$ for which $z$ is some dampening parameter. The program stops if $norm(d^* - d) < tol$ for which $tol$ is some tolerance threshold.

### II.5 Computational Efficiency

For a given calibration and a dimension of $6 * 3 * 9 = 162$ for all basis coefficients related to functional approximates, our model is solved fairly quickly. Running on a PC with a processor of configuration Intel(R) Core(TM) i7-1065G7 CPU @ 1.30GHz along with a 32GB RAM, it takes about 10 seconds to obtain the model solution up to a tolerance criterion of $10^{-3}$.

## III Proof of Proposition 1

We prove Proposition 1 in two steps: first, given prices, the quantities satisfy the household's and the entrepreneurs' optimality conditions; second, the quantities satisfy the market-clearing conditions.

Since the optimization problems of households and firms are all standard convex programming problems, we only need to verify optimality conditions. Equation (II.6) is the household's first-order condition. Equation (II.14) is a normalized version of a resource constraint (16). Both of them are satisfied as listed in Proposition 1.

To verify that the entrepreneur $i$'s allocations $\{N_{i,t}, B_{i,t}, K^d_{i,t}, K^{nd}_{i,t}, L_{i,t}\}$ as constructed in Proposition 1 satisfy the first-order conditions for the optimization problem in equation (10), the first-order condition with respect to $B_{i,t}$ implies:

$$\mu^i_t = E_t\left[\widetilde{M}_{t+1}\right] R_{f,t+1} + \eta^i_t. \tag{III.1}$$

Similarly, the first-order condition for type-$d$ capital $K^d_{i,t+1}$ is:

$$\mu^i_t = E_t\left[\widetilde{M}^i_{t+1} \frac{\Pi_{K^d}\left(\bar{A}_{t+1}, z_{i,t+1}, K^d_{i,t+1}, K^{nd}_{i,t+1}\right) + (1-\delta_d)\, q_{d,t+1}}{q_{d,t}}\right] + \theta(1-\delta_d)\eta^i_t. \tag{III.2}$$

Finally, the optimality with respect to the choice of type-$nd$ capital $K_{i,t+1}^{nd}$ implies:

$$\mu_t^i = E_t\left[\widetilde{M}_{t+1}^i \frac{\Pi_{K^{nd}}\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right) + (1-\delta_{nd})\, q_{nd,t+1}}{q_{d,t}}\right] + \theta(1-\delta_{nd})\eta_t^i. \quad \text{(III.3)}$$

Next, the law of motion of the endogenous state variable $n$ can be constructed from equation (9):

$$\begin{aligned} n' &= (1-\lambda)\left[\begin{array}{c} \alpha\nu A' + \zeta\left(1-\delta_d\right) q_d\left(A', n'\right) + (1-\zeta)\left(1-\delta_{nd}\right) q_{nd}\left(A', n'\right) \\ -\theta\left[\zeta q_d\left(A, n\right) + (1-\zeta)\, q_{nd}\left(A, n\right)\right] R_f\left(A, n\right) \end{array}\right] \\ &\quad + \lambda\chi\frac{n}{\Gamma\left(A, n\right)}. \end{aligned} \quad \text{(III.4)}$$

Using the law of motion of the state variables, we can construct the normalized utility of the household as the fixed point of:

$$u\left(A, n\right) = \left\{(1-\beta)c\left(A, n\right)^{1-\frac{1}{\psi}} + \beta\Gamma\left(A, n\right)^{1-\frac{1}{\psi}}\left(E[u\left(A', n'\right)^{1-\gamma}]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The stochastic discount factors must be consistent with household utility maximization:

$$M' = \beta\left[\frac{c\left(A', n'\right)\Gamma\left(A, n\right)}{c\left(A, n\right)}\right]^{-\frac{1}{\psi}}\left[\frac{u\left(A', n'\right)}{E\left[u\left(A', n'\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}, \quad \text{(III.5)}$$

$$\widetilde{M}' = M'[(1-\lambda)\,\mu\left(A', n'\right) + \lambda]. \quad \text{(III.6)}$$

In our setup, we assume that the idiosyncratic shock $z_{i,t+1}$ is observed before the decisions on $K_{i,t+1}^d$ and $K_{i,t+1}^{nd}$ are made, and thus can construct an equilibrium in which $\mu_t^i$ and $\eta_t^i$ are equalized across all the firms because $\frac{\partial}{\partial K_{i,t+1}^d}\Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right) = \frac{\partial}{\partial K_{i,t+1}^{nd}}\Pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}^d, K_{i,t+1}^{nd}\right)$ are the same for all $i$.

Our next step involves verifying the market-clearing conditions. Given the initial conditions (initial net worth $N_0$, $\frac{K_1^d}{K_1^{nd}} = \frac{\zeta}{1-\zeta}$, $N_{i,0} = z_{i,1}N_0$) and the net worth injection rule for new entrant firms ($N_{t+1}^{entrant} = \chi N_t$ for all $t$), we establish the market-clearing conditions using the following lemma. It's important to note that our model accommodates scenarios in which the collateral constraint occasionally becomes binding. The treatment of cases for which this constraint is binding or not is handled similarly.

**Lemma III.1.** *The optimal allocations* $\{N_{i,t}, B_{i,t}, K_{i,t+1}^d, K_{i,t+1}^{nd}\}$ *constructed as described in*

*Proposition 1 satisfy the market-clearing conditions:*

$$K^d_{t+1} = \int K^d_{i,t+1}\, di, \quad K^{nd}_{t+1} = \int K^{nd}_{i,t+1}\, di, \quad N_t = \int N_{i,t}\, di, \tag{III.7}$$

*for all* $t \geq 0$.

Before proving this lemma, we discuss the timing of the liquidation shock for a firm's entry and exit. As outlined in Section 4.1, the dynamics of the idiosyncratic shock $z_{i,t}$ follow:

$$z_{i,t+1} = z_{i,t}\, e^{\varepsilon_{i,t+1}},$$

in which $\varepsilon_{i,t+1}$ is independently and identically distributed (i.i.d) across firms and over time. Additionally, we assume that $E[e^{\varepsilon_{i,t+1}}] = e^{\mu+\frac{1}{2}\sigma^2}$ for simplicity's sake. It's important to point out that the realization of the liquidation shock $\lambda_{t+1}$ and the idiosyncratic productivity shock $\varepsilon_{i,t+1}$ occur in the morning of $t+1$, before the production takes place.

After the realization of $\lambda_{t+1}$ and $\varepsilon_{i,t+1}$, a fraction of $1-\lambda_{t+1}$ of firms continue to operate in the economy and use their planned $K^d_{i,t+1}$ and $K^{nd}_{i,t+1}$ for production. Simultaneously, a fraction of $\lambda_{t+1}$ firms will liquidate and exit the economy. At the same time, an equal fraction of $\lambda_{t+1}$ of new firms are born. These new firms do not generate any production at time $t+1$ but plan their $K^d_{i,t+2}$ and $K^{nd}_{i,t+2}$ for production at time $t+2$. The initial productivity of these new firms is denoted by $\bar{z}_{t+2}$ and is conditional on not being liquidated at time $t+2$.

