Vague by Design: Performance Evaluation and Learning from Wages

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Introduction

Performance evaluation is a key aspect of labor contracts and organization design

- Many ways to evaluate: Shop floor control, consumer scores, product testing, sales,...
- Digitization and AI provide a growing number of possibilities

Performance evaluations are an important source of information in the workplace

- **Inform the firm** about the worker's performance
  - Necessary basis of incentivizing effort via performance pay
  - Classic results show more information is better Holmström '79, Grossman&Hart '83
- **Inform the worker** about his performance
  - Learn about ability/match with the job
  - Confidence in his capability to succeed and sense of agency

Dual role of performance evaluation: basis of *incentives* and agent *learning*

- How do these two aspects interact?
- How to optimally design performance evaluation when it shapes worker confidence?

- This talk: mostly binary case

Related Literature

- Design of information
  Kolotilin ’18, Kolotilin et al. ’22, Doval&Skreta ’23, ...
  and performance pay:
  Georgiadis&Szentes ’20, Hoffmann et al. ’21, Li&Yang ’20
- Implicit incentives and information design:
  Ely&Szydlowski ’20, Hörner&Lambert ’21, Smolin ’20
- More information can increase the cost of incentives:
  Fang&Moscarini ’05, Jehiel ’14, Meyer&Vickers ’97, Nafziger ’09

General Model

The Model

- Two time periods $t \in \{1, 2\}$, common discount factor δ.
- Agent
  - risk averse with utility index u and reservation utility U
  - observable but nonverifiable effort $e_t \in \{0, 1\}$ at cost $c \cdot e$
  - time-invariant ability $\theta \in \Theta = \{\theta_L, \theta_H\}$ (this talk), with prior μ_0
  - realizes output $y \in Y \subset \mathbb{R}$, compact, according to $F(\cdot|e, \theta)$, mutually a.c.
- Principal
  - risk neutral
  - implements high effort

- At the beginning of each period, the principal commits to a contract (S, p, w) consisting of
  - a signal structure $S, p(s|y_t)$, and
  - wages w as a function the signal.

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  - a signal structure $S, p(s|y_t)$, and
  - wages w as a function the signal.
- Agent observes the contract and makes participation and effort decision
- Output is not observed
- Principal and agent observe the signal realization, wages, and effort
- Update beliefs to $\mu(s)$

The Binary Case

- Output is high or low, $y_t \in \{y_L, y_H\}$, high with probability

type \ effort	$e_t = 0$	$e_t = 1$
$\theta = \theta_L$	a	$a + b$
$\theta = \theta_H$	$a + \Delta a$	$a + b + \Delta a + \Delta b$

- Effort is productive: $b \geq 0$
- Ability is productive: $\Delta a \geq 0$
- Complementarities: Δb
  Log-Supermodular: $\frac{\Delta b}{b} > \frac{\Delta a}{a}$
  Log-Submodular: $\frac{\Delta b}{b} + \frac{\Delta a}{1-a} < 0$

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The Contracting Problem

First Period

$$\Pi_1 = \max_{S,p,w} \iint \left(y - w(s) + \delta\Pi_2(\mu(s))\right)\, dp(s|y)\, dF(y|1,\mu_0) \tag{1}$$

$$\text{s.t.} \iint u(w(s))\, dp(s|y)\, dF(y|1,\mu_0) - c \geq U \tag{P$_1$}$$

$$\iint u(w(s))\, dp(s|y)\, dF(y|1,\mu_0) - c \geq \iint u(w(s))\, dp(s|y)\, dF(y|0,\mu_0) \tag{IC$_1$}$$

Second Period

$$\Pi_2(\mu) = \max_{S,p,w} \iint \left(y - w(s)\right)\, dp(s|y)\, dF(y|1,\mu) \tag{2}$$

$$\text{s.t.} \iint u(w(s))\, dp(s|y)\, dF(y|1,\mu) - c \geq U \tag{P$_2$}$$

$$\iint u(w(s))\, dp(s|y)\, dF(y|1,\mu) - c \geq \iint u(w(s))\, dp(s|y)\, dF(y|0,\mu) \tag{IC$_2$}$$

2^{nd} Period and Continuation Value

The Final Period

- Pure incentive problem, no motive to shape learning
- Classic result:

Proposition

The optimal evaluation in the final period is fully informative.

