Nonlinearities in the Regional Phillips Curve with Labor Market Tightness

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Abstract

This paper is the first to show the presence of nonlinearities in the regional U.S. New Keynesian Phillips curve with the vacancy-to-unemployment ratio to measure labor market tightness. Such nonlinearities contribute to explaining the unexpected and persistent post-COVID inflation surge and have important implications for monetary policy. To guide my empirical exercise, I introduce wage rigidities and search-and-matching frictions in the labor market into a standard multi-sector, two-region New Keynesian model. The model delivers a piecewise log-linear regional Phillips curve, which becomes steeper when labor markets become tight. I estimate the Phillips curve using panel variation in core inflation and a newly imputed measure of vacancy-to-unemployment ratio across U.S. metropolitan areas from December 2000 to July 2024. I instrument the vacancy-to-unemployment ratio with a shift-share instrumental variable to take care of regional supply shocks. The regional Phillips curve has a slope not statistically different from zero in slack labor markets and significantly steepens when labor markets tightens – specifically, when the vacancy-to-unemployment ratio exceeds one. This result suggests that if the monetary authority assumes that the Phillips curve is linear, it will underestimates inflationary pressures in tight labor markets, allowing inflation to surge more than expected.

Keywords: Phillips curve, nonlinearities, inflation, labor market tightness, monetary policy

JEL code: E30, J63, J64

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1 Introduction

Macroeconomists and policy-makers study inflation dynamics through the New Keynesian Phillips curve, a structural equation that relates inflation to measures of real economic activity, supply shocks, and inflation expectations. The relationship between inflation and real economic activity goes through the labor market. The Phillips curve captures the concept that in demand-driven booms, workers ask for higher wages, leading firms to raise prices. Therefore, tight labor market conditions are relevant indicators of inflationary pressures coming from raising labor costs.

The 20 years leading up to the COVID-19 pandemic, characterized by overall slack labor markets[1] and low and stable inflation, fostered a consensus that the U.S. Phillips curve was linear and flat (Hazell et al., 2022). In other words, fluctuations in economic activity would produce limited effects on inflation. The experience of the post-COVID period, however, challenged this consensus. In the aftermath of the pandemic, the U.S. labor market was the tightest since World War II (Michaillat and Saez, 2022). At the same time, the 12-month core CPI inflation rate in the United States began to rise, reaching a 40-year high at seven percent in September 2022 and remaining high well into the beginning of 2024.

In this paper, I investigate whether inflationary pressures in tight labor markets might be stronger than in slack labor markets, potentially leading to a Phillips curve that is nonlinear in the state of the labor market. A major challenge for making this assessment, however, is that time series observations of tight labor markets are limited.[2] For this reason, I turn to panel data, which provides greater variation and more instances of tight labor markets. Figure 1 shows the relationship between the 12-month core CPI inflation rate and the logarithm of the vacancy to-unemployment ratio, known as labor market tightness[3], across 21 U.S. Metropolitan Statistical Areas (MSAs) from December 2000 to July 2024. The scatter plot illustrates a positive correlation between MSA core inflation and labor market tightness, which becomes steeper around a tightness value of 1 – corresponding to the value of logarithm of tightness equal to 0.

This paper is the first to identify and estimate a nonlinear regional Phillips curve (NRPC from

[1]Between 2000 and 2020, the 12-month headline CPI inflation averaged 2.2% in the United States. During the same period, the U.S. labor market was slack, except for a limited period between 2018 and 2019 (Michaillat and Saez, 2022).

[2]Combining data from Barnichon (2010) and Petrosky-Nadeau and Zhang (2021), Michaillat and Saez (2022) show that before 2018-2019 and 2021-2022, the U.S. labor market was inefficiently tight only during World War II, the Korean War, and the Vietnam War.

[3]Labor market tightness is defined as the ratio between vacancies posted by firms and the number of unemployed workers. It measures labor market conditions, taking into account measures of both labor demand (vacancies) and labor supply (unemployed workers looking for a job). From now on, I will refer to the vacancy-to-unemployment ratio as labor market tightness, to be consistent with the search-and-matching literature.

now onward) with labor market tightness as a proxy for real economic activity. To guide my empirical exercise, I introduce two novel features into an otherwise standard multi-sector New Keynesian model of two regions in a monetary union. First, I incorporate search-and-matching frictions in the labor markets, which give rise to a relationship between inflation and tightness. Second, I introduce wage rigidities that generate a kink in this relationship, as in Figure 1. Thanks to these two elements, the model yields a piecewise log-linear regional Phillips curve with labor market tightness as a proxy for real economic activity. To perform the empirical exercise, I impute a novel series of vacancies at the MSA level and use it to measure regional labor market tightness. Thanks to this new variable, I estimate the NRPC derived in the model. Estimation relies on an identification strategy that combines MSA-level panel variation in core inflation and tightness with an instrumental variable approach. I instrument labor market tightness with a shift-share instrumental variable that proxies for sectoral labor demand shocks to deal with unobservable regional supply shocks.

Figure 1: Correlation of inflation and log of labor market tightness at the MSA level, Dec00-Jul24

Notes. The figure shows the scatter plot between the 12-month core inflation rate and the logarithm of labor market tightness (vacancy-to-unemployment ratio) across 21 Metropolitan Statistical Areas (MSAs) in the United States from December 2000 to July 2024. Vacancies across metropolitan areas are imputed by combining state-level vacancies collected by BLS JOLTS with yearly employment weights from the Current Population Survey (CPS).

I find that the regional Phillips curve is nonlinear in labor market tightness. In particular, I document two novel sets of results. First, I estimate that the slope of the regional Phillips curve is positive but not statistically different from zero when labor markets are slack. Second, the regional Phillips curve significantly steepens in tight labor markets. In the baseline specification, I define a tight labor market when the vacancy-to-unemployment ratio exceeds one. This value of the kink is in line with the evidence presented in Figure 1 and in Benigno and Eggertsson (2023). It is also supported by the theoretical argument developed by Michaillat and Saez (2022). This result has important implications for monetary policy. If the central bank assumes that the Phillips curve is linear, it will underestimate inflationary pressures in tight labor markets, allowing inflation to surge more than expected. Moreover, the nonlinearity in the Phillips curve allows the central bank to decrease inflation at a lower economic cost when labor markets are tight, potentially achieving the so-called "soft-landing" during a rate hike.

To guide my empirical exercise, I rely on a New Keynesian general equilibrium model augmented with four key features: two regions in a monetary union, a vertically-linked production structure, search-and-marching frictions, and wage rigidities. The first two components provide the foundation for my empirical strategy, based on panel variation and an instrumental variable approach. MSA-level panel data requires to derive a regional Phillips curve. The instrumental variable exploits labor demand shocks in the intermediate-input industries that affect the pricing-setting decisions of final-goods firms.

Search-and-matching frictions in the labor markets introduce formally into a New Keynesian model the concept of unemployment and generate a relationship between inflation and labor market tightness. I model them in the spirit of the Diamond-Mortensen-Pissarides theoretical framework. Households in each region choose their labor force participation, but only a fraction of the labor force is attached to firms. Employment agencies post firms' job vacancies and match them to unemployed workers searching for a job. Crucially for generating labor market frictions, posting a vacancy is costly.

Wage rigidities give rise to nonlinearities in the regional Phillips curve. Based on the evidence in Figure 1, I model a wage-setting mechanism that introduces a kink in the relationship between inflation and labor market tightness. Following Phillips (1958), I assume that during labor market shortages (in sufficiently tight labor markets), firms bid up wages to attract workers, and wages of new hires rise fast. In this case, I allow wages of new hires to be fully flexible and to be pinned down by the optimal behavior of employment agencies. In normal circumstances (in slack labor markets), new hires are reluctant to accept a wage rate below wages of attached workers. In this case, wages of new hires are equal to wages of attached workers. Wages of attached workers fall slowly and constrain firms' labor demand. In sum, the wages of new hires depends on the level of labor market tightness in each region. As the marginal cost structure of price-setting final-goods firms depends only on the hiring costs and the wages of new hires, their nonlinearity generates a kink in the regional Phillips curve.

To bring the structural NRPC curve to the data, I impute a novel measure of vacancies at the metropolitan area level. Measuring labor market tightness at this level of disaggregation is challenging because a public and representative source of MSA-level vacancy data does not exist. I overcome this problem by imputing the number of vacancies at the city level from state-level vacancies recorded by the BLS's Jop Openings and Labor Turnover Survey (JOLTS), using yearly employment weights from the Current Population Survey (CPS). The main advantage of this new measure is that it is constructed using vacancies posted by a representative sample of firms. The instrumental variable takes care of the measurement error I introduce in labor market tightness, the main explanatory variable.

To estimate the NRPC, I combine panel variation in core inflation and labor market tightness at the MSA level with an instrumental variable approach. Figure 1 reports a simple correlation between core inflation and tightness, which can be driven by aggregate or regional confounders. Examples of these confounders are aggregate or regional supply shocks, such as a rise in the international price of oil or migration inflows into a regional labor market. A negative supply shock decreases labor market tightness and increases inflation, inducing a downward bias in the relationship between tightness and inflation. Using panel data enables me to incorporate regional and time fixed effects into the empirical model. Time fixed effects capture aggregate confounders, such as aggregate supply shocks, but also long-run inflation expectations that depend on the monetary regime in place (Hazell et al., 2022), and endogenous national monetary and fiscal policies (Fitzgerald and Nicolini 2014, McLeay and Tenreyro 2020).

I employ an instrumental variable approach to take care of regional confounders. Guided by the model, I develop a shift-share instrument that combines national labor demand shocks in the tradable intermediate-input industries with the exposure of each metropolitan area to these shocks based on its industrial composition (Bartik, 1991). These aggregate shocks affect labor demand relatively more in those metropolitan areas specialized in such industries. As the labor market is common across sectors within metropolitan areas, a larger change in labor demand will lead to a larger change in wages and, therefore, a larger change in marginal costs of local final-goods firms in those specialized cities, which ultimately will be reflected in higher final-goods prices. For instance, a positive national labor demand shock in the manufacturing sector leads to larger cost increases for final-goods firms located in manufacturing-intensive cities like Detroit. The identifying assumption requires that such cost increases are no larger on average for final-goods firms in Detroit than New York.[4]

I conduct several robustness checks. First, the estimated nonlinearity is robust to different definitions of the kink. Second, using population instead of employment weights to impute vacancies at the MSA level does not affect the results. Third, instead of a single kink, I allow for more flexible nonlinearities and show that my results are unchanged. Finally, I show that the NRPC is robust to the inclusion of a control for final-goods sector productivity shocks, a likely threat to the exclusion restriction for the validity of the shift-share instrument. Overall, I conclude that the finding of nonlinearity in the regional Phillips curve with labor market tightness is robust.

Since the seminal work of Phillips (1958), the Phillips curve has been extensively studied theoretically and empirically. In this vast literature, my paper relates to three strands. First, I contribute to an emerging theoretical literature that seeks to model nonlinearities in the aggregate Phillips curve to explain the post-COVID surge in inflation. In particular, guided by the correlation between MSA-level core inflation and labor market tightness in Figure 1, my work builds on Benigno and Eggertsson (2023). With respect to the literature that incorporates search-and-matching frictions and wage rigidities into the New Keynesian theoretical framework (Blanchard and Galí 2010, Christoffel and Linzert 2005, Trigari 2006, Krause and Lubik 2007, Faia 2008, Michaillat 2014), Benigno and Eggertsson (2023) propose a novel wage-setting mechanism based on the theoretical argument of Phillips (1958) that can generate a kink in the aggregate Phillips curve. The contribution of my paper is to embed search-and-matching frictions and Benigno and Eggertsson (2023)'s wage-setting mechanism into a New Keynesian model of two regions in a monetary union. In addition, I model a vertical production structure. This vertical production structure and the regional nature of my model are the key ingredients that allow me to identify the slope of the Phillips curve by combining fixed effects with an instrumental variable approach. Other relevant contributions in this literature are Harding et al. (2023) and Schmitt-Grohé and Uribe (2022). Harding et al. (2023) generate a nonlinear aggregate Phillips curve through a quasi-kinked demand schedule for goods. Schmitt-Grohé and Uribe (2022) employ heterogeneous downward nominal wage rigidity for individual labor varieties to derive a nonlinear aggregate wage Phillips curve.

Second, my paper adds to the growing literature about the cross-sectional identification of the slope of the Phillips curve. Papers such as Mavroeidis et al. (2014), Fitzgerald and Nicolini (2014), McLeay and Tenreyro (2020), Hazell et al. (2022), and Cerrato and Gitti (2022) show how panel variation can help

[4]Intermediate-input industries' labor demand shocks driven by productivity shocks also have a direct impact on intermediate-input prices, affecting local final-goods firms' marginal costs through this channel as well. As intermediate-input prices are observable, I control for this second channel, preventing the invalidation of my instrumental variable strategy.

overcoming some of the identification challenges that affect the estimation of the slope of the Phillips curve at the aggregate level. Hazell et al. (2022) and Cerrato and Gitti (2022) are the papers closer to mine. With respect to them, I introduce theoretical and empirical innovations. On the theoretical side, I consider search-and-matching frictions and wage rigidities to derive the NRPC with labor market tightness as an explanatory variable. On the empirical side, I impute a novel measure of vacancies at the MSA level to proxy for economic activity with labor market tightness, measured by the ratio of vacancies to unemployment, rather than with the unemployment rate alone. Moreover, I estimate a regional Phillips curve with nonlinearities.

Third, few papers have used regional data to empirically investigate the presence of nonlinearities in the regional Phillips curve before the COVID-19 pandemic. Kiley (2015), Murphy (2017), Babb and Detmeister (2017), Leduc and Wilson (2017), and Hooper et al. (2020) all find evidence of the existence of the Phillips curve at the regional level between 1990 and 2019 and document the presence of nonlinearities. I contribute to this strand of literature by employing an instrumental variable approach to deal with biases coming from regional confounders, which cannot be controlled for by a simple two-way fixed effects model as all the other papers do. Moreover, I am the first to estimate the slope of the regional Phillips curve and to test for nonlinearities using labor market tightness to proxy for economic activity.

The remainder of this paper proceeds as follows. Section 2 describes the model and the derivation of the nonlinear regional Phillips curve. Section 3 details data sources and the construction of the novel measure of MSA-level vacancies. Section 4 discusses the empirical strategy. Section 5 presents the empirical results and their policy implications. Section 6 includes the robustness checks, and Section 7 concludes.

2 Model

I propose a New Keynesian model of two regions in a monetary union, featuring a vertical production structure, search-and-matching frictions in regional labor markets, and wage rigidities. Within the model, I derive an NRPC that I can bring to the data. The model also guides the empirical strategy I use to estimate the NRPC. The regional nature of the model and the vertical production structure are key to developing the empirical strategy. Such empirical strategy is based on two ingredients. First, panel variation across US metropolitan areas requires to estimate a regional Phillips curve. Second, the instrumental variable approach exploits the effects of intermediate-input sectors' labor demand shocks on the pricing-setting decisions of final-goods firms that use the intermediate inputs in their production processes. Search-and-matching frictions in the labor markets formally introduce unemployment into the New Keynesian framework. This feature generates a relation between inflation and labor market tightness. Wage rigidities give rise to the nonlinearity in the Phillips curve.

2.1 Model Setup

The model comprises two regions, i and j, belonging to the same monetary union as in the standard regional New Keynesian framework. The monetary authority sets the common interest rate following a Taylor rule, described in Appendix B. The two regions share the same preferences, market structure, and firm behavior. There is a continuum of representative households of measure ζ in region i and $(1-\zeta)$ in region j. The production side of the economy is composed of three vertically-linked sectors: an international commodity sector, a tradable perfectly-competitive intermediate-input sector, and a non-tradable monopolistically-competitive final-goods sector. Labor is immobile across regions and perfectly mobile across sectors within regions.

Differently from the standard regional New Keynesian framework, the labor market is not competitive. I distinguish between attached workers and new hires and add two frictions: search-and-matching frictions and wage rigidities. The wage-setting mechanism is motivated by the argument developed by Phillips (1958). According to Phillips, wages respond asymmetrically to the state of the labor market: they rise rapidly in tight labor markets and move slowly in slack labor markets. To capture Phillips' idea, I assume that wages of newly hired workers are equal to the maximum between the wages of attached workers and the flexible wage rate. The flexible wage rate is the one that clears the market in the absence of any friction. To pin it down, I introduce a simple model of employment agencies.

Employment agencies oversee the match-and-searching process between workers and firms in each region. They post firms' vacancies subject to a cost charging a fee proportional to the wage offered to the newly hired worker. Employment agencies choose how many vacancies to post in order to maximize real profits. In the optimum, they equate the marginal benefit of posting a vacancy to its marginal cost. The flexible wage rate is pinned down by the problem of the employment agencies. In the following paragraphs, I describe the economy of region i.

