Dominant Currency Paradigm with Input-Output Linkages

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Outline

Outline

Motivations

1. Invoicing currency in global trade
   - In global trade, countries use different invoicing currency, dominantly the US dollar, i.e., DCP PCP LCP DCP
   - Each pricing paradigm has different implication on the exchange rate pass-through (ERPT) to prices and quantities

2. Input-Output (I-O) linkages in global trade
   - Countries engage more in global I-O linkages as the global value chain rises
   - Incorporating I-O linkages in global trade changes ERPT implications

→ **Question**: How US dollar appreciation* affects global trade volume under (exogenous) invoicing currency and I-O linkages?

* Caused by exogenous shocks such as a contractionary US MP shock

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- Claim 1: *Global trade is close to DCP*
  - Close to DCP $\Leftrightarrow$ Expenditure switching present on the import side
  - Data: US dollar's dominance in global trade (Gopinath 2016)
  - Empirics: Dollar exchange rate dominates bilateral exchange rate in pass-through to prices and quantities (Gopinath et al 2020) Detail
- Claim 2: *Global trade is close to LCP*
  - Close to LCP $\Leftrightarrow$ Expenditure switching absent on both import and export
  - As final good prices (supermarket prices) are sticky in local currency, DCP at the border is insignificant

→ **Question**: Is the world trade closer to LCP or DCP?

→ How? Quantitative model!

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Main results of the paper

1. (**Theory**) Global trade response depends on both invoicing currency and I-O linkages through interaction
   - *Direct exposure* on the dollar through the dollar invoicing
   - *Indirect exposure* on the dollar through the I-O linkages
2. (**Quantitative**) Global trade lies in between DCP and LCP in response to dollar appreciation
   - Full calibration close to *Half DCP, Half LCP*
   - I-O linkages can amplify the effect on global trade by $-0.3\% \sim -0.6\%$ per 1% dollar appreciation

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Baseline Model Overview

- **2 country open economy model**: Home (H) and ROW (F)
- **Households**
  - Utility: log-linear
  - Demand structure: Cobb-Douglas between domestic and foreign final goods (Home bias: $1-\gamma$)
  - Cash-in-Advance
- **Producers**
  - Cobb-Douglas Production combining domestic and foreign int. goods (Foreign int. input share of H: $(1-\alpha)\phi$, of F: $(1-\alpha^*)\phi^*$)
- **Calvo sticky prices** Sticky prices
  - Domestic prices: Invoiced in own currency
  - International prices: Invoiced in PCP, LCP, or DCP
  - Dollar invoicing shares:

	H→F	F→H
Final good	θ_D^C	θ_D^{C*}
Int. good	θ_D^X	θ_D^{X*}

→ **Dollar exposure** of consumer/producer in each country

	Home (H)	ROW (F)
Consumer	$\gamma\theta_D^{C*}$	$\gamma\theta_D^C$
Producer	$(1-\alpha)\phi\theta_D^{X*}$	$(1-\alpha^*)\phi^*\theta_D^X$

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Global trade response

Theorem (Fully sticky price)

When prices are fully sticky ($\delta = 0$), then

$$\frac{dx_R}{dm^{\$}} = \left(\frac{w}{\bar{Y}}\psi_{11} + \frac{1-w}{\bar{Y}^*}\psi_{12}\right)\left\{\gamma\theta_D^C + (1-\alpha^*)\phi^*\bar{Y}^*\theta_D^X - (1-\alpha)\phi\bar{Y}\theta_D^{X*}\right\}$$
$$+ \left(\frac{w}{\bar{Y}}\psi_{21} + \frac{1-w}{\bar{Y}^*}\psi_{22}\right)\left\{\gamma\theta_D^{C*} + (1-\alpha)\phi\bar{Y}\theta_D^{X*} - (1-\alpha^*)\phi^*\bar{Y}^*\theta_D^X\right\}$$

where ψ_{ij} is an element of Leontief inverse Ψ

$$\Omega = \begin{bmatrix} (1-\alpha)(1-\phi) & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & (1-\alpha^*)(1-\phi^*) \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} = (I - \Omega)^{-1}$$

Global trade value

Global trade response

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First-round effect ($d\mu \uparrow \rightarrow dx_R \downarrow$)

- Dollar appreciation leads to markup increase of DCP firms by direct exposure on dollar
- Home consumer exposed by $\gamma\theta_D^{C*}$
- Home producer exposed by $(1-\alpha)\phi\theta_D^{X*}$
- Foreign consumer exposed by $\gamma\theta_D^C$
- Foreign producer exposed by $(1-\alpha^*)\phi^*\theta_D^X$

