A Macroeconomic Model of Central Bank Digital Currency

Pascal Paul[1] Mauricio Ulate[1] Cynthia Wu[2]

[1]Federal Reserve Bank of San Francisco

[2]University of Illinois Urbana-Champaign and NBER

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AFA session on Digital Currencies

Motivation

- Introduction of Central Bank Digital Currency (CBDC) for retail consumers is one of the most far-reaching potential innovations in central banking
- A few countries have adopted a CBDC; 19 of the G20 economies are exploring the topic

Research Questions:

1. Is the introduction of a CBDC beneficial for an economy as a whole?
2. What's the optimal interest rate on CBDC, and how does it vary with the level of rates?
3. How does the presence of a CBDC affect the conduct of monetary policy?

This paper: Propose new general equilibrium model with realistic banking sector that is calibrated to empirical evidence

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Outline & Results

1. **Static Partial Equilibrium Model**
    - Cash, deposits & CBDC provide liquidity benefits to HHs, are imperfectly substitutable
    - Banks have deposit market power
    - The deposit spread is endogenous and its level is affected by CBDC

2. **Dynamic General Equilibrium Model** → New-Keynesian DSGE
    - Loan & bond markets, financial frictions → bank capital determines credit supply
    - CBDC: (+) liquidity benefits, (+) curtails market power, (-) credit disintermediation
    - Result #1: Welfare change displays inverted U-shape w.r.t. CBDC rate i^{CBDC}
    - Result #2: Optimal $i^{CBDC} \approx max(0\%, i^{policy} - 1\%)$; large gains at high i^{policy}
    - Result #3: Responses to transitory macro-shocks largely unaffected

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Literature

- **New Monetarist Approach**
  - Keister & Sanches (2022), Williamson (2022), Andolfatto (2021), Chiu et al. (2023)

  Here: NK-DSGE, CBDC & deposits imperfectly substitutable, bank market power in loan and deposit markets, bank profitability matters for lending

- **NK-DSGE Models**
  - Barrdear & Kumhof (2022), Burlon et al. (2023), Abad et al. (2023)

  Here: Bank market power in deposit markets, nonbank lending

- **Optimal Monetary Policy & CBDC Design**
  - Brunnermeier & Niepelt (2019), Davoodalhosseini (2021), Agur et al. (2022)

  Relative to Niepelt (2023): Nominal rigidities, bank market power in loan markets, bank profitability determines credit supply

Static Bank Deposit Model

Deposit Supply

- Bank j faces deposit supply

$$d_j = \left(\frac{1+i_j^d}{1+i^d}\right)^{\varepsilon^d} \frac{d}{n}$$

- where the aggregate deposit rate i^d and deposit amount d are

$$1+i^d = \left(\sum_{j=1}^{n} \frac{1}{n}(1+i_j^d)^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}}$$

$$d = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta} \mathcal{L}$$

- and the gross rate on liquid instruments $i^{\mathcal{L}}$ is defined as

$$1+i^{\mathcal{L}} = \left(\gamma_m + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}}$$

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Bank Problem

- Bank j maximizes

$$\max_{i_j^d, d_j, h_j} \quad (1+i)h_j - (1+i_j^d)d_j$$

$$\text{s.t.} \quad \underbrace{h_j}_{\text{Reserves}} = \underbrace{f_j}_{\text{Equity}} + \underbrace{d_j}_{\text{Deposits}} \quad \text{\& deposit supply}$$

- yielding first-order condition

$$1+i_j^d = \frac{\epsilon_j^d}{\epsilon_j^d+1}\cdot(1+i)$$

- where ϵ_j^d is the endogenous elasticity of deposits. Given symmetry,

$$\epsilon^d = \frac{n-1}{n}\cdot\varepsilon^d + \frac{1}{n}\cdot\theta(1-\omega_{\mathcal{L}}^d)$$

- where $\omega_{\mathcal{L}}^d = \frac{(1+i^d)d}{(1+i^{\mathcal{L}})\mathcal{L}} = \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}$ is the endogenous deposit share

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Effects of CB Rates on Deposit Market

- Define the deposit spread $(i - i^d)/(1 + i^d)$ which satisfies

$$\frac{i - i^d}{1 + i^d} = \frac{1}{\epsilon^d}$$

- Deposit spread is solely driven by endogenous deposit elasticity.

