A Monetary Theory of Bank Balance Sheets*

Saki Bigio† Pierre-Olivier Weill‡ Diego Zúñiga§

December 29, 2025

We present a general equilibrium banking theory where banks supply liabilities used as means of payment (money). Underlying assets vary in risk, and information asymmetries make risky assets poor means of payment. Banks swap assets for their liabilities, which circulate if their payoffs across states are sufficiently stable. Due to the complementarity in the creation of liquidity, in equilibrium, banks acquire all assets, including those that could be used in payments directly. In turn, banks issue a common liability whose risk and liquidity premium varies with the distribution of payoffs across states. The equilibrium allocation is implemented using instruments found in real-world balance sheets.

We further introduce geographical segmentation and liability recognition to study: (I) a tradeoff between using safe assets (outside money) to complement liquidity creation versus using them in non-bank transactions, (II) the incentives to integrate banks into a banking system that issues joint liability (III) the force consolidation, where market power interferes with liquidity creation. Our theory informs historical money-and-banking debates regarding the trade-off between stability and efficiency.

*All errors are ours.

†Department of Economics, University of California Los Angeles and NBER, email sbigio@econ.ucla.edu

‡Department of Economics, University of California Los Angeles, NBER, and CEPR, e-mail: poweill@econ.ucla.edu

§Bank of Canada, email: DZunigaSeminario@bank-banque-canada.ca

1 Introduction

In essence, banking is about swapping assets. When banks make loans or purchase securities, they acquire assets. In exchange, counterparts receive deposits, a bank liability. An evident distinction between the two sides of banks' balance sheets is that deposits circulate as a medium of exchange. Thus, in exchanging their liabilities to acquire assets, banks create money. A classic view of banking is that swapping assets is not mere accounting. Rather, the money creation that comes with it is essential for economic activity.

An essential role of banks in creating money was first translated into formal modeling by Gorton and Pennacchi (1990), henceforth GP. That paper is motivated by the observation that, throughout history, banks provided credit while creating money. The idea in that paper is that trade is frictional due to informational asymmetries. Banks, however, can mitigate those frictions through the design of securities backed by the assets of their clients, providing a security-design service.[1] While security design is a cornerstone of corporate finance, as a theory of money and banking, the theory is incomplete in three important dimensions. The goal of this paper is to complete the GP information-based theory of money and banking along those dimensions.

The first missing element from the information-based theory of money and banking is a market for assets and liabilities. In GP, banks compete by providing services as technical advisers or underwriters on a per-client basis. However, banks don't participate in markets where various assets, liquid and illiquid, are traded in exchange for circulating bank liabilities. As a result, we lack predictions regarding the quantities and prices of these exchanges, the bundling of assets and liabilities in bank balance sheets, and the resulting risk exposures. Considering a market environment is non-trivial: Banks must not only compete by offering security-design services but must offer competitive terms when pooling assets while guaranteeing the circulation of their liabilities.

The second missing element from the information-based theory of money and banking is the competition between bank liabilities and other forms of (outside) money. As a result, we lack predictions regarding how outside money interacts with the liquidity provision function. For example, it is unclear from the theory why, throughout history, substantial amounts of outside money were held as bank assets while simultaneously used in transactions by non-bank agents.

The third missing element from the information-based theory of money and banking is the notion that the general acceptability of bank liabilities requires a lot of information about their liabilities. This gap is particularly significant given the historical evolution of banking systems: For instance, during the Free Banking era in the United States, banks independently would issue bills that did not circulate at par value in distant locations. In the eve of this period, by the mid-1870s, banks began to integrate forming clearing-houses that issued common circulating notes, an institution that persists to this day in the form of the US interbank-market system.[2] In other historical episodes, banks consolidated in the formation of large. While

[1] Security design problems, e.g., those in DeMarzo and Duffie (1999) and Biais and Mariotti (2005), consist of structuring corporate liabilities fully backed by assets prone to adverse selection in ways that mitigate this problem.

[2] In practice, when deposits circulate, they circulate across accounts in different banks. Thus, banks are committed to absorbing

integration and consolidation enhance the general acceptability of bank liabilities, they introduce frictions to the liquidity creation process. For example, integration via a joint liability system is prone to moral hazard, while bank consolidations lead to increased market power. Because the theory lacks forces toward bank integration and consolidation, we lack predictions on how these frictions impact the liquidity creation function.

Completing the theory along these three dimensions enhances our understanding of money and banking. Throughout the paper, we connect the theory's predictions with the historical evolution of banking and classical debates that shaped monetary and financial regulation. The paper proceeds to fill these gaps, one section at a time.

Core Environment. We consider a three-period, two-state economy with an arbitrary distribution of assets in positive supply, differing in risk exposure. Economic activity necessitates trade, but lack of commitment precludes credit use, requiring assets as payment means.[3] A key aspect is that assets must have sufficiently stable payoffs to function as a medium of exchange. Specifically, the difference between high and low-payoff states must not exceed a *liquidity coefficient*. To resolve illiquidity, assets can be sold to banks in exchange for bank liabilities before private information arrives. Banks, following the security design literature, can create liabilities with various payoff structures. Bank liabilities circulate if their payoffs are designed to be sufficiently stable. Unlike traditional security design, we study equilibria in markets where bank assets and liabilities are freely traded.[4]

In a market setting, the information-based theory of money and banking yields some novel predictions: First, without loss of generality, all banks issue a single liability, i.e., liabilities with the same payoff, in exchange for all the assets of agents with trading opportunities. That is, banks not only purchase illiquid assets in exchange for liquid liabilities but will also purchase assets that could be used for trade directly. This shows that liquid assets are complementary in creating liquidity out of illiquid assets. Second, while banks acquire all assets, assets have different liquidity premia relative to their expected payoffs. Third, depending on the initial distribution of assets between banks and non-banks, circulating bank liabilities may be perfectly safe or bear some risk, and bank equity, the riskiest asset in the economy, may be wiped out in low states.

These properties provide a taxonomy of equilibria: If the *aggregate* payoffs across all assets are sufficiently stable, i.e., they fall within the liquidity coefficient bounds, there are no liquidity premia, banks only issue a riskless liability, and banks' equity is moderately risky. When aggregate payoffs are illiquid, outcomes depend on the bank-aggregate initial asset endowment: with sufficient bank wealth, we obtain the same outcome as if *aggregate* payoffs are liquid. If bank wealth is insufficient, banks issue risky liabilities,

each others' liabilities, which does not occur with any other corporate liability.

[3] A continuum of producers with production opportunities own assets and are matched bilaterally with workers. Producers' lack of commitment precludes credit use in labor relations, necessitating asset-based payments. Following Rocheteau (2011) and Bigio (2015), producers receive private information about their assets, rendering risky assets unsuitable for direct trade.

[4] In the simplest case, a banker offers to swap an illiquid asset for a liquid liability. The banker profits by issuing a liability with a lower average payoff than the asset. The producer accepts a payoff reduction in exchange for liquidity, as banks structure the liability's state-contingent payoffs to be safer than the underlying asset.

but still, there are no liquidity premia. Liquidity premia emerge once bank wealth falls under a threshold value in which they issue the riskiest liability possible. As initial bank endowments near zero, banks issue two liabilities: a risky liquid liability and a maximally risky illiquid liability.

While we cast our analysis into an abstract Arrow-Debreu setting, we also show it can be implemented using financial instruments identifiable in actual bank balance sheets: in its greatest generality, the equilibrium trades in Arrow-Debreu space can be implemented through a mix of collateralized loans, outright asset purchases, deposits, and outside equity.[5]

Competition with outside money. To address the second missing piece in the information-based theory of money and banking, we extend our core environment to study the competition between outside money and bank liabilities. To induce a distinguishing role, we introduce geographical separation: bank liabilities are recognized in transactions within the bank's location, while the riskless outside money can be used universally. Non-bank agents can trade directly with their outside money or deposit it at banks, which can use outside money to enhance the creation of circulating liabilities. We study the condition under which outside money is held exclusively by non-bank agents as a competing medium of exchange and when it is deposited at banks. In this environment, banks hold outside money if two conditions are met: the aggregate payoffs of trees are illiquid, and bank liabilities are sufficiently recognizable.

Our analysis reveals that as bank liabilities gain wider acceptance, the safe asset increasingly appears on bank balance sheets. This shift reflects the evolution of the banking system: as bank liabilities become more recognized across locations, trade increasingly relies on inside money relative to outside money, and a larger stock of outside money is utilized in creating inside money. Counterintuitively, the value of the safe asset does not necessarily decrease as bank liabilities become more acceptable.[6]

Consolidation and Integration. Issues like recognizability have created a force toward bank integration through joint liability systems or concentration but have been absent from the theory. To address this third gap in the information-based theory of money and banking, we explore two final extensions to our model.

First, we examine joint liability systems while introducing a moral hazard problem: individual banks can overissue liabilities to acquire goods. This leads to an incentive constraint that limits the value of bank liabilities to a multiple of the value of safe assets. Our analysis reveals two distinct types of liquidity premia: (1) a premium due to aggregate conditions, present even without moral hazard and observable in the liquid reserve assets held by banks, and (2) a premium on bank liabilities that persists even after accounting for the premium on reserve assets, arising from the bank's incentive constraint.

Second, we analyze liquidity provision by a monopolist bank exercising market power. In this scenario, the bank issues the maximum possible liquid liabilities but extracts rents by providing less liquidity than competitive markets would.

[5] The banker provides deposits (loan size) to the producer, who promises repayment (face value) with the asset as collateral. Over-collateralization effectively gives producers an illiquid liability: an option to repurchase the asset at the loan's face value.

[6] This contrasts with the intuition that safe assets and bank liabilities are substitutes as means of payment. In scenarios where bank liabilities are scarce due to insufficient backing assets, the safe asset's value may increase as it serves as an input for bank liability provision.

These extensions illuminate a crucial trade-off in bank integration. Joint liability systems or charters that increase market power both come at a cost: they impair liquidity creation. This analysis demonstrates how frictions in bank integration spill over to the provision of liquidity by banks.

Literature Review. Our paper intersects with three areas: monetary theory, banking theory, and security design. Banking theory has traditionally explored various functions of banks, including delegated monitoring (Diamond, 1984; Williamson, 1987), risk-insurance and liquidity provision (Diamond and Dybvig, 1983), economies of scope and diversification (Boyd and Prescott, 1986; Hellwig, 1991), and information management (Gorton and Pennacchi, 1990; Leland and Pyle, 1977; Allen, 1990). Within this field, several studies have focused on the efficiency of bank deposits (Diamond and Rajan, 2001; Calomiris and Kahn, 1991).[7] Other research has examined how these functions determine banks' optimal asset composition between loans and liquid assets (Kashyap, Rajan and Stein, 1999; Holmström and Tirole, 1997; von Thadden, 1998).[8] Our contribution extends this literature by examining the equilibrium balance sheet of banks that emerges when their primary function is to resolve asymmetric information problems.

Our focus on the emergence of bank liabilities as means of payment connects our work with the monetary theory in Lagos and Wright (2005).[9] A closely related paper is Rocheteau (2011), who examine bilateral trades with multiple assets in the presence of asymmetric information. Our approach differs in that agents in our model do not trade with assets directly. Instead, they find it optimal to sell these assets in banks and trade using bank liabilities.

Historically, money and banking were studied as part of the same subject, but they have since evolved into independent fields. A notable exception is a strand of literature focusing on the elastic supply of money and inflation as mechanisms to prevent runs in a Diamond-Dybvig setting (Champ, Smith and Williamson, 1996; Antinolfi, Huybens and Keister, 2001; Allen, Carletti and Gale, 2014; Robatto, 2019). In these papers, outside money serves as a unit of account and store of value, but bank liabilities are not used as a medium of exchange. Our approach differs as we do not consider these roles; instead, we focus on real liabilities but emphasize their role in trade.

More recently, a branch of literature has emerged where bank liabilities do play a role as a medium of exchange (Gu, Mattesini, Monnet and Wright, 2013; Andolfatto, Berentsen and Martin, 2019; Donaldson and Piacentino, 2022).[10] While our work shares the motivation for the circulation of bank liabilities with these papers, our key contribution lies in examining the equilibrium supply of circulating liabilities and bank asset holdings when many assets differ in their degree of liquidity. This approach allows us to explore the unique of banks in facilitating trade when information asymmetries would otherwise impede the use of

[7] Jacklin (1987) demonstrates that with complete markets, agents in Diamond and Dybvig (1983) could be insured efficiently, highlighting the need for additional frictions to justify the optimality of deposits.

[8] A related strand of literature investigates bank fragility, originating from the model of runs in Bryant (1980) and developed further in works such as Allen and Gale (1998).

[9] Berentsen, Camera and Waller (2007) study the role of banks in helping agents insure against preference shocks: banks reallocate idle balances from agents who want to consume to those who do not in a Lagos and Wright (2005) setting. In contrast, our model does not generate such a demand for insurance.

[10] Li, Li and Sun (2024) present a model of circulation and settlements in a network.

certain assets as direct means of payment.

The use of securities to resolve issues of asymmetric information related to security design models such as DeMarzo and Duffie (1999) and Biais and Mariotti (2005) or more recently Asriyan and Vanasco (2023). Our contribution relative to this literature is to study the emergence of means of payments backed by illiquid assets from different agents in a competitive setting.[11] Farhi and Tirole (2015) consider the trade-off between tranching and bundling an asset in a model of bilateral asset trade with asymmetric information. Tranching and bundling of assets also plays an important role here. We study a competitive setting and demonstrate that the efficient outcome involves a combination of both tranching and bundling.

Finally, the role of providing security-design services in our work closely aligns with recent research by Dang, Gorton, Holmström and Ordonez (2017). Their paper demonstrates that banks optimally maintain opacity about their balance sheets and design debt-like liabilities to reduce agents' incentives to acquire private information. They show that banks preferentially hold safe assets to facilitate the design of securities that minimize information acquisition incentives.

2 Model

2.1 The economic environment

There are three dates, $t \in \{0, 1, 2\}$, and two states, $\omega \in \{\ell, h\}$, with probability $\pi(\omega)$. The assumption of two states is without loss of generality.

Assets. There is a continuum of risky assets, "trees", in positive supply. Trees are heterogeneous and feature state-dependent payoffs that are realized at $t = 2$. Trees are indexed by a vector $R \in \mathbb{R}^2_+$ where the first and second coordinates correspond to the payoff of R in the low, $\omega = \ell$, and high, $\omega = h$, states respectively. We denote the payoff of tree R in state ω as $R(\omega)$. The payoffs, which we call fruit, are always in consumption units. Without loss of generality, we assume that summing across all trees, high-state payoffs exceed low-state, although individually some trees may feature greater low-state payoffs.

Trees represent broad asset classes, such as cash, bonds, fixed-income securities, or equity. Trees are divisible, and property rights over entire trees can be traded.

Agents. Three types of agents populate the economy: a continuum of producers, a continuum of workers, and a continuum of bankers. All agents are risk neutral and only enjoy consumption at $t = 2$.

At $t = 0$, producers are endowed with tree portfolios. At $t = 1$, each producer operates a linear production technology, whereby q labor units yield ρq output units at $t = 2$, regardless of the state. Labor is supplied by workers at a marginal cost, normalized to one. We assume that the producers and workers lack commitment which precludes the use of credit in bilateral trades, even if the tree can be used as collateral.[12]

[11] Bigio and Shi (2021) also study a competitive setting where intermediaries providing funding by pooling collateralized debt contracts. Issues of circulation are not present there.

[12] Using the tree as collateral for state-contingent promises requires a commitment by the worker to return the tree in all states

Lack of commitment induces the use of assets as means of payment as is common in the monetary literature, e.g. Lagos and Wright (2005).

Bankers are also endowed with a portfolio of trees but can neither access the production technology nor supply labor. Instead, unlike producers and workers, a banker can commit to making future state-contingent payments. As a result, the banker is in a unique position to issue liabilities. We can argue that bankers are special because they can commit or, equivalently, have the legal skills to write enforceable contracts.

Importantly, producers have private information about the realization of ω at $t = 1$.

Three stages. The timing of the model is as follows:

- $t = 0$: centralized exchange. Bankers and producers exchange trees for state-contingent securities issued by the banker in a centralized exchange. In doing so, bankers design which liabilities to issue in exchange for trees.

- $t = 1$: bilateral trade. Producers learn the aggregate state, ω, and are bilaterally and anonymously matched with workers. The worker does not know the state. Labor is traded in exchange for trees or banker liabilities.

- $t = 2$: payoff, settlement, and consumption. Output is produced, all trees pay off, contracts with bankers are settled, and all agents consume.

Assumptions. Throughout, we assume: (i) $\rho > 1$ which means that there are always gains from trade between the producer and the worker; (ii) $\pi(h)\rho < 1$ and $\pi(\ell)\rho < 1$, which ensure that some assets suffer from a lemon problem in bilateral trade, in either state. As will become clear, this creates gains from trade between the producers and banks.

2.2 Monetary Exchange

In this section, we study the bilateral trade between the producer and the worker in which the producer makes payment using an arbitrary state-contingent security. The security is either a tree purchased by the producer in a competitive market or a liability issued by the banker backed by trees. This trade is a key building block: it determines the value of using alternative means of payment to purchase labor. Afterward, we study the exchange between bankers and producers of trees for securities.

Workers compete by offering, for each security $D(\omega)$, a menu of quantities of labor input q in exchange for a quantity $n \in [0, 1]$ of the security. Trading is not exclusive; the producer is not restricted to hiring only one worker. Once the producer learns the aggregate state, he chooses among the workers' offers subject to his holdings of the security.[13]

[13]This is the kind of environment studied in Attar, Mariotti and Salanié (2011). There are alternative ways of modeling the bilateral trade, for instance with signaling via retention as in DeMarzo and Duffie (1999) and applying the intuitive criterion. These alternatives do not affect most of our results. Moreover, non-exclusivity is a natural assumption to make in the context of assets that are media of exchange.

In the unique equilibrium of this trading game, the workers post linear price schedules for each security. Specifically, securities with riskier payoffs are priced at their lowest value, while securities with safer payoffs are priced at their expected value.

