Moral Hazard in Peer-to-Peer Insurance with Social Connection

Abstract

As a prominent InsurTech innovation, the emergence of peer-to-peer (P2P) insurance has introduced decentralized insurance arrangements with risk-sharing networks that rely on social connections to mitigate traditional market failures. We develop a theoretical framework to study ex-ante moral hazard in such settings when participants are embedded in social networks and loss-prevention effort is privately chosen. The model combines a risk-transfer rule and a risk-sharing rule and characterizes equilibrium effort under heterogeneous motivations, including selfish, warm-glow, and altruistic preferences. We show that social preferences and network exposure can attenuate moral hazard: stronger pro-social incentives increase equilibrium loss-prevention effort and improve risk outcomes for the pool. These results provide a theoretical foundation for how decentralized risk sharing, when embedded in social connections, can serve as an incentive device in insurance markets with unobservable effort.

Keywords: Moral hazard; Social connection; Peer-to-Peer insurance; InsurTech.
JEL: D82, G22, D85

1 Introduction

Insurance markets play a central role in managing individual exposure to rare but severe risks, yet they are persistently challenged by moral hazard (Cummins and Tennyson, 1996; Abbring et al., 2003; Kim et al., 2009; Spindler et al., 2014). Classic economic theory predicts that insurance coverage weakens incentives for loss prevention when effort is privately chosen and imperfectly observable.[1] To mitigate this incentive distortion, optimal contracts typically rely on partial coverage, deductibles, and monitoring, all of which impose efficiency costs. At the same time, recent technological advances have enabled new forms of decentralized insurance arrangements, often organized within groups whose members are socially connected or interact repeatedly (Fang et al., 2020). This contrast gives rise to a fundamental and unresolved research question: can decentralized and peer-based insurance models mitigate moral hazard, particularly in the absence of centralized enforcement and monitoring mechanisms, and can social connections and decentralized risk sharing discipline effort even when effort is unobservable?

This paper sheds light on this puzzle by studying moral hazard in peer-to-peer (P2P) insurance. First introduced in practice by the German InsurTech start up *Friendsurance* in 2010, P2P insurance has since expanded to many countries, with numerous InsurTech firms adopting similar models. In a typical P2P arrangement, a small group of individuals with similar insurance needs, such as family members or friends, pool risks on a P2P platform while transferring residual risk to a traditional insurer (see, for example, Denuit (2019); Denuit and Robert (2021b); Denuit, Dhaene and Robert (2022); Denuit, Dhaene, Ghossoub and Robert (2023)). Unlike conventional insurance contracts, losses are only partially transferred to an anonymous insurer and are instead partly borne within the group through an internal risk sharing rule. As a result, participants remain residual claimants on part of the aggregate loss, which fundamentally alters the nature of hidden

[1] Economists emphasize that information asymmetry, stemming from insurers' limited ability to observe insured effort, is central to moral hazard (Arrow, 1963; Pauly, 1968; Shavell, 1979; Arnott and Stiglitz, 1991; Research and Markets, 2018; Thakor, 2020; International Association of Insurance Supervisors, 2017; Moenninghoff and Wieandt, 2013; Institute of International Finance, 2015; World Bank Group, 2018). Moral hazard is expected to be weaker when losses are partially shared among socially connected individuals rather than fully borne by a traditional insurer.

action. Loss prevention effort no longer affects only an individual's own expected indemnity, but also the losses borne by peers. Consequently, effort incentives depend jointly on the risk transfer rule, the risk sharing rule, and the social environment of the pool.

These features give rise to two key distinctions between moral hazard in P2P insurance and its traditional counterpart. First, P2P insurance involves multiple participants whose effort choices interact strategically, rather than a single policyholder facing an insurer. An individual's effort can therefore encourage or discourage others' effort, depending on the incentive environment. Second, while effort in traditional insurance has the character of a private good, loss prevention effort in P2P insurance resembles a public good. The cost of effort is borne privately, but the benefit through lower aggregate losses is shared among all participants. Each participant's effort thus generates a positive externality by reducing pooled risk. Together, these features imply that moral hazard in P2P insurance is not merely a bilateral contracting problem, but a multi agent incentive problem with potential coordination frictions. When effort choices are strategic complements, the same contract can sustain both low effort and high effort outcomes. This possibility motivates our focus on equilibrium existence, bounds, and comparative statics rather than on the selection of a unique equilibrium.

To formalize these mechanisms, we develop a theoretical model of P2P insurance that integrates three elements absent from the classical moral hazard framework. First, insured individuals participate in a hybrid risk-transfer and risk-sharing arrangement, in which part of the aggregate loss is transferred to an insurer while the remainder is shared within the group. Second, loss-prevention effort is privately chosen and jointly determines the distribution of aggregate losses, giving rise to strategic interactions among participants. Third, individuals may derive utility not only from their own consumption, but also from exerting effort itself through warm-glow motives and from reducing losses borne by socially connected peers through altruistic concern. These features allow us to analyze how social networks reshape incentive provision in decentralized insurance markets.

The theoretical model delivers three main results. First, we show that moral hazard can be systematically attenuated in P2P insurance when participants exhibit pro-social motivations. Individuals who internalize part of the losses borne by their peers exert higher loss-prevention effort than they would under standard insurance contracts, even though effort remains unobservable. Second, we uncover substantial heterogeneity in equilibrium behavior. Self-interested participants free ride on the efforts of others, whereas individuals with warm-glow or altruistic preferences exert higher effort, thereby stabilizing the pool and amplifying loss prevention through strategic complementarity. As a result, the effectiveness of P2P insurance in mitigating moral hazard depends critically on the composition of social preferences within the group. Third, when risk-transfer and risk-sharing rules are chosen anticipating the induced effort equilibrium, optimal contracts allocate a smaller share of retained losses to more pro-social participants. Intuitively, individuals who exert greater effort generate positive externalities for the group and are compensated through more favorable risk-sharing arrangements. Numerical analysis confirms that these results are robust across a wide range of parameter values and network structures.

This paper contributes to several strands of the literature. It develops a multi agent framework with endogenous effort and heterogeneous social preferences for decentralized arrangements under aggregate risk, building on the classical theory of ex ante moral hazard (Ehrlich and Becker, 1972; Helpman and Laffont, 1975; Arnott and Stiglitz, 1988; Bolton and Dewatripont, 2005). Our framework allows for heterogeneity in preferences toward risk and insurance participation (Maccheroni et al., 2025) and extends this perspective by examining how such preferences interact with social connections and decentralized risk sharing to shape effort incentives and equilibrium outcomes. At the incentive level, the analysis shows that social connections and prosocial incentives generate endogenous discipline that mitigates effort distortions even in the absence of costly monitoring or partial coverage. At the mechanism level, by explicitly modeling social networks, we identify how social preferences and network structure interact with risk sharing to determine equilibrium effort and loss prevention, complementing prior work on mutual insurance (Cabrales et al., 2003; Lee and Ligon, 2001; von Bieberstein et al., 2019). From a design perspective, the framework delivers implications for decentralized risk sharing rules, showing that such rules shape incentives for loss prevention in addition to allocating losses, extending the growing literature on peer to peer and decentralized risk sharing.[2] Beyond incentives, the model also speaks to crisis risk by highlighting how social networks shape risk perception and behavioral responses through peer exposure and information transmission (Scherer and Cho, 2003; Hu, 2022; Wachinger et al., 2013).

From a regulatory perspective, our analysis highlights the economic value of social capital in decentralized insurance markets and informs the design of efficient risk sharing mechanisms. The theoretical results show that social and behavioral mechanisms can lower operating costs by fostering coordination, informal monitoring, and internalization of losses, a prediction supported by industry evidence.[3] These findings suggest that incentive alignment and peer based discipline can partially substitute for formal verification and enforcement across a wide range of risk sharing and financial intermediation settings. By reducing fraud risk and improving loss predictability, social and behavioral incentives may lower monitoring and capital costs and complement traditional prudential regulation in promoting efficiency and stability.

The rest of the paper is organized as follows. Section 2 presents the model and key assumptions. Section 3 characterizes equilibrium behavior and analyzes the role of social networks in mitigating moral hazard. Section 4 examines welfare implications and design considerations. Section 5 discusses regulatory and industry implications. Section 6 concludes.

2 Decentralized Arrangements under Aggregate Risk

This section develops a general framework for decentralized arrangements in which agents share exposure to an aggregate loss and privately choose loss prevention effort. The

[2]See Feng et al. (2022); Feng (2023); Denuit and Dhaene (2012); Denuit and Robert (2021a); Chen et al. (2023); Jiao et al. (2022); Dhaene et al. (2023); Abdikerimova and Feng (2022); Abdikerimova et al. (2024).

[3]For example, Friendsurance reports 20 percent to 40 percent fewer claims than traditional insurers because participants are less likely to defraud their peers, while Lemonade discourages opportunistic claims through its Giveback mechanism, which donates surplus premiums to nonprofit organizations. See `https://www.fastcompany.com/3021024/a-social-network-for-insurance-that-cuts-costs-and-reduces-fraud`.

objective is to clarify how incentives arise in a decentralized pool and how contractual primitives map into participants' loss exposure.

We begin with a general risk environment that applies to a broad class of decentralized arrangements in which agents retain exposure to a common aggregate loss. What distinguishes these decentralized arrangements from standard bilateral insurance is that participants remain residual claimants on a shared aggregate loss. The framework encompasses P2P insurance as a leading application, but also extends to mutual insurance, catastrophe risk pools, joint-liability lending, clearinghouses, margining systems, and financial hedging arrangements in which losses are shared across participants. When individual actions influence the distribution of aggregate risk, this residual exposure implies that such actions generate payoff externalities across participants. The central economic feature of the environment is therefore that privately chosen actions affect the distribution of a shared aggregate outcome, creating incentive externalities that are absent in single-agent insurance settings.

2.1 Risk Transfer, Risk Sharing, and Loss Exposure

Consider a group of n participants, indexed by $\{1, 2, \ldots, n\}$, who face random losses at date $t = 1$ given by the vector

$$\boldsymbol{X}_n = (X_1, X_2, \ldots, X_n).$$

The aggregate loss is

$$S_n = \sum_{k=1}^{n} X_k,$$

with support $[0, \bar{s}]$. We take the aggregate loss S_n as the primitive object for contracting. This modeling choice is standard in environments where losses are pooled or jointly financed, including insurance pools, reinsurance treaties, catastrophe bonds, and centralized clearing mechanisms.

Contractual arrangements in decentralized risk-sharing environments operate through two conceptually distinct but interrelated components. The first contractual component determines how much of the aggregate loss is transferred to an external counterparty, such as an insurer, reinsurer, or financial intermediary.

Definition 1 (Risk-Transfer Rule) *A risk-transfer rule is a continuously differentiable function $t : \mathbb{R}_+ \to \mathbb{R}_+$ that specifies the amount $t(Y)$ of aggregate loss Y ceded to an external counterparty. The corresponding retained loss is*

$$r(Y) := \pi + Y - t(Y), \quad \textit{with } t(0) = 0,$$

where π denotes the premium or upfront payment for the transfer.