The total amount of productivity $z_t$ that is involved in production at time $t+1$ is denoted as $Z_{t+1} = \int z_{i,t} di$. The evolution of $Z_{t+1}$ follows the following steps:

$$\begin{aligned} Z_{t+1} &= (1-\lambda_t)\int z_{i,t+1} di + \lambda_t \bar{z}_{t+1} \\ &= (1-\lambda_t)\int z_{i,t} e^{\varepsilon_{i,t+1}} di + \lambda_t \bar{z}_{t+1} \\ &= (1-\lambda_t)\int z_{i,t} di \int e^{\varepsilon_{i,t+1}} di + \lambda_t \bar{z}_{t+1} \text{ (Independence)} \\ &= (1-\lambda_t) Z_t\, e^{\mu+\frac{1}{2}\sigma^2} + \lambda_t \bar{z}_{t+1} \text{ (Law of Large Number)}. \end{aligned}$$

We normalize the aggregation of productivity to be one in the steady-state (i.e., $Z_{t+1} = Z_t = 1$.) Therefore, the normalized initial productivity denotes:

$$\bar{z}_{t+1} = \frac{1}{\lambda_t}\Big[1 - (1-\lambda_t)e^{\mu+\frac{1}{2}\sigma^2}\Big].$$

To prove Lemma III.1, we will use induction. Let's start with the initial conditions for $t = 0$. We have $N_{i,0} = z_{i,1} N_0$, in which $z_{i,1}$ is chosen from the stationary distribution of

$z$. We will discuss both the binding constraint case and the non-binding constraint case separately.

If the constraint is binding for $t = 0$, then the individual entrepreneur $i$'s capital decisions $K^d_{i,t+1}, K^{nd}_{i,t+1}$ must satisfy the following conditions:

$$N_{i,0} = [1-\theta(1-\delta_d)]\, q_{d,0}K^d_{i,1} + [1-\theta(1-\delta_{nd})]\, q_{nd,0}K^{nd}_{i,1}, \quad \text{(III.8)}$$

$$K^d_{i,1} + K^{nd}_{i,1} = z_{i,1}(K^d_1 + K^{nd}_1). \quad \text{(III.9)}$$

Clearly, we solve $K^d_{i,1}$ and $K^{nd}_{i,1}$ according to the above two equations, in which the solutions for $K^d_{i,1}$ and $K^{nd}_{i,1}$ denote $K^d_{i,1} = z_{i,1}K^d_1$ and $K^{nd}_{i,1} = z_{i,1}K^{nd}_1$. In turn, $B_{i,0} = z_{i,1}B_0$.

Suppose that the constraint is not binding for $t = 0$. The aggregate borrowing constraint can be expressed as:

$$B_0 \leq \theta(1-\delta_d)q_{d,1}K^d_1 + \theta(1-\delta_{nd})q_{nd,1}K^{nd}_1, \quad \text{(III.10)}$$

in which the inequality ensures that the aggregate borrowing does not exceed the fraction of capital investment that can be financed through external borrowing in the first period.

To initiate the recursion process, we assume that the same allocation rule is applied as in the case when the constraint is binding (i.e., $K^d_{i,1}$ and $K^{nd}_{i,1}$.) This allows us to demonstrate that $B_{i,0} = z_{i,1}B_0$ and that the borrowing constraint remains non-binding at the firm level.

Given that $Z_1 = \int z_{i,t}di = 1$, the following conditions hold at the end of period 0:

$$\int K^d_{i,1}\, di = K^d_1, \quad \int K^{nd}_{i,1}\, di = K^{nd}_1, \quad \int N_{i,0}\, di = N_0. \quad \text{(III.11)}$$

At the beginning of period 1, the realization of $\lambda_1$ occurs. A fraction of $1-\lambda_1$ of firms continue to exist in the economy for production, utilizing the planned $K^d_{i,1}$ and $K^{nd}_{i,1}$. After production and repayment of their debt, firm $i$'s net worth is given by:

$$N_{i,1} = \alpha A_1(K^d_{i,1} + K^{nd}_{i,1}) + (1-\delta_d)q_{d,1}K^d_{i,1} + (1-\delta_{nd})q_{nd,1}K^{nd}_{i,1} - R_{f,1}B_{i,0}. \quad \text{(III.12)}$$

On the other hand, a fraction of $\lambda_1$ firms are liquidated and re-enter the economy with an initial net worth of $N^{entrant}_1$, which is given by:

$$N^{entrant}_1 = \chi\left[\alpha A_1(K^d_{i,1} + K^{nd}_{i,1}) + (1-\delta_d)q_{d,1}K^d_{i,1} + (1-\delta_{nd})q_{nd,1}K^{nd}_{i,1}\right]. \quad \text{(III.13)}$$

These newly born firms do not engage in production during period 1. Instead, they wait for the realization of $z_{i,2}$ and plan their capital allocations $K^d_{i,2}$ and $K^{nd}_{i,2}$ for the next period.

To track the aggregation, we consider the total net worth of both existing firms and newly born firms (i.e., new entrants) at $t = 1$:

$$\begin{aligned}(1-\lambda_1)\int N_{i,1}di + \lambda_1 N_1^{entrant} &= (1-\lambda_1)\int \Big[\alpha A_1(K_{i,1}^d + K_{i,1}^n) + (1-\delta_d)q_{d,1}K_{i,1}^d \\ &\quad +(1-\delta_{nd})q_{nd,1}K_{i,1}^{nd} - R_{f,1}B_{i,0}\Big]di \\ &\quad +\lambda_1\chi\Big[\alpha A_1(K_{i,1}^d + K_{i,1}^n) + (1-\delta_d)q_{d,1}K_{i,1}^d + (1-\delta_{nd})q_{nd,1}K_{i,1}^{nd}\Big].\end{aligned}$$

At the end of period 1, each firm, including existing firms and new entrants, will observe $z_{i,2}$ and plan $K_{i,2}^d$ and $K_{i,2}^{nd}$ for period 2 accordingly. After realizing the liquidation shock at $t = 2$, firms generate production without liquidation. Similarly, the productivity of exiting firms is denoted as $z_{i,2} = z_{i,1}\, e^{\varepsilon_{i,2}}$, while the productivity of newly born exiting firms is denoted as $\bar{z}_2$, which is given by $\bar{z}_2 = \frac{1}{\lambda_1}\Big[1-(1-\lambda_1)e^{\mu+\frac{1}{2}\sigma^2}\Big]$. The total productivity at $t = 2$ is calculated as follows:

$$\begin{aligned}Z_2 &= (1-\lambda_1)\int z_{i,2}di + \lambda_1\bar{z}_2 \\ &= (1-\lambda_1)\int z_{i,1}e^{\varepsilon_{i,2}}di + \lambda_1\bar{z}_2 \\ &= (1-\lambda_1)\int z_{i,1}di\int e^{\varepsilon_{i,2}}di + \lambda_1\bar{z}_2 \text{ (Independence)} \\ &= (1-\lambda_t)Z_1\, e^{\mu+\frac{1}{2}\sigma^2} + \lambda_1\bar{z}_2 \text{ (Law of Large Number)}.\end{aligned}$$

