Disagreement, Skewness, and Asset Prices

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We develop a frictionless theory reconciling the apparently distinct puzzles that disagreement and skewness each negatively predict equity returns, while disagreement exhibits varying effects across asset classes. Our key insight is that equilibrium price sensitivity to disagreement depends on the curvature of asset demand, which in turn is proportional to payoff skewness. This mechanism yields an interaction effect: disagreement and skewness have a multiplicative impact on expected returns. Our framework further yields an invariant relationship with Sharpe ratios, independent of volatility, distinguishing disagreement from uncertainty. Robust empirical tests on U.S. equities support our novel predictions.

KEYWORDS: disagreement, divergence of opinions, analyst forecast dispersion, skewness, nonlinear demand
JEL CLASSIFICATIONS: G12, D53, D82

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1 Introduction

Investor disagreement, or divergence of opinions, is pervasive in financial markets and, intuitively, essential for trade. However, its impact on asset prices is less obvious. A large literature starting with Diether et al. (2002) proxies for disagreement using analyst forecast dispersion and finds a *negative* relationship to expected equity returns. In contrast, Güntay and Hackbarth (2010) and Carlin et al. (2014) document a *positive* relationship between disagreement and expected bond returns. In this paper, we develop a parsimonious theory which rationalizes these disparate findings. Importantly, we rely on neither the short-sales frictions mechanism of Miller (1977) nor the limited risk-sharing mechanism of Abel (1989).

Our approach is grounded in two key insights. First, equilibrium price sensitivity to disagreement depends on the curvature of asset demand. Second, the curvature of demand schedules is proportional to payoff skewness. As a result, the model generates novel predictions about the interaction of disagreement and skewness on asset prices, for which we provide robust empirical support. Additionally, we bridge the literature on disagreement with the seemingly distinct literature showing that skewness negatively predicts stock returns (Boyer et al., 2010, among others).

In Section 2, we formally analyze the asset pricing implications of disagreement in a parsimonious neoclassical framework with essentially no parametric assumptions on utility or payoff distributions. Our model considers a two-period financial market with trading in a risk-free asset and a single risky asset, which pays an uncertain liquidating dividend and whose equilibrium price is determined by standard market clearing. There is a unit measure of investors who are price takers, maximize expected utility, have divergence of opinions about the expected dividend, and have a common utility function with the following standard properties: investors strictly prefer more to less, are strictly risk averse, and have non-increasing absolute risk aversion (NARA). Note that the NARA class nests both the constant absolute relative risk aversion (CARA) and constant relative risk aversion (CRRA) preference classes.[1]

[1]NARA ensures that a risky asset with a positive expected return is not an inferior good; i.e., wealthier investors allocate weakly more money to the risky asset. Arrow (1971) argues these are properties of any reasonable utility function. As shown in Arditti (1967), NARA implies a preference for positive skewness.

Via Taylor polynomials of investor demands and the market-clearing price, we obtain the following pricing equations:

$$\text{Expected excess return} \propto -\text{volatility} \times \text{skewness} \times \text{Var(beliefs)}, \quad (1)$$

$$\text{Sharpe ratio} \propto -\text{skewness} \times \text{Var(beliefs)}. \quad (2)$$

Equation (1) predicts an interaction effect: skewness controls the sign and magnitude of the impact of disagreement (variance of beliefs) on asset prices. Because empirically most stocks have positively skewed returns (Boyer et al., 2010), this interaction implies a negative relationship between disagreement and expected returns in equities (the "disagreement effect," as found by Diether et al., 2002). For assets with negatively skewed payoffs, the model predicts a positive relationship, consistent with findings in fixed income markets.[2]

Because variance is non-negative, equation (1) also predicts that expected returns are decreasing in skewness (the "skewness effect," as found by Boyer et al., 2010). Moreover, we obtain a "null" result: when skewness is approximately zero, the effect of disagreement on expected returns should be negligible. Likewise, when disagreement is approximately zero, the effect of skewness on returns should be negligible. Equation (2) demonstrates that disagreement and skewness also interact to influence Sharpe ratios, and this influence remains consistent across different volatility levels. This result highlights a fresh perspective to distinguish effects of disagreement from uncertainty (Diether et al., 2002).

In addition to our primary analysis, we investigate several extensions to further generalize and substantiate the robustness of our theoretical findings. These extensions encompass disagreements among arbitrary investor types, heterogeneous investor preferences, disagreements concerning higher-order moments, and assumptions about asset supply.

In Section 4, we provide economically and statistically significant empirical support for the interaction effect predicted by equation (1) and the volatility-invariance prediction of equation (2) within the universe of U.S. equities. We use an independent double sort to construct a portfolio which is both skew- and disagreement-neutral but exploits the interaction effect. It has economically significant annualized excess return of -6.4% and CAPM alpha

[2] Payoffs of corporate bonds (studied in Güntay and Hackbarth, 2010) and mortgage-backed securities (studied in Carlin et al., 2014) are decidedly negatively skewed.

of -6.6%, both with statistically significant t-statistics greater than 3. Fama and French (2015) and Daniel et al. (2020) alphas exhibit comparable metrics. We repeat the exercise on scaled returns (realized returns divided by ex-ante option-implied volatility) and document evidence consistent with the invariance prediction of equation (2).

Additionally, we address potential alternative explanations to support the conclusion that our findings reflect the proposed frictionless mechanism rather than confounding factors. A key concern is whether disagreement and skewness proxy for investor optimism or reflect asymmetric limits to arbitrage, such as short-selling frictions, which could mimic the demand convexity central to our theory. To address this concern, we examine analyst forecast bias (Kozak et al., 2018) as a measure of optimism following Engelberg et al. (2018) and find no systematic correlation with our variables, distinguishing our effects from sentiment-driven pricing. We assess robustness to trading frictions by investigating market capitalization and volatility proxies for trading frictions, as suggested by Dong et al. (2022), as well as the option-implied measure of shorting fees derived by Muravyev et al. (2025). The interaction effect persists after excluding small-cap and high-shorting-fee firms—where such frictions are most acute. These checks, detailed in Section 4.4, alongside checks under volatility controls in Section 4.3, reinforce that our results stem from the interplay of skewness and disagreement under NARA utility, offering a novel perspective on asset pricing anomalies beyond traditional friction-based explanations.

To understand the mechanism that generates these effects, first consider how demand curvature is governed by payoff skewness. Figure 1 plots demand functions for two assets that are identical except for their skewness, with the thicker curve representing the asset with higher skewness. Because a NARA investor has a preference for skewness, demand for the asset with higher skewness must be weakly higher at any price.[3] Because of risk aversion, however, when price, p, equals the subjective expected payoff, μ_i, both demand

[3]NARA utility implies that an investor's demand schedule is convex (concave) in a neighborhood around the investor's subjective expected payoff if and only if the payoff skewness is positive (negative). As drawn, the second derivatives are positive, but the argument works for any values. We cautiously note that it is generally impossible to change skewness without affecting other moments unless the distribution is unbounded. However, as we show in the proof of Lemma 1, the graphical illustration is correct in a neighborhood around the subjective expected payoff, as the curvature of the demand function locally does not depend on moments higher than the third.

functions must coincide at zero.[4] As a result, the thicker curve is more "curved" than the thinner curve, yet both are anchored to zero at price equal to the expected payoff.

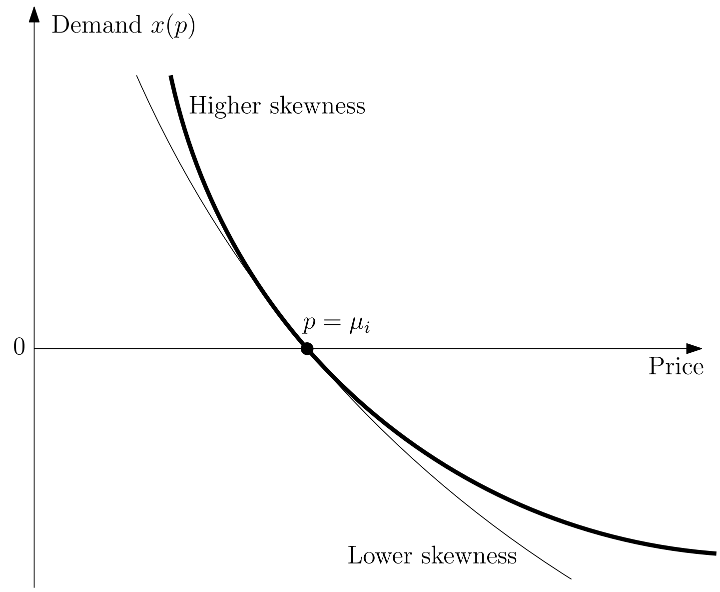

Figure 1: Higher Skewness Leads to a More Convex Demand Curve

Note: This figure illustrates that ceteris paribus, higher skewness leads to a more convex demand curve. The thicker curve represents the demand associated with a higher skewness, while the thinner curve represents the demand associated with a lower skewness. Since a NARA investor has a preference for skewness, the thicker curve is weakly above the thinner curve for any price. Furthermore, when price equals the subjective expected payoff (μ_i), demand is precisely zero, regardless of skewness, because of risk aversion. Put differently, both curves must cross the price axis at the same point, where they are tangent. As a result, the thicker curve is more "curved" than the thinner curve in the neighborhood around the expected payoff.

Next, consider the role of belief heterogeneity. Suppose investors are of two types: optimists (positive type, denoted as +) and pessimists (negative type, denoted as −).[5] Investors disagree about the mean of the dividend distribution but agree about its shape, i.e., volatility and all higher-order central moments.[6] Letting μ and σ be the objective mean and volatilty of the payoff, respectively, the beliefs of the two types are $\mu_+ = \mu + \sigma\delta$ and $\mu_- = \mu - \sigma\delta$ so the

[4] A risk-averse agent rejects any mean-zero lotteries.

[5] In Section 2.3.1, we show our results are robust to an arbitrary number of investor types.

[6] In Section 2.3.3, we show our results are robust to alternative structures of disagreement such as differences of opinion about variance and skewness.

average belief is correct and δ parameterizes the level of disagreement (per unit of volatility).[7]

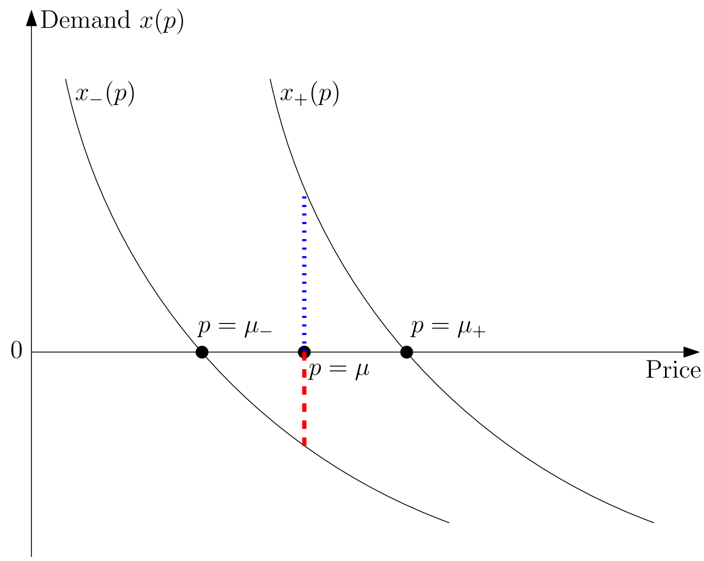

Figure 2: Disagreement Lowers Expected Returns for a Positively Skewed Asset

Note: This figure illustrates the intuition that disagreement lowers returns for a positively skewed asset. Because of positive skewness, an investor's demand is convex in the neighborhood of the asset's expected payoff under the investor's belief. The two curves represent the demand schedules of a positive-type investor and a negative-type investor, respectively. For example, the demand of a positive-type investor crosses zero at price $\mu + \sigma\delta$, which is the risky asset's expected payoff under a positive type's belief. Suppose the price is equal to μ. The blue (upper) dotted line segment represents the shares that a positive-type investor would buy, while the red (lower) dashed line segment represents the shares that a negative-type investor would sell. Because of demand convexity, the length of the dotted-blue line is greater than that of the dashed-red line. In other words, at price μ, there is excess demand above zero. To clear the market, the equilibrium price must be greater than μ.

Figure 2 plots optimist and pessimist demand schedules, respectively, for a positively skewed asset.[8] Suppose the risky asset is in zero supply and consider a candidate equilibrium price equal to the average belief: $p = \mu$.[9] At this price, the positive-type investor perceives the asset as under-priced and would go long while the negative-type investor per-

[7]That is, investors disagree about the asset's Sharpe ratio by $\pm\delta$.

[8]Due to agreement on higher moments, the curves are locally parallel.

[9]We assume that the risk-free asset has infinitely elastic supply with price and payoff normalized to unity. In Section 2.3.4, we show our results are robust to non-zero supply for the risky asset.

ceives it as over-priced and would go short. Can the market clear? The answer is no, because at this price there would be excess demand. Buying a positively skewed asset entails more desirable upside risk whereas shorting the asset involves more downside risk. Convexity implies the positive type would want to buy more shares of the asset than those shorted by the negative type. To clear the market, the equilibrium price must be higher than μ. Similarly, for a negatively-skewed asset, in which case demand schedules are concave, the equilibrium price must be lower than μ. For a symmetric payoff distribution, demand is linear and price equals the average belief, nesting implications of some traditional frameworks. For example, in a simple setting with frictionless trading, zero net supply, and linear demand curves as in the case of a CARA-normal environment or approximate mean-variance preferences, disagreement does not affect price.

Finally, we note that our proposed theory is complementary (rather than contradictory) to existing friction-based explanations of the relationship between disagreement and expected equity returns. Our graphical intuition highlights the importance of nonlinear demand curves and we advance a fresh interpretation of short-sale frictions as another source of demand convexity aside from positive skewness. When short-sale costs are proportional to quantity shorted, demand is kinked at $p = \mu_i$, with a flatter slope in the short-selling region such that the demand schedule is piecewise linear, but convex.[10] The disagreement effect still operates through non-linear demand. Unlike short-sales costs, however, negative skewness is a source of demand *concavity* that further explains the positive relationship between disagreement and expected returns observed in fixed income markets.

Related Literature. Our paper contributes to both the disagreement literature and skewness literature in asset pricing. Disagreement drives trade, as explored in static models (Miller, 1977; Jarrow, 1980; Diamond and Verrecchia, 1987; Chen et al., 2002) and dynamic models of speculative bubbles (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003; Hong et al., 2006; Martin and Papadimitriou, 2022). Studies on trading volume, volatility, and informed trading include Harris and Raviv (1993), Kandel and Pearson (1995), Cao and Ou-Yang (2009), Dumas et al. (2009), Banerjee and Kremer (2010), Ottaviani and Sørensen

[10] When short sales are not allowed, demand is zero for $p \geq \mu_i$, and demand is still convex. In fact, demand convexity obtains if short-sale costs are weakly convex in quantity shorted.

(2015), Atmaz and Basak (2018), Banerjee et al. (2018), and Chabakauri and Han (2020). Surveys by Bielecki et al. (2004), Hong and Stein (2007), Curcuru et al. (2010), and Xiong (2013) summarize this field. The "disagreement effect," where analyst forecast dispersion negatively predicts equity returns (Diether et al., 2002), is further studied by Sadka and Scherbina (2007), Yu (2011), Barinov (2013), Hong and Sraer (2016), Ali et al. (2019), and Daniel et al. (2023), with Chang et al. (2022) providing an overview. Johnson (2004) offers a rational explanation tying dispersion to leverage, though Avramov et al. (2009) find no empirical support. Our frictionless model explains this effect via the interaction of disagreement and skewness, consistent with evidence of varying disagreement effects across asset classes, including a positive relationship in fixed-income markets (Güntay and Hackbarth, 2010; Carlin et al., 2014).

The "skewness effect," where ex-ante idiosyncratic skewness negatively predicts returns, is documented by Boyer et al. (2010), Conrad et al. (2013), Boyer and Vorkink (2014), and Amaya et al. (2015). Theoretical explanations include optimistic beliefs (Brunnermeier and Parker, 2005; Brunnermeier et al., 2007), prospect theory preferences (Barberis and Huang, 2008), and heterogeneous preferences with underdiversification (Kraus and Litzenberger, 1976; Mitton and Vorkink, 2007). Goulding et al. (2023) attribute it to NARA utility and noise traders. Our model generates the skewness effect under NARA utility without noise traders and offers novel interaction predictions with disagreement.

Like Yan (2010), we show that nonlinear demand prevents biases from canceling out, but we derive nonlinearity from NARA utility rather than assuming it, yielding joint pricing implications for skewness and disagreement. In models with heterogeneous priors, Martin and Papadimitriou (2022) link sentiment to price bubbles for skewed assets, while Banerjee et al. (2022) and Banerjee et al. (2023) show skewness affects returns for CARA investors. In noisy rational expectations frameworks (NREE), Goulding (2015) finds an interactive effect of skewness and belief dispersion with binary payoffs and CARA preferences, while Breon-Drish (2015), Chabakauri et al. (2022), and Cianciaruso et al. (2023) explore similar effects under various parametric assumptions. In a non-parametric NREE framework, Albagli et al. (2024) find an interaction effect holds if the equilibrium pricing kernel is log-convex. Unlike these studies, our results do not rely on specific preferences, payoff distributions, or constraints on unobservable variables. Moreover, we offer graphical intuition and quantitative predictions, including a volatility-invariant Sharpe ratio effect, for which we provide robust empirical support.

2 Model

In this section, we offer a theory to study the asset pricing implications of disagreement when a risky asset can have a skewed payoff distribution. We first present a setting without parametric assumptions on the utility function and the payoff distribution. We derive Taylor expansions for demand curves, the market-clearing equilibrium price, and the expected return as functions of investor disagreement and skewness. We also study a parametric version of the model and characterize how the shape of individual demand schedules depends on payoff skewness without resorting to Taylor expansion. We then use the equilibrium prices to the comparative statics with respect to disagreement, skewness, and their interaction. Section 2.3 discusses a variety of model extensions to demonstrate how the insights generalize.

