Don't Ruin the Surprise: Temporal Aggregation Bias in Structural Innovations

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Are Structural Innovations New Information?

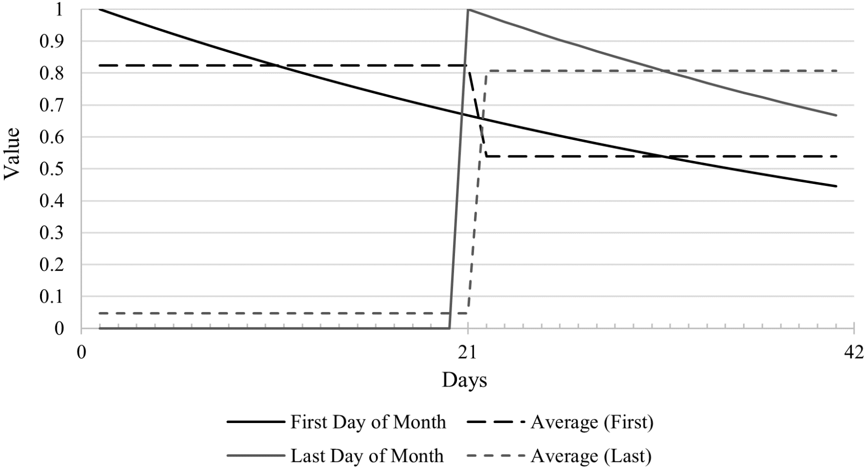
- Temporal aggregation refers to use of sums (IP, GDP) or averages (prices, interest rates) over time (months, quarters)
- **Insight:** use of such data in estimation results in structural shocks being biased and predictable
- **Sufficient conditions:** provided for temporally aggregated and selectively sampled data
- **Propose a test:** for temporal aggregation bias
  - Substantial: over 75% of innovations are predictable
  - Application: over 50% of global crude oil shocks

Literature

- Temporal Aggregation Bias (Christiano and Eichenbaum, 1987)
    1. Loss of information when forecasting (Amemiya and Wu, 1972; Tiao, 1972; Kohn, 1982; Lütkepohl, 1986; Ellwanger and Snudden, 2023)
    2. Information loss reduces model fit (Teles and Sousa, 2017) and influences structural identification (Sims, 1972; Geweke, 1978; Breitung and Swanson, 2002)
- Focus on monthly to quarterly and annual aggregation (Rossana and Seater, 1995; Marcellino, 1999)
- **Contributions:**
    1. Structural bias and forecastability are related
    2. Information loss substantial for aggregation of daily data

Illustrative Example

Illustrative Example: Shock Mistiming



Notes: Impulse response from a daily AR(1) model with $\rho - 0.98$ with monthly average data, $n = 21$.

Example - AR(1)

- Suppose the daily DGP is a AR(1)

$$y_{t,i} = \rho y_{t,i-1} + \epsilon_{t,i}$$

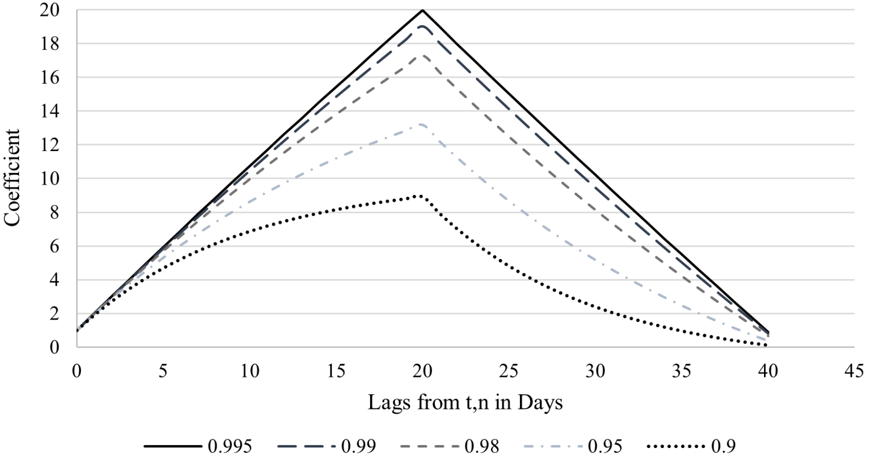
- Two day average approximated by an ARMA(1,1)