2.1.1 Households

The representative household in region i is indexed by h. In each period t, household h chooses how much to consume and how many household members work along the extensive margin in order to maximize the utility flow given by Greenwood–Hercowitz–Huffman (GHH) preferences (Greenwood et al., 1988), defined as

$$u(C_{it}(h), F_{it}(h), \chi_{it}) = \frac{1}{1-\sigma}\left(C_{it}(h) - \chi_{it}\int_0^{F_{it}(h)} f^{\omega}\, df\right)^{1-\sigma}, \tag{1}$$

where $C_{it}(h)$ is total consumption of household h, and $F_{it}(h)$ denotes the number of members of household h who decide to participate in the labor market. Each household member is indexed by f and has fixed disutility f^{ω} from working, as in Galí (2011), with $\omega > 0$. χ_{it} is an exogenous variable governing the intensity of disutility of labor and $\sigma > 0$.

Household members are ordered by their disutility from working, capturing the notion that it may be more costly to have old members in the labor force, rather than young adults. Integrating the cost of labor force participation yields

$$\int_0^{F_{it}(h)} f^{\omega}\, df = \frac{F_{it}(h)^{1+\omega}}{1+\omega}. \tag{2}$$

Households decide labor force participation, but not all the labor force is employed due to the presence of search-and-matching frictions in the labor market. The search-and-matching process is carried out by employment agencies and works as follows. At the beginning of each period t, a fixed share of the labor force supplied by household h is separated from its employment and searches for job. Therefore, the number of unemployed household members at the beginning of the period is equal to

$$U_{it}^{b}(h) = sF_{it}(h), \tag{3}$$

where s denotes the separation rate. The remaining fraction $(1-s)$ of the labor force supplied by household h remains attached to firms. This number is equal to

$$N_{it}^{att}(h) = (1-s)F_{it}(h). \tag{4}$$

Note that, if $s = 0$, we are back to the standard New Keynesian model with perfectly flexible labor markets.

The ability of unemployed workers to find a new job is determined by the employment agencies through the following regional matching function, which determines the total number of new employment matches in region i and period t:

$$M_{it} = m_{it}(U_{it}^b)^{\eta}(V_{it})^{1-\eta}. \tag{5}$$

$m_{it} > 0$ represents the matching efficiency, V_{it} denotes total vacancies posted by employment agencies, U_{it}^b is total beginning-of-period unemployment, and $\eta \in [0, 1]$. Households take V_{it} and U_{it}^b as given, as they are determined at the regional level. Therefore, the number of new hires in region i and period t is:

$$N_{it}^{new} = M_{it}. \tag{6}$$

Defining labor market tightness as the ratio between vacancies and beginning-of-period unemployed workers looking for a job

$$\theta_{it} \equiv \frac{V_{it}}{U_{it}^b}, \tag{7}$$

the job finding probability is

$$f(\theta_{it}) = \frac{N_{it}^{new}}{U_{it}^b} = \frac{M_{it}}{U_{it}^b} = m_{it}\theta_{it}^{1-\eta}. \tag{8}$$

Households take as given the probability of a job seeker finding a job, as it depends on regional variables. The number of newly hired members of household h among the unemployed at the beginning of period t is

$$N_{it}^{new}(h) = f(\theta_{it})U_{it}^b(h) = f(\theta_{it})sF_{it}(h). \tag{9}$$

Therefore, the number of members of household h who are employed at the end of period t is:

$$\begin{aligned} N_{it}(h) &= N_{it}^{att}(h) + N_{it}^{new}(h) \\ &= [1 - s + sf(\theta_{it})]\, F_{it}(h). \end{aligned} \tag{10}$$

The representative household is then subject to the following budget constraint

$$P_{it}C_{it}(h)+B_{it}(h) \leq (1+i_{t-1})B_{it-1}(h)+W_{it}^{att}(1-s)F_{it}(h)+W_{it}^{new}sf(\theta_{it})F_{it}(h)+\int_0^1 \Pi_{it}^F(g)\,dg+\int_0^1 \Pi_{it}^E(l)\,dl, \tag{11}$$

where P_{it} is the price index associated with the consumption basket $C_{it}(h)$, $B_{it}(h)$ denotes the quantity of risk-free nominal bond held in region i at time t, paying a nominal national interest rate i_t in period $t+1$, W_{it}^{att} represents the nominal wage rate of attached workers, while W_{it}^{new} denotes the nominal wage rate of newly hired workers. Finally, $\Pi_{it}^F(g)$ are the profits of the final-goods firm producing variety g

and $\Pi_{it}^{E}(l)$ are the profits of the employment agency l. There is a complete set of financial markets across the two regions. Household h chooses $C_{it}(h)$, $F_{it}(h)$, and $B_{it}(h)$ to maximise utility in Equation (1), subject to the budget constraint in Equation (11), and given the total cost of labor force participation in Equation (2). As households behave all the same in equilibrium, I suppress the superscript h going forward.

Households trade off current consumption, C_{it} and current labor force participation, F_{it}. The optimal labor force participation takes the following form:

$$F_{it} = \left[\frac{w_{it}^{att}(1-s) + w_{it}^{att} s f(\theta_{it})}{\chi_{it}}\right]^{\frac{1}{\omega}}, \tag{12}$$

where $w_{it}^{att} \equiv \frac{W_{it}^{att}}{P_{it}}$ and $w_{it}^{new} \equiv \frac{W_{it}^{new}}{P_{it}}$ denote real wages. The assumption of GHH preferences signifies that the amount of work chosen by households affects the amount of utility they receive from consumption. This implies that income effects are not at play in the optimal choice of labor force participation. I make this assumption for tractability. Households optimally trade off consumption in the current and next periods, as captured by the following Euler equation:

$$\left(C_{it} - \chi_{it}\frac{F_{it}^{1+\omega}}{1+\omega}\right)^{-\frac{1}{\sigma}} = \beta(1+i_t)E_t\left[\left(C_{it+1} - \chi_{it+1}\frac{F_{it+1}^{1+\omega}}{1+\omega}\right)^{-\frac{1}{\sigma}}\frac{P_{it}}{P_{it+1}}\right]. \tag{13}$$

Furthermore, household optimization implies that a standard transversality condition must hold.

I assume that households have constant elasticity of substitution (CES) preferences over varieties, leading to the following final consumption good aggregator:

$$C_{it} = \left[\int_0^1 C_{it}(g)^{\frac{\epsilon-1}{\epsilon}}\, dg\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{14}$$

where $C_{it}(g)$ denotes consumption of final-goods variety g. The parameter $\epsilon > 1$ denotes the elasticity of substitution between different varieties. Households choose how much to purchase of each variety, $C_{it}(g)$, to obtain the desired level of consumption C_{it} at a minimal expense. The minimization problem implies the following demand curve for variety g:

$$C_{it}(g) = C_{it}\left[\frac{P_{it}(g)}{P_{it}}\right]^{-\epsilon}, \tag{15}$$

and the following price index:

$$P_{it} = \left[\int_0^1 P_{it}(g)^{1-\epsilon} dg \right]^{\frac{1}{1-\epsilon}}, \tag{16}$$

where $P_{it}(g)$ is the price of final-goods variety g.

2.1.2 Firms

The production side comprises a vertical supply chain featuring an international commodity market, a national perfectly-competitive intermediate-input sector, and local monopolistically-competitive final-goods sectors. At the first level of the supply chain, a commodity good, O_t, is supplied by an international market at an exogenous price, P_{ot}. The intermediate-input sector is tradable and is characterized by perfect competition. Therefore, the price of the intermediate input, P_{xt}, is common across the two regions. The representative intermediate-input firm in region i uses commodity, O_{it}, and labor, $N_{xit} = N_{xit}^{att} + N_{xit}^{new}$, to produce a homogeneous good, X_{it}, according to the following production function:

$$X_{it} = Z_{xit}(N_{xit})^{\rho}(O_{it})^{1-\rho}, \tag{17}$$

where Z_{xit} denotes local exogenous technology of the intermediate-input sector and $\rho \in (0, 1)$.

The representative firm maximizes its value

$$P_{xt}X_{it} - W_{it}^{att}N_{xit}^{att} - (1 + \gamma_{it}^{b})W_{it}^{new}N_{xit}^{new} - P_{ot}O_{it}, \tag{18}$$

given its production technology in Equation (17) and two additional constraints. First, the intermediate-input firm cannot keep more attached workers at the wage rate W_{it}^{att} than the number of workers attached to the intermediate-input sector:

$$0 \leq N_{xit}^{att} \leq (1 - s)(1 - \delta_{it})F_{it}, \tag{19}$$

where δ_{it} is a region-specific time-varying exogenous variable, capturing shocks to the size of the labor force employed in the intermediate-input and final-goods sectors. Moreover, while attached workers can be fired, new workers can only be added:

$$N_{xit}^{new} \geq 0. \tag{20}$$

For each new worker hired, the intermediate-input firm pays an exogenous fee, γ_{it}^{b}, proportional to W_{it}^{new}, to the employment agencies. As long as the cost of hiring a new worker exceeds that of the existing workforce, $(1+\gamma_{it}^{b})W_{it}^{new} > W_{it}^{att}$, the constraint in Equation (19) is binding, while the constraint in Equation (20) is not. Notably, this implies that the marginal costs of intermediate-input firms depend only on the cost of hiring a new worker, $(1+\gamma_{it}^{b})W_{it}^{new}$. Appendix B shows the first-order conditions of the problem of the representative intermediate-input firm.

The final-goods sector is non-tradable and characterized by monopolistic competition. In region i, there is a continuum of final-goods firms of measure one. Each firm specializes in the production of a differentiated good, g, consumed locally by households. The production function is characterized by constant returns to scale

$$Y_{it}(g) = Z_{yit}[N_{yit}(g)]^{\phi}[X_{it}(g)]^{1-\phi}, \tag{21}$$

where Z_{yit} denotes local productivity of the final-goods sector, $X_{it}(g)$ and $N_{yit}(g) = N_{yit}^{att}(g) + N_{yit}^{new}(g)$ denote, respectively, the quantity of intermediate good and labor used by firm g, and $\phi \in (0,1)$.

Final-goods firm g maximizes the expected discounted value of profits

$$E_t \sum_{k=0}^{\infty} Q_{it,t+k}[P_{it+k}(g)Y_{it+k}(g) - W_{it+k}^{att}N_{yit+k}^{att}(g) - (1+\gamma_{it+k}^{b})W_{it+k}^{new}N_{yit+k}^{new}(g) - P_{xt+k}X_{it+k}(g)] \tag{22}$$

subject to the production technology in Equation (21), the demand for its product

$$Y_{it}(g) = Y_{it}\left(\frac{P_{it}(g)}{P_{it}}\right)^{-\epsilon}, \tag{23}$$

and the two additional constraints on its workforce

$$0 \leq N_{yit}^{att} \leq (1-s)\delta_{it}F_{it} \tag{24}$$

and

$$N_{yit}^{new} \geq 0. \tag{25}$$

$Q_{it,t+k}$ is the stochastic discount factor between period t and $t+k$ and $P_{it}(g)$ is the price set by firm g for its product. In each period, firm g can set its price freely with probability $(1-\alpha)$ as in Calvo (1983). With probability α, the firm must keep its price unchanged. As for the intermediate-input firm, an equilibrium in which the constraint in Equation (24) is binding and the one in Equation (25) is not implies that $(1+\gamma_{it}^{b})W_{it}^{new}$ is the solely relevant cost of labor at the margin also for the final-goods firms. Appendix B shows the solution of the optimization problem of final-goods firms.

2.1.3 Wage Determination

The wage-setting mechanism is motivated by the observation made by Phillips (1958) that the relationship between nominal wage growth and labor market tightness is nonlinear. Phillips (1958) argues that workers are unwilling to take jobs paying below the "prevailing wage rate" even in periods of weak labor demand and high unemployment. On the contrary, workers are perfectly happy to accept jobs paying more than the "prevailing wage rate". Therefore, firms quickly bid up wages to attract workers in periods of sufficiently strong labor demand and low unemployment.

To capture the idea of Phillips (1958), I assume that the wage of new hires in region i at time t is equal to the maximum between the wage of attached workers and the flexible wage. The flexible wage is the one that clears the market in the absence of any frictions. That is:

$$W_{it}^{new} = \max\{W_{it}^{att}, P_{it}w_{it}^{flex}\}, \tag{26}$$

where w_{it}^{flex} denotes the real flexible wage. Such a wage-setting mechanism ensures that the wage of new hires is downward rigid, with the floor being the wage of attached workers. In other terms, new hires cannot be paid less than attached workers. However, when labor market forces push the flexible wage above the wage of attached workers, then workers newly hired are offered the higher wage. Equation (26) can be rewritten in real terms as

$$w_{it}^{new} = \max\{w_{it}^{att}, w_{it}^{flex}\}. \tag{27}$$

In the search-and-matching literature the determination of wages is in general not pinned down, since each worker-firm match generates a surplus. How the surplus is divided between the worker and the firm can be done in different ways, the most common assuming Nash bargaining between the employer and the employee. Search-and-matching models incorporating price rigidities typically assume that the real wage is exogenous. To incorporate Phillips' idea of asymmetric wages' response to the state of the labor market, I follow Benigno and Eggertsson (2023). They propose a simple model of employment agencies that oversee the search-and-matching process in the labor market. The optimization problem of the employment agencies will provide the foundation for the flexible wage rate. Once I derive the flexible wage rate, then I will show how the wage of attached workers depends on the flexible wage rate.

2.1.4 Employment Agencies

Employment agencies carry out the process of search and matching in the labor market. There is a continuum of measure one of employment agencies in region i. Each employment agency is indexed by l. They carry out two actions. First, they post vacancies of intermediate-input and final-goods firms. Second, they match job searchers and firms through the matching function. Since each agency is small, they take as given the wage rate offered to new hires and the probability of filling a vacancy. This probability of defined as

$$q(\theta_{it}) = \frac{N_{it}^{new}}{V_{it}} = \frac{M_{it}}{V_{it}} = m_{it}\theta_{it}^{-\eta} \tag{28}$$

Employment agency l chooses how many vacancy to post, $V_{it}(l)$ to maximize real profits. Agency l charges to the hiring firm a fee γ_{it}^b proportional to the real wage of each newly hired worker and pays a real cost γ_{it}^c to post each vacancy. I assume that employment agencies never post a vacancy that cannot be filled by firms. The number of matches agency l generates is given by $q(\theta_{it})V_{it}(l)$. Therefore, real profits are equal to:

$$\gamma_{it}^b w_{it} m_{it} \theta_{it}^{-\eta} V_{it}^l - \gamma_{it}^c V_{it}^l \tag{29}$$

In the optimum, the employment agency equates the marginal benefit of posting a vacancy to its marginal cost:

$$\underbrace{\gamma_{it}^b w_{it}^{flex} m_{it} \theta_{it}^{-\eta}}_{\text{marginal benefit}} = \underbrace{\gamma_{it}^c}_{\text{marginal cost}} . \tag{30}$$

As long as the marginal benefit is greater than the marginal cost, the agency will post vacancies. As a consequence, the flexible wage rate will decrease and tightness will increase, lowering the number of matches for each vacancy posted until the equilibrium is reached. In general equilibrium, the flexible wage rate and labor market tightness adjust so that such condition is satisfied. Rearranging Equation (30), the expression for the flexible wage rate is:

$$w_{it}^{flex} = \frac{1}{m_{it}} \frac{\gamma_{it}^c}{\gamma_{it}^b} \theta_{it}^{\eta}. \tag{31}$$

Consider now the case in which the real wage of attached workers is higher than the flexible wage rate. In this case, $w_{it}^{new} = w_{it}^{att}$ and firms hire less labor than they would have, had the wage rate been flexible. As employment agencies do not post vacancies that will not be filled by firms, they post only the number of vacancies that satisfy firms' constrained labor demand. The optimal condition indicates that, in this case, the marginal value of posting an additional vacancy for the agency remains positive.

I assume that the real wage of attached workers evolves as follows:

$$w_{it}^{att} = (\bar{w}_i)^{\lambda}(w_{it}^{flex})^{1-\lambda} \tag{32}$$

where $\bar{w}_i$ denotes the steady state level of the real wage in region i and $\lambda \in [0, 1]$. When labor markets are slack, λ determines how quickly the wage of attached workers adjusts to the flexible rate. The flexible wage rate is an anchor towards which the wage of attached workers is pulled by a factor of $(1 - \lambda)$. At the extremes, if $\lambda = 0$, the wage of attached workers is completely flexible. If $\lambda = 1$, the wage of attached workers is fully rigid and equal to the steady state value. I assume such formulation for the wage of attached workers to preserve the forward-looking nature of the Phillips curve and for computational straightforwardness. Richer forms of wage rigidities can be considered, but the intuition does not change.