2.1 Model Setup

We study a two-period financial market, with dates $t \in \{0,1\}$, and a continuum of investors, indexed by $i \in [-1,1]$. Each investor has common utility function $u(w)$ and initial wealth w_0.[11] Investors trade in two assets: a risk-free asset, with both its price and payoff normalized to one, and a single risky asset, with date-0 price p and date-1 payoff $\widetilde{\theta}$.

Investor Preference. The function $u(\cdot)$ is thrice continuously differentiable and exhibits the following properties of any reasonable specification of preferences (Arrow, 1971): investors strictly prefer more to less, which implies $u'(w) > 0$; investors are strictly risk averse, which implies $u''(w) < 0$; and an investor's absolute risk aversion does not increase in wealth—non-increasing absolute risk aversion, or NARA utility—which implies $u'''(w) > 0$. NARA ensures that a risky asset with a positive expected return is not an inferior good; i.e., wealthier investors allocate weakly more money to the risky asset. Note that the NARA

[11] In Section 2.3.2, we study heterogeneous utility functions as well as heterogeneous initial wealth.

utility class nests widely-used parametric utility functions such as constant absolute risk aversion (CARA) utility and constant relative risk aversion (CRRA) utility classes.

Investor Belief. We allow investors to disagree about the date-1 payoff of the risky asset. We use the notation F_z to reflect a horizontal shift of z units; i.e., $F_z(\theta) = F(\theta - \sigma z), \forall\theta$. Note that z is unitless. For simplicity of presentation we consider two types of investors, but we show in Section 2.3.1 that our results generalize to arbitrary number of types. Each investor can be either a positive "optimist" type or a negative "pessimist" type: investors from $(0,1]$ are of positive type, and investors from $[-1,0]$ are of negative type. We assume a positive-type investor believes that the date-1 payoff is drawn from CDF F_δ, while a negative-type investor believes the payoff is drawn from CDF $F_{-\delta}$, such that a higher level of the horizontal shift parameter $\delta \geq 0$ corresponds to a wider dispersion of beliefs between the two investor types.[12] The difference between the optimistic and pessimistic types' beliefs about the risky asset's mean is therefore $2d$, where $d = \delta\sigma$.

Market Clearing. Each investor submits a demand schedule $x_i(p)$, which specifies how many shares of the risky asset she would buy or sell at price p. The price of the risky asset is determined by the market clearing condition:

$$\int_i x_i(p)\,di = \chi,$$

where χ denotes the aggregate supply.

2.2 Non-Parametric Analysis

This section presents nonparametric analysis where we don't impose any parametric assumptions on investor preferences and the risky asset's payoff distribution. For the ease of

[12]We remark that this structure of disagreement may not be realistic if the dividend distribution is bounded and investors can write side contracts. This is because the upper bound of a positive type's perceived distribution is outside the support of the perceived distribution from a negative type. If side contracts are allowed, investors would bet an infinite amount for a contract that only pays outside the support of their respective perceived distribution. In our baseline model, one way to justify our assumption is that side contracts are not allowed or sufficiently costly to write, which is to say that the market is incomplete. We can also justify our assumption by assuming the support of the dividend distribution is the entire real line in which case the above concern is absent. Overall, we choose the simple structure in our baseline model to highlight the mechanism through which disagreement affects asset prices. As we show in Sections 2.3.1 and 2.3.3, our results are robust to general structures of disagreement.

exposition, we assume $\chi = 0$ in this subsection.[13] We first show that the aggregate demand function is convex (concave) when the underlying risky asset has a positively (negatively) skewed payoff. From there, we can prove that the euqilibrium price must be higher (lower) than the average belief. Moreover, through Taylor expansion, we can quantititvely derive how the expected return rely on disagreement, skewness, and their interaction.

2.2.1 Investor Demand

We start with an investor's problem. Formally, the demand schedule $x_i(p)$ solves the following program:

$$x_i(p) = \arg\max_x E_i[u(w_0 + x_i(\widetilde{\theta} - p))], \tag{3}$$

where the expectation is taken under investor i's beliefs. Since u is strictly concave and thrice continuously differentiable, the necessary and sufficient condition which determines $x_i(p)$ is given by the first-order condition (FOC):

$$E_i[u'(w_0 + x_i(p)(\widetilde{\theta} - p))(\widetilde{\theta} - p)] = 0. \tag{4}$$

Note that when the price equals the expected payoff under investor i's belief, $p = E_i[\widetilde{\theta}]$, zero demand ($x_i = 0$) solves the FOC. We write $\mu_i = \mathrm{E}_i[\widetilde{\theta}]$, $\sigma_i^2 = \mathrm{Var}_i[\widetilde{\theta}]$, and $s_i = \mathrm{E}_i[(\widetilde{\theta} - \mu)^3]/\sigma^3$ denote its mean, variance, and skewness, respectively.

It follows that $x_i(\mu_i) = 0$, given that the strict concavity of u guarantees a unique solution of the FOC. We now analyze the properties of optimal demand when p is close to μ_i via a Taylor expansion of $x_i(p)$ at the point $p = \mu_i$. Doing so, we obtain the following lemma:

Lemma 1. *Investor i's demand schedule is given by*

$$x_i(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_i^2}(p - \mu_i) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s_i}{\sigma_i^3}(p - \mu_i)^2 + o(1)(p - \mu_i)^2, \tag{5}$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $p \to \mu_i$.

The slope of the demand function at $p = \mu_i$ is given by $x_i'(\mu_i) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_i^2}$, which is negative because $u'' < 0$. Consider a special case when the utility function is given by a CARA utility

[13] In Section 2.3.4, we discuss the case of non-zero aggregate supply, from which we reach similar conclusions.

function and the payoff of the underlying risky asset $\widetilde{\theta}$ is normally distributed. In this case, skewness is zero, and the quadratic term in equation (5) is zero, consistent with the well-known result that the demand function is linear under CARA and normality.

The quadratic term in equation (5) captures the curvature of the demand function at $p = \mu_i$. It is clear that the sign of $x_i''(\mu_i)$ solely depends on s_i, since the utility function has a negative second derivative and a positive third derivative. Specifically, if $s_i > 0$, then $x_i''(\mu_i) > 0$; if $s_i < 0$, then $x_i''(\mu_i) < 0$. Lemma 1 states that a NARA investor's demand of a positively (negatively) skewed risky asset is strictly convex (concave) at $p = \mu_i$. Figure 3 offers a graphical intuition for the result. Recall that a NARA investor has a preference for positive skewness. This fact implies that when we increase the skewness of the underlying asset, holding everything else equal, the investor would demand weakly more shares of the risky asset, i.e., the demand function is weakly higher, consistent with the observation that the solid curve is above the dashed line in Figure 3.[14] Furthermore, demand has to be zero when $p = \mu_i$ because of risk aversion. As a result, the "curvature" of the demand function of a positively skewed asset at $p = \mu_i$ has to be greater than the "curvature" of a straight line. Since the curvature of a straight line is 0, the curvature of the solid curve at $p = \mu_i$ has to be positive.

2.2.2 Equilibrium Price and Expected Return

Let $E_+[\cdot]$ and $E_-[\cdot]$ denote the expectation operator under positive and negative type beliefs, respectively. From the horizontal shift assumption, we obtain immediately $\mu_+ = \mathrm{E}_+[\widetilde{\theta}] = \mu + \sigma\delta$ and, likewise, $\mu_- = \mu - \sigma\delta$. Accordingly, the two types of investors have mean payoff beliefs corresponding to $\mu \pm \sigma\delta$, where $(+)$ is for the positive type of investors and $(-)$ is for the negative type. Since a horizontal shift in the CDF doesn't affect centered moments, we also have $\sigma_+ = \sigma_- = \sigma$ and $s_+ = s_- = s$; investors agree on the variance and skewness of the payoff distribution.

Using Lemma 1 combined with agreement about centered moments, the demand sched-

[14]We cautiously note that it is generally impossible to increase skewness without affecting other moments. However, as we show in the proof of Lemma 1, the graphical illustration is correct in the neighborhood of $p = \mu_i$, as the curvature of the demand function at $p = \mu_i$ does not depend on moments higher than the third.

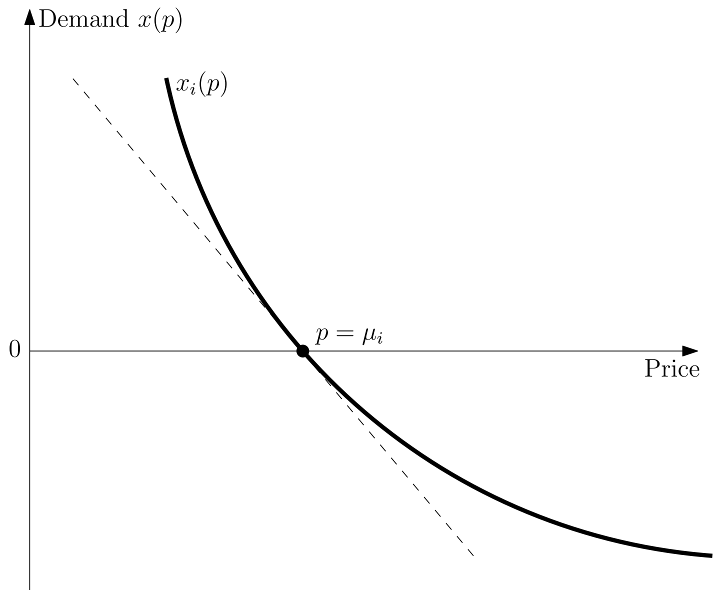

Figure 3: The Demand Schedule of a Positively Skewed Asset

Note: This figure illustrates the shape of the demand schedule. The dashed line is linear demand, which obtains under CARA and normality. The solid curve is the demand schedule when the underlying risky asset is positively skewed. The solid curve and the dashed line are tangent at the point $(p,x) = (\mu_i, 0)$. Since the solid curve has to be weakly higher than the dashed line, the curvature of the solid curve at $p = \mu_i$ has to be positive, consistent with Lemma 1 that the demand function of a positively skewed asset is convex at $p = \mu_i$.

ules of positive and negative-type investors are given by

$$x_+(p) = \frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu_+) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}(p-\mu_+)^2 + o(1)(p-\mu_+)^2, \tag{6}$$

$$x_-(p) = \frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu_-) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}(p-\mu_-)^2 + o(1)(p-\mu_-)^2, \tag{7}$$

respectively, where the little-o notation $o(1)$ is an unknown function that converges to 0 as $p \to \mu_i$.

We use the market-clearing condition to solve for the equilibrium price. Setting the aggregate demand to be zero, price solves the following equation.

$$x_+(p) + x_-(p) = 0. \tag{8}$$

Proposition 1. *There exists a $\overline{\delta} > 0$ such that if $\delta < \overline{\delta}$, then the equilibrium price is given by* *the following equation:*

$$p = \mu + \overline{u}s\sigma\delta^2 + o(1)\delta^2, \tag{9}$$

where

$$\overline{u} = \frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} \tag{10}$$

is a positive constant and where the little-o notation $o(1)$ *is an unknown function that converges to 0 as* $\delta \to 0$.

Bounded disagreement ensures that demand convexity does not change signs between μ_- and μ_+. From zero net supply, the equilibrium price must lie in this interval. The quantity $\overline{u}$ is always positive because the NARA assumption implies $u''' > 0$. Suppose the risky asset payoff is positively skewed. Proposition 1 states that the equilibrium price will be higher than μ, which is the price when there is no disagreement, i.e., $\delta = 0$, or when the marginal investor has the average belief. Proposition 1 implies that a higher level of disagreement biases the equilibrium price upward. Moreover, Proposition 1 provides a functional form for the deviation of the equilibrium price from μ.

Figure 2 provides the intuition. Suppose the underlying risky asset is positively skewed so that an investor's demand is convex in the neighborhood of the asset's expected payoff (Lemma 1). The two curves represent the demand schedules of a positive-type investor and a negative-type investor, respectively. For example, the demand of a positive-type investor crosses zero at price $\mu + \sigma\delta$, which is the risky asset's expected payoff under a positive type's belief. Now suppose the price is equal to μ. The dotted blue line represents the shares that a positive-type investor would buy, while the dashed red line represents the shares that a negative-type investor would sell. Because of convexity, the length of the blue line is greater than that of the red line. Put differently, since buying a positively skewed asset entails more desirable upside risk whereas shorting the asset involves more downside risk, the positive type would buy more shares of the asset than those shorted by the negative type. So, at a candidate equilibrium price μ, we have excess demand for the asset. To clear the market, the equilibrium price must be higher than μ.

Let $\widetilde{r} = (\widetilde{\theta} - p)/p$ denote the net return, $\sigma_r^2 = \mathrm{Var}[\widetilde{r}]$ denote the return variance, and

$\text{Sharpe}[\tilde{r}] = \text{E}[\tilde{r}]/\sigma_r$ denote the Sharpe ratio.[15] Note that $\tilde{\theta}$ is denominated in units of currency, $\tilde{r}$ is unitless, and $\sigma = \sigma_r p$. From Proposition 1, we obtain the following expressions for the dominant terms of the equilibrium expected return and Sharpe ratio.

Corollary 2. *Using the second-order Taylor polynomial for equilibrium price in (9), the corresponding expected return and Sharpe ratio can be expressed as follows:*

$$\text{E}[\tilde{r}] \approx -\overline{u} s \sigma_r \delta^2, \tag{11}$$

$$\text{Sharpe}[\tilde{r}] \approx -\overline{u} s \delta^2. \tag{12}$$

Because $\overline{u} > 0$, for ex-ante positively skewed assets, Corollary 2 indicates that the expected return and Sharpe ratio decrease in the level of disagreement δ. This negative predictive relationship between disagreement and expected returns of individual stocks is consistent with Diether et al. (2002) and a large literature in empirical asset pricing. We arrive at our prediction in a novel way, relying on a frictionless model that accounts for skewness-induced demand convexity, which governs the relationship between investor disagreement and expected returns. For assets with negatively skewed payoffs, Corollary 2 predicts a positive relationship, consistent with evidence from fixed-income markets. Unlike our framework, neither the short-sales frictions mechanism of Miller (1977) nor the limited risk-sharing mechanism of Abel (1989) can explain the empirical evidence across asset classes.

Note that because of the simplifying standard assumption of zero supply, expected returns (11) are zero when there is no skewness. More generally, if net supply were positive, then a positive risk premium would be part of the expected return to induce offsetting investor demand in equilibrium. Therefore, we can interpret our results as the isolated effect of disagreement on expected returns net of the risk premium.[16]

[15] We assume μ is finite and $\sigma_r > 0$ to avoid a trivial solution. Note that net return is well defined if the equilibrium price is positive, which holds, for example, for stocks whose payoffs cannot be negative because of limited liability. Note also that excess return and return are equal because the risk-free asset is normalized to have zero return.

[16] In Section 2.3.4, we show the interactive effect of skewness and disagreement still obtains in the case of non-zero aggregate supply.

2.2.3 Comparative Statics

Corollary 2 allows us to further analyze how various parameters affect the expected return and Sharpe ratio predictions.

Proposition 3. *The expected return and Sharpe ratio expressions of Corollary 2 satisfy the following properties.*

1. *(The skewness effect): Holding disagreement fixed, expected return and Sharpe ratio are decreasing in skewness. When disagreement is zero, the skewness effect should be negligible.*

2. *(The disagreement effect): Holding skewness fixed, expected return and Sharpe ratio are decreasing in disagreement if and only if skewness is positive. If skewness is zero, the disagreement effect is negligible.*

3. *(The interaction effect): Higher skewness amplifies the disagreement effect on expected return and Sharpe ratio and higher disagreement amplifies the skewness effect on expected return and Sharpe ratio.*

Proposition 3 offers empirical predictions that we test in Section 4. The proposition offers predictions on two well-known anomalies: (1) the negative relationship between ex-ante return skewness and expected equity returns and (2) the negative relationship between disagreement and expected equity returns. Moreover, to the best of our knowledge, the interaction effect from our theory is novel to the theoretical asset pricing literature.

Furthermore, our model predicts that the analogous three effects also hold for the Sharpe ratio and, therefore, the effects are invariant to the level of ex-ante return volatility. These Sharpe ratio results affirm and expand on the argument in the literature that the disagreement effect is not a repackaging of uncertainty effects (Diether et al., 2002).

2.3 Extension

We relax simplifying assumptions of the non-parametric analysis and show that our main results are robust to several extensions.

2.3.1 Disagreement with Arbitrary Types

The restriction to two types of investors in Section 2.1 is not an essential assumption. In this subsection, we extend our results to an arbitrary number of types.[17] We assume investor i believes that the date-1 payoff is drawn from CDF F_{δ_i}. The unitless bias parameter δ_i is drawn from a bounded mean-zero random variable $\widetilde{\delta}$, whose variance is denoted as $\mathrm{Var}(\widetilde{\delta})$.[18] In particular, $\widetilde{\delta}$ does not have to be symmetric. In this extended setup, $\mathrm{Var}(\widetilde{\delta})$ captures relative disagreement across investors. Note that the baseline model is a special case in which the random variable $\widetilde{\delta}$ is binary, i.e., only takes two possible values ($\pm\delta$), and relative disagreement reduces to $\mathrm{Var}(\widetilde{\delta}) = \delta$. Proposition 4 summarizes the result.

Proposition 4. *There exists a $\overline{\delta} > 0$ such that if $Var(\widetilde{\delta})$ is bounded by $\overline{\delta}$, then the equilibrium price is given by the following equation.*

$$p = \mu + \overline{u} s \sigma Var(\widetilde{\delta}) + o(1) Var(\widetilde{\delta}), \tag{13}$$

where $\overline{u}$ is a positive constant defined in (10) and the little-o notation $o(1)$ is an unknown function that converges to 0 as $Var(\widetilde{\delta}) \to 0$.