Next, firm $i$ decides the allocation between durable and non-durable capital for production. We note that when a firm's financial constraint is not binding, the specific capital allocation among firms for different capital types is not uniquely determined, given the perfect substitutability of two capital types. A firm, therefore, has different paths of capital financing over time. Concerning this indeterminacy issue, in the following, we present a way to determine $K_{i,2}^d$ and $K_{i,2}^{nd}$ separately by constructing a modified version of equation (III.8). Recalling the non-binding case, the borrowing constraint at the aggregate level is denoted as:

$$B_1 \leq \theta(1-\delta_d)q_{d,1}K_2^d + \theta(1-\delta_{nd})q_{nd,1}K_2^{nd}. \tag{III.14}$$

We take the aggregate measure of constraint slackness in period 1, $\Delta_1 \geq 0$, according to equation (41). It follows that:

$$B_1 = (\theta-\Delta_1)\Big[(1-\delta_d)q_{d,1}K_2^d + (1-\delta_{nd})q_{nd,1}K_2^{nd}\Big], \tag{III.15}$$

and

$$\Delta_1 = \theta - \frac{B_1}{(1-\delta_d)q_{d,1}K_2^d + (1-\delta_{nd})q_{nd,1}K_2^{nd}}. \tag{III.16}$$

$\Delta_1$ equals 0 when the collateral constraint is binding, under which the capital allocation will be uniquely determined at the firm level. By allowing for $\Delta_1 \geq 0$, our capital allocation scheme for the first period is close enough to that of the determinacy case when the aggregate constraint is binding as $\Delta_1 \to 0$. We, therefore, regard our firm-level capital allocation as one of the many possible distributional realizations consistent with the equilibrium at the aggregate level.

We further assume that the borrowing constraint in equation (III.14) holds at the firm level:

$$B_{i,1} = (\theta - \Delta_1)\Big[(1-\delta_d)q_{d,1}K_{i,2}^d + (1-\delta_{nd})q_{nd,1}K_{i,2}^{nd}\Big]. \tag{III.17}$$

Combining the system in equation (III.8) with equation (III.17), we can solve for $K_{i,2}^d$ and $K_{i,2}^{nd}$ simultaneously:

$$\begin{aligned} N_{i,1} &= [1-(\theta-\Delta_1)(1-\delta_d)]\, q_{d,1}K_{i,2}^d + [1-(\theta-\Delta_1)(1-\delta_{nd})]\, q_{nd,1}K_{i,2}^{nd}, \\ K_{i,1}^d + K_{i,1}^{nd} &= z_{i,1}(K_1^d + K_1^{nd}). \end{aligned}$$

The solution for $K_{i,2}^d$ and $K_{i,2}^{nd}$ is given by:

$$\begin{aligned} K_{i,2}^d &= \frac{N_{i,1} - z_{i,2}\,[1-(\theta-\Delta_1)(1-\delta_{nd})]\, q_{nd,1}(K_2^d + K_2^{nd})}{[1-(\theta-\Delta_1)(1-\delta_d)]\, q_{d,1} - [1-(\theta-\Delta_1)(1-\delta_{nd})]\, q_{nd,1}}, \\ K_{i,2}^{nd} &= z_{i,2}(K_2^d + K_2^{nd}) - K_{i,2}^d. \end{aligned}$$

According to the above solution, durable and non-durable capital and net worth among existing firms are no longer proportional to $z_{i,2}$. However, given that $\int N_{i,1}di = N_1$ and $\int z_{i,2}di = 1$, we integrate the solution across $i$ and obtain the result:

$$\begin{aligned} \left\{ \begin{array}{c} [1-(\theta-\Delta_1)(1-\delta_d)]\, q_{d,1}\int K_{i,2}^d di \\ +\,[1-(\theta-\Delta_1)(1-\delta_{nd})]\, q_{nd,1}\int K_{i,2}^{nd} di \end{array} \right\} &= \int N_{i,1}di = N_1, \\ \int K_{i,2}^d di + \int K_{i,2}^{nd} di &= \int z_{i,2}di\left(K_2^d + K_2^{nd}\right) = K_2^d + K_2^{nd}. \end{aligned}$$

It is not necessary to complete the induction argument. If the market-clearing condition holds for $t+1$, then it must hold for $t+2$ and for all rest periods. The following claim characterizes this property:

**Claim 1.** *Suppose* $\int K^d_{i,t+1}\,di = K^d_{t+1}$, $\int K^{nd}_{i,t+1}\,di = K^{nd}_{t+1}$, $\int N_{i,t}\,di = N_t$, *and*

$$N^{entrant}_{t+1} = \chi\Big[\alpha A_{t+1}(K^d_{i,t+1} + K^n_{i,t+1}) + (1-\delta_d)q_{d,t+1}K^d_{i,t+1} + (1-\delta_{nd})q_{nd,t+1}K^{nd}_{i,t+1}\Big] \quad \text{(III.18)}$$

*then*

$$\int K^d_{i,t+2}\,di = K^d_{t+2}, \qquad \int K^{nd}_{i,t+2}\,di = K^{nd}_{t+2}, \qquad \int N_{i,t+1}\,di = N_{t+1} \quad \text{(III.19)}$$

*for all* $t \geq 0$.

1. Using the law of motion for the net worth of existing firms, we can rewrite the total net worth of all surviving firms as follows:

$$\begin{aligned} &(1-\lambda_{t+1})\int N_{i,t+1}\,di \\ = \;& (1-\lambda_{t+1})\int \begin{bmatrix} \alpha A_{t+1}\left(K^d_{i,t+1} + K^{nd}_{i,t+1}\right) + (1-\delta_d)\,q_{d,t+1}K^d_{i,t+1} \\ +(1-\delta_{nd})\,q_{nd,t+1}K^{nd}_{i,t+1} - R_{f,t+1}B_{i,t} \end{bmatrix} di \\ = \;& (1-\lambda_{t+1})\left[\alpha A_{t+1}\left(K^d_{t+1} + K^{nd}_{t+1}\right) + (1-\delta_d)\,q_{d,t}K^d_{t+1} + (1-\delta_{nd})\,q_{nd,t}K^{nd}_{t+1} - R_{f,t+1}B_t\right]. \end{aligned}$$

   Following the assumption $\int K^d_{i,t+1}di = K^d_{t+1}$, $\int K^{nd}_{i,t+1}di = K^{nd}_{t+1}$, and $\int B_{i,t}di = B_t = (\theta - \Delta_t)\left[(1-\delta_d)q_{d,t}K^d_{t+1} + (1-\delta_{nd})q_{nd,t}K^{nd}_{t+1}\right]$, and using the assignment rule for the net worth of new entrants $N^{entrant}_{t+1}$ in equation (III.18), we can demonstrate that the total net worth at the end of period $t+1$ across both survivors and new entrants satisfies $\int N_{i,t+1}di = N_{t+1}$, in which the aggregate net worth $N_{t+1}$ is given by equation (9).
2. At the end of period $t+1$, we have a pool of firms consisting of both existing ones with net worth given by equation (8) and new entrants. All of these firms will observe $z_{i,t+2}$ (for the new entrants $z_{i,t+2} = \bar{z}_{t+2}$) and begin production at the beginning of period $t+1$.
   We compute capital holdings for period $t+2$ for each firm $i$ using equations (4) and (21). At this point in time, capital holdings and net worth of all existing firms will not necessarily be proportional to $z_{i,t+2}$ due to the heterogeneity in the realization of idiosyncratic productivity shocks. However, we know that $\int N_{i,t+1}di = N_{t+1}$ and $\int z_{i,t+2}di = 1$. Similar to the case for period $t+1$, we integrate equations (4) and (21) across all $i$ and obtain the following two equations:

$$\begin{aligned} N_{t+1} \;=\;& \left[1-(\theta-\Delta_{t+1})(1-\delta_d)\right]q_{d,t+1}\int K^d_{i,t+2}\,di \\ &+\left[1-(\theta-\Delta_{t+1})(1-\delta_{nd})\right]q_{nd,t+1}\int K^{nd}_{i,t+2}\,di, \end{aligned} \quad \text{(III.20)}$$

$$K^d_{t+2} + K^{nd}_{t+2} = \int K^d_{i,t+2}\,di + \int K^{nd}_{i,t+2}\,di, \tag{III.21}$$

in which we have used $\int N_{i,t+1}\,di = N_{t+1}$ and $\int z_{i,t+2}\,di = 1$. In turn, this implies $\int K^d_{i,t+2}\,di = K^d_{t+2}$ and $\int K^{nd}_{i,t+2}\,di = K^{nd}_{t+2}$. Hence, the claim is proven.