In sum, the real wage of new hires in region i at time t behaves as follows:

$$w_{it}^{new} = \begin{cases} w_{it}^{flex} & \theta_{it} > \theta_{it}^* \\ (\bar{w}_i)^{\lambda}(w_{it}^{flex})^{1-\lambda} & \theta_{it} \leq \theta_{it}^* \end{cases}. \tag{33}$$

When $\theta_{it} > \theta_{it}^*$, the labor market is sufficiently tight so that firms need to compete among each other to attract workers. In this case, wages are flexible and determined in equilibrium by the optimal behavior of the employment agencies. When $\theta_{it} \leq \theta_{it}^*$, the labor market is slack and the wage of attached workers is higher than the flexible wage rate. As newly hired workers are unwilling to accept wages below the wage of attached workers, wages move only gradually towards the flexible wage rate, at a speed that depends on the value of λ.

To close the model, I determine the value of θ_{it}^*. At θ_{it}^*, the wage of attached workers is equal to the flexible wage rate, yielding:

$$\theta_{it}^* = \left(\frac{1}{m_{it}}\frac{\gamma_{it}^c}{\gamma_{it}^b}\bar{w}_i\right)^{\frac{1}{\eta}}. \tag{34}$$

This formula suggests that θ_{it}^* is region-specific and varies along time. In the empirical exercise I will take a pragmatic approach and approximate it with a value equal to one. This value is in line with the empirical evidence shown in Figure 1 and by Benigno and Eggertsson (2023). Michaillat and Saez (2022) provide theoretical foundation for choosing such a threshold. In Section 6, I test data-driven values of the threshold, allowed to vary across metropolitan areas.

2.2 Regional Nonlinear Phillips Curve

An equilibrium in this economy is an allocation consistent with optimization choices of households, firms, and employment agencies, the interest rate rule, and market clearing conditions. The definition of the equilibrium can be found in Appendix B. Log-linearizing the model around a zero-inflation steady state and combining optimal final-goods pricing, households' labor force participation, and the wage-setting mechanism, I obtain the following expression for the regional Phillips curve in region i:

$$\pi_{it} = \begin{cases} \beta E_t \pi_{it+1} + \kappa^{tight}\hat{\theta}_{it} + \xi\left[\hat{\nu}_{it}^{tight} + (1-\phi)\hat{p}_{xit} - \hat{z}_{yit}\right] & \hat{\theta}_{it} > \hat{\theta}_{it}^{*} \\ \beta E_t \pi_{it+1} + \kappa\hat{\theta}_{it} + \xi\left[\hat{\nu}_{it} + (1-\phi)\hat{p}_{xit} - \hat{z}_{yit}\right] & \hat{\theta}_{it} \leq \hat{\theta}_{it}^{*} \end{cases}, \tag{35}$$

where π_{it} is inflation in region i, $E_t\pi_{it+1}$ denotes regional short-run inflation expectations, and $\hat{\theta}_{it}^{*}$ is such that when $\hat{\theta}_{it} > \hat{\theta}_{it}^{*}$, then $\theta_{it} > \theta_{it}^{*}$. The derivation of equation (35), together with the definitions of the parameters, can be found in Appendix B.

As long as the rate of adjustment of wages when the labor market is slack is positive and less than one, i.e. $\lambda \in (0,1)$, then the regional Phillips curve is nonlinear and $\kappa_\theta \equiv \kappa_\theta^{tight}(1-\lambda) < \kappa_\theta^{tight}$. This means that labor market tightness exerts higher inflationary pressures when labor markets are tight rather than slack, as we observe in the raw data in Figure 1. When $\lambda = 0$, the real wage of new hires is flexible also when labor markets are slack, and the two curves coincide.

Three more terms are present in equation (35) and together they compose the regional cost-push shock. First, $\hat{\nu}_{it}^{tight} = \phi\left[(\hat{\gamma}_{it}^{c} - \hat{m}_{it}) - (1-h_\gamma)\hat{\gamma}_{it}^{b}\right]$ and $\hat{\nu}_{it} = \phi\left[(1-\lambda)(\hat{\gamma}_{it}^{c} - \hat{m}_{it}) - (1-\lambda-h_\gamma)\hat{\gamma}_{it}^{b}\right]$ represent shocks to the regional labor markets' search-and-matching process. Note that this shock have a nonlinear effect on regional inflation as well, with $\hat{\nu}_{it}^{tight} > \hat{\nu}_{it}$. Second, $\hat{p}_{xit} = (\hat{p}_{xt} - \hat{p}_{it})$ denotes the percentage deviation of the regional relative price of intermediate input (i.e., the ratio between the national intermediate-input price, P_{xt}, and the regional price level, P_{it}) from its steady-state value. As one of the factors of production, a change in the price of the intermediate input directly affects the marginal costs of final-goods firms and, consequently, their pricing decision. However, the presence of the relative price of intermediate input captures the notion that, as the price of intermediate input is common across regions, an identical absolute intermediate-input price change has a higher (lower) pass-through on regional inflation rates the lower (higher) the regional price level. Finally, $\hat{z}_{yit}$ captures local shocks to final-goods sector productivity.

To take equation (35) to the data, I follow Hazell et al. (2022) and solve Equation (35) forward. To do so, I make two assumptions. First, I assume that if the labor market is slack in t, then it will remain slack forever. If the labor market is tight in t, then it will remain tight until $t+T-1$. From $t+T$ onward, the labor market becomes slack and will remain slack forever. Second, I assume that $\tilde{\theta}_{it}$ and $\hat{p}_{xit}$ follow AR(1) processes with autocorrelation coefficients ρ_θ and ρ_p.

By doing so, I obtain the following regional Phillips curve:

$$\pi_{it} = \begin{cases} E_t\pi_{t+\infty} + \Psi E_t\hat{\theta}_{it+\infty} + \psi_\theta^{tight}\tilde{\theta}_{it} + \psi_p\hat{p}_{xit} + \varepsilon_{it}^{tight} & \tilde{\theta}_{it} > \tilde{\theta}_{it}^* \\ E_t\pi_{t+\infty} + \psi_\theta^{slack}\tilde{\theta}_{it} + \psi_p\hat{p}_{xit} + \varepsilon_{it} & \tilde{\theta}_{it} \leq \tilde{\theta}_{it}^* \end{cases}, \tag{36}$$

where $\tilde{\theta}_{it} = \hat{\theta}_{it} + E_t\hat{\theta}_{it+\infty}$ represents the transitory component of the variation in labor market tightness, while $E_t\hat{\theta}_{it+\infty}$ is the permanent component. $E_t\pi_{t+\infty}$ denotes long-run inflation expectations, assumed to be common across regions because they depend on the monetary regime in place. This formulation clarifies how regional data helps in dealing with threats to identification coming from controlling for inflation expectations in the estimation of the Phillips curve. Indeed, common long-run inflation expectations are captured by time fixed effects, while $\Psi E_t\hat{\theta}_{it+\infty}$ is captured by region fixed effects.

Crucially, $\psi_\theta^{slack} = \psi_\theta^1$, while $\psi_\theta^{tight} = \psi_\theta^1 + \psi_\theta^2$, where the expressions for ψ_θ^1 and ψ_θ^2 can be found in Appendix B. Therefore, the slope of the regional Phillips curve in tight labor markets is larger than the slope in slack labor markets by a measure equal to ψ_θ^2. The formulas for ψ_p can be found in Appendix B. $\varepsilon_{it} = E_t\sum\limits_{k=0}^{\infty}\beta^k\xi\left(\hat{\nu}_{it+k} - \hat{z}_{yit}\right)$ and $\varepsilon_{it}^{tight} = \xi E_t\left[\sum_{k=0}^{T-1}\beta^k\hat{\nu}_{it+k}^{tight} + \sum_{k=T}^{\infty}\beta^k\hat{\nu}_{it+k} - \sum_{k=0}^{\infty}\beta^k\hat{z}_{yit}\right]$ denote the expected present discounted value of current and future search-and-matching and final-goods firms' productivity shocks in slack and tight labor markets, respectively. Appendix B contains the formal derivation of equation (36).

The interpretation of the slope of the Phillips curve differ between equation (35) and equation (36). κ_θ^{tight} and κ_θ denote the effects of current labor market tightness on current inflation, while ψ_θ^{tight} and ψ_θ denote the effects of current and expected future deviations of labor market tightness from its long-run steady state on current inflation. Depending on the degree of persistence of labor market tightness, ψ_θ^{tight} and ψ_θ can be more or less larger than κ_θ^{tight} and κ_θ. In my empirical exercise, I estimate ψ_θ^{tight} and ψ_θ in equation (36) and not κ_θ^{tight} and κ_θ in equation (35) due to sample size limitations. Estimating ψ_θ^{tight} and ψ_θ allows to abstract from empirically modelling future values of labor market tightness, which increases the sample size. This implies greater statistical power for estimating the coefficients of interest.

3 Data

To carry out my empirical exercise, I draw data from different sources from December 2000 to July 2024. The units of observations are 21 Metropolitan Statistical Areas (MSAs) in the United States. The main dependent variable is inflation, constructed as the 12-month percent difference in the Consumer Price Index (CPI). The Bureau of Labor Statistics (BLS) regularly provides CPI data for 21 MSAs on a monthly or bi-monthly basis. Prices from all categories are collected monthly in the metropolitan areas of Chicago, Los Angeles and New York. In the other MSAs, prices for food and energy items are collected monthly, while prices for other categories are collected every two months. I linearly interpolate the bi-monthly CPI time series to maximize the sample size and to fully exploit the variation in labor market and instrumental variables. Interpolation introduces measurement errors in CPI and, consequently, inflation. However, such measurement errors do not lead to an attenuation bias in the estimates, because they affect the dependent variable only. Crucially, they do not affect the exogenous variation provided by the instrumental variable. The initiation date for CPI data collection varies among the included metropolitan areas, starting from January 1986.

My analysis primarily focuses on core inflation, defined as the growth rate of prices of all items excluding food and energy. The literature suggests to use core inflation to avoid the relative volatility of food and energy prices, which are more connected to global factors. Core inflation, on the other hand, is more related to domestic economic activity. For a more comprehensive understanding of CPI data, I recommend referring to the works of Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008).

I employ a labor market-based variable to measure economic activity: labor market tightness, also known as vacancy-to-unemployment ratio. Labor market indicators are relevant proxies because the mechanism that the Phillips curve captures relies on demand-driven economic fluctuations that affect labor costs for firms. Labor market tightness is the ratio of two elements: the number of vacancies posted by firms in the numerator provides information on labor demand, while the number of unemployed workers in the denominator tracks the supply of workers available in the market. By including a measure of labor demand, tightness provides more accurate information on labor costs than the unemployment rate (Barnichon and Shapiro, 2022). Nonetheless, researchers have primarily used the unemployment rate to proxy for real economic activity in the estimation of the Phillips curve. This is because the availability of administrative data on the unemployment rate is far broader, both at granular levels and back in time.

Recent evidences provide further reasons for using labor market tightness over the unemployment rate as main independent variable. First, Furman and Powell (2021) show that until the outbreak of the

COVID-19 pandemic, the unemployment rate and labor market tightness co-moved well. However, the behavior of the two variables started to diverge in March 2020, with the vacancy-to-unemployment ratio signalling tighter labor markets with respect to the unemployment rate. Hence, an analysis that takes into consideration the COVID and post-COVID periods, such as this one, should be careful on the choice of proxy for economic activity. Furthermore, Barnichon and Shapiro (2022) show that the vacancy-to-unemployment ratio outperforms the unemployment rate in the forecast of both price and wage inflation. For all these reasons, this analysis uses labor market tightness as main explanatory variable, but I check the robustness of my results to the unemployment rate.

As data on vacancies is not publicly available at the MSA level, I impute a new time series of MSA-level vacancies employing state-level data from JOLTS and yearly employment weights from the CPS. JOLTS is a monthly survey of about 21,000 U.S. business establishments, providing representative data on job openings at the national and state levels since December 2000. Yearly employment weights are constructed from the Basic Monthly Samples of the CPS from January 2000 to July 2024. Suppose that metropolitan area i belongs to state x and state y. Vacancies in metropolitan area i in period t are computed as a weighted average of vacancies from states x and y in period t, with weights being the fraction of population of state x living in MSA i and the fraction of population of state y living in MSA i, respectively.

The main advantage of this newly imputed measure of MSA-level vacancies is that it is constructed using publicly available job postings from a representative sample of firms. There exist two private sources of MSA-level vacancies: Lightcast (a merger of Emsi and Burning Glass Technologies) and the Conference Board. Lightcast collects online job postings from 2011. The Conference Board started to collect job advertisements printed in newspapers in the 1950s. This series was discontinued in 2008 and substituted with collection of online job postings since 2005. For a more detailed explanation of the Conference Board data, refer to Barnichon (2010). Since 2019, Lightcast is the only provider of data for the Conference Board. Since 2020, the Conference Board implements an adjustment to bridge the gap between the data from Lightcast and the data from the BLS's Jop Openings and Labor Turnover Survey (JOLTS). The need of such an adjustment underscores the main problem of online job posting data: this data is not representative and might introduce selection bias in the estimates.

The time series of vacancies imputed from JOLTS allows to avoid selection bias, as it comprises job postings from all the sectors of the economy and not from those that recruit online only. However, the distribution of vacancies at the state level might not be similar to the distribution of vacancies in metropolitan areas. The empirical strategy provides for the use of an instrumental variable to instrument labor market tightness. In addition to control for local confounders, the instrumental variable attenuates the measurement error that the imputation of vacancy data introduces in the main independent variable.

In addition to vacancy data, I need to collect unemployment data to construct labor market tightness at the MSA level. I draw monthly MSA-level number of unemployed workers from the BLS's Local Area Unemployment Statistics (LAUS). The LAUS program uses non-survey methodologies to estimate the number of employed and unemployed individuals for sub-national areas, using the national not-seasonally-adjusted estimates from the CPS as controls. From LAUS I also collect monthly MSA-level unemployment rate to be employed in a robustness check.

To construct the shift-share instrument and controls, I need additional data. For the instrument, I collect two types of data. First, I draw monthly, national employment data by industry from the CPS. Second, I collect MSA-level industry employment shares from the 2000 Census, and from the American Community Survey (ACS) from 2006 to 2022. Both types of employment data are taken disaggregated at the two-digit level Census industry. Finally, I measure the relative price of intermediate inputs as the ratio between the monthly producer price index (PPI) for the manufacturing sector and the local all-items CPI. Both variables come from the BLS.

The resulting dataset is a panel of MSA-year-month observations, from December 2001 to July 2024. Figure 2 shows the distributions of the dependent and main independent variables. As you can see, there is a significant degree of variation in core inflation and labor market tightness at the metropolitan area level. Furthermore, Figure A.1 in Appendix A shows a key advantage of employing regional data. Comparing the distributions of MSA-level and national labor market tightness, regional data exhibits more variation and a higher number of tight labor markets episodes with respect to national data. Aggregate labor market tightness reaches the maximum level of 2, while MSA-level labor market tightness reaches 3.44. At the national level, we observe a vacancy-to-unemployment ratio greater than one only in 18.22% of the sample. At the MSA-level, the same proportion is 46.33%. The take-away is that regional data provides more variation and episodes of tight labor markets than national data, increasing the statistical power of the empirical exercise.

4 Empirical Strategy

My empirical exercise aims at testing the presence of nonlinearities in the regional New Keynesian Phillips curve, employing labor market tightness to proxy for real economics activity. To do so, I estimate the NRPC – equation (36) – derived in Section 2. The empirical strategy is based on two ingredients. First, panel variation in inflation and tightness at the MSA level provides higher statistical power than time series variation as shown in Section 3, and takes care of aggregate confounders. Second, the instrumental variable approach deals with regional confounders.

To estimate equation (36), I specify the following empirical model:

$$\pi_{it} = c + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi_\theta^1 \ln(\theta_{it}) + \psi_\theta^2 \ln(\theta_{it}) \times I_{\{\theta_{it}>1\}} + \beta I_{\{\theta_{it}>\theta_i^*\}} + \psi_p p_{xit} + \varepsilon_{it}, \tag{37}$$

where π_{it} denotes the 12-month, core inflation rate in MSA i and year-month t. Constructing the growth rate of prices over 12 months allows me to reduce seasonality. α_i represents MSA fixed effects, absorbing time-invariant characteristics of metropolitan areas, such as differences in long-run economic fundamentals across cities. γ_{t_q} denotes year-quarter fixed effects, absorbing aggregate shocks, such as endogenous fiscal and monetary policies (Fitzgerald and Nicolini 2014, McLeay and Tenreyro 2020), and common beliefs about the long-run monetary policy regime (Hazell et al., 2022). δ_{it_q} is the interaction between MSA and year-quarter fixed effects and addresses the possibility that MSAs are differentially affected by common shocks. I will discuss more about δ_{it_q} in the following paragraphs.