Given a security $D(\omega)$, let $q(\omega), n(\omega)$ be the trade chosen by the producer in state ω. Then, the ex-ante value of the security for the producer is given by

$$\mathbb{U}[D] \equiv \mathbb{E}[\rho q(\omega) + [1 - n(\omega)] D(\omega)] . \tag{1}$$

Liquidity Classification. Securities are segmented into liquid and illiquid securities. Consider a security that pays 1 in state ℓ and ψ in state h. Conditional on knowing the state, the producer would be willing to trade the security at its expected value in either state if the value of keeping the security is lower than the value of trading:

$$\underbrace{\psi \leq \rho\left(\pi(\ell) + \psi\pi(h)\right)}_{\substack{\text{trading at expected value in state } h \\ \text{exceeds payoff}}} \quad \text{and} \quad \underbrace{1 \leq \rho\left(\pi(\ell) + \psi\pi(h)\right).}_{\substack{\text{trading at expected value in state } \ell \\ \text{exceeds payoff}}}$$

These conditions are classic lemons-market conditions. They guarantee that trade doesn't break in either state as long as ψ falls between an interval of *liquidity coefficients*, $\psi \in [\psi_L, \psi_H]$ where where

$$\psi_H \equiv \frac{\rho\pi(\ell)}{1 - \rho\pi(h)} > 1, \qquad \psi_L \equiv \frac{1 - \rho\pi(\ell)}{\rho\pi(h)} < 1.$$

These coefficients are important in the theory, extending liquidity conditions to any security. The following proposition characterizes the marginal value of *any* security in terms of the liquidity coefficients:

Proposition 1 *For a producer, the ex-ante value of a security.* $\mathbb{U}[D]$*, the contracted labor,* $q(\omega)$*, and the pattern of trade,* $n(\omega)$*, will depend on the security's payoffs according to the following classification:*

- *Liquid securities:*

$$\text{if } D(h)/D(\ell) \in [\psi_L, \psi_H] \Rightarrow \{n(h), n(\ell)\} = \{1, 1\}, \quad \mathbb{U}[D] = \mathbb{U}^{\ell}[D] := \rho\mathbb{E}[D].$$

- *Illiquid securities:*

$$\text{if } D(h)/D(\ell) > \psi_H \Rightarrow \{n(h), n(\ell)\} = \{0, 1\}, \quad \mathbb{U}[D] = \mathbb{U}^{i,h}[D] \equiv \pi(h)D(h) + \pi(\ell)\rho D(\ell),$$
$$\text{if } D(h)/D(\ell) < \psi_L \Rightarrow \{n(h), n(\ell)\} = \{1, 0\}, \quad \mathbb{U}[D] = \mathbb{U}^{i,\ell}[D] \equiv \pi(h)\rho D(h) + \pi(\ell)D(\ell).$$

Thus, the ex-ante value of an illiquid security as

$$\mathbb{U}^{i}\left[D\right] := \begin{cases} \mathbb{U}^{i,h}\left[D\right], \textit{ for } D(h) > D(\ell) \\ \mathbb{U}^{i,\ell}\left[D\right], \textit{ for } D(h) < D(\ell) \end{cases}$$

Proposition 1 classifies securities into illiquid and liquid securities, depending on whether they satisfy the liquidity condition that guarantees their trade in all states: $D(h)/D(\ell) \in [\psi_L, \psi_H \cdot]$. It is clear that the condition follows from the homogeneity of payoffs: we can always normalize the payoffs of any security by its low state payoff and the conditions for trade in both states are the same as for the $\{1, \psi\}$ security. Graphically, the liquidity condition implies that the state contingent payoffs $\{D(\ell), D(h)\}$ must fall in the cone blue vertical lines in Figure 1—the complement set is either in the top or bottom regions, marked with red horizontal lines in Figure 1..

Liquid securities satisfy the liquidity condition and are always traded, $n(\omega) = 1$. The optimal bilateral trade pools across states: the producer sells its security in exchange for labor at the expected value $\mathbb{E}[D]$ in both states. The security price is lower than its underlying value in the high state. Hence, the producer can only purchase $\mathbb{E}[D]$ labor units, which is less than the true value of the security, $D(h)$.

Illiquid securities violate the liquidity condition. Bilateral trade breaks down for such payoffs, and $n(h) = 0$. The security does not provide liquidity services . The reason for this breakdown is asymmetric information.[14] The lack of bilateral trade with these securities implies that $\mathbb{U}[D]$ is below $\rho \cdot \mathbb{E}[D]$, the value obtained when trade is efficient.

In summary, securities are liquid if their payoffs do not differ too much across states; their payoffs must be sufficiently safe. Geometrically, their payoffs must fall within a cone of $\mathbb{R}^2_+$ that dictates their "information sensitivity", that is, their use in the bilateral exchange after producers learn the state.

Naturally, we can think of $\mathbb{U}[D]$ as an indirect utility associated with holding a security D. Figure 1 plots an indifference curve associated with $\mathbb{U}[D]$, with respect to the payoffs $\{D(\ell), D(h)\}$. The characterization shows that $\mathbb{U}[D]$ is increasing and concave in payoffs as long as the security is liquid, when $D(h) \in [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]$. In Figure 1, this corresponds to the green region marked with horizontal lines and the blue region marked with vertical lines. When $D(h) \notin [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]$, trade breaks down and $\mathbb{U}[D]$ jumps down discretely. Hence, the indifference condition features a discontinuity at the points at which the security becomes illiquid. At those points, the indifference curve shifts to higher payoffs that compensate the producer for lacking a liquidity service as a medium of exchange.

From now on, we define Λ^{ℓ} the set (the cone) of liquid securities. We denote the set of the remaining, illiquid securities as Λ^{I}, those whose payoffs fall outside of the cone.

Without asymmetric information, the producer would be indifferent between securities with the same expected payoffs, and, in that case, his indifference curves would be flat. With private information, his

[14] After learning the state, the producer's value of holding on to the security is larger than the value of purchasing the worker's labor at a pooling price, $D(h) > \rho \cdot \mathbb{E}[D] \iff D(h) > \psi_H \cdot D(\ell)$, a classic lemons market condition.

indifference curve features discontinuities precisely at the boundaries of the cone that defines liquid securities. The discontinuities indicate that the producer prefers a security that pays slightly less in expectation but satisfies the liquidity condition—less in the high state in the case of the upper boundary and vice versa. Indeed, if he could commit ex-ante, the producer would want to reduce the payoff of his security to exploit the gains from trade. Structuring state-contingent payoffs ex-ante to avoid this commitment problem is the essence of security design.

So far, we have been silent about the actual securities producers use as a medium of exchange. The following section describes an environment where securities are claims on banks fully backed by trees. Thus, banks will provide a security design service.

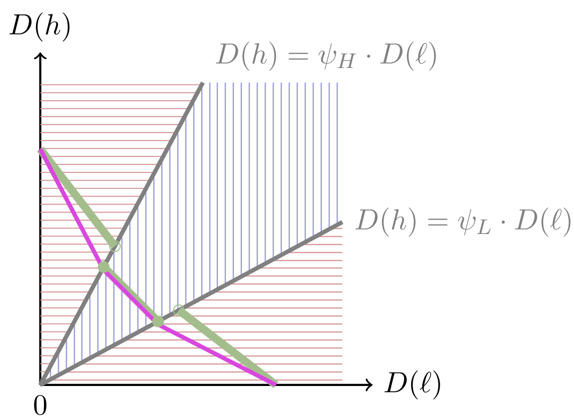
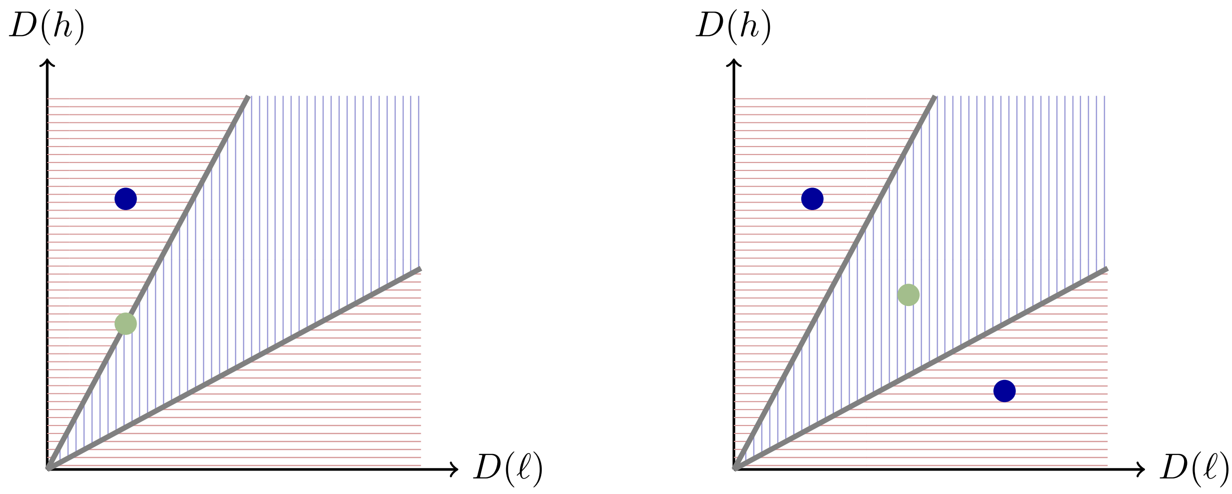


Figure 1: In green, the producer's indifference curve for an individual security. The indifference curve for a portfolio is depicted in purple. It coincides with the individual indifference curve in the liquid region.

The role of banks. As a prelude to the role of banks in the following section, we describe how restructuring of the property rights over a security's payoff could enable its circulation: A security for which $D(h) > \psi_H \cdot D(\ell)$, does not circulate because their payoff in the high state is too large—see Figure 2a. If anyone could dismantle the security into two securities. One that pays $\{0, D(h) - \psi_H \cdot D(\ell)\}$ and one that pays $\{D(\ell), \psi_H \cdot D(\ell)\}$: the first security cannot circulate, but the second one can. This is the idea behind the *tranching* of securities.

Consider, in turn, that instead of branching the security, there is also a second security $D(h) < \psi_L \cdot D(\ell)$, that also cannot circulate because their payoff in the high state is too low. If anyone could combine both securities into a single one with payoffs in the liquidity cone, the combined security would circulate. This is the idea behind the *bundling* of securities.

In our framework, the role of banks is to restructure payoffs. Banks have the ability to tranche payoffs from a single security or bundle payoffs of different securities, creating liabilities that circulate. Moreover, they can tranche and bundle securities of different producers and create liabilities backed by entire portfolios of trees. The next section explains what trees are brought to the bank, what securities are issued, and at what prices.



(a) Tranching **(b)** Bundling

Figure 2: Examples

Information Sensitivity and Liquidity. In this model, an asset can be used as a means of payment if it is relatively safe. Dang, Gorton, Holmström and Ordonez (2017) describe the defining characteristic of such an asset as information (in)sensitivity in the presence of information acquisition. These ideas share more than a passing resemblance. Indeed, if we consider a richer setting in which the investor can acquire information about the state of the world, the investor's belief distribution over payoffs will lie in the liquid region for any liquid security. Therefore, any liquid security isinformation insensitive in the sense of Dang, Gorton, Holmström and Ordonez (2017).

3 Competitive liability design

We now study the exchange of trees for bank liabilities. Producers can sell their trees for bank liabilities, which they can hold to maturity or use in trade. Bank liabilities are also indexed by some $D \in \mathbb{R}^2_+$, with the same interpretation: the first and second coordinates denote $t = 2$ payoffs in the low state and high states. From the previous section, we know that bank liabilities are liquid if they satisfy the liquidity condition: $D(h) \in [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]$ and are illiquid otherwise. We denote prices in terms of goods.

Producer and banker problems. Throughout the paper, we express asset portfolios as measures on $\mathbb{R}^2$—we use the terms portfolio and measures interchangeably. At $t = 0$, the producer and banker enter the period with respective portfolios μ_0^e and μ_0^b, respectively. They exchange trees for bank liabilities in a centralized exchange. The producer sells his assets and chooses portfolios of illiquid and liquid bank liabilities and trees, $\{\mu_1^i, \mu_1^\ell\}$. In turn, the banker buys a portfolio of trees μ_1^+ and issues a portfolio of liabilities μ_1^-. A tree R and bank liability D are traded at at prices $P(R)$ and $P(D)$, respectively.

At this stage, the producer solves:

Problem 1 (Producer Problem)**.**

$$\max_{\{\mu_1^i,\mu_1^\ell\}\geq 0} \int \mathbb{U}^\ell [D]\, d\mu_1^\ell(D) + \int \mathbb{U}^i [D]\, d\mu_1^i(D) \tag{2}$$

subject to:

$$\int P(D)\, d\mu_1^i(D) + \int P(D)\, d\mu_1^\ell(D) \leq \int P(R)\, d\mu_0^e(R). \tag{3}$$

Equation (3) is the producer's budget constraint: The right-hand side is the value of his initial endowment, calculated by integrating the portfolio measure against the price function $P(R)$. These funds are used to purchase a portfolio of illiquid liabilities, μ_1^i, and liquid liabilities μ_1^ℓ. His objective is to maximize the expected payoffs. The illiquid portfolio stays with the producer until maturity. Thus, the expected benefit of holding those securities is lower than that of holding liquid securities: $\mathbb{U}^i [D] < \mathbb{U}^\ell [D]$.

The lack of commitment on the producer's side is encoded in the assumption that he cannot issue securities. This is implicit in that measures, the portfolios, are positive. That assumption means the producer cannot trade using a security he does not own.

The banker's problem is similar to the producers's, except for two important distinctions. First, the banker lacks trading opportunities with workers. Second, the banker can issue liabilities, which allows him to restructure (securitize) trees. However, the banker can issue liabilities but subject to a limited liability constraint.

Problem 2 (Banker problem)**.**

$$\max_{\{\mu_1^+,\mu_1^-,c(\omega)\}\geq 0} \mathbb{E}[c] \tag{4}$$

subject to:

$$\int P(D)\, d\mu_1^+(D) \leq \int P(D)\, d\mu_1^-(D) + \int P(R)\, d\mu_0^b(R). \tag{5}$$

$$\forall\omega, \quad c(\omega) + \int D(\omega)\, d\mu_1^-(D) \leq \int D(\omega)\, d\mu_1^+(D). \tag{6}$$

At $t = 0$, the banker enters with net worth $\int P(R)\, d\mu_0^b(R)$, and raises funds by issuing a portfolio of bank liabilities, $\mu_1^-(D)$. Some liabilities are liquid, and some are illiquid, depending on their payoffs. The budget constraint (5) says that these funds are used to purchase a new menu of trees, $d\mu_1^+(D)$, which will include some trees or possibly other bankers' liabilities. Equation (6) is a budget constraint at $t = 2$ that holds in each state ω. With the payoffs he collects, the banker pays his liabilities and consumes $c(\omega)$. Importantly,

$c(\omega) \geq 0$ means the banker cannot produce goods. This assumption is important to guarantee that resources are not brought to some states artificially to enhance overall liquidity. This condition is also a limited liability constraint.

Competitive Equilibrium Next, we define a competitive equilibrium.

Definition 1. A competitive equilibrium is a price $P : \mathbb{R}_+^2 \to \mathbb{R}_+$ and portfolios for producers and bankers $\left\{\mu_1^i(\cdot), \mu_1^\ell(\cdot)\right\}$ and $\left\{\mu_1^+(\cdot), \mu_1^-(\cdot)\right\}$ which satisfy two conditions,

1. The producer and banker portfolios are solutions to their problems.
2. The market for every security clears: For all Borel sets $B \in \mathbb{R}_+^2$

$$\int_B d\mu_1^i(D) + \int_B d\mu_1^\ell(D) + \int_B d\mu_1^+(D) \leq \int_B d\mu_1^-(D) + \int_B d\mu_0^e(D) + \int_B d\mu_0^b(D). \tag{7}$$

The definition is standard, although the formulation of the market-clearing condition merits discussion. The right-hand side is the supply of securities. The condition says that the supply of security characterized by the vector D is either a tree—part of the banker's and producer's endowments—or issued by a bank. These securities must be held by either banks or producers as part of the liquid or illiquid portfolios. Importantly, by definition, $\mu_1^i(D) = 0$ for any $D \notin \Lambda^I$ and $\mu_1^\ell(D) = 0$ for any $D \notin \Lambda^\ell$.

Notation. It is convenient to introduce notation for two Arrow-Debreu securities—henceforth, A-D securities—that span the space of securities: $e^\ell \equiv \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $e^h \equiv \begin{bmatrix} 0 & 1 \end{bmatrix}$. As is known, any security can be expressed as a linear combination of the two A-D securities. A special security is the normalized security at the liquidity boundary. We call this the *marginally liquid security*, which pays 1 unit of consumption in the low state and ψ_H in the high state:[15] $e^{\psi_H} = \begin{bmatrix} 1 & \psi_H \end{bmatrix} = e^\ell + \psi_H \cdot e^h$. Finally, another special security is the perfectly safe security, which pays 1 unit of consumption in each state: $e^s = \begin{bmatrix} 1 & 1 \end{bmatrix} = e^\ell + e^h$.

It is also convenient to normalize the price of the security that pays a consumption unit in the high state, $P\left(e^h\right) = \pi \equiv \pi(h)$. To further simplify the notation, we denote by $q \equiv P\left(e^\ell\right)$, the price of the A-D security.

Both of the producer's and banker's portfolios imply an overall position in terms of consumption good in the low and high state. We denote the producer's position as $\{\mathbf{D}(\ell), \mathbf{D}(h)\}$ and the banker's position as $\{\mathbf{C}(\ell), \mathbf{C}(h)\}$. Specifically,

$$\mathbf{D}(\ell) = \int D(\ell) d\left(\mu_1^i + \mu_1^\ell\right), \qquad \mathbf{D}(h) = \int D(h) d\left(\mu_1^i + \mu_1^\ell\right),$$
$$\mathbf{C}(\ell) = \int D(\ell) d\left(\mu_1^+ - \mu_1^-\right), \qquad \mathbf{C}(h) = \int D(h) d\left(\mu_1^+ - \mu_1^-\right).$$

[15] Similarly, we can define a marginally liquid security that pays more in the low state. That security would pay 1 unit of consumption in the low state and ψ_L in the high state. Since we assume that the aggregate payoffs will be larger in the high state, this security will not be relevant in equilibrium.

Finally, we denote by $N(\ell)$ and $N(h)$ the aggregate resources in the low and high state, respectively,

$$N(\ell) \equiv \int R(\ell)\left(d\mu_0^e + d\mu_0^b\right) \quad \text{and} \quad N(h) \equiv \int R(h)\left(d\mu_0^e + d\mu_0^b\right). \tag{8}$$

Notice that the market-clearing condition can be written in terms of the overall positions $(\mathbf{D}(\ell), \mathbf{D}(h))$ and $(\mathbf{C}(\ell), \mathbf{C}(h))$. This follows directly from equation (7) specializing the Borel set B to $\mathbb{R}^2$:

$$\mathbf{D}(\ell) + \mathbf{C}(\ell) = N(\ell) \quad \text{and} \quad \mathbf{D}(h) + \mathbf{C}(h) = N(h)$$

With the notation in hand, we now characterize the individual problems.

