# Dominant Currency Paradigm with Input-Output Linkages

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**Abstract**

This paper investigates the relationship between invoicing currency and input-output (I-O) linkages in global trade. Specifically, the paper explores how countries respond to dollar appreciation resulting from contractionary US monetary policy. I propose a theoretical and quantitative framework to analyze this impact, taking into account the exogenous currency of invoicing with I-O linkages. The model suggests that the response to dollar appreciation depends on the interaction between dollar invoicing shares and foreign intermediate input shares. To quantify the effect, a multi-country dynamic general equilibrium model is built with calibrated I-O linkages and invoicing shares. The quantitative results show that the expenditure switching of the calibrated model is muted in half compared to a model with full dollar invoicing. This research sheds light on the importance of invoicing currency and I-O linkages in global trade, and provides implications on the global monetary policy.

**Keywords:** Dominant Currency Paradigm, Input-Output Linkages, Global Trade

**JEL Codes:** E31, E42, F10, F31, F41

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# 1 Introduction

In international trade, countries commonly use different currencies for invoicing their transactions. The currency of invoicing is a critical factor in determining how exchange rate movements affect import prices and quantities of each country, particularly in a sticky price environment. Previous models in international macroeconomics have made various assumptions regarding invoicing currency. The first generation of these models, known as producer currency pricing (PCP), assumes that prices are sticky in the producer's currency.[1] The next generation assumes that prices are sticky in the importer's currency, also known as local currency pricing (LCP).[2] Recently, the literature has focused on the fact that global trade is invoiced in a few dominant currencies, such as the dollar and euro, i.e., dominant currency pricing (DCP).[3]

The impact of DCP on exchange rate pass-through to bilateral prices and quantities, and ultimately on global trade, has been a subject of interest among researchers. To address this, [18] Gopinath et al. (2020) propose the "Dominant Currency Paradigm" as a more comprehensive modeling approach, which includes dominant currency pricing, strategic complementarity in pricing, and the use of imported inputs in production. The authors assume that countries engage in roundabout production by combining domestic and foreign inputs to reduce the value-added content of exports. However, countries may be exposed differently to other countries through global input-output (I-O) linkages.

The paper examines how dollar appreciation affects global trade under DCP with I-O linkages. It presents a theoretical framework based on a static version of the sticky price open economy model proposed by [13] Farhi, Gopinath, and Itskhoki (2014). The model assumes an exogenous share of invoicing currency and features final and intermediate goods trade. The mechanism studied focuses on how dollar appreciation, through foreign intermediate inputs, increases marginal costs. The US is assumed to be exogenous, with contractionary US monetary policy shock causing dollar appreciation. The sticky prices cause markups to adjust, leading to a positive impact on global trade. The paper decomposes the effect of dollar appreciation into a *first-round negative effect* due to expenditure switching and a *second-round positive effect* due to markup adjustment, both amplified by I-O linkages. The interaction between intermediate input use intensity and the dollar invoicing share of the trade linkage determines the global trade response.

Meanwhile, researchers and policymakers are engaged in an active debate regarding the extent to which US monetary policy has externalities for other countries. The policy often

[1] [30] Mundell (1963), [14] Fleming (1962), [32] Obstfeld and Rogoff (1995).

[2] [5] Betts and Devereux (2000), [8] Chari et al. (2002), [11] Devereux and Engel (2003).

[3] [17] Gopinath (2015), [18] Gopinath et al. (2020), [29] Mukhin (2022).

leads to dollar appreciation, which spillovers through prices and quantities in trade, thereby affecting other countries' monetary policies. This can result in an inefficient allocation from a global planner's perspective under DCP. [18] Gopinath et al. (2020) provides empirical evidence that the DCP model is a benchmark framework for describing world trade.[4] However, there is a conflicting opinion among some researchers and policymakers who question whether DCP prices at the border are allocative, given that final goods prices tend to be sticky in local currency (LCP). This raises an important question for policymakers about whether world trade is closer to the DCP or LCP framework, as it will determine how monetary policy should respond to dollar appreciation and its effects on global trade.

The second half of this paper addresses the question of whether world trade is closer to DCP or LCP by using a quantitative macro model calibrated with data. Building on the DCP model proposed by [18] Gopinath et al. (2020), I extend the baseline model to a multi-country dynamic general equilibrium model that includes price stickiness, I-O linkages, and an exogenous share of invoicing currency. The model is calibrated with the World Input-Output Database (WIOD) and dollar invoicing shares of countries observed in the data. The calibrated model is capable of generating empirical patterns and moments in the data as it includes calibrated shock processes.

Based on the simulation results, the calibrated model suggests that the global trade response to dollar appreciation lies between the responses under a full DCP and full LCP model. Specifically, the calibrated model implies a trade invoicing mix that is *half LCP and half DCP*. This suggests that expenditure switching in response to dollar appreciation is muted in *half* compared to a model with full dollar invoicing. Also, I compare the response with the case when final goods trade is locally-priced (LCP) and intermediate goods trade is priced in the dollar (DCP). Moreover, the model indicates that final goods trade invoiced in DCP has a larger impact on global trade response than intermediate goods trade invoiced in DCP. This finding is consistent with the prediction of the baseline model.

To assess the impact of I-O linkages, the study conducts a counterfactual analysis by assuming that the importers are biased towards dollar invoicing exporters for both final and intermediate goods trade. The results indicate that the global trade response can be signif-

[4]Empirical evidence for the DCP model is provided through panel regression analysis of yearly bilateral trade values and exchange rate data. The authors run a regression of bilateral prices and volumes against the bilateral exchange rate and the dollar exchange rate interacted with dollar invoicing share, while controlling for fixed effects. The results show that the dollar exchange rate has more predictive power than the bilateral exchange rate. However, the authors note that these results are reduced-form evidence and require justification with a structural model.

To address this issue, the authors build a small open economy model that trades with the rest of the world and the US under a full DCP counterfactual scenario. They use Colombian custom-level data to match empirical facts and focus on matching exchange rate pass-through to prices, rather than quantities. The fact that Colombian trade is mostly invoiced in dollars makes it a suitable case study for this analysis.

icantly amplified, ranging from 1.5 to 1.8 times larger, as countries switch their expenditure towards dollarized trade. This implies that the I-O linkages play a crucial role in amplifying the impact of dollar appreciation on global trade, and the quantitative importance of the I-O linkages is significant.

**Related Literature** This paper contributes to three strands of literature. First, the paper is related to the recently growing literature on dominant currency pricing.[5] Earlier work by [16] Goldberg and Tille (2008) emphasizes a "coalescing" effect as an incentive for exporters to invoice in vehicle currency. [17] Gopinath (2015) provides empirical evidence of the dollar as a major invoicing currency in global trade and argues that exchange rate pass-through to prices depends on foreign currency invoicing share. A follow-up paper by [6] Boz et al. (2020) collects comprehensive data on invoicing shares from more than 100 countries and confirms the dominant role of the US dollars in global trade. While these works focus on an empirical analysis of how dollar appreciation affects import prices and volumes, this paper addresses the same question with a structural model calibrated by invoicing share data.

Recently, [29] Mukhin (2022) documents how the US dollar maintains its dominant role in global trade, using a general equilibrium model of endogenous invoicing currency choice. Also, [1] Amiti et al. (2022) shows that firms endogenously choose invoicing currency depending on firm-level characteristics and stresses the co-dominance of euros and dollar in Belgium firm-level data. Compared with these works on endogenous currency choice, I calibrate the model with given invoicing shares observed from the data.

The most related paper in this literature is [18] Gopinath et al. (2020), which provides empirical evidence for the dominance of the US dollar with country-level global trade flow data and Colombian detailed firm-product-level data. While they focus on matching moments with a small open economy model of Colombia and have a quantitative implication of intermediate input share, I construct a multi-country model describing the world economy and quantify the importance of I-O linkages in global trade.

The second strand of literature is about the New Keynesian framework in an open economy. [32] Obstfeld and Rogoff (1995) provides a two-country workhorse model with sticky price in PCP to consider how nominal rigidity creates a real effect in an open economy. Since then, extensive research has been conducted based on different assumptions on pricing and market imperfection. [28] Lane (2001) summarizes this literature on new open economy macroeconomics (NOEM) up to the early 2000s. [15] Gali and Monacelli (2005) builds a small open economy model with staggered price-setting à la Calvo to analyze optimal monetary policy under a PCP environment. Based on these seminal researches, this paper

[5] [19] Gopinath and Itskhoki (2022) provides a comprehensive overview of the DCP literature.

contributes to the literature by constructing a sticky price model that incorporates final and intermediate goods trade, and assumes an exogenous currency of invoicing.

The paper is closely related to [20] Huang and Liu (2007), which considers a two-country NOEM model with multiple stages of production and intermediate goods trade. They show that standard staggered price model with international trade in intermediate inputs improves the model performance in explaining empirical facts of international business cycles. While they focus on quantitative implications of intermediate goods trade on business cycle facts, the main focus of this paper is the positive implications on global trade. Also, the baseline model of this paper is based on [13] Farhi, Gopinath, and Itskhoki (2014), which analyzes the effect of fiscal instruments on exchange rate movement with a two-country model under PCP or LCP.

Lastly, the paper is related to the literature on global I-O linkages in an open economy. As intermediate goods trade dominates world trade due to the rise in the global value chain (GVC), researchers try to understand how vertical specialization affects world trade growth. [21] Hummels et al. (2001) argues that growth in the vertical specialization can explain 30% of countries' export growth, using the OECD I-O table. Similarly, [34] Yi (2003) studies the nonlinear effect of tariff reduction on global trade through vertical specialization. Likewise, [4] Bems et al. (2011) shows that the fall in intermediate goods trade accounts for a significant amount of the 2008-09 trade collapse. Recently, [31] Miyamoto and Nguyen (2022) develops a multi-country international business cycle model to show that changes in the global I-O linkages can explain around half of the realized drop in output volatility of all countries between 1970 and 2007.

The closest paper in this literature is [9] Cook and Patel (2022), which analyzes the effect of dollar appreciation on global trade under DCP and GVC with three country model. They show that final and intermediate goods trade respond differently depending on the type of shocks and value-added content of exports. This paper complements their work by focusing on the mechanism that the presence of intermediate goods trade can amplify or attenuate the global trade response to dollar appreciation depending on the relationship between I-O linkages and dollar invoicing shares.

The rest of the paper is organized as follows. Section 2 describes a two-country static model to derive the analytical formulation how dollar appreciation from an exogenous shock affects global trade prices and quantities. Section 3 extends the baseline model into the multi-country model calibrated by the data. The model provides quantitative results by counterfactual analysis to measure the size of expenditure switching in global trade. Section 4 concludes with normative implications of the paper.

## 2 Baseline Model

This section outlines the theoretical framework used to analyze the impact of dollar appreciation on global trade. The model is based on the work of [13] Farhi, Gopinath, and Itskhoki (2014), but without dynamics to facilitate analytical tractability. After introducing the model setup, I explain how key variables respond to shocks using relevant equations and assumptions. By combining these equations, I derive an analytical expression for how global trade value changes in response to a dollar appreciation shock in closed form. The model relies on several crucial elements, including exogenous invoicing shares, price stickiness, and I-O linkages.

In this section, I restrict my description to the Home country, but every notation is symmetric to ROW with an asterisk. To simplify the notation, I denote $dx$ as the log deviation of variable $X$ from its steady state.[6]

### 2.1 Environment

The baseline model is a static open economy consisting of two countries (non-US), the Home and Rest of the World (ROW), denoted as $H$ and $F$, respectively. The United States (US) is the third country, conducting monetary policy by controlling its money supply $M^{\$}$. The US dollar ($) is the dominant currency, and US monetary policy is exogenous to the Home and ROW by changing the dollar exchange rates of the two countries. The main focus of this section is on prices and quantities in international trade between Home and ROW.

In the model, a representative household in each country provides labor and consumes final goods with their labor income. Additionally, producers in each country purchase inputs from domestic intermediate goods producers and foreign intermediate goods producers, and use them together with labor to produce goods. These goods can be either consumed by households as final goods or used by producers as intermediate inputs.

**Household** In the Home country, a representative household consumes the domestic final good $C_H$ and imports foreign final good $C_F$ (Home import of final good) and supplies $L$ units of labor for a nominal wage of $W$. Households solve the following maximization problem

$$\max_{C_H, C_F, L, B'(s)} U(C_H, C_F, L) = \frac{1}{1-\sigma} C^{1-\sigma} - \frac{1}{1+\varphi} L^{1+\varphi} \tag{1}$$

[6] The main analysis in this section focuses on the first-order approximation of a log-linearized model around an efficient equilibrium.

subject to

$$P_H C_H + P_F C_F + \sum_{s \in S} Q(s) B'(s) = WL + B \tag{2}$$

where $C_H$ and $C_F$ are aggregated via a constant elasticity of substitution (CES) function with elasticity $\varepsilon$ and home bias $1-\gamma$

$$C = \left( (1-\gamma)^{\frac{1}{\varepsilon}} C_H^{\frac{\varepsilon-1}{\varepsilon}} + \gamma^{\frac{1}{\varepsilon}} C_F^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

In addition, households have access to a financial market via a complete set of Arrow-Debreu securities (in local currency) on each state $s \in S$ with asset holdings $B$ at the current period and determine asset holdings $B'(s)$ of each state $s \in S$ for the next period. $Q(s)$ is the price of the security that pays one unit of local currency in the next period's state $s$.

I assume that households face the *cash-in-advance* constraint[7]

$$PC \leq M$$

where $P$ is an aggregate price index of aggregate consumption $C$. This condition characterizes aggregate demand of the economy by Home money supply $M$. Similarly, in ROW,

$$P^* C^* = M^*, \text{ where } C^* = \left( (1-\gamma)^{\frac{1}{\varepsilon}} C_F^{*\frac{\varepsilon-1}{\varepsilon}} + \gamma^{\frac{1}{\varepsilon}} C_H^{*\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $C_F^*$ and $C_H^*$ (Home export of final good) are ROW final goods demands on own good and Home good.[8]

**Production** Each firm in the Home country produces unique variety $\omega$ by using labor $L$ and composite intermediate input $X$ by CES production function with elasticity $\rho$

$$Y = A \left( \alpha^{\frac{1}{\rho}} L^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} X^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{3}$$

where $A$ is Hicks-neutral technology shock, $\alpha$ is labor share.[9] In addition, composite intermediate input $X$ is assembled by domestic intermediate input $X_H$ and foreign intermediate

[7] [25] Kehoe and Midrigan (2008) also considers a sticky price open economy model in which total purchase of the households satisfies the cash-in-advance constraint. In their model, the household budget constraint includes money holdings of the adjacent periods and lump-sum transfer of the money supply difference rebated back to the household. To simplify notation and suppress time subscripts, I use a version of (ex-post) budget constraint (2) without money holdings. [22] Itskhoki (2021) reviews a model under this constraint to derive a simple equilibrium solution of the exchange rate.

[8] For simplicity, Home and ROW households have the identical home bias of $1-\gamma$. It is straightforward to generalize the model with heterogeneous home bias.

[9] The variety index $\omega$ is omitted here for notation simplicity.

input $X_F$ (Home import of intermediate good) via CES with elasticity $\theta$ and foreign intermediate input share $\phi$.

$$X = \left((1-\phi)^{\frac{1}{\theta}} X_H^{\frac{\theta-1}{\theta}} + \phi^{\frac{1}{\theta}} X_F^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{4}$$

Similarly, in ROW,

$$Y^* = A^* \left(\alpha^{*\frac{1}{\rho}} L^{*\frac{\rho-1}{\rho}} + (1-\alpha^*)^{\frac{1}{\rho}} X^{*\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{5}$$

$$X^* = \left((1-\phi^*)^{\frac{1}{\theta}} (X_F^*)^{\frac{\theta-1}{\theta}} + \phi^{*\frac{1}{\theta}} (X_H^*)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{6}$$

where $X_F^*$ and $X_H^*$ (Home export of intermediate good) are ROW intermediate goods demands on its good and Home good. Notice here that the ROW production has labor share $\alpha^*$ and foreign intermediate input share $\phi^*$, different from $\alpha$ and $\phi$ of the Home production.

**Exchange rates and Dollar appreciation** Let $\mathcal{E}$ be the bilateral exchange rate between Home and ROW currency, or the price of a ROW currency in a unit of Home currency. Also, $\mathcal{E}_{\$i}$ is the dollar exchange rate of country $i \in \{H, F\}$, or the price of a dollar in a unit of currency of country $i$. Both increases in $\mathcal{E}$ and $\mathcal{E}_{\$H}$ correspond to a depreciation of Home currency relative to ROW currency and the US dollar. In addition, both increases in $\mathcal{E}_{\$H}$ and $\mathcal{E}_{\$F}$ mean a dollar appreciation against Home and ROW currency. By definition, the identity below should satisfy.

$$\mathcal{E} = \frac{\mathcal{E}_{\$H}}{\mathcal{E}_{\$F}} \tag{7}$$

To model dollar appreciation, I assume that dollar exchange rates respond to an exogenous change in US money supply $M^{\$}$. Let $de_{\$i}/dm^{\$}$ be the first-order change in the dollar exchange rate of country $i$ with respect to exogenous US money supply shock $dm^{\$}$. As the dollar appreciates against the weighted average of Home and ROW currency,

$$\omega^{\$} \frac{de_{\$H}}{dm^{\$}} + (1-\omega^{\$}) \frac{de_{\$F}}{dm^{\$}} = -1 \tag{8}$$

where $\omega^{\$}$ (or $1-\omega^{\$}$) is a trade share between the US and Home (or ROW) country.

**Currency of invoicing and Sticky price** All domestic prices are sticky in their currency, whereas international prices are sticky in either producer currency (PCP), local currency (LCP), or dominant currency (DCP). Following [33] Rubbo (2022), I assume $\delta \in [0,1]$ fraction of firms can adjust their prices after observing shocks to model price stickiness in a static setting. Then, the (log) change in Home domestic prices is

$$dp_H = dp_{HX} = \delta dmc \tag{9}$$

where $dmc$ is the log deviation of the marginal cost of Home production. Similarly, change in ROW domestic prices $\{dp^*_F, dp^*_{FX}\}$ is proportional to $dmc^*$.