Proposition 1.

1. The deposit rate increases with the policy rate and the CBDC rate.
2. The deposit spread increases with the policy rate but decreases with the CBDC rate.

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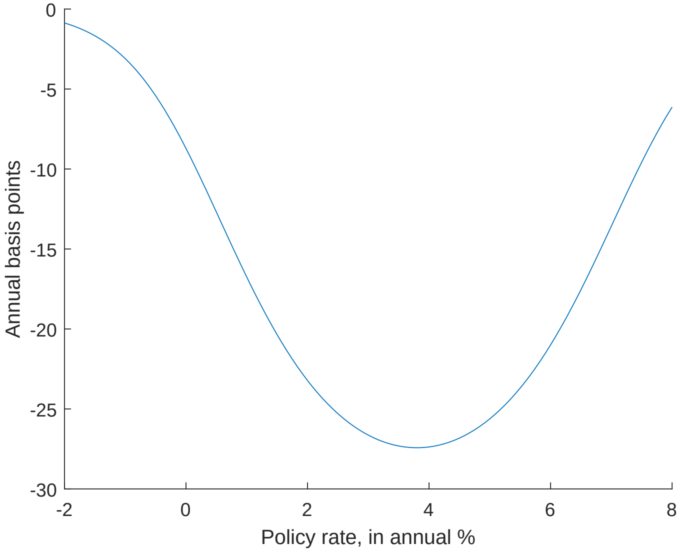
Change in the Deposit Spread due to CBDC Introduction



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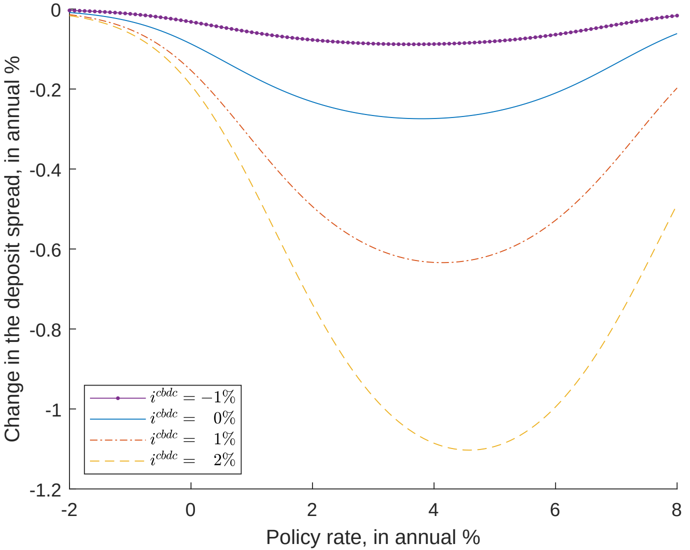

DSGE Model

Representative Household

- Household maximizes lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(C_t) - v(N_t)\right)$$

$$\text{s.t.} \quad \underbrace{P_t C_t}_{\text{Consumption}} + \underbrace{B_t}_{\text{Bonds}} + \underbrace{\Phi(\mathcal{L}_t) P_t}_{\text{Liquidity Costs}} = \underbrace{W_t N_t}_{\text{Income}} + \underbrace{AH_{t-1}}_{\text{Assets in Hand}} + \underbrace{T_t}_{\text{Transfers}}$$

