Structural counterfactual analysis in macroeconomics: theory and inference

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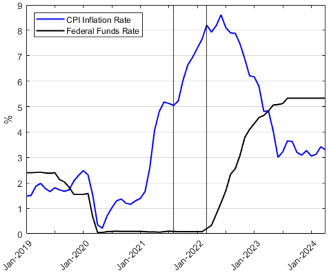
Motivation

- During the pandemic, the CPI inflation rose sharply.
- Since the pandemic, the Federal Reserve implemented its first interest rate hike in March 2022, when CPI inflation had risen to about 8%.



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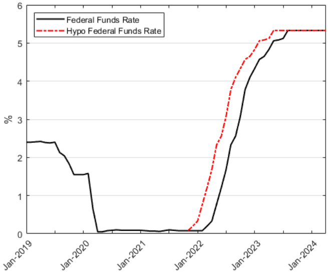

- Question: How would other macro-aggregations have performed if the Fed had hiked the interest rate 3-month earlier?

Outline

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Policy path counterfactuals

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- These parametric assumptions are typically not robust to missing variable, measurement error, or model misspecification.

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- Illustrative example: 3-variate SVAR(1)

$$W_t = A_1 W_{t-1} + \Theta_0 \varepsilon_t$$

where A_1 and Θ_0 are a 3×3 matrices, and ε_t is a 3×1 error.

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- Then, ε_t is NOT recoverable from the current and past observables.

Contribution

- Macroeconomics: Under general linearity assumption (SVMA), we demonstrate IRFs of policy shocks are sufficient for policy-path counterfactuals. (conventional scenario analyses require the knowledge of entire parametric model, e.g., Kilian and Lütkepohl (2017))

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- Empirical studies of the U.S. monetary policy
  1. Hypothetical trajectory: Examine the impact of a counterfactual policy path scenario.
  2. Policy intervention: Analyze the causal effect of an initial shock of interest (e.g., an oil price shock) under the restriction that the policy variable (e.g., the interest rate) exhibits zero response.

Related literature

- "Sufficient statistics" logic for counterfactuals Chetty (2009), Beraja (2023, JPE), McKay and Wolf (2023, Ecta), Barnichon and Mesters (2023, AER)
- Recoverability/Invertibility Stock and Watson (2018, EJ), Nakamura and Steinsson (2018, JEP), Plagborg-Møller and Wolf (2022, JPE), Chahrour and Jurado (2022, RES)
- Conventional methods based on parametric models Kilian and Lütkepohl (2017)
  - Conditional forecasting: Doan, Litterman, and Sims (1984), Waggoner and Zha (1999), Antolin-Diaz, Petrella, and Rubio-Ramirez (2021).
  - Policy intervention (Sims and Zha (1995), Bernanke et al. (1997), Hamilton and Herrera (2004), Sims and Zha (2006), Kilian and Lewis (2011)).
- Mediation analysis (direct and indirect effect) Baron and Kenny (1986), Imai et al. (2010), MacKinnon (2012), VanderWeele (2015), Dufour and Wang (2024b) [impulse response decomposition]
- Lots of literature on IRFs Dufour and Renault (1998), Pelletier (2004), Jordà (2005), Dufour et al. (2006), Plagborg-Møller and Wolf (2021)
- High frequency IV Gertler and Karadi (2015), Ramey (2016), Nakamura and Steinsson (2018), Känzig (2021), Bauer and Swanson (2023)
- "HAC-free" LP Montiel Olea and Plagborg-Møller (2021), Xu (2023), Breitung and Brüggemann (2023), Dufour and Wang (2024a) [low dimen, $h \propto T$, local-to-unity], Dettaa and Wang (2024) [high dimen, de-biased ML]

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for $t \geq 1$, where $\boldsymbol{w}_0$ is an initial condition,

$$\varepsilon_t := (\varepsilon_{x,t}, \varepsilon_{y,t}, \varepsilon_{r,t}, \varepsilon'_{l,t})',$$

ε_t is a $n \times 1$ vector of serially uncorrelated structural shocks, for $n \geq 3$, $\varepsilon_t \sim (0, I)$. Θ_h is a $3 \times n$ impulse response matrix, potentially a wide matrix (more cols than rows).