In summary, we have demonstrated that equilibrium prices and quantities outlined in Proposition 1 adhere to optimality conditions of households and entrepreneurs, and that the quantities also satisfy market-clearing conditions.

Finally, we present a recursive relationship that can be utilized to solve for $\Theta(A, n)$ based on the equilibrium derived in Proposition 1. The recursion (10) implies:

$$\begin{aligned}
\mu_t N_{i,t} + \Theta_t z_{i,t+1}(K^d_t + K^{nd}_t) &= E_t\big[M_{t+1}(1-\lambda_{t+1})\big(\mu_{t+1}N_{i,t+1} + \Theta_{t+1}(K^d_{t+1} + K^{nd}_{t+1})z_{i,t+2}\big) + \lambda_{t+1}N_{i,t+1}\big] \\
&= E_t\big[M_{t+1}\{(1-\lambda_{t+1})\mu_{t+1} + \lambda_{t+1}\}N_{i,t+1}\big] \\
&\quad +(1-\lambda_{t+1})z_{i,t+1}E_t\big[M_{t+1}\Theta_{t+1}(K^d_{t+1} + K^{nd}_{t+1})\big] \\
&= E_t\big[\widetilde{M}_{t+1}N_{i,t+1}\big] + (1-\lambda_{t+1})z_{i,t+1}E_t\big[M_{t+1}\Theta_{t+1}(K^d_{t+1} + K^{nd}_{t+1})\big].
\end{aligned}$$

We next begin by simplifying the term $E_t\left[\widetilde{M}_{t+1}N_{i,t+1}\right]$. We note that an intermittently binding collateral constraint, combined with the entrepreneur's budget constraint (4), leads to the following condition:

$$\left[1 - (\theta - \Delta_t)(1-\delta_d)\right] q_{d,t} K^d_{i,t+1} + \left[1 - (\theta - \Delta_t)(1-\delta_{nd})\right] q_{nd,t} K^{nd}_{i,t+1} = N_{i,t}. \tag{III.22}$$

Equation (III.22), along with the optimality condition (21), determines the functions of $K^d_{i,t+1}$ and $K^{nd}_{i,t+1}$ in terms of $N_{i,t}$ and $z_{i,t+1}$:

$$K^d_{i,t+1} = \frac{N_{i,t} - z_{i,t+1}\left[1 - (\theta - \Delta_t)(1-\delta_{nd})\right] q_{nd,t}(K^d_{t+1} + K^{nd}_{t+1})}{\left[1 - (\theta - \Delta_t)(1-\delta_d)\right] q_{d,t} - \left[1 - (\theta - \Delta_t)(1-\delta_{nd})\right] q_{nd,t}}, \tag{III.23}$$

$$K^{nd}_{i,t+1} = \frac{z_{i,t+1}\left[1 - (\theta - \Delta_t)(1-\delta_d)\right] q_{d,t}(K^d_{t+1} + K^{nd}_{t+1}) - N_{i,t}}{\left[1 - (\theta - \Delta_t)(1-\delta_d)\right] q_{d,t} - \left[1 - (\theta - \Delta_t)(1-\delta_{nd})\right] q_{nd,t}}. \tag{III.24}$$

Utilizing the outcomes from equation (III.23) and the firm $i$'s net worth law of motion in equation (8), we can express $N_{i,t+1}$ as a linear function of $N_{i,t}$ and $z_{i,t+1}$:

$$\begin{aligned} N_{i,t+1} &= z_{i,t+1}\alpha A_{t+1}\left(K^{d}_{t+1}+K^{nd}_{t+1}\right) \\ &+ (1-\delta_d)\, q_{d,t+1}\frac{N_{i,t}-z_{i,t+1}\left[1-(\theta-\Delta_t)(1-\delta_{nd})\right]q_{nd,t}(K^{d}_{t+1}+K^{nd}_{t+1})}{\left[1-(\theta-\Delta_t)(1-\delta_d)\right]q_{d,t}-\left[1-(\theta-\Delta_t)(1-\delta_{nd})\right]q_{nd,t}} \\ &+ (1-\delta_{nd})\, q_{nd,t+1}\frac{z_{i,t+1}\left[1-(\theta-\Delta_t)(1-\delta_d)\right]q_{d,t}(K^{d}_{t+1}+K^{nd}_{t+1})-N_{i,t}}{\left[1-(\theta-\Delta_t)(1-\delta_d)\right]q_{d,t}-\left[1-(\theta-\Delta_t)(1-\delta_{nd})\right]q_{nd,t}} \\ &- R_{f,t+1}(\theta-\Delta_t)\,(1-\delta_d)\, q_{d,t}\frac{N_{i,t}-z_{i,t+1}\left[1-(\theta-\Delta_t)(1-\delta_{nd})\right]q_{nd,t}(K^{d}_{t+1}+K^{nd}_{t+1})}{\left[1-(\theta-\Delta_t)(1-\delta_d)\right]q_{d,t}-\left[1-(\theta-\Delta_t)(1-\delta_{nd})\right]q_{nd,t}} \\ &- R_{f,t+1}(\theta-\Delta_t)\,(1-\delta_{nd})\, q_{nd,t}\frac{z_{i,t+1}\left[1-(\theta-\Delta_t)(1-\delta_d)\right]q_{d,t}(K^{d}_{t+1}+K^{nd}_{t+1})-N_{i,t}}{\left[1-(\theta-\Delta_t)(1-\delta_d)\right]q_{d,t}-\left[1-(\theta-\Delta_t)(1-\delta_{nd})\right]q_{nd,t}}. \end{aligned}$$