- where $\mathcal{L}_t$ is a liquidity aggregator and $\Phi(\mathcal{L}_t) < \mathcal{L}_t$ for small $\mathcal{L}_t \Rightarrow$ convenience benefit

$$\mathcal{L}_t = \left(\gamma_m^{-\frac{1}{\theta}} m_t^{\frac{\theta+1}{\theta}} + \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{\theta+1}{\theta}} + \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{\theta+1}{\theta}}\right)^{\frac{\theta}{\theta+1}} \quad ; \quad d_t = \left(\sum_{j=1}^{n} \alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{\varepsilon^d+1}{\varepsilon^d}}\right)^{\frac{\varepsilon^d}{\varepsilon^d+1}}$$

$$AH_{t-1} = (1 + i_{t-1}) B_{t-1} + M_{t-1} + \sum_{j=1}^{n} (1 + i^d_{j,t-1}) D_{j,t-1} + (1 + i^{cbdc}_{t-1}) CBDC_{t-1}$$

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Household Equilibrium Conditions

- ... yielding first-order conditions

$$x_t = \gamma_x \left(\frac{1 + i_t^x}{1 + i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t \text{ for } x = \{m, cbdc, d\} \; ; \qquad \frac{1 + i_t^{\mathcal{L}}}{1 + i_t} = \Phi'(\mathcal{L}_t)$$

- with remaining deposit supply conditions similar to static model

$$1 + i_t^d = \left(\sum_{j=1}^{n} \alpha_j (1 + i_{j,t}^d)^{\varepsilon^d + 1} \right)^{\frac{1}{\varepsilon^d + 1}} \; ; \qquad d_{j,t} = \alpha_j \left(\frac{1 + i_{j,t}^d}{1 + i_t^d} \right)^{\varepsilon^d} d_t$$

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Corporate Sector & Government

- Intermediate good firm has Cobb-Douglas production function

$$Y_t^m = A_t K_t^\alpha N_t^{1-\alpha}$$

- K_t consists of pledgeable capital K_t^P & nonpledgeable capital K_t^{NP}

$$K_t = \left((1-\psi)^{\frac{1}{\theta^k}} \left(K_t^{NP}\right)^{\frac{\theta^k-1}{\theta^k}} + \psi^{\frac{1}{\theta^k}} \left(K_t^P\right)^{\frac{\theta^k-1}{\theta^k}} \right)^{\frac{\theta^k}{\theta^k-1}} \quad ; \quad K_t^P = \left(\sum_{j=1}^{n} (\alpha_j^l)^{\frac{1}{\epsilon^l}} \left(K_{j,t}^P\right)^{\frac{\epsilon^l-1}{\epsilon^l}} \right)^{\frac{\epsilon^l}{\epsilon^l-1}}$$

- K_t^P is financed with bank loans, while K_t^{NP} is financed with bond borrowing

- Other firms: Retailers subject to nominal rigidities, final good & capital good producers
- Government: Central bank follows Taylor rule, fiscal spending constant fraction of output

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Bank Problem

- The balance-sheet constraint now is

$$\underbrace{L_{j,t}}_{\text{Loans}} + \underbrace{H_{j,t}}_{\text{Reserves}} = \underbrace{F_{j,t}}_{\text{Equity}} + \underbrace{D_{j,t}}_{\text{Deposits}}$$

- Bank faces costs of (i) operation, (ii) issuing loans & deposits, (iii) leverage deviations

$$\underbrace{S_{j,t+1}}_{\text{E.o.P. Res}} = \underbrace{(1+i^l_{j,t}-\mu^l)L_{j,t}}_{\text{Loans}} + \underbrace{(1+i_t)H_{j,t}}_{\text{Reserves}} \underbrace{-(1+i^d_{j,t}+\mu^d)D_{j,t}}_{\text{Deposits}} \underbrace{-\varsigma F_{j,t}}_{\text{Operation}} \underbrace{-\Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t}}_{\text{Leverage}}$$

- Bank pays constant fraction of profits as dividends each period
- And solves $\max \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s+1} DIV_{j,t+s+1}$
- Frictions imply that bank capital is slow-moving & determines credit supply

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Bank Equilibrium Conditions

- Decision separated into a deposit sub-problem and a loan sub-problem, yielding

$$1 + i^d_{j,t} = \frac{\epsilon^d_{j,t}}{\epsilon^d_{j,t} + 1}(1 + i_t - \mu^d)$$