The risk-transfer rule t governs how much aggregate risk is financed externally, while the retained loss $r(Y)$ represents the pool's net burden after accounting for premium and indemnity. This formulation nests a wide range of contracts. Proportional transfer corresponds to coinsurance or linear hedging, stop-loss rules correspond to deductibles and catastrophe reinsurance, and capped rules correspond to policy limits. Similar structures appear in derivative payoffs and risk management contracts, where $t(\cdot)$ maps losses into hedged positions.

The second contractual component allocates the retained loss across participants. The risk-sharing rule determines each participant's marginal exposure to the retained aggregate loss. Such rules are ubiquitous in decentralized arrangements, including mutual insurance, group lending, clearinghouses, and partnership agreements.

Definition 2 (Risk-Sharing Rule) *A risk-sharing rule is a continuously differentiable function*

$$\boldsymbol{h}(Y) = \big(h_1(Y), \ldots, h_n(Y)\big)$$

satisfying the budget-balance condition

$$\sum_{k=1}^{n} h_k(Y) = Y.$$

In many applications, $h_i(Y)$ is implemented via ex ante deposits combined with ex post refunds or assessments.

To prevent ex post manipulation of realized losses and to ensure monotonic exposure to aggregate outcomes, we impose a no-sabotage condition on the risk-transfer rule.

Assumption 1 (No-Sabotage) *A risk-transfer rule t satisfies the no-sabotage condition if $t'(y) \leq 1$ for all $y \in \text{supp}(Y)$.*

This condition ensures that the retained loss $r(y)$ is weakly increasing in the realized aggregate loss, since $r'(y) = 1 - t'(y) \geq 0$.[4] Economically, this condition rules out ex post sabotage or deliberate loss inflation, which would be illegal or contractually infeasible in practice, by ensuring that higher realized losses never mechanically increase net payoffs through the transfer component. In finance terms, Assumption 1 corresponds to no-arbitrage condition on state-contingent payoffs. Similar restrictions appear in asset pricing, derivatives design, and insurance regulation, where payoff schedules must be non-decreasing in the underlying state to rule out arbitrage and perverse incentives.

Additionally, we impose a regularity condition on the risk-sharing rule that ensures participants share exposure to adverse aggregate outcomes and therefore face aligned incentives.

Assumption 2 (Comonotonicity) *A risk-sharing rule $\boldsymbol{h}(y) = (h_1(y), \ldots, h_n(y))$ is comonotonic if each $h_i'(y) \geq 0$ for all $y \in \text{supp}(Y)$ and all $i \in \{1, 2, \ldots, n\}$.*

Assumption 2 implies that all policyholders are "in the same boat": each participant's allocated loss $\boldsymbol{h}\big(r(s)\big) = \big(h_1(r(s)), \ldots, h_n(r(s))\big)$ moves weakly in the same direction as the realized aggregate loss $S_n = s$. Comonotonicity characterizes efficient allocations in a wide class of risk-sharing environments and is standard in insurance theory and cooperative risk sharing. Under standard regularity conditions, a risk-sharing allocation is

[4] The class of no-sabotage risk-transfer rules is general. In addition to proportional transfer $t(S_n) = \alpha S_n$, it includes stop-loss contracts $t(S_n) = (S_n - d)_+$ and capped contracts $t(S_n) = \min(S_n, d)$. Likewise, the family of admissible risk-sharing rules is large. Beyond proportional sharing $\boldsymbol{h}(Y) = (\omega_1 Y, \ldots, \omega_n Y)$, it includes quantile-based and multi-layer allocations. Our analysis does not rely on any specific parametric form for t or $\boldsymbol{h}$ and therefore applies to a wide range of practically relevant contracts.

Pareto optimal if and only if it is comonotonic. This property is particularly appropriate for decentralized arrangements with shared downside risk, including mutual insurance, catastrophe pools, clearinghouses, and peer-based mechanisms. By ruling out designs that shield some participants from losses at the expense of others, comonotonicity ensures that loss-prevention effort mitigates a burden borne jointly by the group and thereby sustains meaningful incentive interaction.

Comonotonicity is also incentive relevant. When each participant's allocated loss increases with the retained loss, all participants remain exposed to adverse pool outcomes. Individual loss-prevention effort therefore reduces a burden borne collectively, generating payoff externalities across participants. This shared exposure creates scope for peer-based discipline and strategic interaction in effort choices, since no participant is insulated from bad realizations by construction of the sharing rule.

2.2 Loss-Prevention Effort under Decentralized Aggregate Risk

We have specified a general environment in which participants share exposure to an aggregate loss, individual effort affects the distribution of that loss. We now introduce loss-prevention effort and specify how individual actions and effort choices are strategically linked and affect the distribution of aggregate losses. The analysis in this section is deliberately general and applies to decentralized risk-sharing environments in which participants retain exposure to a common aggregate outcome.

Participants choose costly, unobservable effort ex ante that influences the distribution of aggregate losses, which is the central moral-hazard margin studied in this paper.[5] Each participant $i \in \{1, 2, \dots, n\}$ selects an effort level $e_i \in \mathbb{R}_+$ at date $t = 0$. Effort is privately chosen and unobservable to other participants and to the contracting intermediary. We interpret e_i broadly as any preventive activity that reduces expected losses, including maintenance, monitoring, precaution, fraud avoidance, or risk-management actions. Let $\boldsymbol{e} = (e_1, \dots, e_n)$ denote the effort profile. Participant i incurs a cost $c_i(e_i)$, where $c_i(0) = 0$

[5] While the no-sabotage condition rules out ex post manipulation of realized losses, it does not eliminate moral hazard in prevention.

and $c_i'(e_i) > 0$. Effort affects outcomes through the distribution of the aggregate loss S_n: individual actions shift the distribution of aggregate losses and therefore the burden borne collectively. Let $F_{S_n}(s \mid \boldsymbol{e})$ denote the cumulative distribution function of S_n conditional on the effort profile.

To formalize how prevention affects risk, we impose two regularity conditions that are standard in models of prevention, public goods, and strategic complementarities.

Assumption 3 (First-Order Stochastic Dominance) *The aggregate loss distribution $F_{S_n}(s \mid \boldsymbol{e})$ satisfies first-order stochastic dominance in the effort vector $\boldsymbol{e} = (e_1, \ldots, e_n)$ if*

$$\frac{\partial F_{S_n}(s \mid \boldsymbol{e})}{\partial e_i} \geq 0 \quad \textit{for all } s \in [0, \bar{s}],\ i \in \{1, 2, \ldots, n\}.$$

Assumption 4 (Supermodularity) *The aggregate loss distribution $F_{S_n}(s; \boldsymbol{e})$ is supermodular in $\boldsymbol{e} = (e_1, \ldots, e_n)$ if*

$$\frac{\partial^2 F_{S_n}(s; \boldsymbol{e})}{\partial e_i\, \partial e_j} \geq 0 \quad \textit{for all } s \in [0, \bar{s}],\ i, j \in \{1, 2, \ldots, n\},\ \textit{and } i \neq j.$$

Assumption 3 requires that higher effort by any participant weakly reduces aggregate losses in the sense of first-order stochastic dominance. It formalizes the idea that loss-prevention effort is effective in expectation and is standard in insurance, prevention, and risk-management models. Assumption 4 captures complementarities in prevention. It requires that the marginal loss-reducing effect of one participant's effort is stronger when others exert higher effort. Economically, coordinated prevention makes each unit of effort more productive. This property is natural in environments with shared infrastructure, correlated risks, or collective safeguards, such as flood protection, cybersecurity, or joint fraud prevention.

To further clarify the economic content of Assumption 4, note that it strengthens Assumption 3. In addition to requiring that higher effort reduces losses, supermodularity requires that the effectiveness of one participant's effort increases with the effort exerted by others. Formally, holding all other efforts fixed, Assumption 4 implies

$$\frac{\partial F_{S_n}(s;\boldsymbol{e})}{\partial e_i}\Big|_{\boldsymbol{e}=(e_i,e_j^H,\boldsymbol{e}_{-i,-j})} \geq \frac{\partial F_{S_n}(s;\boldsymbol{e})}{\partial e_i}\Big|_{\boldsymbol{e}=(e_i,e_j^L,\boldsymbol{e}_{-i,-j})},$$

for all $e_j^H \geq e_j^L$, all $s \in [0, \bar{s}]$, and all $i \neq j$. This condition captures complementarity in prevention technologies: when others invest more in loss prevention, an individual's effort becomes more effective at reducing aggregate losses. Such complementarities generate strategic interaction in effort choices and are the key force underlying the possibility of multiple equilibria in decentralized risk-sharing environments.

Figure 1 provides a graphical illustration of this mechanism.

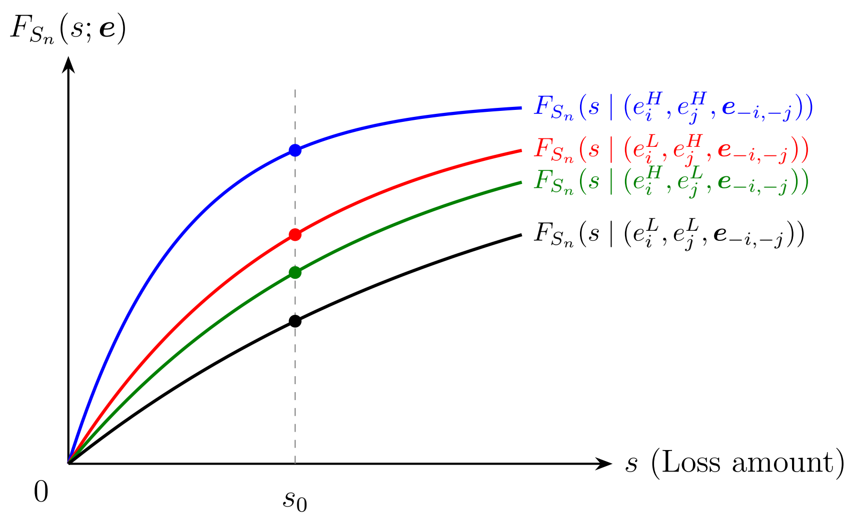

Figure 1: Ordering of Aggregate Loss Distributions under Different Effort Combinations

Together, Assumptions 3 and 4 imply that effort choices are strategic complements. An increase in one participant's effort raises the marginal return to effort for others. As a result, the noncooperative effort game generated by a given contract may admit multiple equilibria, including low-effort and high-effort outcomes under the same contractual primitives. Figure 1 illustrates this mechanism. An increase in participant i's effort from e_i^L to e_i^H shifts the cumulative distribution function $F_{S_n}(s \mid \boldsymbol{e})$ to the top-left, indicating a reduction in aggregate losses in the sense of first-order stochastic dominance. Moreover, this loss-reducing effect is stronger when participant j exerts higher effort (e_j^H rather than e_j^L). Economically, prevention technologies exhibit complementarity: coordinated effort makes each unit of effort more productive at the margin. This complementarity is precisely the force that supports equilibrium multiplicity and renders comparative statics informative in later sections. Additional examples and economic intuition for these assumptions are provided in Appendix A.