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- Note $\varepsilon_{r,t}$ is the monetary policy shock.

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- Interpretation (Kilian and Lütkepohl (2017)): the policymaker has no power to intervene in the economy except through the policy shocks.

Structural interpretation

- Two economics

$$\begin{array}{lll} \text{Baseline economy } (W_{t+h}): & \{\varepsilon_{t,H,(S)} & , \quad U_{t+H}\} \\ \text{Counterfactual economy } (\tilde{W}_{t+h}): & \{\varepsilon_{t,H,(S)} - \delta_H & , \quad U_{t+H}\} \end{array}$$

where U_{t+H} represents the remaining shocks up to period $t+H$, $\{U_{t+H}\} = \{\varepsilon_{1:t+H}\} \backslash \{\varepsilon_{t,H,(S)}\}$.

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- Due to linearity, the difference between two economies at horizon h,

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- If δ_H is known and policy shocks are identified, the differences between two economies can be obtained.

- Once the difference is known, given the baseline economy, the counterfactual economy can be derived.

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- Recall: the linearity ensures a unique mapping from the policy shocks $\boldsymbol{\delta}_H$ to the policy path differences $\boldsymbol{d}_H$ from horizon zero to horizon H.

$$R_{t:t+H} - \tilde{R}_{t:t+H} = \Theta_{re,H}\boldsymbol{\delta}_H, \tag{3}$$

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- Denoted $R_{t:t+H} - \tilde{R}_{t:t+H} := \boldsymbol{d}_H$. One equation, one unknown.

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- If $\Theta_{re,H}$ is NOT invertible, apply Moore Penrose inverse. Econ interpretation

'Sufficient statistics' logic for counterfactuals

- Sufficient information table

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- Extension: If the central bank aims to achieve a specific inflation rate in the future, our method can determine the necessary policy shocks to realize this target, based on the unconditional forecast data (in the absence of policy intervention). Ideal policy path

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- Aggregate output gap $\psi_h := y_{t+h} - \tilde{y}_{t+h}$
- Compute the size of the necessary policy shocks, and map it to the output variable

$$\psi_h = \boldsymbol{\theta}'_{ye,h} \boldsymbol{\delta}_H^{(ht)} \tag{5}$$

where $\boldsymbol{\delta}_H^{(ht)} = \Theta_{re,H}^{-1} \boldsymbol{d}_H^{(ht)}$ is the size of policy shocks necessary for this counterfactual scenario, and $\boldsymbol{\theta}_{ye,h}$ is the IRFs.

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- Given the period-by-period shock choice, $\varepsilon_{t,H,(S)} = (\varepsilon_{r,t}, \varepsilon_{r,t+1}, \cdots, \varepsilon_{r,t+H})'$,

$$\psi_h = \boldsymbol{\theta}'_{ye,h} \boldsymbol{\delta}_H^{(ht)} = \boldsymbol{\theta}'_{ye,h} \Theta_{re,H}^{-1} \boldsymbol{d}_H^{(ht)} = \boldsymbol{\beta}'_h \boldsymbol{d}_h^{(ht)}, \tag{6}$$

  where β_h maps the policy path differences to the output variable at h-period ahead. β_h consists of the first $(h+1)$ elements in the vector $\Theta_{re,H}^{-1'} \boldsymbol{\theta}_{ye,h}$, and $\boldsymbol{d}_h^{(ht)}$ consists of the first $(h+1)$ elements in $\boldsymbol{d}_H^{(ht)}$.

LP-IV: Period-by-period shocks

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$$Y_{t+h} = \beta_h' R_{t:t+h} + u_{t,h}. \tag{7}$$

namely, 'Counterfactual Local Projection'.

