Consumer-Optimal Segmentation in Multi-Product Markets

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Price Discrimination

- Second- and third-degree price discrimination (quality differentiation and market segmentation, respectively) may either benefit or harm consumers
- Most research analyzes these practices in isolation
- Increasingly, these tools are deployed simultaneously, particularly in the data-rich digital economy
    - Student versions of digital products are typically inferior, while enterprise versions have more features
    - Streaming services offer differentiated subscription tiers
- Open question of how these two forms of price discrimination interact and impact consumers

The Paper

- Comprehensive analysis of consumer-optimal segmentation with a monopolist that offers quality-differentiated products

Main results:

1. Completely characterize the value of segmentation
2. Consumption is monotone in valuation across segments (and within segments, by incentive compatibility)
3. Seller offers "essentially" same quality to identical consumers
4. Every segmentation hurts consumer surplus when aggregate demand elasticity is sufficiently high

Today

- Present our model of second- and third-degree price discrimination
- Work through simple example (binary value setting) to introduce concepts used in analysis
- Formally state our main result: value of segmentation

Products and Utilities

- There is a monopolist and a continuum of consumers
- Produces a vertically differentiated good with quality $q \in \mathbb{R}_+$
- Cost of producing good of quality q is given by increasing and convex function $c : \mathbb{R}_+ \to \mathbb{R}_+$

- Each consumer has a privately known willingness to pay v drawn from a finite set $V = \{v_1, \ldots, v_K\}$
- Consumer's gross utility is $v \cdot q$

Markets I

- A *market* $x \in \Delta V$ is a distribution over V, where $x_k \triangleq x(v_k)$ is the probability of value v_k
- Given a *menu* of prices $p(q)$, consumers purchase quality which maximizes their utility:

$$q(v,p) \triangleq \arg\max_q \left[v \cdot q - p(q)\right], \;\; U(v,p) \triangleq \max_q \left[v \cdot q - p(q)\right]$$

- Profits are

$$\Pi(v,p) \triangleq p(q(v)) - c(q(v))$$

Markets II

- The profit-maximizing menu is denoted p^x:

$$p^x \triangleq \arg\max_{p(q)} \sum_{k=1}^{K} x_k \Pi(v_k, p)$$

- This gives us the profit and consumer surplus in a market:

$$U(x) \triangleq \sum_{k=1}^{K} x_k U(v_k, p^x), \quad \Pi(x) = \sum_{k=1}^{K} x_k \Pi(v_k, p^x)$$

Segmentations

- Fix some *aggregate market* $x^* \in \Delta V$
- A *segmentation* σ of x^* is a distribution over ΔV such that

$$\sum_{x \in \Delta V} \sigma(x) x = x^*$$

- Given a segmentation σ of x^*, the seller optimally sets prices p^x for every market x in the support of σ
- We focus on the *consumer-optimal segmentation*, which solves

$$\max_{\sigma \in \Delta(\Delta V)} \left[\sum_x \sigma(x) U(x) \right] \text{ s.t. } \sum_x \sigma(x) x = x^*$$

- This is a Bayesian Persuasion (BP) problem over K states

Binary Values

- Assume $V = \{v_L, v_H\}$ with $x^* = (x_L^*, x_H^*)$
- Define the *hazard rate*

$$h^x \triangleq (v_H - v_L)\frac{1 - x_L}{x_L}, \quad h^* \triangleq (v_H - v_L)\frac{1 - x_L^*}{x_L^*}$$

- Denote by Q the "supply function":

$$Q(v) = (c')^{-1}(v)$$

- Applying standard results (Mussa and Rosen (1978)), total consumer surplus is equal to

$$U(x) = x_H(v_H - v_L)Q(v_L - h^x)$$

Hazard Rate Distributions

- Given a segmentation σ, define the following weights:

$$\lambda^x \triangleq \frac{\sigma(x) x_L}{x_L^*}$$

- λ^x defines a distribution over hazard rates

$$\sum_x \lambda^x = 1$$

which average to the *aggregate hazard rate*

$$\sum_x \lambda^x h^x = h^*$$

Local Information Rent

- Next, define the *local information rent*

$$u_L(h) \triangleq h \cdot Q(v_L - h)$$

- We can rewrite

$$U(x) = x_L u_L(h^x) \implies \sum_x \sigma(x) U(x) = x_L^* \sum_x \lambda^x u_L(h^x)$$

- Consumer surplus is the expected value of the local information rent over the distribution λ^x

Bayesian Persuasion

- Consider the Bayesian persuasion problem where we directly choose a distribution over hazard rates:

$$\max_{\lambda \in \Delta \mathbb{R}_+} \left[x_L^* \sum_h \lambda(h) u_L(h) \right] \text{ s.t. } \sum_h \lambda(h) h = h^* \qquad (1)$$

- Kamenica and Gentzkow (2011): let $\overline{u}_L$ be the concave upper envelope of u, then the solution to (1) is $x_L^* \overline{u}_L(h^*)$
- Already showed: every segmentation σ is feasible in (1)
- Can every feasible λ be generated by a segmentation?