Importantly, these assumptions are not specific to P2P insurance. They apply to any setting in which agents retain exposure to a shared loss and prevention technologies exhibit complementarities. What distinguishes P2P insurance in later sections is how the retained loss is allocated across participants and how social preferences shape incentives.

2.3 Social Exposure, Crisis, and the Perception–Action Gap

A growing empirical literature documents that individuals' responses to crisis risks are shaped not only by their own experiences, but also by social interactions and peer exposure to extreme events. Social networks transmit information, attention, and salience, often increasing perceived risk even when objective local exposure is unchanged.[6] At the same time, heightened awareness of catastrophic risk does not necessarily translate into preventive action. In many disaster contexts, individuals exhibit a well-documented perception–action gap: perceived vulnerability rises, yet investment in mitigation remains limited when responsibility is diffuse or implicitly delegated to institutions.[7]

Building on the general framework of decentralized risk sharing and loss-prevention effort under aggregate risk developed above, this paper provides a structural interpretation of this gap that emphasizes incentives rather than belief formation. We do not model learning, information diffusion, or subjective probability updating explicitly. Instead, we examine how perceived risk maps into incentives for loss-prevention effort in environments where agents share exposure to a common aggregate loss.

In standard insurance and risk-transfer environments, perceived risk affects behavior

[6]For example, households increase insurance purchases when geographically distant friends experience floods even when local risk is unchanged, illustrating how social networks transmit attention and salience (Hu, 2022).

[7]Related work documents a risk perception paradox, whereby heightened awareness of catastrophic risk does not necessarily translate into preventive action when responsibility is diffuse or implicitly delegated to institutions (Wachinger et al., 2013).

only insofar as losses enter individual payoffs. Formally, changes in perceived risk can be represented as changes in the marginal disutility of losses embodied in the utility function, or as increased salience of adverse outcomes. However, when losses are fully transferred to an insurer or hedged away through financial contracts, individual payoffs are independent of realized losses. In such settings, higher perceived risk may increase insurance demand or attention to risk, but it does not alter the marginal return to prevention effort when effort is unobservable. As a result, even highly risk-averse individuals may rationally underinvest in prevention, consistent with the perception–action gap observed in the data.

Decentralized risk-sharing arrangements fundamentally alter this mapping by preserving residual exposure to aggregate outcomes. When agents bear part of the realized loss, changes in perceived risk directly affect the marginal benefit of effort. In our framework, this channel operates whenever an individual's payoff depends on the aggregate loss S_n. In later sections, this dependence arises through the retained loss function $r(S_n)$ and the allocation rule $\boldsymbol{h}$, but the logic applies more generally to any environment in which agents remain residual claimants on collective outcomes. Formally, effort affects incentives through its impact on the distribution $F_{S_n}(s \mid \boldsymbol{e})$. When payoffs depend on S_n, an increase in perceived risk raises the marginal return to effort by increasing the payoff relevance of states with high losses. Thus, perceived risk influences behavior only when contract or institutional design preserves exposure to losses.

Social exposure further strengthens this channel by amplifying the perceived stakes of aggregate losses. When individuals care about how outcomes affect peers, socially transmitted risk salience increases not only perceived vulnerability but also the incentive to reduce losses borne by others. In this sense, social exposure operates through incentives rather than beliefs alone. It magnifies the behavioral response to risk precisely when agents remain exposed to collective outcomes. This perspective helps reconcile empirical findings on crisis risk and social interaction. Social exposure may increase perceived risk without increasing prevention when incentives are weak, such as under full insurance or complete hedging. By contrast, decentralized risk-sharing arrangements can convert socially transmitted risk awareness into preventive action by aligning responsibility with consequences.

This mechanism applies broadly to decentralized risk-sharing institutions in which participants internalize part of the group outcome. P2P insurance offers a particularly transparent setting in which this mechanism operates, because residual loss exposure and social connections are explicit and contractually salient.

2.4 Institutional Structure of Peer-to-Peer Insurance

We now specialize the general environment to P2P insurance and describe the institutional mechanism that maps aggregate losses into individual payoffs. Peer-to-peer insurance is a hybrid system that integrates external risk transfer with internal risk sharing. Participants pool individual risks, transfer part of the aggregate exposure to an external insurer, and allocate the remaining losses within the group. This structure makes residual claimant exposure explicit and provides a concrete setting in which contract design and social preferences jointly shape loss-prevention incentives.

The interaction between the P2P platform and the insurer proceeds as follows. After aggregating individual losses into the pool-level loss S_n, the platform applies a risk-transfer rule. It pays a premium π to an insurer and cedes a portion $t(S_n)$ of the aggregate loss. Reductions in S_n therefore translate into lower expected costs for each participant through both the insurance and sharing components. To ensure enforceability in practice, P2P platforms typically require participants to post a margin deposit. After losses are realized and the risk-sharing process is settled, any remaining balance is refunded. The margin deposit serves as an implementation device to guarantee settlement and prevent default within the pool. The incentive effects in our analysis arise from how effort affects the distribution of S_n and from how retained losses are allocated through the sharing rule $\boldsymbol{h}$, rather than from the existence of the deposit itself.

Definition 3 (Peer-to-Peer Insurance) *A peer-to-peer insurance contract for losses* $\boldsymbol{X}_n = (X_1, X_2, \ldots, X_n)$, *with aggregate loss* $S_n = \sum_{k=1}^{n} X_k$, *is a tuple*

$$\Gamma := (t, \pi, \boldsymbol{h})$$

that specifies a two-stage process:

1. ***Risk Transfer.*** *Participants choose a risk-transfer rule $t(S_n)$ and pay a premium π, resulting in the retained loss*

$$r(S_n) = \pi + S_n - t(S_n).$$

2. ***Risk Sharing.*** *The retained loss is allocated across participants according to a risk-sharing rule*

$$\boldsymbol{h} \circ r(S_n) = \big(h_1 \circ r(S_n), h_2 \circ r(S_n), \dots, h_n \circ r(S_n)\big),$$

with $\sum_{k=1}^{n} h_k \circ r(S_n) = r(S_n)$.

Let $\{w_i\}_{i=1}^n$ denote participants' initial wealth at date $t = 0$. Final wealth at $t = 1$ is given by

$$\big(w_1 - h_1 \circ r(S_n),\ w_2 - h_2 \circ r(S_n),\ \dots,\ w_n - h_n \circ r(S_n)\big),$$

which is contingent on the realization of the aggregate loss S_n.

This definition highlights two incentive-relevant components of a P2P contract. The retained loss function $r(\cdot)$ determines how aggregate losses translate into the pool's net burden, while the allocation rule $\boldsymbol{h}$ determines each participant's marginal exposure to that burden. When loss-prevention effort is endogenous, both components jointly shape incentives because they determine how individual actions affect expected payoffs. Figure 2 presents the mechanism of P2P insurance.

The nodes represent participants, Ⓟ denotes the P2P platform, and Ⓘ denotes the insurer. Straight arrows indicate contractual transfers between entities, while the curved arrows labeled $g_1, g_2, \dots, g_n$ capture how participants evaluate outcomes borne by peers. These links represent social concerns and warm-glow incentives, which are formally introduced in Section 3. Figure 2 highlights that P2P insurance combines formal contractual transfers with social exposure: the platform and insurer determine how aggregate losses are mapped into retained losses and individual allocations through $(t, \pi, \boldsymbol{h})$.

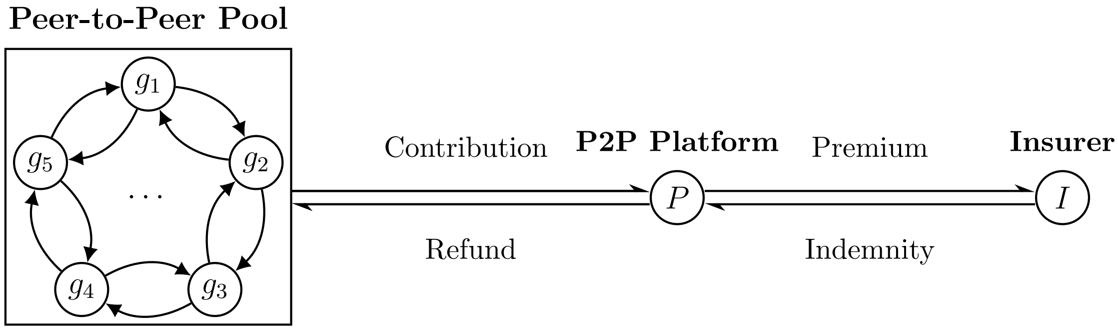

Figure 2: Mechanism of Peer-to-Peer insurance

A defining feature of the P2P environment is that loss-prevention effort has collective consequences. Although effort is privately chosen, its benefits are partially shared through the allocation rule $\boldsymbol{h}$. As a result, participants are partial residual claimants on group outcomes, so individual effort generates spillover benefits for others. This creates strategic interaction in effort choices: one participant's prevention effort affects not only their own expected payoff but also the incentives faced by peers. Such strategic interdependence is absent in standard single-agent insurance models, where effort affects only individual outcomes. In P2P insurance, by contrast, residual exposure and social preferences together shape incentives, complicating the moral-hazard problem and motivating the equilibrium analysis in Section 3.

3 Moral Hazard in Peer-to-Peer Insurance

We now turn from contract primitives to behavior. Given a P2P insurance contract $\Gamma = (t, \pi, \boldsymbol{h})$, which jointly determines how much aggregate loss is transferred to the insurer and how retained losses are allocated across participants, individuals choose loss-prevention effort non-cooperatively and under private information. Because effort affects the distribution of aggregate losses and because retained losses are shared within the pool, individual effort choices are strategically interdependent, in contrast to the single-agent moral-hazard setting of standard insurance models, giving rise to coordination effects and potentially multiple equilibria. This section studies the non-cooperative effort game induced by a given Γ and characterizes the resulting effort equilibria under different motivational structures.

3.1 Effort Equilibrium in Peer-to-Peer Insurance

Each participant $i \in \{1, 2, \ldots, n\}$ with initial wealth w_i can exert an unobservable effort $e_i \in \mathbb{R}_+$ to reduce aggregate losses, at private cost $c_i(e_i)$. We interpret e_i broadly as any privately chosen prevention activity, such as maintenance, monitoring, precaution, or fraud avoidance, that shifts the distribution of the aggregate loss S_n in the sense of first-order stochastic dominance, consistent with Assumption 3. The cost function c_i is continuously differentiable, satisfies $c_i(0) = 0$, and has $c_i' > 0$ (not necessarily $c_i'' > 0$). Let $\boldsymbol{e} := (e_1, \ldots, e_n) \in \mathbb{R}^n_+$ denote the effort profile, decomposable as $\boldsymbol{e} = (e_i, \boldsymbol{e}_{-i})$, where $\boldsymbol{e}_{-i}$ collects others' efforts.[8]

Given a P2P insurance contract $\Gamma = (t, \pi, \boldsymbol{h})$, participant i's payoff depends on the realized aggregate loss through the retained-loss mapping $r(s) = \pi + s - t(s)$ and the allocation rule $h_i(\cdot)$. Utility is additively separable in wealth and effort cost,[9] so participant i's expected utility is

$$
\begin{aligned}
u_i(\Gamma, \boldsymbol{e}) &:= \mathbb{E}\Big[v_i\big(w_i - h_i \circ r(S_n)\big)\Big] - c_i(e_i), \\
&= \int_{[0,\bar{s}]} v_i\bigg(w_i - h_i \circ \Big(\pi + s - t(s)\Big)\bigg)\, dF_{S_n}(s; \boldsymbol{e}) - c_i(e_i), \quad \forall i \in \{1, 2, \ldots, n\}.
\end{aligned}
$$

Here $F_{S_n}(s; \boldsymbol{e})$ is twice continuously differentiable with respect to any pair (e_i, e_j) for $i \neq j$, and v_i is a continuously differentiable von Neumann–Morgenstern utility function with $v_i' > 0$ (not necessarily $v_i'' < 0$).