We are specifically concerned with the coefficients related to $z_{i,t+1}$. Collecting the terms that incorporate $z_{i,t+1}$ on both sides of equation (III.22), we obtain:

$$\Theta_t z_{i,t+1}\left(K^{d}_{t+1}+K^{nd}_{t+1}\right) = z_{i,t+1}\left(K^{d}_{t+1}+K^{nd}_{t+1}\right)\times Term,$$

in which

$$\begin{aligned} Term &= E_t\left[\widetilde{M}_{t+1}\left\{\begin{array}{c} \alpha A_{t+1} \\ +(1-\delta_d)q_{d,t+1}\left(\frac{-[1-(\theta-\Delta_t)(1-\delta_{nd})]q_{nd,t}}{[1-(\theta-\Delta_t)(1-\delta_d)]q_{d,t}-[1-(\theta-\Delta_t)(1-\delta_{nd})]q_{nd,t}}\right) \\ +(1-\delta_{nd})q_{nd,t+1}\left(\frac{[1-(\theta-\Delta_t)(1-\delta_d)]q_{d,t}}{[1-(\theta-\Delta_t)(1-\delta_d)]q_{d,t}-[1-(\theta-\Delta_t)(1-\delta_{nd})]q_{nd,t}}\right) \\ -R_{f,t}\theta q_{d,t}\left(\frac{-[1-(\theta-\Delta_t)(1-\delta_{nd})]q_{nd,t}}{[1-(\theta-\Delta_t)(1-\delta_d)]q_{d,t}-[1-(\theta-\Delta_t)(1-\delta_{nd})]q_{nd,t}}\right) \\ -R_{f,t}\theta q_{nd,t}\left(\frac{[1-(\theta-\Delta_t)(1-\delta_d)]q_{d,t}}{[1-(\theta-\Delta_t)(1-\delta_d)]q_{d,t}-[1-(\theta-\Delta_t)(1-\delta_{nd})]q_{nd,t}}\right) \end{array}\right\}\right] \\ &\quad + (1-\lambda)\, E_t\left[M_{t+1}\Theta_{t+1}\right]. \end{aligned}$$

We can simplify the first term by utilizing the first-order conditions (II.7)-(II.9), which results in:

$$E_t\left[\widetilde{M}_{t+1}\left\{\alpha\left(1-\nu\right)A_{t+1}\right\}\right].$$

Therefore, we arrive at the following recursive relationship for $\Theta\left(A,n\right)$:

$$\begin{aligned} \Theta\left(A,n\right) &= \left[1-\delta+i\left(A,n\right)\right]\Big\{\alpha\left(1-\nu\right)E\left[M'\left\{\lambda+(1-\lambda)\,\mu\left(A',n'\right)\right\}A'\right] \\ &\quad + (1-\lambda)\,E\left[M'\Theta\left(A',n'\right)\right]\Big\}. \end{aligned} \tag{III.25}$$

The term $\alpha(1-\nu) A'$ represents the firm's profit due to decreasing returns to scale. It's clear that $\Theta(A, n)$ can be interpreted as the present value of profit. In the scenario of constant returns to scale, $\Theta(A, n) = 0$.

# IV Data Construction

This section outlines how we (i) form our samples of firms for our empirical analysis and (ii) create firm characteristics to account for underlying fundamentals.

## IV.1 Asset Prices and Accounting Data

Our dataset comprises firms that are common to both Compustat and CRSP (Center for Research in Security Prices). Accounting data are sourced from Compustat, while stock return data are gathered from CRSP. Our chosen firms meet the following criteria: they consist of positive durability data, there are no missing SIC codes, and their domestic common shares (SHRCD = 10 and 11) are traded on NYSE, AMEX, and NASDAQ. We exclude utility firms with four-digit SIC codes between 4900 and 4999, finance firms with SIC codes between 6000 and 6999 (encompassing finance, insurance, trusts, and real estate sectors), as well as public administrative firms with SIC codes between 9000 and 9999. Following Campello and Giambona (2013), we omit firm-year observations with total assets or sales values under $1 million. Additionally, we follow Fama and French (1993) and exclude closed-end funds, trusts, American Depository Receipts, Real Estate Investment Trusts, and units of beneficial interest. To counteract backfilling bias, we require that firms be listed on Compustat for at least two years before being included in our sample. Macroeconomic data are sourced from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve in St. Louis.

# V Additional Empirical Evidence

In this section, we present supplementary empirical findings regarding the connection between asset durability and various other firm characteristics. Additionally, we present a summary of the statistics for asset durability across different industries.

## V.1 More Detailed Firm Characteristics

Table IA.4 provides an overview of the relationship between variations in asset durability among firms and various other firm characteristics. The table presents the average asset durability and corresponding characteristics across five portfolios, which are sorted based on firm-level asset durability, specifically among financially constrained firms.

**[Place Table IA.4 about here]**

In our sample, we have a total of 1,821 firms. These firms are divided into five portfolios based on asset durability, with each portfolio representing a quintile ranging from the lowest to the highest durability. The distribution of firms across these portfolios is relatively even, with the number of firms in each portfolio ranging from 301 to 417 on average.

Asset durability varies significantly across these portfolios, spanning a range from 7.69 to 18.00. Interestingly, the size of firms does not exhibit substantial variation, although it does follow a hump-shaped pattern across the durability portfolios.

Examining other firm characteristics, we observe that firms with lower asset durability tend to have lower book-to-market ratios (B/M) and higher investment rates (I/K) and Tobin's q, indicating a higher potential for investment opportunities. Additionally, firms with lower durability exhibit lower profitability as measured by the return on assets (ROA), along with lower borrowing capacity as measured by book leverage. These firms also appear to be more financially constrained, as evidenced by their lower values of SA and WW indices. These characteristics collectively suggest that firms facing financial constraints, those with limited tangibility and promising investment prospects, tend to opt for less durable assets.

Finally, we note a negative association between asset durability and collateralizability, implying that firms with higher asset durability may have more collateralizable assets compared to those with lower durability, which aligns with our model in equation (5).

## V.2 Summary Statistics across Industries

Table IA.5 presents the average values of asset durability and depreciation by considering tangible and intangible assets separately across various industries based on BEA industry classifications.

Clearly, asset durability and depreciation vary significantly across industries. For instance, industries like educational services and accommodations tend to have higher asset durability and lower depreciation, while other industries might exhibit the opposite pattern. The observed cross-industry variations in asset durability and depreciation are substantial, spanning from 10.84 to 49.49.

These findings highlight the importance of considering industry effects when analyzing the relationship between asset durability and other variables. By controlling for industry fixed effects, we ensure that our results are not influenced by idiosyncratic characteristics of any particular industry but rather capture the broader relationships between asset durability and various characteristics across firms within each industry.

[**Place Table IA.5 about here**]

## Table IA.1: Estimating the Market Price of Risk

In Panel A, we present GMM estimates of parameters of the stochastic discount factor SDF = $1 - b_M \times \widetilde{\text{MKT}} - b \times \text{Macro}$ by using the quintile portfolios sorted on asset durability. Macro refers to the yield spread between Moody's Baa and Aaa corporate bond yields and a GZ credit spread. $\widetilde{\text{MKT}}$ refers to the market factor that is orthogonal to the model-implied financial shocks. $\widetilde{\varepsilon}_A$ denotes the aggregate productivity shocks that are orthogonal to the structural innovations to the liquidation probability, $\widetilde{\varepsilon}_x = \varepsilon_x$. We conduct our normalization such that $E[\text{SDF}] = 1$ (See, e.g., Cochrane (2005)). We report $t$-statistics and computed errors using the Newey-West procedure adjusted for three lags. As a measure of fit, we report the sum of squared errors (SSQE), mean absolute pricing errors (MAPE), and the $J$-statistic of the overidentifying restrictions of our model. Given the Euler equation $\text{E}[\text{SDF} \times R_i^e] = 0$, SSQE and MAPE are based on each testing asset $i$'s moment error $u_i$: $u_i = \frac{1}{T}\sum_{t=1}^{T}\left[\widehat{\text{SDF}} \times R_{i,t}^e\right]$. SSQE and MAPE are defined as $\sum_{i=1}^{N} u_i \times u_i$ and $\frac{1}{N}\sum_{i=1}^{N}|u_i|$, in which $N$ denotes the number of testing assets. In Panel B, we present GMM-implied testing portfolios' risk exposure ($\beta^i_{\text{MKT}}$, $\beta^i_{\text{Yield}}$, and $\beta^i_{\text{GZ}}$) to market factor and financial shocks, together with GMM-implied pricing errors ($\alpha^i$) in percentage.