$$\begin{aligned} \bar{y}_t &= \rho^2 \bar{y}_{t-1} + \epsilon_{t,i} + (1+\rho)\epsilon_{t,i-1} + \rho\epsilon_{t-1,n} \\ &\approx \tilde{\rho}\bar{y}_{t-1} + u_t + \tilde{\alpha} u_{t-1}. \end{aligned} \tag{1}$$

- For arbitrary n:

$$\begin{aligned} \bar{y}_t &= \rho^n \bar{y}_{t-1} + (1 + (1+\rho)L + \cdots + \rho^{n-1} L^{2(n-1)})\epsilon_{t,i} \\ &\approx \tilde{\rho}\bar{y}_{t-1} + u_t + \tilde{\alpha} u_{t-1}. \end{aligned} \tag{2}$$

Coefficients on Lagged Daily Values are Non-constant



Notes: Coefficients on lagged daily values for alternative values of ρ of an AR(1) model with temporal aggregation to monthly data, $n = 21$.

Sufficient Conditions

Theory - Set up

- Define daily structural innovations as $\epsilon_{t,i}$ - "New Information"
- Daily DGP; day i in month t is given by an ARMA(p, q):

$$a(L)y_{t,i} = b(L)\epsilon_{t,i} \quad \text{for } i = 1, 2, ..., n.$$

  - n is the number of days in month
  - $b(L) = (1 + \alpha_1 L + \cdots + \alpha_q L^q)$
  - $a(L) = (1 - \rho_1 L - \cdots - \rho_p L^p)$

- Daily data sampled as non-overlapping intervals of moving averages (i.e. monthly average):

$$\overline{y}_t \equiv \frac{1}{n} \sum_{i=1}^{n} y_{t,i}$$

Implications of Temporal Aggregation

$$h(L)\bar{y}_t = (1 + \cdots + \alpha_q \rho_p^{n-1} L^{(p+1)(n-1)+q})\epsilon_{t,i}$$
$$\approx (1 + \tilde{\alpha}_1 L + \ldots + \tilde{\alpha}_{q^*} L^{q^*}) u_t$$

1. Modifies the structure of the time series
   - $\bar{y}_t$ is an ARMA(p, q^*) with $q^* > q$
   - coefficient values approach zero as n increases
2. Reduction of information:
   - a non-linear polynomial of the structural innovations
   - $covar(u_t, L^j y_{t-1,n}) \neq 0 \; \forall \; j \in \{0, 1, \ldots, p(n-1) + q\}$

Sufficient Conditions

Temporal aggregation bias will be present for:

1. Temporal aggregated data when $p(n-1)+q>0$
   - i.e. present for all ARMA representations
2. Selective sampled data when $(p-1)(n-1)+q>0$
   - present for some representations
   - AR(p) when $p \geq 2$; MA(q) when $q>n-1$

Note: similar conditions hold for VARs (i.e., Marcellino, 1999)

Testing for Aggregation Bias

Test for Aggregation Bias

- **Proposed test:**

$$\hat{u}_t = \sum_{j=0}^{J} \beta_j d_{t-1,n-j} + \nu_t$$

where: $d_{t,i} = y_{t,i} - \bar{y}_t$,
$\hat{u}_t$ is the estimated innovation

- **Null Hypothesis**: Agents cannot predict $E_{t,n}[\hat{u}_{t+1}] = 0$

$$H_0 : \ \beta_j = 0, \ \forall \ j$$

- Test statistics: F-test, UMIDAS, near perfect power

- Parametrization of J: Given by sufficient conditions

Simulation Evidence

Simulation Evidence

- Simulations Procedure
  1. Simulate daily data, $y_{t,i}$. $\epsilon_{t,i} \sim N(0,1)$ (40 years worth, burn 500)
  2. Aggregate, $\bar{y}_t$ to lower frequency, ($n = 5,\ 21$, or 62).
  3. Estimate model on aggregated data, save $\hat{u}_t$
  4. Save estimate of structural shock $\hat{u}_t$
  5. Apply test
- Objects of Interest
  1. Predictability (R^2-adjusted)
  2. Power analysis of different statistics