The strategic linkage across participants arises because effort shapes the distribution of the shared aggregate loss. A participant's marginal return to effort depends on two objects. The risk-transfer rule $t(\cdot)$ governs how strongly aggregate losses translate into retained losses, and therefore the overall strength of prevention incentives. The risk-sharing rule $h_i(\cdot)$ determines how much of that retained loss participant i bears at the

[8] For vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, we write $\boldsymbol{x} \geq \boldsymbol{y}$ if $x_k \geq y_k$ for all k.

[9] Utility is additively separable in wealth and effort cost (Bourgeon and Picard, 2014).

margin. Because effort affects $F_{S_n}(s; \boldsymbol{e})$, and because each participant's exposure is a function of the same retained-loss realization, prevention has a public-good component and induces strategic interdependence.

We study three behavioral environments that differ only in how participants value the consequences of their effort. The selfish benchmark isolates the standard hidden-action logic with residual loss exposure. Warm-glow preferences introduce an intrinsic motive for effort that operates through the cost side, lowering the effective marginal cost of prevention. Altruistic preferences introduce social concern that operates through the benefit side, causing participants to internalize part of the loss borne by their peers. By distinguishing these channels, we isolate how different forms of pro-social motivation discipline moral hazard in P2P insurance through distinct economic mechanisms.

3.1.1 Selfish Effort Equilibrium

In the selfish benchmark, participants care only about their own expected utility and obtain no direct utility from helping others. Given a contract Γ, participants choose efforts simultaneously and non-cooperatively. A selfish effort equilibrium $\boldsymbol{e}(\Gamma) = \big(e_1(\Gamma), \ldots, e_n(\Gamma)\big)$ is a fixed point of the best-response system

$$e_i \in \arg\max_{\hat{e}_i \geq 0} u_i(\Gamma, \big(\hat{e}_i, \boldsymbol{e}_{-i})\big), \text{ given } \boldsymbol{e}_{-i} \text{ for all } i \in \{1, 2, \ldots, n\}. \tag{1}$$

Equivalently, at $\boldsymbol{e}(\Gamma)$ no participant can profitably deviate:

$$e_i(\Gamma) \in \arg\max_{\hat{e}_i \geq 0} u_i\Big(\Gamma, \big(\hat{e}_i, \boldsymbol{e}_{-i}(\Gamma)\big)\Big), \text{ given } \boldsymbol{e}_{-i} = \boldsymbol{e}_{-i}(\Gamma), \ \forall i.$$

This benchmark highlights the basic moral-hazard force in a P2P pool. Participant i bears the private cost $c_i(e_i)$ but benefits only through the reduction in their own allocated share of retained losses. Since a portion of the loss reduction accrues to others via risk sharing, effort is generally under-provided relative to a group-optimal benchmark. The key question is whether social motives and social exposure can partially internalize this externality and thereby raise equilibrium prevention.

3.1.2 Warm-glow Effort Equilibrium

We next consider warm-glow incentives, which affect effort provision through the cost side. Following Andreoni (1989), individuals may derive private utility from the act of contributing itself, even if they do not directly value others' welfare. In the P2P context, warm glow captures motivations such as self-image, reciprocity norms, or reputational benefits within a repeated-interaction environment. Importantly, warm glow does not alter how effort affects losses; instead, it lowers the effective marginal cost of effort.

We capture warm glow by allowing the cost of effort to depend on a type parameter g_i that shifts the effective marginal cost of effort. Formally, let $c_i = c_i(e_i, g_i)$ with $\frac{\partial^2 c_i(e_i,g_i)}{\partial e_i \partial g_i} < 0$, so higher g_i makes exerting effort less costly at the margin.[10] Participant i observes g_i but not others' warm-glow parameters. The utility function becomes

$$\begin{aligned} u_i(\Gamma, g_i, \boldsymbol{e}) &:= \mathbb{E}\Big[v_i\big(w_i - h_i \circ r(S_n)\big)\Big] - c_i(e_i, g_i) \\ &= \int_0^{\bar{s}} v_i\big(w_i - h_i \circ r(s)\big)\, dF_{S_n}(s \mid \boldsymbol{e}) - c_i(e_i, g_i), \quad \forall i \in \{1, 2, \ldots, n\}. \end{aligned}$$

Let $\boldsymbol{g} = (g_1, \ldots, g_n)$ denote the vector of warm-glow parameters. A warm-glow effort equilibrium $\boldsymbol{e}(\Gamma, \boldsymbol{g}) = \big(e_1(\Gamma, \boldsymbol{g}), \ldots, e_n(\Gamma, \boldsymbol{g})\big)$ is a fixed point of

$$e_i \in \arg\max_{\hat{e}_i \geq 0} u_i\big(\Gamma, g_i, (\hat{e}_i, \boldsymbol{e}_{-i})\big), \quad \text{given } \boldsymbol{e}_{-i},\ \forall i \in \{1, 2, \ldots, n\}. \tag{2}$$

A participant with higher g_i has a stronger incentive to exert effort because warm glow lowers their effective marginal cost.

3.1.3 Altruistic Effort Equilibrium

We now turn to altruistic social concerns, which operate on the benefit side of effort by causing participants to internalize part of the losses borne by others in the pool. We allow participants to value peers' welfare in addition to any warm-glow motive. We represent

[10] For example, $c_i(e_i, g_i) = c_i(e_i) - g_i e_i$ satisfies this condition, since $\frac{\partial^2 c_i(e_i,g_i)}{\partial e_i g_i} = -1$. In this case, $u_i(\Gamma, g_i, \boldsymbol{e}) = \mathbb{E}\Big[v_i\big(w_i - h_i \circ r(S_n)\big)\Big] - c_i(e_i) + g_i e_i$ where $g_i e_i$ represents the positive utility from warm glow of giving.

social connections through a directed, weighted graph. For any two distinct participants i and j, let $\delta_{ij} \in \mathbb{R}_+$ denote the degree to which i cares about j. These connections form an $n \times n$ matrix

$$\boldsymbol{\Delta} := \begin{bmatrix} 1 & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & 1 & \cdots & \delta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{n1} & \delta_{n2} & \cdots & 1 \end{bmatrix}. \tag{3}$$

The diagonal elements normalize self-weight to one, while off-diagonal elements capture directed concern, allowing for asymmetric relationships. This structure accommodates pools formed by families or close friends, as well as pools with sparse invitation links or uneven social salience.

Participant i observes only their own concern weights $\boldsymbol{\Delta}_i = (\delta_{i1}, \ldots, \delta_{i,i-1}, 1, \delta_{i,i+1}, \ldots, \delta_{in})$ and their own warm-glow parameter g_i.[11]

Define i's altruistic objective as a weighted sum of the baseline utilities:

$$\begin{aligned} \tilde{u}_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}_i, \boldsymbol{e}) &= \sum_{k=1}^{n} \delta_{ik} u_k(\Gamma, g_k, \boldsymbol{e}) \\ &= u_i(\Gamma, g_i, \boldsymbol{e}) + \sum_{k \neq i} \delta_{ik} u_k(\Gamma, g_k, \boldsymbol{e}). \end{aligned} \tag{4}$$

Under $\tilde{u}_i$, participant i internalizes a fraction of the utility consequences of aggregate losses for others. Since effort reduces S_n and thereby affects all participants' allocated losses, altruism directly converts part of the prevention externality into a private incentive.

An altruistic effort equilibrium $\boldsymbol{e}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) = \big(e_1(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}), \ldots, e_n(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})\big)$ is characterized as the fixed point of

$$e_i \in \arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, (\hat{e}_i, \boldsymbol{e}_{-i})\big), \quad \text{given } \boldsymbol{e}_{-i},\ \forall i \in \{1, 2, \ldots, n\}. \tag{5}$$

A participant with higher concern weights δ_{ik} has a stronger incentive to exert effort

[11] Although $\tilde{u}_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}_i, \boldsymbol{e})$ depends on $\boldsymbol{g} = (g_1, g_2, \ldots, g_n)$, participant i does *not* need to observe $\boldsymbol{g}_{-i} = (g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_n)$ when deciding how much effort should exert, i.e. $e_i \in \arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, (\hat{e}_i, \boldsymbol{e}_{-i})\big)$, because $c_j(e_j, g_j)$ is independent of e_i.

because they internalize more of the positive externality that prevention generates for peers.

3.2 Hyper-rectangle Boundary of Effort Equilibrium

An interior effort equilibrium is determined by participants choosing effort until marginal benefits, which may include internalized spillovers, equal marginal costs, which may be reduced by warm glow. In P2P insurance, however, effort choices interact through the aggregate loss distribution, and strategic complementarity can generate multiple equilibria. Rather than selecting a unique equilibrium, we characterize the equilibrium set and show that it is bounded by extremal equilibria that provide robust predictions.

Let $\overline{\boldsymbol{e}} = (\overline{e}_1, \overline{e}_2, \ldots, \overline{e}_n)$ and $\underline{\boldsymbol{e}} = (\underline{e}_1, \underline{e}_2, \ldots, \underline{e}_n)$ denote the upper and lower bounds of the effort vector, and define the hyper-rectangle

$$[\underline{\boldsymbol{e}}, \overline{\boldsymbol{e}}] = \big\{(e_1, e_2, \ldots, e_n) : \underline{e}_i \leq e_i \leq \overline{e}_i,\ i \in \{1, 2, \ldots, n\}\big\}.$$

For each of the three behavioral environments, we show that the set of equilibria lies within such a hyper-rectangle that contains all possible equilibria. Focusing on the smallest and largest equilibria serves two purposes. First, these extremal equilibria bound all outcomes and therefore yield predictions that are robust to equilibrium selection. Second, under strategic complementarity, comparative statics for extremal equilibria are sharp, allowing us to track how changes in social incentives shift the entire equilibrium set.

Because effort has public good features and marginal benefits depend on others' actions, equilibrium multiplicity is natural. Our goal is therefore not to select a unique equilibrium, but to characterize the entire equilibrium set and how it shifts with social incentives.