| | **Panel A: Price of Risk** | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| $\widetilde{\text{MKT}}$ | 0.69 | 0.25 | 0.59 |
| [t] | 9.33 | 2.67 | 8.25 |
| Yield Spread | | -1.00 | |
| [t] | | -7.25 | |
| GZ Spread | | | -0.29 |
| [t] | | | -2.05 |
| SSEQ (%) | 5.70 | 5.50 | 5.64 |
| MAPE (%) | 5.13 | 5.04 | 5.11 |
| $J$-test | 9.85 | 9.04 | 9.63 |
| p | 0.83 | 0.83 | 0.79 |
| $J$T-Diff | | 31.59 | 21.83 |
| p | | 0.03 | 0.05 |

| | **L** | **2** | **3** | **4** | **H** | **H-L** |
|---|---|---|---|---|---|---|
| | **SDF ($\widetilde{\text{MKT}}$ + Yield Spread)** | | | | | |
| $\beta^i_{\text{MKT}}$ | 27.79 | 24.64 | 21.43 | 21.64 | 22.24 | -5.55 |
| [t] | 10.08 | 11.41 | 9.42 | 6.33 | 4.82 | -1.63 |
| $\beta^i_{\text{Yield}}$ | 6.01 | 3.00 | 1.04 | 1.85 | -0.53 | -6.54 |
| [t] | 1.84 | 0.71 | 0.36 | 0.45 | -0.09 | -1.95 |
| $\alpha^i$ | -3.32 | 0.32 | -2.29 | -1.66 | 2.95 | -1.60 |
| [t] | -1.60 | 0.14 | -0.97 | -0.70 | 1.36 | -0.68 |
| | **SDF ($\widetilde{\text{MKT}}$ + GZ Spread)** | | | | | |
| $\beta^i_{\text{MKT}}$ | 25.81 | 23.96 | 21.13 | 20.96 | 22.51 | -3.30 |
| [t] | 23.13 | 8.47 | 8.09 | 6.50 | 7.37 | -1.51 |
| $\beta^i_{\text{GZ}}$ | 2.18 | 1.65 | 0.49 | 0.44 | -0.03 | -2.20 |
| [t] | 0.85 | 0.62 | 0.15 | 0.11 | -0.01 | -2.03 |
| $\alpha^i$ | -4.03 | 0.26 | -1.71 | -1.91 | 2.96 | 1.91 |
| [t] | -1.73 | 0.11 | -0.73 | -0.81 | 1.27 | 1.24 |

## Table IA.2: Asset Pricing Factor Tests

This table presents asset pricing factor tests for five portfolios sorted on asset durability relative to their industry peers, utilizing NAICS 3-digit industry classifications and rebalancing portfolios at the end of every June. Our results are based on monthly data, spanning from July 1978 to December 2017, and exclude utility, financial, and public administrative industries. The entire sample is divided into financially constrained and unconstrained firms, as classified by the dividend payment dummy (DIV). To account for risk exposure, we conduct time-series regressions of asset-durability-sorted portfolios' excess returns on the Fama-French five-factor model plus the collateralizability factor, which encompasses MKT, SMB, HML, RMW, CMA, LMH, and COL in Panel A. In Panel B, we report portfolio alphas and betas are reported by the HXZ q-factor model plus the collateralizability factor, which includes MKT, SMB, I/A, ROE, and COL. Data sources for the factors are specified accordingly. Betas and alphas are annualized by multiplying by 12. We estimate standard errors using the Newey-West correction, and corresponding t-statistics are reported in parentheses.

| | **L** | **2** | **3** | **4** | **H** | **H-L** |
|---|---|---|---|---|---|---|
| | | | **Panel A: FF5 + COL** | | | |
| $\alpha_{\text{FF5+COL}}$ | -4.13 | 2.51 | 1.55 | 0.43 | 4.02 | 8.14 |
| [t] | -2.06 | 1.44 | 0.94 | 0.29 | 2.52 | 3.38 |
| MKT | 1.28 | 1.14 | 1.15 | 1.13 | 1.17 | -0.11 |
| [t] | 24.57 | 32.69 | 29.01 | 36.65 | 33.10 | -2.22 |
| SMB | 0.51 | 0.46 | 0.36 | 0.46 | 0.43 | -0.08 |
| [t] | 5.97 | 6.35 | 6.22 | 8.25 | 7.54 | -0.91 |
| HML | -0.24 | -0.35 | -0.33 | -0.46 | -0.38 | -0.15 |
| [t] | -2.45 | -4.77 | -4.35 | -6.83 | -4.92 | -1.69 |
| RMW | -0.10 | -0.24 | -0.11 | 0.02 | -0.06 | 0.04 |
| [t] | -0.78 | -2.19 | -1.53 | 0.34 | -0.78 | 0.25 |
| CMA | -0.44 | -0.42 | -0.51 | -0.31 | -0.25 | 0.19 |
| [t] | -3.21 | -4.18 | -4.58 | -3.27 | -2.88 | 1.47 |
| COL | 0.10 | 0.13 | 0.13 | 0.09 | 0.03 | -0.07 |
| [t] | 2.67 | 3.50 | 3.69 | 2.88 | 0.83 | -1.67 |
| | | | **Panel B: HXZ + COL** | | | |
| $\alpha_{\text{HXZ+COL}}$ | -4.71 | 1.65 | 1.60 | -0.30 | 3.82 | 8.54 |
| [t] | -2.36 | 0.86 | 0.79 | -0.17 | 2.26 | 3.48 |
| MKT | 1.31 | 1.18 | 1.17 | 1.15 | 1.18 | -0.13 |
| [t] | 19.40 | 28.08 | 26.40 | 28.47 | 30.62 | -2.20 |
| SMB | 0.42 | 0.37 | 0.26 | 0.37 | 0.37 | -0.06 |
| [t] | 3.30 | 3.96 | 4.37 | 5.74 | 7.01 | -0.42 |
| I/A | -0.62 | -0.77 | -0.88 | -0.80 | -0.69 | -0.08 |
| [t] | -5.18 | -8.05 | -9.03 | -9.30 | -8.59 | -0.64 |
| ROE | -0.03 | -0.08 | -0.04 | 0.12 | 0.01 | 0.04 |
| [t] | -0.34 | -0.98 | -0.55 | 1.92 | 0.17 | 0.62 |
| COL | 0.17 | 0.24 | 0.21 | 0.18 | 0.11 | -0.06 |
| [t] | 3.36 | 6.21 | 6.36 | 6.13 | 3.83 | -1.15 |