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Second-round effect ($d\mu \downarrow \rightarrow dx_R \uparrow$)

- Due to sticky price, marginal cost increase is offset by markup decrease
- Home output increase by $(1-\alpha)\phi\theta_D^{X*}$
- Foreign output increase by $(1-\alpha^*)\phi^*\theta_D^X$
- Two effects work in a opposite direction

Global trade response

Theorem (Fully sticky price)

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$$\frac{dx_R}{dm^{\$}} = \left(\frac{w}{\bar{Y}}\psi_{11} + \frac{1-w}{\bar{Y}^*}\psi_{12}\right)\left\{\gamma\theta_D^C + (1-\alpha^*)\phi^*\bar{Y}^*\theta_D^X - (1-\alpha)\phi\bar{Y}\theta_D^{X*}\right\}$$
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I-O linkages amplification Examples

Outline

Model Extension

	Baseline	**Quantitative**
Countries	Two-country (excl. US)	Multi-country (incl. US)
Utility	Log-linear	CRRA
Sticky price	(Static) Calvo	(Dynamic) Calvo
Demand structure	Cobb-Douglas	Kimball demand with Klenow-Willis
Trade linkages across countries	Heterogeneous home bias (γ) and I-O linkages (ω)	
Invoicing currency across countries	Heterogeneous in final goods (θ^C) and intermediate good trades (θ^X)	
Exogenous shocks	US Money supply ($dm^{\$}$)	US MP shock Shocks to UIP deviation

Quantitative Model

- Household problem Link
  - $\rightarrow$ UIP condition: $i_{j,t} - i^{\$}_{j,t} = \mathbb{E}_t\left[e_{\$j,t+1} - e_{\$j,t}\right] + \boldsymbol{\xi_{j,t}}$
- Production Link
  - $\rightarrow$ Log marginal cost: $mc_{j,t} = \alpha w_{j,t} + (1-\alpha)p^X_{j,t} - \boldsymbol{a_{j,t}}$
- Pricing Link
- Demand Structure Link
- Monetary Policy: Taylor rule
  - $\rightarrow$ $i_{j,t} - i^* = \rho_m(i_{j,t-1} - i^*) + (1-\rho_m)\left(\phi_M \pi_{j,t} + \phi_Y(y_{j,t} - \bar{y}_j)\right) + \boldsymbol{\varepsilon_{j,t}}$

Data and Calibration

- Parameter set 1: Home bias (γ_{ji}) and I-O linkages (ω_{ji})
  - Source: World Input-Output Database
  - Pick 12 countries out of 43 ordered by size of GNE List
- Parameter set 2: Invoicing shares of bilateral trade in final goods ($\theta_{ji}^{C,k}$) and intermediate good ($\theta_{ji}^{X,k}$)
  - Source: Boz et al. (2020) Data
  - Caveat: Data is available in country-level, not in dyad (country pair)
  - Identifying assumption:
    - (**Calibration 1**) Dollar invoicing shares are identical across importers
    - (**Calibration 2**) Dollar invoicing shares are identical across exporters
- Other Parameters: Standard values Link
- Shock Calibration: Match empirical moments Link
- Moment Matching result Link

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Counterfactual 1: World trade LCP or DCP?

Under Calibrated IO linkages, is the world trade close to LCP or DCP?

Scenario	Δ World trade (%p)	
	US MP shock	UIP shocks
Invoicing Calibration 1	-0.56	-0.73
Invoicing Calibration 2	-0.55	-0.72
Full LCP	0.10	-0.02
Final LCP, Int. DCP	-0.23	-0.34
Half LCP, Half DCP	**-0.60**	**-0.72**
Full DCP	-1.29	-1.42

Table 1: World trade response under 1%p dollar appreciation

- (US MP shock) Normalize MP shock size such that dollar appreciates by 1% against all other currencies
- (UIP shocks) Cumulate UIP deviations for each county such that dollar appreciates by 1% against all other currencies

Counterfactual 2: IO linkages

Under Calibrated invoicing shares, what is the role of IO linkages?