I denote $\{\theta^C_P,\ \theta^C_L,\ \theta^C_D\}$ the fractions of Home exports of final goods invoiced in Home currency (PCP), ROW currency (LCP), and dominant currency (DCP), respectively, where three exogenous invoicing shares add up to 1.[10] The notation is similar to the intermediate goods and ROW exports. Denote $\theta^X_k$ ($k \in \{P, L, D\}$) the fraction of Home intermediate goods exports in each pricing paradigm. Similarly, $\theta^{C*}_k$ ($\theta^{X*}_k$) is the fraction of ROW exports of final goods (intermediate goods) in each pricing paradigm.[11]

Since prices are sticky in the currency of invoicing, the (log) change in Home export prices of final goods under PCP, LCP, and DCP in a unit of ROW currency are

$$dp^*_{H,P} = \delta dmc - de \tag{10}$$

$$dp^*_{H,L} = \delta(dmc - de) \tag{11}$$

$$dp^*_{H,D} = \delta(dmc - de_{\$H}) + de_{\$F}. \tag{12}$$

I define *aggregate* Home export price of final good $P^*_H$ to be a Cobb-Douglas aggregate of the three invoiced prices with corresponding invoicing shares as weights: $P^*_H = (P^*_{H,P})^{\theta^C_P}(P^*_{H,L})^{\theta^C_L}(P^*_{H,D})^{\theta^C_D}$. Then, the (log) change in aggregated Home export price of final good is

$$dp^*_H = \theta^C_P dp^*_{H,P} + \theta^C_L dp^*_{H,L} + \theta^C_D dp^*_{H,D} \tag{13}$$

Other aggregate international prices $\{P^*_{HX}, P_F, P_{FX}\}$ are similarly defined as above with corresponding invoicing shares.

**Markups** Markups for domestic prices are prices divided by marginal costs. From equation (9), the (log) change in markups for Home domestic prices is

$$d\mu_H = d\mu_{HX} = (\delta - 1)dmc \tag{14}$$

[10]Notice that subscripts $\{P, L, D\}$ stand for PCP, LCP, and DCP.

[11]Therefore, superscripts $\{C, X, C*, X*\}$ stand for Home exports of final goods, Home exports of intermediate goods, ROW exports of final goods, and ROW exports of intermediate goods, respectively.

Since $\delta - 1 < 0$, a positive productivity shock reduces marginal cost and increases markups of domestic prices. In other words, the response of domestic markups corresponds to sluggish response of domestic prices minus response of marginal cost.

Markups for international prices are prices divided by marginal costs, both of which are denominated in the currency in which they are invoiced. From equation (10), (11) and (12), the (log) change in markups for Home export prices of final goods under PCP, LCP and DCP is

$$d\mu^*_{H,P} = (dp^*_{H,P} + de) - dmc = (\delta - 1)dmc \tag{15}$$

$$d\mu^*_{H,L} = dp^*_{H,L} - (dmc - de) = (\delta - 1)(dmc - de) \tag{16}$$

$$d\mu^*_{H,D} = (dp^*_{H,D} - de_{\$F}) - (dmc - de_{\$H}) = (\delta - 1)(dmc - de_{\$H}). \tag{17}$$

Lastly, I define *aggregate* markup of international prices as aggregate price divided by marginal cost, both of which are denominated in the importing country's currency. Combining equations (10)-(13) with equations (15)-(17),

$$d\mu^*_H = \theta^C_P d\mu^*_{H,P} + \theta^C_L d\mu^*_{H,L} + \theta^C_D d\mu^*_{H,D} \tag{18}$$

Compared with domestic prices, aggregate markups of international prices depend not only on the change in marginal costs but also on the bilateral exchange rate and dollar exchange rates. Therefore, when the dollar appreciates, the dollar-denominated marginal cost decreases while the dollar-denominated price is sticky, so aggregate markups increase by DCP invoicing share.

**Market Clearing** The Home output $Y$ can be either consumed by the Home or ROW household ($C_H$ and $C^*_H$) or used as intermediate inputs by the Home or ROW producer ($X_H$ or $X^*_H$). The market clearing condition is symmetric to ROW output $Y^*$.

$$Y = C_H + C^*_H + X_H + X^*_H \tag{19}$$

$$Y^* = C^*_F + C_F + X^*_F + X_F \tag{20}$$

**Equilibrium** Given exogenous shocks $\{da, dm, da^*, dm^*, dm^\$\}$, in each country $i \in \{H, F\}$, an equilibrium is defined by the following:

- Households maximize utility over domestic and foreign consumption, labor, and asset holdings

- Producers maximize profit over labor, and domestic and foreign intermediate inputs, taking aggregate prices as given
- Goods and labor markets clear

Model equations, steady state, and log-linearized equations from the equilibrium conditions are described in the Model Appendix.

## 2.2 Main Assumptions

The main objective of this section is to understand how global trade (except the US) responds to dollar appreciation. For analytical tractability, I employ several assumptions in the model.

**Assumption 1** *Utility is log in consumption ($\sigma = 1$, i.e. $U(C_H, C_F, L) = \log C - \frac{1}{1+\varphi}L^{1+\varphi}$)*

**Assumption 2** *Utility is linear in labor supply ($\varphi = 0$, i.e., $U(C_H, C_F, L) = \frac{1}{1-\sigma}C^{1-\sigma} - L$)*

**Assumption 3** *The elasticity between labor and intermediate inputs is unitary ($\rho = 1$)*

**Assumption 4** *Elasticities between Home and ROW goods are unitary ($\varepsilon = \theta = 1$)*

These assumptions can simplify the framework without lossing the main message of the model. For example, from a complete market, the Home and ROW Euler equations imply the perfect risk-sharing condition or Backus-Smith condition, i.e., $(C/C^*)^\sigma = Q = \mathcal{E}P^*/P$. Together with Assumption 1 and case-in-advance constraint, the bilateral exchange rate is simply a ratio of Home and ROW money supply.

$$\mathcal{E} = \frac{M}{M^*} \tag{21}$$

Together with equation (7) and (8), dollar exchange rates respond identically to US money supply shock. For example, if US money supply shrinks by $dm^\$ < 0$, dollar exchange rates of Home and ROW increase by $-dm^\$ > 0$, i.e.,

$$de_{\$H} = de_{\$F} = -dm^\$ > 0.$$

On the other hand, if Assumption 3 and 4 are adopted i.e., Cobb-Douglas production, final goods trade value $C_R$ and intermediate goods trade value $X_R$ between Home and ROW in unit of Home currency are

$$C_R = P_F C_F + \mathcal{E}P_H^* C_H^* = \gamma PC + \mathcal{E}\gamma P^* C^* \tag{22}$$

$$X_R = P_{FX}X_F + \mathcal{E}P^*_{HX}X^*_H = (1-\alpha)\phi MC \cdot Y + (1-\alpha^*)\phi^* \mathcal{E} MC^* \cdot Y^* \tag{23}$$

From equation (21) and cash-in-advance constraint, trade value of final goods $C_R$ only depends on $M$ (or $M^*$ when it is denominated in ROW currency). By contrast, trade value of intermediate goods $X_R$ depends on marginal costs $\{MC, MC^*\}$ and outputs $\{Y, Y^*\}$.

For the rest of the section, Assumption 1 and 3 are adopted as they are indispensable for analytical tractability, while Assumption 2 and 4 are adopted as needed, without loss of generality. Model Appendix describes the most general case with proofs. The next subsection describes how marginal costs and outputs respond to the US money supply shock.

## 2.3 Main Equations

The first Lemma shows how marginal costs respond to shocks on impact. This marginal cost equation can also be called the *forward equation*, describing how shocks propagate downstream and affect prices.

**Lemma 1** *(Marginal cost equation) Under Assumption 1, the log change of marginal costs of Home and ROW production satisfy*

$$\begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \left( \begin{bmatrix} dv \\ dv^* \end{bmatrix} - \begin{bmatrix} da \\ da^* \end{bmatrix} \right) + \begin{bmatrix} dm \\ dm^* \end{bmatrix} + \begin{bmatrix} \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{11} & \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{12} \\ \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21} & \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} dy \\ dy^* \end{bmatrix} \tag{24}$$

*where*

$$\begin{bmatrix} dv \\ dv^* \end{bmatrix} = \begin{bmatrix} (1-\alpha)(1-\phi)d\mu_{HX} + (1-\alpha)\phi d\mu_{FX} \\ (1-\alpha^*)(1-\phi^*)d\mu^*_{FX} + (1-\alpha^*)\phi^* d\mu^*_{HX} \end{bmatrix}$$

$$\tilde{\Psi} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} = \frac{1}{\tilde{D}} \begin{bmatrix} \frac{\alpha^*}{1+\varphi} + (1-\alpha^*)\phi^* & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & \frac{\alpha}{1+\varphi} + (1-\alpha)\phi \end{bmatrix}$$

Home marginal cost in this economy moves for three reasons: monetary policy shock $dm$, productivity shock $da$, and the change in markups $dv$. First, when the Home money supply increases, Home currency depreciates due to equation (21). Therefore, foreign intermediate input becomes expensive and the Home marginal cost rises. Second, a productivity shock directly affects its own marginal cost. Lastly, the marginal costs depend on markups of intermediate goods prices. Let $v$ (or $v^*$) be a weighted sum of the (log) markups faced by Home (or ROW) producers. Then, $dv$ acts like a negative productivity shock ($-da$) on Home production. The main difference between $dv$ and $-da$ is that $dv$ is determined endogenously by the markup equation described below, while $-da$ is exogenous.

The last term in equation (24) represents the equilibrium wage response from labor market clearing. As labor supply curve is upward-sloping and the labor demands depend on the outputs, the output response $dy$ can affect marginal cost through the wage response. One more thing that affects Home marginal cost is ROW marginal cost, and vice versa, in the presence of I-O linkages. Matrix $\tilde{\Psi}$ is the Leontief inverse in this regard, where each element $\tilde{\psi}_{ij}$ encodes the direct and indirect response of $i$'s marginal cost to $j$'s marginal cost. In other words, $\tilde{\psi}_{ij}$ is country $i$'s direct and indirect reliance on country $j$'s input.

Under flexible prices, the marginal costs are proportional to their own money supplies, reflecting the idea of *monetary neutrality*. For example, when the Home money supply increases by $dm > 0$, Home nominal wage rises proportionally ($dw = \frac{1}{1+\varphi} dm$) due to upward-sloping labor supply curve. Also, the imported input price from ROW rises ($dp_{FX} = dm$) due to Home currency depreciation. Since labor share is $\alpha$ and foreign input share in Home production is $(1-\alpha)\phi$, Home marginal cost directly increases by $\left(\frac{\alpha}{1+\varphi} + (1-\alpha)\phi\right) dm$. Since ROW production uses Home intermediate goods, increase in Home marginal cost spills over to ROW marginal cost by foreign input share $(1-\alpha^*)\phi^*$ in ROW production. This indirect effect amplifies the effect on marginal costs. Therefore, increase in Home marginal cost equals Home money supply shock ($dmc = dm$) under flexible prices. When all nominal variables are deflated by the level of money supplies, they depend only on the productivity shocks. Later, Lemma 4 shows that Home and ROW outputs stay constant in response to Home and ROW monetary shocks under flexible prices.

The key observation in Lemma 1 is that the invoicing shares do not appear in the marginal cost equation. Here, marginal costs are partial equilibrium objects because $\upsilon$ and $\upsilon^*$ are endogenous. Since prices are sticky in their currencies of invoicing, the change in markups $d\upsilon$ and $d\upsilon^*$ depend on the invoicing shares.

When Assumption 2 is adopted that there is no equilibrium wage response, the last term in equation (24) drops and Leontief inverse $\tilde{\Psi}$ changes to $\Psi$.

**Corollary 1** *Under Assumption 1 and 2, the log change of marginal costs of Home and ROW production satisfy*

$$\begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \left( \begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix} - \begin{bmatrix} da \\ da^* \end{bmatrix} \right) + \begin{bmatrix} dm \\ dm^* \end{bmatrix} \tag{25}$$

*where*

$$\Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} = \frac{1}{D} \begin{bmatrix} \alpha^* + (1-\alpha^*)\phi^* & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & \alpha + (1-\alpha)\phi \end{bmatrix}$$

*and* $D = \alpha\alpha^* + \alpha(1-\alpha^*)\phi^* + \alpha^*(1-\alpha)\phi$ *is some normalizing constant.*

The next Lemma describes how the markups respond to shocks and marginal costs.

**Lemma 2** *(Markup equation) The log change in (aggregate) markups of international prices satisfy*[12]

$$\begin{bmatrix} d\mu_F \\ d\mu_H^* \\ d\mu_{FX} \\ d\mu_{HX}^* \end{bmatrix} = -(1-\delta) \begin{bmatrix} dmc^* + \theta_L^{C*} de - \theta_D^{C*} de_{\$F} \\ dmc - \theta_L^{C} de - \theta_D^{C} de_{\$H} \\ dmc^* + \theta_L^{X*} de - \theta_D^{X*} de_{\$F} \\ dmc - \theta_L^{X} de - \theta_D^{X} de_{\$H} \end{bmatrix} \tag{26}$$

The markups of international prices depend on the change in marginal costs and exchange rates, while markups of domestic prices depend only on marginal costs.[13] In particular, markup responses are asymmetric, depending on which shocks are considered. For example, when Home currency depreciates ($de > 0$), the marginal cost of ROW production in unit of Home currency rises, while LCP prices partially adjust due to sticky prices. Therefore, markups of Home import prices decrease by corresponding LCP invoicing shares. On the other hand, the marginal cost of Home production in unit of ROW currency falls. Hence, markups of ROW import prices increase by corresponding LCP invoicing shares. When the dollar universally appreciates ($de_{\$H}, de_{\$F} > 0$), both marginal costs of Home and ROW productions in unit of the dollar become cheaper. Hence, markups of all international prices increase by DCP invoicing shares.

While equation (24) describes how marginal costs depend on markups, equation (26) shows how the change in markups is driven by the change in marginal costs. These two equations are a linear system of equations given exogenous money supplies that determine exchange rates, and the output responses with endogenous labor supply. Thus, one can solve for change in markups and marginal costs as a function of exogenous shocks and the output responses by combining equations (24) and (26). Since the main interest is the effect of universal dollar appreciation against Home and ROW currency, I focus on the case of $dm = dm^* = 0$, and $dm^\$ < 0$, so that the bilateral exchange rate between Home and ROW stays constant ($de = 0$) and dollar exchange rates of Home and ROW universally increase ($de_{\$H} = de_{\$F} = -dm^\$ > 0$). In addition, fully sticky price ($\delta = 0$) is assumed for simplicity.[14]

**Lemma 3** *(Markup response) Consider a contractionary US monetary shock $dm^\$ < 0$. Un-*

[12] Equation (26) can be derived by combining equations (15)-(18).

[13] From equation (14), the log change in markups of Home domestic prices is $d\mu_H = d\mu_{HX} = -(1-\delta)dmc$. Similarly, $d\mu_F^* = d\mu_{FX}^* = -(1-\delta)dmc^*$ for ROW domestic prices.

[14] Model Appendix describes the markup responses with partially sticky price case under Assumption 2.

*der Assumption 2 and fully sticky prices, markup responses of international prices are*[15]

$$\begin{bmatrix} d\mu_F \\ d\mu_H^* \\ d\mu_{FX} \\ d\mu_{HX}^* \end{bmatrix} = \begin{bmatrix} -\theta_D^{C*} dm^\$ + (1-\alpha^*)\phi^*\theta_D^X dm^\$ \\ -\theta_D^C dm^\$ + (1-\alpha)\phi\theta_D^{X*} dm^\$ \\ -\theta_D^{X*} dm^\$ + (1-\alpha^*)\phi^*\theta_D^X dm^\$ \\ -\theta_D^X dm^\$ + (1-\alpha)\phi\theta_D^{X*} dm^\$ \end{bmatrix} \tag{27}$$

Suppose that the dollar appreciates by a contractionary US monetary shock. Then, the marginal costs in a unit of the dollar decrease, while dollar-denominated prices are sticky under DCP. Therefore, markups for international prices mechanically increase by the DCP invoicing shares as in the first terms of RHS in equation (27). This is the *first-round effect* of mechanical increase in markups due to sticky prices.

As markups of foreign intermediate goods rise, they affect marginal costs through equation (24). Home marginal cost is affected by the markup of foreign intermediate input $\mu_{FX}$ by input share $(1-\alpha)\phi$ and further amplified by $\psi_{11}$. It is also affected by ROW marginal cost, similarly depending on $\mu_{HX}^*$ by $(1-\alpha^*)\phi^*$ and amplified by $\psi_{12}$. However, an increase in marginal costs leads to a reduction in markups under sticky prices, which attenuates the increase in marginal costs. This amplification effect of I-O linkages and attenuation effect of markup reduction on marginal costs are perfectly offset under a fully sticky price. Therefore, marginal costs increase and markups decrease by $(1-\alpha)\phi\theta_D$ as in equation (27). This is the *second-round effect* of a decrease in markups due to the I-O linkages and sticky prices. Since the two effects work in opposite directions, the overall effect on global trade depends on the relative size of the foreign intermediate input shares and DCP invoicing shares.

For example, suppose that all final goods trade is fully invoiced in local currency, i.e., LCP ($\theta_L^C = \theta_L^{C*} = 1$), and intermediate goods trade is in the dollar, i.e., DCP ($\theta_D^X = \theta_D^{X*} = 1$). As the dollar appreciates, the first-round effect implies that markups of intermediate goods prices increase, while markups of final goods prices stay constant. This is because mechanical expenditure switching happens only at the border prices. However, as the marginal costs depend on the markups of foreign intermediate goods, markups of all prices decrease by how intensive foreign inputs are used in production by the second-round effect. If Home production relies more on ROW inputs, the effect on markups of Home goods becomes larger. As markups of final goods prices are subject only to the second-round effect, while markups of intermediate goods prices are subject to both effects, markups of final (intermediate) goods prices decrease (increase). Therefore, the overall effect on global trade is ambiguous.