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  namely, 'Counterfactual Local Projection'.
- IV, $\boldsymbol{z}_{t,h}$, such that $\boldsymbol{z}_{t,h} = (z_t, z_{t+1}, \cdots, z_{t+h})'$, and z_t satisfies 'lead-lag' exogeneity assumption to the policy shock $\varepsilon_{r,t}$.

LP-IV: Period-by-period shocks

- Two methods to obtain β_h
  1. Nonlinear transformation: estimate IRFs and compute (β_h) ▸ Asym theory
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- Identification

$$\beta_h = \mathbb{E}[\boldsymbol{z}_{t,h} R_{t:t+h}']^{-1} \mathbb{E}[\boldsymbol{z}_{t,h} Y_{t+h}]. \tag{8}$$

- β_h has structural interpretation that the movement of the interest rate from period t to $t+h$ only contributes to these monetary policy shocks.

Frame Title

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- Counterfactual scenario and mediation analysis (remove the mediator effect) Dufour and Wang (2024b)
  - Oil price shock if the future interest rate is maintained fixed.
  - Tariff policy shock if the future exchange rate is maintained fixed.
  - ...

LRV estimation

- 2SLS, $\tilde{\beta}_h = \left(\sum_t \mathbf{z}_{t,h} R^{\perp\prime}_{t:t+h}\right)^{-1} \left(\sum_t \mathbf{z}_{t,h} Y^{\perp}_{t+h}\right)$, where $\perp$ is to partial out p-lagged controls of $W_{t-1}, \cdots, W_{t-p}$. Asym theory

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- No free lunch, a little more conditions

HAC-free LP

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- Replace the i-th component in $s_{r,t,h}$ by its $(i-1)$-th lagged value, such as replace $z_{t+i} u^{\perp}_{t,h}$ by $z_t u^{\perp}_{t-i,h}$.

Econometrics intuition

- Recall $z_t = \alpha\varepsilon_{r,t} + \eta_t$ and $s^*_{r,t,h} := z_t(u^{\perp}_{t,h}, u^{\perp}_{t-1,h}, \cdots, u^{\perp}_{t-h,h})'$.
- Thus, we need to show $[\varepsilon_{r,t}u^{\perp}_{t,h}, \varepsilon_{r,t}u^{\perp}_{t-1,h}, \cdots, \varepsilon_{r,t}u^{\perp}_{t-h,h}]'$ is serially uncorrelated.
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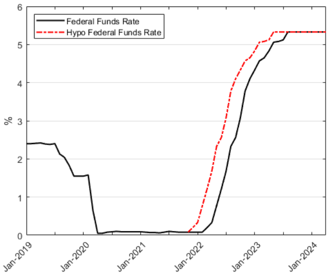
- The variance is identical to LRV

$$\mathrm{Var}(s^*_{r,t,h}) = \Omega_{r,h}. \tag{13}$$

Table of Contents

Historical hypo trajectory

- Question: What if the Fed had raised interest rates earlier during the pandemic?
- Study 3 scenarios:
  1. same slope but 3-month earlier interest rate hike
  2. same slope but 3-month earlier interest rate hike + ceiling rate
  3. less steep slope interest rate hike + ceiling rate



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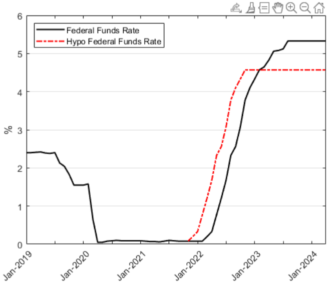

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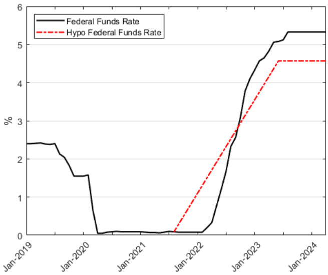