The nonlinearity of the Phillips curve in labor market tightness is specified as follows. $\ln(\theta_{it})$ is the logarithm of labor market tightness in city i and year-month t. I employ the logarithm of labor market tightness for two reasons. First, it mirrors the log-linearized tightness term in equation (36): $\hat{\theta}_{it}$. Second, it enables me to avoid taking a stance about whether I should measure labor market tightness as the vacancy-to-unemployment ratio or the unemployment-to-vacancy ratio. $I_{\{\theta_{it}>1\}}$ is a dummy variable that takes value of 1 when labor market tightness is greater than 1 and 0 otherwise. I select this threshold guided by the empirical evidence shown in Figure 1, in line with Benigno and Eggertsson (2023). I will test for different thresholds allowed to vary across MSAs in Section 6. The interaction between $\ln(\theta_{it})$ and $I_{\{\theta_{it}>1\}}$ generates the nonlinearity. When labor market tightness is less than 1, the slope of the Phillips curve is $\psi_\theta = \psi_\theta^1$ and the intercept is c. When labor market tightness is greater than 1, the slope of the Phillips curve is $\psi_\theta^{tight} = \psi_\theta^1 + \psi_\theta^1$ and the intercept is $c + \beta$.

Finally, guided by equation (36), I add to the main specification one control. p_{xit} is the local relative price of intermediate inputs in MSA i and year-month t, measured as the ratio of national manufacturing PPI in year-month t and all-items CPI in MSA i and year-month t. As explained in Section 2, p_{xit} and the error term ε_{it} constitute together the local cost-push shock. Through the lenses of the model, the error term ε_{it} contains the shocks to the regional labor markets' search-and-matching process and final-goods firms' productivity shocks.

I conduct the empirical exercise employing regional variation for two main reasons. First, regional data provides greater variation and a higher number of episodes of tight labor markets with respect to national data, as shown in Section 3. Second, a growing literature has shown how panel variation helps overcoming three identification problems affecting the estimation of the aggregate Phillips curve. The main challenge for the identification of the slope of the Phillips curve is to distinguish between demand and supply shocks. While demand shocks increase economic activity and inflation, supply shocks depress economic activity and increase inflation, leading to the so-called *simultaneity bias*. The simultaneity bias generates a downward bias in the estimated slope of the Phillips curve. There are two more challenges. First, Fitzgerald and Nicolini (2014) and McLeay and Tenreyro (2020) show that if monetary and fiscal policies react to offset aggregate demand shocks, then the remaining variation in inflation will only be due to supply shocks, leading to a biased estimate of the aggregate slope. Second, the choice of variable to measure inflation expectations affects the estimate of the slope (Mavroeidis et al., 2014).

Panel variation helps in dealing with these aggregate threats to identification because of the introduction of time fixed effects. Time fixed effects absorb any aggregate demand or supply shock, eliminating any bias coming from aggregate fluctuations all at once. Among the aggregate demand shocks, Fitzgerald and Nicolini (2014) and McLeay and Tenreyro (2020) show that time fixed effects capture endogenous monetary and fiscal policies that are set at the national level, solving the problem of omitted variable bias. Hazell et al. (2022) show that time fixed effects are also able to absorb local inflation expectations, if we assume that they are determined by the monetary regime in place.[5] Section 2 and Appendix B discuss and show more in details the derivation of this result.

What is left in the variation of inflation at the regional level comes from both regional demand and regional supply shocks. To solve the simultaneity bias at the regional level, I propose a shift-share instrumental variable that captures labor demand shocks in the tradable intermediate-input sectors,

[5] Moreover, Sargent (1982) shows that common beliefs about the long-run monetary regime in place are a major determinant of sudden fluctuations in inflation.

similar to Cerrato and Gitti (2022). The shift-share instrument takes the following form:

$$z_{it}^{x} = \sum_{k=1}^{N} e_{ki} \times g_{kt},$$

where e_{ki} is the average employment share of industry k in metropolitan area i, and g_{kt} is the three-year growth in national employment of industry k at time t. Industries are identified at the level of the two-digit Census code, and include: agriculture, mining, manufacturing of durable and non-durable goods, wholesale trade of durable and non-durable goods, and financial services. The shifters g_{kt} capture national labor demand shocks in the intermediate-input sectors at business cycle frequencies. The shares e_{ki} measure the regional exposure to such aggregate shocks.

I need an instrumental variable approach to solve the simultaneity bias at the regional level for the following reason. Through the lenses of the model, the regional supply shock is composed of three elements: the regional relative price of intermediate inputs $\hat{p}_{xit}$, the final-goods sector productivity shock $\hat{a}_{it}^{y}$, and the search-and-matching shock $\hat{\nu}_{it}$. While $\hat{p}_{xit}$ is observable and can be controlled for, search-and-matching and final-goods sectors' productivity shocks are not and are contained in the error term. The instrumental variable allows to isolate fluctuations in labor market tightness that are not driven by unobservable search-and-matching shocks and final-goods sectors' productivity shocks.

The shift-share instrument works as follows. A positive labor demand shock in the tradable intermediate-input sector raises labor demand of intermediate-input firms relatively more in those metropolitan areas with a higher degree of specialization in the intermediate-input sector. As a consequence, employment agencies in the specialized metropolitan areas will post more vacancies, leading to higher wages at a speed that depends on labor market tightness. Higher wages represent higher labor costs for final-goods firms, which will ultimately increase final-goods prices. For instance, a positive labor demand shock in the manufacturing sector leads to larger cost increases for final-goods firms located in manufacturing-intensive cities like Detroit. This channel produces an exogenous variation in labor market tightness that the shift-share instrument exploits to identify the slope of the Phillips curve.

The control variable is key to take care of one threat to the exclusion restriction of the shift-share instrument. A positive productivity-driven labor demand shock in the tradable intermediate-input sector decreases the price of the intermediate input, lowering the cost of production for all final-goods firms that use the intermediate input in their production process. Consequently, final-goods firms lower their prices. Such mechanism captures a channel through which tradable intermediate-input labor demand shocks affect inflation that is not through labor market tightness. This represents a violation of the exclusion restriction for the validity of the instrumental variable. However, as prices of intermediate inputs are observable, I can control for their direct incidence on local inflation by including p_{xit} in the specification.

The exogeneity of the instrument, conditional on the control for the relative price of intermediate inputs, stems from the shocks g_{jt}, rather than from the exposure shares e_{ji}. Such a case falls under the framework developed by Borusyak et al. (2022). In this paper, the authors prove that the validity of shift-share instruments can rely on the exogenous variation of the shocks only, allowing the variation in exposure shares to be endogenous. In particular, under some assumptions, shocks can be only as-good-as-randomly assigned, i.e. shocks can be equilibrium objects. An example of this type of shocks are the national industry employment growth rates that this analysis employs, as well as Bartik (1991).

As my instrument is constructed using tradable intermediate-input industries only, I add to the benchmark specification the interaction between MSA and year-quarter fixed effects.[6] Borusyak et al. (2022) argue that cities with more diversified economies tend to have systematically higher shift-share instruments because their sum of exposure shares is higher, and they may have systematically different unobservables. For example, cities with more diversified economies might be more resilient to unobserved shocks. The interaction of MSA and year-quarter fixed effects allows MSAs to be differentially affected by a common shock, overcoming the bias described by Borusyak et al. (2022). The interaction of fixed effects also helps address potential dependencies across MSAs and over time. In this setting, clustering the standard errors at the MSA level is problematic because the sample is composed of only 21 MSAs. To increase the number of clusters, I cluster the standard errors at the MSA-year level. The interaction between MSA and year-quarter fixed effects flexibly accounts for the possibility that groups of MSAs are differentially affected by common shocks due to dependencies within the group.

The identifying assumption is that, conditioning on MSA fixed effects, time fixed effects, their interaction, and p_{xit}, industry-level employment growth rates in the intermediate-input sectors capture labor demand shocks plausibly uncorrelated with industry-level aggregates of regional labor supply shocks. Related to the example of a labor demand shock in the manufacturing sector illustrated before, the identifying assumption requires that the labor cost increases generated by such a shock are no larger on average for final-goods firms in Detroit than New York.

[6] All the variables are measured at monthly frequency, so the interaction between MSA and year-quarter fixed effects does not saturate the regression. The variation used to identify the parameters of interest stems from within-MSA-year-quarter variation.

The nonlinearity of the main independent variable requires that I instrument not only the logarithm of labor market tightness $\ln(\theta_{it})$, but also the interaction term between the logarithm of labor market tightness and the dummy signalling when labor markets are tight $\ln(\theta_{it}) \times I_{\theta_{it}>1}$. I instrument the interaction term with an interaction between my shift-share instrument and the dummy that takes value of one when the labor market tightness exceeds one, $z_{it}^x \times I_{\theta_{it}>1}$. As p_{xit} is measured as the ratio between the national manufacturing PPI and MSA-level, all-items CPI, the denominator is mechanically correlated to core inflation on the right-hand side. To deal with this, I instrument p_{xit} with the ratio between the national PPI in the manufacturing sector and the MSA-level, all-items CPI measured 24 months before.

Table A.1 in Appendix A shows the first-stage coefficients and F-statistics. Column 1 shows that the shift-share instrument z_{it}^x strongly predicts $\ln(\theta_{it})$, while column 2 shows that the instrument for the interaction $z_{it}^x \times I_{\theta_{it}>1}$ strongly predicts the interaction term $\ln(\theta_{it}) \times I_{\theta_{it}>1}$. All instruments are strong, as the F-statistics are greater than 10.

5 Results

5.1 Main Results

I find two novel sets of results. First, the regional Phillips curve in labor market tightness if flat when labor markets are slack. Second, I find evidence of nonlinearities in the regional Phillips curve: its slope increases significantly when labor market tightness exceeds one. Table 1 shows these results. Column (1) reports the results of my benchmark specification, where equation (37) is estimated by instrumenting labor market tightness and the interaction term with the shift-share instrument and its interaction. I estimate the slope of the Phillips curve not to be statistically different from zero when labor market tightness is less than one and 1.95% when labor market tightness is greater than one. That is, a 1% increase in labor market tightness does not lead to a significant increase in inflation when regional labor markets are slack and leads to a 1.95% increase when regional labor markets become tight.

Comparing the estimation of equation (37) by IV and OLS is instructive to understanding the role played by the instrumental variables. Column (2) of Table 1 reports a simple OLS regression that controls only for the local relative price of intermediate inputs p_{xit} and MSA-fixed effects. This simple OLS regression yields a nonlinear regional Phillips curve with a slope of 0.84% when labor market tightness is below one and 4.12% above one. Column (3) shows that the slope of the Phillips curve decreases to 0.20% below the threshold and 0.74% above the threshold when I add time fixed effects. Taken together, these results reveal that aggregate shocks play a role in reinforcing stronger inflationary pressures in tight labor markets. When aggregate shocks are absorbed by time fixed effects, the nonlinearity diminishes because the presence of regional supply shocks weakens the effect of tight labor markets on inflation. Once I control for them with the instrumental variables, the regional Phillips curve flattens in slack labor markets and the nonlinearity strengthens in tight labor markets. It is noteworthy noticing that the local relative price of the intermediate inputs exerts a positive and larger effect on inflation when it is instrumented to address the mechanical correlation with the dependent variable.

Figure 3 shows the goodness-of-fit of the regional Phillips curve estimated in Table 1, column (1). Inflation deviations are defined as the difference between the 12-month core inflation rates and the estimated controls and fixed effects. What is left is the variation due to labor market tightness and the error terms. Such variation is plotted against the logarithm of labor market tightness, my main independent variable. The red line shows the curve estimated in Table 1, column (1). As you can see, when labor market tightness becomes greater than 1, the slope of the regional Phillips curve increases, confirming the pattern that we see in the raw data in Figure 1.

5.2 Policy Implications

Ignoring nonlinearities in the Phillips curve can lead the monetary authority to allow inflation to rise more than expected. Panel (a) of Figure 4 provides a visual representation of this dynamic. Suppose that the economy is at E_1, with low inflation and a slack labor market – labor market tightness below one. Following a shock, labor market tightness increases, exceeding one. If the central bank believes that the Phillips curve is linear and flat, such an increase in labor market tightness is expected to induce a fixed increase in inflation, no matter the level of labor market tightness. The central bank expects the economy to move along the dashed blue line to E_2, where inflationary pressures are limited.

However, if the Phillips curve is nonlinear, after the kink the economy lies on the steeper solid blue line and reaches E_2'. Hence, a given increase in labor market tightness will spur larger inflationary pressures than expected, leading to an unexpected surge in inflation. The surprise increase in inflation equals to the difference between the solid and dashed lines. Through the lenses of the model, the increase in the slope of the Phillips curve in tight labor markets is due to firms bidding up wages to attract workers. Wages become flexible and do not constraint anymore firms' demand, leading to a larger change in marginal costs and consequently prices of final-goods firms.

The nonlinearity of the Phillips curve allows the central bank to reduce inflation at a lower economic cost, as long as labor markets are tight. Panel (b) of Figure 4 shows this scenario. Suppose that the economy is on the steep part of the Phillips curve at E_1 with high inflation and tight labor markets. The steeper slope of the Phillips curve allows a restrictive monetary policy stance to reduce inflation at a lower economic cost because the economy moves to E_2 instead of E_2', limiting the decline of labor market tightness. Such possibility of "soft landing" has been much discussed recently, as the central banks around the world are aggressively trying to bring down inflation without causing much economic recession. As I have argued, a nonlinear Phillips curve in labor market tightness can explain this puzzle.

6 Robustness Checks

In this section, I perform four robustness exercises to evaluate the stability of my results. First, I find that the nonlinearity of the regional Phillips curve is robust to different definitions of the kink. Second, I evaluate whether the results change when imputing vacancies at the MSA level using population weights, rather than employment weights. I find that the nonlinearity of the regional Phillips curve is robust to the use of different weights for imputation across different definitions of the kink. Third, instead of a single kink, I show that the nonlinearity of the regional Phillips curve is robust to more flexible functional forms of nonlinearity. Fourth, I estimate the presence of a kink even after the introduction of a proxy for final-goods sector productivity.

The first robustness exercise checks whether the existence of the NRPC is robust to different definitions of the kink. As explained before, in the benchmark specification I define the kink to be at one, driven by the raw evidence shown in Figure 1. Moreover, such a definition of the kink is in line with the aggregate evidence presented by Benigno and Eggertsson (2023). Michaillat and Saez (2022) provide a theoretical foundation for the kink to be equal to one, as they show that labor markets are inefficiently tight when there are more vacancies than unemployed workers looking for a job. Nonetheless, the value of the kink, as well as its existence, are empirical questions.

Table A.2 shows that the regional Phillips curve is nonlinear when using different definitions of the kink. I evaluate two definitions. First, I select the value of the vacancy-to-unemployment ratio in each MSA that maximizes the fit of the relationship between core inflation and the logarithm of labor market tightness. To select the best-fit value, I run the following algorithm. For each value that labor market tightness takes in one MSA, I regress core inflation on the logarithm of labor market tightness and its interaction with a dummy that takes value of 1 when labor market tightness exceeds the value I am considering. I compute the root mean squared error (RMSE) for each regression and I select the value of labor market tightness that minimizes the RMSE in each MSA. Second, I define the kink to be equal to the average value of labor market tightness in each MSA. Both these categories of kinks are allowed to vary across MSAs, providing a more flexible and data-driven definition. The estimates of the slope of the Phillips curve are presented in Table A.2. The Phillips curve is flat when labor markets are slack and steepens significantly across all definitions of the kink. The coefficient on the interaction term defined using the best-fit kink is similar in magnitude to the benchmark estimate, although it is estimated with less precision. When using the average kink, the magnitude of the coefficient on the interaction term decreases in magnitude to 1.18%. The local relative price of the intermediate inputs affects positively core inflation across the three specifications, without significant difference in the magnitude of the coefficient.

The second robustness exercise reveals that the results are robust to the use of different weights for imputing vacancies at the MSA level. As explained in Section 3, there is no public and representative source of vacancy data at the MSA level. To overcome this challenge, I impute MSA-level vacancies from state-level vacancies recorded in JOLTS. In the benchmark specification, I present results where vacancies are imputed using employment weights measured at the year frequency from 2000 to 2024 from the CPS. However, metropolitan areas might differ in the number of people not in the labor force, and vacancies might correlate better with the size of the working-age population, rather than the share of the labor force that is employed. Therefore, I construct working-age population weights from the 2000 Census and the ACS from 2006 to 2022 and use these weights to impute MSA-level vacancies from state-level JOLTS data. Table A.3 shows that the existence of the kink is robust to the weights used to impute vacancies across all the definitions of the kink. The magnitude of the coefficients on the interaction term is lower when the kink is equal to one and similar when the kink equals to the best-fit value and the average of labor market tightness in each MSA. The coefficients are estimated with slightly less precision, supporting the use of employment weights as the preferred method to impute vacancies.

The third robustness exercise shows that the nonlinearity of the regional Phillips curve is robust to other nonlinear functional forms. I test the following functional forms: logarithmic, quadratic-logarithmic, piecewise-linear, quadratic-linear, and inverse. All these specifications are linear in parameters and non-linear in the main independent variables. Table A.4 in Appendix A finds evidence of an NRPC when labor market tightness is subject to logarithmic, quadratic-logarithmic, and inverse transformations. The coefficients on such transformations of labor market tightness, instrumented with the shift-share instrument, are significantly different from zero. The quadratic term of the quadratic-logarithmic specification is instrumented with the square of the shift-share instrument.