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$$\int \Pi_2(\mu)\,\mathrm{d}m(\mu)$$

- What determines the shape of the continuation value?
- Easy to compute, but hard to characterize in general.
- Important special case: binary

The Binary Case: The Impact of Information

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   } scales with Δa: impact of ability

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   } scales with Δb: interaction of effort and ability

The Binary Case: The Impact of Agent Learning

- Second-period IC:

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- Required bonus inversely proportional to a linear function of beliefs
    - Agent with high impact $(b + \mu \Delta b)$ cheaper to motivate
    - Uncertain agent is cheaper to motivate
    - Given change in belief: larger effect at low impact

The Binary Case: Learning is Costly

Proposition

In the binary case (under a bound on $u^{-1'''}$):

If the technology is log-supermodular, Π_2 is strictly concave and it is more concave at low posteriors, $\Pi_2''' > 0$.

If the technology is log-submodular, Π_2 is strictly concave and it is more concave at high posteriors, $\Pi_2''' < 0$.

- Strong interaction of effort and ability: Agent learning dominates
- Principal has an incentive to conceal information
- Avoid agents who think they have no impact: pessimism and complacency

The Optimal Evaluation Structure

Solving the Full Problem

- First period: Incentives and learning
  - Incentives: More informative evaluation *decreases* agency cost *this period*
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- First period: Incentives and learning
  - Incentives: More informative evaluation *decreases* agency cost *this period*
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- Transform problem to an information design problem, with:
  - Endogenous payoffs (wages are designed)
  - Additional constraints (participation and incentive compatibility)
- Maintained assumptions:
  - MLRP
  - no incentives at infinity: $\frac{u^{-1}(x)}{x} \to \infty$ as $x \to \infty$

Posterior Space

Proposition (Linear Distribution Function)

Suppose that the distribution over output $F(\cdot|e,\mu) \in \Delta Y$ can be decomposed as

$$F(\cdot|e,\mu) = F(\cdot|0,0) + (\Delta a\mu + e\,(b + \Delta b\mu))\,\Delta F(\cdot).$$

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- Rewrite the program as a choice of $m \in \Delta\Delta\Theta$
- $\bar{m}$: distribution of posterior with fully informative evaluation

$$\Pi_1 = \max_{w,m\in\Delta[0,1]} \mathbb{E}_m\left[y - w(\mu) + \delta\Pi_2(\mu)\right] \tag{3}$$

$$\text{s.t.}\ \mathbb{E}_m\left[u(w(\mu))\right] - c \geq U \tag{P_1}$$

$$\mathbb{E}_m\left[x(\mu)u(w(\mu))\right] \geq c \tag{IC_1}$$

$$m \leq_{MPS} \bar{m} \tag{BP}$$

- An evaluation features **lower censorship** if realizations below a cutoff are pooled and those above the cutoff fully revealed.

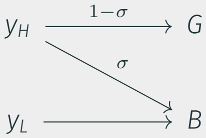
Theorem

Suppose that F satisfies the LCDF condition and let $v = u^{-1}$.

- If $\Pi_2''' > 0$ and v'' is decreasing, the optimal evaluation structure features lower censorship.
- If $\Pi_2''' < 0$ and v'' is increasing, the optimal evaluation structure features upper censorship.