- policy shocks/IRF ▸ appendix:IRFs

Historical hypo trajectory

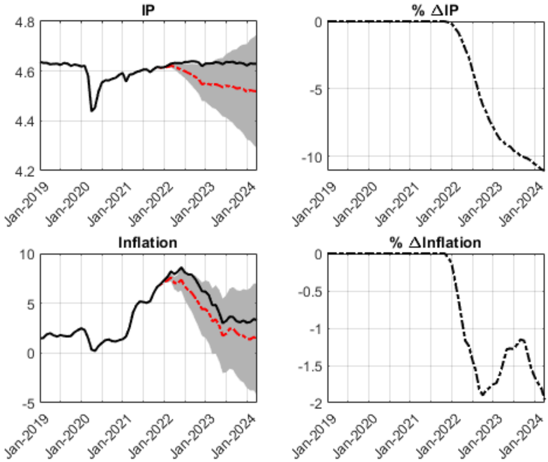


Figure: Same slope but 3-month earlier interest rate hike

Historical hypo trajectory

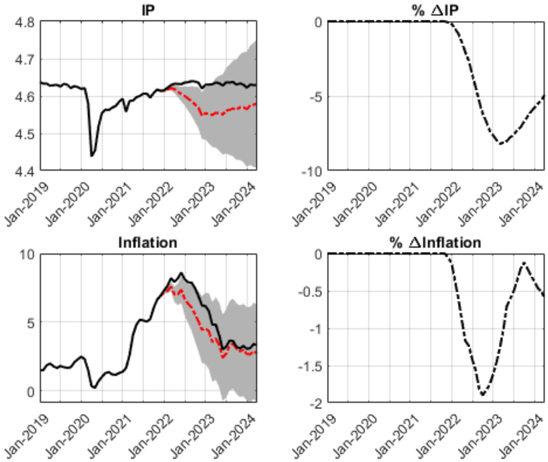


Figure: Same slope but 3-month earlier interest rate hike + ceiling rate

Historical hypo trajectory

Message: An earlier interest rate hike could have reduced the inflation rate by 1.5%, at the cost of a temporary 6% decline in industrial production. [During the pandemic, the Fed prioritized output over inflation.]

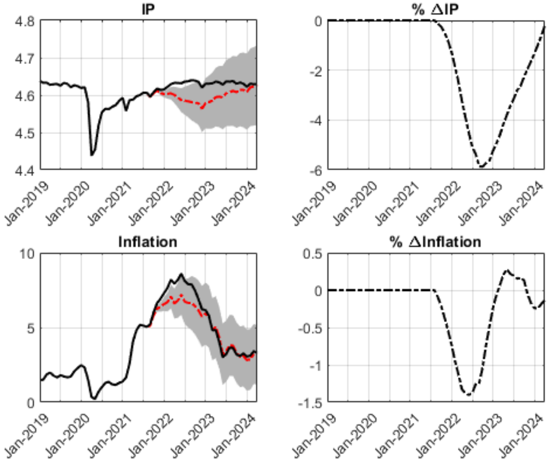
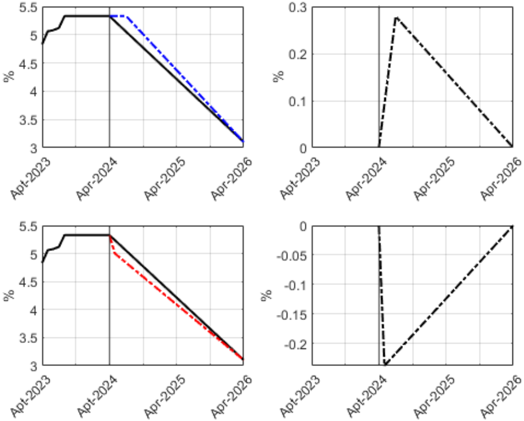