From Proposition 4 and the identity $\sigma = \sigma_r p$, we obtain the following expressions for the dominant terms of the equilibrium expected return and Sharpe ratio, consistent with the implications of the main model.

Corollary 5. *Using the second-order Taylor polynomial for equilibrium price in (13), the corresponding expected return and Sharpe ratio can be expressed as follows:*

$$\mathrm{E}[\widetilde{r}] \approx -\overline{u} s \sigma_r Var(\widetilde{\delta}), \tag{14}$$

$$\mathrm{Sharpe}[\widetilde{r}] \approx -\overline{u} s Var(\widetilde{\delta}). \tag{15}$$

[17] For technical convenience, we restrict attention to a finite number of types. It is straightforward to extend to countable or even uncountable types at the cost of cumbersome notation.

[18] Because of the boundedness assumption, the variance is finite.

2.3.2 Heterogeneous Utility Functions

So far, our analysis assumes homogeneous utility functions. In this subsection, we show that this assumption does not drive our results. To simplify the analysis, we assume each investor has one of the two utility functions: $u_1(\cdot)$ and $u_2(\cdot)$, both in the NARA class. We return to our baseline model and assume that each investor can be either a positive "optimist" type or a negative "pessimist" type.

First, we consider the case that each investor can be one of the following four types with equal probability: u_1 and optimist; u_2 and optimist; u_1 and pessimist; u_2 and pessimist. Put differently, utility function being u_1 or u_2 and belief being positive or negative are independent. In this case, it is clear that the equilibrium price is higher than the expected payoff under the average belief, μ. This is because at the price μ, there is excess demand among the two belief types with the same utility function: u_k and pessimist, and u_k and optimist, for $k = 1,2$. Summing over k still gives excess aggregate demand. In all, the equilibrium price has to be higher than μ to clear the market.

Second, we consider the case in which each investor can be one of the following two types with equal probability: u_1 and optimist; u_2 and pessimist. In other words, investors with positive belief necessarily have the utility function u_1, while investors with negative belief necessarily have u_2. We define the benchmark price as the price that obtains when investors have mean-variance preferences, in which case demand schedules are linear. We denote the benchmark price as p_0, which is given by the following equation

$$p_0 = \frac{\frac{u_1'(w_0)}{-u_1''(w_0)}\mu_+ + \frac{u_2'(w_0)}{-u_2''(w_0)}\mu_-}{\frac{u_1'(w_0)}{-u_1''(w_0)} + \frac{u_2'(w_0)}{-u_2''(w_0)}}. \tag{16}$$

That is, the benchmark price is the weighted average of the expected payoff under each type's belief, where the weight is given by the reciprocal of each type's absolute risk aversion. Intuitively, a less risk-averse investor, trading more aggressively, gets a larger weight in determining the benchmark price.

We are now ready to present the result.

Proposition 6. *There exists a $\overline{\delta} > 0$ such that if $\delta < \overline{\delta}$, then the equilibrium price is given by* *the following equation:*

$$p = p_0 + \overline{u}_1 s\sigma\delta^2 + o(1)\delta^2,$$

where $\overline{u}_1 > 0$ is a constant and is given in the proof. In particular, the price is greater than the benchmark price p_0 if and only if $s > 0$.

We make a final remark. Our baseline model assumes that all investors have the same initial wealth w_0. If investors have different initial wealth, effectively, investors have different risk aversion and prudence. As we can see from the analysis in this subsection, Proposition 6 still holds. Hence, the same initial wealth assumption is not crucial.

2.3.3 Disagreement On Other Moments

Thus far, we have assumed that investors disagree about the mean payoff but agree on the variance and skewness. In this subsection, we discuss the possibility of disagreement on other moments. For the ease of exposition, we assume there are two types of investors, type A and type B. The probability of each type is one-half. The two types of investors can disagree about the distribution of the time-1 payoff, but have common utility function $u(w)$. We assume that a type j investor believes that the expected payoff is μ_j, the standard deviation is σ_j, and the skewness is s_j, for $j = A, B$.

First, we note that if $\mu_A = \mu_B = \mu$, then the equilibrium price is also $p = \mu$. This is because each type investor demands zero shares of the asset so the market is cleared at $p = \mu$, given our zero aggregate supply assumption. Therefore we assume $\mu_A \neq \mu_B$ so that disagreement "matters."

Now, we can write each type of investors' demand schedule in a neighborhood around μ_j as the following.

$$x_A(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_A^2}(p - \mu_A) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s_A}{\sigma_A^3}(p - \mu_A)^2 + o(1)(p - \mu_A)^2, \tag{17}$$

$$x_B(p) = \frac{u'(w_0)}{u''(w_0)} \frac{1}{\sigma_B^2}(p - \mu_B) - \frac{1}{2} \frac{u'''(w_0)}{u''(w_0)} \left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s_B}{\sigma_B^3}(p - \mu_B)^2 + o(1)(p - \mu_B)^2, \tag{18}$$

where $x_j(p)$ denotes type j's demand, for $j = A, B$.

Similar to Section 2.3.2, we define the benchmark price as the price that obtains when investors have mean-variance preferences, in which case the demand schedules are linear. Given zero net supply, the benchmark price is given by the following expression

$$p_0 = \frac{\frac{\mu_A}{\sigma_A^2} + \frac{\mu_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}}. \tag{19}$$

Intuitively, the benchmark price is the weighted average of the expected payoff under each type's belief, where the weight is given by the reciprocal of each type of investor's belief about the variance of the payoff. This intuition holds because an investor who believes the variance is smaller would trade more aggressively, implying a higher weight in determining the benchmark price. We are now ready to present the result.

Proposition 7. *There exists a $\bar{\epsilon} > 0$ such that if $|\mu_A - \mu_B| \le \bar{\epsilon}$, the equilibrium price is given by*

$$p = p_0 + \bar{u}\frac{(\mu_A - \mu_B)^2}{\left(\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}\right)^3}\frac{\sigma_A s_A + \sigma_B s_B}{\sigma_A^4 \sigma_B^4} + o(1)(\mu_A - \mu_B)^2,$$

where $\bar{u} = \frac{1}{2}\frac{u'''(w_0)}{-u''(w_0)}\left(\frac{u'(w_0)}{-u''(w_0)}\right)$ is a positive constant. In particular, the equilibrium price is greater than the benchmark price p_0 if and only if $\sigma_A s_A + \sigma_B s_B > 0$.

When $\sigma_A s_A + \sigma_B s_B = 0$, it is ambiguous whether the equilibrium price is higher or lower than the benchmark price. Proposition 7 shows that the volatility-weighted average subjective skewness, $\bar{s} := (\sigma_A s_A + \sigma_B s_B)/(\sigma_A + \sigma_B)$, is a “sufficient statistic" that plays the role of objective skewness in Proposition 1. Furthermore, suppose we parameterize $\mu_A - \mu_B = t \times \delta$, $\sigma_A = \sigma_{A0} t$, and $\sigma_B = \sigma_{B0} t$ so that t plays the role of standard deviation in the benchmark model. Then, the dominant term of $p - p_0$ from Proposition 7 is proportional to $t \times \bar{s} \times \delta^2$, which is consistent with the findings in the benchmark model.

2.3.4 Non-Zero Aggregate Supply

So far our analysis assumes zero aggregate supply. In this subsection, we discuss the case when aggregate supply is not zero. Denote the aggregate supply as χ. To be clear, we return to the baseline model and only change the aggregate supply from zero to χ. In particular, we still assume investors have the same utility function and there are positive-type and negative-type investors.

Similar to Sections 2.3.2 and 2.3.3, the benchmark price is defined to be the equilibrium price where we used the mean-variance preference, which implies a linear demand function. Define p_+ and p_- such that $x_+(p_+) = x_-(p_-) = \frac{\chi}{2}$, and the benchmark price clears the market when we use the linear approximation of demand schedules:

$$x_+(p_+) + x'_+(p_+)(p_0 - p_+) + x_-(p_-) + x'_-(p_-)(p_0 - p_-) = \chi. \tag{20}$$

That is, the benchmark price p_0 is given by

$$p_0 = \frac{x'_+(p_+)p_+ + x'_-(p_-)p_-}{x'_+(p_+) + x'_-(p_-)}. \tag{21}$$

As the level of disagreement approaches zero, p_+ and p_- converge to p_1, which is defined as the solution of

$$x_0(p_1) = \frac{\chi}{2},$$

where $x_0(\cdot)$ denotes the demand function when $\delta \to 0$. We have the following proposition when the aggregate supply is non-zero.

Proposition 8. *Suppose the risky asset's payoff, $\widetilde{\theta}$, is bounded under both types of investor beliefs, and there is non-zero aggregate supply χ. Let p_0 denote the benchmark price as defined in (21). Then, there exists thresholds $\overline{\delta} > 0$, such that if $\delta < \overline{\delta}$, the equilibrium price is given by the following equation:*

$$p = p_0 + \overline{u}_2 \frac{x''_0(p_1)}{-x'_0(p_1)} \delta^2 + o(1)\delta^2, \tag{22}$$

where $\overline{u}_2 > 0$ is a constant. Moreover, there exist $\overline{s} > \underline{s}$, such that the following properties hold.

- *The term $\frac{x''_0(p_1)}{-x'_0(p_1)}$ is monotonically increasing in s for $s \in (-\infty, \underline{s}) \cup (\overline{s}, +\infty)$.*
- *If $s > \overline{s}$, the equilibrium price is greater than p_0.*
- *If $s < \underline{s}$, the equilibrium price is smaller than p_0.*

From Proposition 8, it follows that our results are robust to introducing non-zero supply. Either higher skewness or higher δ leads to a higher price, which in turn implies a lower return, and there is an interaction effect between them. We remark that Proposition 8 covers the case $s \in (-\infty, \underline{s}) \cup (\overline{s}, +\infty)$. This is because the term $\frac{x_0''(p_1)}{-x_0'(p_1)}$ is a cubic function of skewness, which is monotonic for extreme values.

3 A Parametric Model: CARA with Skew-Normal Payoffs

In parallel to the non-parametric analysis in Section 2, we now show that the same interaction mechanism arises in a parametric version of the model. The economic environment is identical: a two-date market with a risk-free asset, a single risky asset, and two investor types (optimists and pessimists) who disagree only about the mean of the risky payoff but agree on its higher-order moments. The only differences are that we now impose CARA utility and a Skew-Normal family for the perceived payoff distribution, which allows us to characterize individual demand without relying on Taylor expansions.

For each investor type $i \in \{+,-\}$, the perceived date-1 payoff $\tilde{\theta}_i$ is Skew-Normal:

$$\tilde{\theta}_i \sim SN(\xi_i, \omega^2, \tau),$$

with common scale parameter $\omega > 0$ and common shape parameter τ. We define $\lambda = \frac{\tau}{\sqrt{1+\tau^2}} \in (-1,1)$, and λ and τ have the same sign. The location parameters ξ_i capture disagreement about the mean while keeping the shape of the distribution fixed, in the same spirit as the horizontal shifts in Section 2.1. For $d > 0$, we write

$$\xi_+ = \xi + d, \qquad \xi_- = \xi - d,$$

so $2d$ is the difference in subjective means.[19] In our parametric setup, λ *parameterizes payoff skewness*, and disagreement is captured by the parameter d.

Investors have CARA utility

$$u(w) = -\exp(-\gamma w), \qquad \gamma > 0.$$

Let $x_i(p)$ denote the number of shares of the risky asset chosen by type i at price p. Given

[19]Recall the mean of a Skew-Normal distribution $SN(\xi, \omega^2, \tau)$ is given by $\xi + \omega\lambda\sqrt{\frac{2}{\pi}}$.

belief (ξ_i, ω, λ), investor i's problem is

$$\max_x \mathbb{E}_i\left[-e^{-\gamma\left(w_0 + x(\tilde{\theta}_i - p)\right)}\right].$$

The following lemma characterizes the shape of $x_i(p)$.

Lemma 2. *For a CARA investor facing a Skew-Normal distributed risky payoff, $SN(\xi_i, \omega^2, \tau)$, the demand function $x_i(p)$ is convex if and only if $\lambda > 0$, and concave if and only if $\lambda < 0$.*

Lemma 2 states that the convexity or concavnity of the demand schedule solely depends on the shape parameter, λ, of the Skew-Normal distribution. We remark that Lemma 1 characterizes the shape of the demand schedule locally, while Lemma 2 shows a global result with CARA and Skew-Normal distribution.

Finally, price is determined by the market's clearing condition, which states that $x_+(p) + x_-(p) = \chi$. The effect of disagreement on the equilibrium price depends on whether the demand schedule is convex or concave, which in turn is determined by the sign of the Skew-Normal shape parameter. The following proposition summarizes this result.

Proposition 9. *$\partial p / \partial d > 0$ when $\lambda > 0$, and $\partial p / \partial d < 0$ when $\lambda < 0$. For $\lambda = 0$, $\partial p / \partial d = 0$.*

Consistent with Proposition 1, Proposition 9 shows that disagreement can affect equilibrium price through non-linear demand schedule. It states that when the risky asset has a positive (negative) skewed payoff, then the equilibrium price is an increasing (decreasing) function of disagreement d. When $\lambda = 0$, the Skew-Normal distribution reduces to a Gaussian distribution. In this case, demand schedules are linear and the equilibrium price does not depend on the level of disagreement.

Proposition 10. *The equilibrium price p satisfies*

$$\frac{\partial^2 p}{\partial d\, \partial \lambda} > 0.$$

unless $\lambda = 0$, in which case, the cross-partial derivative is also zero.

Consistent with Proposition 3, there is an interaction mechanism in the parametric setting: higher skewness (larger λ) amplifies the effect of disagreement (d) on the equilibrium price, and conversely, higher disagreement magnifies the impact of skewness.

3.1 Multiple Risky Assets

We now extend the model to multiple risky assets with a potential factor structure. There are N risky assets in supply χ with date-1 payoff

$$\tilde{\theta}_n = \beta_n^{'} F + \epsilon_n, \tag{23}$$

where F is a vector of common factors and $\epsilon_n \sim SN(\xi_n, \omega^2, \tau_n)$ are jointly independent. We define $\lambda_n = \frac{\tau_n}{\sqrt{1+\tau_n^2}} \in (-1,1)$ so it parameterizes ϵ_n's skewness. Independence of the idiosyncratic terms is a simplification in the spirit of the APT (Ross, 1976) and also used by Barberis and Huang (2008). For each asset n, half the investors are optimists endowed with belief $\xi_{n+} = \xi_n + d_n$ and half are pessimists with belief $\xi_{n-} = \xi_n - d_n$. We make no assumption on the cross-sectional distribution of d_n except that it has finite variance. Consistent with our two-way independent sorting methodology in Section 4, we assume d_n is cross-sectionally independent of λ_n.[20] We make no assumption about the common factors F except that they have a finite covariance matrix and can be traded via ETFs.

Because the asset and factor payoffs have finite second moments, in any equilibrium (with finite prices) the minimum-variance SDF of Hansen and Jagannathan (1991) exists; prices exhibit no-arbitrage and the law of one price (LOOP). Therefore, we can define synthetic "idiosyncratic" assets with payoffs ϵ_n. By LOOP, the actual price of the security will be $p_n = \beta_n^{'} p_F + P(\epsilon_n)$, where $P(\cdot)$ is the pricing operator and p_F is the vector of factor ETF prices.

Now recall that the ϵs are jointly independent and also independent of the factor payoffs. As is well-known for CARA utility, independence of payoffs implies that portfolio decisions are separable: investors separately determine their demand for the factors, and for each of the idiosyncratic payoffs. Therefore, demand for and pricing of each ϵ_n simply follows from Lemma 2 and Propositions 9 and 10. Recall, they imply $P(\epsilon_n) = E(\epsilon_n) + g(\lambda_n, d_n)$ where $g(\cdot,\cdot)$ has the following properties: (1) $-sign\left(\frac{\partial g}{\partial d}\right) = sign(\lambda)$ and is 0 if $\lambda = 0$ and (2) $\frac{\partial^2 g}{\partial \lambda \partial d} > 0$. We define the dollar alpha of asset n as $\alpha_n = E(\epsilon_n) - P(\epsilon_n)$. Because d_n is cross-sectionally

[20] This assumption is not strictly necessary, but simplifies the presentation.

independent of λ_n, the positive cross-partial derivative for price will manifest as a negative interaction effect in the cross-section of expected returns, α.[21]

4 Empirical Tests

In this section, we test the prediction that skewness and disagreement have an interactive pricing effect, summarized in Section 2.2.3, on a sample of U.S. stocks.

4.1 Data

Our sample is formed as the intersection of the Center for Research in Security Prices (CRSP) and Institutional Brokers Estimate System (IBES) universes. From CRSP we obtain daily stock returns for universe of U.S. firms (adjusted for delisting bias)[22], prices, volume, and shares outstanding. We drop firm-month observations with missing prior-month market capitalization (price×shares outstanding).

Analysts' forecasts data are taken from the Unadjusted U.S. Statistics Summary History dataset of IBES. Our primary proxy for forecast dispersion, DISP, is the same as that used by Diether et al. (2002) and many others: the month-end standard deviation of current-fiscal-year earnings per share (EPS) estimates scaled by the absolute value of the mean forecast across analysts tracked by IBES, from December 1983 to December 2023:

$$\mathrm{DISP} := \frac{\mathrm{stdF}_t}{|\mathrm{meanF}_t|}, \tag{24}$$

where stdF_t is the month-end standard deviation of next-fiscal-year earnings estimates across analysts tracked by IBES, and meanF_t is the average of those forecasts.[23] We define the next fiscal year as the closest upcoming within 170 to 550 days of the IBES statistical period (date of aggregation). We exclude firm-months where $|\mathrm{meanF}_t| < 0.1$, lagged price one week prior is below \$5/share, or where the price/earnings ratio, $P_t/|\mathrm{meanF}_t| < 1$.[24] In

[21] Note that even if d_n and λ_n are correlated, the interaction effect will obtain when performing an independent sort.

[22] This adjustment is suggested by Shumway (1997). Our results are not sensitive to this adjustment. We restrict to common stock (share codes 10 or 11).