Proposition 1 *Under Assumptions 1, 2, and 4, there exists at least one effort equilibrium for each of the three types, and all effort equilibria are contained in a unique hyper-rectangle:*

1. The selfish effort equilibrium $\boldsymbol{e}(\Gamma) \in [\underline{\boldsymbol{e}}(\Gamma), \overline{\boldsymbol{e}}(\Gamma)]$,

2. The warm-glow effort equilibrium $\boldsymbol{e}(\Gamma, \boldsymbol{g}) \in [\underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}), \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g})]$,

3. The altruistic effort equilibrium $\boldsymbol{e}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \in [\underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}), \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})]$,

where $\underline{\boldsymbol{e}}$ *and* $\overline{\boldsymbol{e}}$ *denote the smallest and largest effort equilibria, respectively.*

Proof: See Appendix B.1. ■

Proposition 1 implies that even when multiple equilibria exist, effort outcomes are disciplined. For each participant i, any equilibrium effort e_i^* must satisfy $\underline{e}_i \leq e_i^* \leq \overline{e}_i$, and the equilibrium set is non-empty. Economically, the contract primitives and the prevention technology jointly determine a well-defined range of prevention outcomes: coordination can move the pool within this range, but incentives cannot generate arbitrarily extreme effort levels once the regularity conditions hold.

Figure 3 illustrates the hyper-rectangle for a three-participant example. The axes e_1, e_2, e_3 represent the effort levels of participant 1, 2, and 3, respectively. The coordinate $\underline{e}_i$ ($\overline{e}_i$) denotes the effort of participant i in the smallest (largest) equilibrium, for $i = 1, 2, 3$. By Proposition 1, every equilibrium point $\boldsymbol{e}^*$ lies inside the rectangular cuboid $[\underline{\boldsymbol{e}}, \overline{\boldsymbol{e}}]$.

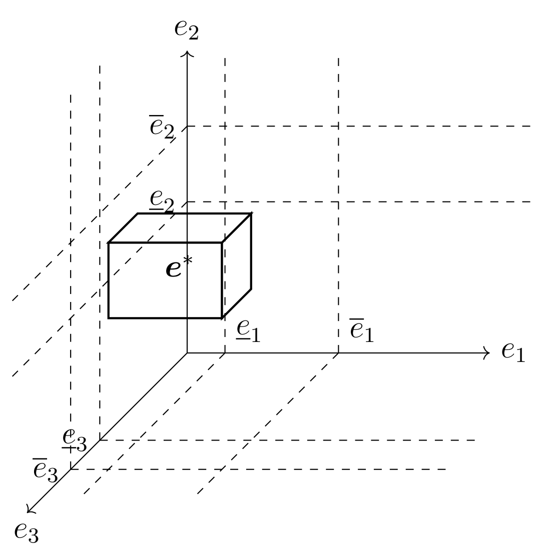

Figure 3: Hyper-rectangle for the three-dimension case

The economic force behind Proposition 1 is strategic complementarity in prevention. Under Assumptions 1, 2, and 4, a higher effort by one participant increases the marginal effectiveness of others' effort in reducing losses, so best responses are increasing. To illustrate, consider the altruistic equilibrium and start from $\boldsymbol{e}^0 = \mathbf{0}$. Let

$$e_i^1 = \arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, (\hat{e}_i, \boldsymbol{e}_{-i}^0)\big), \quad i \in \{1, 2, \ldots, n\},$$

be the first-round best responses. Since $\boldsymbol{e}^1 \geq \boldsymbol{e}^0$, complementarity implies that best responses weakly increase, so iterating

$$e_i^k = \arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, (\hat{e}_i, \boldsymbol{e}_{-i}^{k-1})\big), \quad i \in \{1, 2, \ldots, n\},$$

yields a monotone sequence $\boldsymbol{e}^k \geq \boldsymbol{e}^{k-1}$. The smallest fixed point of this iteration is the smallest Nash equilibrium $\underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$. An analogous construction starting from a high effort vector generates the largest Nash equilibrium $\overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$. This monotone best-response logic clarifies why social incentives can have amplified effects: by shifting some participants' best responses upward, they trigger higher efforts by others, which further feeds back into the original incentives.

3.3 Comparative Statics with Social Incentives

We now study how warm-glow incentives and social concerns shift the equilibrium set. Write $\boldsymbol{g}^H \geq \boldsymbol{g}^L$ to mean $g_i^H \geq g_i^L$ for all i, and $\boldsymbol{\Delta}^H \geq \boldsymbol{\Delta}^L$ to mean $\delta_{ij}^H \geq \delta_{ij}^L$ for all i, j. Warm glow operates on the cost side by lowering the effective marginal cost of prevention. Social concern operates on the benefit side by internalizing part of the prevention externality. Under strategic complementarity, both channels are magnified through others' best responses.

Proposition 2 *Under Assumptions 1, 2, 3 and 4, the largest and smallest effort equilibria are monotonically increasing with respect to the warm-glow and altruism parameters.*

Specifically, if $\boldsymbol{g}^H \geq \boldsymbol{g}^L$ *and* $\boldsymbol{\Delta}^H \geq \boldsymbol{\Delta}^L$*, then*

$$\overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}^H, \boldsymbol{\Delta}^H) \geq \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}^L, \boldsymbol{\Delta}^L) \quad \textit{and} \quad \underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}^H, \boldsymbol{\Delta}^H) \geq \underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}^L, \boldsymbol{\Delta}^L),$$

where the inequalities hold component-wise.

Proof: See Appendix B.2. ■

Proposition 2 shows that stronger pro-social incentives expand the equilibrium set upward in a strong sense: both extremal equilibria increase, so every equilibrium is weakly higher than before. The intuition has two layers. First, higher g_i reduces marginal effort costs, and higher δ_{ik} raises marginal effort benefits by internalizing peers' gains. Second, because best responses are increasing, any direct increase in one participant's effort makes others' optimal efforts higher, and this indirect effect feeds back into the original participant. Strategic complementarity therefore turns heterogeneous pro-social motives into a pool-wide increase in prevention. Proposition 2 directly implies the following corollary, which orders extremal equilibria across the three behavioral environments.

Corollary 1 *Under Assumptions 1, 2, 3, and 4, the extremal effort equilibria satisfy the component-wise ordering*

$$\overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \geq \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}) \geq \overline{\boldsymbol{e}}(\Gamma) \quad \textit{and} \quad \underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \geq \underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}) \geq \underline{\boldsymbol{e}}(\Gamma).$$

The ordering follows from two applications of Proposition 2. Adding social concerns $\boldsymbol{\Delta} \geq \mathbf{0}$ (holding $\boldsymbol{g}$ fixed) increases marginal benefits of effort and shifts best responses upward, raising both extremal equilibria. Setting $\boldsymbol{\Delta} = \mathbf{0}$ yields the warm-glow case, and with $\boldsymbol{g} \geq \mathbf{0}$ warm glow lowers marginal costs and again raises both extremal equilibria. Economically, altruism and warm glow mitigate moral hazard through distinct channels, but both are amplified by strategic complementarity in prevention.

4 Peer-to-Peer Insurance Market Equilibrium

Having characterized the effort equilibrium under a fixed P2P insurance contract Γ, we now study the optimal design of Γ itself, accounting for the induced effort equilibrium. The key distinction relative to standard insurance design is that the contract simultaneously determines (i) how much aggregate risk is transferred to the insurer and (ii) how the retained risk is allocated across participants, thereby shaping both risk sharing and effort incentives within the pool. In this sense, the P2P platform faces a joint design problem: risk-transfer and risk-sharing rules are not merely about allocating losses, but also about governing incentive provision when effort is unobservable and generates externalities.

We model the market equilibrium as a Stackelberg–Cournot game with three stages:

Stage 1 (Cooperative contract choice): Participants cooperatively choose the P2P insurance contract $\Gamma = (t, \pi, \boldsymbol{h})$.

Stage 2 (Non-cooperative effort choice): Given $(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$, each participant non-cooperatively selects an unobservable effort level e_i, leading to the effort profile $\boldsymbol{e}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$.

Stage 3 (Loss realization and settlement): Losses are realized, and the risk-transfer and risk-sharing rules determine each participant's final payoff:

$$\big(u_1(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, \boldsymbol{e}), u_2(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, \boldsymbol{e}), \ldots, u_n(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, \boldsymbol{e})\big).$$

This timing captures a natural institutional interpretation. In practice, P2P groups (or platforms acting on their behalf) specify the contractual terms ex ante, anticipating how members will respond through their loss-prevention effort. Effort is then chosen non-cooperatively and remains privately observed, and only afterward are losses realized and settled. The game therefore combines a *sequential* (Stackelberg) structure between Stages 1 and 2 with a *simultaneous* (Cournot) interaction among participants in Stage 2.

4.1 Equilibrium concept and design problem

We assume a competitive P2P insurance market in which the insurer earns zero profit. Under competition, the premium equals the actuarially fair expected transferred loss, conditional on the induced effort profile. The market equilibrium $\Gamma^* = (t^*, \pi^*, \boldsymbol{h}^*)$ and $\boldsymbol{e}(\Gamma^*, \boldsymbol{g}, \boldsymbol{\Delta})$ are characterized as the solution to:

$$\max_{\Gamma, \boldsymbol{e}} \sum_{i=1}^{n} u_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, \boldsymbol{e}) \tag{6}$$

$$\text{s.t.} \quad \pi = \mathbb{E}\big[t(S_n); \boldsymbol{e}\big],$$

$$e_i \in \arg\max_{\hat{e}_i \geq 0} \tilde{u}_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, \big(\hat{e}_i, \boldsymbol{e}_{-i}\big)), \text{ given } \boldsymbol{e}_{-i} \text{ for all } i \in \{1, 2, \ldots, n\}.$$

This formulation nests the three behavioral environments studied in Section 3. The warm-glow equilibrium corresponds to $\boldsymbol{\Delta} = \boldsymbol{0}$, while the purely selfish equilibrium corresponds to $\boldsymbol{g} = \boldsymbol{0}$ and $\boldsymbol{\Delta} = \boldsymbol{0}$. Importantly, the design problem highlights a central P2P trade-off: increasing risk transfer (higher insurance coverage) improves risk sharing but weakens prevention incentives, while allocating retained losses across participants can be used as an internal mechanism to reward or discipline effort when preferences are heterogeneous.

4.2 Contract design with heterogeneous social incentives

We solve for the optimal P2P insurance contract $\Gamma^* = (t^*, \pi^*, \boldsymbol{h}^*)$ and its induced effort $\boldsymbol{e}(\Gamma^*, \boldsymbol{g}, \boldsymbol{\Delta})$ using the functional forms and parameters in Table 1. The goal of this exercise is not to calibrate a specific market, but to illustrate economically interpretable implications of the model that mirror the comparative statics of Section 3 and clarify how optimal risk-transfer and risk-sharing respond to prevention technology and heterogeneity in social incentives.