Lastly, the Lemma below shows how outputs respond to shocks.[16] This output equation

[15]For domestic prices, $d\mu_H = d\mu_{HX} = (1-\alpha)\phi\theta_D^{X*} dm^\$$ and $d\mu_F^* = d\mu_{FX}^* = (1-\alpha^*)\phi^*\theta_D^X dm^\$$

[16]Here, I denote *output* to be (real) gross output including final and intermediate goods consumption.

can also be called a *backward equation*, describing how shocks propagate upstream and affect demands and outputs.

**Lemma 4** *(Output equation) Under Assumption 2 and 4,*[17] *the log change of Home and ROW output satisfy*

$$\begin{bmatrix} dy \\ dy^* \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{21}\frac{\bar{Y}^*}{\bar{Y}} \\ \psi_{12}\frac{\bar{Y}}{\bar{Y}^*} & \psi_{22} \end{bmatrix} \begin{bmatrix} du \\ du^* \end{bmatrix} - \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} dv - da \\ dv^* - da^* \end{bmatrix} \tag{28}$$

*where*

$$\begin{bmatrix} du \\ du^* \end{bmatrix} = - \begin{bmatrix} \frac{1-\gamma}{\bar{Y}} d\mu_H + \frac{\gamma}{\bar{Y}} d\mu_H^* + (1-\alpha)(1-\phi) d\mu_{HX} + (1-\alpha^*)\phi^* \frac{\bar{Y}^*}{\bar{Y}} d\mu_{HX}^* \\ \frac{1-\gamma}{\bar{Y}^*} d\mu_F^* + \frac{\gamma}{\bar{Y}^*} d\mu_F + (1-\alpha^*)(1-\phi^*) d\mu_{FX}^* + (1-\alpha)\phi \frac{\bar{Y}}{\bar{Y}^*} d\mu_{FX} \end{bmatrix}$$

As discussed, the outputs stay constant in response to Home and ROW monetary shocks when all markups are constant under flexible prices. Since all demands of final and intermediate goods are Cobb-Douglas under Assumption 4, nominal final expenditures and total costs of production are deflated by prices, or marginal costs under flexible prices. Therefore, Home and ROW monetary shocks have no real effect.

In general, output falls when markups increase, not only markups of their own products but also foreign products. Here $du$ (or $du^*$) is a weighted sum of the (log) markups of Home (or ROW) producers with weights on steady-state relative demands.[18] As markups of Home products increase, demand for Home products shrinks, so Home output falls. Like the marginal costs, ROW output can also affect Home output through foreign intermediate input demand, i.e., I-O linkages. Therefore, the adjusted Leontief inverse[19] is pre-multiplied with the markup vector in the first term of RHS in equation (28). The second term corresponds to the marginal cost component in prices, which is also negatively related to demands through prices.

Lemma 4 is a special case of inefficient economy in [3] Baqaee and Farhi (2020) with endogenous wedges coming from sticky prices. They provide a general framework of a CES economy with a representative consumer and producers characterized by a single elasticity of substitution less or bigger than 1. While Lemma 4 abstracts from imperfect substitution by assuming a Cobb-Douglas demand structure, endogenous markup responses from dollar appreciation have a real effect on outputs. Denote $\tilde{\Omega}$ and $\Omega$ as cost-based and revenue-based

[17] General case by relaxing these assumptions is described in the Model Appendix

[18] For example, $\frac{1-\gamma}{\bar{Y}} = \frac{\bar{C}_H}{\bar{Y}}$ and $(1-\alpha^*)\phi^* = \frac{\bar{X}_H^*}{\bar{Y}}$.

[19] This adjusted Leontief inverse matrix is slightly different from $\Psi$ introduced in Lemma 1 in two ways. First, it is transposed because markups affect downstream demand, not upstream. Second, off-diagonal elements are multiplied by relative output to keep track of the log changes in outputs.

input-output matrices, as in [3] Baqaee and Farhi (2020). Since $\tilde{\Omega}$ is constant from Assumption 4, a markup increase of downstream producer $i$ ($d\mu_i > 0$) causes less expenditure on upstream inputs $j$ ($d\Omega_{ij} = -\Omega_{ij} d\mu_i < 0$), where initial expenditure share $\Omega_{ij}$ corresponds to the weight on each markup in $du$ and $du^*$. Therefore, output falls for each upstream producer. The last part of this section discusses the general case under a CES economy.

## 2.4 Global Trade Response

Armed with the above Lemmas, I analyze how dollar appreciation affects global trade value in the first order. Recall that universal dollar appreciation ($de_{\$H}, de_{\$F} > 0$) is characterized by a reduction in US money supply ($dm^{\$} < 0$). Therefore, the main objective is to derive the elasticity of global trade value $X_R$ with respect to the US money supply $M^{\$}$.[20] By log-linearizing equation (23),

$$dx_R = w(dmc + dy) + (1 - w)(dmc^* + dy^*) \tag{29}$$

where $w$ is the intermediate goods trade share from Home to ROW. From the Lemmas, $dm^{\$}$ does not directly affect marginal costs and outputs, but indirectly through markups. In other words, by chain rule,

$$\frac{dmc}{dm^{\$}} = \frac{dmc}{d\mathcal{M}} \frac{d\mathcal{M}}{dm^{\$}}$$

where $d\mathcal{M}$ is a vector of the log change in all markups. It it possible to obtain $\frac{dmc}{d\mathcal{M}}$ from Lemma 1, and $\frac{d\mathcal{M}}{dm^{\$}}$ from Lemma 3. Applying the chain rule for other variables, below is the main result of this section.

**Proposition 1** *(Global trade response)*

$$\frac{dx_R}{dm^{\$}} = \left(\frac{w}{\bar{Y}}\psi_{11} + \frac{1-w}{\bar{Y}^*}\psi_{12}\right)\left\{\underbrace{\gamma\theta_D^C + (1-\alpha^*)\phi^*\bar{Y}^*\theta_D^X}_{\text{1st round } (H \to F)} - \underbrace{(1-\alpha)\phi\bar{Y}\theta_D^{X*}}_{\text{2nd round } (dy)}\right\}$$
$$+ \left(\frac{w}{\bar{Y}}\psi_{21} + \frac{1-w}{\bar{Y}^*}\psi_{22}\right)\left\{\underbrace{\gamma\theta_D^{C*} + (1-\alpha)\phi\bar{Y}\theta_D^{X*}}_{\text{1st round } (F \to H)} - \underbrace{(1-\alpha^*)\phi^*\bar{Y}^*\theta_D^X}_{\text{2nd round } (dy^*)}\right\}$$

Conventionally, it is expected $\frac{dx_R}{dm^{\$}}$ to be positive. When the dollar appreciates ($dm^{\$} < 0$), bilateral import prices increase under DCP so that expenditure switches toward domestic goods. Therefore, global trade shrinks ($dx_R < 0$).

[20] Since trade value of final goods $C_R$ is only a function of Home money supply, I focus on trade value of intermediate goods $X_R$, a function of marginal costs and outputs. Mathematically, the elasticity is $\frac{dx_R}{dm^{\$}}$.

The first two terms in curly brackets correspond to the *first-round effect* of increase in markups interacting with direct exposure on imported goods. As markups of international prices increase mechanically by DCP invoicing shares $\theta_D$, import demands shrink, and output falls to the extent that consumers or producers are exposed to imported goods. For example, as $\theta_D^C$ fraction of the Home final goods exports are invoiced in the dollar, ROW consumers with home bias $1-\gamma$ are directly exposed to dollar appreciation by $\gamma\theta_D^C$. Similarly, ROW producers with foreign input share $(1-\alpha^*)\phi^*$ and output $\bar{Y}^*$ are exposed to dollar appreciation by $(1-\alpha^*)\phi^*\bar{Y}^*\theta_D^X$. While the first line in Proposition 1 is about the change in global trade from Home to ROW, the second line is one from ROW to Home. Overall, this standard expenditure-switching mechanism shrinks global trade after dollar appreciation.

On the other hand, the last term in curly brackets is about the *second-round effect* of a reduction in all markups that boosts output and global trade. Since all markups decline in response to a rise in marginal costs, changes in markups are identical within countries as in Lemma 3. Therefore, all demands for goods and outputs rise proportionally from market clearing conditions. While the first line in Proposition 1 is related to Home output $dy$, the second line is about ROW output $dy^*$. This effect offsets the standard expenditure switching, where the key mechanism comes from the I-O linkages and sticky prices.

Lastly, each term outside the curly bracket amplifies the effect through the I-O linkages. While the terms inside the curly bracket of the first (or second) line are equal to the change in Home (or ROW) output, the output of one country affects the output of another country through foreign intermediate input demands. $\psi_{ij}$ can be interpreted as the direct and indirect response of a change in markups of country $i$'s prices on country $j$'s output, multiplied by initial trade shares[21] and divided by initial output to keep track of the log changes.

Proposition 1 shows that interaction between the currency of invoicing and the I-O linkages is a key mechanism to understanding global trade response to dollar appreciation. From the Home importers' point of view, not only how they are exposed to foreign goods but also how much foreign exporters invoice their goods in the dollar matters. In addition, the I-O linkages create another second-round effect that affects every producer's marginal costs, which can positively impact global trade. The examples below demonstrate this mechanism under simplifying assumptions.

**Simple examples** First, it is evident that global trade is not responsive to dollar appreciation when each country invoices prices in their currency or opponent's. This case is a convex combination of PCP and LCP environment, while $\theta_D^i = 0$ for all $i \in \{C, X, C*, X*\}$.

On the other extreme, suppose that all trades are invoiced only in the dollar ($\theta_D^i = 1$),

[21] $w$ for Home output and $1-w$ for ROW output

and Home and ROW are symmetric ($\alpha = \alpha^*$, $\phi = \phi^*$). Then, Proposition 1 implies that $dx_R = \gamma dm^\$$. In other words, global trade is more responsive to dollar appreciation when final goods trade is more open (higher $\gamma$). The response of marginal costs depends on whether prices of intermediate goods are invoiced in the dollar. Therefore, marginal costs increase by foreign input share $(1-\alpha)\phi$ under a full DCP environment. On the other hand, export demands of final and intermediate goods shrink due to a rise in marginal costs. From market clearing conditions, outputs decrease by $(1-\alpha)\phi+\gamma$. In aggregation by equation (29), global trade value reduces by $\gamma$.

Now suppose that final goods trade is in LCP ($\theta_L^C = \theta_L^{C*} = 1$) and intermediate goods trade is in DCP ($\theta_D^X = \theta_D^{X*} = 1$). The response of marginal costs is the same as in the full DCP case because intermediate goods trade is still invoiced in the dollar. Nevertheless, export demands of final goods are not responsive because prices of final goods are sticky in local currency. Therefore, outputs solely move with intermediate goods demands by $(1-\alpha)\phi$. Since the rise in marginal costs is offset by a reduction in outputs, global trade value stays constant after dollar appreciation. Table 1 summarizes the results in two simple examples.

These two examples support the earlier works arguing that dollar appreciation leads to a reduction in global trade, depending on the currency in which final goods trade is invoiced. If both final and intermediate goods trades are in DCP, global trade shrinks in quantity and value, as in [18] Gopinath et al. (2020). However, as final goods trade becomes LCP, global trade shrinks in quantity but not in value. So it is important to correctly calibrate invoicing shares in a trade and to determine whether it is a final or intermediate goods trade. The next section revisits this question using a quantitative model calibrated by the invoicing currency data.

| Environment: | | | | Global trade response in log of: | | |
|---|---|---|---|---|---|---|
| Final | Int. | Price | Utility | Value ($P \times Q$) | Price ($P$) | Volume ($Q$) |
| LCP | LCP | Fully | Log | 0 | 0 | 0 |
| LCP | DCP | Fully | Log | 0 | $(1-\alpha)\phi$ | $-(1-\alpha)\phi$ |
| DCP | DCP | Fully | Log | $-\gamma$ | $(1-\alpha)\phi$ | $-(1-\alpha)\phi-\gamma$ |
| LCP | DCP | Fully | CRRA | 0 | $(1-\alpha)\phi$ | $-(1-\alpha)\phi$ |
| DCP | DCP | Fully | CRRA | $-\sigma\gamma$ | $\alpha(1-\sigma)\gamma$ $+(1-\alpha)\phi$ | $-\alpha(1-\sigma)\gamma$ $-(1-\alpha)\phi-\sigma\gamma$ |
| LCP | DCP | Partially | CRRA | $(1-\alpha)\phi\frac{(\delta-1)\sigma\delta}{\alpha\sigma\delta-(\delta-1)}$ | $(1-\alpha)\phi\frac{-(\delta-1)}{\alpha\sigma\delta-(\delta-1)}$ | $(1-\alpha)\phi\frac{(\delta-1)(\sigma\delta+1)}{\alpha\sigma\delta-(\delta-1)}$ |

Table 1: Global trade response under symmetric economy

**Extension** The main analysis in this section relies on the assumptions in Section 2.2 with fully sticky prices for simplicity. Nevertheless, it is straightforward to extend the model with more general results. Here I describe three main extensions summarized in Table 1, while detailed derivation is in Model Appendix.

First, if prices are partially sticky, $0 < \delta < 1$, a change in markups does not fully offset marginal costs movement, thereby a function of $\delta$. Since this change is universal for all markups through the second-round effect, global trade response shifts by a constant fraction compared to the case of fully sticky prices.[22]

Consider a general class of utility functions by relaxing Assumption 1 and 2 ($\sigma > 1, \varphi \neq 0$). In this case, the nominal wage is a function of the aggregate price index and labor supply. Therefore, the marginal cost equation in Lemma 1 includes additional terms from a general equilibrium wage response which depends on $\sigma$ and $\varphi$. Table 1 generalizes the simple examples with partially sticky price and CRRA utility.

Finally, consider a constant elasticity of substitution production function by relaxing Assumption 4 ($\varepsilon \neq 1, \theta \neq 1$). Especially from [2] Atalay (2017), intermediate goods tend to be more complimentary than final goods ($\varepsilon \approx 1, \theta \approx 0$). While this change does not affect the marginal cost equation in the log changes, the output equation in Lemma 4 includes additional expenditure switching terms between domestic and foreign intermediate inputs. This effect attenuates global trade response since dollar appreciation increases the price of foreign inputs, so producers should increase their expenditure toward foreign inputs when intermediate inputs are close to complementary goods.

# 3 Quantitative Analysis

This section presents a quantitative model that examines the impact of dollar appreciation resulting from exogenous shocks on trade prices and quantities, building upon the work of [18] Gopinath et al. (2020) and incorporating explicit I-O linkages. The model is motivated by the baseline model presented in Section 2. After outlining the key equations of the model, the paper demonstrates how to calibrate the model using data and evaluates its performance against empirical moments. Using the calibrated model, the paper provides the key finding of the paper that global trade responds differently than in a full DCP model, even though it is primarily invoiced in dollars. The counterfactual analyses quantify the significance of

[22] For example, markup responses of foreign intermediate goods in Lemma 3 become

$$d\mu_{FX} = (\delta - 1)\theta_D^{X*} dm^\$ + \frac{(\delta-1)^2}{1-\delta(1-\alpha^*)}(1-\alpha^*)\phi^*\theta_D^X dm^\$$$

$$d\mu_{HX}^* = (\delta - 1)\theta_D^X dm^\$ + \frac{(\delta-1)^2}{1-\delta(1-\alpha)}(1-\alpha)\phi\theta_D^{X*} dm^\$$$

invoicing currency and I-O linkages.

## 3.1 Quantitative model

The quantitative model is a multi-country large open economy where each country $j \in \{1, \cdots, I\}$ trades final and intermediate goods bilaterally. Each country has a continuum of representative household $h \in [0, 1]$ maximizing lifetime utility with discount factor $\beta$. The per-period utility is the same as in equation (1) with relative risk aversion $\sigma_c$.

$$U(C_{j,t}, N_{j,t}) = \frac{1}{1-\sigma_c} C_{j,t}^{1-\sigma_c} - \frac{\kappa}{1+\varphi} N_{j,t}^{1+\varphi}$$

Compared to the budget constraint equation (2) in the baseline model, the same equation in this model includes dollar bond holdings $B_{j,t+1}^{\$}$ with dollar bond interest rate $i_{j,t}^{\$}$. Following [24] Jiang et al. (2021), I assume that there is a convenience yield $e^{\xi_{j,t}}$ on dollar bond holdings of country $j$ as a safe asset. Optimal conditions of the bonds imply an uncovered interest rate parity (UIP) condition between the dollar and country $j$'s currency with a shock on UIP deviation $\xi_{j,t}$.[23]

$$i_{j,t} - i_{j,t}^{\$} = \mathbb{E}_t \left[ e_{\$j,t+1} - e_{\$j,t} \right] + \xi_{j,t} \tag{30}$$

$$\xi_{j,t} = \rho_\xi \xi_{j,t-1} + \varepsilon_{j,t}^{\xi}$$

where $i_{j,t}$ is the domestic interest rate of country $j$ and $e_{\$j,t}$ is the log dollar exchange rate of country $j$ defined in Section 2. Here, UIP shock $\xi_{j,t}$ follows AR(1) process with autoregressive coefficient $\rho_\xi$ and exogenous innovation $\varepsilon_{j,t}^{\xi} \sim (0, \sigma_\xi)$.