Structural Innovations are Highly Predictable: R^2

	AR(1)		AR(2)		Persistent AR(2)	
ρ \ Sampling	EoM	Average	EoM	Average	EoM	Average
1.00	0.000	0.388	0.000	0.740	0.000	0.740
	(0.013)	(0.035)	(0.019)	(0.019)	(0.019)	(0.019)
0.995	0.000	0.375	0.000	0.708	0.000	0.739
	(0.013)	(0.035)	(0.019)	(0.021)	(0.019)	(0.019)
0.95	-0.001	0.271	-0.003	0.486	0.000	0.730
	(0.013)	(0.034)	(0.019)	(0.03)	(0.019)	(0.02)
0.50	-0.002	0.016	-0.003	0.071	0.013	0.665
	(0.013)	(0.017)	(0.018)	(0.028)	(0.021)	(0.024)
0.25	-0.002	0.003	-0.003	0.042	0.043	0.646
	(0.013)	(0.014)	(0.018)	(0.025)	(0.026)	(0.025)
0.00	-0.002	0.000	-0.004	0.028	0.390	0.634
	(0.013)	(0.013)	(0.019)	(0.024)	(0.036)	(0.025)

Notes: Mean R^2-adjusted values from 5000 Monte Carlo simulations using UMIDAS. Daily DGP: $y_{t,i} = \rho y_{t,i-1} + \gamma y_{t,i-2} + \epsilon_{t,i}$ for 40 years of daily data. Monthly average and end-of-month (EoM) sampling. Standard deviation in parentheses.

Similar Predictability in SVARs

Daily DGP:

$$y_{t,i} = \rho_{11} y_{t,i-1} + \rho_{12} x_{t,i-1} + \epsilon_{t,i}$$
$$x_{t,i} = \rho_{21} y_{t,i-1} + \rho_{22} x_{t,i-1} + \eta_2 \epsilon_{t,i} + \nu_{t,i}$$

Predictability of Monthly Shocks:

ρ_{11}	ρ_{22}	ρ_{12}	ρ_{21}	η_2	R^2
0.99	0.45	0.00	-0.50	0.50	0.351
0.99	0.45	0.00	-0.50	0.10	0.230
0.99	0.45	0.05	0.10	0.10	0.439
0.99	0.45	-0.05	0.50	0.50	0.098

Notes: Mean R^2-adjusted values of structural shocks from equation $\bar{y}_t$. 5000 Monte Carlo simulations using UMIDAS. Monthly average sampling, 40 years of daily data.

Empirical Application

Application: Shocks to Global Market for Crude Oil

- SVAR models: Kilian (2009) and Baumeister & Hamilton (2019)
  1. Global oil supply (total monthly production)
  2. Aggregate demand (monthly average BDI)
  3. Precautionary demand (end of month inventories)
  4. Oil-specific demand (monthly average WTI)

- Test predictability of shocks:

$$\tilde{u}_t^{shk} = \sum_{j=0}^{J} \beta_j^p d_{t-1,n-j}^p + \sum_{k=0}^{K} \beta_k^d d_{t-1,n-k}^d + \nu_t^{shk}$$

  where

  - $d_{t,i}^p = ln(p_{t,i}) - ln(\bar{p}_t)$ for WTI
  - $d_{t,i}^d = ln(bdi_{t,i}) - ln(\bar{bdi}_t)$ for Baltic Dry Index (BDI)
  - both AR(1) $\Rightarrow$ J=21, K=20