In particular, we adopt a proportional risk-sharing rule $h_i(Y) = \omega_i Y$ (with $\sum_{i=1}^{n} \omega_i = 1$) and a proportional risk-transfer rule $t(S_n) = \alpha S_n$. Each participant is assumed to have a CRRA utility function $v_i(w) = \frac{w^{1-\gamma}}{1-\gamma}$ with $\gamma = 2$, initial wealth $w_i = 100$, and a

Table 1: Functional forms and parameter values

Parameter	**Meaning**	**Value/Form**
$h_i(Y)$	Risk-sharing rule	$\omega_i Y$, $\sum_i \omega_i = 1$
$t(S_n)$	Risk-transfer rule	αS_n
$v_i(w)$	Utility function	$\frac{w^{1-\gamma}}{1-\gamma}$
$c(e_i, g_i)$	Effort cost function	$\frac{1}{2}\kappa e_i^2 - g_i e_i$
S_n	Aggregate loss	$U/T(\boldsymbol{e})$
U	Loss shock	$\text{Uniform}(0, s_{\max})$
$T(\boldsymbol{e})$	Loss reduction function	$\exp\Big(\sum_{i=1}^n a_i e_i + \sum_{i=1}^n \sum_{j \neq i} \beta_{ij} e_i e_j\Big)$
$s_{\max}$	Maximum possible loss	300
$\boldsymbol{g}$	Warm-glow parameters	$(g_1, g_2, g_3) = (0.003, 0.003, 0.0015)$
$\boldsymbol{\Delta}$	Social-concern matrix	$\begin{bmatrix} 1 & \delta_{12} & \delta_{13} \\ \delta_{21} & 1 & \delta_{23} \\ \delta_{31} & \delta_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.9 & 0.9 \\ 0.3 & 1 & 0.3 \\ 0.3 & 0.3 & 1 \end{bmatrix}$
a_i	Individual loss-prevention efficiency	$a_i = 0.01$
β_{ij}	Cross-effort synergy	$\beta_{ij} = 0.005$ for $i \neq j$
n	Number of participants	3
w_i	Initial wealth	100
γ	Coefficient of relative risk aversion	2
κ	Cost function coefficient	Varying from 0.0008 to 0.004

quadratic cost function

$$c(e_i, g_i) = \frac{1}{2}\kappa e_i^2 - g_i e_i.$$

The aggregate loss is modeled as $S_n = U/T(\boldsymbol{e})$, where $U \sim \text{Uniform}(0, s_{\max})$ and

$$T(\boldsymbol{e}) = \exp\Big(\sum_{i=1}^n a_i e_i + \sum_{i=1}^n \sum_{j \neq i} \beta_{ij} e_i e_j\Big).$$

Consequently,

$$F_{S_n}(s \mid \boldsymbol{e}) = \begin{cases} 0, & s < 0, \\ \dfrac{s\,T(\boldsymbol{e})}{s_{\max}}, & 0 \leq s \leq \dfrac{s_{\max}}{T(\boldsymbol{e})}, \\ 1, & s > \dfrac{s_{\max}}{T(\boldsymbol{e})}. \end{cases}$$

We set $n = 3$ participants[12] with warm-glow parameters

$$\boldsymbol{g} = (0.003,\ 0.003,\ 0.0015),$$

[12] The framework can be extended to an arbitrary finite number of participants.

and a social-concern matrix

$$\mathbf{\Delta} = \begin{bmatrix} 1 & \delta_{12} & \delta_{13} \\ \delta_{21} & 1 & \delta_{23} \\ \delta_{31} & \delta_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.9 & 0.9 \\ 0.3 & 1 & 0.3 \\ 0.3 & 0.3 & 1 \end{bmatrix}.$$

For each effort cost κ, we compute the optimal P2P insurance contract $\Gamma^* = (t^*, \pi^*, \boldsymbol{h}^*)$, characterized by: $t^*(S_n) = \alpha^* S_n$, $\pi^* = \alpha^* \mathbb{E}[S_n]$, $r^*(S_n) = \pi^* + (1-\alpha^*)S_n$, and $\boldsymbol{h}^*\big(r^*(S_n)\big) = \big(\omega_1^* \cdot r^*(S_n), \omega_2^* \cdot r^*(S_n), \omega_n^* \cdot r^*(S_n)\big)$.

4.3 Design implications and comparative statics

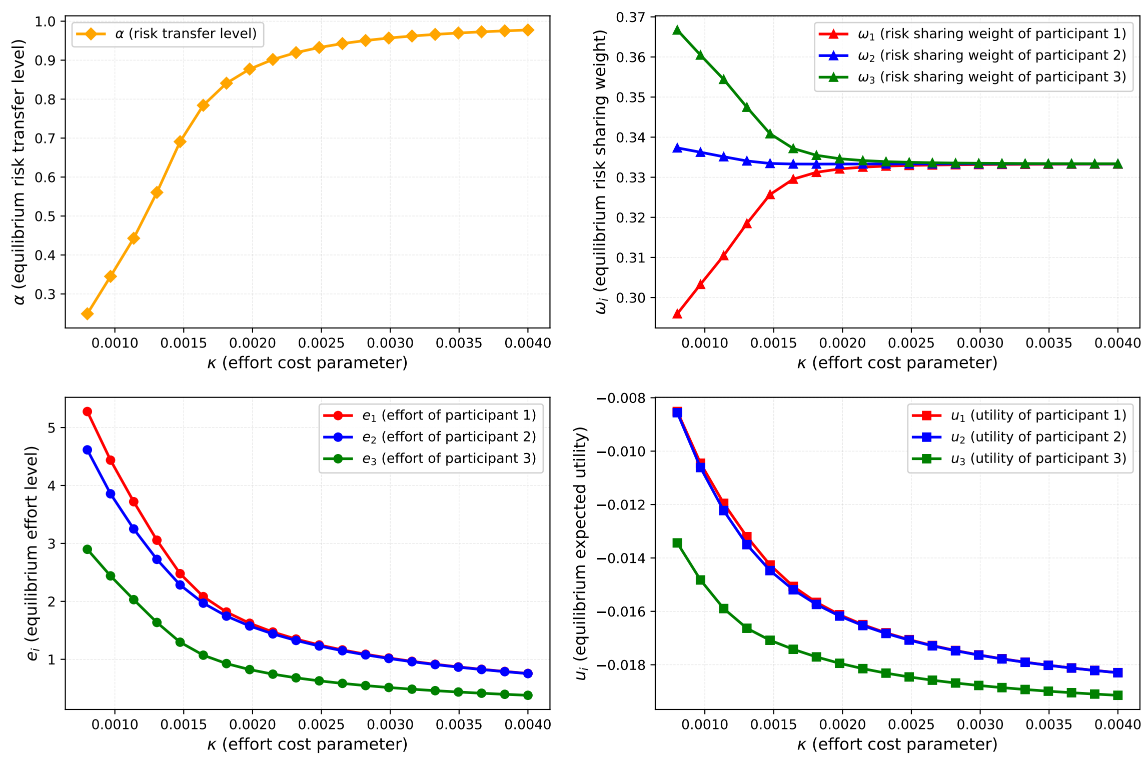

Figure 4: Comparative static analysis results for varying effort cost κ.

Figure 4 reports comparative statics with respect to the marginal effort cost κ. The results highlight two distinct design margins. The first margin is external risk transfer, governed by the optimal coverage parameter α^*, which determines how much of the aggregate loss is ceded to the insurer. The second margin is internal risk allocation, governed by the loss-sharing weights $\boldsymbol{\omega}^*$, which determine how retained losses are distributed among participants. In standard insurance with a single insured, contract design primarily operates through the first margin, reflecting the familiar trade-off between coverage and incentives. In contrast, P2P insurance introduces the second margin as a powerful additional instrument, because retained losses can be redistributed across participants in a way that responds to heterogeneity in prevention incentives.

Figure 4 illustrates how P2P insurance design operates through two distinct but interacting channels: the choice of external risk transfer in response to prevention technology, and the allocation of retained losses across participants to align incentives and compensate those who contribute most to collective loss reduction. This two-margin design is central to the economic distinctiveness of P2P insurance relative to standard insurance contracts with a single insured. The top-left panel shows that optimal insurance coverage α^* increases with the effort cost κ. When effort is relatively inexpensive, prevention is an effective technology for reducing expected losses, and it is optimal for the pool to retain more risk in order to preserve incentives. As effort becomes more costly, prevention becomes less effective, and the optimal contract shifts toward transferring a larger share of aggregate risk to the insurer. This pattern mirrors the canonical moral hazard logic: as loss prevention becomes harder, it is optimal to rely more on risk transfer and less on incentive provision through retained exposure.

The top-right panel illustrates how retained losses are allocated across participants. When effort costs are low and some risk is optimally retained, the most pro-social participant (Participant 1, with the highest warm-glow parameter g_1 and strongest social concerns δ_{1k}) is assigned the smallest share of retained loss, while the least pro-social participant (Participant 3) is assigned the largest share. This allocation is a distinctive implication of P2P insurance design. The contract uses internal loss allocation to reward participants who generate positive externalities through higher prevention effort. In effect, the pool endogenously creates an incentive-compensation mechanism through risk-sharing weights. Such targeted allocation of loss exposure is not available in standard bilateral insurance contracts.

The bottom-left panel confirms that equilibrium effort is increasing in pro-social incentives. The most pro-social participant exerts the highest effort despite bearing the smallest share of retained losses, while the least pro-social participant exerts the lowest effort despite bearing the largest share. This reflects the interaction of two opposing forces. On the one hand, a smaller retained loss share weakens direct prevention incentives. On the other hand, warm-glow motives and social concerns increase the effective marginal benefit and reduce the effective marginal cost of effort. In the parameterization considered here, the latter effect dominates, so the optimal contract both induces higher prevention from pro-social participants and compensates them through reduced exposure.

The bottom-right panel shows that utility is increasing in pro-social incentives. The most pro-social participant achieves the highest utility, while the least pro-social participant achieves the lowest. Economically, this outcome reflects the joint benefits of lower effective effort costs and more favorable risk-sharing terms. More broadly, these results highlight the welfare logic of internal risk allocation in P2P insurance: when some participants generate positive externalities through prevention, efficient contracts reward them via favorable loss shares. This provides a mechanism-design rationale for why social capital and pro-social incentives can improve risk outcomes in decentralized insurance arrangements.

4.4 Additional Discussion

Although the analysis is theoretical, it delivers a set of sharp, testable predictions that can be taken directly to data from P2P insurance platforms and related decentralized risk-sharing arrangements. First, conditional on risk type, coverage level, and baseline exposure, participants with stronger social exposure, such as denser network connections, higher interaction frequency, or greater visibility within the pool, should exhibit lower claim frequency and lower realized loss severity. This prediction reflects higher equilibrium loss-prevention effort induced by residual loss exposure combined with social incentives. Empirically, this implication can be tested using within-pool variation in network structure, holding contract terms fixed.

Second, when platforms allow heterogeneity in internal loss allocation or refunds, for example through experience rating, group-level rebates, or differential refund rules, participants with stronger pro-social indicators should, in equilibrium, bear a smaller share of retained losses while exerting higher prevention effort. Observable proxies for pro-sociality may include network centrality, past cooperative behavior, or measures of social engagement on the platform. The model therefore predicts a negative relationship between individual loss-sharing weights and pro-social characteristics, alongside a positive relationship between those same characteristics and prevention outcomes.

Third, exogenous increases in prevention costs, such as higher local repair costs, tighter mitigation constraints, increased hazard intensity, or regulatory changes affecting prevention technology, should induce a systematic shift in contract design toward greater external risk transfer and weaker reliance on retained-loss incentives. In the data, this implies an increase in coverage levels or reinsurance intensity accompanied by a flattening of internal loss-sharing schedules. These responses mirror the model's comparative statics: when prevention becomes more costly, optimal contracts substitute away from incentive provision toward risk transfer. These effects should be strongest in environments with large effort spillovers, such as correlated climate and catastrophe risks where one participant's prevention reduces expected losses for the entire pool.