Individual Problems. We begin with an important equilibrium property of security prices: no-arbitrage.

Lemma 1 (Non-Arbitrage pricing) *In any equilibrium, prices satisfy a non-arbitrage property: any security is priced at its replication cost (in the Arrow-Debreu basis):*

$$P(D) = qD(\ell) + \pi D(h).$$

The proposition establishes that equilibrium prices are arbitrage free and it follows from the banker's problem. If this property didn't hold, the banker could always increase its consumption in some state, contradicting the market-clearing condition. As a result, liability design is provided competitively, without benefits to the banker in the creation of securities. The following result is immediate:

Proposition 2 *Given non-arbitrage prices, the banker's problem can be solved as if the banker choose directly its aggregate payoffs $\{\mathbf{C}(\ell), \mathbf{C}(h)\}$. In particular, the banker solves*

$$\max_{\{\mathbf{C}(\ell),\mathbf{C}(h)\}\geq 0} (1-\pi)\mathbf{C}(\ell) + \pi\mathbf{C}(h)$$

subject to a single budget constraint:

$$q\mathbf{C}(\ell) + \pi\mathbf{C}(h) = n^b \equiv \int P(R)\, d\mu_0^b = qN^b(\ell) + \pi N^b(h).$$

This result implies that we can solve the problem as if the banker decides only to purchase quantities of the two A-D securities given the value of their assets. This proposition is convenient to characterize the banker's optimal consumption as a function of the relative price of the two A-D security prices.

Proposition 3 *The banker's optimal portfolio holdings $\{\mathbf{C}(\ell), \mathbf{C}(h)\}$ satisfy:*

- *Under-priced liquidity $(1-\pi > q)$:* $\quad \mathbf{C}(\ell) = N^b(\ell) + \frac{\pi}{q}N^b(h), \quad \mathbf{C}(h) = 0$
- *Fairly priced liquidity $(1-\pi = q)$):* $\quad q\mathbf{C}(\ell) + \pi\mathbf{C}(h) = qN^b(\ell) + \pi N^b(h)$

- *Over-priced liquidity $(1-\pi < q)$:* $\mathbf{C}(\ell) = 0, \quad \mathbf{C}(h) = N^b(h) + \frac{q}{\pi} N^b(\ell)$

Since the banker is risk-neutral, the marginal rate of substitution for the banker is constant and equal to $\pi/(1-\pi)$. The banker's optimal overall portfolio must be at corner solution unless the value of the high-state A-D is fairly priced $q = 1-\pi$, in which case the banker is indifferent. In all other cases, the banker concentrates its wealth on the cheaper A-D security.

Next, we study a modified version of the producer's problem, one where we are explicit about prices. Denote by

$$n^e \equiv \int P(R)\, d\mu_0^e = qN^e(\ell) + \pi N^e(h),$$

the wealth of the producer. In the previous section, we encountered the producer's indifference curve for any given security $D = \{D(\ell), D(h)\}$ depicted in the green broken line in Figure 1. This indifference curve was obtained from the producer's indirect utility, which incorporates the value of trading with the security. Given a producer's wealth n^e, the producer's optimal security portfolio would have to fall at the tangency points of this indifference curve with a budget set. When there is no arbitrage—i.e., we can price each security in terms of their replication costs, the budget set will be a line.

For the sake of simplicity, let's temporarily focus on the case where $q \geq 1-\pi$.[16] From the geometry of the indifference curve, in Figure 1 it should be clear that given n^e the price of the low-state A-D security, q, the producer's desired portfolio may be at indifference, among the set of liquid securities or at corners. When $q \geq 1-\pi$, there are two relevant corners: one at a marginally liquid security and the one that pays only in the high state h.

Given the shape of the indifference curve, an important price is $q^\star$, the price that makes a producer indifferent between holding e^h, the illiquid security, and e^{ψ_H}, the marginally liquid security:

$$\underbrace{\rho\left(\pi \cdot \psi^h + (1-\pi)\right)}_{\text{trading value}} \cdot \underbrace{\frac{\pi}{\pi \cdot \psi^h + q^\star}}_{\text{exchange rate}} = \pi.$$

Given that we normalize the price of the high state by π, this condition says that selling e^h for π and buying marginally liquid securities at the replication price $\pi \cdot \psi^h + q^\star$, which in turn have a trading value of $\rho\left(\pi \cdot \psi^h + (1-\pi)\right)$ should yield the same as the expected payoff of the security, π. Solving this condition, we obtain that $q^\star \equiv (1-\pi)\psi_H$. We can characterize the producer's holdings as follows:

Proposition 4 *The optimal security holdings of the producer are as follows:*

- *No liquidity premium $(1-\pi = q)$: Any portfolio of only liquid securities, $\mu_1^i(D) = 0, \mu_1^\ell(D) \geq 0$, such that $n^e = \int p(D)\, d\mu_1^\ell(D)$ is a solution.*

[16] Given the symmetry of the environment, describing the case with $q \leq 1-\pi$ would be superfluous. Moreover, given the assumption that there are more payoffs in the high-state $N(h) \geq N(\ell)$, the equilibrium price will satisfy this condition.

- *Positive liquidity premium $(1-\pi<q)$: Only the marginally liquid security and the maximally illiquid securities are held: $\mu_1^{\ell}=\mu_1^{i}=0, \quad \forall D\notin\{e^{\psi},e^{h}\}$. Moreover,*
  - ▷ *No illiquid assets, $q\in(1-\pi,q^{\star})$: Only the marginally liquid security is held:*

    $$\mu_1^{\ell}\left(e^{\psi}\right)=n^e/\left(q+\pi\psi\right).$$

  - ▷ *Some illiquid assets, $q=q^{\star}$: Both the marginally liquid security and the maximally illiquid securities are held. Any, $\left\{\mu_1^{\ell}\left(e^{\psi}\right),\mu_1^{i}\left(e^{h}\right)\right\}$ is a solution if:*

    $$\left(q+\pi\psi\right)\mu_1^{\ell}\left(e^{\psi}\right)+\pi\mu_1^{i}\left(e^{h}\right)=n^e.$$

  - ▷ *Only illiquid assets, $q^{\star}<q$: Only the maximally illiquid security is held:*

    $$\mu_1^{i}\left(e^{h}\right)=n^e/\pi.$$

The proposition can be explained intuitively: if there is no liquidity premium, i.e., if low state payoffs are priced at their actuarial fair value of $q=1-\pi$, the producer should only buy liquid securities. Otherwise, she would miss the opportunity to trade with securities that cost the same. Indeed, any liquid security with a given expected payoff yields the same trading value.

If the liquidity premium is positive, $q>1-\pi$, low-state payoffs are more expensive. If the producer intends to hold any liquid securities, she should only hold marginally liquid securities, $D=e^{\psi_H}$. Indeed, marginally liquid securities are the cheapest way to buy hours. The marginally liquid security contains the least amount of low-state payoffs that guarantee the same amount of trade with the worker. Likewise, if the producer holds any illiquid securities, it should only hold the most illiquid security $D=e^{h}$. This is the cheapest way to obtain expected payoffs at $t=1$.

Given that the producer will hold either the marginally liquid security or the most illiquid security, the question is which among the two types of securities will she hold. Proposition 4 establishes that she will be indifferent between the maximally illiquid security in the high state and the "marginally liquid" security when the price is $q^{\star}$. Naturally, if $q>q^{\star}$, purchasing liquid assets to trade is too expensive relative to holding high-state A-D securities. If that is the case, the producer concentrates all her wealth on high-state payoffs. Otherwise, if $q<q^{\star}$, the producer buys only the marginally liquid security: It is expensive from an actuarial point of view, but the additional cost is more than compensated by the value of trading.

We covered the case where the liquidity premia go in one direction $q\geq 1-\pi$. In the general case where we consider prices such that $q<1-\pi$. By symmetry, the producer will be indifferent between the maximally illiquid security in the low state and the "marginally liquid security" in the low state when the price is $q_{\star}=(1-\pi)\cdot\psi_L$. Thus, in the general case, the producer will only hold liquid securities if the price $q\in[q_{\star},q^{\star}]$, only extreme illiquid securities if $q>q^{\star}$ or $q<q_{\star}$.

Competitive Equilibrium. Given that we've characterized the individual decisions, we next characterize the equilibrium using an Edgeworth box analysis. For that, we seek to re-write the producer's indirect utility, originally written in terms of the portfolio (measures) of securities, of an indirect utility over the state contingent payoffs of security. Recall that $\{\mathbf{D}(\ell), \mathbf{D}(h)\}$ represents aggregate payoffs in each state of any given portfolio. The utility only level that can be achieved through a portfolio with those payoffs only depends on the aggregate payoffs and is given by:

$$\tilde{\mathbb{U}}(\mathbf{D}) \equiv \begin{cases} q^{\star}\mathbf{D}(\ell) + \pi\mathbf{D}(h), & \text{if } \mathbf{D} \in \Lambda^{I}, \mathbf{D}(h) > \mathbf{D}(\ell) \\ \rho\left[(1-\pi)\mathbf{D}(\ell) + \pi\mathbf{D}(h)\right], & \text{if } \mathbf{D} \in \Lambda^{\ell} \\ q_{\star}\mathbf{D}(\ell) + \pi\mathbf{D}(h), & \text{if } \mathbf{D} \in \Lambda^{I}, \mathbf{D}(h) < \mathbf{D}(\ell). \end{cases}$$

This indirect utility function maximizes the amount of trade with the worker, conditional on holding a portfolio with payoffs $\{\mathbf{D}(\ell), \mathbf{D}(h)\}$. The purple line in Figure 1 depicts the indifference curve corresponding to the function $\tilde{\mathbb{U}}$. If payoffs fall in the liquidity cone, all securities in the portfolio must be used for trade. If the payoffs fall outside, an indifference curve must cross the points $\{e^{\psi}, e^{h}\}$ since it must yield the same indirect utility level. The slope at this indifference must be exactly $q^{\star}$, the price that creates indifference between trading the marginally liquid and the maximally illiquid security.[17] The virtue of this function is that it allows us to convexify the indifference curves and allows us to rewrite the producer's problem as:

Problem 3. Producers solve:

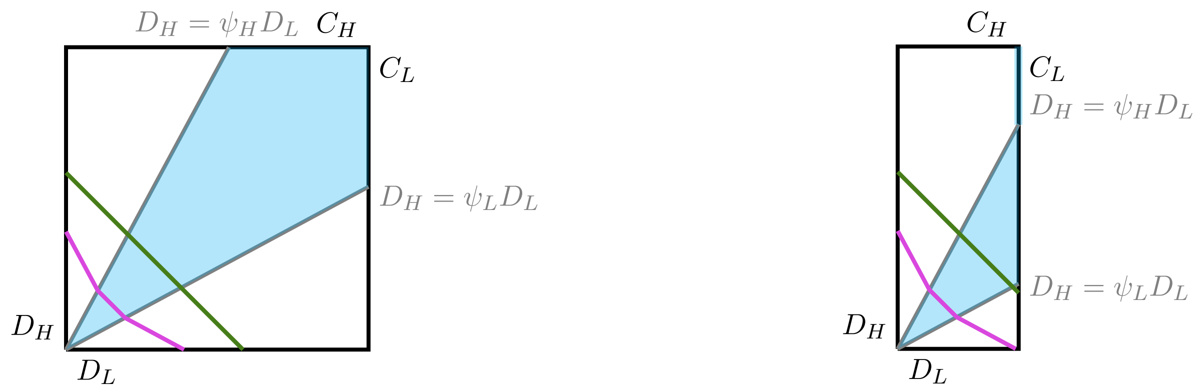
$$\max_{\{\mathbf{D}(\ell),\mathbf{D}(h)\}\geq 0} \tilde{\mathbb{U}}(\mathbf{D}) \quad \text{s.t.} \quad q\mathbf{D}(\ell) + \pi\mathbf{D}(h) \leq qN^{e}(\ell) + \pi N^{e}(h).$$

This representation says that e can treat the producer's problem as if his portfolio decision consisted only of choosing its overall payoffs. When the solution involves $\mathbf{D} \in \Lambda^{I}$, this portfolio will be composed only of the maximally illiquid and the "marginally liquid" security. From the perspective of the overall portfolios, the environment is akin to a two-agents two-goods economy, where overall payoffs across states play the role of different goods. Importantly, we don't need to make any reference to tree endowments and portfolios holdings, but only to final holdings of state-contigent payoffs.

The following representation (Figures 3a and 3b) display the producer and banker's indifference curves within an Edgeworth Boxes. The size of endowments is represented in the axis of the figure. The contract "curve" or set of Pareto efficient points is represented as the shaded blue area. At any point in the shaded blue area, the marginal rate of substitution of the banker and producer are equal— implying no liquidity premia and producers' holding only liquid portfolios. Importantly, the Edgeworth Box can look differently depending on the size of aggregate resources across states, $\{N(\ell), N(h)\}$. Figure 3a shows a situation where the aggregate endowment of payoffs is liquid, a situation we label "abundant aggregate liquidity." In

[17] This is because for any payoffs, the entrepreneur could maximize the value of trade by restructuring payoffs into a portfolio of maximillay illiquid and the most illiquid security.

turn, Figure 3b displays the Edgeworth Box where the aggregate portfolio is illiquid, a situation we label "scarce aggregate liquidity."



(a) Edgeworth box. Liquid Aggregate Portfolio, $N(h) \in (\psi_L N(\ell), \psi_H N(\ell))$.

(b) Edgeworth box. Illiquid Aggregate Portfolio, $N(h) \notin (\psi_L N(\ell), \psi_H N(\ell))$.

Figure 3: Edgeworth boxes. Liquid and Illiquid Aggregate Portfolios. Green line is a banker portfolio indifference curve. Purple line is an producer portfolio indifference curve. Shaded blue area is the set of all Pareto efficient points. Notice this set includes some points at the edge of the box.

When aggregate liquidity is abundant, Pareto efficient allocations include allocations of payoffs where the producer only holds liquid securities, and, for that reason, there is no liquidity premium. However, while scarce aggregate liquidity is necessary to have a positive liquidity premium, it is not a sufficient condition:a premium reflects not only the lack of aggregate liquidity but also the lack of banker wealth. The equilibrium price of the low-state A-D security q takes the following form:

Theorem 1 *There always exists an equilibrium with low-state price $q \in [(1-\pi), q^\star]$. In particular,*

- *Abundant Liquidity $N(h) \in [\psi_L N(\ell), \psi_H N(\ell)]$:* $\quad q = 1 - \pi.$
- *Scarce Liquidity $N(h) > \psi_H N(\ell)$:*

$$q = \min\left\{q^\star, \max\left\{\pi \frac{N^e(h) - \psi_H N(\ell)}{N^b(\ell)}, 1-\pi\right\}\right\}.$$

Under abundant liquidity, regardless of bank wealth, banks will structure illiquid trees into liquid securities, and each A-D security will trade at its expected value. When liquidity is scarce, the scarcity of low-state payoffs and the banker's wealth determine prices. When banker wealth $N^b(\ell)$ is high, relative to the producer's excess holdings of the illiquid asset ($N^e(h) - \psi_H N(\ell)$), again, there isn't a liquidity premium, $q = 1 - \pi$. Intuitively, even though society lacks enough low-state payoffs to make every security liquid, banks are wealthy enough to buy illiquid trees in exchange for a small amount of circulating securities. Both the producer and the banker are the marginal investor. When banker wealth is scarce, the liquidity premium is the highest possible, $q^\star = q$. This occurs because, at that price, the producer buys some illiquid assets and marginally liquid assets. Bankers, on the other hand, only hold illiquid trees but can't afford more at those prices.

The intermediate case of scarce liquidity occurs when producers only buy marginally liquid assets and bankers only by the high-state A-D security. In that case, the relative price q/π equals the ratio of the producer's excess holdings of the illiquid asset $(N^e(h) - \psi_H N(\ell))$ to the banker's excess holdings of the liquid asset $(N^b(\ell))$. This must be so since, at these intermediate prices, the producer only wants to hold the "marginally liquid" security, and the banker will sell all of his liquid assets.

Figure 4 allows us to explain the formula for q. We can observe the set of endowments that would result in $q = 1 - \pi$: those such that the producer's desired portfolio at that price is feasible. Similarly, if the banker lacks wealth, the price must be $q^\star$ because that is the price at which the producer is willing to hold both illiquid and liquid securities. Figure 4 also shows the determination of the price in the intermediate case. In this example, the initial endowment will result in an intermediate price $q \in (1 - \pi, q^\star)$. If the price were $q = 1 - \pi$, the producer would like to buy more liquid securities than can be created. The equilibrium relative price q/π is the slope of the red line connecting points A and B. This slope is given by the ratio of the sides A to C and C to B. The first side is the producer's excess holdings of illiquid securities: $N^e(h) - \psi N(\ell)$. The second one is the endowment of liquid assets by the banker: $N^b(\ell)$.

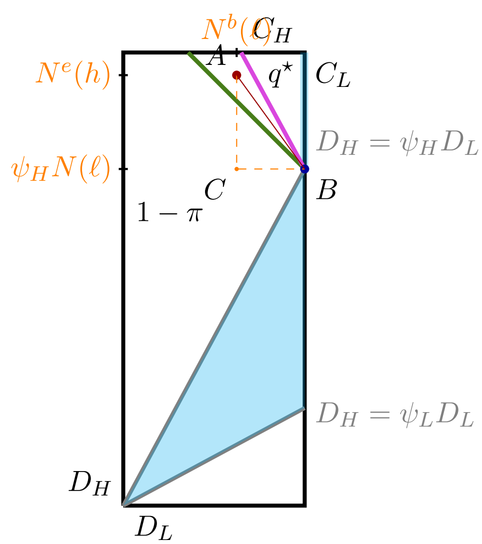


Figure 4: Edgeworth Box with Scarce Liquidity $N(h) > \psi_H N(\ell)$ - Example with $q \in (1 - \pi, q^\star)$. Point A represents the initial endowment. Point B is the equilibrium allocation: all liquid assets are used to create the "marginally" liquid security, and the producer holds no illiquid securities.

Special Case: No banker wealth. In the extreme case where the bankers have no wealth, the equilibrium price will depend only on the aggregate liquidity only. In particular, in each case, the endowment (and equilibrium allocation) will be at the top-right corner of each Edgeworth box. Therefore, the equilibrium price will be

$$q=\begin{cases}q_{\star}, & \text{if } N(h)<\psi_L N(\ell)\,,\\ 1-\pi, & \text{if } N(h)\in[\psi_L N(\ell)\,,\psi_H N(\ell)),\\ q^{\star}, & \text{if } N(h)>\psi_H N(\ell)\,,\end{cases}$$

with multiplicity when the endowment is exactly at the boundary. Without banker wealth, the equilibrium price reflects the aggregate liquidity: there is a premium if liquidity is scarce and actuarially fair prices if liquidity is abundant.