Besides, the production function is also the same as in equation (3). Therefore, the log of nominal marginal cost $mc_{j,t}$ is

$$mc_{j,t} = \alpha w_{j,t} + (1-\alpha) p_{j,t}^{X} - a_{j,t}$$

where $w_{j,t}$ is the log nominal wage and $p_{j,t}^{X}$ is the log price index of composite intermediate goods of country $j$. Productivity shock $a_{j,t}$ follows AR(1) process with autoregressive coefficient $\rho_a$ and exogenous innovation $\varepsilon_{j,t}^{a} \sim (0, \sigma_a)$.

**Pricing** When a firm producing a variety $w$ in country $j$ sells final (intermediate) goods to households (producers) of country $i$ in currency $k$, prices $P_{ji,t}^{C,k}(w)$ $\left(P_{ji,t}^{X,k}(w)\right)$ are sticky

[23] This paper assumes that UIP shocks are exogenous as in [10] Devereux and Engel (2002). Meanwhile, [23] Itskhoki and Mukhin (2021) endogenizes the UIP deviation by introducing segmented financial markets subject to financial shocks. They explain that financial shocks can match empirical patterns of UIP deviations, and further movement of exchange rates.

in currency $k$ à la Calvo with price stickiness parameter $\delta_p$. Also, prices are subject to variable markup due to strategic complementarity with steady-state markup elasticity $\Gamma$. Let $\theta_{ji}^{C,k}$ $\left(\theta_{ji}^{X,k}\right)$ be the fraction of exports in final (intermediate) goods from $j$ to $i$ invoiced in currency $k$.[24] Then, the log deviation of reset prices $\bar{p}_{ji,t}^{C,k}$ and $\bar{p}_{ji,t}^{X,k}$ satisfy

$$\bar{p}_{ji,t}^{C,k} = (1-\beta\delta_p)\left(\frac{1}{1+\Gamma}(mc_{j,t} - e_{kj,t} + \bar{\mu}) + \frac{\Gamma}{1+\Gamma}(p_{i,t} - e_{ki,t})\right) + \beta\delta_p\mathbb{E}_t\left[\bar{p}_{ji,t+1}^{C,k}\right] \quad (31)$$

$$\bar{p}_{ji,t}^{X,k} = (1-\beta\delta_p)\left(\frac{1}{1+\Gamma}(mc_{j,t} - e_{kj,t} + \bar{\mu}) + \frac{\Gamma}{1+\Gamma}(p_{i,t}^{X} - e_{ki,t})\right) + \beta\delta_p\mathbb{E}_t\left[\bar{p}_{ji,t+1}^{X,k}\right] \quad (32)$$

where $\bar{\mu}$ is the log of steady state markup. Under Calvo pricing, the log prices evolve

$$p_{ji,t}^{C,k} - p_{ji,t-1}^{C,k} = (1-\delta_p)(\bar{p}_{ji,t}^{C,k} - p_{ji,t-1}^{C,k})$$

$$p_{ji,t}^{X,k} - p_{ji,t-1}^{X,k} = (1-\delta_p)(\bar{p}_{ji,t}^{X,k} - p_{ji,t-1}^{X,k}).$$

Combining prices with corresponding invoicing shares as in equation (13), import prices of final and intermediate goods from $j$ to $i$ are defined by

$$p_{ji,t}^{C} = \sum_k \theta_{ji}^{C,k}(p_{ji,t}^{C,k} + e_{ki,t})$$

$$p_{ji,t}^{X} = \sum_k \theta_{ji}^{X,k}(p_{ji,t}^{X,k} + e_{ki,t})$$

**Demand structure** Aggregate consumption $C_{i,t}$ and composite intermediate goods $X_{i,t}$ are defined by [26] Kimball (1995) aggregator $\Upsilon(\cdot)$, calibrated by [27] Klenow and Willis (2016) specification with elasticity parameters $(\sigma, \varepsilon)$. Then, import demands of final goods and intermediate goods from $j$ to $i$ are

$$C_{ji,t}^{k}(\omega) = \gamma_{ji}\psi\left(D_{i,t}^{C}\frac{P_{ji,t}^{C,k}(\omega)}{P_{i,t}^{k}}\right)C_{i,t}$$

$$X_{ji,t}^{k}(\omega) = \omega_{ji}\psi\left(D_{i,t}^{X}\frac{P_{ji,t}^{X,k}(\omega)}{P_{i,t}^{X,k}}\right)X_{i,t}$$

[24] If final goods trade from $j$ to $i$ is invoiced in the producer currency (PCP), $\theta_{ji}^{C,j} = 1$. Similarly, $\theta_{ji}^{C,i} = 1$ for LCP and $\theta_{ji}^{C,\$} = 1$ for DCP. Assume that countries price their goods only in PCP, LCP, or DCP, so that $\sum_k \theta_{ji}^{C,k} = \theta_{ji}^{C,j} + \theta_{ji}^{C,i} + \theta_{ji}^{C,\$} = 1$. The same identity also holds for invoicing shares of intermediate goods trade $\theta_{ji}^{X,k}$.

where $\psi(\cdot) = \Upsilon'^{-1}(\cdot)$, $P^k_{i,t}$ is a final good price index of country $i$ denominated in currency $k$, and $D^C_{i,t}$ and $D^X_{i,t}$ are the demand index of each aggregator. Home bias parameters $\gamma_{ji}$ for final goods and $\omega_{ji}$ for intermediate goods are separately calibrated by WIOD data described below. Combining demands across countries, the per-period nominal profit of a firm in country $j$ is

$$\Pi_{j,t} = \sum_{i=1}^{I} \sum_{k} \mathcal{E}_{kj} \left( \theta^{C,k}_{ji} P^{C,k}_{ji,t} C^k_{ji,t} + \theta^{X,k}_{ji} P^{X,k}_{ji,t} X^k_{ji,t} \right) - MC_{j,t} Y_{j,t}. \tag{33}$$

**Equilibrium** The goods market clearing condition for variety $w$ in country $j$ is

$$\begin{aligned} Y_{j,t}(w) &= \sum_{i=1}^{I} \left( C_{ji,t}(w) + X_{ji,t}(w) \right) \\ &= \sum_{i=1}^{I} \sum_{k} \left( \theta^{C,k}_{ji} C^k_{ji,t}(w) + \theta^{X,k}_{ji} X^k_{ji,t}(w) \right) \end{aligned}$$

The bonds market clearing conditions are $B_{j,t} = 0$ and $\sum_{j=1}^{I} B^{\$}_{j,t} = 0$.

To close the model, I adopt a monetary policy to follow a Taylor rule for each country.

$$i_{j,t} - i^* = \rho_m (i_{j,t-1} - i^*) + (1 - \rho_m) \left( \phi_M \pi_{j,t} + \phi_Y (y_{j,t} - \bar{y}_j) \right) + \varepsilon_{j,t} \tag{34}$$

$$\varepsilon_{j,t} = \rho_\varepsilon \varepsilon_{j,t-1} + \varepsilon^m_{j,t}$$

where $i^*$ is steady state interest rate and $\bar{y}_j$ is the log output of country $j$ under flexible price. Here, $\varepsilon^m_{j,t}$ follows AR(1) process autoregressive coefficient $\rho_i$ and exogenous innovation $\varepsilon^i_{j,t} \sim (0, \sigma_i)$.

## 3.2 Data and Calibration

The main parameters of interest in this study are the home bias parameters, denoted as $(\gamma_{ji}, \omega_{ji})$, which are used in the demand structure, as well as the invoicing shares, denoted as $(\theta^{C,k}_{ji}, \theta^{X,k}_{ji})$. The former is calibrated using data from WIOD, while the latter is calibrated based on invoicing share data from [6] Boz et al. (2020). The remaining parameters are calibrated using standard values from the literature, with quarterly frequency, following [18] Gopinath et al. (2020).

WIOD is a database that provides information about inter-country and inter-sector input-output linkages for intermediate goods demands and final goods expenditure. It describes the input-output relationship of 44 countries, including 15 major countries, 28 EU countries, and the rest of the world, across 56 sectors classified in ISIC Rev.4 between 2000 and 2014. To focus on a steady state, the year 2014 is selected for analysis. Since the model is country-level, sectors within each country are aggregated. To address computational feasibility[25], 44 countries are ordered by the size of their gross national expenditure (GNE) implied by final expenditure, and the top 12 countries are selected for analysis. The remaining 32 countries are aggregated by the 13th country, the rest of the world (ROW).[26] Table 2 provides a list of the 13 countries with their export and import invoicing shares in the US dollar, Euro, and other currencies.

To calibrate the invoicing shares of bilateral trade, the invoicing share database collected by [6] Boz et al. (2020) is used, which provides unbalanced panel data for more than 100 countries since 1990. This database contains information about each country's shares of exports and imports invoiced in US dollars (USD), euros (EUR), and other currencies. The year 2014 is fixed, consistent with the WIOD dataset, except for Canada, as invoicing data is only available for a single observation in 2001. Due to data limitations, invoicing shares for China and Mexico are unavailable. For China, it is assumed that half of exports and imports are invoiced in USD, while the other half is invoiced in other currencies, including the Renminbi. In contrast, Mexico is assumed to engage in fully dollarized trade.[27] Lastly, the invoicing shares of the ROW countries are calculated by taking a weighted average of the remaining countries in the sample. Table 2 below presents the invoicing shares for the selected 13 countries. As shown in the table, the majority of countries predominantly invoice their trade in USD, while Eurozone countries mainly use EUR for invoicing. Japan and the United Kingdom extensively use their own currencies, rather than USD or EUR, for their trade.

Using the datasets described above, ($\gamma_{ji}$, $\omega_{ji}$) and ($\theta_{ji}^{C,k}$, $\theta_{ji}^{X,k}$) are calibrated as below. First, home bias parameters are calculated by expenditure shares of final and intermediate goods. As Kimball demand is specified by [27] Klenow and Willis (2016) specification, the final goods expenditure share of country $i$ on country $j$'s goods is equal to $\gamma_{ji}$ at steady state.[28] Similarly, intermediate input share of country $i$ on country $j$'s goods is equal to $\omega_{ji}$

[25] Since the model solves for bilateral prices and quantities and invoicing currency, the number of variables increases by an order of 2 with the number of countries. If I include all 44 countries, the total number of variables is more than $21 \times 44^2 \approx 40,000$.

[26] The sum of GNE weight except ROW before aggregation (among the 44 countries) is 0.846, which becomes 0.696 after aggregation (among the 13 countries). On the other hand, the total value of final and intermediate goods trade in US dollars is \$4.7 trillion and \$13.3 trillion before aggregation. In comparison, they are \$3.8 trillion and \$10.5 trillion after aggregation, which is around 20% less than before. Therefore, WIOD after aggregation still represents global trade.

[27] Changing these assumptions does not affect the main results.

[28] $\gamma_{ji,t} = \frac{\text{Final goods expenditure of } i \text{ on } j}{\text{Total final goods expenditure of } i} = \frac{\sum_k \theta_{ji}^{C,k} \mathcal{E}_{ki,t} P_{ji,t}^{C,k} C_{ji,t}^k}{P_{i,t} C_{i,t}} \to \gamma_{ji}$ at steady state.

| No | Country | ISO Code | GNE weight | Export invoicing share | | | Import invoicing share | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | USD | EUR | Others | USD | EUR | Others |
| 1 | United States | USA | 0.284 | 95.9 | 1.1 | 2.9 | 95.6 | 2.2 | 2.2 |
| 2 | Brazil | BRA | 0.032 | 95.5 | 3.4 | 1.0 | 84.9 | 10.1 | 4.9 |
| 3 | Canada | CAN | 0.021 | 70.0 | 7.0 | 23.0 | 70.0 | 7.0 | 23.0 |
| 4 | China | CHN | 0.087 | 50 | | 50 | 50 | | 50 |
| 5 | Germany | DEU | 0.042 | 16.8 | 78.1 | 5.1 | 19.7 | 78.3 | 2.1 |
| 6 | France | FRA | 0.032 | 22.9 | 72.6 | 4.5 | 23.2 | 75.4 | 1.4 |
| 7 | United Kingdom | GBR | 0.039 | 26.4 | 29.7 | 43.9 | 40.1 | 34.1 | 25.9 |
| 8 | India | IND | 0.031 | 86.8 | 7.7 | 5.5 | 89.4 | 7.2 | 3.5 |
| 9 | Italy | ITA | 0.028 | 14.3 | 82.9 | 2.8 | 28.0 | 69.3 | 2.7 |
| 10 | Japan | JPN | 0.061 | 53.0 | 6.0 | 41.1 | 73.8 | 3.6 | 22.7 |
| 11 | Mexico | MEX | 0.019 | 100 | | | 100 | | |
| 12 | Russia | RUS | 0.022 | 76.0 | 8.4 | 15.6 | 39.6 | 28.1 | 32.3 |
| 13 | Rest of the World | ROW | 0.304 | 49.1 | 42.4 | 8.5 | 47.5 | 42.0 | 10.5 |

Table 2: List of countries with export and import invoicing shares (%)
Source: WIOD, [6] Boz et al. (2020)

at steady state.[29]

To calibrate the dollar invoicing shares ($\theta_{ji}^{C,\$}, \theta_{ji}^{X,\$}$) in bilateral trade, an identifying assumption needs to be imposed that assumes identical dollar invoicing shares across either importers or exporters. The available data on dollar invoicing shares only pertains to each country's exports and imports, not to bilateral trade. Suppose that the number of countries in the model is denoted by $C$. In that case, the number of dollar invoicing shares in the data is $2C$, which includes the export invoicing shares of exporting country $j$ ($\theta_{j\cdot}^{\$}$) and import invoicing shares of importing country $i$ ($\theta_{\cdot i}^{\$}$). By contrast, the number of dollar invoicing share parameters to be calibrated is $2C^2$, which are the dollar invoicing shares of final goods trade from $j$ to $i$ ($\theta_{ji}^{C,\$}$) and of intermediate goods trade from $j$ to $i$ ($\theta_{ji}^{X,\$}$). To identify the parameters, two types of identifying assumptions are employed: (i) dollar invoicing shares are assumed to be identical across importers for each exporter $j$ (Calibration 1: $\theta_{ji}^{C,\$} = \theta_{ji}^{X,\$} = \theta_{j\cdot}^{\$}$), or (ii) dollar invoicing shares are assumed to be identical across exporters for each importer $i$ (Calibration 2: $\theta_{ji}^{C,\$} = \theta_{ji}^{X,\$} = \theta_{\cdot i}^{\$}$). The model is separately calibrated under these two assumptions, and the quantitative results of each calibration are displayed side by side.[30]

[29] $\omega_{ji,t} = \frac{\text{Intermediate goods expenditure of } i \text{ on } j}{\text{Total intermediate goods expenditure of } i} = \frac{\sum_k \theta_{ji}^{X,k} \mathcal{E}_{ki,t} P_{ji,t}^{X,k} X_{ji,t}^k}{P_{i,t}^X X_{i,t}} \to \omega_{ji}$ at steady state.

[30] Given that the primary aim of this section is to assess the impact of dollar appreciation, the focus is on calibrating the dollar invoicing shares. With respect to PCP and LCP invoicing shares, and given the dollar invoicing share $\theta_{ji}^{C,\$}$, a fraction $\zeta$ of trade is conducted using PCP, such that $\theta_{ji}^{C,j} = \zeta(1-\theta_{ji}^{C,\$})$, and the remaining $1-\zeta$ fraction of trade is conducted using LCP, such that $\theta_{ji}^{C,i} = (1-\zeta)(1-\theta_{ji}^{C,\$})$. This way, the three invoicing shares add up to 1. For the baseline calibration, a value of $\zeta = 0.5$ is set, and the model

To calibrate the shock processes, I match the model-implied moments with empirical moments. The model has three exogenous shocks for each country: productivity shock $a_{j,t}$, UIP shock $\xi_{j,t}$, and monetary policy shock $\varepsilon_{j,t}$, all of which follow an AR(1) process. I assume that the shocks are uncorrelated and that each shock $x_{j,t}$ has the same autocorrelation $\rho_x$ and standard deviation of innovation $\sigma_x$ across countries. To begin, I calibrate the parameters of the productivity shock $(\rho_a, \sigma_a)$ to match the average of (detrended) multi-factor productivity $(\hat{\rho}_a, \hat{\sigma}_a)$ across OECD countries provided by OECD Statistics.[31] Next, I assume $\rho_\xi = 0.9$ and use the relative volatility $\sigma_\xi/\sigma_a$ to match the average volatility of the dollar exchange rate growth $\hat{\sigma}(\Delta e_U)$ across countries.[32] I also assume $\rho_\varepsilon = 0.9$ and use the relative volatility $\sigma_\varepsilon/\sigma_a$ to match the average volatility of the GDP growth $\hat{\sigma}(\Delta y)$ across countries.

Once the shock processes are calibrated to match the model-implied moments with empirical moments, I conduct a bilateral panel regression using the same variables as in [18] Gopinath et al. (2020). The aim is to examine whether the calibrated model can replicate the empirical pattern of trade volume of countries in the rest of the world (ROW), except for the United States, in response to dollar appreciation. The panel regression equation is presented below:

$$\Delta y_{ij,t} = \sum_{k=0}^{2} \left(\beta_k + \eta_k S_j\right) \Delta e_{ij,t-k} + \sum_{k=0}^{2} \left(\beta_k^{\$} + \eta_k^{\$} S_j^{\$}\right) \Delta e_{\$j,t-k} + \lambda_{ij} + \alpha' X_{ij,t} + \varepsilon_{ij,t}$$

where $\Delta y_{ij,t}$ is the annualized growth of trade volume from $i$ to $j$, and $\Delta e_{ij,t}$ and $\Delta e_{\$j,t}$ are the annualized growth of the bilateral exchange rate and dollar exchange rate of importing country $j$. Each exchange rate interacts with the importer's invoicing share in terms of its own currency $S_j$ and the US dollar $S_j^{\$}$. $\lambda_{ij}$ is a dyad fixed effect, and $X_{ij,t}$ includes annualized GDP growth of the importing country with lag 0 to 2. Let $w_j$ be the import share of country $j$ from all countries, so that $\sum_{j\neq\$} w_j = 1$. Then, ROW import volume response $\sum_{j\neq\$} w_j \Delta y_{ij,t}$ from any exporter $i$ to 1% dollar appreciation is $\beta_k^{\$} + \eta_k^{\$} \sum_{j\neq\$} w_j S_j^{\$}$ after $k$ years. Let $\hat{\beta}_{row}^{\$}$ be the ROW trade response to dollar appreciation on impact $(k = 0)$. Table 3 below summarizes moment matching results.

