Mosaics of Predictability

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Motivation: Return Predictability

- Return predictability is well-documented empirically:

  —— Aggregate market return (e.g., Campbell and Thompson, 2008).

  —— Individual stock return (e.g., Fama and French, 2008; Rapach, Strauss, and Zhou, 2013; Lewellen, 2015; Han, He, Rapach, and Zhou, 2024).

- Studies regard predictability as an attribute of predictors or models.

  —— Agg. predictors (e.g., dividend yield) and char. (e.g., size or value).

  —— Models include historical average (e.g., Welch and Goyal, 2008) and machine learning (e.g., Gu, Kelly, and Xiu, 2020).

- We find that predictability is heterogeneous for stocks and varies over time.

  —— Does high predictability imply high return?

  —— It might a characteristic and an anomaly!

Motivation: Heterogeneous Predictability

- Some ad-hoc empirical evidence that predictability is not homogeneous:

  —— Some stocks (small-cap, distressed) are more predictable than others (e.g., Avramov, Cheng, and Metzker, 2023).

  —— Return predictability might be time-varying (e.g., Henkel et al., 2011).

- However, predictability is

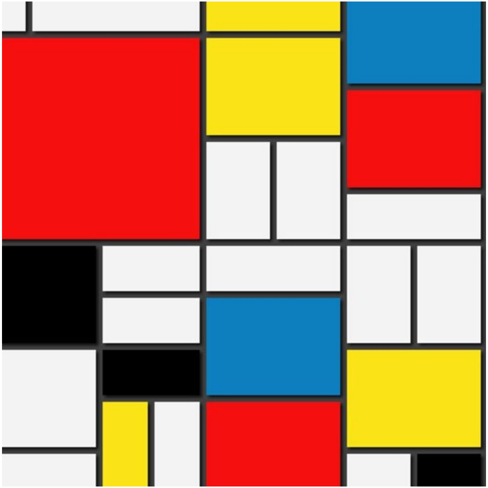
  —— An unobservable characteristic.

  —— Even not well-defined (e.g., anomaly average return, predictor significance, out-of-sample R^2, forecast-implied portfolio).

- Before exploiting heterogeneous predictability, we need to measure them.

- Separate predictable observations from less predictable ones —— clustering.

Mosaics of Predictability: Mondrian



We partition the panel of returns by their heterogeneous predictability!

Our Clustering Solution

- Tree-based clustering (self-supervised) to separate and group asset returns
  $\Longrightarrow$ **Mosaics of Predictability**.

- Objective: Maximize differences in predictability across groups.

- Interpretable: a decision tree based on firm char. and/or agg. predictors

- NOT a horse race of return prediction accuracy!
  —— We revisit a classic problem from a new angle!
  —— We study heterogeneous return predictability and the cross section.

Clustering and Split Criterion

- Never know the true label for stocks (see K-means applications, e.g., Ahn et al., 2009; Patton and Weller, 2022; Evgeniou et al., 2023)
    - "Unsupervised" subsample analysis: char. quintiles or industry classifications
    - "Goal-oriented" clustering by a decision tree: economic model objective
- Heterogeneous predictability between clusters: (in-sample) signal / noise

$$R^2_{\text{leaf}_j} = 1 - \frac{\sum_{\{i,t\}\in\text{leaf}_j}(r_{i,t} - \widehat{r}_{i,t})^2}{\sum_{\{i,t\}\in\text{leaf}_j}(r_{i,t} - 0)^2},$$

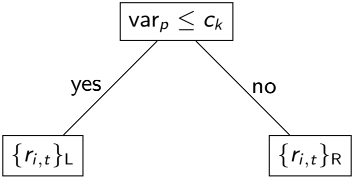
which is impossible to calculate for each stock!

- $\widehat{r}_{i,t}$: volatility-weighted Ridge regression (avoid dominance of microcaps).

Panel tree (e..g, Cong et al., 2024) for clustering observations by predictability.

$\Longrightarrow$ maximizing between-cluster R^2 difference to differentiate predictability.