Taking Stock The setting, which collapses an environment with an arbitrary distribution of assets into a simple Edgeworth box analysis, has rich implications for the balance between bank stability and efficient liquidity provision. The section demonstrates how banks create liquidity efficiently within the economy. To do so, banks effectively take on all assets in the economy and, without loss of generality, issue a single liability that circulates.

There are important implications for the desirability of risk in bank equity and bank liabilities. To guarantee the circulation of their liabilities, banks must make their liabilities sufficiently safe. For that, they absorb a lot of residual risk—losing all their wealth in low states. Hence, the model shows that risk-taking by banks and the resulting instability of their wealth is, in fact, fundamental for the efficient creation of bank money. When bank wealth is ample and there are no liquidity premia, bank liabilities can be made perfectly safe. However, when bank wealth is limited, it carries some risk, and this risk is efficient. The scarcity of safety is shown in liquidity premia.

Importantly, banks enhance liquidity creation not only by bearing risk but also by buying safe assets that could circulate. Perfectly safe assets, such as government bonds or currency, would be acquired by banks to enhance the liquidity of their liabilities, showcasing the complementary nature of liquid and illiquid assets in liquidity creation.

The findings challenge narrow banking proposals, such as the Chicago Plan, which aimed to enhance bank stability by separating money creation from lending. In our model, that would be akin to taking safe assets to issue safe liabilities. Our model suggests that such separation would unduly limit liquidity provision in the economy.

Of course, the model lacks any externality imposed by banks. It also misses that, in a dynamic setting, banks' risk-bearing would likely be more limited, impairing liquidity provision. All in all, the model challenges the classic Aristotelean view that "banks charge a lot for doing very little." Here, they earn nothing for doing a lot.

4 Safest-Deposit Implementation

So far, we have described banking activities in the abstract space A-D securities. Here, we describe how to implement the resulting A-D equilibrium with a mix of non-recourse collateralized loans and outright security purchases as bank assets and a mix of deposits and outside equity as bank liabilities. The implementation is not always unique, so we focus on the implementation with the safest deposits. We present the details of the implementation and the corresponding asset prices.

Bank liabilities and assets. The A-D equilibria of the previous section feature allocations where banks hold all trees and issue at most two securities: a single liquid deposit with $\psi^D \equiv D(h)/D(\ell) \in \left[\psi^\ell, \psi^\ell\right]$ and one illiquid security that pays only in the high state e^h. We implement the A-D equilibrium payoffs by implementing the purchases of illiquid trees such that $R(h)/R(\ell) > \psi^h$ with collateralized debt. The purchase of the remainder trees is implemented via outright purchases. To implement the payoffs of the bank's liabilities in the A-D equilibrium, banks issue two liabilities: deposits and outside equity. Finally, to implement the payoffs for the banker, we use inside equity.

The implementation of the liability side is straightforward: The payoffs of liquid bank liabilities are implemented via deposit contracts: Deposits promise to deliver consumption goods at $t = 2$—normalizing one unit of deposits to one unit of promised consumption at $t = 2$. Deposits have a face value of $D(h)$ but default in the low state. In particular, in the low state, bank asset payoffs are only enough to partially honor the value of deposits to the value $D(\ell) < D(h)$. Thus, in the implementation, deposits mimic the payoffs of the liquid liability, $D(\omega)$, and consequently, can also be used as a means of payment.

Recall that when bank equity and liquidity are scarce, the A-D equilibrium features a positive issuance of illiquid liabilities that only pay in the state $\omega = h$. We implement the issuance of illiquid liabilities with outside bank equity shares. Since outside equity is junior to deposits, it does not pay in the low state, mimicking the payoffs of the illiquid security. Finally, to implement the payoffs for the banker, we consider inside equity. Inside equity mimics the banker's consumption of the banker, $c(\omega)$, in the A-D allocation: the banker is insolvent in the low state.

To implement the payoffs of the asset side in the A-D equilibrium, bankers hold two securities: outright purchases of trees or loans collateralized by trees. A loan collateralized by a tree $\{R(\ell), R(h)\}$ specifies a loan size and a face value for the loan repayment. We denominate both the loan size and the face value in terms of deposits, i.e., consumption units in the high state.

- Face Value. The face value for the loan, denoted by $F(R) \equiv R(h)$, are consumption goods promised to the banker at $t = 2$.
- Loan size. The loan size, denoted by $L(R)$, is the amount of deposits the banker makes in exchange for the promise $F(R)$.
- Default and Liquidation. The producer has the option to default on the loan. Upon default, the bank seizes the proceeds of the tree. Default is rational since $R(\ell) < F(R)$.

By construction, the payoff for the bank is $\min\{F(R), R(\omega)\} = R(\omega)$—i.e., the payoff is the same as acquiring the illiquid tree in the original setting. We use outright purchases to implement the payoffs of liquid trees or trees with lower payoffs in the high state; outright purchases are an exchange of trees for deposits.

Rates and asset prices. Next, we present the asset prices that emerge in the implementation, treating deposits as a unit of account. The loan face value for collateralized loans is $F(R)$. Let $\gamma^R = R(h)/R(\ell)$, the implicit rate $1 + i(R) \equiv F(R)/L(R)$ is:

$$1 + i(R) = \frac{\pi \cdot \psi^D + q}{\pi \cdot \gamma^{(R)} + q} \cdot \frac{\gamma^{(R)}}{\psi^D} > 1.$$

This rate is obtained from the prices in the A-D, which we use to price the securities we employ in the implementation. Thus, the gross rate is given by the product of the exchange rate between the tree and liquid security times, normalized by each security's units of high-state payoffs. We further re-arrange terms to obtain a different interpretation:

$$(1 + i(R)) \cdot \left(1 - \underbrace{(1-\pi)\frac{\gamma^R - 1}{\gamma^R}}_{\text{default}} + \underbrace{\frac{q - (1-\pi)}{\gamma^R}}_{\text{liquidity}} \right) = 1 - \underbrace{(1-\pi)\frac{\psi^D - 1}{\psi^D}}_{\text{default}} + \underbrace{\frac{q - (1-\pi)}{\psi^D}}_{\text{liquidity}}.$$

This interest rate has a natural interpretation: a collateralized loan contract swaps a loan for a deposit. The interest rate compensates for the default risk relative to a liquidity premium. The default compensation is the recovery value relative to the promised value of the security. The liquidity premium is the difference between the price of low-state payoffs and their fair value relative to the promised value of the security.

Regarding outright purchases, these are bought at a premium:

$$p(R) = R(h)\frac{\pi \cdot \gamma^R + q}{\pi \cdot \psi^D + q} \cdot \frac{\psi^D}{\gamma^R} = R(h) \cdot \frac{1}{1 + i(R)}.$$

Intuitively, their price is above one relative to their face value since these assets have a negative beta. Moreover, there is a connection between the price of an asset in an outright purchase and the interest rate on a collateralized loan: for an asset R with payoff $R(h) = 1$, we have that $(1 + i(R))p(R) = 1$, a condition reflects that all assets are priced in the same way.

Regimes. There are four regions of the endowment space with distinct properties regarding the safest deposit implementation. These regions are depicted in Figure 5 and feature the following taxonomy:

Proposition 5 *I. In Region I, liquidity is abundant, $q = 1 - \pi$, and banks have enough net worth to buy the producer's trees issuing riskless deposits, $\psi^D = 1$.*

II. In Region II, liquidity is abundant, $q = 1 - \pi$. However, banks cannot afford to buy the producers' trees *by only issuing riskless deposits. Thus, $\psi^D \in (1, \psi_H)$. The interest rates (asset prices) is lower (higher) than in Region I.*

III. *In Region III, liquidity is scarce, $q \in (1-\pi, (1-\pi)\psi_H)$ and bank deposits are marginally liquid, $\psi^D = \psi_H$. In this region, interest rates and asset prices are higher than in the Region II.*

IV. *In Region IV, the liquidity premium attains its highest value $q = (1-\pi)\psi_H$. Deposits are marginally liquid, $\psi^D = \psi_H$ and banks issue outside equity at a price $p(\{0,1\}) = \pi$.*

This taxonomy explains the classes of securities issued by banks. As banks are wealthier and liquidity is more abundant, banks can issue safe deposits (Region I). As their endowment falls to Region II, deposits have to be riskier because bank equity is insufficient to insure deposits in all states. Thus, the value of deposits is penalized with a risk premium that reflects in lower rates for the collateralized loans and higher prices for outright purchases. As aggregate liquidity becomes scarce and bank equity limited, Region III, the liquidity premium rises, increasing the interest rate again due to the lower liquidity value of collateralized loans. In turn, the prices of outright purchases are higher because securities bought have a lower "beta" than deposits. In Reion IV, banks have the least net worth and liquidity is scarcest. Interest rates and prices are constant in this region and equal to their value in the boundary of Region III. As we reduce the initial bank endowment in this region, banks issue greater amounts of illiquid outside equity to afford their asset holdings.

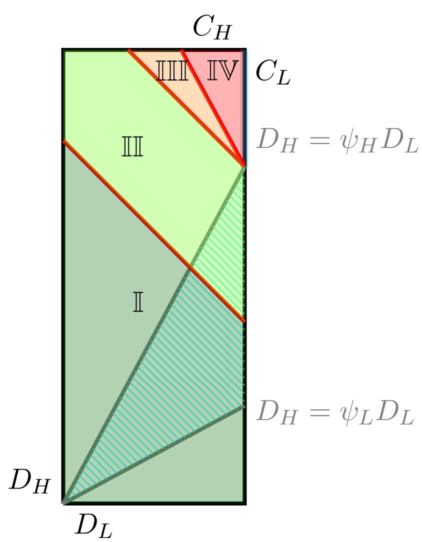


Figure 5: Regimes for safest implementation. Only region I features perfectly safe deposits. Regions I and II have no liquidity premia. In region IV, the bank issues outside equity.

5 Geographical segmentation and competition against outside money

So far, bank liabilities are the only medium of exchange. Historically, bank notes have co-existed alongside outside money, such as gold or fiat government money, both as media of exchange and in bank balance sheets. Furthermore, we assumed substantial knowledge about bank liabilities: all trading partners know their payoffs. In this section, we study how outside money affects the creation of circulating bank liabilities as we vary the extent of information about bank liabilities. Specifically, we obtain conditions under which banks hold outside money in their balance sheets and determine liquidity premia as we vary a parameter that captures the recognizability of bank liabilities.

Environment. We assume that besides trees, there is a special riskless asset: outside money. It should be clear from the previous section that, absent additional frictions, banks would acquire all the outside money to create more liabilities, just as they would with riskless trees in the previous section. Thus, to introduce a special role to outside money, we introduce "geographical" segmentation and relax the assumption of universal recognizability of bank liabilities. In particular, trades between workers can be either of two types: local or interregional. The key assumption is that bank liabilities are recognized only in local transactions. Instead, outside money is recognized across locations, a property that we associate with "safety" as opposed to risklessness.[18] Thus, from now on, we label outside money and safe assets interchangeably. As a result of these distinct properties, risklessness and safety, there's a tradeoff between using safe assets to trade across locations versus enhancing the creation of liabilities.

Banks, producers, and workers are now assigned to a specific location among a continuum. There are no differences in the gains from trade across locations. Banks only trade with local producers. In turn, the match between producers and workers can happen within and across locations, with probabilities ν and $1-\nu$, respectively. The key assumption is that workers only accept liquid securities of their locations, regardless of whether these are bank liabilities or trees. These assumptions are justified if the workers in one location cannot recognize the issuer bank and evaluate their liabilities in other locations to the point that they decide not to accept those payments, as in the model of fraudulent assets in Li, Rocheteau and Weill (2012). By contrast, safe assets are universally accepted.

The timing is the same as in the previous section. We denote the price of the safe asset by p^s. To provide sharp descriptions, we assume symmetric holdings of trees by banks and producers in each location.

Modified Bilateral Trading. Given the recognition of bank liabilities with probability ν and the presence of safe assets, the value of bringing an asset to the bilateral trade will differ from the one in the previous section. In particular, the ex-ante value of a (non-safe) asset D for the producer will be given by $\tilde{\mathbb{U}}[D] = \nu\mathbb{U}[D] + (1-\nu)\mathbb{E}[D]$ where $\mathbb{U}[D]$ is given by Proposition 1. We write $\tilde{\mathbb{U}}^\ell[D] = \nu\mathbb{U}^\ell[D] + (1-\nu)\mathbb{E}[D]$ and $\tilde{\mathbb{U}}^i[D] = \nu\mathbb{U}^i[D] + (1-\nu)\mathbb{E}[D]$ for liquid and illiquid assets, respectively. The ex-ante expected gross return of trading with a liquid asset is $\tilde{\rho} = \nu\rho + (1-\nu)$.

[18] We refer to risk as a property of the assets payoffs, whereas safety regards how safely a worker can transact without being transferred a fraudulent asset.

As before, securities used in local trades are segmented into liquid and illiquid securities according to the set of *liquidity coefficients* (ψ_L, ψ_H)—these coefficients do not depend on the probability of a local match, ν, because the liquidity of an asset is determined after the match is realized. The value of trading with the safe asset is given by $\mathbb{U}[(1,1)] = \rho$, given that it is universally accepted.

Individual Problems. Agent problems now consider the assets' recognizability. Let N_s^e and N_s^b denote the endowment of safe assets and S^e and S^b denote their final holdings, respectively.

Problem 4 (Producer Problem)**.**

$$\max_{\{\mu_1^i, \mu_1^\ell, S^e\} \geq 0} \int \tilde{\mathbb{U}}^\ell [D] \, d\mu_1^\ell(D) + \int \tilde{\mathbb{U}}^i [D] \, d\mu_1^i(D) + \rho \cdot S^e$$

subject to:

$$p^s \cdot S^e + \int P(D) \, d\mu_1^i(D) + \int P(D) \, d\mu_1^\ell(D) \leq p^s \cdot N_s^e + \int P(R) \, d\mu_0^e(R).$$

Two differences are evident between this and the problem encountered above: the budget constraint accounts for safe assets, and the objective function features $\tilde{\mathbb{U}}$ rather than $\mathbb{U}$ to account for the recognizability of assets while the objective scale the holdings of save assets by their return, ρ.

The banker problem retains its objective function, but the presence of safe assets modifies its budget and resource constraints. Their budget constraint is now

$$p^s \cdot S^b + \int P(D) \, d\mu_1^+(D) \leq p^s \cdot N_s^b + \int P(D) \, d\mu_1^-(D) + \int P(R) \, d\mu_0^b(R),$$

whereas, given the riskless nature of the safe asset, their resource constraint is given by:

$$\forall \omega, \quad c(\omega) + \int D(\omega) \, d\mu_1^-(D) \leq \int D(\omega) \, d\mu_1^+(D) + S^b. \tag{9}$$

Finally, the banker's safe asset holdings are restricted to be non-negative, $S^b \geq 0$.

The definition of competitive equilibrium now includes one more unknown, the price of safe assets, and one more equation, the market clearing condition for safe assets, $S^e + S^b \leq N_s^e + N_s^b$.

Characterization. As a preliminary observation, we note that since banks can issue riskless liabilities equal in payoffs to the safe asset, the safe asset's price, p^s, cannot fall under its replication cost $q + \pi$ with a riskless asset. In turn, because the bank can always sell the tranches of any security, it will never hold a safe asset when its price is above its replication cost. These observations lead to the following non-arbitrage condition:

Lemma 2 *In equilibrium, $p^s \geq q + \pi$. Moreover, $S^b = 0$ if the inequality is strict.*

Following the same arguments laid out in Section 3, the problems of the agents can be simplified to choices of portfolios of state payoffs with the addition of their holdings of safe assets. Moreover, as in that environment, banks issue at most two securities when liquidity is scarce, the maximally illiquid and the marginally liquid securities: e^h and e^{ψ_H}. The characterization boils down to finding the prices of low-state payoffs q and the safe asset p^s that solve the market clearing conditions for safe assets and low- and high-state payoffs.

With limited recognizability, the producers' return from holding these securities are respectively:

$$\mathcal{R}^h = 1, \quad \mathcal{R}^{\psi_H} \equiv \tilde{\rho}\frac{\pi \cdot \psi + (1-\pi)}{\pi \cdot \psi + q},$$

which are similar to the returns encountered above replacing ρ for $\tilde{\rho}$. In turn, the return of holding the safe asset is: $\mathcal{R}^s \equiv \rho/p^s$.

The critical insight to characterize the equilibrium is that since safe assets have different uses for bankers and producers, either will be at a corner in their safe asset holdings, or both are indifferent in their safe asset holdings, and no illiquid assets are issued. Next, we characterize the equilibrium when the banker has no wealth (no initial endowments), simplifying the exposition without changing the essence of the characterization. Hence, from now on, $\mu_0^b = 0 = N_s^b$. A taxonomy of equilibria as a function of parameters is provided by the branches of the scenarios depicted in Figure 6. The taxonomy organizes the equilibria by their predictions regarding the safe asset holdings by producers and bankers.

Safe Assets Only Held as Means of Payment. We start by characterizing conditions such that the safe asset is exclusively held as a means of payment by producers—and banks are at a corner solution concerning their holdings. Since the value of the safe asset for a bank is derived from the bank's issuance of liabilities, whether the bank holds a safe asset depends on the recognizability of bank liabilities, its trading value, and the liquidity of the aggregate portfolio, which determines its value in enhancing liquidity creation.

The next lemma shows that when the aggregate payoffs of trees are liquid, as in the previous section, are liquid banks do not hold safe assets:

Lemma 3 *If (tree) liquidity is abundant, $N_H < \psi_H N_L$, banks do not hold safe assets, and there is no liquidity premia: $q = 1 - \pi$. The safe asset's price is $p^s = \rho/\tilde{\rho}$.*

Proof. See Appendix B. ■

Intuitively, when the aggregate portfolio is liquid, banks do not need safe assets to create liquid liabilities from illiquid trees, and there is no liquidity premium. However, a bank liability is a worse mean of payment, $\tilde{\rho} < \rho$, than the safe asset, so the latter is more expensive than its replication cost with trees. Because the banker has no trade use for the safe assets, it will be at a corner.