The quantitative model appears to be successful in generating both the targeted empirical moments, such as the productivity process, volatility of dollar exchange rates growth, and

is simulated with varying values of $\zeta$, which consistently produce the same results.

[31] Specifically, I fit the log productivity process of each country after detrending with an AR(1) process to obtain the autocorrelation and standard deviation of innovation $(\hat{\rho}_{a,j}, \hat{\sigma}_{a,j})$. Then, I take the average across countries to obtain $(\hat{\rho}_a, \hat{\sigma}_a)$.

[32] Provided by BIS, the dollar exchange rates are from 1999Q1 to 2022Q2, relative to the Brazilian real, Canadian dollar, China Renminbi, Euro, Pound (Sterling), Indian rupee, Japanese Yen, Mexican Peso, and Russian rouble.

| | Data | Model | |
|---|---|---|---|
| | | Calibration 1 | Calibration 2 |
| *Matched* | | | |
| $\hat{\rho}_a$ | 0.786 | 0.774 | 0.774 |
| $\hat{\sigma}_a$ | 0.012 | 0.012 | 0.012 |
| $\hat{\sigma}(\Delta e_\$)$ | 0.044 | 0.040 | 0.041 |
| $\hat{\sigma}(\Delta y)$ | 0.019 | 0.021 | 0.021 |
| *Implied* | | | |
| $\hat{\beta}_{row}$ | -0.600 | -0.657 | -0.542 |

Table 3: Moment matching result

GDP growth, as well as the implied moment. Notably, the estimated regression results for the ROW trade response to dollar appreciation, for both calibrations, are similar to the empirical results found in [18] Gopinath et al. (2020), even though the model does not directly target the estimated coefficient.

## 3.3 Main Quantitative Results

Using the calibrated model, I conduct simulations with exogenous shocks that appreciate the US dollar, and analyze the resulting impulse-response of bilateral trade volume. These shocks can take the form of either a contractionary US monetary policy shock or a bundle of UIP shocks for each country except the US. In both cases, the shocks are normalized such that the US dollar appreciates by 1% against every other currency. The primary outcome variable is global trade volume (excluding the US), which is calculated as a weighted sum of bilateral trade volume response with steady-state trade share weights. First, I present the results of the full calibration, which includes both calibrated I-O linkages and calibrated invoicing shares, and then I examine how the results vary with different specifications of invoicing shares.

| Scenario | $\Delta$ World trade (%p) | |
|---|---|---|
| | US MP shock | UIP shocks |
| Invoicing Calibration 1 | −0.56 | −0.73 |
| Invoicing Calibration 2 | −0.55 | −0.72 |
| Full LCP | 0.10 | −0.02 |
| Final LCP, Int. DCP | −0.23 | −0.34 |
| Half LCP, Half DCP | −0.60 | −0.72 |
| Full DCP | −1.29 | −1.42 |

Table 4: World trade response under 1%p dollar appreciation

**Global Trade Volume Response** Table 4 displays the percentage response of global trade volume on impact to exogenous shocks that appreciate the US dollar by 1% against every other currency. The first column represents the response under a contractionary US monetary policy shock only, and the second column shows the response under UIP shocks of all countries except the US. The first two rows present global trade responses under the full calibration. As expected, global trade responses under invoicing shares of Calibration 1 and 2 are very similar. This is because each country's export and import invoicing patterns are similar, as shown in Table 2. Additionally, the empirical moment from the regression in Table 3 ($\hat{\beta}^{\$}_{row} = -0.6$) falls between the responses under the two shocks. Although the numbers are not directly comparable, this suggests that empirical evidence is a combination of structural impulse-responses under two exogenous shocks.

The next part of Table 4 examines the effect of counterfactual invoicing shares, ranging from a full LCP to full DCP model, while still using calibrated I-O linkages. As the name suggests, for example, "Final LCP, Int. DCP" means that final goods are invoiced only in LCP ($\theta_{ji}^{C,i} = 1$) and intermediate goods are invoiced only in DCP ($\theta_{ji}^{X,\$} = 1$). The table shows that the global trade response increases as trade is more invoiced in the US dollar, which is intuitive. Under a full LCP, the global trade response to dollar appreciation is negligible or slightly positive, depending on the shock.[33] As intermediate goods trade is invoiced more in DCP, the global trade response becomes quantitatively larger, yet still smaller than the calibrated invoicing cases.

The results for the "Half LCP, Half DCP" case, where final and intermediate goods are invoiced half in LCP ($\theta_{ji}^{C,i} = \theta_{ji}^{X,i} = 0.5$) and half in DCP ($\theta_{ji}^{C,\$} = \theta_{ji}^{X,\$} = 0.5$), show that the global trade response is quantitatively close to the full calibration. In other words, global trade responds to dollar appreciation like a mixture of LCP and DCP with equal weight. Quantitatively, the numbers fall between those of the full LCP and full DCP cases. This finding suggests that even though world trade is predominantly invoiced in DCP, global trade lies somewhere between LCP and DCP.

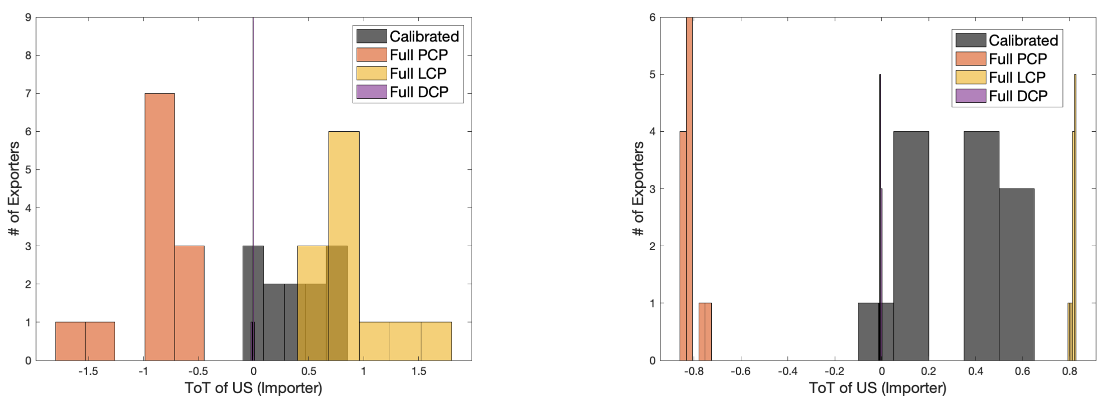
Furthermore, the results highlight that the invoicing of final goods in DCP has a significant impact on the global trade response to dollar appreciation. Specifically, the global trade response under "Half LCP, Half DCP" is more than twice that of the "Final LCP, Int. DCP" case. This observation is consistent with the interpretation presented in section 2, where the invoicing of final goods in DCP has a first-round effect only, while the invoicing of intermediate goods in DCP has first-round and second-round effects that offset each other.

[33] Theoretically, import prices are stable under a full LCP, so quantities should also be stable. However, the dollar exchange rates of some big countries like China or ROW move more than other countries under US monetary policy shock. Therefore, exports from China rise with a high trade share, resulting in a slightly positive global trade response.

**Terms-of-Trade Response** So far, I have argued that global trade lies between LCP and DCP in terms of quantity responses, and this conclusion is supported by price responses as well, specifically by changes in the terms-of-trade (ToT). The ToT is defined as the ratio of the import price to the export price in the same currency. The log change of ToT between exporter $i$ and importer $j$ is denoted

$$\Delta tot_{ij,t} = \Delta p_{ij,t} - (\Delta p_{ji,t} + \Delta e_{ij,t}).$$

Suppose that the dollar exchange rate of country $i$ increases ($\Delta e_{\$i,t} > 0$) under a fully sticky price environment. Theoretically, ToT between exporter $i$ and the US as importer declines under PCP ($\Delta tot_{iU,t} = -\Delta e_{\$i,t} < 0$). On the other hand, the ToT increases under LCP ($\Delta tot_{iU} = \Delta e_{\$i,t} > 0$), and is stable under DCP ($\Delta tot_{iU,t} = 0$). Since three pricing paradigms have stark predictions on ToT, it is possible to identify which pricing paradigm is close to global trade indirectly.



(a) US MP shock (b) UIP shocks

Figure 1: ToT response under 1%p dollar appreciation

Figure 1 depicts histograms of ToTs for each exporter with the US as an importer under various invoicing calibrations and shocks. The left panel corresponds to a US monetary policy shock that results in a 1% appreciation of the US dollar, while the right panel depicts UIP shocks. The histograms are color-coded to indicate the distribution of the number of exporters with corresponding ToTs on the X-axis under different invoicing calibrations. As expected, the ToTs are negative under a full PCP (red bar), positive under a full LCP (yellow bar), and highly concentrated around zero under a full DCP (blue bar). Notably, the ToTs under the full calibration (black bar) are distributed between zero and positive area, providing further evidence that world trade lies somewhere between LCP and DCP.

**Role of I-O linkages** The subsequent counterfactual analysis aims to evaluate the significance of I-O linkages in global trade. As discussed in section 2, the baseline model suggests that the global trade response to dollar appreciation critically depends on the interplay between I-O linkages and DCP invoicing shares. The response becomes pronounced when linkages with a substantial trade share are more invoiced in US dollars. More precisely, since import dollar invoicing shares differ among exporters for each importer, the effect of dollar appreciation will be greater when imports are biased toward high dollar invoicing shares.[34]

To implement this, compared with the full calibration, the home bias matrix $\gamma = [\gamma_{ji}]$ and input-output matrix $\Omega = [\omega_{ji}]$ are reordered such that the magnitude order of $(\gamma, \Omega)$ and dollar invoicing shares $\theta^{\$}_{j\cdot}$ are aligned for each importer. In other words, this considers a situation where countries switch their import expenditure towards exporters with higher dollar invoicing shares and away from exporters with lower dollar invoicing shares, reaching a new steady state.

| Scenario | Δ World trade (%p) | |
|---|---|---|
| | US MP shock | UIP shocks |
| Calibrated IO | -0.56 | -0.73 |
| High Corr(IO,Invoicing) | -0.86 | -1.31 |
| Low Corr(IO,Invoicing) | -0.12 | -0.26 |

Table 5: World trade response under 1%p dollar appreciation

Table 5 presents the results of counterfactual analysis on the response of global trade volume to different shocks under alternative calibrations of $\gamma$ and $\Omega$. The first row shows the results under the full calibration, which is identical to the first row of Calibration 1 in Table 4. The next two rows show the responses after reordering the matrices such that the magnitude orders of $(\gamma, \Omega)$ and import dollar invoicing shares $\theta^{\$}_{j\cdot}$ are aligned ("High Corr(IO,Invoicing)") or opposite ("Low Corr(IO,Invoicing)") for each importer.

The results indicate that reordering $\gamma$ and $\Omega$ has a significant impact on the response of global trade to dollar appreciation. In the case where the magnitude orders are aligned, global trade responds more to the shock by 0.30-0.58 percentage points, depending on the shock. On the other hand, when the magnitude orders are opposite, global trade responds less to the shock by 0.44-0.47 percentage points, depending on the shock. The magnitudes of these effects are quantitatively significant, ranging from 50% to 80% compared to the full

[34]Recall that I have assumed in Calibration 1 that dollar invoicing shares are the same across importers for each exporter, whereas they are the same across exporters for each importer in Calibration 2. Thus, import invoicing shares are heterogeneous in Calibration 1, but homogeneous in Calibration 2. Since the results of Table 4 are quantitatively similar for the two calibrations, I will focus on Calibration 1 to examine the variation in the importer's invoicing shares.

calibration. These findings suggest that I-O linkages are important in shaping the response of global trade to exchange rate shocks, and their interaction with dollar invoicing shares can lead to significant amplification or dampening effects.

# 4 Conclusion

This paper investigates the impact of dollar appreciation resulting from exogenous shocks on global trade, taking into account the input-output linkages under the dominant currency paradigm. The baseline model provides an analytical framework for understanding how dollar appreciation spills over to import prices and quantities of final and intermediate goods. One of the main insights from the model is that the interaction between dollar invoicing shares and the intensity of foreign goods is a sufficient statistic for importers' direct exposure to the dollar. Furthermore, the model shows that the presence of intermediate goods increases marginal costs, leading to a reduction in markups due to sticky prices. This indirect exposure to the dollar generates a second-round effect, boosting output and global trade under dollar appreciation.

The quantitative model calibrated using WIOD and invoicing share data suggests that the global trade volume response is close to a model with half dominant currency paradigm (DCP) and half local currency paradigm (LCP). This result is also supported by the terms of trade (ToT) responses of the US, indicating that the calibrated model lies between a full LCP and full DCP. Additionally, the study quantifies the importance of input-output linkages by reordering home bias and input-output matrix. The counterfactual analysis shows that switching imports toward high dollar invoicing exporters can amplify global trade response to 1% dollar appreciation by 50% to 80%.

Although this paper primarily focuses on the positive implications of dollar appreciation on global trade, it is possible to extend the analysis to welfare and normative implications. The study provides implications on trade policy, monetary policy, and exchange rate policy. For instance, suppose that countries with dollar-invoiced trade are subject to dollar appreciation. In that case, it is worth considering alternative hedging strategies other than directly affecting the home currency through foreign exchange intervention. One option is to incentivize firms and consumers to change their import invoicing currency from the US dollar to other currencies. However, this approach is not efficient or plausible since there is a fixed cost associated with switching invoicing currency, and dollar invoicing has historical dependence and anchor currency advantage, as emphasized by [29] Mukhin (2022).

Another option to hedge against dollar appreciation is to incentivize firms and consumers to change their trade partners with less dollar invoicing, which can be implemented through import tax policy. In other words, if they can switch the exporting sources such that their input-output linkages are less correlated with the given dollar invoicing shares, it is possible to ex-ante hedge their trade against dollar appreciation.

Meanwhile, there are extensive and ongoing discussions on monetary and exchange rate policy under dominant currency environment in international trade and financial market. Gita Gopinath, the first deputy managing director of the International Monetary Fund (IMF), gave a number of speeches including the Jackson Hole Economic Symposium to introduce the new foundations of international macro policy, called the IMF's Integrated Policy Framework (IPF). This policy framework emphasizes that a floating exchange rate and domestic inflation target cannot achieve first-best allocation, which was the case under Mundell-Fleming model. This is because Mundell-Fleming model assumes that prices are fully invoiced in producer currency (PCP), so that expenditure switching happens symmetrically both on export and import sides. So if policymakers target domestic inflation, domestic output gap and export gap are closed simultaneously.

However, as empirical evidence supports DCP that expenditure switching takes place mostly on the import side, a floating exchange rate regime cannot achieve first-best allocation. According to IMF's IPF and [12] Egorov and Mukhin (2022), inflation targeting is still optimal under a floating exchange rate, but cannot close the output gap. On the other hand, [11] Devereux and Engel (2003) argues that the optimal monetary policy supports a fixed exchange rate regime under LCP. This is because policymakers know exchange rate flexibility cannot affect relative prices between home and foreign goods since expenditure switching is absent both on export and import sides under LCP. As the paper suggests that expenditure switching of the calibrated model lies in between LCP and DCP, managed float can be supported in a global planner's point of view. This aligns with the idea of "Fear of Floating" coined by [7] Calvo and Reinhart (2002) that emerging market countries optimally choose to move away from free floating.

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# Model Appendix

This appendix describes the baseline model in detail and provides proofs.

## Environment

**Household** A household in the Home country solves utility maximization problem subject to budget constraint

$$\max_{C_H, C_F, L, B'(s)} U(C_H, C_F, L) = \frac{1}{1-\sigma} C^{1-\sigma} - \frac{1}{1+\varphi} L^{1+\varphi}$$

subject to

$$P_H C_H + P_F C_F + \sum_{s \in S} Q(s) B'(s) = WL + B$$

$$PC \leq M$$

where $C = \left( (1-\gamma)^{\frac{1}{\varepsilon}} C_H^{\frac{\varepsilon-1}{\varepsilon}} + \gamma^{\frac{1}{\varepsilon}} C_F^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$.

Similarly, for a household in ROW,

$$\max_{C_F^*, C_H^*, L^*, B^{*'}(s)} U(C_F^*, C_H^*, L^*) = \frac{1}{1-\sigma} C^{*1-\sigma} - \frac{1}{1+\varphi} L^{*1+\varphi}$$

subject to

$$P_F^* C_F^* + P_H^* C_H^* + \sum_{s \in S} Q^*(s) B^{*'}(s) = W^* L^* + B^*$$

$$P^* C^* \leq M^*$$

where $C^* = \left( (1-\gamma)^{\frac{1}{\varepsilon}} C_F^{*\frac{\varepsilon-1}{\varepsilon}} + \gamma^{\frac{1}{\varepsilon}} C_H^{*\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$.

**Production** A firm in the Home country solves cost minimization problem

$$\min_{L, X_H, X_F} WL + P_{HX} X_H + P_{FX} X_F$$

where $Y = A \left( \alpha^{\frac{1}{\rho}} L^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} X^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$ and $X = \left( (1-\phi)^{\frac{1}{\theta}} X_H^{\frac{\theta-1}{\theta}} + \phi^{\frac{1}{\theta}} X_F^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$.

Similarly, for a firm in ROW,

$$\min_{L^*, X_F^*, X_H^*} W^* L^* + P_{FX}^* X_F^* + P_{HX}^* X_H^*$$

where $Y^* = A^* \left( \alpha^{*\frac{1}{\rho}} L^{*\frac{\rho-1}{\rho}} + (1-\alpha^*)^{\frac{1}{\rho}} X^{*\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$ and $X^* = \left( (1-\phi^*)^{\frac{1}{\theta}} X_F^{*\frac{\theta-1}{\theta}} + \phi^{*\frac{1}{\theta}} X_H^{*\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$.