## Table IA.3: Fama-Macbeth Regressions

This table presents the results of Fama-MacBeth regressions, in which we analyze individual stock excess returns based on their asset durability and alternative variables that are relevant in the literature. We conduct our regressions in a cross-sectional manner for each month, spanning from July of year $t$ to June of year $t+1$. Specifically, in each month, we regress the monthly excess returns of individual stocks (annualized by multiplying by 12) on the asset durability value from year $t-1$, various sets of control variables known by the end of June of year $t$, and industry fixed effects. Industry categories are defined using NAIC 3-digit industry classifications. To mitigate the influence of outliers, all independent variables are normalized to have a zero mean and one standard deviation, after winsorization at the 1st and 99th percentiles. Our reported t-statistics are computed based on standard errors that we estimated using the Newey-West correction. The sample period for the analysis spans from July 1978 to December 2017.

| | **(1)** | **(2)** | **(3)** | **(4)** | **(5)** | **(6)** | **(7)** | **(8)** | **(9)** | **(10)** | **(11)** | **(12)** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Durability | 2.13 | 3.62 | 2.76 | 1.74 | 2.31 | 2.09 | 1.56 | 1.95 | 1.84 | 1.29 | 1.46 | 1.93 |
| [t] | 3.44 | 5.24 | 4.28 | 2.35 | 3.23 | 3.37 | 2.74 | 3.10 | 3.08 | 2.13 | 2.86 | 3.14 |
| Colllateralizability | | -3.07 | | | | | | | | | | |
| [t] | | -3.87 | | | | | | | | | | |
| Operating Lev. | | | 1.46 | | | | | | | | | 2.18 |
| [t] | | | 2.86 | | | | | | | | | 3.79 |
| Log Inflex | | | | -0.51 | | | | | | | | 0.98 |
| [t] | | | | -1.25 | | | | | | | | 2.60 |
| Redeployability | | | | | -0.49 | | | | | | | 0.31 |
| [t] | | | | | -0.66 | | | | | | | 0.37 |
| Durable Output | | | | | | -5.01 | | | | | | -5.88 |
| [t] | | | | | | -3.10 | | | | | | -2.85 |
| O | | | | | | | -2.71 | | | | | 0.49 |
| [t] | | | | | | | -2.79 | | | | | 0.42 |
| Z | | | | | | | | -2.07 | | | | 0.22 |
| [t] | | | | | | | | -1.46 | | | | 0.19 |
| DD | | | | | | | | | -1.59 | | | -1.87 |
| [t] | | | | | | | | | -1.51 | | | -1.47 |
| FP | | | | | | | | | | -17.55 | | -18.49 |
| [t] | | | | | | | | | | -1.37 | | -3.85 |
| Log ME | | | | | | | | | | | -0.75 | 0.36 |
| [t] | | | | | | | | | | | -0.67 | 0.29 |
| Log B/M | | | | | | | | | | | 4.82 | 4.77 |
| [t] | | | | | | | | | | | 8.73 | 5.66 |
| ROA | | | | | | | | | | | 6.36 | 6.91 |
| [t] | | | | | | | | | | | 8.98 | 6.72 |
| I/K | | | | | | | | | | | -1.13 | -1.62 |
| [t] | | | | | | | | | | | -2.78 | -2.37 |
| OC/AT | | | | | | | | | | | 1.03 | 1.66 |
| [t] | | | | | | | | | | | 2.29 | 2.42 |
| R&D/AT | | | | | | | | | | | 5.71 | 6.11 |
| [t] | | | | | | | | | | | 7.05 | 6.97 |
| Book Lev. | -1.89 | -0.57 | -2.02 | -1.66 | -1.55 | -1.85 | -0.75 | -2.57 | -1.71 | -1.40 | -0.99 | -0.32 |
| [t] | -4.17 | -1.09 | -4.48 | -3.33 | -2.94 | -4.11 | -1.32 | -5.52 | -3.62 | -2.99 | -2.28 | -0.43 |
| Observations | 846,277 | 632,464 | 778,893 | 725,608 | 737,897 | 846,277 | 819,508 | 841,335 | 608,519 | 750,884 | 806,449 | 476,878 |
| R-squared | 0.09 | 0.10 | 0.09 | 0.11 | 0.09 | 0.09 | 0.09 | 0.09 | 0.10 | 0.09 | 0.11 | 0.14 |
| Industry FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |

## Table IA.4: Firm Characteristics

This table presents time-series averages of the cross-sectional median values of firm characteristics across five portfolios. These portfolios are sorted based on asset durability relative to their industry peers, and industry classifications are based on NAICS 3-digit codes. The portfolios are rebalanced at the end of every June. The sample used for this analysis covers the years from 1977 to 2016, and it excludes industries in the financial, utility, and public administrative sectors. To differentiate between financially constrained and unconstrained firms, we classify the entire sample into these two categories at the end of each June. This classification is based on the dividend payment dummy as indicated by the dividend payment dummy (DIV), following the approach outlined in Farre-Mensa and Ljungqvist (2016). We report the results for the five portfolios that are part of the financially constrained subsample. For a detailed understanding of the variables and their definitions, please refer to Table IA.6 in the Internet Appendix.

| **Variables** | **L** | **2** | **3** | **4** | **H** |
|---|---|---|---|---|---|
| Asset Durability | 7.69 | 9.99 | 11.45 | 14.24 | 18.00 |
| Depreciation | 0.19 | 0.16 | 0.15 | 0.13 | 0.11 |
| Log ME | 4.88 | 5.13 | 5.16 | 5.22 | 5.07 |
| B/M | 0.48 | 0.51 | 0.53 | 0.60 | 0.67 |
| I/K | 0.37 | 0.30 | 0.28 | 0.24 | 0.22 |
| q | 1.65 | 1.54 | 1.48 | 1.37 | 1.27 |
| ROA | 0.07 | 0.09 | 0.10 | 0.11 | 0.11 |
| ROE | 0.12 | 0.17 | 0.18 | 0.22 | 0.23 |
| OC/AT | 0.36 | 0.25 | 0.21 | 0.17 | 0.13 |
| R&D/AT | 0.03 | 0.03 | 0.03 | 0.00 | 0.00 |
| Collateralizability | 0.21 | 0.25 | 0.27 | 0.37 | 0.51 |
| Book Lev. | 0.13 | 0.19 | 0.21 | 0.28 | 0.32 |
| Short-term Lev. | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 |
| Long-term Lev. | 0.04 | 0.09 | 0.11 | 0.17 | 0.21 |
| TANT | 0.08 | 0.13 | 0.17 | 0.25 | 0.34 |
| SA | -2.47 | -2.68 | -2.80 | -2.91 | -2.92 |
| WW | -0.16 | -0.18 | -0.19 | -0.20 | -0.20 |
| Number of Firms | 365 | 345 | 301 | 393 | 417 |

## Table IA.5: Asset Durability and Depreciation across BEA Industries

This table provides summary statistics for the average asset durability and depreciation associated with tangible and intangible assets across various industries. The industries are categorized according to the BEA industry classifications. The data cover the period from 1977 to 2016.