The previous result shows that safe assets can only be helpful as bank assets if there is liquidity scarcity among trees. In that case, there is no liquidity premium and the safe tree is expensive. The next lemma shows that when liquidity is scarce among trees, the converse is true about the liquidity premium, even if the aggregate portfolio, including safe assets, is liquid:

Lemma 4 *If (tree) liquidity is scarce, $N_H > \psi_H N_L$, there is a liquidity premium.*

Proof. See Appendix B. ■

The lemma shows that while banks can use safe assets to enhance liquidity creation, they will never go all the way to eliminate the liquidity premium. While scarce liquidity guarantees a liquidity premium, it is not a sufficient condition to observe safe assets being held by banks. The next result shows that banks will not hold safe assets, even under scarce liquidity, if the recognizability of bank liabilities is too low.

Lemma 5 *Suppose (tree) liquidity is scarce, $N_H > \psi_H N_L$. The bank must hold safe assets if recognizability is high, $\nu > ((1-\pi)+\pi\psi_H)^{-1}$. Conversely, the bank won't hold safe assets if recognizability is low, $\nu < ((1-\pi)+\pi\psi_H)^{-1}$.*

Proof. See Appendix B. ■

Intuitively, when liquidity among trees is scarce, a bank can benefit by bundling safe assets with illiquid trees. However, this advantage will not be enough if bank liabilities are not very valuable as a medium of exchange relative to safe assets. The relative value depends on the deposit's recognizability. Namely, whether its recognizability exceeds a threshold.[19]

To understand why the recognizability threshold takes the form given in the proposition, we compare the societal value of using a safe asset as a means of payment versus using it as a bank asset. Since banks must issue the maximally illiquid liability when liquidity is scarce, it suffices to compare the return to holding the safe asset against bundling it with the maximally illiquid liability. Thus, take one unit of the safe asset and $(\psi_H - 1)$ units of an asset paying only in the high state. Since the price of both bundles is the same, we just need to compare the independent values versus their joint value. If a bank bundles the two, it can issue a liquid liability $(1, \psi_H)$ whose trading value is $\tilde{\rho}$ times the expected payoff of the bundle. If the two assets are not bought by the bank, the safe asset is used for payment, so its value is ρ. Since the illiquid asset is not traded, its value is only its expected payoff. It is better to keep the safe asset outside the bank if

$$\underbrace{\underbrace{\rho}_{\text{payment}} + \underbrace{\pi \cdot (\psi_H - 1)}_{\text{holding}}}_{\text{independent values}} > \underbrace{\underbrace{\tilde{\rho} \cdot (1 + \pi \cdot (\psi_H - 1))}_{\text{liquidity value}}}_{\text{bundled value}}.$$

Substituting out $\tilde{\rho}$ and clearing the expression yields the recognizability threshold.

With these results, we can characterize the equilibrium outcome with scarce liquidity and low recognizability.

[19] The threshold has properties intuitive capture that intuition: if $\psi_H = 1$, safe assets do not enhance liquidity creation. Hence, the recognizability requirement is extreme. Conversely, as ψ_H increases, the recognizability threshold falls.

Lemma 6 *With scarce liquidity and low recognizability the bank holds no safe assets. Moreover, equilibrium prices equalize the return to safe assets, liquid liabilities, and illiquid liabilities. That is,* $q = q^*(\nu)$ *and* $p^s = \rho$ *where*

$$q^*(\nu) \equiv \nu(1-\pi)\psi_H + (1-\nu)(1-\pi),$$

Proof. The asset prices follow directly from indifference conditions. As shown before, in the special case without banker wealth, with scarce liquidity, the producer must be indifferent between holding the bank's marginally liquid liability and the maximally illiquid liability. Moreover, the producer must also be willing to hold the safe asset. In summary,

$$\underbrace{\frac{\pi}{\pi}}_{\text{Illiquid liability}} = \underbrace{\frac{q + \pi \cdot \psi_H}{\tilde{\rho} \cdot \bar{\psi}_H}}_{\text{Liquid liability}} = \underbrace{\frac{p^s}{\rho}}_{\text{Safe asset}}. \implies \begin{cases} q & = q^*(\nu), \\ p^s & = \rho, \end{cases}$$

where $q^*(\nu) \equiv \nu(1-\pi)\psi_H + (1-\nu)(1-\pi)$ is an average of the benchmark price (equivalent to $\nu = 1$) and the actuarially fair price, $1 - \pi$. ■

The proposition uses that since recognizability is low and liquidity is scarce, producers will hold all three types of securities. The equality of their returns yields the equilibrium prices. Having analyzed the cases where banks do not hold safe assets, we move on to those in which they do.

Safe Assets at the Bank. When recognizability is high and liquidity is scarce, the bank will hold safe assets. However, we need to establish the quantity of safe assets held by banks and the corresponding prices. We establish that the relevant taxonomy for equilibrium allocation and prices depend on whether, once we we include safe assets, the aggregate portfolio is liquid (abundant safety) or not (scarce safety).

Proposition 6 *With scarce liquidity and high recognizability banks hold safe assets (*$p^s = q + \pi$ *and* $S^b > 0$*). Furthermore,*

- *If there is Scarce Safety,* $N(h) + S > \psi_H \cdot (N(\ell) + S)$*, the bank holds all the safe assets* $S^b = S$*, issues illiquid liabilities, and* $q = q^*(\nu)$ *equalizes the return of liquid and illiquid liabilities.*
- *Otherwise, there is Abundant Safety, the bank holds only enough safe assets to make the portfolio of trees marginally liquid:*

$$N(h) + S^b = \psi_H \cdot \left(N(\ell) + S^b\right),$$

do not issue illiquid liabilities, and $q = q^{SL}(\nu)$*, where* q^{SL} *equalizes the return of safe assets and* *liquid liabilities:*

$$\frac{\rho}{q^{SL}(\nu)+\pi} = \frac{\tilde{\rho}\bar{\psi}_H}{q^{SL}(\nu)+\pi\bar{\psi}_H}.$$

Proof. See Appendix B. ■

With scarce safety and high recognizability, it is valuable to use safe assets to enhance liquidity. Since the overall portfolio is illiquid, the producer must hold the maximally illiquid liability, along with the bank's marginally liquid liability, so the price must equalize the return to liquid and illiquid liabiliites.

With abundant safety, safe asset are useful to enhance the liquidity of liabilities up until the point where the bank portfolio becomes liquid. This sets a value for the bank holdings of safe assets. In this case, the producer must be willing to hold the safe asset and the bank's marginally liquid liability, which pins down the price of q when safety is abundant.

Figure 6 displays the prices under different scenarios for recognizability—and the resulting balance sheets. It is interesting to note that the price of the safe asset is increasing in the recognition probability under scarce safety, but decreasing under abundant safety. With scarce safety, the safe asset is used to enhance liquidity: a higher recognition makes bank liabilities more valuable which, in turn, makes the safe asset more valuable. On the other hand, with abundant safety, the safe asset is used as means of payment by the producer and is, therefore, a substitute for bank liabilities. As recognizability rises, the return of the bank liability increases, so the price of the safe asset must decrease to induce the same rate of return. This dichotomy shows that when safety is scarce, safe assets are a complement to bank liability creation, but when it is abundant, they are a substitute means of payment.

Taking stock.

6 Bank integration and consolidation

In the previous section, we discussed how the lack of recognition of bank liabilities across geographies could limit their use in transactions. The lack of recognition of bank liabilities has a historical motivation: the Free Banking Era of the United States and Scotland, where circulating liabilities issued by private banks, in the form of bills, would trade at a discount across locations. In this section, we study two natural solutions to the recognition problem: bank integration and bank consolidation. We interpret bank integration as the formation of clearinghouses, clubs of banks that would jointly accept their liabilities. This again is motivated by their historical emergence in Scotland and New England.[20] In turn, we interpret bank consolidation as a wave of mergers leading to bank monopolies, as occurred, in Canada where only a small number of banks survive today. Next, we study how joint liability and monopoly power impact liquidity creation.

[20]Kroszner and Cowen (1989) for example, describes how a clearinghouse system emerged in Scotland and New England during their Free banking periods.

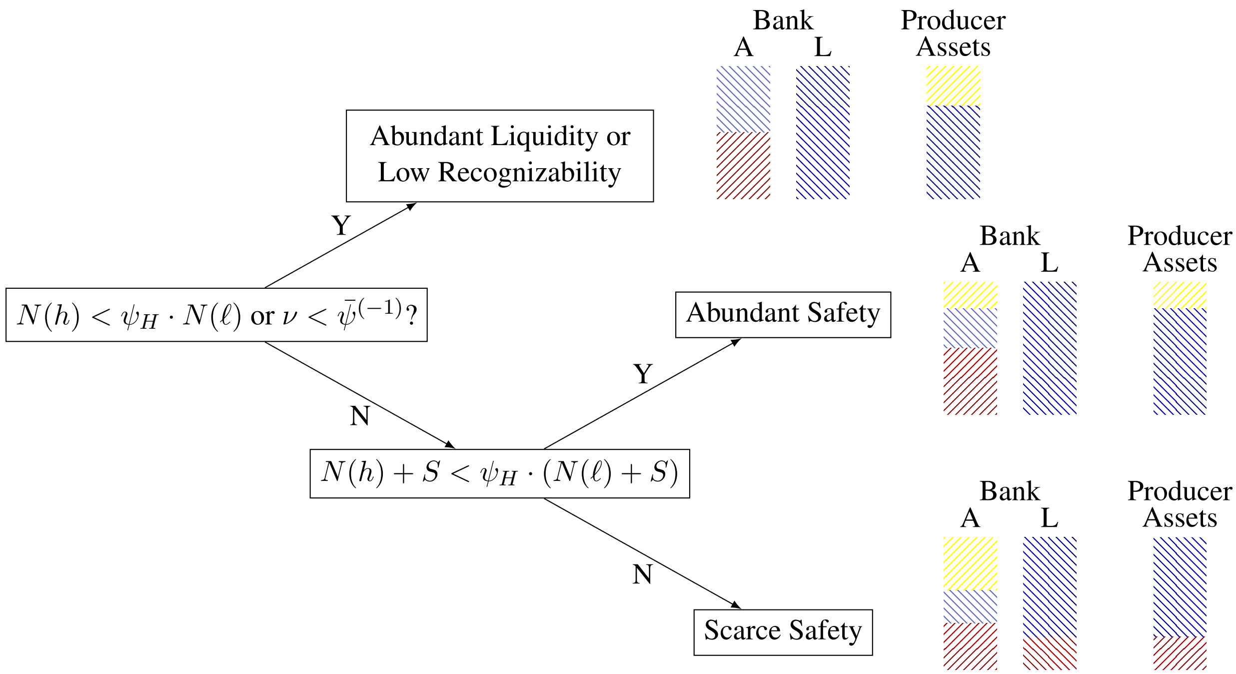
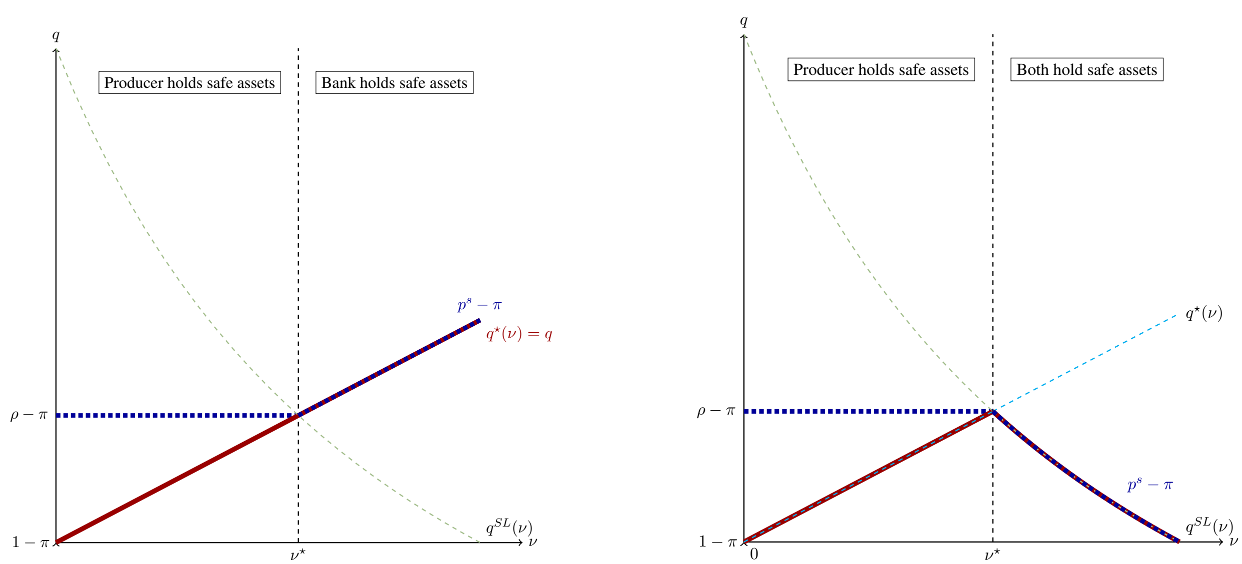


Figure 6: Scenarios with Safe Assets. The equilibrium balance sheets display liquid securities in blue, illiquid in red, and safe assets in yellow. The top balance sheet corresponds to abundant liquidity; with low recognizability, the bank issues some illiquid liabilities for the producer to hold.



(a) Scarce Safe Assets.

(b) Abundant Safe Assets.

Figure 7: Scarce Liquidity with Safe Assets. Equilibrium price of the L-state A-D security, q, highlighted in yellow, as function of recognition probability ν.

6.1 Integration: joint liability and moral hazard

Internal Note. SB: notation changes. $e^x \equiv [1, x]$ vector. Then, $\mathbb{E}[e] = 1 - \pi + \pi x$.

While we maintain the regional segmentation and liability-recognition problem, we allow for bank integration in the form of a joint liability system. Under a joint liability system, banks issue liabilities that are claims against other banks. Thus, although the issuance happens at the individual bank level, claims are against the system. Because liabilities are accepted across locations, workers can deposit the liability of any producer, including those of different locations, in accounts in banks in their location. Once the liability is deposited, workers have claims against the assets of their own banks. Banks then settle the movement of liabilities with cross-bank liabilities. In doing so, this institution solves the liability-recognition problem outlined in the previous section: a worker no longer needs to know anything about any other bank but her's. All she needs to know is that the deposit held by a producer is accepted by her bank.

While joint liability is a solution to the liability-recognition problem, it is natural to assume it is a system prone to moral hazard. Namely, if other banks accept an individual bank's liabilities, the banker can overissue liabilities, i.e., it may issue liabilities to itself, exchanging these for goods at a profit. Thus, to study how liquidity creation under a joint liability system in a meaningful way, we thus add the possibility overissuance.[21]

Environment. The introduction of a joint liability system, per se, creates no differences with the environment in Section 3. Recall that liabilities circulated, so the identity of the ultimate holder does not matter. When liabilities circulate between banks, it is as if a claim was created against another bank, and between another bank and the worker.

Moral hazard changes things and interacts with liquidity creation. To highlight this point, we introduce a simple form of moral hazard. Whereas banks can issue trees to buy assets from producers, we also allow bankers to issue liquid liabilities without buying trees in order to consume. In particular, banks can issue deposits to themselves at time $t = 0$ to hire workers who produce goods with a technology that produces one unit of consumption per unit of hours worked. To do so, the bank has to spend resources window dressing or "altering" the accounting books, so that it appears to other banks in the system that he indeed bought trees. Considering the inferior technology and window dressing costs, the banker can turn the fraction $\kappa < 1$ of the expected value of the liabilities it issues into consumption goods.[22] Since workers can deposit liabilities onto other banks who, in turn, guarantee their payoffs, workers don't care about the originating source of those liabilities.

For a joint liability system to operate, the system must limit the value of liability issuances so that acquiring assets instead of issuing liabilities is incentive-compatible. Hence, while the producer's problem and the equilibrium definition remain unchanged, the bank's problem now involves an additional incentive-

[21] See also Cavalcanti and Wallace (1999) for a related study of overissuances.

[22] Thus, we can think of $1 - \kappa$ as the costs associated with faking the purchase of assets to acquire goods from workers in exchange for the liabilities.

compatibility constraint:

$$\int \mathbb{E}(D)\, d\left(\mu_1^+(D) - \mu_1^-(D)\right) \geq \kappa \int \mathbb{E}(D)\, d\mu_1^-(D) \mathbb{1}(\mathbf{D} \in \Lambda^L). \tag{10}$$

The interpretation of this constraint is that the profit from the tree purchase and issuance of liabilities must exceed the benefit from mimicking the same tree portfolio while issuing the liabilities to itself. This constraint resembles a standard leverage constraint under limited commitment, say a là Kiyotaki and Moore, except for a subtle difference: the IC constraint only affects the issuance of liquid liabilities. That is, banks are not prevented from leveraging up by issuing illiquid liabilities, they are only limited by how many liquid liabilities they can issue against the rest of the system.

Because the incentive-compatibility constraint requires some profitability from issuing liabilities, the following assumption ensures that the producer's problem is profitable enough that some exchange of trees for liquid bank liabilities can happen in equilibrium:

$$1 + \kappa \leq \min\left\{\frac{1}{1-(\rho-1)\pi}, \quad \frac{1}{1-(\rho-1)(1-\pi)}\right\} < \rho. \tag{11}$$

The assumption requires that the technology of producers be sufficiently productive to cover the bank's incentives and the producer's relative trading values. We adopt this assumption for the rest of the section.

Characterization. As in the previous section, we characterize the equilibrium outcomes in the limit as the banker's endowment approaches zero. This assumption ensures that the incentive constraint binds and allows to explain the effects of this constraint in equilibrium behavior and allocations in a simple way.

We can anticipate that several results encountered in previous sections no longer hold. For example, recall that producers must be indifferent between holding liquid bank liabilities and holding liquid trees since they can trade with these directly. For that, their price must be equalized to equalize their holding returns. However, to issue liquid liabilities that circulate in the system, banks must profit from the creation of liquid liabilities. The consequence of this observation is twofold: First, the return of liquid liabilities and liquid assets will not be equalized. As a result, prices cannot be linear in high and low-state payoffs. Formally,

Lemma 7 *If banks issue liquid liabilities, asset prices are not linear in payoffs.*

Proof. We prove that if asset prices are linear in payoffs, the banks make zero profits from issuing liabilities. In turn, the bank cannot issue liquid liabilities. To see this, we note that a bank must hold at least enough assets to back their liabilities. Since prices are linear, the bank's revenue from issuing a liability is exactly equal to its cost of purchasing the backing assets. Thus, profits are zero. Since the left-hand side of the incentive constraint is equal to the bank's profits, the bank's liquid liabilities must be equal to zero. ■

We explain this feature through an example. Consider an economy with only two illiquid assets $(0, 1)$, $(1, 0)$ held by the producer. If prices would be linear in payoffs, bank profits would be zero. Therefore, the bank's incentive constraint would inhibit the issuance of liquid securities.