Figure: more gentle slope + ceiling rate

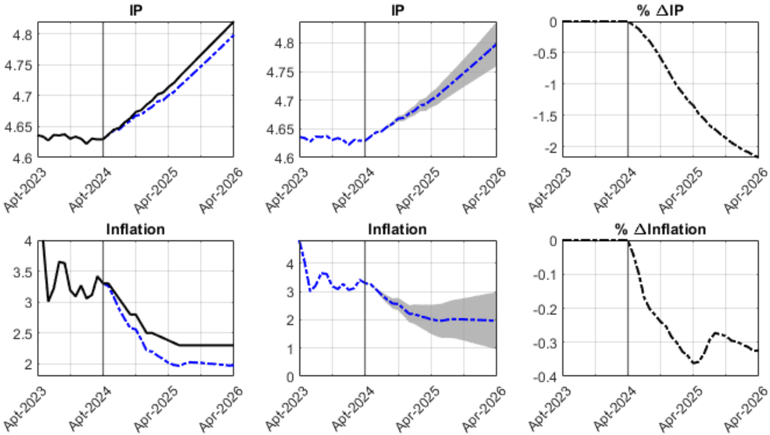
Future hypo trajectory

- Question: What if the Fed implements a more hawkish (or dovish) policy in the future (May 2024 and ongoing)? (baseline: The SPF)
- Hawkish: stay at 5.25% - 5.5% range for another 3 months and cut the rate at August
- Dovish: cut the rate to 5% - 5.25% range at May 2024



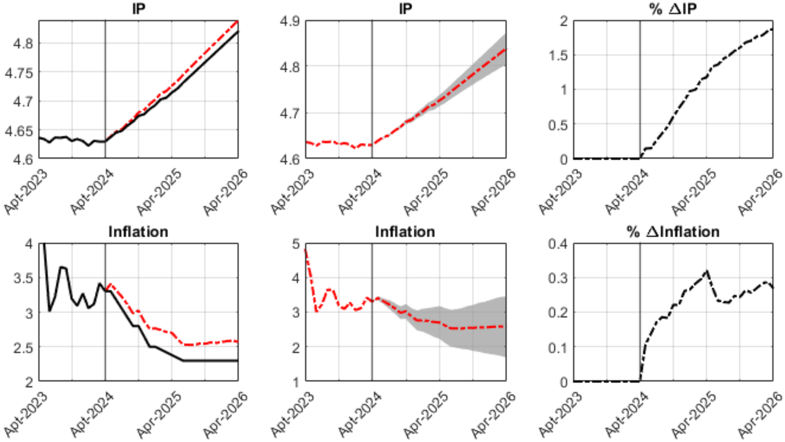
Future hypo trajectory

Message: Our hawkish policy analysis shows the inflation rate can be brought down to 2% in April 2025 with 1.4% cost on IP. [The recent policy decision shows that the Fed is now more focused on controlling inflation.]



Future hypo trajectory

Our dovish policy path, with the first cut in May, could boost industrial production by 1.2% within a year, but it would come at the cost of a 0.3% increase in the inflation rate. [Dovish policy is not on the Fed's table now.]



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Macroeconometrics take-away

- Under linearity, we demonstrate IRFs are sufficient to conduct macro policy path counterfactuals. through the computation of the necessary policy shocks to realize the targeted counterfactual policy path.
- We bring counterfactual interpretation on local projection and provide a "HAC free" LP-IV to implement robust policy counterfactuals.

Monetary policy take-away

- Historical context: Our findings suggest that the Fed could have reduced the inflation rate by 1.5% with an earlier and more gentle rate increase, although at the cost of a temporary 6% decline in industrial production.
- Future context: Our hawkish policy analysis shows the inflation rate can be brought down to 2% in April 2025 with 1.4% cost on IP.
- Future researches on econometrics
  - LP-IV with GMM (more high frequency IVs for monetary policy shocks).
  - Semiparametric/Nonparametric IV estimation.