In the last robustness check, I show that the NRPC is robust to the inclusion of a proxy for final-goods sector productivity. From the lenses of the model, we know that the error term contains the productivity shock common across final-goods firms. If productivity shocks in the tradable intermediate-input sectors captured by the shift-share instrument are correlated with productivity shocks in the final-goods sectors, the exclusion restriction for the validity of the shift-share instrument would be violated. Such correlation has likely been in place during and after the COVID-19 pandemic (Guerrieri et al., 2022), when local economies experienced robust labor demand recoveries across all sectors. If this is the case, the instrument affects final-goods prices though the correlation with productivity shocks of regional final-goods sectors. To deal with this threat to the exclusion restriction, I follow Borusyak et al. (2022) and include as control a shift-share variable proxying for productivity shocks in the local final-goods sectors, z^y_{it}. The structure of this variable mirrors the one of the shift-share instrument, employing two-digit, non-tradable, final-goods Census industries.[7] Table A.5 shows that the correlation between the productivity of the intermediate-input and final-goods sectors is not a concern for the validity of the shift-share instrument. The coefficient on the interaction term is estimated precisely and is similar in magnitude to the one of the benchmark specification. Moreover, the coefficient on z^y_{it} is positive but not statistically different from zero and the F-statistics of the two instruments are largely unaffected.

7 Conclusion

This is the first paper that show the presence of nonlinearities in the regional Phillips curve with labor market tightness as proxy for economic activity. In doing so, this paper provides both theoretical and empirical contributions. From a theoretical point of view, I introduce search-and-matching frictions and wage rigidities in an otherwise standard multi-sector, New Keynesian model of two regions in a monetary union. Search-and-matching frictions give rise to unemployment in the New Keynesian model formally and to a relationship between inflation and labor market tightness. Wage rigidities generate a kink in the regional Phillips curve. Augmented with these features, the model delivers a piecewise log-linear regional Phillips curve in labor market tightness. From an empirical point of view, I impute a novel measure of vacancies across metropolitan areas. This newly imputed variable allows me to measure labor market tightness at the MSA level and to estimate the NRPC derived in the model. Moreover, I enrich the empirical strategy based on MSA-level panel variation with an instrumental variable approach to deal

[7] Two-digit, non-tradable, final-goods Census sectors used to construct z^y_{it}: construction, retail trade, information and communication, professional and business services, educational services, health care and social assistance, arts, entertainment and recreation, accommodation and food services, other services.

with unobservable regional supply shocks.

I find that the regional Phillips curve is flat in slack labor markets and significantly steepens when labor market tightness exceeds one. Hence, I provide evidence that the Phillips curve is piecewise log-linear at the MSA level. This result has two implications for the monetary authorities. First, if the central bank assumes that the Phillips curve is linear, this might lead them to underestimate the inflationary pressures in tight labor markets, allowing inflation to surge more than expected. Second, the nonlinearity of the Phillips curve allows the central bank to bring down inflation at a lower economic cost. More work is needed to relate the regional Phillips curve to the aggregate one, in order to infer more detailed implications for monetary policy. I believe that this is an exciting path for future research.

References

Babb, Nathan R. and Alan K. Detmeister, "Nonlinearities in the Phillips Curve for the United States : Evidence Using Metropolitan Data," Finance and Economics Discussion Series 2017-070, Board of Governors of the Federal Reserve System (U.S.) June 2017.

Barnichon, Regis, "Building a Composite Help-Wanted Index," *Economics Letters*, 2010, *109* (3), 175–178.

— **and Adam H. Shapiro**, "What's the Best Measure of Economic Slack?," Economic Letter 4, Federal Reserve Bank of San Francisco 2022.

Bartik, Timothy J., "Who Benefits from State and Local Economic Development Policies?," WE Upjohn Institute for Employment Research 1991.

Benigno, Pierpaolo and Gauti B Eggertsson, "It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve," Working Paper 31197, National Bureau of Economic Research April 2023.

Blanchard, Olivier and Jordi Galí, "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American economic journal: macroeconomics*, 2010, *2* (2), 1–30.

Borusyak, Kirill, Peter Hull, and Xavier Jaravel, "Quasi-experimental Shift-Share Research Designs," *The Review of Economic Studies*, 2022, *89* (1), 181–213.

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Cerrato, Andrea and Giulia Gitti, "Inflation Since COVID: Demand or Supply," *Available at SSRN 4193594*, 2022.

Christoffel, Kai Philipp and Tobias Linzert, "The Role of Real Wage Rigidity and Labor Market Frictions for Unemployment and Inflation Dynamics," 2005.

Faia, Ester, "Optimal Monetary Policy Rules with Labor Market Frictions," *Journal of Economic dynamics and control*, 2008, *32* (5), 1600–1621.

Fitzgerald, Terry J. and Juan Pablo Nicolini, "Is There a Stable Relationship between Unemployment and Future Inflation?," Federal Reserve Bank of Minneapolis Working Paper 2014.

Furman, Jason and Wilson Powell, "What is the Best Measure of Labor Market Tightness?," Peterson Institute for International Economics 2021.

Galí, Jordi, *Unemployment Fluctuations and Stabilization Policies: a New Keynesian Perspective*, MIT press, 2011.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *The American Economic Review*, 1988, pp. 402–417.

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning, "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?," *American Economic Review*, 2022, *112* (5), 1437–74.

Harding, Martin, Jesper Lindé, and Mathias Trabandt, "Understanding post-COVID inflation dynamics," *Journal of Monetary Economics*, 2023, *140*, S101–S118.

Hazell, Jonathon, Juan Herreño, Emi Nakamura, and Jón Steinsson, "The Slope of the Phillips Curve: Evidence from US States," *The Quarterly Journal of Economics*, 2022, *137* (3), 1299–1344.

Hooper, Peter, Frederic S. Mishkin, and Amir Sufi, "Prospects for Inflation in a High Pressure Economy: Is the Phillips Curve Dead or is it just Hibernating?," *Research in Economics*, 2020, *74* (2), 26–52.

Kiley, Michael T., "An Evaluation of the Inflationary Pressure Associated with Short- and Long-Term Unemployment," *Economics Letters*, 2015, *137*, 5–9.

Klenow, Peter J. and Oleksiy Kryvtsov, "State-Dependent or Time-Dependent Pricing: Does it Matter for Recent US Inflation?," *The Quarterly Journal of Economics*, 2008, *123* (3), 863–904.

Krause, Michael U and Thomas A Lubik, "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions," *Journal of Monetary Economics*, 2007, *54* (3), 706–727.

Leduc, Sylvain and Daniel Wilson, "Has the wage Phillips curve gone dormant?," *FRBSF Economic Letter*, 2017.

Mavroeidis, Sophocles, Mikkel Plagborg-Møller, and James H. Stock, "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature*, 2014, *52* (1), 124–188.

McLeay, Michael and Silvana Tenreyro, "Optimal Inflation and the Identification of the Phillips Curve," *NBER Macroeconomics Annual*, 2020, *34* (1), 199–255.

Michaillat, Pascal, "A Theory of Countercyclical Government Multiplier," *American Economic Journal: Macroeconomics*, 2014, *6* (1), 190–217.

— **and Emmanuel Saez**, "u*=$\sqrt{uv}$," Working Paper 30211, NBER 2022.

Murphy, Anthony, "Is the US Phillips Curve Convex? Some Metro Level Evidence," *Manuscript*, 2017.

Nakamura, Emi and Jón Steinsson, "Five Facts about Prices: a Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, 2008, *123* (4), 1415–1464.

Petrosky-Nadeau, Nicolas and Lu Zhang, "Unemployment crises," *Journal of Monetary Economics*, 2021, *117*, 335–353.

Phillips, Alban W., "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957," *Economica*, 1958, *25* (100), 283–299.

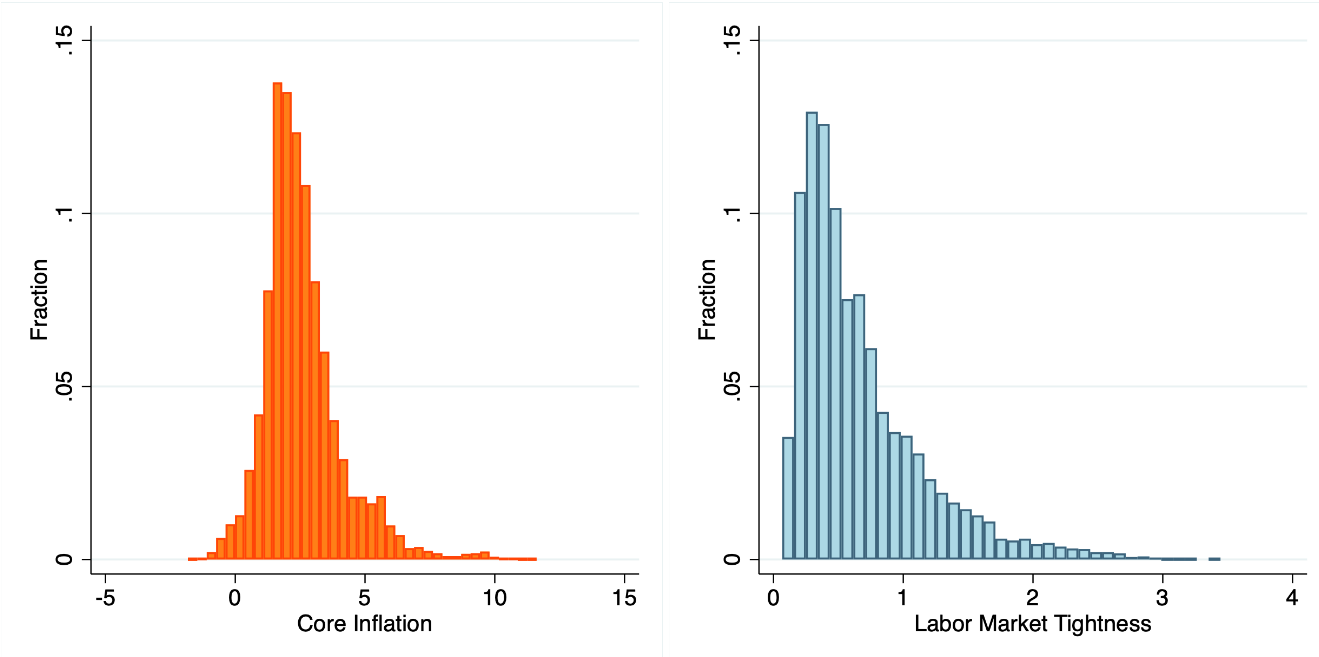
Sargent, Thomas J., "The Ends of Four Big Inflations," in "Inflation: Causes and effects," University of Chicago Press, 1982, pp. 41–98.

Schmitt-Grohé, Stephanie and Martín Uribe, "Heterogeneous Downward Nominal Wage Rigidity: Foundations of a Static Wage Phillips Curve," Technical Report, National Bureau of Economic Research 2022.

Trigari, Antonella, "The Role of Search Frictions and Bargaining for Inflation Dynamics," 2006.

Main Figures and Tables

Figure 2: Distributions of MSA-level Inflation and Labor Market Tightness, Dec01-Jul24

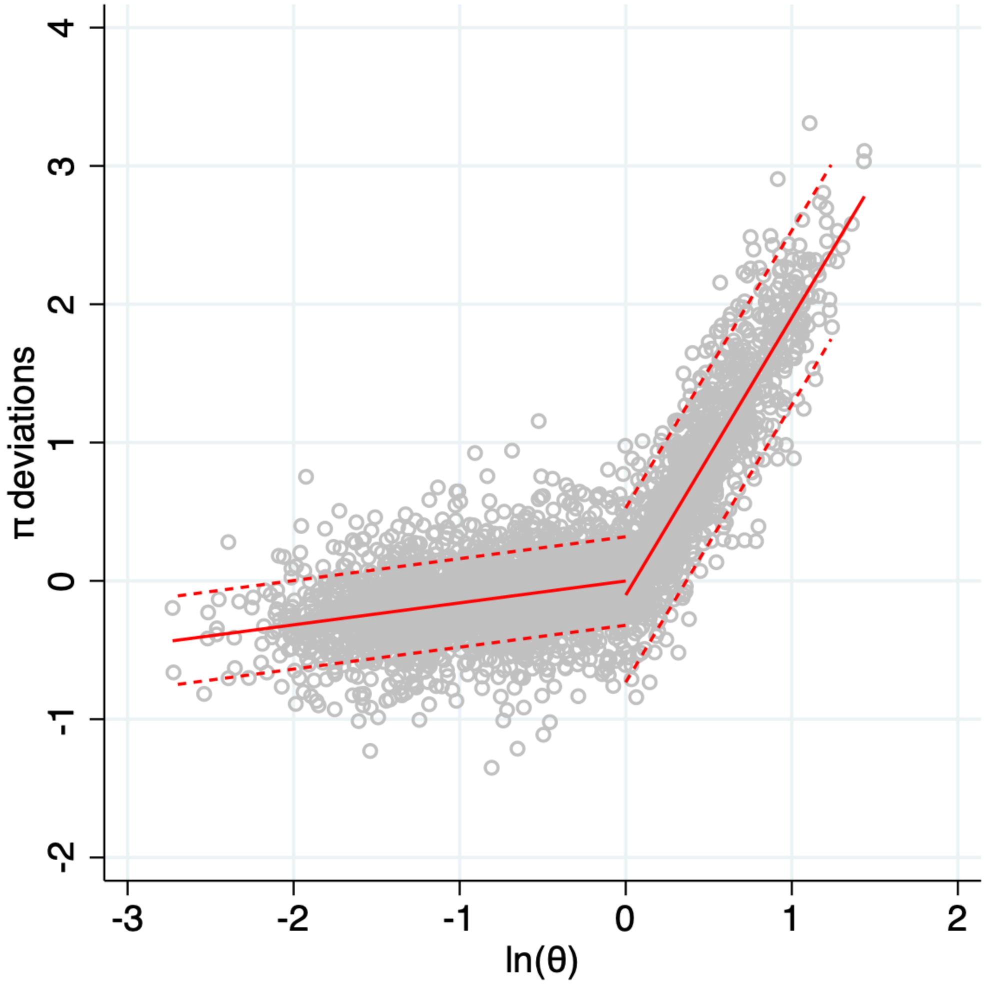


(a) Core Inflation

(b) Labor Market Tightness

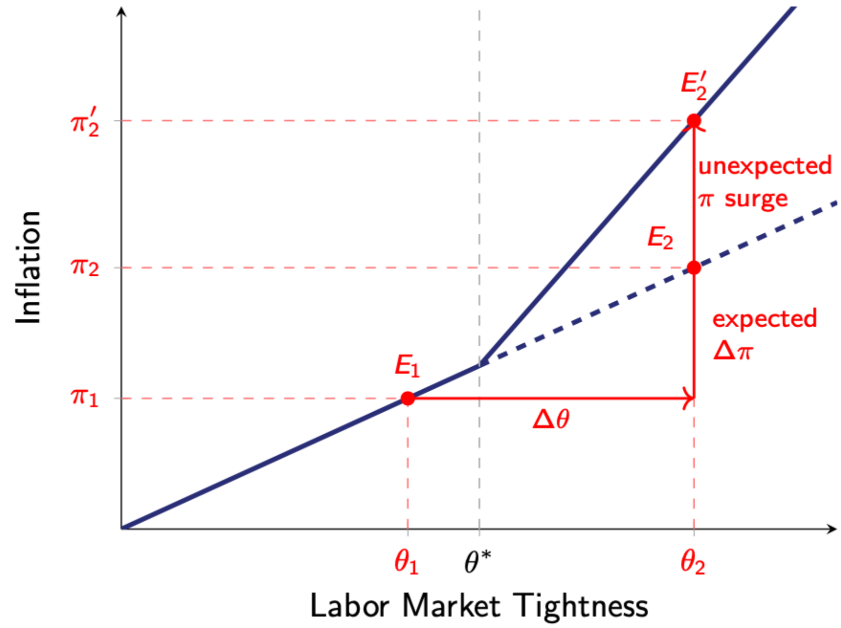
Notes. The figure shows the distribution of core inflation rates (2a) and of labor market tightness (2b) across 21 U.S. metropolitan areas from December 2000 to July 2024. Vacancies across metropolitan areas are imputed combining state-level vacancies collected by BLS JOLTS with yearly employment weights from the CPS.