Second, aggregate state payoffs are no longer a sufficient statistic to characterize the equilibrium allocation: two economies with the same aggregate payoffs can lead to very different allocations. Consider, for instance, one economy with a single liquid tree with payoffs $(1, 1)$ and another one with only illiquid assets, as encountered in earlier sections. Whereas in previous environments, a liquid aggregate tree portfolio guarantees the same allocation, here the latter cannot achieve the same outcome because issuing liabilities must produce bank profits. While knowledge about aggregate payoffs is insufficient to characterize the equilibrium, it is enough to know the aggregate payoffs of liquid and illiquid trees.

Since the incentive constraint is tightened by the purchase of liquid trees, banks avoid acquiring liquid assets unless the benefits of enhancing liquid exceed the incentive cost. Thus, similar to our characterization of the equilibrium with safe assets, Section 5, here we provide a taxonomy of equilibria based on whether liquid trees are used as a means of payment directly or purchased by banks to enhance liquidity.

Liquid Trees as Means of Payment. If liquid trees are only used as direct means of payments by producers and not to back bank liabilities, bank liabilities must only be backed by illiquid assets. We can assume, without loss of generality, that banks hold all illiquid assets because the incentive constraint does not apply to illiquid liabilities.[23] If liquid trees are not held by banks, an equilibrium with liquid liabilities is only feasible if the bank can indeed issue liquid liabilities backed by the stock of illiquid assets, without bundling these with any liquid assets. This requires the portfolio of illiquid assets to be sufficiently liquid. Toward a characterization of this scenario, we denote the aggregate payoffs of the portfolio of illiquid trees as $\{N^{illiq}(\ell), N^{illiq}(h)\}$. The following proposition summarizes a sufficient condition for observing an equilibrium where banks do not acquire liquid trees:

Proposition 7 *There is abundant liquidity in a joint-liability system if the portfolio of illiquid assets satisfies:* $N^{illiq}(h) < \phi_H \cdot N^{illiq}(\ell)$*, where* ϕ_H *solves:*

$$\mathbb{E}[e^{\phi_H}] = (\kappa + 1)\,\mathbb{E}[e^{\psi_H}]. \tag{12}$$

In that case, the bank holds all the illiquid assets and no liquid assets. Moreover, asset prices are given by:

$$P(D) = \begin{cases} \mathbb{E}[D(\omega)], & \text{if } D \text{ is illiquid,} \\ (1+\kappa)\mathbb{E}[D(\omega)], & \text{if } D \text{ is liquid.} \end{cases}$$

We can explain the proposition intuitively: In the previous section, we observed that if the portfolio of illiquid assets is liquid, $N^{illiq}(h) < \psi_H \cdot N^{illiq}(\ell)$, banks could bundle all the illiquid assets to create a liquid

[23] For any equilibrium allocation where the producer holds some of the illiquid assets, there exists another equilibrium allocation where the bank holds those illiquid assets and issues equivalent illiquid liabilities. Both allocations are payoff equivalent and do not affect the bank's incentive constraint.

liability without safe assets. The additional liquidity that can be created with safe assets has to compensate for the universal trading use of safe assets. Thus, bringing safe assets to the bank has no purpose if the portfolio of trees is liquid. A similar logic here, but we should think of liquid assets as being expensive because part of their value has to go to the bank. If the aggregate portfolio of illiquid trees is liquid, banks can maximize the amount of liquidity created from the illiquid portfolio, respecting the IC constraint, without acquiring liquid trees.

Different from the previous section, the relevant liquidity condition is more relaxed. Since $\phi_H > \psi_H$, portfolios of trees that are illiquid in previous sections can be liquid here once we consider the moral hazard problem. The reason for this is that with moral hazard, banks must profit from bundling illiquid trees. The cheapest way to do so is by structuring liabilities such that they obtain payoffs in the high state. Thus, while the portfolio of illiquid trees may be illiquid in the sense of previous sections, it is once we net out the banks' required profits.

The liquidity abundance condition, (12), indicates that for a portfolio of illiquid assets (normalized payoffs) equal to e^{ϕ_H} and the bank must be able to issue a marginally liquid liability to buy the portfolio e^{ψ_H} and make enough profits to satisfy the incentive constraint. Furthermore, notice that prices are such that the return of holding on to illiquid trees is dominated by the return of buying a liquid tree and trading:

$$1 = \mathcal{R}^i < \mathcal{R}^\ell = \frac{\rho}{1+\kappa},$$

whereas for the bank, buying a liquid tree to issue a liquid liability with same payoffs generates no profits.

Another feature is that asset prices are proportional to asset payoffs. This follows because the liquidity premium is not associated with the producers' private information problem; there is no shortage of payoffs in the low state. Instead, the premium is associated with the arbitrage between liquid and illiquid assets needed to provide incentives in a joint liability system.

Notice that since the liquidity abundance condition is more relaxed here than earlier, $N^{illiq}(h) < \psi_H \cdot N^{illiq}(\ell)$, banks don't acquire liquid trees even in scenarios where, without moral hazard, they would. This doesn't reflect an improvement of the private information problem. On the contrary, the amount of trade is lower than without moral hazard. The condition reflects that given that banks must be compensated with high-state payoffs, bringing liquid assets is not necessary to enhance the liquidity of those payoffs. All in all, this scenario highlights that a joint liability system is a way to enhance the universal circulation of bank liabilities, but this system comes at a cost of incentive provisions which ultimately limit liquidity creation.

Liquid Assets Acquired by the Bank. When the aggregate payoffs of illiquid trees do not feature liquidity abundance, $N^{illiq}(h) \geq \phi_H \cdot N^{illiq}(\ell)$, high-state payoffs are excessive even after subtracting the bank profits needed to satisfy the incentive constraints. In that scenario, the banker has to either issue some of that excess of high-state payoffs as illiquid liabilities or acquire liquid assets to enhance their liquidity.

Again, there are similarities to the scenario of liquidity scarcitiy of Section 5 where safe assets play a dual role as a medium of exchange and enhancing liquidity: the bank can benefit of acquiring liquid assets only when the portfolio of illiquid assets features excessive payoffs in the high state. In that is the case, bundling a maximally illiquid assets with liquid asset may be profitable if they can form a liquid liability that overcomes the incentive cost. Clearly, if the producer holds a marginally liquid asset with normalized payoffs, e^{ψ}, the bank cannot create liquid securities by bundling this asset with a maximally illiquid security. Consequently, there is a cutoff liquid asset such that all liquid asset with higher high-state payoffs remain with the producer, as the following Lemma shows. held by the producer, while the others may be held by the bank.

Lemma 8 *In any equilibrium, banks do not acquire liquid trees with normalized high-state payoffs above $e^{z^\star}$ where*

$$\kappa \mathbb{E}\left[e^{\psi_H}\right] = (\rho - 1)\mathbb{E}\left[e^{\psi_H} - e^{z^\star}\right]. \tag{13}$$

Proof. The producer could use this liquid asset as a means of payment for a total payoff of $\rho \bar{z}$, where $\bar{z} = (1-\pi) + \pi z$ is the expected payoff of this liquid asset. Alternatively, the producer could give this asset to the banker and receive in return the marginally liquid liability $\{1, \psi_H\}$. The producer would then obtain a payoff of $\rho\bar{\psi}_H$. However, to back this liability, the banker must obtain payoffs $\psi_H - z$ in the high state; this costs the producer $\pi(\psi_H - z) = \bar{\psi}_H - \bar{z}$. Moreover, since the bank is issuing an extra liability, if the incentive constraint is binding, the bank must also obtain extra profits $\kappa\bar{\psi}_H$. Using the liquid asset $\{1, z\}$ as a reserve asset is worthwhile then if

$$\underbrace{\rho \cdot \bar{z}}_{\text{value as payment}} \leq \underbrace{\underbrace{\rho \cdot \bar{\psi}_H}_{\text{value of bank liability}} - \underbrace{\left(\bar{\psi}_H - \bar{z}\right)}_{\text{resource cost}} - \underbrace{\kappa\bar{\psi}_H}_{\text{incentive cost}}}_{\text{value as reserve asset}}$$

■

We dub liquid assets with high-state payoff relative to low state $z \leq z^\star$ as *reserve assets*, to distinguish them from liquid assets that must be used for trading directly.[24] The marginal reserve asset, that is, the one whose normalized high payoff $z^\star$, is given by a resource condition: note that $\left[e^{\psi_H} - e^{z^\star}\right]$ is the amount of maximally illiquid needed to make the marginal reserve asset the marginally liquid assets. Multiplying by $(\rho - 1)$ yields the value of liquidity creation of maximally illiquid assets. That value must exceed the necessary compensation for banks to issue the marginally liquid liability.

Similarly to Section 5, under scarce liquidity, equilibrium outcomes depend on whether the payoffs of all reserve assets are sufficient to eliminate the liquidity scarcity. If the portfolio of illiquid and reserve assets is illiquid, the bank holds all the reserve assets. If this portfolio is liquid, the bank will hold all the reserve assets below a certain level of risk.

Toward the characterization, we denote by $\{N^\star(\ell), N^\star(h)\}$ the aggregate payoffs of reserve assets.

[24] Notice that the set of reserve assets may be empty if the moral hazard problem is severe, i.e., if κ is large enough, $z^\star < \psi_L$.

Proposition 8 *If liquidity is scarce in a joint-liability system, we have the following outcome:*

$$P(D) = \begin{cases} \pi D(h) + qD(\ell), & \textit{if } D \textit{ is illiquid or a reserve asset held by banks},\\ p^{\ell}\left[\pi D(h) + (1-\pi)D(\ell)\right], & \textit{otherwise.} \end{cases}$$

- *Reserve Asset Scarcity: If $N^{illiq}(h) + N^{\star}(h) > \phi_H \cdot (N^{illiq}(\ell) + N^{\star}(\ell))$ banks hold all illiquid and all reserve assets and issue the marginally liquid and maximally illiquid liability. Moreover, $p^{\ell} = \rho$ and $q = q^{\star}$ where $q^{\star} = (1-\pi) + [\rho - 1 - \kappa]\,\mathbb{E}\left[e^{\psi_H}\right]$.*
- *Reserve Asset Abundance: If $N^{illiq}(h) + N^{\star}(h) \leq \phi_H \cdot (N^{illiq}(\ell) + N^{\star}(\ell))$ banks hold all illiquid assets and all reserve assets whose normalized high-state payoff is below some threshold value z such that $N^{illiq}(h) + N^{z}(h) = \phi_H \cdot (N^{illiq}(\ell) + N^{z}(\ell))$ where $\{N^{z}(\ell), N^{z}(h)\}$ are the aggregate payoffs of the reserve assets held by banks. Moreover, banks only issue the marginally liquid liability and the prices are*

$$p^{\ell} = 1 + \kappa \cdot \frac{\mathbb{E}\left[e^{\psi_H}\right]}{\mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}[e^{z}]}, \qquad q = (1-\pi) + \kappa \frac{\mathbb{E}\left[e^{\psi_H}\right]}{\mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}\left[e^{z}\right]}\mathbb{E}\left[e^{z}\right].$$

The proposition shows that the equilibrium taxonomy resonates the one encountered that applied to safe assets. However, the concept of reserve asset scarcity regards the modified liquidity condition that accounts for the resources needed to satisfy bank incentives. When even with the inclusion of all reserve assets liquidity is scarce in this modified sense, all illiquid and reserve assets are held by the bank. The low-state price equals $q^{\star}$ that guarantees that the bank's incentive constraint holds when issuing the marginally liquid liability. Since the bank issues illiquid liabilities producers are indifferent between holding any liquid and the maximally illiquid liability:

$$1 = \mathcal{R}^{i} = \mathcal{R}^{\ell},$$

but strictly prefer to sell reserve assets:

$$\mathcal{R}^{z} = \frac{\rho \mathbb{E}\left[e^{z}\right]}{\pi z + q^{\star}} \leq 1.$$

Under reserve abundance, instead....

Taking Stock. While it might be tempting to call them safe asset, the special role of these assets in this model comes from its ability to expand the the issuance of means of payments. In other words, safer assets feature a "safety" premium or convenience yield, that is ultimately due to their role in liquidity provision.

With moral hazard, liquid assets are separated into two types: those used as means of payments by the producer and those used as reserve assets by the banker. Since the cutoff $z^{\star}$ is below 1, those liquid reserve assets are those with negative beta, those that appreciate in the low state. The convenience yields vary across

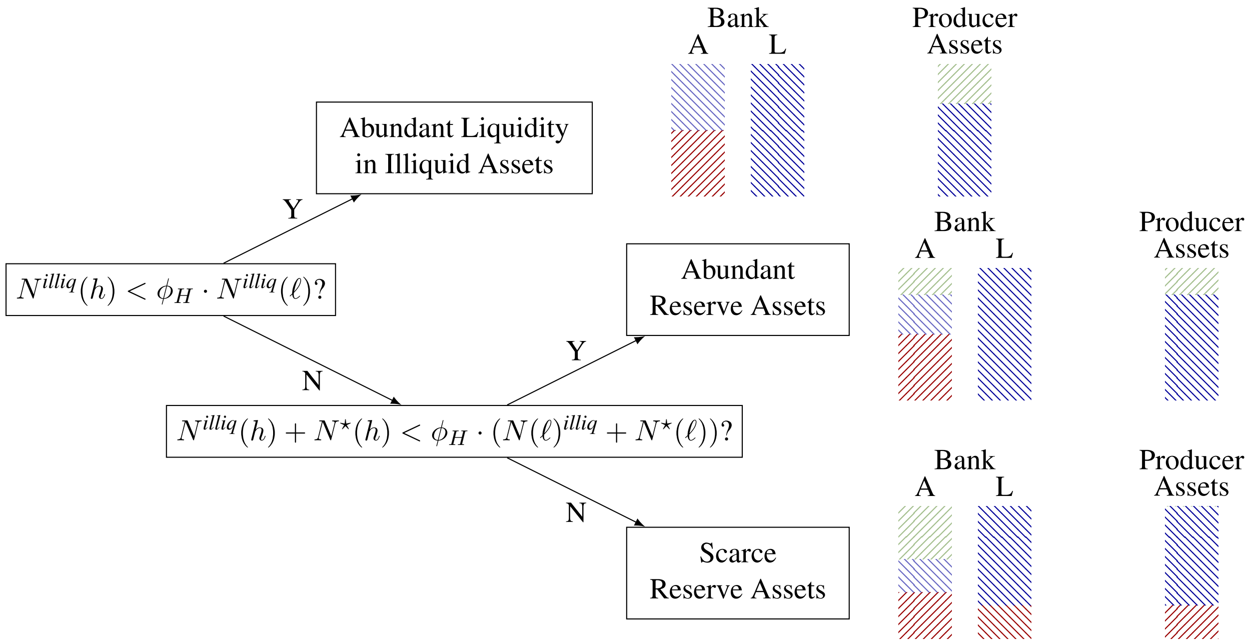


Figure 8: Scenarios with Moral Hazard. The equilibrium balance sheets display illiquid securities in red, liquid *reserve* assets in green, and other liquid securities in blue.

these liquid assets. The expected return rate, in terms of expected payoff, for a liquid security held by the producer is $\left(1+p^L\right)^{-1}$. The expected return rate for a liquid asset held by the banker is necessarily higher than those for the producer, in particular the maximum return rate for the banker is $\left(1+p^L z^*/\psi_L\right)^{-1}$. If we interpret the convenience yield as the difference to the 'fair' return on assets, 1, then we can see that both the producer and the banker pay convenience yields, but these have different origins. The producer pays convenience yields for securities that can be used as means of payment (checking accounts), while the banker pays convenience yields for securities (government bonds) that allow it to expand its issuance of means of payments.

6.2 Consolidation: market power

The previous section discussed how joint liability systems are impaired by moral hazard. This limits the creation of liabilities which, to circulate, must be accepted by all banks. One solution to the moral-hazard between banks is to grant a banking charter to a small number of banks that are recognized in every location. Of course, providing charter grants market power. We now study an extreme version of market power. We study the problem of a monopolist bank. To do so, we recast some of the results from the previous sections where we characterized the preferences of the producer over different securities.

In particular, a monopolist bank faces a unit-continuum of identical producers. Each producer holds a single asset $D = (N^e(\ell), N^e(h))$. To keep the notation consistent with the previous sections, we denote the producer's asset portfolio as the measure μ_0^e, without loss of generality. The bank holds a portfolio of assets μ_0^b. The banker makes a take-it-or-leave-it offer to each producer: the producer, if he accepts, obtains a portfolio of securities $\{\mu_1^i, \mu_1^\ell\}$ in exchange for the producer's assets μ_0^e. The producer then decides whether to accept or reject this offer. The bank is constrained in terms of the securities it can issue. In particular, the bank's constraint is

$$\int D(\omega)d(\mu_1^i + \mu_1^\ell) \leq \int D(\omega)d(\mu_0^e + \mu_0^b), \text{ for } \omega \in \{\ell, h\}.$$

That is, the bank must be able to honor the payments of the securities using either the producer's or its own assets.

Before proceeding to the analysis, we highlight the differences between this setting and the competitive one studied in the previous section. First, in this setting, the bank can set the terms of the exchange. Second, the producer, were he not to participate, can only trade using his asset, as it is. The first point implies that the profit-maximizing bank will offer the producer a portfolio of securities which give him the same utility as his original asset. The second point implies that the producer's utility over his original asset is the utility defined over an individual security (see Proposition 1 or Problem 1), rather than the utility defined over a portfolio (as in Problem 3).

The gains from trade between the bank and the producer depend on the type of asset that the producer holds. In particular, there will be no gains from trade if the producer's asset is liquid. Notice that among the liquid portfolios, the marginal rates of substitution of the bank and the producer are both equal, with value $\pi/(1-\pi)$. Moreover, the section of the producer's indifference curve corresponding to illiquid portfolios lies behind the bank's initial indifference curve. That is, any illiquid portfolio that the producer would find acceptable would leave the bank worse off.

The counterpart of the previous statement is that there could be gains from trade if the producer's initial asset is illiquid or liquid-separating. In that case, the bank would like to offer a liquid-pooling portfolio that delivers the same utility. The only limitation to such an exchange comes from the available assets. Specifically, the liquid-pooling portfolios that would exhaust the gains from trade might not be feasible for lack of assets. We analyze the bank's problem for those types of assets next.