Shocks to the Global Market for Crude Oil are Predictable

	Kilian (2009)			Baumeister & Hamilton (2019)			
Method	Aggregate Demand	Oil Demand	Oil Supply	Aggregate Demand	Oil Demand	Oil Supply	Inventory Demand
UMIDAS	0.505	0.307	0.083	0.182	0.199	0.198	0.024
J=N-1	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.045)
MIDAS	0.456	0.242	0.015	0.107	0.184	0.151	0.046
Expon.	(0.000)	(0.000)	(0.207)	(0.222)	(0.000)	(0.018)	(0.033)

Notes: R^2-adj estimates using MIDAS. Robust p-values in parentheses using F-tests for UMIDAS, and DM-test for MIDAS.

Evidence of Moving Average in Shocks

- Rejected $E_{t,n}[\hat{u}_{t+1}] = 0$ but what about $\bar{E}_t[\hat{u}_{t+1}] = 0$?
- Test for a moving average

$$\hat{u}_t^{shk} = e_t + \alpha e_{t-1}$$

		Kilian (2009)			Baumeister & Hamilton (2019)			
Estimate	Method	Aggregate Demand	Oil Demand	Oil Supply	Aggregate Demand	Oil Demand	Oil Supply	Inventory Demand
$\hat{\alpha}$	MA(1)	0.006	-0.002	-0.004	-0.013	0.038	0.089	-0.089
		(0.848)	(0.932)	(0.906)	(0.566)	(0.322)	(0.016)	(0.045)

Have Agents Already Responded to Predictable information?

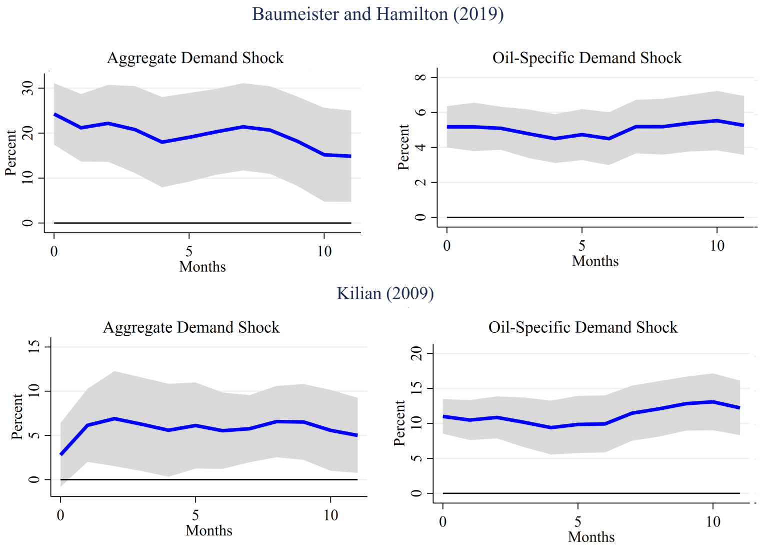
- The test provides an estimate of the new information, $\hat{\nu}_t$
- ... and the expected component $\hat{e}^{shk}_{t-1}$:

$$\hat{e}^{shk}_{t-1} = \sum_{j=0}^{n} \hat{\beta}^p_j d^p_{t-1,n-j}$$

- IRFs via local projection

$$y_{t+h,n} = \beta^u_h \hat{\nu}^{shk}_t + \beta^e_h \hat{e}^{shk}_t + \boldsymbol{\beta(L) Y_t} + e_{t+h}, \quad \forall h$$

Markets Have Already Responded to Expected Component



Notes: IRFs via local projections, 1983M6–2022M6. 90 percent confidence intervals

Conclusion

Take Aways

- Models estimated with temporally aggregated data suffer from temporal aggregation bias
    - shocks are highly predictable
    - worse when daily data is persistent
- Agents likely to have already responded
- Selective sampling can only correct for bias under special cases
- Test if structural innovations are unexpected

Calls for a major re-examination of existing findings in models estimated with aggregated data