[23] In Section 4.4, we show our results are robust to alternative proxies used in the literature such as scaling by lagged price or absolute median forecast instead of $|\mathrm{meanF}_t|$.

[24] These filters remove microcap stocks and observations that are clearly data errors.

order to compute stdF_t, there must be at least two analyst forecasts, which tilts our sample towards larger firms compared to the CRSP universe. Note that DISP is a unitless measure that corresponds to our modeled unitless relative disagreement δ in Section 2.

Our primary proxy for skewness, SKEW, is the monthly expected idiosyncratic skewness measure for each stock as in Boyer et al. (2010), provided by the authors from July 1969 to December 2023. Using total skewness gives essentially identical results. Expected (or ex-ante) skewness is difficult to measure. As opposed to means, variances and covariances, skewness is not stable over time. Moreover, lagged skewness alone does not adequately forecast skewness (Harvey and Siddique, 1999; Boyer et al., 2010). Instead, Boyer et al. (2010) (hereafter BMV) use firm-level variables to predict skewness (following the approach of Chen et al., 2001). Specifically, BMV develop measures of skewness each month that predict skewness of the return distribution over the next 60 months, based on firm characteristics in the prior 60 months, including lagged skewness, idiosyncratic volatility, momentum, turnover, size, exchange, and industry. BMV point out that although other variables, such as accounting variables, could be useful in predicting skewness, limiting variables to this collection allows the measure to be computed for every stock in CRSP with available history. As a result, using BMV's proxies for skewness maintains a large sample. Moreover, BMV demonstrate that their measures of skewness exhibit a negative cross-sectional relationship with expected returns—the skewness effect. This measure of skewness implicitly requires at least 250 days of daily returns in the prior 60-month period. In our sample, the 5th and 95th percentiles of expected idiosyncratic skewness are 0.03 and 1.62 , respectively; it is typically positive. Note that SKEW is a measure of return skewness that corresponds to our modeled return skewness s in Section 2.

After merging and applying filters, our sample consists of 918,572 firm-months. There are 10,548 unique firms, for an average of 1,667 firms per month. Our data sample is comparable to those used in prior studies of the forecast dispersion effect, with the main innovation being the inclusion of ex-ante skewness. Finally, we obtain monthly "factor" returns for the Fama and French (2015) model, henceforth FF5, from WRDS (Wharton Research Data Service) and for the Daniel et al. (2020) model, henceforth DMRS, from Kent Daniel's website.[25]

[25] We thank Kent for making this data available at http://www.kentdaniel.net/data.php.

The DMRS data ends in March 2023. Therefore, our sample covers all months in the period from December 1983 through March 2023.

4.2 Expected Return Predictions and Double Sorts

Our primary test of the model implications is via an independent double-sort. To eliminate potential confounding effects of firm size (and other characteristics correlated with size), we first sort observations each month into four quartiles based on prior month market capitalization. Within each quartile, we then perform an independent two-way sort on SKEW and DISP. Finally, we collapse the size dimension, yielding 16 portfolios. First, we show that our independent double-sort conforms to desirable properties. Each month, we compute the median of SKEW and DISP across stocks in each portfolio. We then average these values across months. We report the resulting values for SKEW and DISP in Tables 1 and 2, respectively.

A "test" of a good independent sort is cross-sectional variation in each characteristic that is approximately orthogonal with the other: for a given quartile of SKEW, SKEW itself should be approximately constant across quartiles of DISP, which obtains in Table 1; and, similarly, for a given quartile of DISP, DISP itself should be approximately constant across quartiles of SKEW, which obtains in Table 2. Moreover, our sorting procedure produces large spreads in SKEW and DISP, which are approximately orthogonal to firm size. Furthermore, Table 1 indicates that the average level of SKEW is positive in all 16 portfolios, which supports our focus in Section 2 on results pertaining to the case of positive skewness, $s > 0$. Also, Table 2 indicates that the average level of DISP is well below one in all 16 portfolios, reinforcing consistency of DISP with δ's interpretation in our model.[26]

Next, we adapt predictions from Corollary 2 (and the more general analogous predictions from Corollary 5) to form the following hypotheses:

H1: When DISP ≈ 0, the effect of SKEW on expected returns should be negligible.

H2: The effect of SKEW on expected returns should be increasing (in magnitude) in DISP.

H3: When SKEW ≈ 0, the effect of DISP on expected returns should be negligible.

[26] For example, under the baseline model of Section 2, the variance of mean return beliefs is $\sigma_r^2\delta^2$. It is reasonable to expect that mean beliefs are less variable than returns themselves so that $\sigma_r^2\delta^2 < \sigma_r^2$, which would imply $\delta < 1$.

Table 1: Average SKEW by Portfolio

	LowSkew	2	3	HighSkew
LowDisp	0.42	0.67	0.87	1.15
2	0.42	0.67	0.88	1.14
3	0.41	0.67	0.89	1.18
HighDisp	0.40	0.67	0.90	1.22

Note: This table reports the sample average of median monthly SKEW for stocks independently double-sorted into DISP and SKEW quartiles, where double sorting is performed within size quartiles and aggregated. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

Table 2: Average DISP by Portfolio

	LowSkew	2	3	HighSkew
LowDisp	0.02	0.02	0.02	0.02
2	0.04	0.04	0.04	0.04
3	0.08	0.08	0.08	0.08
HighDisp	0.19	0.19	0.20	0.21

Note: This table reports the sample average of median monthly DISP for stocks independently double-sorted into DISP and SKEW quartiles, where double sorting is performed within size quartiles and then aggregated. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

H4: The effect of DISP on expected returns should be increasing (in magnitude) in SKEW.

Table 3 reports annualized average excess returns by portfolio and high–low spreads, with absolute Newey and West (1987) t-statistics in parentheses. The top row shows that ignoring DISP, our sample displays a weak unconditional SKEW effect: monotonically lower expected returns for higher quartiles of skewness. The first column likewise shows the unconditional DISP effect: monotonically lower expected returns for higher DISP quartiles. Turning to the predictions of the model, we first look at DISP spreads (last row) and obtain the predicted pattern: the DISP spread has substantially larger magnitude for high SKEW assets (**H2**). Moreover, the spread in average returns is essentially zero for the lowest quartile of SKEW stocks (**H1**). Looking at SKEW spreads (last column) we observe a similar pattern: the skewness spread has larger magnitude for high dispersion assets (**H4**). Moreover, the spread in average returns is essentially zero for the lowest quartile of DISP stocks (**H3**).

Table 3: Average Annualized Excess Return by Portfolio

	All	LowSkew	2	3	HighSkew	H-L
All	-	11.4	11.5	9.9	9.3	-2.1
		(4.22)	(4.29)	(3.66)	(3.26)	(1.52)
LowDisp	12.2	11.4	13.0	11.6	12.8	1.4
	(5.20)	(4.85)	(5.35)	(4.87)	(5.15)	(1.15)
2	11.3	11.7	12.1	10.9	10.5	-1.1
	(4.45)	(4.55)	(4.68)	(4.14)	(3.84)	(0.88)
3	10.5	11.8	11.5	9.4	9.2	-2.6
	(3.77)	(4.13)	(4.00)	(3.37)	(3.15)	(1.78)
HighDisp	8.1	10.9	9.3	7.7	5.9	-5.0
	(2.51)	(3.25)	(2.92)	(2.30)	(1.72)	(2.39)
H-L	-4.1	-0.5	-3.7	-4.0	-6.9	-6.4
	(2.68)	(0.27)	(2.28)	(2.40)	(3.71)	(3.39)

Note: This table presents the (annualized) average monthly percent excess returns by portfolio. Portfolios are formed from an independent double-sort on DISP, analyst forecast dispersion (Diether et al., 2002), and SKEW, expected idiosyncratic skewness (Boyer et al., 2010), as in Tables 1 and 2. The first column shows results from a univariate sort on DISP while the first row is from a univariate sort on SKEW. The bottom-right element, -6.4%, is the interaction effect (S4D4+S1D1-S1D4-S4D1). Heteroskedasticity and autocorrelation consistent absolute (Newey and West, 1987) *t*-statistics are in parentheses.

Finally, the bottom-right corner is calculated as the return on the long-short portfolio (S4D4+S1D1-S1D4-S4D1). Essentially, it is the average main-diagonal return minus the average off-diagonal return. Equivalently, it is the High-Low DISP spread for HighSkew (-6.9%) minus the High-Low DISP spread for LowSkew (-0.5%). Therefore, it tests whether the dispersion effect depends on skewness. Furthermore, it is the High-Low SKEW spread for HighDisp (-5%) minus the High-Low SKEW spread for LowDisp (1.4%). Therefore, it tests whether the skewness effect spread depends on dispersion. Hence, it is simultaneously a test of **H2** and **H4**. As the model predicts, this return is negative, with economically and statistically significant magnitude: -6.4%, absolute *t*-statistic 3.39.

Our theory is about alphas, not simply excess returns. Patterns of factor loadings could possibly overturn the conclusions drawn from Table 3. However, this brings up the age-old "joint-hypothesis problem" of which factors to include? Assuming no arbitrage, there exists some factor model that "explains" all patterns in expected returns. Besides the CAPM, most common factor models are loosely (if at all) microfounded. Fama and French (2015) argue that their five factor model (FF5) is almost tautological given the present value relation and clean surplus accounting. Still, it is commonly used in current research and does “absorb” substantial cross-sectional variation in average returns. Additionally, Daniel et al. (2020) (DMRS) argue that the FF5 model can be improved by their procedure to “remove unpriced risk” from characteristic-sorted factors “using covariance information estimated from past returns.” Their procedure nearly doubles the Sharpe ratio of the resulting mean-variance efficient combination.

Using time-series regressions, we compute alphas relative to these three factor models: CAPM, FF5, DMRS. To save space, we present the alphas for three key portfolios corresponding to Table 3: in the bottom row of Table 4 we present the interaction effect, corresponding to the bottom-right element of Table 3, and, for comparison, we also include the unconditional DISP and SKEW effects in the first two rows, corresponding to the bottom-left and top-right elements of Table 3, respectively. Recall, the interaction effect is measured by the return on the long-short portfolio (S4D4+S1D1-S1D4-S4D1). Table 4 shows that the interaction effect is robust to adjustment for popular risk factor models, having essentially the same sample estimate and statistical strength. It seems, however, that the unconditional DISP and SKEW effects are less robust to such risk factor adjustments.

Table 4: DISP, SKEW, and Interaction Effects

	Excess	CAPM	FF5	DMRS
DISP H-L	-4.1	-6.9	-4.5	-2.4
	(2.68)	(5.58)	(4.27)	(1.56)
SKEW H-L	-2.1	-2.6	-1.6	-2.1
	(1.52)	(1.78)	(1.09)	(1.37)
Interaction	-6.4	-6.6	-5.5	-7.4
	(3.39)	(3.30)	(2.82)	(3.64)

Note: This table presents the (annualized) average monthly excess return and alphas under the CAPM, Fama and French (2015, FF5), and Daniel et al. (2020, DMRS) models for the DISP, SKEW, and interaction effects, corresponding to the bottom left, top right, and bottom right portfolios of Table 3, respectively. Heteroskedasticity and autocorrelation consistent absolute (Newey and West, 1987) t-statistics are in parentheses.

4.3 Sharpe Ratio Predictions and the Role of Volatility

Our theoretical results, as articulated in Corollary 2 and extended in Corollary 5, predict that the interaction of disagreement (DISP) and skewness (SKEW) influences both expected returns and Sharpe ratios, with the latter effect invariant with return volatility (σ_r). Specifically, Equation (12) states that the Sharpe ratio decreases with the product of skewness and the variance of beliefs, regardless of volatility levels. This volatility-invariant prediction is a novel implication of our frictionless framework, distinguishing the disagreement effect from uncertainty-based explanations and offering a testable hypothesis. In this section, we empirically test this Sharpe ratio prediction and address potential confounding effects of volatility that could obscure the sources of the interaction effect.

To conduct these analyses, we require an *ex-ante* volatility measure, $\sigma_{i,t}$, for each stock month, for which we use the ex-ante at-the-money option-implied volatility from OptionMetrics.[27] Although using option-implied volatility restricts our sample to stocks with traded options, starting from 1996, it avoids disadvantages associated with ex-post volatility measures while offering the advantage of partially mitigating short-selling frictions (as such stocks tend to have less trading frictions), which reinforces alignment with our frictionless framework. We use this measure to construct *scaled* returns as

$$\widetilde{r}_{i,t} = \frac{\overline{\sigma}_t \times r_{i,t}}{\sigma_{i,t}}, \tag{25}$$

where $\overline{\sigma}_t$ is the cross-sectional median of $\sigma_{i,t}$. The scaling factor of $\overline{\sigma}_t$ aligns the distribution of scaled returns with unscaled returns, facilitating comparison, and scaled returns provide a natural empirical counterpart to Sharpe ratios in the model.

Table 5 reports the interaction effect for Sharpe ratios, paralleling the tests reported in Table 4, but for scaled returns. Table 5 shows that the interaction effect, measured as the scaled return on the long-short portfolio (S4D4 + S1D1 - S1D4 - S4D1), is economically and statistically significant across factor models, and closely aligns with the results of Table 4 in both magnitude and statistical strength despite the shorter sample and restriction to stocks with traded options. These results support the model's prediction that the interaction of

[27] Implied monthly volatilities are winsorized at the 5th and 95th percentiles each month.

skewness and disagreement negatively predicts Sharpe ratios. The weaker unconditional DISP and SKEW effects underscore the interaction's distinct predictive power.

Table 5: DISP, SKEW, and Interaction Effects for Scaled Returns

	Excess	CAPM	FF5	DMRS
DISP H-L	-3.4	-4.4	-1.0	-0.4
	(1.99)	(2.64)	(0.77)	(0.32)
SKEW H-L	-4.0	-4.2	-2.9	-3.3
	(2.12)	(2.07)	(1.49)	(1.51)
Interaction	-7.6	-8.1	-7.4	-7.9
	(3.14)	(3.21)	(2.98)	(3.05)

Note: This table presents the (annualized) average monthly DISP, SKEW, and interaction effects for returns scaled by option-implied volatility, paralleling the results reported in Table 4 but for scaled returns. Heteroskedasticity and autocorrelation consistent absolute (Newey and West, 1987) *t*-statistics are in parentheses.

A key concern, however, is that volatility could drive our results through alternative mechanisms, such as the low-volatility anomaly (Falkenstein, 1994), arbitrage limits (Dong et al., 2022), or a statistical artifact in which DISP and SKEW affect Sharpe ratios via their correlations with volatility, creating an apparent interaction effect on expected returns that is not economically meaningful (a "mechanical interaction"). By assessing the robustness of the interaction effect on Sharpe ratios across various volatility controls, we offer sharper evidence that our findings reflect the model's proposed mechanism rather than these potential confounders.

To test the volatility-invariance prediction as well as assess the influence of the low-volatility anomaly, we split the sample each month at the median implied volatility, creating Low Vol and High Vol subsamples—volatility is approximately 50% higher in the High Vol subsample (Appendix B, Tables A1, A2, and A3). If the interaction effect is invariant to volatility, as predicted by Equation (12), it should remain consistent across high and low volatility regimes, ruling out volatility-driven confounders like the low-volatility anomaly.

The first three rows of Table 6 report the interaction effect for scaled returns, each paralleling the last row of Table 5 but applied to all firms and for each volatility subsample separately. In order to make comparisons meaningful, we use the same DISP and SKEW breakpoints even when restricting to subsamples, and, as before, we adjust for the CAPM, FF5, and DMRS factor models. The consistency evident across subsamples in Table 6 supports the volatility-invariance prediction, suggesting that the interaction effect is not driven by volatility differences or the low-volatility anomaly. The last row of Table 6 reports the difference between High and Low volatility samples. The remarkable takeaway is that for scaled returns (Sharpe ratios), the interaction effect is not only essentially the same across high and low volatility subsamples, but also further unaffected by controlling for factor exposures.

Table 6: Interaction Effect for Scaled Returns: By Implied Volatility

	Excess	CAPM	FF5	DMRS
All Firms	-7.6	-8.1	-7.4	-7.9
	(3.14)	(3.21)	(2.98)	(3.05)
Low Vol	-8.2	-8.3	-7.9	-7.5
	(3.23)	(3.07)	(2.86)	(2.76)
High Vol	-5.9	-6.3	-5.7	-6.7
	(2.25)	(2.36)	(2.10)	(2.55)
High - Low	2.3	2.0	2.2	0.7
	(0.79)	(0.68)	(0.75)	(0.29)

Note: The first three rows of this table report the (annualized) average monthly interaction effect for returns scaled by option-implied volatility, paralleling the results reported in the last row of Table 5, with the first row including all firms, as in Table 5, but the second and third rows, respectively, including stocks partitioned by below-median or above-median option-implied volatility. The last row tests the difference between the second and third rows of the table. Heteroskedasticity and autocorrelation consistent absolute (Newey and West, 1987) t-statistics are in parentheses.

Implied volatility exhibits cross-sectional correlation with DISP, however, both in the full sample as well as some residual correlation within the high and low volatility subsamples (Appendix B, Tables A1, A2, and A3).[28] Although when compared to the full sample there is substantially reduced volatility variation within each subsample, the residual DISP-volatility correlation in the volatility subsamples prompts further scrutiny. This correlation could indicate that DISP proxies for volatility-related factors, such as arbitrage limits or a mechanical interaction where DISP independently affects Sharpe ratios, and its correlation

[28] Implied volatility exhibits essentially zero correlation with SKEW in our samples.

with volatility creates an apparent interaction effect.

To address this concern, we adopt an alternative sorting procedure that controls for volatility. Each month, we sort stocks into four quartiles based on implied volatility, then perform an independent double-sort on DISP and SKEW within each quartile, and finally collapsing the volatility dimension to yield 16 portfolios. By ensuring volatility is nearly constant across portfolios (Appendix B, A4), we neutralize its influence, isolating the interaction effect and ruling out mechanical interactions or arbitrage-related confounders. Tables A5 and A6 (Appendix B) provide evidence that the SKEW and DISP distributions remain comparable to the full sample (Tables 1 and 2).