Producer's asset is illiquid. Now, we study the case in which the producer's asset has payoffs $(N^e(\ell), N^e(h))$ with $N^e(h) > \psi_H N^e(\ell)$. While the solution to the previous case required only a single security, the bank could in principle issue multiple securities. In fact, in the current case, if liquidity is scarce, the bank would like to give the producer two securities: the maximally illiquid security and the "marginally liquid" security. This ability of the bank implies that the bank can convexify the producer's indifference curve. In other words,, any asset $(N^e(\ell), N^e(h))$ can be split into two securities $(D(\ell)^A, D(h)^A)$ and $(D(\ell)^B, D(h)^B)$ with proportions λ and $1-\lambda$ such that $\lambda D^A + (1-\lambda)D^B = (N^e(\ell), N^e(h))$. In our analysis, this amounts to the producer evaluating his overall securities portfolio by the portfolio utility as in Problem 3.

The main difference between this case and the previous one lies on the discontinuity of the producer's security indifference curve. In particular, the bank can, via securitization, use this discontinuity to provide an acceptable portfolio to the producer at a much lower cost. In fact, for illiquid securities that are close to being liquid, the bank can create an equally-valuable "marginally liquid" security by *lowering* payoffs. This observation motivates the following classification of illiquid assets.

Definition 2 (Structurally and Fundamentally Illiquid Assets.) *An asset* $(D(\ell), D(h))$ *is structurally illiquid if*

- *It is illiquid,* $D(h) > \psi_H D(\ell)$ *or* $D(h) < \psi_L D(\ell)$ *but*
- *not illiquid enough,* $D(h) \leq \rho\psi_H D(\ell)$ *or* $D(h) \geq \frac{\psi_L}{\rho} D(\ell)$.

An asset is fundamentally illiquid if it is illiquid, but not structurally illiquid.

Proposition 9 *If the producer holds a structurally illiquid asset, the bank's solution is to offer a "marginally liquid" security that makes the producer indifferent. This security is always feasible, even if the bank has no assets.*

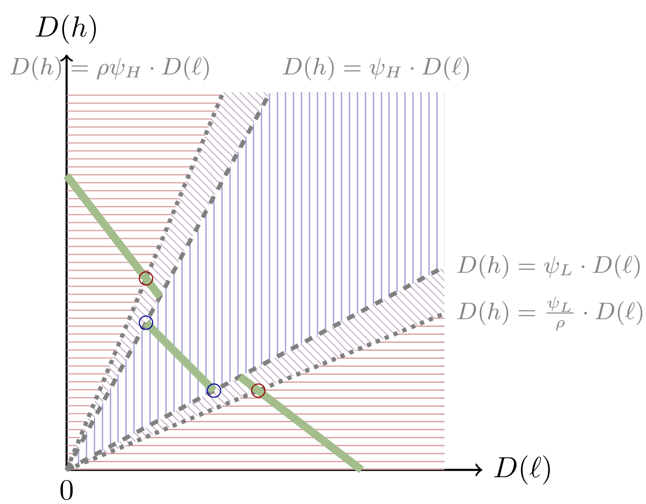


Figure 9: Assets are classified according to their ratio of payoffs $N^e(h)/N^e(\ell)$ as (red) fundamentally illiquid, (purple) structurally illiquid, (blue) liquid. The red circles are structurally illiquid assets; the producer is indifferent between them and their liquid tranche, the blue circles.

In summary, a structurally illiquid asset does not require extra assets to be *structured* into a liquid asset that has the same value to the producer.

In contrast, the bank cannot do this with a fundamentally illiquid asset. In that case, the "marginally liquid" security that would make the producer indifferent requires the bank to combine the producer's illiquid asset with other liquid assets. If there are not enough liquid assets, the best the bank can do is to offer the largest feasible amount of "marginally liquid" securities and make up the difference with the maximally illiquid security.

7 Conclusion

Our paper extends to the information-based theory of money and banking along three dimensions with historical bearing: competition for bank money, co-existence of private and public money, and joint liability. In each variation of the core model, there is a tension between the liquidity creation process and some agency friction.

Policies such as capital requirements, reserve requirements, central bank discounts, deposit insurance, last-resort lending, separation of banking activities, etc., are all about regulating banks' balance sheet. Good policy prescriptions identify an externality that markets cannot correct. Extensions to our theory should speak to those externalities and the best way to correct them.

One potential route are relaxing the commitment assumption on the side of banks: A critical aspect of our model is that while producers have information about the assets on the banker's balance sheet, banks commit to issuing securities with known payoffs across states—even though states are unknown. A natural extension of our theory would allow the banker to deceive workers about their choice of assets. What regulatory mechanisms could be employed to guarantee incentives and what are the costs for liquidity creation? The answer to this question is left for future work.

References

Allen, Franklin, "The market for information and the origin of financial intermediation," *Journal of Financial Intermediation*, 1990, *1* (1), 3–30. 1

— **and Douglas Gale**, "Optimal Financial Crises," *Journal of Finance*, 1998, *53* (4), 1245–1284. 8

— **, Elena Carletti, and Douglas Gale**, "Money, financial stability and efficiency," *Journal of Economic Theory*, 2014, *149*, 100–127. Financial Economics. 1

Andolfatto, David, Aleksander Berentsen, and Fernando M Martin, "Money, Banking, and Financial Markets," *The Review of Economic Studies*, 10 2019, *87* (5), 2049–2086. 1

Antinolfi, Gaetano, Elisabeth Huybens, and Todd Keister, "Monetary Stability and Liquidity Crises: The Role of the Lender of Last Resort," *Journal of Economic Theory*, 2001, *99* (1), 187–219. 1

Asriyan, Vladimir and Victoria Vanasco, "Security Design in Non-Exclusive Markets with Asymmetric Information," *The Review of Economic Studies*, 04 2023, *91* (2), 690–719. 1

Attar, Andrea, Thomas Mariotti, and François Salanié, "Nonexclusive competition in the market for lemons," *Econometrica*, 2011, *79* (6), 1869–1918. 13

Berentsen, Aleksander, Gabriele Camera, and Christopher Waller, "Money, Credit, and Banking," *Journal of Economic Theory*, 2007, *135*, 171–195. 9

Biais, Bruno and Thomas Mariotti, "Strategic Liquidity Supply and Security Design," *Review of Economics Studies*, 2005, *72*, 615–649. 1, 1

Bigio, Saki, "Endogenous Liquidity and the Business Cycle," *American Economic Review*, June 2015, *105* (6), 1883–1927. 3

— **and Liyan Shi**, "Repurchase Options in the Market for Lemons," 2021. revise and resubmit Restud. 11

Boyd, John H and Edward C Prescott, "Financial intermediary-coalitions," *Journal of Economic Theory*, 1986, *38* (2), 211–232. 1

Bryant, John, "A Model of Reserves, Bank Runs, and Deposit Insurance," *Journal of Banking & Finance*, 1980, *4* (4), 335–344. 8

Calomiris, Charles W. and Charles M. Kahn, "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review*, 1991, *81* (3), 497–513. 1

Champ, Bruce, Bruce D. Smith, and Stephen D. Williamson, "Currency Elasticity and Banking Panics: Theory and Evidence," *Canadian Journal of Economics*, 1996, *29* (4), 828–864. 1

Dang, Tri Vi, Gary Gorton, Bengt Holmström, and Guillermo Ordonez, "Banks as secret keepers," *American Economic Review*, 2017, *107* (4), 1005–1029. 1, 2.2

de O. Cavalcanti, Ricardo and Neil Wallace, "A model of private bank-note issue," *Review of Economic Dynamics*, 1999, *2*, 104–136. 21

DeMarzo, Peter and Darrell Duffie, "A Liquidity-Based Model of Security Design," *Econometrica*, 1999, *67*, 65–99. 1, 1, 13

Diamond, Douglas W., "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 1984, *51*, 393–414. 1

_ **and Philip H. Dybvig**, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 1983, *91*, 401–417. 1, 7

_ **and Raghuram G. Rajan**, "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," *American Economic Review*, 2001, *91* (2), 422–425. 1

Donaldson, Jason Roderick and Giorgia Piacentino, "Money runs," *Journal of Monetary Economics*, 2022, *126*, 35–57. 1

Farhi, Emmanuel and Jean Tirole, "Liquid Bundles," *Journal of Economic Theory*, 2015, *158*, 6434–655. 1

Gorton, Gary B. and George Pennacchi, "Financial Intermediaries and Liquidity Creation," *Journal of Finance*, 1990, *45* (1), 49–71. 1, 1

Gu, Chao, Fabrizio Mattesini, Cyril Monnet, and Randall Wright, "Banking: A new monetarist approach," *Review of Economic Studies*, 2013, *80* (2), 636–662. 1

Hellwig, Martin, "Banking, Financial Intermediation, and Corporate Finance," in Alberto Giovannini and Colin Mayer, eds., *European Financial Integration*, Cambridge University Press, 1991, pp. 35–63. 1

Holmström, Bengt and Jean Tirole, "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics*, 1997, *112* (3), 663–691. 1

Jacklin, Charles J., "Demand Deposits, Trading Restrictions, and Risk Sharing," in Edward C. Prescott and Neil Wallace, eds., *Contractual Arrangements for Intertemporal Trade*, University of Minnesota Press, 1987, pp. 26–47. 7

Kashyap, Anil K., Raghuram G. Rajan, and Jeremy C. Stein, "Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-Taking," *Journal of Finance*, 1999, *54* (1), 33–73. 1

Kroszner, Randall S. and Tyler Cowen, "Free Banking: The Scottish Experience as a Model for Transition to a Laissez-Faire Banking System," *Journal of Money, Credit and Banking*, 1989, *21* (2), 221–231. 20

Lagos, Ricardo and Randall Wright, "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 2005, *113*, 463–484. 1, 9, 2.1

Leland, Hayne E. and David H. Pyle, "Informational Asymmetries, Financial Structure, and Financial Intermediation," *The Journal of Finance*, 1977, *32* (2), pp. 371–387. 1

Li, Ye, Yi Li, and Huijun Sun, "The Network Structure of Money Multiplier," 2024. Mimeo. 10

Li, Yiting, Guillaume Rocheteau, and Pierre-Olivier Weill, "Liquidity and the threat of fraudulent assets," *Journal of Political Economy*, 2012, *120* (5), 815–846. 5

Robatto, Roberto, "Systemic banking panics, liquidity risk, and monetary policy," *Review of Economic Dynamics*, 2019, *34*, 20–42. 1

Rocheteau, Guillaume, "Payment and Liquidity under Adverse Selection," *Journal of Monetary Economics*, 2011, *58* (191-205). 3, 1

von Thadden, Ernst-Ludwig, "Intermediated versus Direct Investment: Optimal Liquidity Provision and Dynamic Incentive Compatibility," *Journal of Financial Intermediation*, 1998, *7* (2), 177–197. 1

Williamson, Stephen D., "Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing," *The Quarterly Journal of Economics*, 1987, *102* (1), pp. 135–146. 1

Appendix

A Trading Game

A.1 Description of Trading Game

- **Agents:**

  ▷ **Producer:** A producer is endowed with a unit of a state-contingent, divisible security D. The security pays $D(\omega)$ in state ω and its expected payoff is denoted $\mathbb{E}[D]$. The producer seeks to exchange (a fraction of) this security for labor.

  ▷ **Workers:** There is a finite number (at least two) of workers. Each worker can provide labor q at a marginal cost normalized to 1 in exchange for a fraction n of the producer's security D.

- **States of the World and Information Asymmetry:**

  ▷ There are two aggregate states of the world, $\omega \in \{\ell, h\}$.

  ▷ The producer privately observes the realized state ω before trading. This is the producer's private information

  ▷ Workers do not observe ω when making or accepting offers. They only know the ex-ante distribution $\pi(\omega)$.

- **Sequence of Actions at $t = 1$:**

  1. **Workers' Offers (Menus):** Each worker j simultaneously and nonexclusively offers a menu of contracts M_j. A menu is a *compact* set of pairs (n, q), where n is the fraction of the security D the worker demands, and q is the amount of labor the worker offers. For simplicity of exposition, we require each menu to include the no-trade option $(0, 0)$.
  2. **Producer Observes Menus and State:** The producer observes all offered menus M_j from all workers. The producer then privately learns the realized state ω.
  3. **Producer's Trading Decision (Nonexclusive):** Conditional on ω and the observed menus, the producer chooses a set of contracts $\{(n_j(\omega), q_j(\omega))\}_{j\in J}$ where $(n_j(\omega), q_j(\omega)) \in M_j$ for each worker j, subject to the constraint that the total fraction of the security traded does not exceed their endowment: $\sum_{j\in J} n_j(\omega) \leq 1$. In this way, trading is nonexclusive, the producer can hire more than one worker at a time. We denote the aggregate allocation, $\{N(\omega), Q(\omega)\}$ for $\omega \in \{,\}$, where $N(\omega) = \sum_j n_j(\omega)$ is the total fraction of the security traded and $Q(\omega) = \sum_j q_j(\omega)$ is the total labor hired.

- **Payoffs (for state ω):**

  ▷ **Producer:** The producer's utility from the trade in state ω is $\rho Q(\omega) + (1 - N(\omega))D(\omega)$, where $\rho > 1$ is the productivity of labor (, p.7).

  ▷ **Worker j:** If worker j provides $q_j(\omega)$ units of labor for $n_j(\omega)$ units of security D, their payoff is $n_j(\omega)D(\omega) - q_j(\omega)$ (since labor cost is normalized to 1). Workers are risk-neutral and aim to maximize expected profit.

- **Equilibrium concept.** Perfect Bayesian Equilibrium.
- **Bilateral vs Multilateral Trading.** We call this trading bilateral because a producer enters into a separate contract with each worker and the workers cannot coordinate among themselves or offer contracts contingent in the offers of other workers.

A.2 Equilibrium

To simplify the exposition, we assume that $D_H \geq D_L$ in the following results. The case of $D_H \leq D_L$ is symmetric and we provide the results for that case at the end.

A.2.1 Equilibrium Characterization

Lemma 9 *In any equilibrium, $Q_L \geq Q_H$ and $N_L \geq N_H$.*

Proof. This result is a direct consequence of the incentive compatibility conditions for the producer. The producer in state ω must prefer the allocation $N(\omega), Q(\omega)$ to the allocation in the other state $N(\omega'), Q(\omega')$.

$$\begin{aligned} &\rho Q_H + (1 - N_H)D_H \geq \rho Q_L + (1 - N_L)D_L, \\ &\quad \to \rho(Q_H - Q_L) \geq D_H(N_H - N_L), \end{aligned}$$

and

$$\begin{aligned} &\rho Q_L + (1 - N_L)D_L \geq \rho Q_H + (1 - N_H)D_L, \\ &\to D_L(N_H - N_L) \geq \rho(Q_H - Q_L). \end{aligned}$$

Combining the two inequalities, we get that

$$\begin{aligned} D_L(N_H - N_L) &\geq D_H(N_H - N_L), \\ \to (D_L - D_H)(N_H - N_L) &\geq 0, \\ \to N_H &\leq N_L. \end{aligned}$$

Combining this result with the -producer, we see that $Q_H \leq Q_L$. ∎

Our main result is the characterization of the aggregate allocation of this game.

Proposition 10 *In any equilibrium of the bilateral trading game between a producer and workers:*

(a) For a given security D and realized state ω:

- *If D is a "liquid security" (i.e., $D_H \leq \psi_H \cdot D_L \iff \rho \cdot D_H \leq [D]$), the producer trades the entire fraction $N_H = N_L = 1$ of the security for $Q_H = Q_L = [D]$ units of labor in both states and .*
- *If D is an "illiquid security" (e.g., $D_H > \psi_H \cdot D_L \iff \rho \cdot D_H > [D]$), the producer trades $N_L = 1$ for $Q_L = D_L$ labor in state , and $N_H = 0$ (no trade, $Q_H = 0$) in state .*

(b) The expected payoff of each worker is zero.

Proof. **Deviations.** In this proposition, we characterize the aggregate allocation in any equilibrium of the game. To do this, we consider deviations by a worker where a worker changes its menu to add more contracts. When considering a deviation from a candidate equilibrium allocation, we say that a worker "adds a trade $\{n^*, q^*\}$ to its menu." This should be interpreted in the following way. In a candidate equilibrium, there will be, for each worker j, a menu M_j and a selected contract (n_j, q_j) (possibly the trivial contract). If the deviation is made by worker j, he will offer (n^*, q^*) and $(n_j + n^*, q_j + q^*)$ in addition to his original menu M_j. Alternatively, we could assume that there is always an inactive worker k–meaning his selected contract is the trivial one $n_k = 0 = q_k$–and consider the deviation as this worker adding the contract (n^*, q^*) to his menu.

Liquid security. Suppose that $N_H < 1$. Then, a worker can add to each option in its menu the trade $\{1 - N_H, \tilde{Q}\}$, with $\tilde{Q}/[1 - N_H] \in [D_H/\rho, [D]]$. This additional trade is profitable for the producer in both states. Since the producer would accept this extra offer in both states, the offer is also profitable for the worker because he offers to work less than the expected value of the asset $[D]$. Thus, there is a profitable deviation which reveals that this cannot be an equilibrium outcome. We must have that $N_H = 1$. By monotonicity, as shown in the previous lemma, $N_L = 1$ so any equilibrium must be pooling.

Given that the equilibrium is pooling, we must have $\{N(\omega), Q(\omega)\} = \{1, [D]\}$. Notice that Q cannot be higher because then the workers would prefer to not offer labor. If Q were lower, then any worker can deviate by offering labor $Q + \epsilon < [D]$ choosing $\{n_j, q_j\} = (1, Q + \epsilon)$ and obtain all of the asset. Therefore, $Q = [D]$.

Illiquid Security. We can see that a pooling equilibrium is ruled out because a producer in state is worse off by hiring the workers.

$N_L = 1$**.** Suppose $N_L < 1$. Then, consider a worker adding to each offer in his menu the trade $\{1 - N_L, \tilde{Q}\}$ with $\tilde{Q}/[1 - N_L] \in [D_L/\rho, D_L]$. Notice that the worker is better off because he is working less than the value of the asset in state , which is less than the value in state ω_H, so it does not matter whether the producer in state ω_H chooses this contract as well. The producer in state is also better off because his profit is $\rho\tilde{Q} - [1 - N_L]D_L \geq 0$. Therefore, there cannot be a separating equilibrium with $N_L < 1$.

$N_H = 0, Q_H = 0$**.** Since there cannot be a pooling equilibrium, we must have that $N_H < 1$. Therefore, given an equilibrium, there will be an allocation for each state $\{1, Q()\}$ and $\{N(, Q()\}$. Let us consider an offer (possibly off-equilibrium) to top up the ω_H-state allocation. This offer is $\{1 - N_H, \tilde{Q}\}$.