Figure 3: Fit of estimated Phillips curve

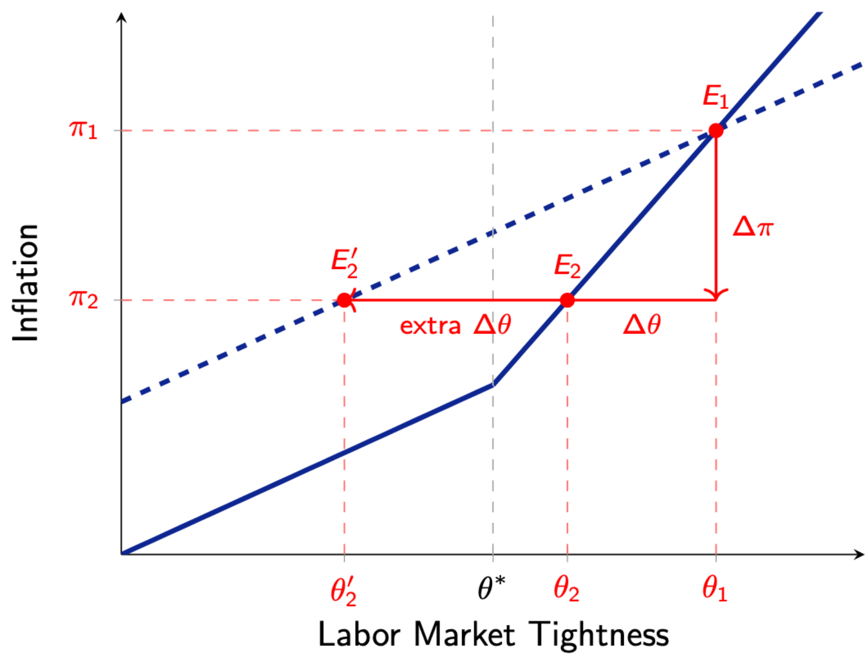


Notes. The figure shows the scatter plot of core inflation deviations and the log of labor market tightness across 21 U.S. metropolitan areas. Inflation deviations are defined as the difference between the 12-month core inflation rate and the estimated controls and fixed effects in column (1) of Table 1. The red lines plot the Phillips curve estimated in column (1) of Table 1.

Figure 4: Implications for monetary policy



(a) Unexpected increase in inflation



(b) Reduction of inflation at lower economic costs

Notes. The figure shows the implication for monetary policy of the Phillips curve being piecewise log-linear. The blue solid line represents the real Phillips curve, while the blue dashed line denotes the Phillips curve believed to be in place by the monetary authority. Panel (a) shows that inflation increases more than expected when the economy moves from E_1 to $E_2^{'}$ if the Phillips curve is nonlinear. Panel (b) shows that the economic costs of reducing inflation are lower when the economy moves from E_1 to E_2 if the Phillips curve is nonlinear.

Table 1: Estimates of ψ_θ^1 and ψ_θ^2

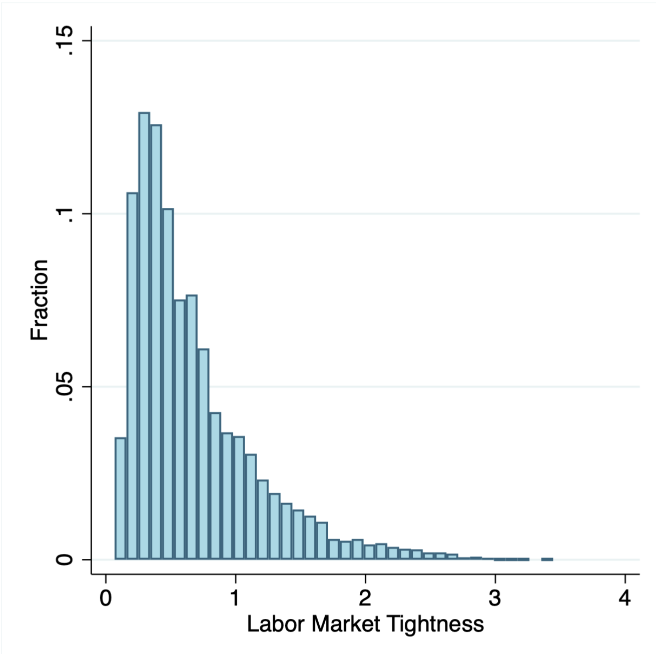
	(1) IV	(2) OLS	(3) OLS
$\ln(\theta_{it})$	0.16	0.84***	0.20**
	(0.32)	(0.10)	(0.09)
$\ln(\theta_{it}) \times I_{\{\theta_{it}>1\}}$	1.79***	3.28***	0.54***
	(0.63)	(0.51)	(0.18)
$I_{\{\theta_{it}>1\}}$	-0.15**	-0.32*	0.00
	(0.06)	(0.17)	(0.04)
$\ln(p_{it}^x)$	6.09***	2.75***	-0.13
	(1.22)	(1.03)	(0.98)
Observations	4334	4456	4418
MSA FE	✓	✓	✓
Year-Quarter FE	✓		✓
MSA × Year-Quarter FE	✓		✓
F-stat θ	54.54		
F-stat $\theta \times I$	43.23		

Notes. This table presents estimates of ψ_θ^1 and ψ_θ^2 from equation (37) from December 2001 to July 2024. All specifications feature the 12-month core inflation rate as dependent variable and two independent variables: the logarithm of labor market tightness $\ln(\theta_{it})$, and its interaction with a dummy that takes value of 1 when labor market tightness exceeds one $\ln(\theta_{it}) \times I_{\theta_{it}>1}$. Column (1) displays IV coefficients, while columns (2) to (3) display OLS coefficients. Column (1) displays IV estimates of ψ_θ^1 and ψ_θ^2 obtained by instrumenting $\ln(\theta_{it})$ with the shift-share instrument z_{it}^x. The interaction term is instrumented with the interaction of the shift-share instrument with the same dummy that takes value of 1 when labor market tightness exceeds one $z_{it}^x \times I_{\theta_{it}>1}$. Column (2) features MSA fixed effects. Column (3) additionally controls for year-quarter fixed effects and their interaction with MSA fixed effects. All specifications control for the logarithm of local relative price of intermediate input $\ln(p_{xit})$, measured as the ratio between the aggregate PPI of the manufacturing sector and the local all-items CPI. In column (1), $\ln(p_{xit})$ is instrumented with the ratio between the aggregate PPI of the manufacturing sector and the two-year lag of the local all-items CPI. Standard errors in parentheses are clustered at the MSA-year level. In column (1), first-stage F-statistics from Table A.1 are reported. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

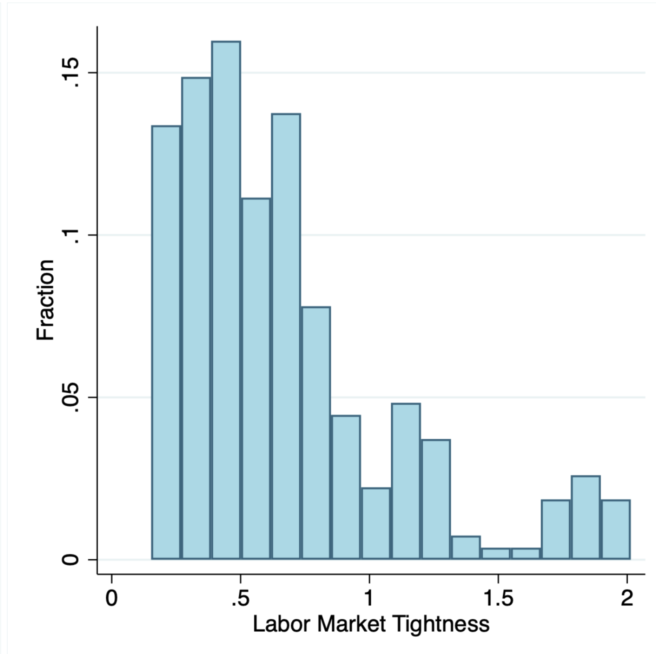
Appendix

A Figures and Tables

Figure A.1: Distributions of MSA-level and national labor market tightness, Dec01-Jul24



(a) MSA-level Labor Market Tightness



(b) National Labor Market Tightness

Notes. The figure shows the distribution of labor market tightness across 21 U.S. metropolitan areas (A.1a) and the distribution of labor market tightness at the national level (A.1b) from December 2000 to July 2024. Vacancies across metropolitan areas are imputed combining state-level vacancies collected by BLS JOLTS with yearly employment weights from the CPS.

Table A.1: First stage coefficients of Equation (37)

	(1)	(2)	(3)
	$\ln(\theta_{it})$	$\ln(\theta_{it}) \times I_{\{\theta_{it}>1\}}$	$\ln(p_{it}^x)$
z_{it}^x	9.40***	1.42***	-0.11***
	(1.31)	(0.29)	(0.03)
$z_{it}^x \times I_{\{\theta_{it}^x>1\}}$	3.00**	8.95***	-0.13***
	(1.43)	(1.16)	(0.05)
$I_{\{\theta_{it}>1\}}$	0.19***	0.08***	-0.00
	(0.01)	(0.01)	(0.00)
$\ln(p_{it-24}^x)$	1.30***	0.58***	0.68***
	(0.19)	(0.11)	(0.01)
Observations	4334	4334	4334
MSA FE	✓	✓	✓
Year-Quarter FE	✓	✓	✓
MSA × Year-Quarter FE	✓	✓	✓
F-stat	54.53	43.22	1314.25

Notes. This table presents the first stage regression coefficients for IV estimation of equation (37). In column (1), the dependent variable is the logarithm of labor market tightness $\ln(\theta_{it})$. In column (2), the dependent variable is the logarithm of labor market tightness interacted with a dummy that takes value of 1 when labor market tightness exceeds one $\ln(\theta_{it}) \times I_{\theta_{it}>1}$. In column (3), the dependent variable is the logarithm of the local relative price of intermediate inputs $\ln(p_{xit})$, measured as the ratio between the aggregate PPI of the manufacturing sector and the local all-items CPI. The main independent variables are the shift-share instrument constructed with 2-digits, tradable, intermediate-input Census industries z_{it}^x, the interaction between the shift-share instrument and the dummy that takes value of 1 when labor market tightness exceeds one $z_{it}^x \times I_{\theta_{it}>1}$, and the logarithm of the two-year lagged local relative price of intermediate inputs $\ln(p_{it-24}^x)$, constructed as the ratio between the aggregate PPI of the manufacturing sector and the two-year lag of the local all-items CPI. All columns control for the dummy that takes value of 1 when labor market tightness exceeds one $I_{\theta_{it}>1}$, MSA fixed effects, year-quarter fixed effects, and their interaction. Standard errors in parentheses are clustered at the MSA-year level. F-statistics are reported for each column. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.2: IV estimates of NRPC with different kinks

	(1) $k = 1$	(2) $k = \theta_i^{\text{best-fit}}$	(3) $k = \bar{\theta}_i$
$\ln(\theta_{it})$	0.16	0.20	0.17
	(0.32)	(0.43)	(0.36)
$\ln(\theta_{it}) \times I_{\{\theta_{it}>k\}}$	1.79***	1.89*	1.18**
	(0.63)	(1.09)	(0.55)
$I_{\{\theta_{it}>k\}}$	-0.15**	-0.27*	0.24
	(0.06)	(0.16)	(0.16)
$\ln(p^x_{it})$	6.09***	5.89***	6.10***
	(1.22)	(1.23)	(1.21)
Observations	4334	4334	4334
MSA FE	✓	✓	✓
Year-Quarter FE	✓	✓	✓
MSA x Year-Quarter FE	✓	✓	✓
F-stat θ	54.54	46.11	47.06
F-stat $\theta \times I$	43.23	15.91	41.98

Notes. This table presents IV estimates of equation (37) from December 2000 to July 2024 with different thresholds for defining tight labor markets. All specifications feature the 12-month core inflation rate as dependent variable. They feature the logarithm of labor market tightness $\ln(\theta_{it})$ and its interaction with a dummy that indicates when labor market tightness is above the kink $\ln(\theta_{it}) \times I_{\theta_{it}>k}$ as independent variables. Column (1) presents the main specification with the kink at 1. In column (2), the kink is set at the value of labor market tightness that minimizes the root mean squared error of an OLS regression in which core inflation is regressed on labor market tightness and its interaction term for each MSA. In column (3), the kink is set at the average value of labor market tightness in each MSA. All specifications control for the dummy indicating when labor market tightness exceeds the kink $I_{\theta_{it}>k}$, the logarithm of the local relative intermediate-input prices $\ln(p_{xit})$, MSA fixed effects, year-quarter fixed effects, and their interaction. All columns display IV estimates of the coefficients obtained by instrumenting $\ln(\theta_{it})$ with the shift-share instrument z^x_{it}. The interaction term is instrumented with the interaction of the shift-share instrument with the dummy indicating when labor market tightness exceeds the kink $z^x_{it} \times I_{\theta_{it}>k}$. $\ln(p_{xit})$ is instrumented with the logarithm of the two-year lagged local relative price of intermediate inputs $\ln(p^x_{it-24})$. Standard errors in parentheses are clustered at the MSA-year level. For each column, first-stage F-statistics are reported. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.3: IV estimates of NRPC with vacancies imputed with population weights

	(1) $k = 1$	(2) $k = \theta_i^{\text{best-fit}}$	(3) $k = \bar{\theta}_i$
$\ln(\theta_{it})$	0.32	0.24	0.19
	(0.34)	(0.40)	(0.34)
$\ln(\theta_{it}) \times I_{\{\theta_{it}>k\}}$	1.39**	1.72*	1.17**
	(0.69)	(1.01)	(0.55)
$I_{\{\theta_{it}>k\}}$	-0.08	-0.24	0.28
	(0.06)	(0.16)	(0.18)
$\ln(p^x_{it})$	6.12***	6.01***	6.06***
	(1.20)	(1.20)	(1.22)
Observations	4334	4334	4334
MSA FE	✓	✓	✓
Year-Quarter FE	✓	✓	✓
MSA x Year-Quarter FE	✓	✓	✓
F-stat θ	55.44	46.00	47.54
F-stat $\theta \times I$	39.50	19.00	41.75

Notes. This table presents IV estimates of equation (37) from December 2000 to July 2024 when population weights are used to impute state-level vacancies at the MSA level. All specifications feature the 12-month core inflation rate as dependent variable. They feature the logarithm of labor market tightness $\ln(\theta_{it})$ and its interaction with a dummy that indicates when labor market tightness is above the kink $\ln(\theta_{it}) \times I_{\theta_{it}>k}$ as independent variables. Column (1) presents the main specification with the kink at 1. In column (2), the kink is set at the value of labor market tightness that minimizes the root mean squared error of an OLS regression in which core inflation is regressed on labor market tightness and its interaction term for each MSA. In column (3), the kink is set at the average value of labor market tightness in each MSA. All specifications control for the dummy indicating when labor market tightness exceeds the kink $I_{\theta_{it}>k}$, the logarithm of the local relative intermediate-input prices $\ln(p_{xit})$, MSA fixed effects, year-quarter fixed effects, and their interaction. All columns display IV estimates of the coefficients obtained by instrumenting $\ln(\theta_{it})$ with the shift-share instrument z^x_{it}. The interaction term is instrumented with the interaction of the shift-share instrument with the dummy indicating when labor market tightness exceeds the kink $z^x_{it} \times I_{\theta_{it}>k}$. $\ln(p_{xit})$ is instrumented with the logarithm of the two-year lagged local relative price of intermediate inputs $\ln(p^x_{it-24})$. Standard errors in parentheses are clustered at the MSA-year level. For each column, first-stage F-statistics are reported. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.4: IV estimates of Phillips curve with different nonlinear functional forms

	(1) Piecewise-log	(2) Log	(3) Quadratic-log	(4) Piecewise-lin	(5) Inverse	(6) Piecewise-inverse
$\ln(\theta_{it})$	0.16	0.78***	1.05***			
	(0.32)	(0.22)	(0.28)			
$\ln(\theta_{it}) \times I_{\theta_{it}>1}$	1.79***					
	(0.63)					
$\ln(\theta_{it})^2$			0.29**			
			(0.14)			
θ_{it}				0.48		
				(0.86)		
$\theta_{it} \times I_{\theta_{it}>1}$				0.59		
				(0.94)		
$\frac{1}{\theta_{it}}$					-0.34***	-0.04
					(0.11)	(0.09)
$\frac{1}{\theta_{it}} \times I_{\frac{1}{\theta_{it}}<1}$						-3.21***
						(0.84)
Observations	4334	4334	4334	4334	4334	4334
MSA FE	✓	✓	✓	✓	✓	✓
Year-Quarter FE	✓	✓	✓	✓	✓	✓
MSA x Year-Quarter FE	✓	✓	✓	✓	✓	✓
F-stat θ	56.99	57.95	41.25	51.14	38.73	28.14
F-stat $\theta \times I/\theta^2$	45.17		14.59	35.57		43.85