- where endogenous deposit elasticity $\epsilon^d_{j,t}$ takes similar form as in static model, and

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Calibration

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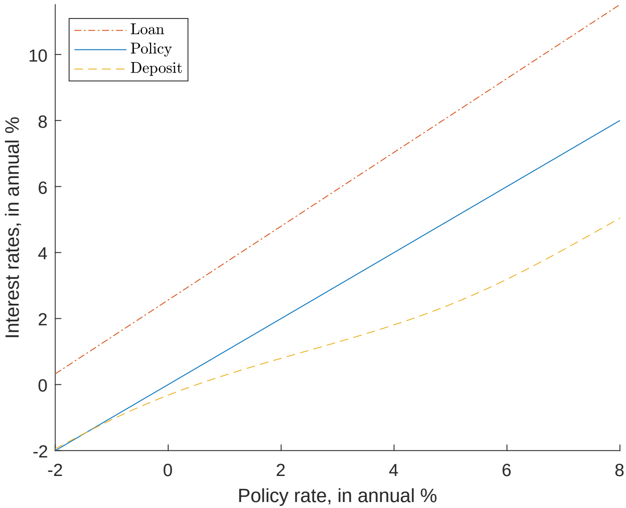
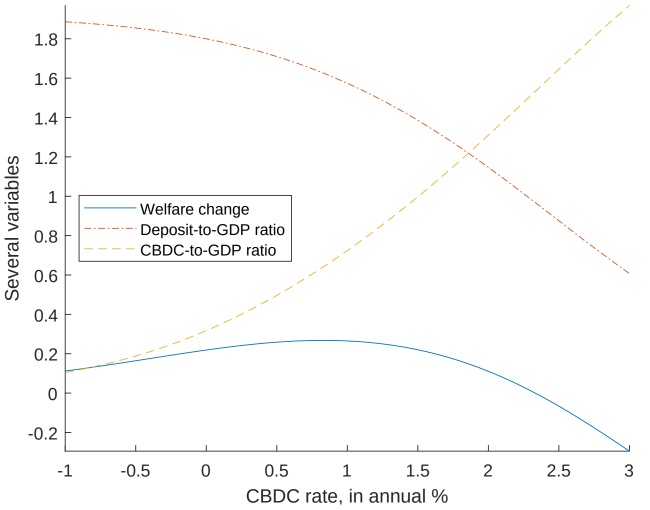
Param.	Value	Description	Target or source
Deposit side			
γ_m	0.3005	Importance of cash in liquidity	$\gamma_m + \gamma_d + \gamma_{cbdc} = 1$
γ_d	0.3990	Importance of deposits in liquidity	$D/\mathcal{L} = 0.8$ at $i = 2\%$
γ_{cbdc}	0.3005	Importance of CBDC in liquidity	$\gamma_{cbdc} = \gamma_m$ (Bidder et al.)
n	1.1685	Number of banks	Deposit rate target #1
θ	554.21	E.o.S. between instruments in liquidity	Deposit rate target #2
ε^d	661.36	E.o.S. between banks in deposits	Deposit rate target #3
μ^d	-0.20%	Cost of issuing deposits	Deposit rate target #4
Loan side			
ψ	0.3000	Importance of pledgeable capital	Crouzet (2021)
ϱ	0.70%	Extra cost of corporate-bond borrowing	Schwert (2020)
μ^l	0.35%	Cost of issuing loans	Schwert (2020)