Split Criterion and Tree Growth

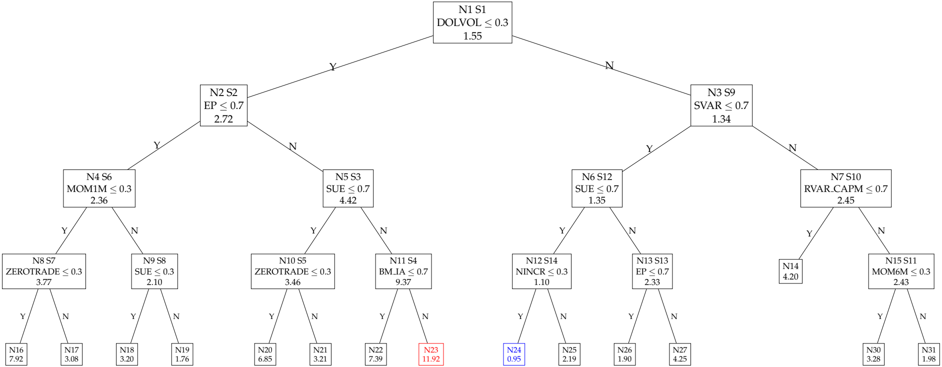


- Candidate cutpoints: $\{0.3, 0.7\}$ among standardized range $[0, 1]$.
  — monthly cross-sectional ranked char. or agg. predictor.
  — For example, small-cap on the left and non-small-cap on the right.

- Calculate the model objective for a splitting candidate (var_p, c_k) on the R^2 difference, which differentiates predictability:

$$S_{\{\text{leaf}_L, \text{leaf}_R\}}(\text{var}_p, c_k) = |R^2_{\text{left}} - R^2_{\text{right}}|$$

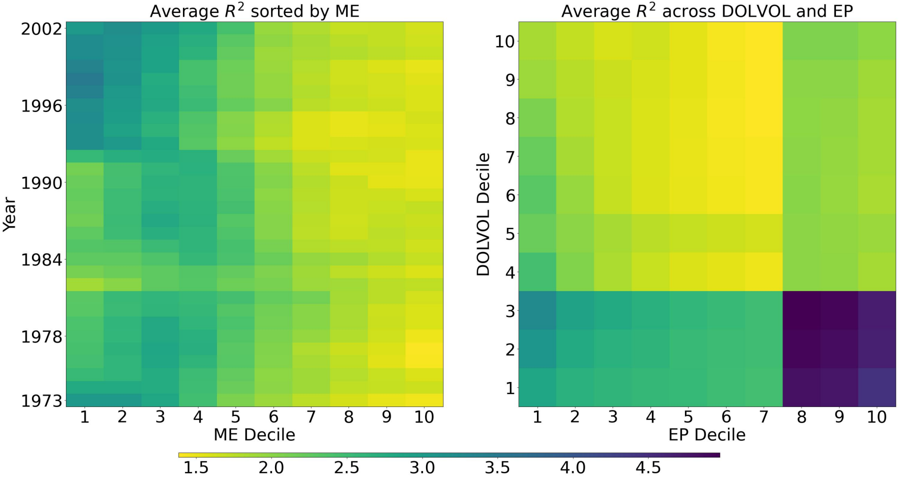
- Greedy algorithm: leaf-wise tree growth, search all combinations, and find the best splitting candidate to partition the subsample into two.



- Highly Predictable: N23 (11.92%) about 0.7%
  $\mathbb{1}\{\text{DOLVOL} \leq 0.3\}\mathbb{1}\{\text{EP} > 0.7\}\mathbb{1}\{\text{SUE} > 0.7\}\mathbb{1}\{\text{BM\_IA} > 0.7\}$

- Less Predictable: N24 (0.95%) about 35.9%
  $\mathbb{1}\{\text{DOLVOL} > 0.3\}\mathbb{1}\{\text{SVAR} \leq 0.7\}\mathbb{1}\{\text{SUE} \leq 0.7\}\mathbb{1}\{\text{NINCR} \leq 0.3\}$

Mosaics of Predictability - Chars Sorting



Evaluations of Predictability (R^2, %)

- **Cluster-wise heterogeneous models $>$ Global homogeneous model**
- **Highly predictable clusters show persistently high R^2s out of sample.**

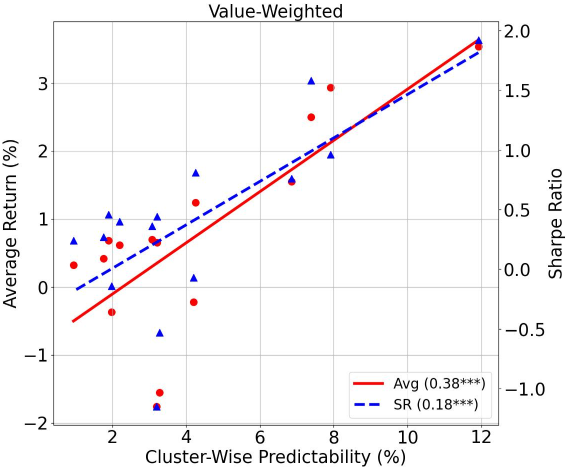
	1973 - 2002				2003 - 2022			
Forecasts	All	High	Medium	Low	All	High	Medium	Low
Global	1.00	2.02	1.01	0.77	0.47	1.62	0.45	0.32
CW	1.60	**6.50**	1.66	0.43	0.62	**2.05**	0.63	0.34

— Global: Homogeneous predictive model (similar to GKX2020).