Corollary

In the binary case with log-complements, the optimal evaluation is binary ($S = \{G, B\}$) and tough. The optimal contract consists of



- a good evaluation and associated high wage, only if output was good,
- a bad evaluation and associated low wage: always after output was bad, with prob. σ after output was good.

$$\mathcal{L}(w, m; \underbrace{(\lambda_P, \lambda_{IC})}_{\lambda})$$

Lagrangian of the contracting problem including (P) and (IC)

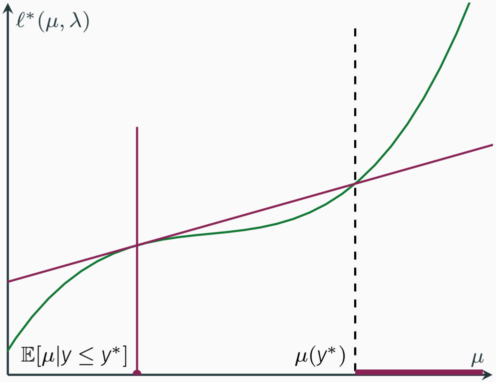
Information design on the partially maximized Lagrangian (Georgiadis&Szentes '20)

$$\mathcal{L}(w, m; \lambda)$$

Optimal Wages given m, λ: Standard moral hazard problem $\mapsto w^*(\hat{\mu}; \lambda)$
objective is an expectation given λ: $\mathcal{L}(w^*(\hat{\mu}; \lambda), m; \lambda) = \int \ell^*(\hat{\mu}; \lambda) m(\hat{\mu})\, \mathrm{d}\hat{\mu}$

Information Design given λ: Shape of $\ell^* \mapsto m^*(\hat{\mu}; \lambda)$
$\frac{\partial^3}{\partial \hat{\mu}^3} \ell^*(\hat{\mu}; \lambda) = \lambda_{IC}^3[\cdot]\rho''(\lambda_P + \lambda_{IC}[\cdot](\hat{\mu} - \mu)) + \delta \Pi_2'''(\hat{\mu})$

Duality: $\mapsto$ Solution exists and features of m^* hold in the optimal contract



- Unconstrained information design with $\ell^*(\mu; \lambda)$
- New difficulty: $\overline{m}$ with atoms and gaps in support $\implies$ generalize KMZ '22

Theorem 2

Suppose $V''' > 0$. Then, generalized lower censorship is the essentially unique solution to $\max_{H \leq_{MPS} F} \int_0^1 V(s)\, \mathrm{d}H(s)$.

- $\frac{\partial^3}{\partial \hat{\mu}^3} \ell^*(\hat{\mu}; \lambda) = \lambda_{IC}^3[\cdot]\rho''(\lambda_P + \lambda_{IC}[\cdot](\hat{\mu} - \mu)) + \delta\Pi_2'''(\hat{\mu})$
  - Convex $\implies$ m fully informative
  - Concave-convex $\implies$ lower-censorship
- This for given λ, but $\lambda(m)$!

Optimal Evaluation: Discussion

- Noisy evaluation can be optimal
  - Preserve agent's uncertainty
- Complements:
  - Base wage + substantial, tailored bonuses for high performance / tough evaluation
  - Binary case: "Drill-sergeant mentality" is part of optimal organization design – avoid unwarranted praise, embrace unwarranted reprimand
- Substitutes:
  - Capped performance pay (rich Y) / lenient evaluation
- Prevent very low expected impact of effort
  - Costly to motivate, change in posterior has a large effect
- Result of joint design of evaluation and wages

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  - Full-commitment difficult: belief-manipulation & belief-dependent costs of delay
- Many periods
  - Not analytically tractable: lack of control over shape of continuation value
  - Numerically: Same structure within period; noisier evaluation early in the relationship

Conclusion

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- Tension between those two channels (learning about the importance of effort)
  - As much information as possible about effort
  - Often as little information as possible about ability
- Optimal Performance Evaluation
  - Noisy, even though wages could condition on true y
  - Strong complementarity: avoid very low posterior beliefs (tough/lower-censorship)

Outlook

- Preference across given information sources: conduct, not results!
  - Salary differences between workers: mostly driven by types, so should be concealed
- Affects task design: Harder/easier to keep agents motivated
- Career Concerns: informationally opposite forces
  - information about effort and ability inseparably intertwined
  - here: source of friction; CC: source of incentives

Thank You!