**Currency of Invoicing** First, Home domestic final good price $P_H$ and intermediate good price $P_{HX}$ are sticky in Home currency. Therefore, $P_H = \mu_H MC$ and $P_{HX} = \mu_{HX} MC$ are denominated in Home currency.

Under PCP, Home export final good price $P^*_{H,P}$ and intermediate good price $P^*_{HX,P}$ are sticky in Home currency. Therefore, $P_{H,P} = \mu^*_{H,P} MC$ and $P_{HX,P} = \mu^*_{HX,P} MC$ are denominated in Home currency, and $P^*_{H,P} = \frac{P_{H,P}}{\mathcal{E}} = \mu^*_{H,P} \frac{MC}{\mathcal{E}}$ and $P^*_{HX,P} = \frac{P_{HX,P}}{\mathcal{E}} = \mu^*_{HX,P} \frac{MC}{\mathcal{E}}$ are denominated in ROW currency. Under LCP, Home export prices $P^*_{H,L}$ and $P^*_{HX,L}$ are sticky in ROW currency. Therefore, $P^*_{H,L} = \mu^*_{H,L} \frac{MC}{\mathcal{E}}$ and $P^*_{HX,L} = \mu^*_{HX,L} \frac{MC}{\mathcal{E}}$ are denominated in ROW currency. Under DCP, Home export prices $P^*_{H,D}$ and $P^*_{HX,D}$ are sticky in the dollar. Therefore, $P^{\$}_{H,D} = \mu^*_{H,D} \frac{MC}{\mathcal{E}_{\$H}}$ and $P^{\$}_{HX,D} = \mu^*_{HX,D} \frac{MC}{\mathcal{E}_{\$H}}$ are denominated in the dollar. If denominated in ROW currency,

$$P^*_{H,D} = P^{\$}_{H,D} \mathcal{E}_{\$F} = \mu^*_{H,D} MC \frac{\mathcal{E}_{\$F}}{\mathcal{E}_{\$H}} = \mu^*_{H,D} \frac{MC}{\mathcal{E}}$$

$$P^*_{HX,D} = P^{\$}_{HX,D} \mathcal{E}_{\$F} = \mu^*_{HX,D} MC \frac{\mathcal{E}_{\$F}}{\mathcal{E}_{\$H}} = \mu^*_{HX,D} \frac{MC}{\mathcal{E}}$$

Home export final good price $P^*_H$ and intermediate good price $P^*_{HX}$ are Cobb-Douglas aggregates of the three invoiced prices as below.

$$\begin{aligned} P^*_H &= (P^*_{H,P})^{\theta^C_P} (P^*_{H,L})^{\theta^C_L} (P^*_{H,D})^{\theta^C_D} = \left( \mu^*_{H,P} \frac{MC}{\mathcal{E}} \right)^{\theta^C_P} \left( \mu^*_{H,L} \frac{MC}{\mathcal{E}} \right)^{\theta^C_L} \left( \mu^*_{H,D} \frac{MC}{\mathcal{E}} \right)^{\theta^C_D} \\ &= \left( \mu^*_{H,P} \right)^{\theta^C_P} \left( \mu^*_{H,L} \right)^{\theta^C_L} \left( \mu^*_{H,D} \right)^{\theta^C_D} \frac{MC}{\mathcal{E}} = \mu^*_H \frac{MC}{\mathcal{E}} \end{aligned}$$

$$\begin{aligned} P^*_{HX} &= (P^*_{HX,P})^{\theta^X_P} (P^*_{HX,L})^{\theta^X_L} (P^*_{HX,D})^{\theta^X_D} = \left( \mu^*_{HX,P} \frac{MC}{\mathcal{E}} \right)^{\theta^X_P} \left( \mu^*_{HX,L} \frac{MC}{\mathcal{E}} \right)^{\theta^X_L} \left( \mu^*_{HX,D} \frac{MC}{\mathcal{E}} \right)^{\theta^X_D} \\ &= \left( \mu^*_{HX,P} \right)^{\theta^X_P} \left( \mu^*_{HX,L} \right)^{\theta^X_L} \left( \mu^*_{HX,D} \right)^{\theta^X_D} \frac{MC}{\mathcal{E}} = \mu^*_{HX} \frac{MC}{\mathcal{E}} \end{aligned}$$

Similarly, ROW domestic prices $P^*_F$ and $P^*_{FX}$ and ROW export prices in each currency of invoicing are

$$P^*_F = \mu^*_F MC^*, \quad P^*_{FX} = \mu^*_F MC^*$$

$$P^*_{F,P} = \mu_{F,P} MC^*, \quad P^*_{FX,P} = \mu_{FX,P} MC^*$$

$$P_{F,P} = P^*_{F,P} \mathcal{E} = \mu_{F,P} MC^* \mathcal{E}, \quad P_{FX,P} = P^*_{FX,P} \mathcal{E} = \mu_{FX,P} MC^* \mathcal{E}$$

$$P_{F,L} = \mu_{F,L} MC^* \mathcal{E}, \quad P_{FX,L} = \mu_{FX,L} MC^* \mathcal{E}$$

$$P^{\$}_{F,D} = \mu_{F,D} \frac{MC^*}{\mathcal{E}_{\$F}}, \quad P^{\$}_{FX,D} = \mu_{FX,D} \frac{MC^*}{\mathcal{E}_{\$F}}$$

$$P_{F,D} = P^{\$}_{F,D} \mathcal{E}_{\$H} = \mu_{F,D} MC^* \frac{\mathcal{E}_{\$H}}{\mathcal{E}_{\$F}} = \mu_{F,D} MC^* \mathcal{E}$$

$$P_{FX,D} = P^{\$}_{FX,D} \mathcal{E}_{\$H} = \mu^*_{HX,D} MC^* \frac{\mathcal{E}_{\$H}}{\mathcal{E}_{\$F}} = \mu_{FX,D} MC^* \mathcal{E}$$

In aggregation, ROW export prices $P_F$ and $P_{FX}$ are

$$P_F = (P_{F,P})^{\theta_P^{C*}} (P_{F,L})^{\theta_L^{C*}} (P_{F,D})^{\theta_D^{C*}} = \mu_F MC^* \mathcal{E}$$

$$P_{FX} = (P_{FX,P})^{\theta_P^{X*}} (P_{FX,L})^{\theta_L^{X*}} (P_{FX,D})^{\theta_D^{X*}} = \mu_{FX} MC^* \mathcal{E}$$

**Sticky Prices** In response to an exogenous shock, the (log) change in Home domestic prices are $dp_H = dp_{HX} = \delta dmc$ as they are sticky in own currency.

Under PCP, the change in Home export prices are $dp_{H,P} = dp_{HX,P} = \delta dmc$ denominated in Home currency, and $dp^*_{H,P} = dp^*_{HX,P} = \delta dmc - de$ denominated in ROW currency. Under LCP, the change in Home export prices are $dp^*_{H,L} = dp^*_{HX,L} = \delta(dmc - de)$ denominated in ROW currency. Under DCP, the change in Home export prices are $dp^{\$}_{H,D} = dp^{\$}_{HX,D} = \delta(dmc - de_{\$H})$ denominated in the dollar and $dp^*_{H,D} = dp^*_{HX,D} = \delta(dmc - de_{\$H}) + de_{\$F}$ denominated in ROW currency. By Cobb-Douglas aggregates, the change in Home export prices are

$$dp^*_H = \theta_P^C dp^*_{H,P} + \theta_L^C dp^*_{H,L} + \theta_D^C dp^*_{H,D}$$

$$dp^*_{HX} = \theta_P^X dp^*_{HX,P} + \theta_L^X dp^*_{HX,L} + \theta_D^X dp^*_{HX,D}$$

Similarly, the change in ROW domestic prices and ROW export prices in each currency of invoicing are

$$dp^*_F = dp^*_{FX} = \delta dmc^*$$

$$dp^*_{F,P} = dp^*_{FX,P} = \delta dmc^*, \quad dp_{F,P} = dp_{FX,P} = \delta dmc^* + de$$

$$dp_{F,L} = dp_{FX,L} = \delta(dmc^* + de)$$

$$dp^{\$}_{F,D} = dp^{\$}_{FX,D} = \delta(dmc^* - de_{\$F}), \quad dp_{F,D} = dp_{FX,D} = \delta(dmc^* - de_{\$F}) + de_{\$H}$$

In aggregation, the change in ROW export prices are

$$dp_F = \theta_P^{C*} dp_{F,P} + \theta_L^{C*} dp_{F,L} + \theta_D^{C*} dp_{F,D}$$

$$dp_{FX} = \theta_P^{X*} dp_{FX,P} + \theta_L^{X*} dp_{FX,L} + \theta_D^{X*} dp_{FX,D}$$

**Markups** The (log) change in markups for Home domestic prices and Home export prices in each currency of invoicing are

$$d\mu_H = d\mu_{HX} = (\delta - 1)dmc$$

$$d\mu^*_{H,P} = d\mu^*_{HX,P} = (\delta - 1)dmc$$

$$d\mu^*_{H,L} = d\mu^*_{HX,L} = (\delta - 1)(dmc - de)$$

$$d\mu^*_{H,D} = d\mu^*_{HX,D} = (\delta - 1)(dmc - de_{\$H})$$

In aggregation, the change in markups for Home export prices are

$$d\mu^*_H = \theta_P^C d\mu^*_{H,P} + \theta_L^C d\mu^*_{H,L} + \theta_D^C d\mu^*_{H,D}$$

$$d\mu^*_{HX} = \theta_P^X d\mu^*_{HX,P} + \theta_L^X d\mu^*_{HX,L} + \theta_D^X d\mu^*_{HX,D}$$

The (log) change in markups for ROW domestic prices and ROW export prices in each currency of invoicing are

$$d\mu^*_F = d\mu^*_{FX} = (\delta - 1)dmc^*$$

$$d\mu_{F,P} = d\mu_{FX,P} = (\delta - 1)dmc^*$$

$$d\mu_{F,L} = d\mu_{FX,L} = (\delta - 1)(dmc^* + de)$$

$$d\mu_{F,D} = d\mu_{FX,D} = (\delta - 1)(dmc^* - de_{\$F})$$

In aggregation, the change in markups for ROW export prices are

$$d\mu_F = \theta_P^{C*} d\mu_{F,P} + \theta_L^{C*} d\mu_{F,L} + \theta_D^{C*} d\mu_{F,D}$$

$$d\mu_{FX} = \theta_P^{X*} d\mu_{FX,P} + \theta_L^{X*} d\mu_{FX,L} + \theta_D^{X*} d\mu_{FX,D}$$

## Model Equations

### Household

Final goods demand for Home household: $C_H = (1-\gamma)\left(\frac{P_H}{P}\right)^{-\varepsilon} C$

$C_F = \gamma \left(\frac{P_F}{P}\right)^{-\varepsilon} C$

Final goods demand for ROW household: $C^*_F = (1-\gamma)\left(\frac{P^*_F}{P^*}\right)^{-\varepsilon} C^*$

$C_H^* = \gamma \left(\frac{P_H^*}{P^*}\right)^{-\varepsilon} C^*$

Labor supply: $W/P = C^\sigma L^\varphi$

$W^*/P^* = C^{*\sigma} L^{*\varphi}$

Case-in-advance: $PC = M$

$P^* C^* = M^*$

Aggregate price: $P = \left\{(1-\gamma) P_H^{1-\varepsilon} + \gamma P_F^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}$

$P^* = \left\{(1-\gamma) P_F^{*1-\varepsilon} + \gamma P_H^{*1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}$

**Production**

Labor demand: $L = \alpha A^{\rho-1} \left(\frac{W}{MC}\right)^{-\rho} Y$

$L^* = \alpha^* A^{*\rho-1} \left(\frac{W^*}{MC^*}\right)^{-\rho} Y^*$

Intermediate goods demand for Home firm: $X_H = (1-\alpha)(1-\phi) A^{\rho-1} \left(\frac{P_{HX}}{P_X}\right)^{-\theta} \left(\frac{P_X}{MC}\right)^{-\rho} Y$

$X_F = (1-\alpha)\phi A^{\rho-1} \left(\frac{P_{FX}}{P_X}\right)^{-\theta} \left(\frac{P_X}{MC}\right)^{-\rho} Y$

Intermediate goods demand for ROW firm: $X_F^* = (1-\alpha^*)(1-\phi^*) A^{*\rho-1} \left(\frac{P_{FX}^*}{P_X^*}\right)^{-\theta} \left(\frac{P_X^*}{MC^*}\right)^{-\rho} Y^*$

$X_H^* = (1-\alpha^*)\phi^* A^{*\rho-1} \left(\frac{P_{HX}^*}{P_X^*}\right)^{-\theta} \left(\frac{P_X^*}{MC^*}\right)^{-\rho} Y^*$

Home marginal cost: $MC = \frac{1}{A} \left\{\alpha W^{1-\rho} + (1-\alpha) P_X^{1-\rho}\right\}^{\frac{1}{1-\rho}}$

where $P_X = \left\{(1-\phi) P_{HX}^{1-\theta} + \phi P_{FX}^{1-\theta}\right\}^{\frac{1}{1-\theta}}$

ROW marginal cost: $MC^* = \frac{1}{A^*} \left\{\alpha^* W^{*1-\rho} + (1-\alpha^*) P_X^{*1-\rho}\right\}^{\frac{1}{1-\rho}}$

where $P_X^* = \left\{(1-\phi^*) P_{FX}^{*1-\theta} + \phi^* P_{HX}^{*1-\theta}\right\}^{\frac{1}{1-\theta}}$

**Exchange Rate**

Bilateral Exchange rate: $\mathcal{E} = \frac{\mathcal{E}_{\$H}}{\mathcal{E}_{\$F}}$

Dollar appreciation: $\omega^\$ \frac{de_{\$H}}{dm^\$} + (1-\omega^\$) \frac{de_{\$F}}{dm^\$} = -1$

### Sticky Prices and Markups

| | |
|---|---|
| Markups for Home domestic prices: | $d\mu_H = d\mu_{HX} = (\delta - 1)dmc$ |
| Markup for Home export final good price: | $d\mu_H^* = (\delta{-}1)\left(dmc - \theta_L^C de - \theta_D^C de_{\$H}\right)$ |
| Markup for Home export intermediate good price: | $d\mu_{HX}^* = (\delta{-}1)\left(dmc - \theta_L^X de - \theta_D^X de_{\$H}\right)$ |
| Markups for ROW domestic prices: | $d\mu_F^* = d\mu_{FX}^* = (\delta - 1)dmc^*$ |
| Markup for ROW export final good price: | $d\mu_F = (\delta{-}1)\left(dmc^* + \theta_L^{C*} de - \theta_D^{C*} de_{\$F}\right)$ |
| Markup for ROW export intermediate good price: | $d\mu_{FX} = (\delta{-}1)\left(dmc^* + \theta_L^{X*} de - \theta_D^{X*} de_{\$F}\right)$ |

### Market clearing

| | |
|---|---|
| Home goods market: | $Y = C_H + C_H^* + X_H + X_H^*$ |
| ROW goods market: | $Y^* = C_F^* + C_F + X_F^* + X_F$ |

## Log-linearized Equations

### Household

| | |
|---|---|
| Final goods demand for Home household: | $dc_H = -\varepsilon(dp_H - dp) + dc$ |
| | $dc_F = -\varepsilon(dp_F - dp) + dc$ |
| Final goods demand for ROW household: | $dc_F^* = -\varepsilon(dp_F^* - dp^*) + dc^*$ |
| | $dc_H^* = -\varepsilon(dp_H^* - dp^*) + dc^*$ |
| Labor supply: | $dw - dp = \sigma dc + \varphi dl$ |
| | $dw^* - dp^* = \sigma dc^* + \varphi dl^*$ |
| Aggregate price: | $dp = (1 - \gamma)dp_H + \gamma dp_F$ |
| | $dp^* = (1 - \gamma)dp_F^* + \gamma dp_H^*$ |

**Production**

Labor demand: $dl = -\rho\,(dw - dmc) + dy$

$dl^* = -\rho\,(dw^* - dmc^*) + dy^*$

Int. goods demand for Home firm: $dx_H = -\theta\,(dp_{HX} - dp_X) - \rho\,(dp_X - dmc) + dy$

$dx_F = -\theta\,(dp_{FX} - dp_X) - \rho\,(dp_X - dmc) + dy$

Int. goods demand for ROW firm: $dx_F^* = -\theta\,(dp_{FX}^* - dp_X^*) - \rho\,(dp_X^* - dmc^*) + dy^*$

$dx_H^* = -\theta\,(dp_{HX}^* - dp_X^*) - \rho\,(dp_X^* - dmc^*) + dy^*$

Marginal cost: $dmc = -da + \alpha dw + (1-\alpha)(1-\phi)dp_{HX} + (1-\alpha)\phi dp_{FX}$

$dmc^* = -da^* + \alpha^* dw^* + (1-\alpha^*)(1-\phi^*)dp_{FX}^* + (1-\alpha^*)\phi^* dp_{HX}^*$

**Market Clearing**

Home goods: $dy = \frac{1-\gamma}{\bar{Y}}dc_H + \frac{\gamma}{\bar{Y}}dc_H^* + (1-\alpha)(1-\phi)dx_H + \frac{(1-\alpha^*)\phi^*\bar{Y}^*}{\bar{Y}}dx_H^*$

ROW goods: $dy^* = \frac{1-\gamma}{\bar{Y}^*}dc_F^* + \frac{\gamma}{\bar{Y}^*}dc_F + (1-\alpha^*)(1-\phi^*)dx_F^* + \frac{(1-\alpha)\phi\bar{Y}}{\bar{Y}^*}dx_F$