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Asymptotic Normality

Assumption (Asymptotic Normality of Impulse Response estimates)

Suppose

$$\sqrt{T}\begin{pmatrix} \hat{\theta}_{yx,h} - \theta_{yx,h} \\ \hat{\boldsymbol{d}}_H^{(po)} - \boldsymbol{d}_H^{(po)} \\ \text{vec}(\hat{\Theta}_{re,H} - \Theta_{re,H}) \\ \hat{\boldsymbol{\theta}}_{ye,h} - \boldsymbol{\theta}_{ye,h} \end{pmatrix} \xrightarrow{d} N(\mathbf{0}, \Omega_{h,H}). \quad (14)$$

Let

$$\begin{aligned} \check{\psi}_h &= \check{\beta}'_{h,H}\boldsymbol{d}_H^{(ht)}, \\ \check{\phi}_h &= \hat{\theta}_{yx,h} - \check{\beta}'_{h,H}\hat{\boldsymbol{d}}_H^{(po)}, \end{aligned} \quad (15)$$

where $\check{\beta}_{h,H} = \hat{\Theta}_{re,H}^{-'}\hat{\boldsymbol{\theta}}_{ye,h}$.

Analytical solution

Go back

Proposition

Suppose Assumption 1 holds, then

$$\sqrt{T}(\check{\psi}_h - \psi_h) \xrightarrow{d} N(0, \text{AVar}(\check{\psi}_h)), \tag{16}$$

$$\sqrt{T}(\check{\phi}_h - \phi_h) \xrightarrow{d} N(0, \text{AVar}(\check{\phi}_h)), \tag{17}$$

where

$$\text{AVar}(\check{\psi}_h) = \mathbf{d}_H^{(ht)'} G\Omega_{e,h,H} G' \mathbf{d}_H^{(ht)}, \quad \text{AVar}(\check{\phi}_h) = J\Omega_{h,H} J',$$

such that $\sigma_{\psi,h,k} = \mathbf{b}_k' G\Omega_{e,h,k} G' \mathbf{b}_k$ *and* $\sigma_{\phi,h,k} = J\Omega_{x,e,h,k} J'$, *and* G, J *are Jacobian matrices, such that* $J = \left[1, -\vartheta'_{yr,h,k}, -\boldsymbol{\theta}'_{rx,k} G\right]$, *and* G *depends on the form of Moore–Penrose inverse,*

1. *if* $\Theta^{-}_{re,k} = (\Theta'_{re,k}\Theta_{re,k})^{-1}\Theta'_{re,k}$, *then*

$$G = [(\boldsymbol{\theta}'_{ye,h,k}(\Theta'_{re,k}\Theta_{re,k})^{-1}) \otimes (I_{k+1} - \Theta^{-'}_{re,k}\Theta'_{re,k}) - (\boldsymbol{\theta}'_{ye,h,k}\Theta^{-}_{re,k} \otimes \Theta^{-'}_{re,k})K_{(k+1),n_e}, \Theta^{-'}_{re,k}];$$

2. *if* $\Theta^{-}_{re,k} = \Theta^{-1}_{re,k}$, *then* $G = [-(\boldsymbol{\theta}'_{ye,h,k}\Theta^{-1}_{re,k}) \otimes \Theta^{-1'}_{re,k} \quad , \quad \Theta^{-'}_{re,k}]$;

3. *if* $\Theta^{-}_{re,k} = \Theta'_{re,k}(\Theta_{re,k}\Theta'_{re,k})^{-1}$

$$G = [(\boldsymbol{\theta}'_{ye,h,k}(I_{n_e} - \Theta^{-}_{re,k}\Theta_{re,k})) \otimes (\Theta_{re,k}\Theta'_{re,k})^{-1} - ((\boldsymbol{\theta}'_{ye,h,k}\Theta^{-}_{re,k}) \otimes \Theta^{-'}_{re,k})K_{(k+1),n_e}, \Theta^{-'}_{re,k}];$$

where $K_{n,m}$ is the commutation matrix, $\text{vec}(A') = K_{n,m}\text{vec}(A)$, for a $m \times n$ A matrix A.