Table 7 reports the interaction effect for scaled returns under this alternative sorting, paralleling the tests of Table 5. The robust interaction effect, remarkably consistent in magnitude and strength with earlier results despite tightly controlling for volatility, supports the conclusion that the DISP-SKEW interaction is not a statistical artifact of volatility scaling or arbitrage limits. The diminished unconditional effects further highlight the interaction's predictive power, aligning with the model's prediction of skewness and disagreement as joint drivers.

Table 7: DISP, SKEW, and Interaction Effects for Scaled Returns: Alternative Sorting

	Excess	CAPM	FF5	DMRS
DISP H-L	-2.1	-2.7	-1.8	-1.2
	(1.92)	(2.60)	(1.92)	(1.06)
SKEW H-L	-1.9	-1.4	-1.5	-2.1
	(1.53)	(1.12)	(1.09)	(1.51)
Interaction	-7.5	-7.3	-7.2	-7.3
	(3.59)	(3.36)	(3.73)	(3.53)

Note: This table presents the (annualized) average monthly DISP, SKEW, and interaction effects for returns scaled by option-implied volatility, paralleling the results of Table 5, but here portfolios are formed using the alternative sorting procedure which controls for implied volatility. Heteroskedasticity and autocorrelation consistent absolute Newey and West (1987) t-statistics are in parentheses.

4.4 Robustness of the Mechanism

We conduct robustness checks to ensure that the interaction effect of DISP and SKEW reflects the frictionless mechanism of skewness-induced demand convexity under NARA utility, rather than alternative explanations. Potential concerns are that the empirical findings could be driven by asymmetric limits to arbitrage in the form of short-selling frictions (Miller, 1977), by investor optimism (Brunnermeier et al., 2007), or by measurement issues with DISP (Cen et al., 2017), which could mimic the interaction effect. By testing for these alternatives, we strengthen the paper's claim of providing a unified, preference-based explanation for the disagreement and skewness effects across asset classes.

It can be reasonably argued that markets feature numerous significant frictions, and although our model assumes no trading frictions, we do not claim that such frictions lack importance. In fact, as we outlined in the introduction, short-sale constraints could induce demand convexity akin to that driven by positive skewness under NARA utility, potentially confounding our results as both effects could operate together with the same directional influence on expected returns. Although distinguishing these mechanisms empirically can be challenging without a definitive measure of arbitrage limits, the recent innovation of a theoretically-founded option-implied measure of shorting fees derived by Muravyev et al. (2025), in addition to market capitalization and volatility as proxies for trading frictions (as suggested by Dong et al., 2022), aids sharper exploration of this distinction. Short-selling constraints are most acute for small firms, with highly volatile returns, and those with high shorting fees, so excluding these should weaken the interaction effect if frictions are the primary driver.

Building on our analyses in Tables 6 and 7 that show, respectively, that the interaction effect is present in the sample of low volatility stocks and unchanged even after tightly controlling for volatility, we further address the issue of short-sale constraints by dropping the highest quartile of observations each month based on option-implied shorting fee. Muravyev et al. (2025) show theoretically that the total stock lending fee over the life of an options contract is proportional to the difference between the implied volatilities of at-the-money call and put options

$$FEE_{implied} \approx -\frac{\sigma_C - \sigma_P}{\sqrt{2\pi T}}, \tag{26}$$

where T is the time to expiration. They state unambiguously, "a researcher who wants to construct a proxy for the borrow fee can apply [Equation (26)] to the at-the-money implied volatilities from the OptionMetrics volatility surfaces, which are widely available."

In Table 8, we parallel the analyses presented in Table 7 except we drop the highest 25% of observations each month based on the implied shorting fee. For additional robustness, we also exclude "small firms", firm-months with lagged market capitalization in the bottom quartile of the CRSP universe. The magnitude and strength of each estimate are essentially unchanged relative to the full sample despite the restrictive filters and using the same sorting procedure as applied in Table 7, which tightly controls for volatility. The robustness of the interaction effect suggests it is not driven primarily by short-selling frictions, supporting the frictionless mechanism we propose as being important.

Table 8: DISP, SKEW, and Interaction Effects for Scaled Returns: No Small or High Fee

	Excess	CAPM	FF5	DMRS
DISP H-L	-2.0	-2.7	-1.7	-1.0
	(1.75)	(2.36)	(1.67)	(0.82)
SKEW H-L	-0.5	-0.1	-0.2	-0.8
	(0.36)	(0.10)	(0.14)	(0.53)
Interaction	-7.2	-7.0	-6.8	-6.6
	(3.10)	(2.89)	(3.16)	(2.88)

Note: This table presents the (annualized) average monthly DISP, SKEW, and interaction effects for returns scaled by option-implied volatility, paralleling the results of Table 7 in which portfolios are formed using the alternative sorting procedure which controls for implied volatility, but portfolios exclude small or high-fee stocks. Specifically, the sample excludes firms with market capitalization in the bottom quartile and also excludes the top quartile of observations each month based on option-implied shorting fee (Muravyev et al., 2025). Heteroskedasticity and autocorrelation consistent absolute Newey and West (1987) t-statistics are in parentheses.

Another potential concern is that our measures of disagreement (DISP) and skewness (SKEW) might proxy for investor optimism rather than capturing the frictionless mechanism we propose. Optimism could inflate prices and lower expected returns independently of skewness and disagreement, even in a frictionless setting. To address this possibility, we follow Engelberg et al. (2018), who argue that analyst earnings forecasts should exhibit bias—underestimating earnings for stocks with high expected returns (long-side anomaly portfolios) and overestimating for those with low expected returns (short-side portfolios)—if optimism drives returns.

Table 9: Average Forecast Bias by Portfolio

	All	LowSkew	2	3	HighSkew	H-L
All	-	-1.4	-1.2	-0.6	-0.3	1.1
		(2.72)	(3.36)	(2.71)	(0.70)	(1.65)
LowDisp	-0.1	-0.6	-0.4	-0.0	0.4	1.0
	(0.45)	(1.28)	(1.12)	(0.07)	(1.01)	(1.34)
2	-0.8	-1.3	-1.0	-0.5	-0.4	0.9
	(2.70)	(2.07)	(2.20)	(1.74)	(0.90)	(1.39)
3	-1.6	-2.2	-2.1	-1.3	-0.8	1.4
	(3.38)	(3.48)	(4.05)	(2.29)	(1.19)	(2.23)
HighDisp	-0.9	-1.9	-1.1	-0.5	-0.0	1.9
	(2.17)	(2.60)	(2.44)	(0.99)	(0.04)	(1.89)
H-L	-0.8	-1.3	-0.7	-0.5	-0.4	0.9
	(1.32)	(1.85)	(1.38)	(0.65)	(0.52)	(1.53)

Note: This table presents the sample average of monthly average analyst forecast bias (in basis points), the negative of the time-series average of the quarterly portfolio-level earnings surprise series, for stocks independently double-sorted into DISP and SKEW quartiles, where double sorting is performed within size quartiles and aggregated. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010). Heteroskedasticity and autocorrelation consistent absolute Newey and West (1987) t-statistics are in parentheses.

We estimate analyst forecast bias by portfolio, following Kozak et al. (2018, KNS), as the negative of the time-series average of quarterly earnings surprises. Specifically, each calendar quarter we compute realized bias as the negative of the average earnings surprise by portfolio.[29] By the law of iterated expectations, this quantity equals the ex-ante actual bias plus mean-zero forecast error. Averaging it over time gives an unbiased and consistent estimate of the actual bias. As noted in KNS, positive bias indicates analyst optimism, whereas negative bias reflects pessimism.

Table 9 reports these results. Contrary to an optimism-driven explanation, we find no

[29] Following KNS, we use information from quarter $t-1$ to assign firms to portfolios to prevent possible look-ahead biases. Earnings surprise is constructed as in DellaVigna and Pollet (2009).

systematic pattern in forecast bias across DISP or SKEW dimensions, nor along the interaction diagonal (S4D4+S1D1-S1D4-S4D1). The absence of correlation between bias and our key variables suggests that the observed effects of skewness and disagreement on returns are distinct from optimism or pessimism, consistent with our preference-based mechanism.

Table 10: Interaction Effect for Scaled Returns: By Implied Volatility, using DISP_{med}

	Excess	CAPM	FF5	DMRS
All Firms	-6.5	-6.8	-6.5	-7.0
	(3.13)	(3.18)	(3.01)	(3.16)
Low Vol	-6.4	-6.5	-6.1	-5.8
	(2.59)	(2.49)	(2.37)	(2.34)
High Vol	-6.9	-7.0	-6.7	-7.7
	(2.95)	(2.97)	(2.79)	(3.26)
High - Low	-0.4	-0.5	-0.6	-1.8
	(-0.14)	(-0.15)	(-0.21)	(-0.67)

Note: This table presents the (annualized) average monthly interaction effect for returns scaled by option-implied volatility, paralleling Table 6 but with alternative analyst forecast dispersion measure DISP_{med}, in which the standard deviation of analyst EPS forecasts are scaled by the median forecast. Heteroskedasticity and autocorrelation consistent absolute Newey and West (1987) t-statistics are in parentheses.

Table 11: Interaction Effect for Scaled Returns: By Implied Volatility using DISP_p

	Excess	CAPM	FF5	DMRS
All Firms	-5.6	-5.6	-6.0	-5.1
	(2.92)	(2.84)	(2.90)	(2.40)
Low Vol	-6.9	-7.2	-7.7	-6.7
	(3.01)	(3.01)	(3.06)	(2.69)
High Vol	-6.6	-6.4	-6.1	-5.8
	(3.17)	(2.97)	(2.81)	(2.49)
High - Low	0.3	0.8	1.6	0.9
	(0.12)	(0.27)	(0.58)	(0.32)

Note: This table presents the (annualized) average monthly interaction effect for returns scaled by option-implied volatility, paralleling Table 6 but with alternative analyst forecast dispersion measure DISP_p, in which the standard deviation of analyst EPS forecasts are scaled by lagged price. Heteroskedasticity and autocorrelation consistent absolute Newey and West (1987) t-statistics are in parentheses.

Finally, we address a concern raised in the literature regarding our disagreement measure. Some have claimed that scaling by $|\text{meanF}_t|$ introduces noise in the measure, or, even more concerning, that cross-sectional variation in the denominator itself is responsible for the disagreement effect (Cen et al., 2017). To alleviate such concerns, we use two alternative denominators motivated by the literature: (1) $|\text{medF}_t|$, the absolute median analyst forecast, and (2) P_t, the share price one week before the I/B/E/S statistical period, defined respectively as follows:[30]

$$\text{DISP}_{\text{med}} := \frac{\text{stdF}_t}{|\text{medF}_t|}, \tag{27}$$

$$\text{DISP}_p := \frac{\text{stdF}_t}{\text{P}_t}. \tag{28}$$

We use price rather than book equity per share to improve data availability, even though price may bias against finding a disagreement effect.[31]

The estimated interaction effects, presented in Tables 10 and 11, which parallel Table 6 but with alternative measures of analyst forecast dispersion, are consistent with our main specification.

5 Conclusion

We develop a unifying frictionless theory that reconciles apparently conflicting empirical findings about the relationship between analyst forecast dispersion and expected returns—a negative relationship to equity returns (Diether et al., 2002), which tend to be positively skewed, and a positive relationship to fixed-income returns (Güntay and Hackbarth, 2010; Carlin et al., 2014), which are negatively skewed. It further generates the negative relationship between ex-ante skewness and equity returns (Boyer et al., 2010). Our approach leverages skewness-driven demand curvature and its interplay with heterogenous beliefs, offering testable predictions about how disagreement and skewness jointly determine ex-

[30] Studies employing standard deviation of EPS forecasts scaled by absolute median forecast include Banerjee (2011) and Sheng and Thevenot (2012). Studies using standard deviation of EPS forecasts scaled by lagged price include: Danielsen and Sorescu (2001); Doukas and McKnight (2005); Zhang (2006a); Zhang (2006b); Garfinkel (2009); Rees and Thomas (2010); Banerjee (2011); Cheong and Thomas (2011); Chatterjee et al. (2012); and Ge et al. (2016).

[31] As noted by (Cen et al., 2017), "since stock prices will be relatively high in the presence of the high divergence of opinion, scaling the standard deviation of forecasted earnings by price will mechanically reduce the spread in the disagreement measures and lead to weakened return predictability."

pected returns and Sharpe ratios. Our model is parsimonious yet flexible, requiring minimal restrictions on utility or payoff distributions, enhancing its generality, and is amenable to intuitive graphical interpretations of its mechanisms. We demonstrate its robustness through several extensions.

Empirical tests on U.S. equities provide robust support for our predictions. A portfolio that is neutral to skewness and disagreement but exploits their interaction effect yields an annualized CAPM alpha of -6.6% with a t-statistic exceeding 3, with comparable results under Fama and French (2015) and Daniel et al. (2020) factor models. The model's prediction of a volatility-invariant interaction effect on Sharpe ratios is also robust in empirical tests, distinguishing disagreement from uncertainty-driven effects. Robustness checks, including controls for firm size, implied volatility, short-selling frictions, investor optimism, and alternative disagreement measures, reinforce that our findings likely stem from the proposed frictionless mechanism.

References

Abel, A. B. (1989). Asset prices under heterogeneous beliefs: implications for the equity premium.

Albagli, E., C. Hellwig, and A. Tsyvinski (2024). Information aggregation with asymmetric asset payoffs. *Journal of Finance 79*(4), 2715–2758.

Ali, A., M. Liu, D. Xu, and T. Yao (2019). Corporate disclosure, analyst forecast dispersion, and stock returns. *Journal of Accounting, Auditing & Finance 34*(1), 54–73.

Amaya, D., P. Christoffersen, K. Jacobs, and A. Vasquez (2015). Does realized skewness predict the cross-section of equity returns? *Journal of Financial Economics 111*(1), 135–167.

Arditti, F. D. (1967). Risk and the required return on equity. *Journal of Finance 22*(1), 19–36.

Arrow, K. J. (1971). *Essays in the Theory of Risk-Bearing*. Markham Chicago.

Atmaz, A. and S. Basak (2018). Belief dispersion in the stock market. *Journal of Finance 73*(3), 1225–1279.

Avramov, D., T. Chordia, G. Jostova, and A. Philipov (2009). Dispersion in analysts' earnings forecasts and credit rating. *Journal of Financial Economics 91*(1), 83–101.

Banerjee, S. (2011). Learning from prices and the dispersion in beliefs. *The Review of Financial Studies 24*(9), 3025–3068.

Banerjee, S., B. M. Breon-Drish, and K. Smith (2022). Asymmetric information, disagreement, and the valuation of debt and equity. *Available at SSRN 4278165*.

Banerjee, S., J. Davis, and N. Gondhi (2018). When transparency improves, must prices reflect fundamentals better? *Review of Financial Studies 31*(6), 2377–2414.

Banerjee, S. and I. Kremer (2010). Disagreement and learning: Dynamic patterns of trade. *Journal of Finance 65*(4), 1269–1302.

Banerjee, S., I. Marinovic, and K. Smith (2023). Disclosing to informed traders. *Journal of Finance 79*(2), 1513–1578.

Barberis, N. and M. Huang (2008). Stocks as lotteries: The implications of probability weighting for security prices. *American Economic Review 98*(5), 2066–2100.

Barinov, A. (2013). Analyst disagreement and aggregate volatility risk. *Journal of Financial and Quantitative Analysis 48*(6), 1877–1900.

Bekaert, G., M. Bergbrant, and H. Kassa (2025). Expected idiosyncratic volatility. *Journal of financial economics 167*, 104023.

Bielecki, T. R., T. Björk, M. Jeanblanc, M. Rutkowski, J. A. Scheinkman, and W. Xiong (2004). Heterogeneous beliefs, speculation and trading in financial markets. *Paris-Princeton Lectures on Mathematical Finance 2004*, 217–250.

Boyer, B., T. Mitton, and K. Vorkink (2010). Expected idiosyncratic skewness. *Review of Financial Studies 23*(1), 169–202.

Boyer, B. and K. Vorkink (2014). Stock options as lotteries. *Journal of Finance 69*(4), 1485–1527.

Breon-Drish, B. (2015). On existence and uniqueness of equilibrium in a class of noisy rational expectations models. *Review of Economic Studies 82*(3), 868–921.

Brunnermeier, M. K., C. Gollier, and J. A. Parker (2007). Optimal beliefs, asset prices, and the preference for skewed returns. *American Economic Review 97*(2), 159–165.

Brunnermeier, M. K. and J. A. Parker (2005). Optimal expectations. *American Economic Review 95*(4), 1092–1118.

Cao, H. H. and H. Ou-Yang (2009). Differences of opinion of public information and speculative trading in stocks and options. *Review of Financial Studies 22*(1), 299–335.

Carlin, B. I., F. A. Longstaff, and K. Matoba (2014). Disagreement and asset prices. *Journal of Financial Economics 114*(2), 226–238.

Cen, L., K. J. Wei, and L. Yang (2017). Disagreement, underreaction, and stock returns. *Management Science 63*(4), 1214–1231.

Chabakauri, G. and B. Y. Han (2020). Collateral constraints and asset prices. *Journal of Financial Economics 138*(3), 754–776.

Chabakauri, G., K. Yuan, and K. E. Zachariadis (2022). Multi-asset noisy rational expectations equilibrium with contingent claims. *Review of Economic Studies 89*(5), 2445–2490.

Chang, Y.-C., P.-J. Hsiao, A. Ljungqvist, and K. Tseng (2022). Testing disagreement models. *Journal of Finance 77*(4), 2239–2285.

Chatterjee, S., K. John, and A. Yan (2012). Takeovers and divergence of investor opinion. *Review of Financial Studies 25*(1), 227–277.

Chen, J., H. Hong, and J. C. Stein (2001). Forecasting crashes: Trading volume, past returns, and conditional skewness in stock prices. *Journal of Financial Economics 61*(3), 345–381.