The analysis also clarifies the organizational limits of decentralized insurance. Social and peer-based incentives are most effective in relatively small, well-connected groups, where individual actions are salient and accountability is preserved. As pools grow larger or networks become sparse, individual responsibility becomes diluted, weakening the incentive effects of residual loss exposure and social preferences. This trade-off between scale and incentive provision helps explain why many P2P platforms rely on segmentation, small-group pooling, or layered designs that combine decentralized mutualization with traditional insurance or reinsurance coverage.

More broadly, the framework contributes to understanding how social structure shapes responses to risk. Social connections affect not only beliefs and attention, as emphasized in the empirical literature, but also equilibrium incentives and effort provision. By making participants partial residual claimants on shared aggregate risk, P2P insurance provides an institutional mechanism that helps translate heightened risk awareness into preventive action, a channel that is particularly relevant for climate adaptation and disaster risk management, where collective prevention and correlated losses are central features.

5 Conclusion

This paper provides a theoretical foundation for understanding when and why decentralized, socially embedded insurance arrangements can mitigate ex ante moral hazard when loss-prevention effort is privately chosen and unobservable. We show that P2P insurance can discipline prevention behavior even in the absence of centralized monitoring or enforcement. In contrast to standard insurance contracts, where effort affects only an individual's own expected indemnity, effort in P2P insurance has socially shared consequences through internal risk sharing. This feature fundamentally reshapes incentives and equilibrium behavior, allowing social interactions to play an economically meaningful role in risk management.

The model delivers three main insights. First, pro-social motivations attenuate moral hazard. Participants with warm-glow motives or altruistic concern for peers exert higher loss-prevention effort despite the absence of observability or direct enforcement. Second, because prevention efforts are strategic complements, individual incentives are amplified at the group level. Increases in pro-social incentives raise not only an individual's own effort, but also the effort of others, generating a multiplier effect that improves aggregate risk outcomes. Third, P2P insurance introduces an additional contract-design margin that is absent from standard insurance. Beyond choosing the extent of external risk transfer, optimal contracts can allocate retained losses across participants in a way that rewards those who contribute most to collective loss reduction.

These results provide a mechanism-design rationale for several practices observed on P2P insurance platforms, including group-level refunds, experience-based rebates, and an emphasis on social accountability. By combining residual claimant exposure with social incentives, P2P insurance expands the set of feasible institutional responses to hidden action in environments characterized by correlated risks and collective prevention technologies. The framework also offers guidance for the design of decentralized risk-sharing institutions beyond insurance, including mutual aid systems and other settings in which individual actions jointly determine aggregate risk.

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A Illustrative Examples and Economic Intuition

This appendix provides examples that illustrate the key primitives and assumptions used in the main analysis. Across these examples, individual effort shifts the distribution of aggregate losses, and complementarities imply that effort choices are strategically linked. Combined with residual claimant exposure induced by risk-transfer and risk-sharing rules, these properties generate a multi-agent moral-hazard problem with strategic interaction.

A.1 Risk Transfer and Risk Sharing

We begin by illustrating a class of risk-transfer and risk-sharing rules that satisfy the no-sabotage and comonotonicity assumptions.

Example 1 (Proportional Risk Transfer and Risk Sharing) *Consider the proportional risk-transfer rule* $t(Y) = \alpha Y$ *with* $0 \leq \alpha \leq 1$*, combined with an actuarially fair premium* $\pi = \mathbb{E}[\alpha Y]$*. Let the internal risk-sharing rule be proportional,*

$$\boldsymbol{h}(Y) = (\omega_1 Y, \ldots, \omega_n Y),$$

where

$$(\omega_1, \ldots, \omega_n) \in \Big\{ \boldsymbol{\omega} \in \mathbb{R}_+^n : \sum_{k=1}^{n} \omega_k = 1 \Big\}.$$

The resulting P2P insurance contract $\Gamma := (t, \pi, \boldsymbol{h})$ *yields the retained loss*

$$r(S_n) = \mathbb{E}[\alpha S_n] + (1-\alpha) S_n,$$

and individual loss allocations

$$\boldsymbol{h} \circ r(S_n) = \big(\omega_1 r(S_n), \ldots, \omega_n r(S_n)\big).$$

This example highlights the two fundamental design margins of P2P insurance. The parameter α governs the extent of external risk transfer, while the weights $(\omega_1, \ldots, \omega_n)$ determine how retained losses are allocated internally. The risk-transfer rule satisfies the no-sabotage condition because $t'(Y) = \alpha \leq 1$, and the risk-sharing rule is comonotonic because each participant's loss share increases with the retained loss. When prevention effort is endogenous, these two margins interact with incentives: greater external insurance weakens residual exposure, while internal allocation shapes which participants bear marginal risk and therefore where prevention incentives are concentrated.

A.2 First-Order Stochastic Dominance, and Supermodularity

We next provide examples of loss distributions that satisfy the first-order stochastic dominance and supermodularity assumptions imposed on the prevention technology.

Example 2 (Continuous Distribution with Complementary Effort) *Let*

$$S_n = \frac{U}{T(\boldsymbol{e})},$$

where $U \sim \text{Uniform}(0,1)$ *and*

$$T(\boldsymbol{e}) = \exp\Big(\sum_{i=1}^{n} a_i e_i + \sum_{i=1}^{n}\sum_{j\neq i} \beta_{ij} e_i e_j\Big),$$

with $a_i \geq 0$ *and* $\beta_{ij} \geq 0$ *for all* $i \neq j$*. The cumulative distribution function of* S_n *conditional on effort is*

$$F_{S_n}(s \mid \boldsymbol{e}) = s\,T(\boldsymbol{e}), \quad s \in \Big[0, \frac{1}{T(\boldsymbol{e})}\Big].$$

For all $s \in [0, \bar{s}]$ *and* $i \neq j$*,*

$$\frac{\partial F_{S_n}(s \mid \boldsymbol{e})}{\partial e_i} = s\,T(\boldsymbol{e})\Big(a_i + \sum_{k\neq i}(\beta_{ik} + \beta_{ki})e_k\Big) \geq 0,$$

$$\frac{\partial^2 F_{S_n}(s \mid \boldsymbol{e})}{\partial e_i \partial e_j} = s\,T(\boldsymbol{e})\Big((\beta_{ij} + \beta_{ji}) + \big(a_i + \sum_{k\neq i}(\beta_{ik} + \beta_{ki})e_k\big)\big(a_j + \sum_{k\neq j}(\beta_{jk} + \beta_{kj})e_k\big)\Big) \geq 0.$$

This example shows how prevention effort can both reduce aggregate losses in expectation and exhibit complementarities. Higher effort by one participant not only shifts the loss distribution toward lower realizations, but also increases the effectiveness of others' effort. The structure is flexible and can be extended to any continuous baseline distribution F_{S_0} with $F''_{S_0} \geq 0$, combined with a scaling function $T(\boldsymbol{e})$ that is increasing and supermodular in effort.

Example 3 (Discrete Distribution with Complementary Effort) *Let the aggregate loss* S_n *take three values* $\{0, L_1, L_2\}$ *with* $0 < L_1 < L_2 = \bar{s}$ *and probabilities*

$$\begin{aligned} \Pr(S_n = 0) &= \gamma_{0,0}, \\ \Pr(S_n = L_1) &= \gamma_{1,0} + a_{1,1}e_1 + a_{1,2}e_2 + \beta_{1,12}e_1e_2, \\ \Pr(S_n = L_2) &= 1 - \Pr(S_n = L_1) - \Pr(S_n = 0), \end{aligned}$$

where $a_{1,1}, a_{1,2}, \beta_{1,12} \geq 0$*. The corresponding cumulative distribution function satisfies*

$$\frac{\partial F_{S_n}(s \mid \boldsymbol{e})}{\partial e_i} \geq 0 \quad \text{and} \quad \frac{\partial^2 F_{S_n}(s \mid \boldsymbol{e})}{\partial e_1 \partial e_2} \geq 0 \quad \text{for all } s.$$

This discrete example illustrates that the assumptions do not rely on continuity or smoothness of losses. The construction extends naturally to finite-support distributions with arbitrary numbers of loss states, provided effort enters probabilities with nonnegative first and cross-partial effects.

B Proofs of Propositions

B.1 Proposition 1

First, note that the selfish and warm-glow equilibria are degenerate cases of the altruistic equilibrium. Specifically:

- When $\boldsymbol{\Delta} = \text{diag}(1, 1, \ldots, 1)$, the altruistic equilibrium $\boldsymbol{e}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$ reduces to the warm-glow equilibrium $\boldsymbol{e}(\Gamma, \boldsymbol{g})$.
- When $\boldsymbol{\Delta} = \text{diag}(1, 1, \ldots, 1)$ and $\boldsymbol{g} = (0, 0, \ldots, 0)$, the altruistic equilibrium further reduces to the selfish equilibrium $\boldsymbol{e}(\Gamma)$.

Thus, it suffices to prove the existence of a hyper-rectangle for the most general altruistic equilibrium:

$$\boldsymbol{e}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \in [\underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}), \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})].$$

Recall the altruistic utility function:

$$\tilde{u}_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}_i, \boldsymbol{e}) = u_i(\Gamma, g_i, \boldsymbol{e}) + \sum_{k \neq i} \delta_{ik} u_k(\Gamma, g_k, \boldsymbol{e}),$$

where $u_i(\Gamma, g_i, \boldsymbol{e}) = \mathbb{E}[v_i(w_i - h_i \circ r(S_n)) \mid \boldsymbol{e}] - c_i(e_i, g_i)$.

Using Lebesgue-Stieltjes integration by parts, we express the utility function as:

$$\begin{aligned}
u_i(\Gamma, g_i, \boldsymbol{e}) &:= \int_{[0,\bar{s}]} v_i\big(w_i - h_i \circ r(s)\big)\, dF_{S_n}(s; \boldsymbol{e}) - c_i(e_i, g_i) \\
&= v_i\big(w_i - h_i \circ r(\bar{s})\big) F_{S_n}(\bar{s}) - v_i\big(w_i - h_i \circ r(0^-)\big) F_{S_n}(0^-) \\
&\quad - \int_{[0,\bar{s}]} F_{S_n}(s^-; \boldsymbol{e}) dv_i\big(w_i - h_i \circ r(s)\big) - c(e_i, g_i) \\
&= v_i\big(w_i - h_i \circ r(\bar{s})\big) - \int_{[0,\bar{s}]} F_{S_n}(s^-; \boldsymbol{e}) dv_i\big(w_i - h_i \circ r(s)\big) - c(e_i, g_i) \\
&= v_i\big(w_i - h_i \circ r(\bar{s})\big) + \int_{[0,\bar{s}]} F_{S_n}(s^-; \boldsymbol{e}) v_i'\big(w_i - h_i \circ r(s)\big) h_i'\big(r(s)\big) r'(s) ds - c_i(e_i, g_i).
\end{aligned}$$

The second equality follows from $F_{S_n}(0^-) = 0$ and $F_{S_n}(\bar{s}) = 1$.