Value of Segmentation

Proposition

The consumer-optimal segmentation achieves the concavification bound (1) *with equality:*

$$\max_{\sigma}\left[\sum_{x}\sigma(x)U(x)\right] = x_L^*\overline{u}_L(h^*).$$

- Segmentation increases consumer surplus if and only if

$$\overline{u}_L(h^*) > u_L(h^*)$$

- We can read off the (unique) consumer-optimal segmentation from the solution to (1)

Example: Isoelastic Costs I

- Suppose that
$$c(q) = \frac{q^\gamma}{\gamma}, \quad \gamma > 1$$
- u_L and $\overline{u}_L$ plotted below

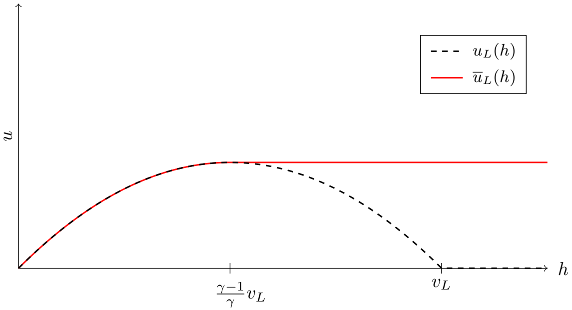

Figure 1

Example: Isoelastic Costs II

- Segmentation improves consumer surplus if and only if

$$h^* > \frac{\gamma - 1}{\gamma} v_L \iff \eta^* < \frac{\gamma}{\gamma - 1}$$

- If the optimal segmentation is non-trivial, v_L consumes quality

$$q(v_L) = \left(\frac{v_L}{\gamma} \right)^{\frac{1}{\gamma - 1}}$$

Generalized Objects

- Denote by D^x the demand associated with market x:

$$D_k^x \triangleq D^x(v_k) = \sum_{i \geq k} x_i$$

- Generalize objects from before to higher dimensions:

$$h_k^x \triangleq (v_{k+1} - v_k)\frac{D_{k+1}^x}{x_k}, \quad u_k(h) = h \cdot Q(v_k - h)$$

- In a regular market,[1]

$$U(x) = \sum_{k=1}^{K-1} x_k u_k(h_k^x) \tag{2}$$

- Repeat the analysis of the two-value case $K - 1$ times, but now we need an aggregate constraint

[1] It is without loss to restrict to distributions supported on regular markets.

Pointwise Concavification

- Let $D \prec D^*$ denote the majorization constraint ▸ Definition
- Define the hazard rate associated with D:

$$h_k^D \triangleq (v_{k+1} - v_k)\frac{D_{k+1}}{x_k^*}$$

Theorem

The consumer-optimal σ attains the concavification bound:

$$\max_{\sigma} \left[\sum_x \sigma(x) U(x) \right] = \max_{D \prec D^*} \sum_{k=1}^{K-1} x_k^* \overline{u}_k(h_k^D) \qquad (3)$$

Discussion

Theorem

The consumer-optimal σ attains the concavification bound:

$$\max_{\sigma}\left[\sum_{x}\sigma(x)U(x)\right]=\max_{D\prec D^*}\sum_{k=1}^{K-1}x^*(v_k)\overline{u}_k(h_k^D)$$

- Original problem is a maximization over $\Delta(\Delta V)$; this is constrained maximization over $\mathbb{R}_+^K$ (c.f. Kleiner et al. (2021))
- In paper: frequently, $D=D^*$ is optimal

Proof Idea

Results in Paper

- Across segments, quality consumed across types is monotone, and we can bound quality dispersion within types
- Qualitative description of optimal segmentation under a broad class of cost functions and aggregate markets
    - Low types consume higher quality than under no segmentation
    - Segmentation has no benefit when demand elasticities in aggregate market are sufficiently high
- Extend methodology to Pareto frontier of surplus divisions

Related Literature

Market segmentation and consumer surplus: Pigou (1920); Robinson (1969); Schmalensee (1981); Varian (1985); Aguirre et al. (2010); Cowan (2016)

Second degree price discrimination: Mussa and Rosen (1978); Maskin and Riley (1984); Johnson and Myatt (2003)

Single good benchmark: Bergemann et al. (2015)

Segmentation with multi-product monopolist: Haghpanah and Siegel (2022, 2023); today's results are orthogonal

Conclusion

- Characterize how market segmentation affects consumer surplus when monopolist can engage in quality differentiation
- Suggests that, for competition authorities, segmentation should be evaluated based on market characteristics
- Further work may be able to develop empirical tests

Thank you!

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Quasi-Markets

- We say that $D \in \mathbb{R}_+^K$ is *majorized* by D^*, $D \prec D^*$, if

$$\sum_{i=k}^{K-1} (v_{i+1} - v_i) D_{i+1} \leq \sum_{i=k}^{K-1} (v_{i+1} - v_i) D^*_{i+1}$$

  for all $k \in \{1, \ldots, K-1\}$
- D is a "quasi-market": may be increasing at points
- If D were a market, equivalent to MPS of D^*

Back

Proof Idea

- Majorization constraint arises from analyzing the way support gaps affect the average hazard rate across markets
- From this, upper bound is straightforward as a double relaxation: u to $\overline{u}$, and D^* to $D \prec D^*$
- Showing that the upper bound is tight is convoluted; proof explicitly constructs a segmentation achieving it

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