Chen, J., H. Hong, and J. C. Stein (2002). Breadth of ownership and stock returns. *Journal of Financial Economics 66*(2-3), 171–205.

Cheong, F. S. and J. Thomas (2011). Why do EPS forecast error and dispersion not vary with scale? implications for analyst and managerial behavior. *Journal of Accounting Research 49*(2), 359–401.

Cianciaruso, D., I. Marinovic, and K. Smith (2023). Asymmetric disclosure, noise trade, and firm valuation. *The Accounting Review 98*(5), 215–240.

Conrad, J., R. F. Dittmar, and E. Ghysels (2013). Ex ante skewness and expected stock returns. *Journal of Finance 68*(1), 85–123.

Curcuru, S., J. Heaton, D. Lucas, and D. Moore (2010). Heterogeneity and portfolio choice:

Theory and evidence. In *Handbook of Financial Econometrics: Tools and Techniques*, pp. 337–382. Elsevier.

Daniel, K., A. Klos, and S. Rottke (2023). The dynamics of disagreement. *Review of Financial Studies 36*(6), 2431–2467.

Daniel, K., L. Mota, S. Rottke, and T. Santos (2020). The cross section of risk and return. *Review of Financial Studies 33*(5), 1927–1979.

Danielsen, B. R. and S. M. Sorescu (2001). Why do option introductions depress stock prices? a study of diminishing short sale constraints. *Journal of Financial and Quantitative Analysis 36*(4), 451–484.

DellaVigna, S. and J. M. Pollet (2009). Investor inattention and Friday earnings announcements. *Journal of Finance 64*(2), 709–749.

Diamond, D. W. and R. E. Verrecchia (1987). Constraints on short-selling and asset price adjustment to private information. *Journal of Finance 46*(2), 427–466.

Diether, K. B., C. J. Malloy, and A. Scherbina (2002). Difference of opinion and the cross section of stock returns. *Journal of Finance 57*(5), 2113–2141.

Dong, X., Y. Li, D. E. Rapach, and G. Zhou (2022). Anomalies and the expected market return. *Journal of Finance 77*(1), 639–681.

Doukas, J. A. and P. J. McKnight (2005). European momentum strategies, information diffusion, and investor conservatism. *European Financial Management 11*(3), 313–338.

Dumas, B., A. Kurshev, and R. Uppal (2009). Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility. *Journal of Finance 64*(2), 579–629.

Engelberg, J., R. D. McLean, and J. Pontiff (2018). Anomalies and news. *Journal of Finance 73*(5), 1971–2001.

Falkenstein, E. G. (1994). *Mutual funds, idiosyncratic variance, and asset returns*. Northwestern University.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Garfinkel, J. A. (2009). Measuring investors' opinion divergence. *Journal of Accounting Research 47*(5), 1317–1348.

Gasull, A. and F. Utzet (2014). Approximating mills ratio. *Journal of Mathematical Analysis and Applications 420*(2), 1832–1853.

Ge, L., T.-C. Lin, and N. D. Pearson (2016). Why does the option to stock volume ratio predict stock returns? *Journal of Financial Economics 120*(3), 601–622.

Goulding, C. L. (2015). *Opposite Sides of a Skewed Bet: Implications and Evidence for Forecast Dispersion and Returns*. Ph. D. thesis, University of Pennsylvania.

Goulding, C. L., S. Santosh, and X. Zhang (2023). Pricing implications of noise. *Review of Financial Studies 36*(6), 2468–2508.

Güntay, L. and D. Hackbarth (2010). Corporate bond credit spreads and forecast dispersion. *Journal of Banking & Finance 34*(10), 2328–2345.

Hansen, L. P. and R. Jagannathan (1991). Implications of security market data for models of dynamic economies. *Journal of Political Economy 99*(2), 225–262.

Harris, M. and A. Raviv (1993). Differences of opinion make a horse race. *Review of Financial Studies 6*(3), 473–506.

Harrison, J. M. and D. M. Kreps (1978). Speculative investor behavior in a stock market with heterogeneous expectations. *Quarterly Journal of Economics 92*(2), 323–336.

Harvey, C. R. and A. Siddique (1999). Autoregressive conditional skewness. *Journal of Financial and Quantitative Analysis 34*(4), 465–487.

Hong, H., J. Scheinkman, and W. Xiong (2006). Asset float and speculative bubbles. *Journal of Finance 61*(3), 1073–1117.

Hong, H. and D. A. Sraer (2016). Speculative betas. *Journal of Finance 71*(5), 2095–2144.

Hong, H. and J. C. Stein (2007). Disagreement and the stock market. *Journal of Economic Perspectives 21*(2), 109–128.

Jarrow, R. (1980). Heterogeneous expectations, restrictions on short sales, and equilibrium asset prices. *Journal of Finance 35*(5), 1105–1113.

Johnson, T. C. (2004). Forecast dispersion and the cross section of expected returns. *Journal of Finance 59*(5), 1957–1978.

Kandel, E. and N. D. Pearson (1995). Differential interpretation of public signals and trade in speculative markets. *Journal of Political Economy 103*(4), 831–872.

Kozak, S., S. Nagel, and S. Santosh (2018). Interpreting factor models. *Journal of Finance 73*(3), 1183–1223.

Kraus, A. and R. H. Litzenberger (1976). Skewness preference and the valuation of risk assets. *Journal of Finance 31*(4), 1085–1100.

Martin, I. and D. Papadimitriou (2022). Sentiment and speculation in a market with heterogeneous beliefs. *American Economic Review 112*(8), 2465–2517.

Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. *Journal of Finance 32*(4), 1151–1168.

Mitton, T. and K. Vorkink (2007). Equilibrium underdiversification and the preference for skewness. *Review of Financial Studies 20*(4), 1255–1288.

Muravyev, D., N. D. Pearson, and J. M. Pollet (2025). Why does options market information predict stock returns. *Journal of Financial Economics*.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–708.

Ottaviani, M. and P. N. Sørensen (2015). Price reaction to information with heterogeneous beliefs and wealth effects: Underreaction, momentum, and reversal. *American Economic Review 105*(1), 1–34.

Rees, L. and W. Thomas (2010). The stock price effects of changes in dispersion of investor beliefs during earnings announcements. *Review of Accounting Studies 15*(1), 1–31.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory 13*(3), 341–360.

Sadka, R. and A. Scherbina (2007). Analyst disagreement, mispricing, and liquidity. *Journal of Finance 62*(5), 2367–2403.

Scheinkman, J. A. and W. Xiong (2003). Overconfidence and speculative bubbles. *Journal of Political Economy 111*(6), 1183–1220.

Sheng, X. and M. Thevenot (2012). A new measure of earnings forecast uncertainty. *Journal of Accounting and Economics 53*(1-2), 21–33.

Shumway, T. (1997). The delisting bias in crsp data. *Journal of Finance 52*(1), 327–340.

Welch, I. (2017). Stock return outliers. *Available at SSRN 3068347*.

Xiong, W. (2013). Bubbles, crises, and heterogeneous beliefs. Technical report, National Bureau of Economic Research.

Yan, H. (2010). Is noise trading cancelled out by aggregation? *Management Science 56*(7), 1047–1059.

Yu, J. (2011). Disagreement and return predictability of stock portfolios. *Journal of Financial Economics 99*(1), 162–183.

Zhang, X. F. (2006a). Information uncertainty and analyst forecast behavior. *Contemporary Accounting Research 23*(2), 565–590.

Zhang, X. F. (2006b). Information uncertainty and stock returns. *Journal of Finance 61*(1), 105–137.

Appendix A Proofs

Proof of Lemma 1. Recall, an investor's FOC is given by

$$E[u'(w_0 + x_i(p)(\tilde{\theta} - p))(\tilde{\theta} - p)] = 0. \tag{A1}$$

Our goal is to derive $x_i(p)$ when p is close to μ_i. By Taylor expansion,

$$x_i(p) = x_i(\mu_i) + x_i'(\mu_i)(p - \mu_i) + \frac{x_i''(\mu_i)}{2}(p - \mu_i)^2 + o((p - \mu_i)^2).$$

In the following, we solve all the coefficients, $x(\mu_i)$, $x_i'(\mu_i)$, and $x_i''(\mu_i)$ explicitly.

First, from the FOC (A1), $x_i(\mu_i) = 0$ clearly solves the FOC. Together with the fact that the utility function is concave, which implies the solution to (A1) is unique, we conclude that $x_i(\mu_i) = 0$.

Second, taking the derivative with respect to p on both sides of (A1) gives the following equation:

$$0 = E\left[[u''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i'(p)(\tilde{\theta} - p) - x_i(p)](\tilde{\theta} - p)\right] \tag{A2}$$

$$- E[u'(w_0 + x_i(p)(\tilde{\theta} - p))]. \tag{A3}$$

Plugging in $p = \mu_i$ and using the fact that $x_i(\mu_i) = 0$, we derive

$$x_i'(\mu_i) = \frac{u'(w_0)}{u''(w_0)\sigma^2}. \tag{A4}$$

Third, taking the derivative with respect to p on both sides of (A2) gives to the following equation:

$$\begin{aligned}
0 =& E[u'''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i'(p)(\tilde{\theta} - p) - x_i(p)]^2(\tilde{\theta} - p) \\
&+ E[u''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i''(p)(\tilde{\theta} - p) - 2x_i'(p)](\tilde{\theta} - p) \\
&- E[u''(w_0 + x_i(p)(\tilde{\theta} - p))][x_i'(p)(\tilde{\theta} - p) - x_i(p)]
\end{aligned}$$

$$-E[u''(w_0+x_i(p)(\tilde{\theta}-p))][x_i'(p)(\tilde{\theta}-p)-x_i(p)]. \tag{A5}$$

Plugging in $p=\mu_i$, we derive

$$x_i''(\mu_i)=-\frac{u'''(w_0)E(\tilde{\theta}-\mu_i)^3x_i'(\mu_i)^2}{u''(w_0)\sigma^2}=-\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s}{\sigma^3}, \tag{A6}$$

where the last equality uses Equation (A4).

So we obtain the Taylor series of $x(p)$ around $p=\mu_i$:

$$x_i(p)=\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu_i)-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s}{\sigma^3}(p-\mu_i)^2+o(1)(p-\mu_i)^2, \tag{A7}$$

where the little-o notation $o(1)$ is an unknown function that converges to zero when $p\to\mu$.

Proofs of Proposition 1 and Proposition 4. Since Proposition 1 is a special case of Proposition 4 when the random variable $\tilde{\delta}$ is binary, we proceed to prove the more general case, i.e., Proposition 4.

Since $\tilde{\delta}$ is discrete, we write the possible realizations as δ_i for $i=1,\cdots,n$ with the probability of δ_i being q_i. With some abuse of notations, we write $x_{\delta_i}(p)$ as the demand schedule for the investor with δ_i. Following the same argument in Lemma 1, we have the Taylor expansion for $x_{\delta_i}(p)$ as given by the following equation.

$$x_{\delta_i}(p)=\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu_{\delta_i})-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s}{\sigma^3}(p-\mu_{\delta_i})^2+\epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2, \tag{A8}$$

where $\epsilon_{\delta_i}(p)$ is a function that converges to 0 as $p\to\mu_{\delta_i}$.

The market's clearing condition states that

$$\sum_{i=1}^{n}q_ix_{\delta_i}(p)=0. \tag{A9}$$

Substituting $x_{\delta_i}(p)$ from equation (A8), it leads to the following equation.

$$\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}\sum_{i=1}^{n}q_i(p-\mu_{\delta_i})-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s}{\sigma^3}\sum_{i=1}^{n}q_i(p-\mu_{\delta_i})^2$$

$$+\sum_{i=1}^{n} q_i \epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2 = 0. \tag{A10}$$

Notice that $\sum_{i=1}^n q_i(p-\mu_{\delta_i}) = \sum_{i=1}^n q_i(p-\mu-\delta_i\sigma) = p-\mu$, where the last equality uses the fact that $\widetilde{\delta}$ is a mean-zero random variable, and $\sum_{i=1}^n q_i(p-\mu_{\delta_i})^2 = \sum_{i=1}^n q_i(p-\mu-\delta_i\sigma)^2 = \sum_{i=1}^n q_i[(p-\mu)^2 - 2(p-\mu)\delta_i\sigma + \delta_i^2\sigma^2] = (p-\mu)^2 + Var(\widetilde{\delta})\sigma^2$. We write $v^2 := Var(\widetilde{\delta})$. It follows that (A10) is equivalent to the following equation.

$$\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p-\mu) - \frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2 \frac{s}{\sigma^3}\left((p-\mu)^2 + v^2\sigma^2)\right) + \sum_{i=1}^{n} q_i\epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2 = 0. \tag{A11}$$

We vary v to study the behavior of the equilibrium price when v is small. To emphasize that the price changes when v changes, we write the price as a function of v: $p(v)$. By Taylor expansion, it follows that $p(v) = p(0) + p'(0)v + p''(0)/2v^2 + o(1)v^2$ for small v.

First, $p(0) = \mu$. This can be seen by setting $v = 0$ in (A11). Second, taking derivative with respect to v on both sides in (A11) gives to

$$\frac{\partial(p-\mu)}{\partial v} = \frac{1}{2}\frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2}\frac{s}{\sigma}\left[2(p-\mu)\frac{\partial(p-\mu)}{\partial v} + 2v\sigma^2\right] - \frac{\partial\mathscr{E}(v)}{\partial v}, \tag{A12}$$

where

$$\mathscr{E}(v) = \frac{u''(w_0)\sigma^2}{u'(w_0)}\sum_{i=1}^{n} q_i\epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2. \tag{A13}$$

We claim that $\frac{\partial\mathscr{E}(v)}{\partial v}|_{v=0} = 0$. It is sufficient to show that $\frac{\partial\mathscr{E}_{\delta_i}(v)}{\partial v}|_{v=0} = 0$, where $\mathscr{E}_{\delta_i}(v) = \epsilon_{\delta_i}(p)(p-\mu_{\delta_i})^2$. Note that $\frac{\partial\mathscr{E}_{\delta_i}(v)}{\partial v} = \epsilon'_{\delta_i}(p)p'(v)(p-\mu-v\delta_i)^2 + \epsilon_{\delta_i}(p)2(p-\mu-v\delta_i)(p'(v)-\delta_i)$, which goes to zero as $v \to 0$.

Evaluating equation (A12) at $v = 0$ implies that $\frac{\partial(p-\mu)}{\partial v}|_{v=0} = 0$, i.e., $p'(0) = 0$. Taking derivative with respect to v on both sides in (A12) gives to

$$\frac{\partial^2(p-\mu)}{\partial v^2} = \frac{1}{2}\frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2}\frac{s}{\sigma}\left[2\left(\frac{\partial(p-\mu)}{\partial v}\right)^2 + 2(p-\mu)\frac{\partial^2(p-\mu)}{\partial v^2} + 2\sigma^2\right] - \frac{\partial^2\mathscr{E}(v)}{\partial v^2}. \tag{A14}$$

We claim that $\frac{\partial^2\mathscr{E}(v)}{\partial^2 v}|_{v=0} = 0$. It is sufficient to show that $\frac{\partial^2\mathscr{E}_{\delta_i}(v)}{\partial v^2}|_{v=0} = 0$. Note that $\frac{\partial^2\mathscr{E}_{\delta_i}(v)}{\partial v^2} = \epsilon''_{\delta_i}(p)p'(v)^2(p-\mu-v\delta_i)^2 + \epsilon'_{\delta_i}(p)p''(v)(p-\mu-v\delta_i)^2 + \epsilon'_{\delta_i}(p)p'(v)2(p-\mu-v\delta_i)(p'(v)-\delta_i^*) + \epsilon'_{\delta_i}(p)p'(v)2(p-\mu-v\delta_i)(p'(v)-\delta_i) + \epsilon_{\delta_i}(p)2(p'(v)-\delta_i)(p'(v)-\delta_i) + \epsilon_{\delta_i}(p)2(p-\mu-v\delta_i)p''(v)$, which goes to zero as $v \to 0$ because every single term goes to zero as $v \to 0$. Evaluating (A14) at $v = 0$ implies that

$$\frac{\partial^2 (p-\mu)}{\partial v^2}\Big|_{v=0} = \frac{u'''(w_0)u'(w_0)}{(u''(w_0))^2} s\sigma. \tag{A15}$$

Using Taylor expansion, we have

$$p - \mu = \frac{1}{2}\frac{\partial^2 (p-\mu)}{\partial v^2}\Big|_{v=0} v^2 + o(1)v^2 = \frac{u'''(w_0)u'(w_0)}{2(u''(w_0))^2} s\sigma Var(\widetilde{\delta}) + o(1)Var(\widetilde{\delta}), \tag{A16}$$

where the little-o notation $o(1)$ is an unknown function that converges to 0 as $v \to 0$.

Proof of Proposition 3. It is straightforward to see that $\frac{\partial \mathrm{E}[\widetilde{r}]}{\partial s} = -\overline{u}\sigma\delta^2 < 0$, $\frac{\partial \mathrm{E}[\widetilde{r}]}{\partial \delta} = -2\overline{u}s\sigma\delta < 0 \Leftrightarrow s > 0$, and $\frac{\partial^2 \mathrm{E}[\widetilde{r}]}{\partial s \partial \delta} = -2\overline{u}\sigma\delta < 0$. The analogous results for the Sharpe ratio hold because the Sharpe ratio is the expected return divided by a positive constant, σ_r.