Notes. This table presents estimates of the slope of the Phillips curve from December 2000 to July 2024. All specifications feature the 12-month, core inflation rate as dependent variable and labor market tightness as the main independent variable, in different functional forms. Column (1) presents the main specification, where labor market tightness is modeled according to a piecewise log-linear function with a kink at one. Columns (2) to (6) display the following nonlinear transformations: log, quadratic-log, piece-wise linear with kink at 1, quadratic-linear, and inverse, respectively. All specifications control for the logarithm of the local relative intermediate-input prices $\ln(p_{xit})$, MSA fixed effects, year-quarter fixed effects, and their interaction. All columns display IV estimates of the coefficients obtained by instrumenting θ_{it} and its transformations with the shift-share instrument z^x_{it}. The coefficients on the interaction terms in columns (1) and (4) are obtained by using the interaction of the shift-share instrument with the dummy indicating when labor market tightness exceeds one $z^x_{it} \times I_{\theta_{it}>1}$. The coefficients on the quadratic terms in columns (3) and (5) are are obtained by using the square of the shift-share instrument ${z^x_{it}}^2$. $\ln(p_{xit})$ is instrumented with the logarithm of the two-year lagged local relative price of intermediate inputs $\ln(p^x_{it-24})$. Standard errors in parentheses are clustered at the MSA-year level. For each column, first-stage F-statistics are reported. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.5: IV estimates of Phillips curve with control for final-goods sector productivity

	(1) Benchmark	(2) Final-goods sector productivity
$\ln(\theta_{it})$	0.16	0.14
	(0.32)	(0.33)
$\ln(\theta_{it}) \times I_{\theta_{it}>1}$	1.79***	1.68***
	(0.63)	(0.62)
$I_{\theta_{it}>1}$	-0.15**	-0.13*
	(0.06)	(0.07)
$\ln(p^x_{it})$	6.09***	6.08***
	(1.22)	(1.20)
z^y_{it}		1.41
		(1.65)
Observations	4334	4334
MSA FE	✓	✓
Year-Quarter FE	✓	✓
MSA x Year-Quarter FE	✓	✓
F-stat θ	56.99	54.35
F-stat $\theta \times I$	45.17	36.88

Notes. This table presents estimates of the slope of the Phillips curve from December 2000 to July 2024 with and without the control for final-goods sector productivity. All specifications feature the 12-month, core inflation rate as dependent variable. They feature the logarithm of labor market tightness $\ln(\theta_{it})$ and its interaction with a dummy that indicates when labor market tightness is above one $\ln(\theta_{it}) \times I_{\theta_{it}>1}$ as independent variables. Column (1) presents the benchmark specification without the control for final-goods sector productivity z^y_{it}. Column (2) includes z^y_{it}. z^y_{it} is constructed as the shift share instrument using only 2-digits, non-tradable, final-goods Census industries. All specifications control for the dummy indicating when labor market tightness exceeds one $I_{\theta_{it}>1}$, the logarithm of the local relative intermediate-input prices $\ln(p_{xit})$, MSA fixed effects, year-quarter fixed effects, and their interaction. All columns display IV estimates of the coefficients obtained by instrumenting $\ln(\theta_{it})$ with the shift-share instrument z^x_{it}. The interaction term is instrumented with the interaction of the shift-share instrument with the dummy indicating when labor market tightness exceeds one $z^x_{it} \times I_{\theta_{it}>1}$. $\ln(p_{xit})$ is instrumented with the logarithm of the two-year lagged local relative price of intermediate inputs $\ln(p^x_{it-24})$. Standard errors in parentheses are clustered at the MSA-year level. For each column, first-stage F-statistics are reported. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

B Model

B.1 FOCs of Intermediate-Input and Final-Goods Firms

The optimal commodity demand from the representative intermediate-input firm is

$$P_{ot} O_{it} = (1 - \rho) X_{it} MC_{xit}, \tag{B.1}$$

where MC_{xit} denotes the marginal cost of the intermediate-input firm and takes the following form:

$$MC_{xit} = \frac{1}{Z_{xit}} \left[\frac{(1+\gamma_{it}^{b})W_{it}^{new}}{\rho} \right]^{\rho} \left[\frac{P_{ot}}{1-\rho} \right]^{1-\rho}. \tag{B.2}$$

The optimal labor demand from the intermediate-input firm is

$$(1+\gamma_{it}^{b})W_{it}^{new}[N_{xit}^{att} + N_{xit}^{new}] = \rho X_{it} MC_{xit}. \tag{B.3}$$

From the optimization problem of the final-goods firm, the optimal intermediate-input and labor demand are, respectively,

$$P_{xt}X_{it} = (1-\phi)Y_{it}MC_{yit} \tag{B.4}$$

and

$$(1+\gamma_{it}^{b})W_{it}^{new}[N_{yit}^{att} + N_{yit}^{new}] = \phi Y_{it} MC_{yit}, \tag{B.5}$$

where MC_{yit} denotes the marginal cost and takes the following form:

$$MC_{yit} = \frac{1}{Z_{yit}} \left[\frac{(1+\gamma_{it}^{b})W_{it}^{new}}{\phi} \right]^{\phi} \left[\frac{P_{xt}}{1-\phi} \right]^{1-\phi}. \tag{B.6}$$

The optimal pricing condition is

$$E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} Y_{it+k}(g) \left[\frac{P_{it}^{*}(g)}{P_{it-1}} - \frac{\epsilon}{\epsilon-1} \frac{MC_{yit+k}}{P_{it+k}} \frac{P_{it+k}}{P_{it-1}} \right] = 0. \tag{B.7}$$

B.2 Monetary Authority

The monetary authority implements a common monetary policy across the two regions following the Taylor rule

$$r_t^n = \phi_\pi(\pi_t - \pi_t^*) - \phi_\theta(\hat{\theta}_t - \theta_t^*) + \varepsilon_{rt}, \tag{B.8}$$

where hatted variables represent deviations from a zero-inflation steady state and lower-case variables are logs of upper-case variables. $\pi_t = \zeta\pi_{it} + (1-\zeta)\pi_{jt}$ denotes economy-wide inflation, where $\pi_{it} = p_{it} - p_{it-1}$ is consumer price inflation in region i and π_{jt} is the counterpart in region j. $\hat{\theta}_t = \zeta\hat{\theta}_{it} + (1-\zeta)\hat{\theta}_{jt}$ denotes the deviation of aggregate labor market tightness from its steady-state value. Finally, π_t^* represents a time-varying inflation target. We assume that the monetary authority targets a value for labor market tightness consistent with its long-run inflation target, i.e. $\theta_t^* = \frac{(1-\beta)}{\kappa_\theta}\pi_t^*$. Finally, ϕ_π and ϕ_u ensure a unique locally bounded equilibrium, and ε_{rt} denotes a transitory monetary shock, assumed to follow an AR(1) process. The model in its simplest form abstracts from fiscal policy, as the government does not tax, spend, nor issues debt, and monetary policy has no fiscal implications.

B.3 Equilibrium

An equilibrium in a collection of stochastic processes for $\{C_{it}, F_{it}, \theta_{it}, \theta^*_{it}, V_{it}, w^{att}_{it}, w^{new}_{it}, w^{flex}_{it}, O_{it}, X_{it}, Y_{it}, N^{new}_{xit}, N^{new}_{yit}, MC_{xit}, MC_{yit}, P^*_{it}, P_{it}, Q_{it,t+1}, C_{jt}, F_{jt}, \theta_{jt}, \theta^*_{jt}, V_{jt}, w^{att}_{jt}, w^{new}_{jt}, w^{flex}_{jt}, O_{jt}, X_{jt}, Y_{jt}, N^{new}_{xjt}, N^{new}_{yjt}, MC_{xjt}, MC_{yjt}, P^*_{jt}, P_{jt}, Q_{jt,t+1}, i_t, O_t, P_{xt}\}_{t=0}^{\infty}$ that satisfy Equations (12), (13), (B.1), (B.3), (B.2), (B.4), (B.5), (B.6), (B.7), (B.22), (31), (32), (33), (34) and their counterparts in region j, together with the following final-goods and labor market clearing conditions in region i and their counterparts in region j:

$$Y_{it} = C_{it} + \gamma^c_{it} V_{it} \tag{B.9}$$

and

$$\left(1 - s + s m_{it}\theta_{it}^{1-\eta}\right) F_{it} = \frac{1}{(1+\gamma^b_{it})W^{new}_{it}} \left(\rho X_{it} MC_{xit} + \phi Y_{it} MC_{yit}\right); \tag{B.10}$$

Equation (B.8) and the aggregate commodity and intermediate-input market clearing conditions:

$$O_t = \frac{1-\rho}{P_{ot}} \left(X_{it} MC_{xit} + X_{jt} MC_{xjt}\right) \tag{B.11}$$

and

$$X_{it} + X_{jt} = \frac{1-\phi}{P_{xt}} \left(Y_{it} MC_{yit} + Y_{jt} MC_{yjt}\right); \tag{B.12}$$

given exogenous processes for $\{\chi_{it}, \gamma^b_{it}, \gamma^c_{it}, m_{it}, z_{xit}, z_{yit}, \delta_{it}, \chi_{jt}, \gamma^b_{jt}, \gamma^c_{jt}, m_{jt}, z_{xjt}, z_{yjt}, \delta_{jt}, P_{ot}\}_{t=0}^{\infty}$.

B.4 Steady State

Consider a steady state in which: $\chi_{it} = \bar{\chi}_i$, $P_{ot} = \bar{P}_o$, $\gamma^b_{it} = \bar{\gamma}^b_i$, $\gamma^c_{it} = \bar{\gamma}^c_i$, $m_{it} = \bar{m}_i$, $z_{xit} = \bar{z}_{xi}$, $z_{yit} = \bar{z}_{yi}$, and $\delta_{it} = \bar{\delta}_i$. Barred variables denote steady-state levels. Note that the same applies in region j.

I assume that the labor market is slack in the steady state. Therefore, $\bar{w}^{flex}_i < \bar{w}^{att}_i$ and $\bar{w}^{new}_i = \bar{w}^{att}_i = \bar{w}_i$. Equation (B.7) pins down the steady-state level of wages:

$$\bar{w}_i = \frac{\phi}{1+\bar{\gamma}^b_i} \left(\frac{\epsilon - 1}{\epsilon} \bar{z}_{yi}\right)^{\frac{1}{\phi}} (1-\phi)^{\frac{1-\phi}{\phi}}. \tag{B.13}$$

In this steady state, $\bar{\theta}_i < \bar{\theta}_i^*$. Imposing $\bar{w}_i^{flex} = \bar{w}_i^{att} = \bar{w}_i$, I find $\bar{\theta}_i^*$:

$$\bar{\theta}_i^* = \left[\frac{\phi \bar{\gamma}_i^b \bar{m}_i}{\bar{\gamma}_i^c (1 + \bar{\gamma}_i^b)} \left(\frac{\epsilon - 1}{\epsilon} \bar{z}_{yi} \right)^{\frac{1}{\phi}} (1 - \phi)^{\frac{1-\phi}{\phi}} \right]^{\frac{1}{\eta}}. \tag{B.14}$$

In steady state, Equation (31) becomes

$$\bar{w}_i^{flex} = \frac{\bar{\gamma}_i^c}{\bar{\gamma}_i^b \bar{m}_i} \bar{\theta}_i \tag{B.15}$$

and Equation (32)

$$\bar{w}_i^{att} = (\bar{w}_i)^{\lambda} (\bar{w}_i^{flex})^{1-\lambda}. \tag{B.16}$$

Substituting Equation (B.15) into Equation (B.16) and using $\bar{w}_i^{att} = \bar{w}_i$, I pin down $\bar{\theta}_i$:

$$\bar{\theta}_i = \left(\frac{\bar{\gamma}_i^b \bar{m}_i}{\bar{\gamma}_i^c} \bar{w}_i \right)^{\frac{1}{\eta}}. \tag{B.17}$$

Substituting Equation (B.17) into the steady-state version of Equation (12), I find $\bar{F}_i$:

$$\bar{F}_i = \left[\frac{\bar{w}_i}{\bar{\chi}_i} \left(1 - s + s \bar{m}_i \bar{\theta}^{1-\eta} \right) \right]^{\frac{1}{\omega}}. \tag{B.18}$$

Finally, I substitute Equations (B.17) and (B.18) into the steady-state version of the labor market clearing condition to pin down $\bar{N}_i$:

$$\bar{N}_i = \left(1 - s + s \bar{m}_i \bar{\theta}^{1-\eta} \right) \bar{F}_i. \tag{B.19}$$

B.5 Derivation of Regional Phillips Curve

Log-linearazing Equation (B.7) around the zero inflation steady state yields

$$\hat{p}_{it}^*(g) - \hat{p}_{it-1} = (1 - \alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^{\kappa} E_t \left[(\hat{mc}_{it+k} - \hat{p}_{it+k}) - (\hat{p}_{it+k} - \hat{p}_{it-1}) \right],$$

where

$$\hat{mc}_{it} = -\hat{z}_{yit} + \phi(h_\gamma \hat{\gamma}_{it}^b + \hat{w}_{it}^{new}) + (1 - \phi)(\hat{p}_{xt} - \hat{p}_{it}) \tag{B.20}$$

and $h_\gamma = \frac{\bar{\gamma}_i^b}{1+\bar{\gamma}_i^b}$. Rearranging the equation, I obtain

$$\hat{p}_{it}^*(g) - \hat{p}_{it-1} = \alpha\beta E_t \left[\hat{p}_{it+1}^*(g) - \hat{p}_{it} \right] + (1 - \alpha\beta)(\hat{mc}_{it} - \hat{p}_{it}) + \pi_{it}, \tag{B.21}$$

where π_{it} is derived from the definition of the price index in Equation (16). Indeed, only $(1-\alpha)$ firms are able to reset their price, and since they are faced by the same probability of changing price in the future and the same current and expected same marginal costs, they will choose the same price P^*_{it}. Hence, the price index becomes

$$P_{it}^{1-\epsilon} = \alpha P_{it-1}^{1-\epsilon} + (1-\alpha) P_{it}^{*1-\epsilon}. \tag{B.22}$$

Taking a log-linear approximation of this last expression yields

$$\hat{p}_{it} = \alpha \hat{p}_{it-1} + (1-\alpha)\hat{p}^*_{it}, \tag{B.23}$$

which implies

$$\pi_{it} = (1-\alpha)(\hat{p}^*_{it} - \hat{p}_{it}). \tag{B.24}$$

Substituting Equation (B.24) in Equation (B.21), after some manipulations I obtain

$$\pi_{it} = \beta E_t \pi_{it+1} + \xi(\hat{mc}_{it} - \hat{p}_{it}), \tag{B.25}$$

where

$$\xi = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}.$$

Combining Equations (B.20) and (B.25), I get

$$\pi_{it} = \beta E_t \pi_{it+1} + \xi\phi(h_\gamma \hat{\gamma}^b_{it} + \hat{w}^{new}_{it}) + \xi(1-\phi)(\hat{p}_{xt} - \hat{p}_{it}) - \xi \hat{x}_{yit}. \tag{B.26}$$

The log-linearized expression for the wage norm is

$$\hat{w}^{new}_{it} = \begin{cases} \hat{\gamma}^c_{it} - \hat{\gamma}^b_{it} - \hat{m}_{it} + \eta\hat{\theta}_{it} & \hat{\theta}_{it} > \hat{\theta}^*_i \\ (1-\lambda)(\hat{\gamma}^c_{it} - \hat{\gamma}^b_{it} - \hat{m}_{it} + \eta\hat{\theta}_{it}) & \hat{\theta}_{it} \leq \hat{\theta}^*_i \end{cases}. \tag{B.27}$$

Substituting Equation (B.27) into Equation (B.26), I obtain the regional Phillips curve

$$\pi_{it} = \begin{cases} \beta E_t \pi_{it+1} + \kappa^{tight}\hat{\theta}_{it} + \xi\left[\hat{\nu}^{tight}_{it} + (1-\phi)\hat{p}_{xit} - \hat{z}_{yit}\right] & \hat{\theta}_{it} > \hat{\theta}^*_{it} \\ \beta E_t \pi_{it+1} + \kappa\hat{\theta}_{it} + \xi\left[\hat{\nu}_{it} + (1-\phi)\hat{p}_{xit} - \hat{z}_{yit}\right] & \hat{\theta}_{it} \leq \hat{\theta}^*_{it} \end{cases}, \tag{B.28}$$

where

- $\kappa^{tight} = \xi\phi\eta$,

- $\kappa = (1-\lambda)\kappa^{tight}$,

- $\hat{\nu}_{it}^{tight} = \phi[(\hat{\gamma}_{it}^{c} - \hat{m}_{it}) - (1-h_\gamma)\hat{\gamma}_{it}^{b}]$

- $\hat{\nu}_{it} = \phi[(1-\lambda)(\hat{\gamma}_{it}^{c} - \hat{m}_{it}) - (1-\lambda-h_\gamma)\hat{\gamma}_{it}^{b}]$

- $\hat{p}_{xit} = \hat{p}_{xt} - \hat{p}_{it}$.