## Steady State

At steady state, assume that

- Prices are fully flexible, i.e. All markups are eqaul to 1
- $\bar{\mathcal{E}} = 1$ (or $\bar{M} = \bar{M}^*$)
- $\bar{A} = \bar{A}^* = 1$
- $\overline{MC} = \overline{MC}^*$
- $\bar{M} = \bar{M}^*$ are set such that $\bar{C} = \bar{C}^* = 1$

Under these assumptions, steady state prices and wages are all identical to steady state marginal costs.

$$\bar{P}_H = \bar{P}_H^* = \bar{P}_{HX} = \bar{P}_{HX}^* = \overline{MC}$$

$$\bar{P}_F^* = \bar{P}_F = \bar{P}_{FX}^* = \bar{P}_{FX} = \overline{MC}^*$$

$$\bar{P} = \bar{P}^* = \bar{P}_X = \bar{P}_X^* = \overline{MC} = \overline{MC}^*$$

$$\bar{W} = \bar{W}^* = \overline{MC}$$

From market clearing conditions,

$$\begin{aligned}
Y &= Y_H + Y_H^* = C_H + C_H^* + X_H + X_H^* \\
&= (1-\gamma)\left(\frac{P_H}{P}\right)^{-\varepsilon} C + \gamma \left(\frac{P_H^*}{P^*}\right)^{-\varepsilon} C^* \\
&+ (1-\alpha)(1-\phi)\left(\frac{P_{HX}}{P_X}\right)^{-\theta}\left(\frac{P_X}{MC}\right)^{-1} Y + (1-\alpha^*)\phi^*\left(\frac{P_{HX}^*}{P_X^*}\right)^{-\theta}\left(\frac{P_X^*}{MC^*}\right)^{-1} Y^*
\end{aligned}$$

where $Y_H = C_H + X_H$ and $Y_H^* = C_H^* + X_H^*$, and

$$\begin{aligned}
Y^* &= Y_F^* + Y_F = C_F^* + C_F + X_F^* + X_F \\
&= (1-\gamma)\left(\frac{P_F^*}{P^*}\right)^{-\varepsilon} C^* + \gamma \left(\frac{P_F}{P}\right)^{-\varepsilon} C \\
&+ (1-\alpha^*)(1-\phi^*)\left(\frac{P_{FX}^*}{P_X^*}\right)^{-\theta}\left(\frac{P_X^*}{MC^*}\right)^{-1} Y^* + (1-\alpha)\phi\left(\frac{P_{FX}}{P_X}\right)^{-\theta}\left(\frac{P_X}{MC}\right)^{-1} Y
\end{aligned}$$

where $Y_F^* = C_F^* + X_F^*$ and $Y_F = C_F + X_F$. At steady state,

$$\bar{Y} = (1-\gamma)\bar{C} + \gamma\bar{C}^* + (1-\alpha)(1-\phi)\bar{Y} + (1-\alpha^*)\phi^*\bar{Y}^*$$

$$\bar{Y}^* = (1-\gamma)\bar{C}^* + \gamma\bar{C} + (1-\alpha^*)(1-\phi^*)\bar{Y}^* + (1-\alpha)\phi\bar{Y}$$

As $\bar{C} = \bar{C}^* = 1$,

$$\begin{aligned}
\begin{bmatrix} \bar{Y} \\ \bar{Y}^* \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} (1-\alpha)(1-\phi) & (1-\alpha^*)\phi^* \\ (1-\alpha)\phi & (1-\alpha^*)(1-\phi^*) \end{bmatrix} \begin{bmatrix} \bar{Y} \\ \bar{Y}^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \Omega' \begin{bmatrix} \bar{Y} \\ \bar{Y}^* \end{bmatrix} \\
&= \Psi' \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{21} \\ \psi_{12} & \psi_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \psi_{11} + \psi_{21} \\ \psi_{12} + \psi_{22} \end{bmatrix}
\end{aligned}$$

## Partially Sticky Prices Case

**Markup Equation** When prices are partially sticky ($\delta > 0$), the analytical expression for the markup response as in Lemma 3 is very complicated. Instead, it can be derived sequentially by marginal and markup equations as below.

1. (Markup equation) Since foreign prices are sticky in DCP, dollar appreciation ($de_{\$F} =$

$de_{\$H} = -dm^{\$} > 0$) has (direct) positive impact on the markups for foreign prices

$$d\mu_F = (\delta - 1)\theta_D^{C*}(-de_{\$F}) = (\delta - 1)\theta_D^{C*} dm^{\$} > 0$$

$$d\mu_H^* = (\delta - 1)\theta_D^{C}(-de_{\$H}) = (\delta - 1)\theta_D^{C} dm^{\$} > 0$$

$$d\mu_{FX} = (\delta - 1)\theta_D^{X*}(-de_{\$F}) = (\delta - 1)\theta_D^{X*} dm^{\$} > 0$$

$$d\mu_{HX}^* = (\delta - 1)\theta_D^{X}(-de_{\$H}) = (\delta - 1)\theta_D^{X} dm^{\$} > 0$$

$$\begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix} = (\delta - 1)\begin{bmatrix} (1-\alpha)\phi\theta_D^{X*} dm^{\$} \\ (1-\alpha^*)\phi^*\theta_D^{X} dm^{\$} \end{bmatrix}$$

2. (Marginal cost equation) The marginal costs change by

$$\begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix} = (\delta - 1)\tilde{\Psi} \begin{bmatrix} (1-\alpha)\phi\theta_D^{X*} dm^{\$} \\ (1-\alpha^*)\phi^*\theta_D^{X} dm^{\$} \end{bmatrix}$$

3. (Markup equation) The markups change by

$$d\mu_H = d\mu_H^* = d\mu_{HX} = d\mu_{HX}^* = (\delta - 1)dmc$$

$$d\mu_F^* = d\mu_F = d\mu_{FX}^* = d\mu_{FX} = (\delta - 1)dmc^*$$

$$\begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix} = \Omega \begin{bmatrix} (\delta-1)dmc \\ (\delta-1)dmc^* \end{bmatrix} = (\delta - 1)^2\Omega\tilde{\Psi} \begin{bmatrix} (1-\alpha)\phi\theta_D^{X*} dm^{\$} \\ (1-\alpha^*)\phi^*\theta_D^{X} dm^{\$} \end{bmatrix}$$

4. (Marginal cost equation) The marginal costs change by

$$\begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} d\upsilon \\ d\upsilon^* \end{bmatrix} = (\delta - 1)^2\tilde{\Psi}\Omega\tilde{\Psi} \begin{bmatrix} (1-\alpha)\phi\theta_D^{X*} dm^{\$} \\ (1-\alpha^*)\phi^*\theta_D^{X} dm^{\$} \end{bmatrix}$$

Therefore, the response of marginal costs are the infinite sum derived sequentially.

$$\begin{aligned} \begin{bmatrix} dmc \\ dmc^* \end{bmatrix} &= \left\{(\delta - 1)\tilde{\Psi} + (\delta - 1)^2\tilde{\Psi}\Omega\tilde{\Psi} + (\delta - 1)^3\tilde{\Psi}\Omega\tilde{\Psi}\Omega\tilde{\Psi} + \cdots\right\} \begin{bmatrix} (1-\alpha)\phi\theta_D^{X*} dm^{\$} \\ (1-\alpha^*)\phi^*\theta_D^{X} dm^{\$} \end{bmatrix} \\ &= \left(I - (\delta - 1)\tilde{\Psi}\Omega\right)^{-1} (\delta - 1)\tilde{\Psi} \begin{bmatrix} (1-\alpha)\phi\theta_D^{X*} dm^{\$} \\ (1-\alpha^*)\phi^*\theta_D^{X} dm^{\$} \end{bmatrix} \end{aligned}$$

The response of markups are

$$\begin{bmatrix} d\mu_H \\ d\mu_F^* \\ d\mu_F \\ d\mu_H^* \\ d\mu_{HX} \\ d\mu_{FX}^* \\ d\mu_{FX} \\ d\mu_{HX}^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ (\delta-1)\theta_D^{C*} dm^{\$} \\ (\delta-1)\theta_D^{C} dm^{\$} \\ 0 \\ 0 \\ (\delta-1)\theta_D^{X*} dm^{\$} \\ (\delta-1)\theta_D^{X} dm^{\$} \end{bmatrix} + (\delta-1) \begin{bmatrix} dmc \\ dmc^* \\ dmc^* \\ dmc \\ dmc \\ dmc^* \\ dmc^* \\ dmc \end{bmatrix}$$

## CES Demands Case

**Output Equations** Lemma 5 is a generalized version of Lemma 4 that labor supply is endogenous and final and intermediate goods have CES demand structure. In other words, Lemma 4 is a corollary of Lemma 5 if Assumption 2 and 4 are imposed.

**Lemma 5** *(Output equation) The log change of Home and ROW output satisfy*

$$\begin{bmatrix} dy \\ dy^* \end{bmatrix} = \begin{bmatrix} d\tilde{u} \\ d\tilde{u}^* \end{bmatrix} + \begin{bmatrix} (1-\alpha)(1-\phi) & (1-\alpha^*)\phi^* \frac{\bar{Y}^*}{\bar{Y}} \\ (1-\alpha)\phi \frac{\bar{Y}}{\bar{Y}^*} & (1-\alpha^*)(1-\phi^*) \end{bmatrix} \begin{bmatrix} dy \\ dy^* \end{bmatrix}$$
$$- \begin{bmatrix} \frac{1}{\bar{Y}} + (1-\alpha^*)\phi^* \frac{\bar{Y}^*}{\bar{Y}} - \xi & -(1-\alpha^*)\phi^* \frac{\bar{Y}^*}{\bar{Y}} + \xi \\ -(1-\alpha)\phi \frac{\bar{Y}}{\bar{Y}^*} + \xi \frac{\bar{Y}}{\bar{Y}^*} & \frac{1}{\bar{Y}^*} + (1-\alpha)\phi \frac{\bar{Y}}{\bar{Y}^*} - \xi \frac{\bar{Y}}{\bar{Y}^*} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} d\upsilon - da + \frac{\alpha\varphi}{1+\varphi} dy \\ d\upsilon^* - da^* + \frac{\alpha^*\varphi}{1+\varphi} dy^* \end{bmatrix}$$

*where*

$$\begin{bmatrix} d\tilde{u} \\ d\tilde{u}^* \end{bmatrix} = - \begin{bmatrix} \frac{1-\gamma}{\bar{Y}} \left\{ (1-(1-\varepsilon)\gamma)\, d\mu_H + (1-\varepsilon)\gamma d\mu_F \right\} \\ \frac{1-\gamma}{\bar{Y}^*} \left\{ (1-(1-\varepsilon)\gamma)\, d\mu_F^* + (1-\varepsilon)\gamma d\mu_H^* \right\} \end{bmatrix}$$
$$- \begin{bmatrix} \frac{\gamma}{\bar{Y}} \left\{ (1-\varepsilon)(1-\gamma) d\mu_F^* + (1-(1-\varepsilon)(1-\gamma))\, d\mu_H^* \right\} \\ \frac{\gamma}{\bar{Y}^*} \left\{ (1-\varepsilon)(1-\gamma) d\mu_H + (1-(1-\varepsilon)(1-\gamma))\, d\mu_F \right\} \end{bmatrix}$$
$$- \begin{bmatrix} (1-\alpha)(1-\phi) \left\{ (1-(1-\theta)\phi)\, d\mu_{HX} + (1-\theta)\phi d\mu_{FX} \right\} \\ (1-\alpha^*)(1-\phi^*) \left\{ (1-(1-\theta)\phi^*)\, d\mu_{FX}^* + (1-\theta)\phi^* d\mu_{HX}^* \right\} \end{bmatrix}$$
$$- \begin{bmatrix} (1-\alpha^*)\phi^* \frac{\bar{Y}^*}{\bar{Y}} \left\{ (1-\theta)(1-\phi^*) d\mu_{FX}^* + (1-(1-\theta)(1-\phi^*))\, d\mu_{HX}^* \right\} \\ (1-\alpha)\phi \frac{\bar{Y}}{\bar{Y}^*} \left\{ (1-\theta)(1-\phi) d\mu_{HX} + (1-(1-\theta)(1-\phi))\, d\mu_{FX} \right\} \end{bmatrix}$$

*and*

$$\xi = (1-\theta)\left( (1-\alpha)(1-\phi)\phi + (1-\alpha^*)\phi^*(1-\phi^*)\frac{\bar{Y}^*}{\bar{Y}} \right)$$

**Global Trade Response** Under CES demands, final goods trade value in domestic currency is

$$\begin{aligned} C_R &= P_F C_F + \mathcal{E} P_H^* C_H^* = P_F \gamma \left(\frac{P_F}{P}\right)^{-\varepsilon} C + \mathcal{E} P_H^* \gamma \left(\frac{P_H^*}{P^*}\right)^{-\varepsilon} C^* \\ &= \gamma \left(\frac{P_F}{P}\right)^{1-\varepsilon} M + \gamma \left(\frac{P_H^*}{P^*}\right)^{1-\varepsilon} M \end{aligned}$$

In log deviation from the steady state,

$$\begin{aligned} dc_R &= \frac{1}{2}\left((1-\varepsilon)(dp_F - dp) + dm\right) + \frac{1}{2}\left((1-\varepsilon)(dp_H^* - dp^*) + dm\right) \\ &= dm + \frac{(1-\varepsilon)(1-\gamma)}{2}(dp_F - dp_H) + \frac{(1-\varepsilon)(1-\gamma)}{2}(dp_H^* - dp_F^*) \\ &= dm + \frac{(1-\varepsilon)(1-\gamma)}{2}(d\mu_F - d\mu_H) + \frac{(1-\varepsilon)(1-\gamma)}{2}(d\mu_H^* - d\mu_F^*) \end{aligned}$$

On the other hand, intermediate good trade value in domestic currency is

$$X_R = P_{FX} X_F + \mathcal{E} P_{HX}^* X_H^* = (1-\alpha)\phi \left(\frac{P_{FX}}{P_X}\right)^{1-\theta} MCY + (1-\alpha^*)\phi^* \left(\frac{P_{HX}^*}{P_X^*}\right)^{1-\theta} \mathcal{E} MC^* Y^*$$

In log deviation from the steady state,

$$\begin{aligned} dx_R &= \frac{(1-\alpha)\phi\bar{Y}}{(1-\alpha)\phi\bar{Y} + (1-\alpha^*)\phi^*\bar{Y}^*}\left\{(1-\theta)(dp_{FX} - dp_X) + dmc + dy\right\} \\ &+ \frac{(1-\alpha^*)\phi^*\bar{Y}^*}{(1-\alpha)\phi\bar{Y} + (1-\alpha^*)\phi^*\bar{Y}^*}\left\{(1-\theta)(dp_{HX}^* - dp_X^*) + de + dmc^* + dy^*\right\} \\ &= w\,(dmc + dy) + (1-w)\,(de + dmc^* + dy^*) \\ &+ w(1-\theta)(1-\phi)(dp_{FX} - dp_{HX}) + (1-w)(1-\theta)(1-\phi^*)(dp_{HX}^* - dp_{FX}^*) \\ &= w\,(dmc + dy) + (1-w)\,(de + dmc^* + dy^*) \\ &+ (1-\theta)\left\{w(1-\phi)(d\mu_{FX} - d\mu_{HX}) + (1-w)(1-\phi^*)(d\mu_{HX}^* - d\mu_{FX}^*)\right\} \\ &+ (1-\theta)\left((1-w)(1-\phi^*) - w(1-\phi)\right)(dmc - de - dmc^*) \end{aligned}$$

If $\varepsilon = 1$, then $dc_R = dm$, i.e., final goods trade value only depends on the money supply. If $\varepsilon > 1$, i.e. Home and ROW goods are substitutes, then increase in foreign prices (or markups) shrinks global trade to the extent that how substitutable the goods are, and how open the economies are. The sign is opposite when $\varepsilon < 1$, i.e. Home and ROW goods are complements. Same logic holds for intermediate goods trade value for elasticity $\theta$.