Scenario	Δ World trade (%p)	
	US MP shock	UIP shocks
Calibrated IO	-0.56	-0.73
High Corr(IO, Invoicing)	-0.86	-1.31
Low Corr(IO, Invoicing)	-0.12	-0.26

Table 2: World trade response under 1%p dollar appreciation

Reorder (γ, Ω) matrix such that:

- (High Corr) Importers switch expenditures *toward* dollar invoicing exporters
- (Low Corr) Importers switch expenditures *away* from dollar invoicing exporters

Conclusion

- Dollar exchange rate affects global trade depending on how invoicing currency and IO linkages are different across countries
  - Invoicing currency relates direct exposure on dollar
  - IO linkages relates indirect exposure on dollar
- Quantitatively, world trade is close to Half LCP/Half DCP in final good trade and int. good trade

Three types of Currency of Invoicing

- **Producer Currency Pricing (PCP)**: Export prices are invoiced in currency of exporters (producers)
  - Mundell (1963), Fleming (1962), Obstfeld and Rogoff (1995)
- **Local Currency Pricing (LCP)**: Export prices are invoiced in currency of importers
  - Betts and Devereux (2000), Devereux and Engel (2003), Bacchetta and van Wincoop (2000)
- **Dominant Currency Pricing (DCP)**: Both export and import prices are invoiced in dominant currency
  - Gopinath (2016), Gopinath et al. (2020), Mukhin (2021)
- Each paradigm is equivalent to in which currency prices are sticky

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Differential predictions

- Consider two countries, H and F, engaging in trade under **fully** sticky price
- If Home currency depreciates,

	PCP	LCP	DCP
Home import price	$+$	0	$+$
Foreign import price	$-$	0	0
Home import volume	$-$	0	$-$
Foreign import volume	$+$	0	0
Total trade	$\pm$	0	$-$

Table 3: Price and Quantity response under Home currency depreciation

- Dollar appreciation only affects under DCP

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Different predictions under I-O linkages

- Home currency depreciation under fully sticky price and **PCP**
- When countries use imported inputs in production,
- Short-run: Imported input prices $\uparrow \rightarrow$ Import volume $\downarrow$
- Long-run: Marginal cost $\uparrow \rightarrow$ Pass-through to export prices $\uparrow \rightarrow$ Export volume $\downarrow$

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Gopinath et al. (2020) "Dominant Currency Paradigm"

- Question: Which paradigm is close to data?
- Empirical results (reduced-form evidence)
  - Prices and quantities are not responsive to bilateral exchange rate, but responsive to dollar exchange rate
  - 1% dollar appreciation predicts 0.6% reduction in global trade except US
- Model results
  - SOE model calibrated by Colombian firm-level data

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Exchange rate

- Exchange rate definitions
  - $\mathcal{E}$: Bilateral exchange rate between Home and ROW currency
  - $\mathcal{E}_{\$H}$: Home currency relative to dollar
  - $\mathcal{E}_{\$F}$: Foreign currency relative to dollar

$$\rightarrow \mathcal{E} = \frac{\mathcal{E}_{\$H}}{\mathcal{E}_{\$F}}$$

- Backus-Smith condition $Q = C/C^*$ implies

$$\mathcal{E} = \frac{PC}{P^*C^*} = \frac{M}{M^*}$$

- Dollar appreciates against weighted average of Home and Foreign currency

$$\omega \frac{de_{\$H}}{dm^{\$}} + (1-\omega)\frac{de_{\$F}}{dm^{\$}} = -1$$

$\rightarrow$ A contractionary US monetary policy ($dm^{\$} < 0$) leads to dollar appreciation

$$de_{\$H} = de_{\$F} = -dm^{\$} > 0$$

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Marginal cost equation

Given that $dm = dm^* = 0$, log change in marginal costs are

$$\begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \begin{bmatrix} \alpha dw + (1-\alpha)(1-\phi)dp_{HX} + (1-\alpha)\phi dp_{FX} \\ \alpha dw^* + (1-\alpha^*)(1-\phi^*)dp^*_{FX} + (1-\alpha^*)\phi^* dp^*_{HX} \end{bmatrix}$$

$$= \begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix} + \Omega \begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \Psi \begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix}$$

where

$$\begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix} = \begin{bmatrix} (1-\alpha)(1-\phi)d\mu_{HX} + (1-\alpha)\phi d\mu_{FX} \\ (1-\alpha^*)(1-\phi^*)d\mu^*_{FX} + (1-\alpha^*)\phi^* d\mu^*_{HX} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} = \begin{bmatrix} (1-\alpha)(1-\phi) & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & (1-\alpha^*)(1-\phi^*) \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} = (I-\Omega)^{-1}$$

- ω_{ij}: Country i's direct reliance on country j's input
- ψ_{ij}: Country i's indirect reliance on country j's input