Model-Implied Loan and Deposit Rates

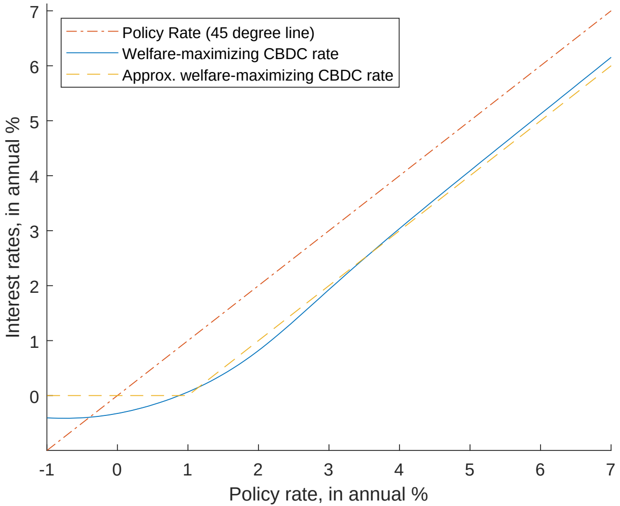


Results

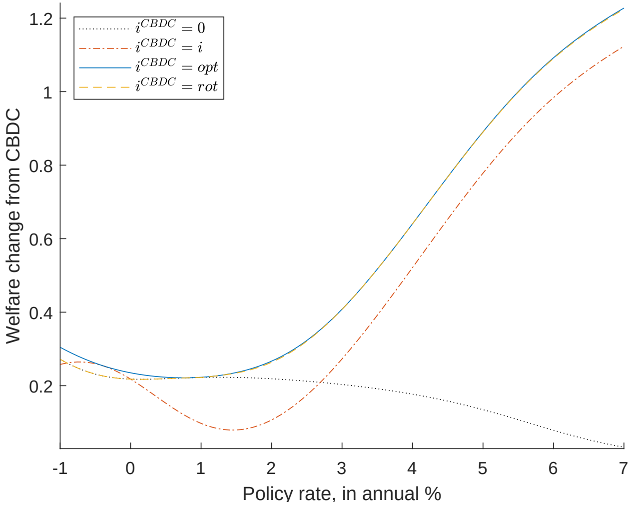
Result 1: CBDC Introduction Across CBDC Rates



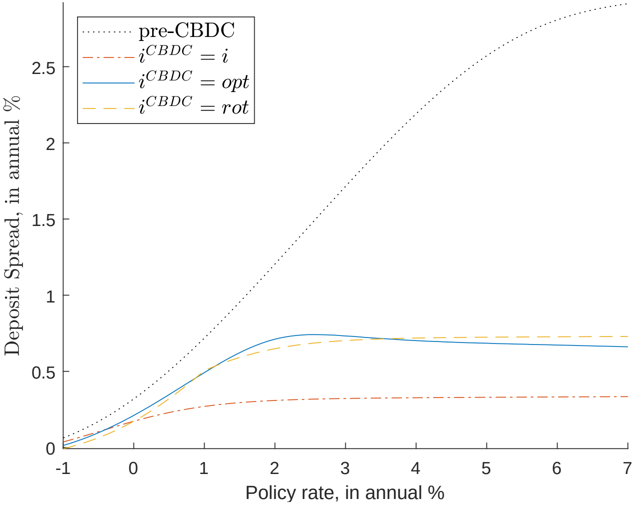
Result 2: Welfare-Maximizing CBDC Rate Across Policy Rates