Assumptions

Assumption (Regularity conditions)

For all $t \geq 1$ and $h \geq 0$, let c_0, c_1 are positive constant, suppose

1. $\eta_t, W^{\perp}_{t+h}$ *are covariance stationary.*
2. $\varepsilon_t, \eta_t, W^{\perp}_{t+h}$ *are strong mixing (α-mixing) processes with mixing size $-r/(r-2)$, for $r > 2$ and all finite integer $h \geq 0$.*
3. $\mathbb{E}\|\varepsilon_t\|^{4r+\delta}$, $\mathbb{E}\|\eta_t\|^{4r+\delta}$, *and* $\mathbb{E}\|W^{\perp}_{t+h}\|^{4r+\delta} < c_0 < \infty$, *for r defined in (ii), and* $\delta > 0$.
4. $\lambda_{\min}\left(\Omega_{i,h}\right) > c_1 > 0$, *for* $i \in \{r, x\}$.

Asymptotic normality

Go back

Proposition

Let W_t follows a SVMA and Assumption 3 is satisfied. Then

$$\sqrt{T}\sigma_{\psi,h}^{-1}\left(\tilde{\psi}_h - \psi_h\right) \xrightarrow{d} N(0,1), \tag{18}$$

where $\sigma_{\psi,h}^2 = (\boldsymbol{d}_h^{(ht)'}\Sigma_{zr,h}^{-1}\Omega_{r,h}\Sigma_{zr,h}^{'-1}\boldsymbol{d}_h^{(ht)})^{1/2}$ *and* $\Sigma_{zr,h} := \mathbb{E}[\boldsymbol{z}_{t,h}R_{t:t+h}^{\perp'}]$. *And*

$$\sqrt{T}\sigma_{\phi,h}^{-1}\left(\tilde{\phi}_h - \phi_h\right) \xrightarrow{d} N(0,1), \tag{19}$$

where $\sigma_{\phi,h} = (v(1)'\Sigma_{zx,h}^{-1}\Omega_{x,h}\Sigma_{zx,h}^{'-1}v(1))^{1/2}$ *and* $\Sigma_{zx,h} := \mathbb{E}[\boldsymbol{z}_{x,t,h}R_{x,t,h}^{\perp'}]$.

Policy shocks

Go back

1. Too few shocks ($col(\Theta_{re,H}) < (H+1)$): The policymaker manipulates the available shocks as effectively as possible to replicate the counterfactual path. However, because of the limited number of shocks they can control, they cannot perfectly replicate the path and must instead select the closest possible approximation.
2. Exact shocks ($col(\Theta_{re,H}) = (H+1)$: The policy maker has just enough policy tools to replicate the counterfactual path exactly.
3. Too many shocks ($col(\Theta_{re,H}) > (H+1)$): The policy maker has more than sufficient tools. They replicate the counterfactual path precisely in the least "surprising" way to the private sector. This "least surprising" approach is characterized by minimizing the Euclidean norm of the shock sizes.

Choices of structural shocks

Go back

Choices of structural shocks

Go back

- Period-by-period shocks, $\varepsilon_{t,H,(S)} = (\varepsilon_{r,t}, \varepsilon_{r,t+1}, \cdots, \varepsilon_{r,t+H})'$. e.g., Bernanke et al. (1997)
  - $\varepsilon_{r,t}$ is a scalar.
  - Private sector has no prior information about future intervention. They receive and respond to the surprise period after period.
  - No ex-ante expectations of the private sector.

Choices of structural shocks ▸ Go back

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Choices of structural shocks

Go back

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- Multi-shock at initial date $\varepsilon_{t,H,(S)} = \varepsilon_{r,t}$, McKay and Wolf (2023)
  - The Lucas critique: the ex-ante expectations of the private sector. Multi-shock at initial date McKay and Wolf (2023)
  - $\varepsilon_{r,t}$ is a vector, e.g., multiple monetary shocks Inoue and Rossi (2021)
  - Not nested in SVAR model
  - Private sector receives all information about future intervention and responses at the initial period.