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Output equation

From market clearing conditions, log change in outputs are

$$\begin{bmatrix} dy \\ dy^* \end{bmatrix} = \begin{bmatrix} \frac{1-\gamma}{\bar{Y}} dc_H + \frac{\gamma}{\bar{Y}} dc_H^* + (1-\alpha)(1-\phi)dx_H + (1-\alpha^*)\phi^* \frac{\bar{Y}^*}{\bar{Y}} dx_H^* \\ \frac{1-\gamma}{\bar{Y}^*} dc_F^* + \frac{\gamma}{\bar{Y}^*} dc_F + (1-\alpha^*)(1-\phi^*)dx_F^* + (1-\alpha)\phi \frac{\bar{Y}}{\bar{Y}^*} dx_F \end{bmatrix}$$

$$= \begin{bmatrix} \psi_{11} & \psi_{21} \frac{\bar{Y}^*}{\bar{Y}} \\ \psi_{12} \frac{\bar{Y}}{\bar{Y}^*} & \psi_{22} \end{bmatrix} \begin{bmatrix} du \\ du^* \end{bmatrix} - \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix}$$

where

$$\begin{bmatrix} du \\ du^* \end{bmatrix} = - \begin{bmatrix} \frac{1-\gamma}{\bar{Y}} d\mu_H + \frac{\gamma}{\bar{Y}} d\mu_H^* + (1-\alpha)(1-\phi)d\mu_{HX} + (1-\alpha^*)\phi^* \frac{\bar{Y}^*}{\bar{Y}} d\mu_{HX}^* \\ \frac{1-\gamma}{\bar{Y}^*} d\mu_F^* + \frac{\gamma}{\bar{Y}^*} d\mu_F + (1-\alpha^*)(1-\phi^*)d\mu_{FX}^* + (1-\alpha)\phi \frac{\bar{Y}}{\bar{Y}^*} d\mu_{FX} \end{bmatrix}$$

- ω_{ij}: Technical coefficient (Value of input j on i's output/i's total output)
- ψ_{ij}: Production inducement coefficient (If final demand of i increases by 1 unit, output of j increases by ψ_{ij})

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Examples

- Assume symmetry $(\alpha = \alpha^*, \phi = \phi^*)$
  - $dx_R = \gamma dm^\$$ under DCP/DCP (Full DCP)
  - $dx_R = 0$ under LCP/LCP (Full LCP)
  - $dx_R = 0$ under LCP/DCP
- How is it connected to Gopinath et al. (2020) and Engel's comments?
  - (Full DCP) global trade responds as in Gopinath et al. (2020)
  - (Full LCP) global trade mechanically does not respond
  - (LCP/DCP) global trade does not respond as in Engel's comments
- Under LCP/DCP, first-round effect from DCP is cancelled by second-round effect by IO linkages

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Household

- Household problem

$$\max_{C_{j,t},W_{j,t},B_{j,t+1},B^{\$}_{j,t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma_c} C_{j,t}^{1-\sigma_c} - \frac{\kappa}{1+\varphi} N_{j,t}^{1+\varphi} \right)$$

subject to

$$P_{j,t} C_{j,t} + \mathcal{E}_{\$j,t}(1 + i^{\$}_{j,t-1}) e^{\boldsymbol{\xi_{j,t-1}}} B^{\$}_{j,t} + B_{j,t}$$
$$= W_{j,t} N_{j,t} + \Pi_{j,t} + \mathcal{E}_{\$j,t} B^{\$}_{j,t+1} + \sum_{s' \in S} Q_{j,t}(s') B_{j,t+1}(s')$$

- Euler equations

$$C_{j,t}^{-\sigma_c} = \beta(1 + i_{j,t}) \mathbb{E}_t \left[C_{j,t+1}^{-\sigma_c} \frac{P_{j,t}}{P_{j,t+1}} \right]$$

$$C_{j,t}^{-\sigma_c} = \beta(1 + i^{\$}_{j,t}) e^{\boldsymbol{\xi_{j,t}}} \mathbb{E}_t \left[C_{j,t+1}^{-\sigma_c} \frac{P_{j,t}}{P_{j,t+1}} \frac{\mathcal{E}^{\$}_{j,t+1}}{\mathcal{E}^{\$}_{j,t}} \right]$$

→ UIP condition

$$i_{j,t} - i^{\$}_{j,t} = \mathbb{E}_t \left[e_{\$j,t+1} - e_{\$j,t} \right] + \boldsymbol{\xi_{j,t}}$$