Since we are in an equilibrium, this offer cannot be profitable for a worker and the producer. We focus on the case where this offer would be accepted only by the -producer. This offer is profitable for the worker if

$$\tilde{Q} \leq D_L(1 - N_H)$$

and profitable for the producer if

$$\rho(\tilde{Q} + Q_H) \geq \rho Q_L \implies \tilde{Q} \geq Q_L - Q_H.$$

Here, we have written the condition for the producer being better off topping up the allocation with this additional offer. Therefore, to rule out this deviation, we must have

$$D_L \cdot (1 - N_H) \leq Q_L - Q_H.$$

Notice that the workers' aggregate payoff is given by

$$
\begin{aligned}
[D(\omega)N(\omega) - Q(\omega)] &= \pi_L(D_L - Q_L) + \pi_H(D_H N_H - Q_H), \\
&= \pi_L(D_L - (Q_L - Q_H)) + \pi_H D_H N_H - Q_H, \\
&\leq \pi_L(D_L - (1 - N_H)D_L) + \pi_H D_H N_H - Q_H, \\
&= N_H[D] - Q_H.
\end{aligned}
$$

Yet, the participation constraint for the -producer is $\rho Q_H \geq N_H D_H$, which allows us to say

$$
\begin{aligned}
[D(\omega)N(\omega) - Q(\omega)] &\leq N_H[D] - Q_H, \\
&\leq N_H\left([D] - \frac{D_H}{\rho}\right).
\end{aligned}
$$

Since the workers' participation constraint implies that their aggregate payoff is non-negative and the security being illiquid implies that the term in parenthesis is negative, we must have $N_H = 0$. From the participation constraint for the worker, $Q_H = 0$.

$Q_L = D_L$. In the previous argument, we showed that

$$D_L \cdot (1 - N_H) \leq Q_L - Q_H.$$

Given that $N_H = Q_H = 0$, this implies that $D_L \leq Q_L$. The workers' participation constraint says that $Q_L \leq D_L$. Therefore, $Q_L = D_L$. ■

Notice that this proof shows that the equilibrium of this game is essentially unique. That is, the aggregate allocations are uniquely determined in equilibrium, although there might be many equilibria where different workers are hired or where different menus of contracts are offered.

A.2.2 Equilibrium Existence

The next result shows that there exists an equilibrium. Specifically, we show that there is an equilibrium where all workers offer a linear-pricing menu.

Proposition 11 *An equilibrium exists in which each worker offers a linear price schedule for labor in terms of the security D. Specifically, each worker offers a menu consisting of a continuum of contracts such that for each $(n, q) \in M$, q/n is constant.*

(a) If D is a liquid security, each worker offers his labor at a rate $n/q = [D]$..

(b) If D is an illiquid security, each worker offers his labor at a rate $n/q = D_L$.

Proof. **Liquid Security.** Consider a deviation by a worker to offer a new trade (n, q). For the producer to select this trade, it must be that $q > n \cdot [D]$. This deviation will not be profitable for the worker if the producer selects it in every state. Therefore, the trade can only be profitable if it is selected only by the producer in the state .

Suppose the -producer is willing to accept this trade, then

$$\rho q + (1 - n)D_H \geq \rho[D].$$

But then, the -producer would also be willing to accept this trade *along with a trade of* $(1-n, [D](1-n))$ from another worker. Indeed,

$$\begin{aligned}\rho[D] &\leq \rho q + D_H(1-n),\\ &\leq \rho q + \rho[D](1-n),\end{aligned}$$

where we used the definition of liquid security $\rho[D] \geq D_H$.

This result shows how cream-skimming deviations do not work with non-exclusivity. Since the producer in the low state can always spend the rest of its security in hiring additional workers, there is no signaling value in conducting a small trade.

Illiquid Security. Consider again a deviation with an offer (n, q). For this offer to be selected, it must be that $q > n \cdot D_L$. This implies that a low-state producer will select this offer. Notice that a producer in the high state will select this if

$$\rho q \geq n \cdot D_H.$$

Thus, this offer is profitable for the worker if

$$n \cdot [D] \geq q.$$

Combining this condition with the previous one, we get that

$$\rho[D] \geq D_H,$$

which means that this deviation is not feasible for illiquid securities. ■

B Safe Assets

Lemma 10 *Assume the bank has no initial endowment. In any equilibrium, the producer must hold some bank liquid liabilities.*

Proof. Suppose the producer holds no bank liabilities. Since the bank has no initial endowment, the bank will hold no assets. Therefore, the producer must hold all the assets which include illiquid assets. However, since the price of an asset is linear $P(D) = q \cdot D(\ell) + \pi \cdot D(h)$, the return on an illiquid asset must be lower than the return on a liquid asset. Thus, this cannot be an equilibrium outcome. ■

Lemma 11 *For any equilibrium where the bank does not hold all the trees, there exists a payoff-equivalent equilibrium where the bank holds all the trees and prices are the same.*

Proof. Let [notation for holdings and prices here] be an equilibrium where [condition that bank holdings are different from aggregate tree endowments]. Then, we claim that [alternative allocation with bank holding all trees and with same prices] is an equilibrium. Notice that since prices are the same, the first-order conditions hold. Since producers are indifferent between holding trees or bank liabilities, the producers' holdings in the new equilibrium are optimal and the producer achieves the same payoff. Since prices are linear, the new allocation does not change the bank's payoff. ■

Lemma 12 *If (tree) liquidity is abundant, $N_H < \psi_H N_L$, banks do not hold safe assets, and there is no liquidity premia: $q = 1 - \pi$. The safe asset's prices is: $p^s = \rho/\tilde{\rho}$.*

Proof. Even with safe assets, under abundant liquidity, there cannot be a liquidity premium: $q = 1 - \pi$. We prove this by contradiction by showing that if there were a liquidity premium, the market clearing condition for low-state payoffs would not hold. By Lemma [ref to bankbuysalltrees], it is without loss of generality to assume that the bank buys all the trees. The bank's portfolio of holdings will be liquid, whether the bank acquires safe assets or not: $N_H/N_L \in [\psi_L, \psi_H] \implies (N_H+S)/(N_L+S) \in [\psi_L, \psi_H]$. Assuming that there is a liquidity premium, $q > 1 - \pi$, the return of a bank's liquid liability $\{1, z\}$ is $\mathcal{R}(\{1, z\}) = \tilde{\rho}\frac{(1-\pi)+\pi \cdot z}{q+\pi \cdot z}$ which is strictly increasing in z. Thus, among liquid liabilities, the producer would only be willing to hold the marginally liquid one, e^{ψ_H}. A similar argument shows that among illiquid liabilities, the producer would only be willing to hold the maximally illiquid one that pays only in the high state, e^h. Thus, the producer's portfolio of bank liabilities has a ratio of payoffs (high-state over low-state) greater than ψ_H. However, since the bank makes zero profits, the producer's portfolio of bank liabilities must match the portfolio of bank assets. Since the latter consists of all trees, it has a ratio of payoffs $N_H/N_L < \psi_H$ which is a contradiction.

Now we show that the bank cannot hold safe assets. By Lemma 2, banks hold the safe asset if and only if $p^s = q+\pi = 1$. However, without a liquidity premium and at this price for the safe asset, the producer would strictly prefer to hold the safe asset over a liquid bank liability $\mathcal{R}^{\psi_H} = \tilde{\rho} < \rho = \mathcal{R}^s$, contradicting market clearing in the issuance of liquid liabilities. Thus, the price must be higher to make the producer indifferent between holding the safe asset and the liquid liability. Since the return of the bank's marginally liquid liability is $\tilde{\rho}$, the price of the safe asset must be such that it equalizes returns $p^s = \frac{\rho}{\tilde{\rho}} > 1 = q + (1-\pi)$. ∎

Lemma 13 *If (tree) liquidity is scarce, $N_H > \psi_H N_L$, there is a liquidity premium.*

Proof. Suppose aggregate liquidity is scarce and there is no liquidity premium. As shown in the previous proof, the bank cannot hold the safe asset: the bank would only hold the safe asset if the price is $p^s = q + \pi = 1$, then $\mathcal{R}^s = \rho > \tilde{\rho} = \mathcal{R}^{\psi_H}$. But if the bank won't hold safe assets, the bank must be issuing illiquid liabilities. However, without a liquidity premium the return on the bank's marginally liability is $\mathcal{R}^h = 1 < \tilde{\rho} = \mathcal{R}^{\psi_H}$ and the producer is not willing to hold those securities. ∎

Lemma 14 (Recognizability 1/2) *Suppose (tree) liquidity is scarce, $N_H > \psi_H N_L$. The bank must hold safe assets if recognizability is high, $\nu > ((1-\pi) + \pi\psi_H)^{-1}$.*

Proof. High recognizability implies bank holds safe assets. Suppose recognizability is high. Assume, toward contradiction, that there is an equilibrium where the bank holds no safe assets. Therefore the producer must hold the safe assets and also, since liquidity is scarce, illiquid securities. Thus, the producers' returns for each of these securities must be equal.

$$\mathcal{R}^s = \mathcal{R}^h = \mathcal{R}^{\psi_H} \implies p^s = \rho, q = \tilde{\rho}\bar{\psi}_H - \pi\psi_H.$$

We now show that, at these prices, the bank would strictly prefer to buy the safe asset. That is, we will show that $q + \pi > p^s$. Notice that we can conveniently write $q = (\tilde{\rho} - 1)\bar{\psi}_H + 1 - \pi$, by adding and subtracting $\bar{\psi}_H$. Then, since $\tilde{\rho} - 1 = \nu(\rho - 1)$, we can say that $q + \pi > \rho = p^s$. Therefore, the bank's choice not to hold safe assets is suboptimal and this cannot be an equilibrium. ∎

Lemma 15 (Recognizability 2/2) *Suppose (tree) liquidity is scarce, $N_H > \psi_H N_L$. The bank won't hold safe assets if recognizability is low, $\nu < ((1-\pi) + \pi\psi_H)^{-1}$.*

Proof. Low recognizability implies bank does not safe assets. Since the producer must be willing to hold the bank's marginally liquid liability, we must have

$$\max\{\mathcal{R}^s, \mathcal{R}^h\} \le \mathcal{R}^{\psi_H}.$$

We will have $\mathcal{R}^{\psi_H} \ge \mathcal{R}^h$ if

$$\begin{aligned}
&\tilde{\rho}\frac{(1-\pi) + \pi \cdot \psi_H}{q + \pi \cdot \psi_H} && \ge 1, \\
\implies &\tilde{\rho}\left[(1-\pi) + \pi \cdot \psi_H\right] && \ge q - (1-\pi) + [(1-\pi) + \pi \cdot \psi_H], \\
\implies &(\tilde{\rho} - 1)\left[(1-\pi) + \pi \cdot \psi_H\right] && \ge q - (1-\pi), \\
\implies &(\rho - 1)\nu\left[(1-\pi) + \pi \cdot \psi_H\right] && \ge q - (1-\pi), \\
\implies &(\rho - 1) && > q - (1-\pi), \\
\implies &\rho && > q + \pi.
\end{aligned}$$

The second line simply rearranges the inequality. The third inequality groups the terms involving $[(1-\pi) + \pi \cdot \psi_H]$. The fourth inequality uses the identity $\tilde{\rho} - 1 = \nu(\rho - 1)$. The last inequality follows from the assumption of low recognizability.

Given that we must have $q < \rho - \pi$, we use this condition to check if $\mathcal{R}^{\psi_H} \ge \mathcal{R}^s$. Since we are assuming that the bank holds the safe asset, the price $p^s = q + \pi$.

$$\begin{aligned}
&\tilde{\rho}\frac{(1-\pi) + \pi \cdot \psi_H}{q + \pi \cdot \psi_H} && \ge \frac{\rho}{q+\pi}, \\
\implies &\tilde{\rho}\frac{(1-\pi) + \pi \cdot \psi_H}{\rho} && \ge \frac{q + \pi \cdot \psi_H}{q + \pi} = 1 + \frac{\pi(\psi_H - 1)}{q+\pi} > 1 + \frac{\pi(\psi_H - 1)}{\rho}, \\
\implies &\tilde{\rho}[(1-\pi) + \pi \cdot \psi_H] && > \rho + \pi(\psi_H - 1) = \rho - 1 + [(1-\pi) + \pi \cdot \psi_H], \\
\implies &(\tilde{\rho} - 1)[(1-\pi) + \pi \cdot \psi_H] && > \rho - 1, \\
\implies &\nu[(1-\pi) + \pi \cdot \psi_H] && > 1.
\end{aligned}$$

The second line rearranges the inequality and uses the strict bounds on the value of q derived above. The third line eliminates the denominator and constructs the term $[(1-\pi) + \pi \cdot \psi_H]$. The fourth line groups this term. The fifth line follows from the identity $\tilde{\rho} - 1 = \nu(\rho - 1)$.

Notice that the last line contradicts our assumption of low recognizability. Thus, it is not possible for the bank to hold safe assets when recognizability is low. ■

Proposition 12 *With scarce liquidity and high recognizability banks hold safe assets ($p^s = q + \pi$ and $S^b > 0$). Furthermore,*

- *If there is Scarce Safety, $N(h) + S > \psi_H \cdot (N(\ell) + S)$, the bank holds all the safe assets $S^b = S$, issues illiquid liabilities, and $q = q^*(\nu)$ equalizes the return of liquid and illiquid liabilities.*

- *Otherwise, there is Abundant Safety, the bank holds only enough safe assets to make the portfolio of trees marginally liquid:*

$$N(h) + S^b = \psi_H \cdot \left(N(\ell) + S^b\right),$$

do not issue illiquid liabilities, and $q = q^{SL}(\nu)$, where q^{SL} equalizes the return of safe assets and liquid liabilities:

$$\frac{\rho}{q^{SL}(\nu) + \pi} = \frac{\tilde{\rho}\bar{\psi}_H}{q^{SL}(\nu) + \pi\bar{\psi}_H}.$$

Proof. The previous lemma showed that banks hold safe assets and $p^s = q + \pi$.

Scarce Safety. Because safety is scarce, even if the bank holds all the safe assets, the producer must hold the illiquid security. Therefore, $\mathcal{R}^{\psi_H} = \mathcal{R}^h$. This implies $q = q^*(\nu)$. We next show that the producer will not hold the safe asset $\mathcal{R}^s < 1$, specifically $q^*(\nu) + \pi > \rho$. From the indifference condition $\mathcal{R}^{\psi_H} = \mathcal{R}^h$, we get

$$\begin{aligned}
\frac{\tilde{\rho} \cdot \bar{\psi}_H}{q + \pi\psi_H} &= 1, \\
\implies q - (1 - \pi) &= (\tilde{\rho} - 1)\bar{\psi}_H, \\
\implies q + \pi &= (\rho - 1)\nu\bar{\psi}_H + 1 > \rho, \\
\implies q + \pi &> \rho,
\end{aligned}$$

where the third line uses the assumption that recognizability is high. This shows that the producer will not hold the safe asset, therefore the bank holds all of them.

Abundant Safety. We show the bank will hold only enough safe assets to have a liquid portfolio of assets. If the bank holds less safe assets than this, the producer must hold the illiquid security. But then, by the arguments above the producer would not be willing to hold the safe asset. Since the bank is not holding all of the assets, this contradicts market clearing. If the bank holds more safe assets than this, the bank's portfolio of assets is strictly in the liquid region. Therefore, the bank's liquid liability is not the marginally liquid liability. However, the producer will only be willing to hold this liability if there is no liquidity premium $q = 1 - \pi$. But, as a previous lemma showed, under scarce liquidity there must be a liquidity premium. ■

C Moral Hazard

C.1 Liquid Assets Acquired by the Bank

When there are abundant reserve assets, the value of the price of the assets are

$$P(D) = \begin{cases} \pi D(h) + qD(\ell), & \text{if } D \text{ is illiquid or a reserve asset held by banks,} \\ p^\ell \cdot [\pi D(h) + (1 - \pi)D(\ell)], & \text{otherwise.} \end{cases}$$

The threshold z is determined by $N^{illiq}(h) + N^z(h) = \phi_H \cdot (N^{illiq}(\ell) + N^z(\ell))$ where $\{N^z(\ell), N^z(h)\}$ are the aggregate payoffs of the reserve assets held by banks.

The prices p^ℓ and q can be solved by combining the budget constraint of the banker along with the producer's indifference between holding the threshold reserve asset e^z or not.

In particular, the latter condition is

$$p^\ell \mathbb{E}[e^z] = q + \pi \cdot z \rightarrow (p^\ell - 1)\mathbb{E}[e^z] = q - (1 - \pi) =: Q$$

The budget constraint from the bank says

$$p^\ell D = q \cdot (N^{illiq}(\ell) + N^z(\ell)) + \pi \cdot (N^{illiq}(h) + N^z(h)).$$

Use the fact that $N^{illiq}(h) + N^z(h) = \phi_H \cdot (N^{illiq}(\ell) + N^z(\ell))$ to simplify this expression to get

$$p^\ell D = (q + \pi \cdot \phi_H) \cdot (N^{illiq}(\ell) + N^z(\ell)).$$

Then, from the feasibility constraint, we know that

$$D = \mathbb{E}\left[e^{\psi_H}\right] \cdot (N^{illiq}(\ell) + N^z(\ell)),$$

because we know that the bank issues the marginally liquid liability.

Substituting, we obtain

$$p^\ell \mathbb{E}\left[e^{\psi_H}\right] = (q + \pi \cdot \phi_H) = Q + \mathbb{E}\left[e^{\phi_H}\right].$$

Then, we combine this condition with the one we had before (repeated here for exposition)

$$(p^\ell - 1)\mathbb{E}[e^z] = Q.$$

Combining these expressions, we can solve for p^ℓ and get

$$p^\ell = \frac{\mathbb{E}\left[e^{\phi_H}\right] - \mathbb{E}[e^z]}{\mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}[e^z]}.$$

The latter expression can be rewritten in many other forms involving κ.

$$\begin{aligned} p^\ell &= \frac{\mathbb{E}\left[e^{\phi_H}\right] - \mathbb{E}[e^z]}{\mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}[e^z]}, \\ &= \frac{(1+\kappa) \cdot \mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}[e^z]}{\mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}[e^z]}, \\ &= 1 + \kappa \cdot \frac{\mathbb{E}\left[e^{\psi_H}\right]}{\mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}[e^z]}. \end{aligned}$$

The value of q is given by

$$q = Q + (1 - \pi),$$
$$= \kappa \left[\frac{\mathbb{E}\left[e^{\psi_H}\right]}{\mathbb{E}\left[e^{\psi_H}\right] - \mathbb{E}[e^{z}]} \right] \cdot \mathbb{E}[e^{z}] + (1 - \pi).$$

Notice that this is consistent with the previous results because as $z \to z^*$, $p^{\ell} \to \rho$. Then, the price $q \to q^* = (\rho - 1 - \kappa)\mathbb{E}\left[e^{\psi_H}\right]$.