Proof of Proposition 6. Given the two types of investors and Lemma 1, we can write the market's clearing condition as the following equation.

$$\begin{aligned} &\frac{u_1'(w_0)}{u_1''(w_0)}\frac{1}{\sigma^2}(p-\mu_+) - \frac{1}{2}\frac{u_1'''(w_0)}{u_1''(w_0)}\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2 \frac{s}{\sigma^3}(p-\mu_+)^2 + o(1)(p-\mu_+)^2 \\ &+ \frac{u_2'(w_0)}{u_2''(w_0)}\frac{1}{\sigma^2}(p-\mu_-) - \frac{1}{2}\frac{u_2'''(w_0)}{u_2''(w_0)}\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2 \frac{s}{\sigma^3}(p-\mu_-)^2 + o(1)(p-\mu_-)^2 = 0. \end{aligned} \tag{A17}$$

With loss, we assume $s > 0$. (When $s < 0$, all analysis carries over). In order to show that the equilibrium price is greater than the benchmark price p_0, we need to show that at price p_0, there is excess demand. In other words, we need to show that the left-hand-side of the above market's clearing condition is strictly positive when $p = p_0$.

Note that because

$$p_0 = \frac{\frac{u_1'(w_0)}{-u_1''(w_0)}\mu_+ + \frac{u_2'(w_0)}{-u_2''(w_0)}\mu_-}{\frac{u_1'(w_0)}{-u_1''(w_0)} + \frac{u_2'(w_0)}{-u_2''(w_0)}}, \tag{A18}$$

by algebraic manipulation, it is straightforward to see that

$$\frac{u_1'(w_0)}{u_1''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_+) + \frac{u_2'(w_0)}{u_2''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_-) = 0. \tag{A19}$$

In order to show there is excess demand p_0, it is sufficient to show that

$$-\frac{1}{2}\frac{u_1'''(w_0)}{u_1''(w_0)}\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_+)^2-\frac{1}{2}\frac{u_2'''(w_0)}{u_2''(w_0)}\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_-)^2>0. \tag{A20}$$

Plugging into p_0, note that

$$\begin{aligned}&-\frac{1}{2}\frac{u_1'''(w_0)}{u_1''(w_0)}\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_+)^2-\frac{1}{2}\frac{u_2'''(w_0)}{u_2''(w_0)}\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2\frac{s}{\sigma^3}(p_0-\mu_-)^2\\&=\frac{1}{2}\frac{(\mu_+-\mu_-)^2}{\left(\frac{u_1'(w_0)}{-u_1''(w_0)}+\frac{u_2'(w_0)}{-u_2''(w_0)}\right)^2}\left[\frac{u_1'''(w_0)}{-u_1''(w_0)}+\frac{u_2'''(w_0)}{-u_2''(w_0)}\right]\left(\frac{u_1'(w_0)}{u_1''(w_0)}\right)^2\left(\frac{u_2'(w_0)}{u_2''(w_0)}\right)^2\frac{s}{\sigma^3},\end{aligned} \tag{A21}$$

which is positive as long as $s>0$. Solving (A17) implies that

$$p-p_0\approx\frac{\frac{1}{2}\frac{(\mu_+-\mu_-)^2}{\left(\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}\right)^2}\left[\frac{u_1'''}{-u_1''}+\frac{u_2'''}{-u_2''}\right]\left(\frac{u_1'}{u_1''}\right)^2\left(\frac{u_2'}{u_2''}\right)^2\frac{s}{\sigma^3}}{\frac{u_1'}{-u_1''}\frac{1}{\sigma^2}+\frac{u_2'}{-u_2''}\frac{1}{\sigma^2}-\frac{u_1'''}{u_1''}\left(\frac{u_1'}{u_1''}\right)^2\frac{s}{\sigma^3}(p_0-\mu_+)-\frac{u_2'''}{u_2''}\left(\frac{u_2'}{u_2''}\right)^2\frac{s}{\sigma^3}(p_0-\mu_-)} \tag{A22}$$

$$=\frac{\frac{1}{2}\frac{(\mu_+-\mu_-)^2}{\left(\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}\right)^2}\left[\frac{u_1'''}{-u_1''}+\frac{u_2'''}{-u_2''}\right]\left(\frac{u_1'}{u_1''}\right)^2\left(\frac{u_2'}{u_2''}\right)^2\frac{s}{\sigma}}{\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}+\frac{\mu_+-\mu_-}{\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}}\left(\frac{u_1'''}{u_1''}\left(\frac{u_1'}{u_1''}\right)^2\frac{u_2'}{-u_2''}-\frac{u_2'''}{u_2''}\left(\frac{u_2'}{u_2''}\right)^2\frac{u_1'}{-u_1''}\right)\frac{s}{\sigma}} \tag{A23}$$

$$=\frac{\frac{2}{\left(\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}\right)^2}\left[\frac{u_1'''}{-u_1''}+\frac{u_2'''}{-u_2''}\right]\left(\frac{u_1'}{u_1''}\right)^2\left(\frac{u_2'}{u_2''}\right)^2 s\sigma\delta^2}{\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}+\frac{2}{\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}}\left(\frac{u_1'''}{u_1''}\left(\frac{u_1'}{u_1''}\right)^2\frac{u_2'}{-u_2''}-\frac{u_2'''}{u_2''}\left(\frac{u_2'}{u_2''}\right)^2\frac{u_1'}{-u_1''}\right)s\delta} \tag{A24}$$

$$\approx\frac{\frac{2}{\left(\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}\right)^2}\left[\frac{u_1'''}{-u_1''}+\frac{u_2'''}{-u_2''}\right]\left(\frac{u_1'}{u_1''}\right)^2\left(\frac{u_2'}{u_2''}\right)^2 s\sigma\delta^2}{\frac{u_1'}{-u_1''}+\frac{u_2'}{-u_2''}}=:\overline{u}_1 s\sigma\delta^2, \tag{A25}$$

where the first line comes from replacing the quadratic terms in (A17) with 0, the second line comes from plugging into p_0 from (A18), the third line comes from $\mu_+-\mu_-=2\sigma\delta$, and the last line comes from fact that the second term in the denominator is dominated by the first term due to small δ. This completes the proof.

Proof of Proposition 7. we can write the market's clearing condition as the following equation.

$$\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma_A^2}(p-\mu_A)-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_A}{\sigma_A^3}(p-\mu_A)^2+o(1)(p-\mu_A)^2$$
$$+\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma_B^2}(p-\mu_B)-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_B}{\sigma_B^3}(p-\mu_B)^2+o(1)(p-\mu_B)^2=0. \tag{A26}$$

Similar to the proof of Proposition 6, we need to show that there is excess demand at price p_0 if and only if $\sigma_A s_A+\sigma_B s_B>0$.

Note that when

$$p_0=\frac{\frac{\mu_A}{\sigma_A^2}+\frac{\mu_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2}+\frac{1}{\sigma_B^2}}, \tag{A27}$$

it follows that

$$\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_A)+\frac{u'(w_0)}{u''(w_0)}\frac{1}{\sigma^2}(p_0-\mu_B)=0, \tag{A28}$$

and

$$-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_A}{\sigma_A^3}(p_0-\mu_A)^2-\frac{1}{2}\frac{u'''(w_0)}{u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{s_B}{\sigma_B^3}(p_0-\mu_B)^2$$
$$=\frac{1}{2}\frac{u'''(w_0)}{-u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{(\mu_A-\mu_B)^2}{\left(\frac{1}{\sigma_A^2}+\frac{1}{\sigma_B^2}\right)^2}\frac{\sigma_A s_A+\sigma_B s_B}{\sigma_A^4\sigma_B^4} \tag{A29}$$

which is positive if and only if $\sigma_A s_A+\sigma_B s_B>0$. Following similarly from the proof of Proposition 6, we can solve (A26) for an approximate p by ignoring the quadratic terms,

$$p-p_0\approx\frac{\frac{1}{2}\frac{u'''(w_0)}{-u''(w_0)}\left(\frac{u'(w_0)}{u''(w_0)}\right)^2\frac{(\mu_A-\mu_B)^2}{\left(\frac{1}{\sigma_A^2}+\frac{1}{\sigma_B^2}\right)^2}\frac{\sigma_A s_A+\sigma_B s_B}{\sigma_A^4\sigma_B^4}}{\frac{u'(w_0)}{-u''(w_0)}\frac{1}{\sigma_A^2}+\frac{u'(w_0)}{-u''(w_0)}\frac{1}{\sigma_B^2}} \tag{A30}$$
$$=\frac{1}{2}\frac{u'''(w_0)}{-u''(w_0)}\left(\frac{u'(w_0)}{-u''(w_0)}\right)\frac{(\mu_A-\mu_B)^2}{\left(\frac{1}{\sigma_A^2}+\frac{1}{\sigma_B^2}\right)^3}\frac{\sigma_A s_A+\sigma_B s_B}{\sigma_A^4\sigma_B^4}. \tag{A31}$$

This completes the proof.

Proof of Proposition 8. The market's clearing condition states that $x_+(p)+x_-(p)=\chi$. Using Taylor expansion, it follows that

$$\begin{aligned}\chi =& x_+(p_+)+x'_+(p_+)(p-p_+)+\frac{1}{2}x''_+(p_+)(p-p_+)^2+o(1)(p-p_+)^2\\ &+x_-(p_-)+x'_-(p_-)(p-p_-)+\frac{1}{2}x''_-(p_-)(p-p_-)^2+o(1)(p-p_-)^2.\end{aligned} \tag{A32}$$

The definition of the benchmark price states that $x_+(p_+)+x'_+(p_+)(p_0-p_+)+x_-(p_-)+x'_-(p_-)(p_0-p_-)=\chi$. Thus, the above equation becomes

$$\begin{aligned}0 =& x'_+(p_+)(p-p_0)+\frac{1}{2}x''_+(p_+)(p-p_0+p_0-p_+)^2+o(1)(p-p_+)^2\\ &+x'_-(p_-)(p-p_0)+\frac{1}{2}x''_-(p_-)(p-p_0+p_0-p_-)^2+o(1)(p-p_-)^2.\end{aligned} \tag{A33}$$

Solving for $(p-p_0)$ when ignoring the quadratic terms and higher, we obtain

$$p-p_0 \approx \frac{\frac{1}{2}x''_+(p_+)(p_0-p_+)^2+\frac{1}{2}x''_-(p_-)(p_0-p_-)^2}{-(x'_+(p_+)+x'_-(p_-)+x''_+(p_+)(p_0-p_+)+x''_-(p_-)(p_0-p_-))}. \tag{A34}$$

Plugging p_0 from equation (21), it follows that the numerator of the above equation can be simplified as

$$\frac{1}{2}x''_+(p_+)(p_0-p_+)^2+\frac{1}{2}x''_-(p_-)(p_0-p_-)^2=\frac{1}{2}\frac{\left(x''_+(p_+)(x'_-)^2+x''_-(p_-)(x'_+)^2\right)}{(x'_+(p_+)+x'_-(p_-))^2}(p_+-p_-)^2.$$

Writing p_+ and p_- as functions of δ and using Taylor expansion around $\delta=0$, it follows that

$$p_+(\delta)-p_-(\delta)=p_+(0)-p_-(0)+\left(\frac{\partial p_+}{\partial \delta}|_{\delta=0}-\frac{\partial p_-}{\partial \delta}|_{\delta=0}\right)\delta+o(1)\delta.$$

As $\delta\to 0$, both p_+ and p_- approach to the solution of $2x_0(p)=\chi$. Thus,

$$p_+-p_-=\left(\frac{\partial p_+}{\partial \delta}|_{\delta=0}-\frac{\partial p_-}{\partial \delta}|_{\delta=0}\right)\delta+o(1)\delta.$$

Thus, $x''_+(p_+)(p_0-p_+)+x''_-(p_-)(p_0-p_-)=\frac{\left(x''_+(p_+)(x'_-)+x''_-(p_-)(x'_+)\right)}{(x'_+(p_+)+x'_-(p_-))}(p_+-p_-)$ is of order δ. Returning to equation (A34),

$$p - p_0 = \frac{1}{2}\left(\frac{\partial p_+}{\partial \delta}|_{\delta=0} - \frac{\partial p_-}{\partial \delta}|_{\delta=0}\right)^2 \frac{x_+''(p_+)(x_-'(p_-))^2 + x_-''(p_-)(x_+'(p_+))^2}{-(x_+'(p_+)+x_-'(p_-))^3}|_{\delta=0}\delta^2 + o(1)\delta^2 \quad \text{(A35)}$$

$$= \overline{u}_2 \frac{x_0''(p_1)}{-x_0'(p_1)}\delta^2 + o(1)\delta^2, \quad \text{(A36)}$$

where $\overline{u}_2 = \frac{1}{8}\left(\frac{\partial p_+}{\partial \delta}|_{\delta=0} - \frac{\partial p_-}{\partial \delta}|_{\delta=0}\right)^2 > 0$.

For the second part, we utilize a result from the proof of Proposition 3 in Goulding et al. (2023), in which they show that the demand function $x(p)$ is convex for a sufficiently large skewness and the demand function is concave for a sufficiently small (potentially negative) skewness. Let $\overline{\theta}$ be the bound of $\tilde{\theta}$. Following their procedure,

$$x_0''(p_1) = \frac{E[u'''(w_0 + \frac{\chi}{2}(\tilde{\theta} - p_1))][x_0'(p_1)^2(\tilde{\theta} + \overline{\theta})^3] + \text{terms unrelated to skewness}}{E[-u''(w_0 + \frac{\chi}{2}(\tilde{\theta} - p_1))][(\tilde{\theta} - p_1)^2]}. \quad \text{(A37)}$$

Note that we add $\overline{\theta}$ to $\tilde{\theta}$ to ensure that the cubic term $(\tilde{\theta} + \overline{\theta})^3$ is always nonnegative, and we omit all the terms unrelated to skewness because we are interested in the limit of $x_0''(p)$ as skewness goes to infinity. So, skewness affects the numerator in a cubic manner, moreover the coefficient of the cubic term is positive. It follows that for skewness sufficiently large or sufficiently small, $x_0''(p_1)$ is monotonic increasing in skewness. We note that $(-x_0'(p_1))$ does not rely on skewness. Thus, the term $\frac{x_0''(p_1)}{-x_0'(p_1)}$ is monotonically increasing in s for extreme values of s, for example when $s \in (-\infty, \underline{s}) \cup (\overline{s}, +\infty)$.

Recall a cubic function with a positive coefficient on the cubic term goes to infinity as its argument goes to infinity. Thus, as skewness goes to positive infinity, the numerator of (A37) goes to positive infinity as well; as skewness goes to negative infinity, the numerator of (A37) goes to negative infinity as well. So there exist thresholds $\overline{s}$ and $\underline{s}$ such that $\frac{x_0''(p_1)}{-x_0'(p_1)} > 0$ for $s > \overline{s}$ and $\frac{x_0''(p_1)}{-x_0'(p_1)} < 0$ for $s < \underline{s}$. This completes the proof.

Proof of Lemma 2. The perceived date-1 payoff $\tilde{\theta}_i$ is a Skew-Normal distribution:

$$\tilde{\theta}_i \sim SN(\xi_i, \omega^2, \lambda).$$

The moment-generating function (mgf) of $\tilde{\theta}_i$ is

$$M_{\tilde{\theta}_i}(t) = \mathbb{E}_i[e^{t\tilde{\theta}_i}] = 2\exp\left(\xi_i t + \frac{1}{2}\omega^2 t^2\right)\Phi(\lambda\omega t),$$

where ϕ and Φ denote the standard normal pdf and cdf, respectively.

Using the mgf, an investor's expected utility can be written as

$$\mathbb{E}_i[e^{-\gamma w}] = e^{-\gamma w_0} e^{\gamma x p}\mathbb{E}_i\left[e^{-\gamma x\tilde{\theta}_i}\right] = e^{-\gamma w_0} e^{\gamma x p} M_{\tilde{\theta}_i}(-\gamma x).$$

Maximizing expected utility is equivalent to maximizing the certainty equivalent

$$CE_i(x) = -\frac{1}{\gamma}\log\mathbb{E}_i[e^{-\gamma W}] = w_0 - xp - \frac{1}{\gamma}\log M_{\tilde{\theta}_i}(-\gamma x).$$

Substituting the mgf and using the belief parameters (ξ_i, ω, λ) yields

$$CE_i(x) = w_0 - xp - \frac{1}{\gamma}\log 2 + \xi_i x - \frac{1}{2}\omega^2\gamma x^2 - \frac{1}{\gamma}\log\Phi(-\lambda\omega\gamma x).$$

Differentiating with respect to x gives the first order condition

$$\frac{d}{dx}CE_i(x) = -p + \xi_i - \omega^2\gamma x + \frac{1}{\gamma}\frac{\phi(-\lambda\omega\gamma x)(\lambda\omega\gamma)}{\Phi(-\lambda\omega\gamma x)} = 0,$$

so the optimal demand $x_i(p)$ solves

$$\xi_i - p = \omega^2\gamma x_i - \lambda\omega\frac{\phi(-\lambda\omega\gamma x_i)}{\Phi(-\lambda\omega\gamma x_i)}. \tag{A38}$$

We write

$$f(x) := \omega^2\gamma x - \lambda\omega M(-\lambda\omega\gamma x), \quad \text{where} \quad M(x) := \frac{\phi(x)}{\Phi(x)}. \tag{A39}$$

We reference the following established properties of the inverse Mills Ratio function:[32]

Lemma 3. *1.* $-1 < M'(z) < 0$*, based on the result that the variance of a standard normal truncated on* $(-\infty, z]$ *is* $Var(Y) = 1 + M'(z) > 0$.

[32]See Gasull and Utzet (2014) among others.

2. *$M''(z) > 0$, i.e., $M(z)$ is strictly convex (a known property of the reciprocal of the standard Mills Ratio).*

Proof. Recall

$$M'(x) = \frac{-x\phi(x)}{\Phi(x)} - \frac{[\phi(x)]^2}{[\Phi(x)]^2}.$$

Substituting $M(x) = \frac{\phi(x)}{\Phi(x)}$ back into the equation:

$$M'(x) = -M(x)[x + M(x)].$$

Since $M(x) > 0$ and it is known that $x + M(x) > 0$ (the Mills ratio implies $M(x) > -x$), $M'(x)$ is always negative.