| BEA Industries | Tangible | | Intangible | |
|---|---|---|---|---|
| **Industry Name** | **Durability** | **Depreciation** | **Durability** | **Depreciation** |
| Farms | 27.92 | 0.07 | 2.58 | 0.40 |
| Forestry, fishing, and related activities | 24.43 | 0.09 | 2.38 | 0.43 |
| Oil and gas extraction | 14.98 | 0.07 | 4.33 | 0.23 |
| Mining, except oil and gas | 20.56 | 0.07 | 4.50 | 0.23 |
| Support activities for mining | 13.67 | 0.09 | 3.40 | 0.30 |
| Utilities | 40.49 | 0.03 | 3.38 | 0.31 |
| Construction | 20.13 | 0.10 | 3.95 | 0.26 |
| Wood products | 22.67 | 0.07 | 4.61 | 0.23 |
| Nonmetallic mineral products | 20.65 | 0.07 | 5.90 | 0.17 |
| Primary metals | 21.28 | 0.07 | 5.73 | 0.17 |
| Fabricated metal products | 19.36 | 0.08 | 5.68 | 0.18 |
| Machinery | 20.94 | 0.07 | 5.68 | 0.18 |
| Computer and electronic products | 22.97 | 0.07 | 3.44 | 0.29 |
| Electrical equipment, appliances, and components | 23.98 | 0.06 | 5.89 | 0.17 |
| Motor vehicles, bodies and trailers, and parts | 17.97 | 0.08 | 3.19 | 0.31 |
| Other transportation equipment | 24.09 | 0.06 | 4.47 | 0.22 |
| Furniture and related products | 23.05 | 0.06 | 5.37 | 0.19 |
| Miscellaneous manufacturing | 22.33 | 0.07 | 5.86 | 0.17 |
| Food, beverage, and tobacco products | 21.90 | 0.07 | 5.55 | 0.18 |
| Textile mills and textile product mills | 22.65 | 0.06 | 5.46 | 0.18 |
| Apparel and leather and allied products | 26.52 | 0.06 | 5.73 | 0.17 |
| Paper products | 18.12 | 0.08 | 5.38 | 0.19 |
| Printing and related support activities | 19.06 | 0.08 | 5.02 | 0.21 |
| Petroleum and coal products | 21.09 | 0.07 | 5.86 | 0.17 |
| Chemical products | 22.25 | 0.07 | 8.09 | 0.12 |
| Plastics and rubber products | 18.44 | 0.08 | 5.72 | 0.18 |
| Wholesale trade | 24.93 | 0.08 | 4.13 | 0.25 |
| Retail trade | 33.63 | 0.05 | 4.05 | 0.26 |
| Air transportation | 19.23 | 0.07 | 3.28 | 0.31 |
| Railroad transportation | 44.31 | 0.03 | 4.30 | 0.25 |
| Water transportation | 18.99 | 0.06 | 4.08 | 0.26 |
| Truck transportation | 11.49 | 0.14 | 4.19 | 0.26 |
| Transit and ground passenger transportation | 35.17 | 0.05 | 3.50 | 0.30 |
| Pipeline transportation | 39.5 | 0.03 | 3.12 | 0.32 |
| Other transportation and support activities | 30.07 | 0.06 | 3.50 | 0.31 |
| Warehousing and storage | 37.45 | 0.04 | 3.88 | 0.28 |
| Publishing industries (including software) | 23.51 | 0.07 | 6.39 | 0.16 |
| Motion picture and sound recording industries | 29.43 | 0.05 | 7.86 | 0.13 |
| Broadcasting and telecommunications | 34.89 | 0.04 | 5.42 | 0.19 |
| Information and data processing services | 22.86 | 0.10 | 4.50 | 0.23 |
| Federal Reserve banks | 34.66 | 0.05 | 3.25 | 0.31 |
| Credit intermediation and related activities | 26.75 | 0.07 | 2.99 | 0.34 |
| Securities, commodity contracts, and investments | 35.37 | 0.04 | 3.12 | 0.32 |
| Insurance carriers and related activities | 33.83 | 0.05 | 3.10 | 0.33 |
| Funds, trusts, and other financial vehicles | 40.54 | 0.03 | 3.02 | 0.33 |
| Real estate | 40.04 | 0.03 | 2.89 | 0.35 |
| Rental and leasing services and lessors of intangible assets | 10.84 | 0.12 | 2.87 | 0.35 |
| Legal services | 31.14 | 0.06 | 2.57 | 0.40 |
| Computer systems design and related services | 31.76 | 0.07 | 2.83 | 0.35 |
| Miscellaneous professional, scientific, and technical services | 26.62 | 0.07 | 5.41 | 0.19 |
| Management of companies and enterprises | 35.71 | 0.04 | 3.23 | 0.31 |
| Administrative and support services | 29.09 | 0.07 | 2.79 | 0.36 |
| Waste management and remediation services | 48.14 | 0.05 | 3.91 | 0.26 |
| Educational services | 49.49 | 0.03 | 4.80 | 0.21 |
| Ambulatory health care services | 34.39 | 0.06 | 4.86 | 0.21 |
| Hospitals | 45.77 | 0.04 | 4.39 | 0.24 |
| Nursing and residential care facilities | 39.67 | 0.04 | 5.05 | 0.20 |
| Social assistance | 37.26 | 0.04 | 3.18 | 0.32 |
| Performing arts, spectator sports, museums, and related activities | 36.87 | 0.04 | 6.10 | 0.16 |
| Amusements, gambling, and recreation industries | 30.35 | 0.05 | 3.95 | 0.26 |
| Accommodation | 48.59 | 0.03 | 4.07 | 0.25 |
| Food services and drinking places | 27.15 | 0.07 | 4.16 | 0.24 |
| Other services, except government | 43.02 | 0.04 | 5.24 | 0.19 |

**Table IA.6: Definition of Variables**

| Variables | Definition | Sources |
|---|---|---|
| Durability | Details refer to Section 2.1 | BEA; Compustat |
| Depreciation | Details refer to Section 2.1 | BEA; Compustat |
| ME (real) | Market capitalization deflated by CPI at the end of June in year t. | CRSP |
| B/M | The ratio of book equity of fiscal year ending in year t-1 to market equity at the end of year t-1. | Compustat |
| Tobin's q | The sum of market capitalization at the end of the year and book value of preferred shares deducting inventories over total assets (AT). | CRSP; Compustat |
| I/K | The ratio of investment (CAPX) to purchased capital (PPENT). | Compustat |
| ROA | The ratio of operating income before depreciation (OIBDP) over total assets (AT). | Compustat |
| ROE | The ratio of operating income before depreciation (OIBDP) over book equity. | Compustat |
| OC/AT | Following Peters and Taylor (2017). | Compustat |
| R&D Intensity | Following Peters and Taylor (2017). | Compustat |
| Tangibility | The ratio of purchased capital (PPENT) to total assets (AT). | Compustat |
| Book Lev. | The sum of long-term liability (DLTT) and current liability (DLCT) divided by total assets (AT). | Compustat |
| Short-term Lev. | Current liability (DLCT) divided by total assets (AT). | Compustat |
| Long-term Lev. | Long-term liability (DLTT) divided by total assets (AT). | Compustat |
| DIV | Following Farre-Mensa and Ljungqvist (2016). | Compustat |
| SA Index | Following Hadlock and Pierce (2010). | Compustat |
| Credit Rating | The entire list of credit ratings is as follows: AA+, AA, and AA- = 6, A+, A, and A- = 5, BBB+, BBB, BBB- = 4, BB+, BB, BB- = 3, B+, B, and B- = 2, rating below B- or missing is 0. | Compustat |
| WW Index | Following Whited and Wu (2006). | Compustat |

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