B.6 Solving Forward

To derive equation (36), I solve equation (B.28) forward. I assume that if the labor market is slack in t, then it will remain slack forever. If the labor market is tight in t, it will remain tight until $t+T-1$. From period $t+T$, the labor market becomes slack and will remain slack forever. Hence, the long-run component of the variation of labor market tightness corresponds to a slack labor market. I make this assumption because labor markets in the U.S. are generally slack. The episodes of tight labor markets are indeed quite rare.

Consider the case in which the labor market in region i at time t is slack. Equation (B.28) implies that inflation is equal to

$$\pi_{it}^{slack} = \beta E_t \pi_{it+1}^{slack} + \kappa\hat{\theta}_{it} + \xi\left[\hat{\nu}_{it} + (1-\phi)\hat{p}_{xit} - \hat{z}_{yit}\right]. \tag{B.29}$$

Solving equation (B.29) forward, I obtain

$$\pi_{it}^{slack} = E_t \sum_{k=0}^{\infty} \beta^k \kappa \hat{\theta}_{it+k} + E_t \sum_{k=0}^{\infty} \beta^k \xi\left[\hat{\nu}_{it+k} + (1-\phi)\hat{p}_{xit+k} - \hat{z}_{yit+k}\right]. \tag{B.30}$$

Following Hazell et al. (2022), I decompose the variation of future labor market tightness $\hat{\theta}_{it+k}$ into a transitory and a permanent component. The transitory component is defined as $\tilde{\theta}_{it} = \hat{\theta}_{it} + E_t\hat{\theta}_{it+\infty}$, where $E_t\hat{\theta}_{it+\infty}$ is the permanent component of variation in labor market tightness. Applying the decomposition to equation (B.30), I get

$$\pi_{it}^{slack} = E_t \sum_{k=0}^{\infty} \beta^k \kappa \tilde{\theta}_{it+k} + \frac{\kappa}{1-\beta} E_t\hat{\theta}_{it+\infty} + E_t \sum_{k=0}^{\infty} \beta^k \xi\left[\hat{\nu}_{it+k} + (1-\phi)\hat{p}_{xit+k} - \hat{z}_{yit+k}\right]. \tag{B.31}$$

Assuming that the shocks $\hat{\nu}_{it}$, $\hat{p}_{xit}$, and $\hat{z}_{yit}$ are transitory, and the labor market is slack in the long run, equation (B.28) implies that $E_t\pi_{it+\infty} = \frac{\kappa}{1-\beta}E_t\theta_{it+\infty}$. Moreover, as $E_t\pi_{it+\infty}$ represents the long-run belief about the monetary policy regime and such regime is common between the two regions, then $E_t\pi_{it+\infty} = E_t\pi_{jt+\infty} = E_t\pi_{t+\infty}$. Substituting, I obtain

$$\pi_{it}^{slack} = E_t\pi_{t+\infty} + E_t\sum_{k=0}^{\infty}\beta^k\kappa\tilde{\theta}_{it+k} + E_t\sum_{k=0}^{\infty}\beta^k\xi\left[\hat{\nu}_{it+k} + (1-\phi)\hat{p}_{xit+k} - \hat{z}_{yit+k}\right]. \tag{B.32}$$

Assuming that $\tilde{\theta}_{it}$ and $\hat{p}_{xit}$ follow AR(1) processes with autocorrelation coefficients ρ_θ and ρ_p, the regional Phillips curve when the labor market is slack at time t takes the following form:

$$\pi_{it}^{slack} = E_t\pi_{t+\infty} + \psi_\theta^1\tilde{\theta}_{it} + \psi_p\hat{p}_{xit} + \varepsilon_{it}, \tag{B.33}$$

where

- $\psi_\theta^1 = \frac{\kappa}{1-\beta\rho_\theta}$,
- $\psi_p = \frac{\xi(1-\phi)}{1-\beta\rho_p}$,
- $\varepsilon_{it} = E_t\sum\limits_{k=0}^{\infty}\beta^k\xi\left[\hat{\nu}_{it+k} - \hat{z}_{yit+k}\right]$.

Let's turn to the case in which the labor market in region i at time t is tight. I assume that the labor market remains tight until $t+T-1$. At $t+T$ the labor market becomes slack and remains slack forever. From equation (B.28), inflation in region i at time $t+T$ is equal to

$$\pi_{it+T}^{slack} = \beta E_{t+T}\pi_{it+T+1}^{slack} + \kappa\hat{\theta}_{it+T} + \xi\left[\hat{\nu}_{it+T} + (1-\phi)\hat{p}_{xit+T} - \hat{z}_{yit+T}\right]. \tag{B.34}$$

Solving equation (B.34) forward, I obtain

$$\pi_{it+T}^{slack} = E_{t+T}\sum_{k=T}^{\infty}\beta^{k-T}\kappa\hat{\theta}_{it+k} + E_{t+T}\sum_{k=T}^{\infty}\beta^{k-T}\xi\left[\hat{\nu}_{it+k} + (1-\phi)\hat{p}_{xit+k} - \hat{z}_{yit+k}\right]. \tag{B.35}$$

Decomposing the variation of labor market tightness in the transitory and permanent components, I get

$$\pi_{it+T}^{slack} = E_{t+T}\sum_{k=T}^{\infty}\beta^{k-T}\left\{\kappa\tilde{\theta}_{it+k} + \xi\left[\hat{\nu}_{it+k} + (1-\phi)\hat{p}_{xit+k} - \hat{z}_{yit+k}\right]\right\} + \sum_{k=T}^{\infty}\beta_{t+T}^{k-T}\hat{\theta}_{it+\infty}. \tag{B.36}$$

Finally, I assume that $\tilde{\theta}_{it}$ and $\hat{p}_{xit}$ follow AR(1) processes with autocorrelation coefficients ρ_θ and ρ_p.

Equation (B.36) becomes

$$\pi_{it+T}^{slack} = \sum_{k=T}^{\infty}(\beta\rho_\theta)^{k-T}\kappa\tilde{\theta}_{it+T} + \sum_{k=T}^{\infty}\beta^{k-T}\kappa E_{t+T}\hat{\theta}_{it+\infty} + \sum_{k=T}^{\infty}(\beta\rho_p)^{k-T}\xi(1-\phi)\hat{p}_{it+T}^{x} + E_{t+T}\sum_{k=T}^{\infty}\beta^{k-T}\xi(\hat{\nu}_{it+k} - \hat{z}_{yit+k}). \tag{B.37}$$

From period t to $t+T-1$, the labor market is tight. From equation (B.28), inflation in region i at time $t+T-1$ is equal to

$$\pi_{it+T-1}^{tight} = \beta E_{t+T-1}\pi_{it+T}^{slack} + \kappa^{tight}\hat{\theta}_{it+T-1} + \xi\hat{\nu}_{it+T-1}^{tight} + \xi(1-\phi)\hat{p}_{it+T-1}^{x} - \xi\hat{z}_{yit+T-1}. \tag{B.38}$$

Solving backward, I obtain

$$\begin{aligned}
\pi_{it+T-2}^{tight} =& \beta E_{t+T-2}\pi_{it+T-1}^{tight} + \kappa^{tight}\hat{\theta}_{it+T-2} + \xi\hat{\nu}_{it+T-2}^{tight} + \xi(1-\phi)\hat{p}_{it+T-2}^{x} - \xi\hat{z}_{yit+T-2} \\
=& \beta E_{t+T-2}\left[\beta E_{t+T-1}\pi_{it+T}^{slack} + \kappa^{tight}\hat{\theta}_{it+T-1} + \xi\hat{\nu}_{it+T-1}^{tight} + \xi(1-\phi)\hat{p}_{it+T-1}^{x} - \xi\hat{z}_{yit+T-1}\right] \\
& + \kappa^{tight}\hat{\theta}_{it+T-2} + \xi\hat{\nu}_{it+T-2}^{tight} + \xi(1-\phi)\hat{p}_{it+T-2}^{x} - \xi\hat{z}_{yit+T-2} \\
=& \beta^2 E_{t+T-2}\pi_{it+T}^{slack} + \kappa^{tight}\left(\beta E_{t+T-2}\hat{\theta}_{it+T-1} + \hat{\theta}_{it+T-2}\right) + \xi\left(\beta E_{t+T-2}\hat{\nu}_{it+T-1}^{tight} + \hat{\nu}_{it+T-2}^{tight}\right) \\
& + \xi(1-\phi)\left(\beta E_{t+T-2}\hat{p}_{it+T-1}^{x} + \hat{p}_{it+T-2}^{x}\right) - \xi\left(\beta E_{t+T-2}\hat{z}_{yit+T-1} + \hat{z}_{yit+T-2}\right) \\
& \ldots \\
\pi_{it}^{tight} =& \beta^T E_t\pi_{it+T}^{slack} + E_t\sum_{k=0}^{T-1}\beta^k\left\{\kappa^{tight}\hat{\theta}_{it+k} + \xi\left[\hat{\nu}_{it+k} + (1-\phi)\hat{p}_{xit+k} - \hat{z}_{yit+k}\right]\right\}.
\end{aligned} \tag{B.39}$$

Decomposing the variation of labor market tightness into the transitory and permanent components, I get

$$\begin{aligned}
\pi_{it}^{tight} =& \beta^T E_t\pi_{it+T}^{slack} + E_t\sum_{k=0}^{T-1}\beta^k\left\{\kappa^{tight}\tilde{\theta}_{it+k} + \xi\left[\hat{\nu}_{it+k} + (1-\phi)\hat{p}_{xit+k} - \hat{z}_{yit+k}\right]\right\} \\
& + \sum_{k=0}^{T-1}\beta^k\kappa^{tight}E_t\hat{\theta}_{it+\infty}.
\end{aligned} \tag{B.40}$$

Assuming that $\tilde{\theta}_{it}$ and $\hat{p}_{xit}$ follow AR(1) processes with autocorrelation coefficients ρ_θ and ρ_p, I obtain

$$\begin{aligned}\pi_{it}^{tight} =& \beta^T E_t \pi_{it+T}^{slack} + \sum_{k=0}^{T-1} \left(\beta\rho_\theta\right)^k \kappa^{tight} \tilde{\theta}_{it} + \sum_{k=0}^{T-1} \beta^k \kappa^{tight} E_t \hat{\theta}_{it+\infty} \\ &+ \sum_{k=0}^{T-1} (\beta\rho_p)^k \xi(1-\phi)\hat{p}_{xit+k} + \sum_{k=0}^{T-1} \beta^k \xi \left(\hat{\nu}_{it+k}^{tight} - \hat{z}_{yit+k}\right). \end{aligned} \tag{B.41}$$

Finally, I substitute equation (B.37) into equation (B.41) and obtain

$$\begin{aligned}\pi_{it}^{tight} =& \beta^T E_t \Bigg[\sum_{k=T}^{\infty} (\beta\rho_\theta)^{k-T} \kappa \tilde{\theta}_{it+T} + \sum_{k=T}^{\infty} \beta_{t+T}^{k-T} \hat{\theta}_{it+\infty} + \sum_{k=T}^{\infty} (\beta\rho_p)^{k-T} \xi(1-\phi)\hat{p}_{it+T}^{x} \\ &+ \sum_{k=T}^{\infty} \beta^{k-T} \xi \left(\hat{\nu}_{it+k} - \hat{z}_{yit+T}\right) \Bigg] \\ &+ \sum_{k=0}^{T-1} \left(\beta\rho_\theta\right)^k \kappa^{tight} \tilde{\theta}_{it} + \sum_{k=0}^{T-1} \beta^k \kappa^{tight} E_t \hat{\theta}_{it+\infty} + \sum_{k=0}^{T-1} (\beta\rho_p)^k \xi(1-\phi)\hat{p}_{xit} \\ &+ \sum_{k=0}^{T-1} \beta^k \xi \left(\hat{\nu}_{it+k}^{tight} - \hat{z}_{yit+k}\right) \\ =& \sum_{k=0}^{T-1} \left(\beta\rho_\theta\right)^k \kappa^{tight} \tilde{\theta}_{it} + \sum_{k=T}^{\infty} (\beta\rho_\theta)^{k-T} \beta^T \kappa E_t \tilde{\theta}_{it+T} + \left[\kappa^{tight} \sum_{k=0}^{T-1} \beta^k + \kappa \sum_{k=T}^{\infty} \beta^k \right] E_t \hat{\theta}_{it+\infty} \\ &+ \sum_{k=0}^{T-1} \left(\beta\rho_p\right)^k \xi(1-\phi)\hat{p}_{xit} + \sum_{k=T}^{\infty} (\beta\rho_p)^{k-T} \beta^T \xi(1-\phi) E_t \hat{p}_{xit+T} \\ &+ E_t \left[\sum_{k=0}^{T-1} \beta^k \xi \left(\hat{\nu}_{it+k}^{tight} - \hat{z}_{yit+k}\right) + \sum_{k=T}^{\infty} \beta^k \xi \left(\hat{\nu}_{it+k} - \hat{z}_{yit+k}\right) \right]. \end{aligned} \tag{B.42}$$

Assuming that $\tilde{\theta}_{it}$ and $\hat{p}_{xit}$ follow AR(1) processes with autocorrelation coefficients ρ_θ and ρ_p, equation (B.42) becomes

$$\begin{aligned}\pi_{it}^{tight} =& \left[\frac{1-\beta^T}{1-\beta}\kappa^{tight} + \frac{\beta^T}{1-\beta}\kappa\right] E_t \hat{\theta}_{it+\infty} + \left[\frac{1-\left(\beta\rho_\theta\right)^T}{1-\beta\rho_\theta}\kappa^{tight} + \frac{\left(\beta\rho_\theta\right)^T}{1-\beta\rho_\theta}\kappa\right] \tilde{\theta}_{it} \\ &+ \frac{\xi(1-\phi)}{1-\beta\rho_p}\hat{p}_{xit} + \varepsilon_{it}^{tight}, \end{aligned} \tag{B.43}$$

where $\varepsilon_{it}^{tight} = E_t \left[\sum_{k=0}^{T-1} \beta^k \xi \hat{\nu}_{it+k}^{tight} + \sum_{k=T}^{\infty} \beta^k \xi \hat{\nu}_{it+k} - \sum_{k=0}^{\infty} \beta^k \xi \hat{z}_{yit+k}\right]$. Manipulating the coefficients on

$E_t\hat{\theta}_{it+\infty}$ and $\tilde{\theta}_{it}$, I obtain

$$\begin{aligned}
\pi_{it}^{tight} =& \left[\frac{\kappa}{1-\beta} + \frac{1-\beta^T}{1-\beta}\left(\kappa^{tight}-\kappa\right)\right] E_t\hat{\theta}_{it+\infty} + \left[\frac{\kappa}{1-\beta\rho_\theta} + \frac{1-(\beta\rho_\theta)^T}{1-\beta\rho_\theta}\left(\kappa^{tight}-\kappa\right)\right]\tilde{\theta}_{it} \\
&+ \frac{\xi(1-\phi)}{1-\beta\rho_p}\hat{p}_{xit} + \varepsilon_{it}^{tight} \\
=& E_t\pi_{t+\infty} + \frac{1-\beta^T}{1-\beta}\lambda\kappa^{tight}E_t\hat{\theta}_{it+\infty} + \left[\psi_\theta^1 + \frac{1-(\beta\rho_\theta)^T}{1-\beta\rho_\theta}\lambda\kappa^{tight}\right]\tilde{\theta}_{it} + \psi_p\hat{p}_{xit} + \varepsilon_{it}^{tight} \\
=& E_t\pi_{t+\infty} + \Psi E_t\hat{\theta}_{it+\infty} + \left(\psi_\theta^1 + \psi_\theta^2\right)\tilde{\theta}_{it} + \psi_p\hat{p}_{xit} + \varepsilon_{it}^{tight},
\end{aligned} \tag{B.44}$$

where

- $\Psi = \frac{1-\beta^T}{1-\beta}\lambda\kappa^{tight}$
- $\psi_\theta^2 = \frac{1-(\beta\rho_\theta)^T}{1-\beta\rho_\theta}\lambda\kappa^{tight}$
- $\psi_p = \frac{\xi(1-\phi)}{1-\beta\rho_p}$

Putting together equations B.33 and B.44 the regional Phillips curve takes the following form:

$$\pi_{it} = \begin{cases} E_t\pi_{t+\infty} + \Psi E_t\hat{\theta}_{it+\infty} + \psi_\theta^{tight}\tilde{\theta}_{it} + \psi_p\hat{p}_{xit} + \varepsilon_{it}^{tight} & \tilde{\theta}_{it} > \tilde{\theta}_{it}^* \\ E_t\pi_{t+\infty} + \psi_\theta^{slack}\tilde{\theta}_{it} + \psi_p\hat{p}_{xit} + \varepsilon_{it} & \tilde{\theta}_{it} \leq \tilde{\theta}_{it}^* \end{cases}, \tag{B.45}$$

where

- $\psi_\theta^{slack} = \psi_\theta^1$
- $\psi_\theta^{tight} = \psi_\theta^1 + \psi_\theta^2$