Let Z be a standard normal random variable. Consider the variance of Z truncated to the interval $(-\infty, x]$.The variance of this truncated distribution, denoted $V(x) = \text{Var}(Z \mid Z \leq x)$, is given by the formula:

$$V(x) = 1 - xM(x) - M(x)^2$$

Notice that this expression relates directly to our first derivative $M'(x) = -xM(x) - M(x)^2$. Substituting this into the variance equation gives: $V(x) = 1 + M'(x)$. Since $V(x) > 0$, it immediately follows that $M'(x) > -1$.

Now, we differentiate the variance $V(x)$ with respect to x:

$$V'(x) = \frac{d}{dx}(1 + M'(x)) = M''(x).$$

Note that $V(x)$ represents the variance of a standard normal variable truncated at x.As x increases, the truncation constraint is relaxed (we include more of the normal distribution's tail and eventually the bell itself). Relaxing the truncation on a standard normal distribution strictly increases the variance (it moves from a near-zero variance at the far left tail to a variance of 1 as $x \to \infty$). Therefore, $V(x)$ is a strictly increasing function of x. Together with $V'(x) = M''(x)$, it follows that $M''(x) > 0$. □

Returning to the proof of Lemma 2, we show that f is an increasing convex (concave) function of x for negative (positive) λ. First, $f'(x) = \omega^2\gamma + \lambda^2\omega^2\gamma M'(-\lambda\omega\gamma x) = \omega^2\gamma(1 + \lambda^2 M'(-\lambda\omega\gamma x)) >$

$\omega^2\gamma(1+M'(-\lambda\omega\gamma x)) > 0$, where the first inequality comes from $\lambda < 1$ and $M' < 0$, and the second inequality is a result of $M' > -1$.

Next, the second order derivative is $f''(x) = -\lambda^3\omega^3\gamma^2 M''(-\lambda\omega\gamma x)$. Given $M'' > 0$, it follows that $f''(x) > 0$ if and only if $\lambda < 0$, and $f''(x) < 0$ if and only if $\lambda > 0$.

Equation (A38) reduces to

$$\xi_i - p = f(x_i).$$

Taking derivative with respect to p:

$$-1 = f'(x_i)\frac{\partial x_i}{\partial p}.$$

Given $f' > 0$, it implies that $\frac{\partial x_i}{\partial p} < 0$. Taking another derivative with respect to p:

$$0 = f''(x_i)\left(\frac{\partial x_i}{\partial p}\right)^2 + f'(x_i)\frac{\partial^2 x_i}{\partial p^2}.$$

Thus,

$$\frac{\partial^2 x_i}{\partial p^2} = -\frac{f''(x_i)\left(\frac{\partial x_i}{\partial p}\right)^2}{f'(x_i)}.$$

Given that $f''(x)$ has the same sign of $(-\lambda)$, it follows that $\frac{\partial^2 x_i}{\partial p^2}$ has the same sign of λ, which has the same sign of α. This completes the proof.

Proof of Proposition 9. Let g denote the inverse of f: $g(\cdot) := f^{-1}(\cdot)$. An investor's demand schedule is given by

$$x_i(p) = g(\xi_i - p).$$

So the equilibrium price $p = p(d,\lambda)$ satisfies

$$g(\xi + d - p) + g(\xi - d - p) = \chi.$$

Differentiating with respect to d, it follows that

$$\frac{\partial p}{\partial d}(d,\lambda) = \frac{g'(\xi+d-p) - g'(\xi-d-p)}{g'(\xi+d-p) + g'(\xi-d-p)}. \tag{A40}$$

Recall from the proof of Lemma 2 that $f'(x) > 0$, it follows its inverse g is strictly increasing. Therefore the denominator of the above equation is positive. Recall from the proof of Lemma 2 that $f''(x) > 0$ if any only if $\lambda < 0$ and $f''(x) < 0$ if any only if $\lambda > 0$. Since the inverse function flips the convexity/concavity of the original function, it follows that g is convex (concave) if and only if $\lambda > 0$ ($\lambda < 0$).

- If $\lambda > 0$: g is convex, so g' is an increasing function. It follows that $g'(\xi + d - p) > g'(\xi - d - p)$, which implies that $\frac{\partial p}{\partial d}(d, \lambda) > 0$.
- If $\lambda < 0$: g is concave, so g' is a decreasing function. It follows that $g'(\xi + d - p) < g'(\xi - d - p)$, which implies that $\frac{\partial p}{\partial d}(d, \lambda) < 0$.

Finally, when $\lambda = 0$, the Skew-Normal distribution reduces to a Gaussian distribution. In this case, demand schedules are linear and the equilibrium price is given by the average belief and does not depend on the level of disagreement.

Proof of Proposition 10. We first consider $\lambda > 0$. We follow the notation from the proof of Proposition 9. First, $f(g(x)) = x$ implies that $g'(x) = \frac{1}{f'(g(x))} = \frac{1}{\omega^2\gamma(1+\lambda^2 M'(-\lambda\omega\gamma x)}$. Plugging into equation (A40) leads to

$$Q(z) := \frac{\partial p}{\partial d} = \frac{\lambda^2[M'(z) - M'(-z)]}{2 + \lambda^2[M'(z) + M'(-z)]},$$

where $z = \lambda\omega\gamma x(\lambda, d)$, which is positive since $\lambda > 0$. The total derivative of Q with respect to λ is given by the chain rule:

$$\frac{dQ}{d\lambda} = \underbrace{\frac{2(M'(z) - M'(-z))}{(2 + \lambda^2(M'(z) + M'(-z)))^2} 2\lambda}_{\text{Explicit dependence}} + \underbrace{\frac{\partial Q}{\partial z}\frac{dz}{d\lambda}}_{\text{Implicit dependence via } x} \tag{A41}$$

We show that all components of this sum are positive. Since $M'(z)$ is strictly increasing and $z > 0$, we have $M'(z) > M'(-z)$, which implies the explicit dependence term is strictly positive. We then check the sign of the numerator of $\frac{\partial Q}{\partial z}$ (the denominator is always positive by the quotient rule).

Numerator of $\left(\frac{\partial Q}{\partial z}\right)$

$$= \lambda^2 \left[(2 + \lambda^2(M'(z) + M'(-z)))(M''(z) + M''(-z)) - \lambda^2(M'(z) - M'(-z))(M''(z) - M''(-z)) \right]$$
$$= 2\lambda^2 \left[M''(z)(1 + \lambda^2 M'(-z)) + M''(-z)(1 + \lambda^2 M'(z)) \right].$$

Since $M'' > 0$ (from convexity), and $1 + \lambda^2 M' \geq 1 + M' > 0$ (from bounds), every factor and term in the sum is strictly positive. Thus, $\frac{\partial Q}{\partial z} > 0$.

Finally, we inspect the sign of $\frac{dz}{d\lambda}$. The equilibrium condition is $2d = f_\lambda(x) - f_\lambda(-x)$. Substituting f_λ from equation (A39), it follows

$$\frac{2d}{\omega} = H(z, \lambda) := \frac{2z}{\lambda} + \lambda[M(z) - M(-z)].$$

By the Implicit Function Theorem, $\frac{dz}{d\lambda} = -\frac{\partial H / \partial \lambda}{\partial H / \partial z}$.

We note that

$$\frac{\partial H}{\partial z} = \frac{2}{\lambda} + \lambda[M'(z) + M'(-z)] = \frac{2}{\lambda} + \lambda\Sigma.$$

Since $\frac{\partial H}{\partial z} = \frac{2+\lambda^2\Sigma}{\lambda}$ and $2 + \lambda^2\Sigma = \left(1 + \lambda^2 M'(z)\right) + \left(1 + \lambda^2 M'(-z)\right) > 0$, the denominator is positive when $\lambda > 0$.

Similarly,

$$\frac{\partial H}{\partial \lambda} = -\frac{2z}{\lambda^2} + [M(z) - M(-z)]$$

For $z > 0$, so $M(z) - M(-z) < 0$. Thus, $\frac{\partial H}{\partial \lambda} < 0$, and the numerator term $-\frac{\partial H}{\partial \lambda}$ is positive. Since $\frac{dz}{d\lambda} = -\frac{\text{Negative}}{\text{Positive}}$, we conclude that $\frac{dz}{d\lambda} > 0$.

Substituting the signs back into the total derivative (Equation A41), we observe that the total derivative is a sum of strictly positive terms for positive λ. Thus, $\frac{dQ}{d\lambda} > 0$ for $\lambda > 0$. Due to the symmetry of the problem (Q is odd in λ), the derivative is strictly positive for all $\lambda \neq 0$. When $\lambda \to 0$, both the explicit dependence term of (A41) and Numerator of $\left(\frac{\partial Q}{\partial z}\right)$ approaches zero, so $\frac{dQ}{d\lambda} \to 0$.

Appendix B Supplementary Tables

Table A1: Average Implied Volatility by Portfolio (All Stocks)

	LowSkew	2	3	HighSkew
LowDisp	10.3	10.3	10.2	10.5
2	10.9	11.0	11.1	11.5
3	11.7	11.9	12.1	12.6
HighDisp	13.0	13.2	13.7	14.4

Note: This table reports the sample average of monthly average option-implied volatility (in percent) for stocks independently double-sorted into DISP and SKEW quartiles, where double sorting is applied within size quartiles and aggregated. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

Table A2: Average Implied Volatility by Portfolio (Low Vol)

	LowSkew	2	3	HighSkew
LowDisp	8.9	8.9	8.8	8.5
2	9.2	9.3	9.2	9.1
3	9.6	9.8	9.7	9.5
HighDisp	10.0	10.3	10.3	10.1

Note: This table reports the sample average of monthly average option-implied volatility (in percent) for stocks independently double-sorted into DISP and SKEW quartiles, where double sorting is applied within size quartiles and aggregated. The sample includes firms with below-median implied volatility. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

Table A3: Average Implied Volatility by Portfolio (High Vol)

	LowSkew	2	3	HighSkew
LowDisp	13.7	13.5	13.4	13.6
2	13.6	13.6	13.6	13.9
3	13.9	14.0	14.2	14.5
HighDisp	14.6	14.7	15.2	15.7

Note: This table reports the sample average of monthly average option-implied volatility (in percent) for stocks independently double-sorted into DISP and SKEW quartiles, where double sorting is applied within size quartiles and aggregated. The sample includes firms with above-median implied volatility. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

Table A4: Average Implied Volatility by Portfolio: Alternative Sorting

	LowSkew	2	3	HighSkew
LowDisp	12.0	11.8	11.6	11.6
2	11.7	11.9	11.9	12.2
3	11.7	11.9	12.2	12.6
HighDisp	11.7	12.0	12.5	13.0

Note: This table reports the sample average of monthly average option implied volatility (in percent) for stocks independently double-sorted into DISP and SKEW quartiles, where each month double sorting is performed within implied volatility quartiles and then aggregated. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

Table A5: Average SKEW by Portfolio: Alternative Sorting

	LowSkew	2	3	HighSkew
LowDisp	0.32	0.58	0.77	1.07
2	0.31	0.58	0.77	1.10
3	0.30	0.57	0.79	1.12
HighDisp	0.28	0.58	0.79	1.15

Note: This table reports the sample average of median monthly SKEW for stocks independently double-sorted into DISP and SKEW quartiles, where each month double sorting is performed within implied volatility quartiles and then aggregated. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

Table A6: Average DISP by Portfolio: Alternative Sorting

	LowSkew	2	3	HighSkew
LowDisp	0.02	0.02	0.02	0.02
2	0.04	0.04	0.04	0.04
3	0.07	0.07	0.07	0.07
HighDisp	0.16	0.17	0.18	0.19

Note: This table reports the sample average of median monthly DISP for stocks independently double-sorted into DISP and SKEW quartiles, where each month double sorting is performed within implied volatility quartiles and then aggregated. DISP is analyst forecast dispersion (cf. Diether et al., 2002). SKEW is expected idiosyncratic skewness (Boyer et al., 2010).

B.1 Predicted Volatility

Welch (2017)

As in Bekaert et al. (2025), we define realized variance in month m as

$$RV_{i,m} = \sum_d r_{i,d}^2, \tag{A42}$$

where the summation is over trading days. We restrict to firm-months with at least 15 days with actual trading ($prc > 0$ since CRSP uses negative prices to indicate a midpoint when there is no closing price) and where the minimum closing price during the month is greater than $5/share. Further, we drop firm-months where more than 50% of returns are identically zero (this is the 99th percentile).

We then take logs, $rv_{i,m} = log\left(RV_{i,m}\right)$ and estimate the following predictive regression using the full CRSP universe (restricted to common equity of U.S. firms)

$$rv_{i,m+1} = b_0 + b_1 \times rv_{i,m} + b_2 \times rv_{agg,m} + b_3 \times \log(\text{Mktcap})_{i,m} + b_4 \times \log(\text{Turnover})_{i,m} + \nu_{i,m+1}, \tag{A43}$$

where $rv_{agg,m}$ is the cross-sectional average of $rv_{i,m}$, $\log(\text{Mktcap})_{i,m}$ is log of the actual market capitalization of firm i divided by the total market capitalization of all firms in the sample, and $\log(\text{Turnover})_{i,m}$ is total trading volume scaled by shares outstanding. This is essentially an AR(1) model for realized volatility, augmented with additional predictors, and with pooled estimation. Bekaert et al. (2025) argue that modeling realized variances as an ARMA(1,1) with separate parameters for each firm is theoretically reasonable and empirically superior to other methods they consider. However, their methodology only includes firm-months with at least 36 trailing monthly observations, reducing its usefulness in our context. They show, however, that a simpler AR(1) specification performs nearly as well. To address the issue of parameter heterogeneity, we also estimate two extensions. First, we include industry fixed-effects since the level of volatility definitely varies across business types. Second, we also interact the industry identifiers with $rv_{i,m}$ to allow heterogeneity in the AR(1) persistence parameter across business types.

Table A7 presents estimates of $100 \times b$ as well as the regression R^2 (t-statistics are in parentheses and standard errors are clustered by month). As shown in previous work, there is substantial predictability of volatility, mainly through past volatility. The second row shows that including industry fixed effects has negligible impact on the estimated coefficients as well as R^2. Finally, the third row shows estimates when allowing for separate b_1 for each industry. Instead of displaying 49 estimates (48 industries plus a category for SIC=9999), we display the average coefficient and t-stat. Amazingly, there is almost no improvement in R^2 even after allowing for subtstantial heterogeneity. Therefore, for reasons of parsimony and statistical efficiency, we use the first model with no industry-specific variables.

Exponentiating Equation A43, taking expectations and using the definition of covariance, we have

$$E\left(RV_{i,m+1}\right) = \exp\left(\widehat{rv}_{i,m+1}\right) \times E\left(\exp\left(\nu_{i,m+1}\right)\right) + \text{cov}\left[\exp\left(\widehat{rv}_{i,m+1}\right),\ \exp\left(\nu_{i,m+1}\right)\right] \tag{A44}$$

where we estimate $E\left(\exp\left(\nu_{i,m+1}\right)\right) \approx 1.5$ and $\text{cov}[\cdot,\cdot] \approx 17$ b.p from the full sample of predictions and forecast errors. Finally, we define the predictive standard deviation as $\widehat{\sigma}_{i,m} = \sqrt{E\left(RV_{i,m+1}\right)}$. We then proceed as in Section 4.3. For comparison, when predicting realized standard deviation on the subsample of firms with option-implied volatility, the predictive model and and option-implied vol both predict realized standard deviaton with $\approx 40\%$ R^2.

Table A7: Volatility Forecasting

	$rv_{i,m-1}$	$rv_{agg,m-1}$	log(Mktcap)	log(Turnover)	Adj. R^2 (%)
No FE	57.8	17.3	-8.6	6.6	47.0
	(120.62)	(5.80)	(31.10)	(11.33)	
Industry FE	53.6	21.7	-8.8	4.1	48.4
	(114.74)	(7.28)	(31.67)	(7.34)	
Interacted	51.4	22.0	-9.0	4.3	48.4
	(56.40)	(7.40)	(-32.41)	(7.61)	

Note: This .

Table A8 is a repeat of Table 6 except we scale returns by predicted (instead of option-implied) volatility. We also repeat the alternative sorting procedure: each month, we sort stocks into four quartiles based on predicted volatility, then perform an independent double-sort on DISP and SKEW within each quartile, and finally collapsing the volatility dimension to yield 16 portfolios. In Table A9 we report the interaction effect separately for all firms (first row) and excluding the bottom quartile of firms based on market capitalization (second row).

Table A8: Interaction Effect for Scaled Returns: By Predicted Volatility

	Excess	CAPM	FF5	DMRS
All Firms	-3.9	-4.7	-4.5	-4.7
	(1.98)	(2.37)	(2.24)	(2.30)
Low Vol	-1.3	-1.9	-1.6	-1.5
	(0.56)	(0.81)	(0.67)	(0.61)
High Vol	-3.8	-4.3	-4.1	-4.5
	(1.60)	(1.84)	(1.77)	(1.95)
High - Low	-2.5	-2.4	-2.5	-3.0
	(-0.99)	(-0.91)	(-0.93)	(-1.10)

Note: The first three rows of this table report the (annualized) average monthly interaction effect for returns scaled by predicted volatility. The first row includes all firms, but the second and third rows, respectively, include stocks partitioned by below-median or above-median predicted volatility. The last row tests the difference between the second and third rows of the table. Heteroskedasticity and autocorrelation consistent absolute (Newey and West, 1987) t-statistics are in parentheses.

Table A9: Interaction Effect for Scaled Returns: Alternative Sorting

	Excess	CAPM	FF5	DMRS
All Firms	-4.5	-4.7	-5.0	-4.8
	(2.48)	(2.65)	(2.77)	(2.56)
No Small	-5.2	-5.6	-5.8	-5.5
	(2.79)	(3.04)	(3.06)	(2.80)

Note: This table report the (annualized) average monthly interaction effect for returns scaled by predicted volatility. Portfolios are formed using the alternative sorting procedure which controls for predicted volatility. The first row includes all firms, but the second row excludes stocks in the bottom quartile of market capitalization. Heteroskedasticity and autocorrelation consistent absolute (Newey and West, 1987) t-statistics are in parentheses.