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Production

- Production function

$$Y_{j,t} = e^{\boldsymbol{a}_{j,t}} L_{j,t}^{1-\alpha} X_{j,t}^{\alpha}$$

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Pricing

- Reset price equations

$$\bar{p}^{C,k}_{ji,t} = (1-\beta\delta_p)\left(\frac{1}{1+\Gamma}(mc_{j,t} - e_{kj,t} + \bar{\mu}) + \frac{\Gamma}{1+\Gamma}(p_{i,t} - e_{ki,t})\right) + \beta\delta_p \mathbb{E}_t\left[\bar{p}^{C,k}_{ji,t+1}\right]$$

$$\bar{p}^{X,k}_{ji,t} = (1-\beta\delta_p)\left(\frac{1}{1+\Gamma}(mc_{j,t} - e_{kj,t} + \bar{\mu}) + \frac{\Gamma}{1+\Gamma}(p^{X}_{i,t} - e_{ki,t})\right) + \beta\delta_p \mathbb{E}_t\left[\bar{p}^{X,k}_{ji,t+1}\right]$$

- Price evolution

$$p^{C,k}_{ji,t} - p^{C,k}_{ji,t-1} = (1-\delta_p)(\bar{p}^{C,k}_{ji,t} - p^{C,k}_{ji,t-1})$$

$$p^{X,k}_{ji,t} - p^{X,k}_{ji,t-1} = (1-\delta_p)(\bar{p}^{X,k}_{ji,t} - p^{X,k}_{ji,t-1}).$$

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Demand structure

- Import demands of final goods and intermediate goods from j to i

$$C^k_{ji,t}(\omega) = \gamma_{ji}\psi\left(D^C_{i,t}\frac{P^{C,k}_{ji,t}(\omega)}{P^k_{i,t}}\right)C_{i,t}$$

$$= \gamma_{ji}\left(1-\varepsilon\log\left(\frac{\sigma}{\sigma-1}D^C_{i,t}\frac{P^{C,k}_{ji,t}(\omega)}{P^k_{i,t}}\right)\right)^{\frac{\sigma}{\varepsilon}}C_{i,t}$$

$$X^k_{ji,t}(\omega) = \omega_{ji}\psi\left(D^X_{i,t}\frac{P^{X,k}_{ji,t}(\omega)}{P^{X,k}_{i,t}}\right)X_{i,t}$$

$$= \omega_{ji}\left(1-\varepsilon\log\left(\frac{\sigma}{\sigma-1}D^X_{i,t}\frac{P^{X,k}_{ji,t}(\omega)}{P^{X,k}_{i,t}}\right)\right)^{\frac{\sigma}{\varepsilon}}X_{i,t}$$

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List of countries

No	Country	Code	Invoicing data Available	EU	GNE weight
1	Brazil	BRA	Y		0.032
2	Canada	CAN	Y		0.021
3	China	CHN			0.087
4	Germany	DEU	Y	Y	0.042
5	France	FRA	Y	Y	0.032
6	United Kingdom	GBR	Y		0.039
7	India	IND	Y		0.031
8	Italy	ITA	Y	Y	0.028
9	Japan	JPN	Y		0.061
10	Mexico	MEX			0.019
11	Russia	RUS	Y		0.022
12	United States	USA	Y		0.284
13	Rest of the World	ROW	Y		0.304

Table 4: List of countries

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Data on Invoicing shares

Code	Export invoicing share			Import invoicing share		
	USD	EUR	Others	USD	EUR	Others
USA	95.9	1.1	2.9	95.6	2.2	2.2
BRA	95.5	3.4	1.0	84.9	10.1	4.9
CAN	70.0	7.0	23.0	70.0	7.0	23.0
CHN	50		50	50		50
DEU	16.8	78.1	5.1	19.7	78.3	2.1
FRA	22.9	72.6	4.5	23.2	75.4	1.4
GBR	26.4	29.7	43.9	40.1	34.1	25.9
IND	86.8	7.7	5.5	89.4	7.2	3.5
ITA	14.3	82.9	2.8	28.0	69.3	2.7
JPN	53.0	6.0	41.1	73.8	3.6	22.7
MEX	100			100		
RUS	76.0	8.4	15.6	39.6	28.1	32.3
ROW	49.1	42.4	8.5	47.5	42.0	10.5