IRF to 25bps monetary policy shocks

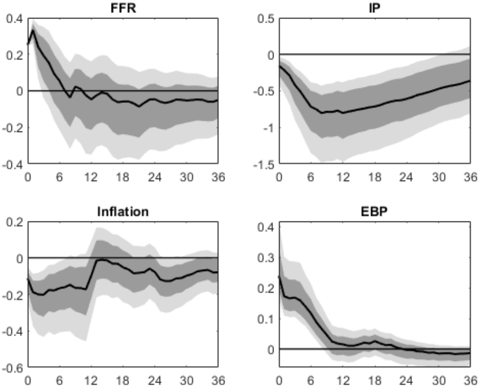

Figure: The data: January 1973 to December 2019 on a monthly basis. Estimation model: $\Phi(L)W_t = \Theta_0 \varepsilon_t$, 4-variate SVAR-IV model with 14 (AIC) lags, W_t includes the FFR, industrial production, inflation rate, and excess bond premium. The monetary policy column in Θ_0 is identified through an IV, the orthogonalized monetary policy surprise (MPS ORTH) measure published by Bauer and Swanson (2023).

Go back

IRF to 10% oil price supply shock

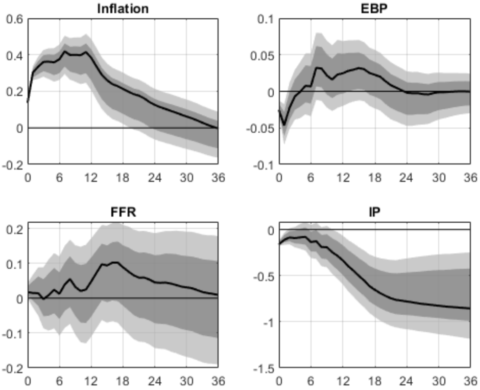

Figure: Five-variable SVAR-IV model with twelve lags, $\Phi(L)W_t = \Theta_0 \varepsilon_t$, W_t includes the WTI oil price (log), industrial production (log), the federal funds rate (FFR), the CPI inflation rate, and the excess bond premium (EBP), spanning from January 1975 to December 2019. The external instrument for the oil price shock is the monthly "Oil Supply News Shocks" provided by Känzig (2021).

Go back

Ideal policy path ▸ back

- Given a baseline economy, suppose the policy maker would like to see an ideal path for aggregate output $\tilde{\boldsymbol{y}}^*_{t:t+H}$.

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Ideal policy path ▸ back

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- How should the policymaker intervene in the market to achieve the specified aggregate output path, and what would be the corresponding values of the interest rates?
- The only tool that the policy maker has is the policy shocks. They will intervention the market with the policy shocks of size $\boldsymbol{\delta}^*_H$ to achieve the ideal aggregate output path,

$$\Theta_{ye,H}\boldsymbol{\delta}^*_H = (\boldsymbol{y}_{t:t+H} - \tilde{\boldsymbol{y}}^*_{t:t+H}), \tag{20}$$

where $\Theta_{ye,H} := \partial Y_{t:t+H}/\partial \varepsilon'_{t,H,(S)}$.

Ideal policy path ▸ back

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where $\Theta_{ye,H} := \partial Y_{t:t+H}/\partial \varepsilon'_{t,H,(S)}$.

- The desired policy path $\tilde{\boldsymbol{r}}^*_{t:t+H}$ can be computed as follows,

$$\tilde{\boldsymbol{r}}^*_{t:t+H} = \boldsymbol{r}_{t:t+H} - \Theta_{re,H}\boldsymbol{\delta}^*_H. \tag{21}$$

- Specify an ideal path of aggregate output ⇒ suggested policy shocks ⇒ suggested policy path