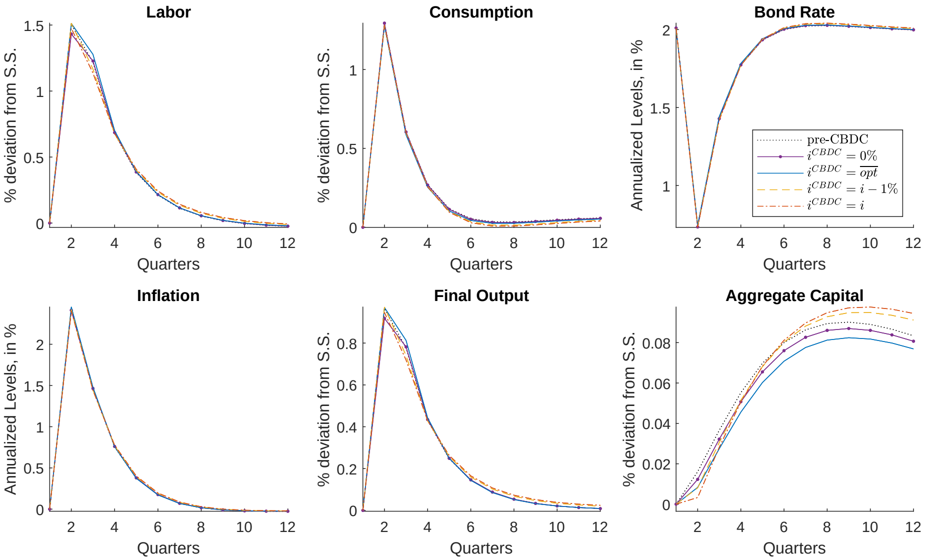
Result 2: Welfare Changes



Result 2: Deposit Spread



Result 3: Responses to Monetary Policy Shock



Conclusion

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- Introduction of CBDC debated worldwide, but practical experience remains scarce
  $\Rightarrow$ analysis based on theoretical models needed
- **This paper**: provides such guidance and delivers a simple practical message
- Substantial welfare improvements from introducing CBDC based on a rule-of-thumb: optimal CBDC rate $i^{CBDC} \approx max(0\%, i^{policy} - 1\%)$
- Can be easily communicated to the public and avoids political-economy concerns related to paying negative rates on CBDC
- Introduction of CBDC most beneficial for economies with high interest rates
  $\Rightarrow$ bank market power in deposit markets sharply curtailed

Extra Slides

Full Endogenous Elasticities

- $\epsilon^d_{j,t}$ is the endogenous elasticity of deposits. Given symmetry,

$$\epsilon^d_t = \frac{n-1}{n} \cdot \varepsilon^d + \frac{1}{n}\left[(1-\omega^d_{\mathcal{L},t})\theta + \omega^d_{\mathcal{L},t}\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i^{\mathcal{L}}_t)}\right]$$

- where $\omega^d_{\mathcal{L},t} \equiv \frac{(1+i^d_t)d_t}{(1+i^{\mathcal{L}}_t)\mathcal{L}_t} = \gamma_d \left(\frac{1+i^d_t}{1+i^{\mathcal{L}}_t}\right)^{\theta+1}$ is the endogenous deposit share
- ϵ^l_j is the endogenous loan elasticity. Given symmetry,

$$\epsilon^l_t = \left\{\frac{n-1}{n} \cdot \varepsilon^l + \frac{1}{n}\left[(1-\omega^{K_p}_{K,t})\theta + \frac{\omega^{K_p}_{K,t}}{1-\alpha}\right]\right\}\frac{Q_t}{P_t}\frac{1+i^l_t}{1+i_t}\frac{1}{z^P_t}$$

- where $\omega^{K_p}_{K,t} \equiv \frac{z^p_t K^p_t}{z_t K_t} = \psi\left(\frac{z^p_t}{z_t}\right)^{1-\theta^k}$ is the endogenous loan share

Full Intermediate Good Producer Setup

$$
\begin{aligned}
Y_t^m &= A_t K_t^{\alpha} N_t^{1-\alpha} \\
K_t &= \left((1-\psi)^{\frac{1}{\theta^k}} (K_t^P)^{\frac{\theta^k-1}{\theta^k}} + \psi^{\frac{1}{\theta^k}} (K_t^{NP})^{\frac{\theta^k-1}{\theta^k}} \right)^{\frac{\theta^k}{\theta^k-1}} \\
K_t^{NP} &= \left(\sum_{j=1}^{n} (\alpha_j^l)^{\frac{1}{\varepsilon^l}} (K_{j,t}^{NP})^{\frac{\varepsilon^l-1}{\varepsilon^l}} \right)^{\frac{\varepsilon^l}{\varepsilon^l-1}} \\
\Pi_t^m &= P_t^m Y_t^m - W_t N_t + (1-\delta) Q_t \sum_{j=1}^{n} K_{j,t}^{NP} + (1-\delta) Q_t K_t^P \\
&- \sum_{j=1}^{n} (1+i_{j,t-1}^l) Q_{t-1} K_{j,t}^{NP} - (1 + i_{t-1} + \varrho_{t-1}) Q_{t-1} K_t^P
\end{aligned}
$$