Table 5: Export and Import invoicing shares

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Baseline calibration

	Parameter	Value	Source
Relative risk aversion	σ_c	2	
Inverse Frisch elasticity	φ	2	
VA share	α	2/3	
Home bias	γ_{ij}		WIOD
Int. input share	ω_{ij}		WIOD
Price rigidity	δ_p	0.75	
Demand elasticity	σ	2	
Super elasticity	ε	1	

Table 6: Baseline Calibration

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Shock calibration

- **Productivity shock** $a_{j,t}$: AR(1) process with (ρ_a, σ_a)
  - Data Source: OECD Statistics
  - Target moments: (De-trended) Multi-factor productivity for OECD countries
- **UIP shock** $\xi_{j,t}$: AR(1) process with $(\rho_\xi = 0.9, \sigma_\xi)$
  - Target moment: $\hat{\sigma}(\Delta e_\$)$ (Average volatility of dollar exchange rate growth)
- **MP shock** $\varepsilon_{j,t}$: AR(1) process with with $(\rho_\varepsilon = 0.9, \sigma_\varepsilon)$
  - Target moment: $\hat{\sigma}(\Delta y)$ (Average volatility of GDP growth)
- Implied moment: ROW trade regression in Gopinath et al. (2020)
  - RoW trade response to dollar appreciation $\hat{\beta}_{row}$ Equation

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ROW trade panel regression in Gopinath et al. (2020)

$$\Delta y_{ij,t} = \sum_{k=0}^{2} (\beta_k + \eta_k S_j) \Delta e_{ij,t-k} + \sum_{k=0}^{2} \left(\beta_k^{\$} + \eta_k^{\$} S_j^{\$}\right) \Delta e_{\$j,t-k}$$
$$+ \lambda_{ij} + \alpha' X_{ij,t} + \varepsilon_{ij,t}$$

ROW import volume response to 1% dollar appreciation

$$= \sum_{j \neq \$} w_j \Delta y_{ij,t}$$
$$= \beta_k^{\$} + \eta_k^{\$} \sum_{j \neq \$} w_j S_j^{\$} = \beta_{row,k}$$

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Moment Matching result

	Data	Model	
		Calibration 1	Calibration 2
Matched			
$\hat{\rho}_a$	0.786	0.774	0.774
$\hat{\sigma}_a$	0.012	0.012	0.012
$\hat{\sigma}(\Delta e_{\$})$	0.044	0.040	0.041
$\hat{\sigma}(\Delta y)$	0.019	0.021	0.021
Implied			
$\hat{\beta}_{row}$	-0.600	-0.657	-0.542

Table 7: Moment Matching result

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Sticky prices

Calvo sticky pricing: Probability of price adjustment δ

Domestic prices

- Invoiced in own currency $P_H = \mathcal{M}_H MC$

→ Log change of P_H & $\mathcal{M}_H$ $dp_H = \delta dmc$

$d\mu_H = (\delta - 1)dmc$

International prices Exchange rates

- Invoiced in PCP (Ideal price) $dp^*_{H,P} = \delta dmc - de$

$d\mu^*_{H,P} = (\delta - 1)dmc$

- Invoiced in LCP (Ideal price) $dp^*_{H,L} = \delta(dmc - de)$

$d\mu^*_{H,L} = (\delta - 1)(dmc - de)$

- Invoiced in DCP (Ideal price) $dp^*_{H,D} = \delta(dmc - de_{\$H}) + de_{\$F}$

$d\mu^*_{H,D} = (\delta - 1)(dmc - de_{\$H})$

→ dp^*_H & $d\mu^*_H$ (Actual price) $\theta^C_P dp^*_{H,P} + \theta^C_L dp^*_{H,L} + \theta^C_D dp^*_{H,D}$

$\theta^C_P d\mu^*_{H,P} + \theta^C_L d\mu^*_{H,L} + \theta^C_D d\mu^*_{H,D}$

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Global trade response

- Final good trade value in domestic currency:

$$C_R = P_F C_F + \mathcal{E} P_H^* C_H^* = \gamma PC + \mathcal{E}\gamma P^* C^*$$
$$= \gamma M + \mathcal{E}\gamma M^* = 2\gamma M$$

- Intermediate good trade value in domestic currency:

$$X_R = P_{FX} X_F + \mathcal{E} P_{HX}^* X_H^* = (1-\alpha)\phi MC \cdot Y + (1-\alpha^*)\phi^* \mathcal{E} MC^* \cdot Y^*$$

- In log difference:

$$dx_R = w(dmc + dy) + (1-w)(dmc^* + dy^*)$$

Marginal cost equation Output equation Back to slide

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