— CW: Cluster-wise heterogeneous models.

Characteristic: Predictability v.s. Return

- Cluster-wise value-weighted average return v.s. $R^2 \Longrightarrow$ monotonic trend.
- high predictability $\Longrightarrow$ high average return



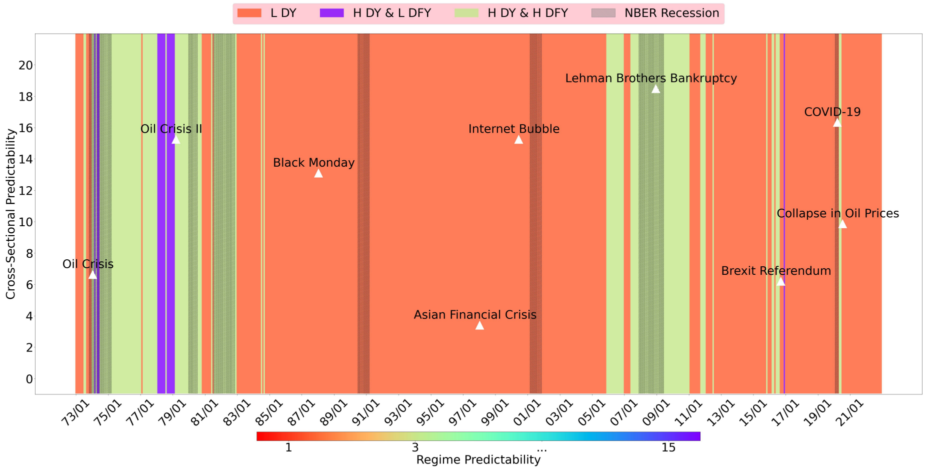
Heterogeneous Predictability Anomaly

	1973 - 2002 (in-sample)			2003 - 2022 (out-of-sample)		
	L5	S1	L5-S1	L5	S1	L5-S1
			Panel A: Performance			
Avg (%)	2.35	0.32	2.03	1.81	0.73	1.08
Ann. SR	1.35	0.24	1.85	1.03	0.58	1.13
			Panel B: Unexplained monthly alphas (%)			
CAPM	1.95***	-0.08	2.03***	0.87***	-0.06	0.92***
FF3+MOM	1.67***	-0.09**	1.76***	0.98***	-0.04	1.01***
FF5	1.42***	-0.19***	1.61***	1.00***	-0.08**	1.07***
FF5+MOM+IVOL	1.59***	-0.12***	1.72***	1.05***	-0.07**	1.12***
Q5	1.59***	-0.04	1.64***	1.03***	-0.06	1.09***
BS6	1.35***	-0.14***	1.49***	0.91***	-0.06*	0.98***

Percentage of stocks in the sorted portfolio:

- L5: 7.7%
- S1: 35.9%

Heterogeneous Predictability and Regime Switches



- Regimes partitioned by S&P 500 dividend yield (DY) and default yield (DFY).
- Time-series partitions display larger predictability heterogeneity (color bar).
- Numerous events trigger regime changes (e.g., Oil Crisis, COVID-19).

- A by-product: cluster-wise heterogeneous predictive model
  —— global models fail to account for modeling heterogeneity (Feng and He, 2022; Evgeniou et al., 2023)
- Forecast-Weighted portfolio (based on the normalized predictions)
- Highly predictable clusters show the highest investment gains!

	In-Sample (1973 - 2002)					Out-of-Sample (2003 - 2022)				
	Avg	Std	SR	Alpha	MDD	Avg	Std	SR	Alpha	MDD
Global	2.28	3.57	2.21	2.16***	17.84	1.44	4.43	1.13	0.75***	19.36
Aggregate	3.09	3.86	2.77	2.95***	12.24	1.83	4.37	1.45	1.22***	19.46
High	4.46	8.72	1.77	3.99***	27.71	3.24	6.87	1.63	2.34***	19.73
Medium	3.10	3.99	2.69	3.02***	23.14	1.95	4.61	1.46	1.54***	19.98
Low	1.18	3.08	1.33	1.02***	13.71	0.86	4.77	0.62	0.06	21.80

Summary

- Mosaics of predictability — heterogeneity of return predictability.
- Unexplained anomaly: high predictability $\Longrightarrow$ high average return
- Tree-based clustering approach — based on firm char. and agg. predictors.
- **All comments are welcome! Thank you!**

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