Intermediate Good Producer Equilibrium Conditions

$$\frac{W_t}{P_t} = (1-\alpha)\frac{P_t^m}{P_t}\frac{Y_t^m}{N_t} \qquad z_t = \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_{t+1}^m}{P_{t+1}}\frac{Y_{t+1}^m}{K_{t+1}}\right)$$

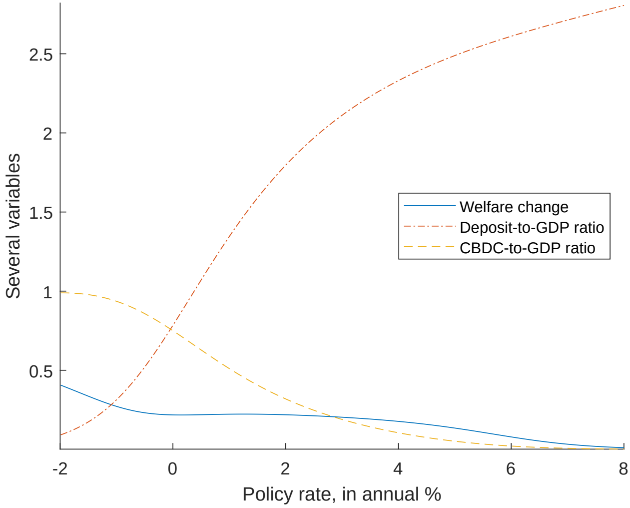
$$z_t = \left(\psi(z_t^{NP})^{1-\theta^k} + (1-\psi)(z_t^P)^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}} \qquad z_t^{NP} = \left(\sum_{j=1}^{n}\alpha_j^l(z_{j,t}^{NP})^{1-\varepsilon^l}\right)^{\frac{1}{1-\varepsilon^l}}$$

$$z_t^P = \frac{Q_t}{P_t}\frac{1+i_t+\varrho_t}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right)$$

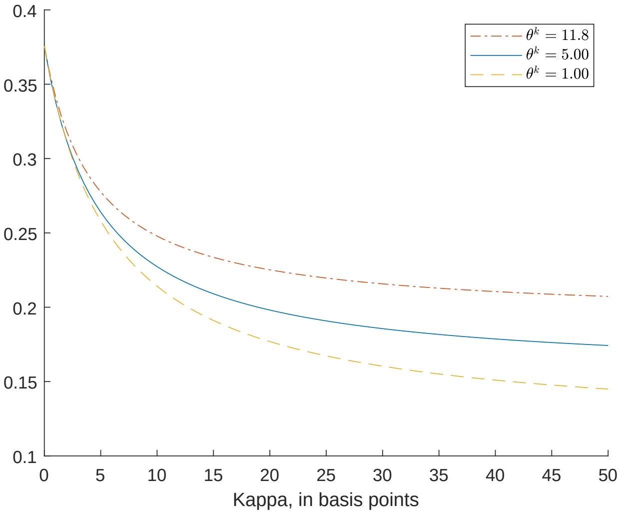
$$z_{j,t}^{NP} = \frac{Q_t}{P_t}\frac{1+i_{j,t}^l}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) \qquad K_{j,t+1}^{NP} = \alpha_j^l\left(\frac{z_{j,t}^{NP}}{z_t^{NP}}\right)^{-\varepsilon^l} K_{t+1}^{NP}$$

$$K_{t+1}^{NP} = \psi\left(\frac{z_t^{NP}}{z_t}\right)^{-\theta^k} K_{t+1} \qquad K_{t+1}^P = (1-\psi)\left(\frac{z_t^P}{z_t}\right)^{-\theta^k} K_{t+1}$$

0% Interest Rate CBDC for Different Policy Rates



Welfare Across κ



Recalibrating Additional Parameters