Proof that $h_i'(r(s)) \geq 0$: By Assumption 2, h_i is non-decreasing, so $h_i'(y) \geq 0$ for all y. With $y = r(s)$, we have $h_i'(r(s)) \geq 0$.

Proof that $r'(s) \geq 0$: By definition, $r(s) = \pi + s - t(s)$. Since the premium π is paid ex-ante and is constant with respect to the realized loss s, we have:

$$r'(s) = 1 - t'(s) \geq 0,$$

where the inequality follows from Assumption 1.

Proof that $\frac{\partial^2 \tilde{u}_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}_i, \boldsymbol{e})}{\partial e_i \partial e_j} \geq 0$: Differentiating twice yields:

$$\begin{aligned}\frac{\partial^2 \tilde{u}_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}_i, \boldsymbol{e})}{\partial e_i \partial e_j} &= \sum_{k=1}^{n} \delta_{ik} \int_0^{\bar{s}} \frac{\partial^2 F_{S_n}(s^- \mid \boldsymbol{e})}{\partial e_i \partial e_j} v_k'(w_k - h_k \circ r(s)) h_k'(r(s)) r'(s)\, ds \\ &\geq 0,\end{aligned}$$

where $\delta_{ii} = 1$ by convention. The inequality holds because all terms in the integrand are non-negative: $\delta_{ik} \geq 0$ by construction, $v_k' > 0$ by strict increasing utility, $h_k'(r(s)) \geq 0$ and $r'(s) \geq 0$ as shown above, and $\frac{\partial^2 F_{S_n}(s^- \mid \boldsymbol{e})}{\partial e_i \partial e_j} \geq 0$ by Assumption 4.

Since $\frac{\partial^2 \tilde{u}_i(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}_i, \boldsymbol{e})}{\partial e_i \partial e_j} \geq 0$ for all i, j, the game is supermodular. Let $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$ be the largest element in $\arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, (\hat{e}_i, \boldsymbol{e}_{-i})\big)$, and let $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$ be the smallest element in $\arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}, (\hat{e}_i, \boldsymbol{e}_{-i})\big)$. Denote

$$\overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) = \big(\overline{B}_1(\boldsymbol{e}_{-1}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}), \ldots, \overline{B}_n(\boldsymbol{e}_{-n}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta})\big)$$

and

$$\underline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) = \big(\underline{B}_1(\boldsymbol{e}_{-1}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}), \ldots, \underline{B}_n(\boldsymbol{e}_{-n}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta})\big)$$

as the corresponding vector-valued best-response mappings. By Tarski's fixed point theorem, Nash equilibrium exists, and the set of Nash equilibria forms a complete lattice. In particular, there exist largest and smallest equilibria

$$\overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) = \sup\{\boldsymbol{e} \in [\boldsymbol{0}, e_{\max}\boldsymbol{1}] \mid \overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \geq \boldsymbol{e}\}.$$

and

$$\underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) = \inf\{\boldsymbol{e} \in [\boldsymbol{0}, e_{\max}\boldsymbol{1}] \mid \underline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \leq \boldsymbol{e}\}.$$

such that all equilibria lie in the hyper-rectangle:

$$\boldsymbol{e}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \in [\underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}), \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta})].$$

B.2 Proposition 2

We prove that $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ is increasing with respect to both g_i and δ_{ik} for $i \in \{1, 2, \ldots, n\}$, i.e.

$$\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i') \geq \overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$$

if $\boldsymbol{g}' \geq \boldsymbol{g}$ and $\boldsymbol{g}' \geq \boldsymbol{g}$. Taking derivatives, we obtain

$$\begin{aligned}\frac{\partial \tilde{u}_i(\Gamma, g_i, \boldsymbol{\Delta}_i, \boldsymbol{e})}{\partial e_i \partial \delta_{ik}} &= \int_0^{\bar{s}} \frac{\partial F_{S_n}(s^- \mid \boldsymbol{e})}{\partial e_i} \cdot v_k'(w_k - h_k \circ r(s)) h_k'(r(s)) r'(s)\, ds \geq 0, \\ \frac{\partial \tilde{u}_i(\Gamma, g_i, \boldsymbol{\Delta}_i, \boldsymbol{e})}{\partial e_i \partial g_i} &= -\frac{\partial c_i(e_i, g_i)}{\partial e_i \partial g_i} \geq 0.\end{aligned}$$

Therefore, for any $e_i^H \geq e_i^L$

$$\tilde{u}_i\big(\Gamma, g_i', \boldsymbol{\Delta}_i', (e_i^H, \boldsymbol{e}_{-i})\big) - \tilde{u}_i\big(\Gamma, g_i', \boldsymbol{\Delta}_i', (e_i^L, \boldsymbol{e}_{-i})\big) \geq \tilde{u}_i\big(\Gamma, g_i, \boldsymbol{\Delta}_i, (e_i^H, \boldsymbol{e}_{-i})\big) - \tilde{u}_i\big(\Gamma, g_i, \boldsymbol{\Delta}_i, (e_i^L, \boldsymbol{e}_{-i})\big).$$

Let $e_i^H = \overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$, and $e_i^L = \overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$. Assume by contradiction that $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i') < \overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$, then,

$$\begin{aligned} &\tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \\ &\geq \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \end{aligned} \tag{7}$$

Moreover, because $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ and $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$ are maximizer under $(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ and $(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$, we have:

$$\tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \geq \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) \tag{8}$$

$$\tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\overline{B}_i\big(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i\big), \boldsymbol{e}_{-i}\big)\Big) \geq \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \tag{9}$$

By the Equation (9) and (7), we obtain:

$$\begin{aligned} &\tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \\ &\geq \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \geq 0 \end{aligned} \tag{10}$$

Moreover, Equation (10) and (8) together imply:

$$\tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) = \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big).$$

Therefore, under $(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$, both $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ and $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$ are maximizer. However, $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i) > \overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$, which implies that $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$ is *not* the largest element in $\arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, g_i', \boldsymbol{\Delta}_i', (\hat{e}_i, \boldsymbol{e}_{-i})\big)$. This creates a contradiction. Therefore, $\overline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ is increasing with respect to $\boldsymbol{e}_{-i}$, g_i and δ_{ik} for $i \in \{1, 2, \ldots, n\}$. This implies that $\overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$ is an increasing function with respect to $\boldsymbol{e}$, $\boldsymbol{g}$ and $\boldsymbol{\Delta}$.

By Tarski's fixed point theorem, we know that

$$\overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) = \sup\{\boldsymbol{e} \in [\boldsymbol{0}, e_{\max}\boldsymbol{1}] \mid \overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \geq \boldsymbol{e}\}.$$

Because $\overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$ is an increasing function with respect to $\boldsymbol{e}$, $\boldsymbol{g}$ and $\boldsymbol{\Delta}$, we know that

$$\{\boldsymbol{e} \in [\boldsymbol{0}, e_{\max}\boldsymbol{1}] \mid \overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \geq \boldsymbol{e}\} \subset \{\boldsymbol{e} \in [\boldsymbol{0}, e_{\max}\boldsymbol{1}] \mid \overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}', \boldsymbol{\Delta}') \geq \boldsymbol{e}\}$$

and

$$\overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) = \sup\{\boldsymbol{e} \in [\boldsymbol{0}, e_{\max}\boldsymbol{1}] \mid \overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \geq \boldsymbol{e}\}$$

$$\leq \sup\{\boldsymbol{e} \in [\mathbf{0}, e_{\max}\mathbf{1}] \mid \overline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma', \boldsymbol{g}', \boldsymbol{\Delta}) \geq \boldsymbol{e}\} = \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}', \boldsymbol{\Delta}').$$

Similarly, we prove that $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ is increasing with respect to both g_i and δ_{ik} for $i \in \{1, 2, \ldots, n\}$. Assume by contradiction that $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i') < \underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$, then,

$$\begin{aligned}
&\tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \qquad (11)\\
&\geq \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big)
\end{aligned}$$

Moreover, because $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ and $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$ are maximizer under $(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ and $(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$, we have:

$$\tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \geq \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) \qquad (12)$$

$$\tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i\big(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i\big), \boldsymbol{e}_{-i}\big)\Big) \geq \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \qquad (13)$$

By the Equation (12) and (11), we obtain:

$$\begin{aligned}
0 &\geq \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i', \boldsymbol{\Delta}_i', \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big) \quad (14)\\
&\geq \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) - \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big)
\end{aligned}$$

Moreover, Equation (14) and (13) together imply:

$$\tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i), \boldsymbol{e}_{-i}\big)\Big) = \tilde{u}_i\Big(\Gamma, g_i, \boldsymbol{\Delta}_i, \big(\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i'), \boldsymbol{e}_{-i}\big)\Big).$$

Therefore, under $(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$, both $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ and $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$ are maximizer. However, $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_, \boldsymbol{\Delta}_i) > \underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$, which implies that $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i', \boldsymbol{\Delta}_i')$ is *not* the smallest element in $\arg\max_{\hat{e}_i \geq 0} \tilde{u}_i\big(\Gamma, g_, \boldsymbol{\Delta}_i, (\hat{e}_i, \boldsymbol{e}_{-i})\big)$. This creates a contradiction. Therefore, $\underline{B}_i(\boldsymbol{e}_{-i}, \Gamma, g_i, \boldsymbol{\Delta}_i)$ is increasing with respect to $\boldsymbol{e}_{-i}$, g_i and δ_{ik} for $i \in \{1, 2, \ldots, n\}$. This implies that $\underline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta})$ is an increasing function with respect to $\boldsymbol{e}$, $\boldsymbol{g}$ and $\boldsymbol{\Delta}$.

By Tarski's fixed point theorem again, we know that

$$\{\boldsymbol{e} \in [\mathbf{0}, e_{\max}\mathbf{1}] \mid \underline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}', \boldsymbol{\Delta}') \geq \boldsymbol{e}\} \subset \{\boldsymbol{e} \in [\mathbf{0}, e_{\max}\mathbf{1}] \mid \underline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \geq \boldsymbol{e}\}$$

and

$$\begin{aligned}
\underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) &= \inf\{\boldsymbol{e} \in [\mathbf{0}, e_{\max}\mathbf{1}] \mid \underline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \leq \boldsymbol{e}\}\\
&\leq \inf\{\boldsymbol{e} \in [\mathbf{0}, e_{\max}\mathbf{1}] \mid \underline{\boldsymbol{B}}(\boldsymbol{e}, \Gamma, \boldsymbol{g}', \boldsymbol{\Delta}') \leq \boldsymbol{e}\} = \underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}', \boldsymbol{\Delta}').
\end{aligned}$$

Therefore,

$$\overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}', \boldsymbol{\Delta}') \geq \overline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}) \quad \text{and} \quad \underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}', \boldsymbol{\Delta}') \geq \underline{\boldsymbol{e}}(\Gamma, \boldsymbol{g}, \boldsymbol{\Delta}).$$

for $\boldsymbol{g}' \geq \boldsymbol{g}$ and $\boldsymbol{\Delta}' \geq \boldsymbol{\Delta}$.