# Proofs

## Lemma 1

The log changes of marginal costs are

$$\begin{aligned} dmc &= -da + \alpha dw + (1-\alpha)(1-\phi)dp_{HX} + (1-\alpha)\phi dp_{FX} \\ &= -da + \alpha dw + (1-\alpha)(1-\phi)(d\mu_{HX} + dmc) + (1-\alpha)\phi(d\mu_{FX} + dmc^* + de) \\ &= \{(1-\alpha)(1-\phi)d\mu_{HX} + (1-\alpha)\phi d\mu_{FX}\} - da + \alpha dw \\ &+ (1-\alpha)(1-\phi)dmc + (1-\alpha)\phi dmc^* + (1-\alpha)\phi(dm - dm^*) \end{aligned}$$

$$\begin{aligned} dmc^* &= -da^* + \alpha^* dw^* + (1-\alpha^*)(1-\phi^*)dp^*_{FX} + (1-\alpha^*)\phi^* dp^*_{HX} \\ &= -da^* + \alpha^* dw^* + (1-\alpha^*)(1-\phi^*)(d\mu^*_{FX} + dmc^*) + (1-\alpha^*)\phi^*(d\mu^*_{HX} + dmc - de) \\ &= \{(1-\alpha^*)(1-\phi^*)d\mu^*_{FX} + (1-\alpha^*)\phi^* d\mu^*_{HX}\} - da^* + \alpha^* dw^* \\ &+ (1-\alpha^*)(1-\phi^*)dmc^* + (1-\alpha^*)\phi^* dmc - (1-\alpha^*)\phi^*(dm - dm^*) \end{aligned}$$

In a matrix form,

$$\begin{aligned} \begin{bmatrix} dmc \\ dmc^* \end{bmatrix} &= \begin{bmatrix} (1-\alpha)(1-\phi)d\mu_{HX} + (1-\alpha)\phi d\mu_{FX} \\ (1-\alpha^*)(1-\phi^*)d\mu^*_{FX} + (1-\alpha^*)\phi^* d\mu^*_{HX} \end{bmatrix} - \begin{bmatrix} da \\ da^* \end{bmatrix} + \begin{bmatrix} \alpha & 0 \\ 0 & \alpha^* \end{bmatrix} \begin{bmatrix} dw \\ dw^* \end{bmatrix} \\ &+ \begin{bmatrix} (1-\alpha)(1-\phi) & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & (1-\alpha^*)(1-\phi^*) \end{bmatrix} \begin{bmatrix} dmc \\ dmc^* \end{bmatrix} + \begin{bmatrix} (1-\alpha)\phi & -(1-\alpha)\phi \\ -(1-\alpha^*)\phi^* & (1-\alpha^*)\phi^* \end{bmatrix} \begin{bmatrix} dm \\ dm^* \end{bmatrix} \end{aligned}$$

The log changes of consumer prices are

$$\begin{aligned} dp &= (1-\gamma)dp_H + \gamma dp_F = (1-\gamma)\left(d\mu_H + dmc\right) + \gamma\left(d\mu_F + dmc^* + de\right) \\ &= \{(1-\gamma)d\mu_H + \gamma d\mu_F\} + (1-\gamma)dmc + \gamma dmc^* + \gamma(dm - dm^*) \end{aligned}$$

$$\begin{aligned} dp^* &= (1-\gamma)dp^*_F + \gamma dp^*_H = (1-\gamma)\left(d\mu^*_F + dmc^*\right) + \gamma\left(d\mu^*_H + dmc - de\right) \\ &= \{(1-\gamma)d\mu^*_F + \gamma d\mu^*_H\} + (1-\gamma)dmc^* + \gamma dmc - \gamma(dm - dm^*) \end{aligned}$$

In a matrix form,

$$\begin{bmatrix} dp \\ dp^* \end{bmatrix} = \begin{bmatrix} (1-\gamma)d\mu_H + \gamma d\mu_F \\ (1-\gamma)d\mu^*_F + \gamma d\mu^*_H \end{bmatrix} + \begin{bmatrix} \gamma & -\gamma \\ -\gamma & \gamma \end{bmatrix} \begin{bmatrix} dm \\ dm^* \end{bmatrix} + \begin{bmatrix} 1-\gamma & \gamma \\ \gamma & 1-\gamma \end{bmatrix} \begin{bmatrix} dmc \\ dmc^* \end{bmatrix}$$

By labor demand and supply relations, The log change of wages are

$$\begin{aligned} dw &= dp + dc + \varphi dl = dp + (dm - dp) + \varphi dl = dm + \varphi dl \\ &= dm + \varphi(dmc + dy - dw) \\ &= \frac{1}{1+\varphi} dm + \frac{\varphi}{1+\varphi}(dmc + dy) \end{aligned}$$

$$\begin{aligned} dw^* &= dp^* + dc^* + \varphi dl^* = dp^* + (dm^* - dp^*) + \varphi dl^* = dm^* + \varphi dl^* \\ &= dm^* + \varphi(dmc^* + dy^* - dw^*) \\ &= \frac{1}{1+\varphi} dm^* + \frac{\varphi}{1+\varphi}(dmc^* + dy^*) \end{aligned}$$

In a matrix form,

$$\begin{bmatrix} dw \\ dw^* \end{bmatrix} = \frac{1}{1+\varphi} \begin{bmatrix} dm \\ dm^* \end{bmatrix} + \frac{\varphi}{1+\varphi} \begin{bmatrix} dmc \\ dmc^* \end{bmatrix} + \frac{\varphi}{1+\varphi} \begin{bmatrix} dy \\ dy^* \end{bmatrix}$$

Combining these,

$$\begin{aligned} \begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \begin{bmatrix} dv \\ dv^* \end{bmatrix} - \begin{bmatrix} da \\ da^* \end{bmatrix} + \begin{bmatrix} \frac{\alpha\varphi}{1+\varphi} + (1-\alpha)(1-\phi) & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & \frac{\alpha^*\varphi}{1+\varphi} + (1-\alpha^*)(1-\phi^*) \end{bmatrix} \begin{bmatrix} dmc \\ dmc^* \end{bmatrix} \\ + \begin{bmatrix} \frac{\alpha}{1+\varphi} + (1-\alpha)\phi & -(1-\alpha)\phi \\ -(1-\alpha^*)\phi^* & \frac{\alpha^*}{1+\varphi} + (1-\alpha^*)\phi^* \end{bmatrix} \begin{bmatrix} dm \\ dm^* \end{bmatrix} + \begin{bmatrix} \frac{\alpha\varphi}{1+\varphi} & 0 \\ 0 & \frac{\alpha^*\varphi}{1+\varphi} \end{bmatrix} \begin{bmatrix} dy \\ dy^* \end{bmatrix} \end{aligned}$$

Denote input-output matrix $\tilde{\Omega}$ and Leontief inverse $\tilde{\Psi}$

$$\tilde{\Omega} = \begin{bmatrix} \frac{\alpha\varphi}{1+\varphi} + (1-\alpha)(1-\phi) & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & \frac{\alpha^*\varphi}{1+\varphi} + (1-\alpha^*)(1-\phi^*) \end{bmatrix}$$

$$\tilde{\Psi} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} = (I - \tilde{\Omega})^{-1} = \frac{1}{\tilde{D}} \begin{bmatrix} \frac{\alpha^*}{1+\varphi} + (1-\alpha^*)\phi^* & (1-\alpha)\phi \\ (1-\alpha^*)\phi^* & \frac{\alpha}{1+\varphi} + (1-\alpha)\phi \end{bmatrix}$$

where $\tilde{D} = \frac{\alpha\alpha^*}{(1+\varphi)^2} + \frac{\alpha(1-\alpha^*)\phi^* + \alpha^*(1-\alpha)\phi}{1+\varphi}$. Solving for marginal costs vector, equation (24) is derived.

$$\begin{bmatrix} dmc \\ dmc^* \end{bmatrix} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \left( \begin{bmatrix} dv \\ dv^* \end{bmatrix} - \begin{bmatrix} da \\ da^* \end{bmatrix} \right) + \begin{bmatrix} dm \\ dm^* \end{bmatrix} + \begin{bmatrix} \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{11} & \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{12} \\ \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21} & \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} dy \\ dy^* \end{bmatrix}$$

## Lemma 3

As the marginal cost equation from Lemma 1 and the markup equation from Lemma 2 are the linear system, I plug equation (24) into equation (26). For example, for the markup for Home import price of final good,

$$
\begin{aligned}
d\mu_F &= (\delta-1)\left(dmc^* + \theta_L^{C*}de - \theta_D^{C*}de_{\$F}\right) \\
&= (\delta-1)\left(\tilde{\psi}_{21}(d\upsilon - da) + \tilde{\psi}_{22}(d\upsilon^* - da^*) + dm^*\right) \\
&+ (\delta-1)\left(\frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22}dy^* + \theta_L^{C*}de - \theta_D^{C*}de_{\$F}\right) \\
&= (\delta-1)\tilde{\psi}_{21}\left\{(1-\alpha)(1-\phi)d\mu_{HX} + (1-\alpha)\phi d\mu_{FX}\right\} \\
&+ (\delta-1)\tilde{\psi}_{22}\left\{(1-\alpha^*)(1-\phi^*)d\mu_{FX}^* + (1-\alpha^*)\phi^* d\mu_{HX}^*\right\} \\
&+ (\delta-1)\left\{-\tilde{\psi}_{21}da - \tilde{\psi}_{22}da^* + \theta_P^{C*}dm^* + \theta_L^{C*}dm + \theta_D^{C*}dm^{\$} + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22}dy^*\right\}
\end{aligned}
$$

Stacking all markups in a vector and substituting marginal costs with equation (24),

$$
\frac{1}{\delta-1}\begin{bmatrix} d\mu_H \\ d\mu_F^* \\ d\mu_F \\ d\mu_H^* \\ d\mu_{HX} \\ d\mu_{FX}^* \\ d\mu_{FX} \\ d\mu_{HX}^* \end{bmatrix} = \begin{bmatrix}
-\tilde{\psi}_{11}da - \tilde{\psi}_{12}da^* + dm + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{11}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{12}dy^* \\
-\tilde{\psi}_{21}da - \tilde{\psi}_{22}da^* + dm^* + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22}dy^* \\
-\tilde{\psi}_{21}da - \tilde{\psi}_{22}da^* + \theta_P^{C*}dm^* + \theta_L^{C*}dm + \theta_D^{C*}dm^{\$} + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22}dy^* \\
-\tilde{\psi}_{11}da - \tilde{\psi}_{12}da^* + \theta_P^{C}dm + \theta_L^{C}dm^* + \theta_D^{C}dm^{\$} + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{11}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{12}dy^* \\
-\tilde{\psi}_{11}da - \tilde{\psi}_{12}da^* + dm + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{11}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{12}dy^* \\
-\tilde{\psi}_{21}da - \tilde{\psi}_{22}da^* + dm^* + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22}dy^* \\
-\tilde{\psi}_{21}da - \tilde{\psi}_{22}da^* + \theta_P^{X*}dm^* + \theta_L^{X*}dm + \theta_D^{X*}dm^{\$} + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{21}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{22}dy^* \\
-\tilde{\psi}_{11}da - \tilde{\psi}_{12}da^* + \theta_P^{X}dm + \theta_L^{X}dm^* + \theta_D^{X}dm^{\$} + \frac{\alpha\varphi}{1+\varphi}\tilde{\psi}_{11}dy + \frac{\alpha^*\varphi}{1+\varphi}\tilde{\psi}_{12}dy^*
\end{bmatrix}
$$

$$
+\begin{bmatrix}
\tilde{\psi}_{11}(1-\alpha)(1-\phi) & \tilde{\psi}_{12}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{11}(1-\alpha)\phi & \tilde{\psi}_{12}(1-\alpha^*)\phi^* \\
\tilde{\psi}_{21}(1-\alpha)(1-\phi) & \tilde{\psi}_{22}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{21}(1-\alpha)\phi & \tilde{\psi}_{22}(1-\alpha^*)\phi^* \\
\tilde{\psi}_{21}(1-\alpha)(1-\phi) & \tilde{\psi}_{22}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{21}(1-\alpha)\phi & \tilde{\psi}_{22}(1-\alpha^*)\phi^* \\
\tilde{\psi}_{11}(1-\alpha)(1-\phi) & \tilde{\psi}_{12}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{11}(1-\alpha)\phi & \tilde{\psi}_{12}(1-\alpha^*)\phi^* \\
\tilde{\psi}_{11}(1-\alpha)(1-\phi) & \tilde{\psi}_{12}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{11}(1-\alpha)\phi & \tilde{\psi}_{12}(1-\alpha^*)\phi^* \\
\tilde{\psi}_{21}(1-\alpha)(1-\phi) & \tilde{\psi}_{22}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{21}(1-\alpha)\phi & \tilde{\psi}_{22}(1-\alpha^*)\phi^* \\
\tilde{\psi}_{21}(1-\alpha)(1-\phi) & \tilde{\psi}_{22}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{21}(1-\alpha)\phi & \tilde{\psi}_{22}(1-\alpha^*)\phi^* \\
\tilde{\psi}_{11}(1-\alpha)(1-\phi) & \tilde{\psi}_{12}(1-\alpha^*)(1-\phi^*) & \tilde{\psi}_{11}(1-\alpha)\phi & \tilde{\psi}_{12}(1-\alpha^*)\phi^*
\end{bmatrix}\begin{bmatrix} d\mu_{HX} \\ d\mu_{FX}^* \\ d\mu_{FX} \\ d\mu_{HX}^* \end{bmatrix}
$$

First I take the last four elements of the markup vector and solve for them. Under Assumption 2 ($\varphi = 0$) and fully sticky prices ($\delta = 0$),

$$\begin{bmatrix} d\mu_{HX} \\ d\mu_{FX}^* \\ d\mu_{FX} \\ d\mu_{HX}^* \end{bmatrix} = \begin{bmatrix} -\alpha-(1-\alpha)\phi & 0 & (1-\alpha)\phi & 0 \\ 0 & -\alpha^*-(1-\alpha^*)\phi^* & 0 & (1-\alpha^*)\phi^* \\ 0 & (1-\alpha^*)(1-\phi^*) & -1 & (1-\alpha^*)\phi^* \\ (1-\alpha)(1-\phi) & 0 & (1-\alpha)\phi & -1 \end{bmatrix}$$
$$\times \begin{bmatrix} -\tilde{\psi}_{11}da - \tilde{\psi}_{12}da^* + dm \\ -\tilde{\psi}_{21}da - \tilde{\psi}_{22}da^* + dm^* \\ -\tilde{\psi}_{21}da - \tilde{\psi}_{22}da^* + \theta_P^{X*}dm^* + \theta_L^{X*}dm + \theta_D^{X*}dm^\$ \\ -\tilde{\psi}_{11}da - \tilde{\psi}_{12}da^* + \theta_P^{X}dm + \theta_L^{X}dm^* + \theta_D^{X}dm^\$ \end{bmatrix}$$

When all shocks are muted except $dm^\$ < 0$,

$$\begin{bmatrix} d\mu_{HX} \\ d\mu_{FX}^* \\ d\mu_{FX} \\ d\mu_{HX}^* \end{bmatrix} = \begin{bmatrix} (1-\alpha)\phi\theta_D^{X*}dm^\$ \\ (1-\alpha^*)\phi^*\theta_D^{X}dm^\$ \\ -\theta_D^{X*}dm^\$ + (1-\alpha^*)\phi^*\theta_D^{X}dm^\$ \\ -\theta_D^{X}dm^\$ + (1-\alpha)\phi\theta_D^{X*}dm^\$ \end{bmatrix}$$

Plugging this into the original equation,

$$\begin{bmatrix} d\mu_{H} \\ d\mu_{F}^* \\ d\mu_{F} \\ d\mu_{H}^* \end{bmatrix} = \begin{bmatrix} (1-\alpha)\phi\theta_D^{X*}dm^\$ \\ (1-\alpha^*)\phi^*\theta_D^{X}dm^\$ \\ -\theta_D^{C*}dm^\$ + (1-\alpha^*)\phi^*\theta_D^{X}dm^\$ \\ -\theta_D^{C}dm^\$ + (1-\alpha)\phi\theta_D^{X*}dm^\$ \end{bmatrix}$$

## Lemma 4

If Assumption 2 and 4 are imposed, the output equation in Lemma 5 becomes

$$\begin{bmatrix} dy \\ dy^* \end{bmatrix} = \begin{bmatrix} du \\ du^* \end{bmatrix} + \begin{bmatrix} (1-\alpha)(1-\phi) & (1-\alpha^*)\phi^*\frac{\bar{Y}^*}{\bar{Y}} \\ (1-\alpha)\phi\frac{\bar{Y}}{\bar{Y}^*} & (1-\alpha^*)(1-\phi^*) \end{bmatrix} \begin{bmatrix} dy \\ dy^* \end{bmatrix}$$
$$- \begin{bmatrix} \frac{1}{\bar{Y}} + (1-\alpha^*)\phi^*\frac{\bar{Y}^*}{\bar{Y}} & -(1-\alpha^*)\phi^*\frac{\bar{Y}^*}{\bar{Y}} \\ -(1-\alpha)\phi\frac{\bar{Y}}{\bar{Y}^*} & \frac{1}{\bar{Y}^*} + (1-\alpha)\phi\frac{\bar{Y}}{\bar{Y}^*} \end{bmatrix} \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} dv - da \\ dv^* - da^* \end{bmatrix}$$

As $\frac{1}{\bar{Y}} + (1-\alpha^*)\phi^*\frac{\bar{Y}^*}{\bar{Y}} = 1-(1-\alpha)(1-\phi)$ and $\frac{1}{\bar{Y}^*} + (1-\alpha)\phi\frac{\bar{Y}}{\bar{Y}^*} = 1-(1-\alpha^*)(1-\phi^*)$, solving for the output vector, equation (28) is derived.

## Lemma 5

By log-linearizing the market clearing conditions,

$$dy = \frac{1-\gamma}{\bar{Y}} dc_H + \frac{\gamma}{\bar{Y}} dc_H^* + (1-\alpha)(1-\phi)dx_H + \frac{(1-\alpha^*)\phi^*\bar{Y}^*}{\bar{Y}} dx_H^*$$

$$dy^* = \frac{1-\gamma}{\bar{Y}^*} dc_F^* + \frac{\gamma}{\bar{Y}^*} dc_F + (1-\alpha^*)(1-\phi^*)dx_F^* + \frac{(1-\alpha)\phi\bar{Y}}{\bar{Y}^*} dx_F$$

Stacking all demands in a vector and substituting marginal costs with equation (24),

$$\begin{bmatrix} dc_H \\ dc_F^* \\ dc_F \\ dc_H^* \\ dx_H \\ dx_F^* \\ dx_F \\ dx_H^* \end{bmatrix} = \begin{bmatrix} -\varepsilon\left(dp_H - dp\right) + dc \\ -\varepsilon\left(dp_F^* - dp^*\right) + dc^* \\ -\varepsilon\left(dp_F - dp\right) + dc \\ -\varepsilon\left(dp_H^* - dp^*\right) + dc^* \\ -\theta\left(dp_{HX} - dp_X\right) - \left(dp_X - dmc\right) + dy \\ -\theta\left(dp_{FX}^* - dp_X^*\right) - \left(dp_X^* - dmc^*\right) + dy^* \\ -\theta\left(dp_{FX} - dp_X\right) - \left(dp_X - dmc\right) + dy \\ -\theta\left(dp_{HX}^* - dp_X^*\right) - \left(dp_X^* - dmc^*\right) + dy^* \end{bmatrix}$$

$$= \begin{bmatrix} -\left(1-(1-\varepsilon)\gamma\right) d\mu_H - (1-\varepsilon)\gamma d\mu_F \\ -\left(1-(1-\varepsilon)\gamma\right) d\mu_F^* - (1-\varepsilon)\gamma d\mu_H^* \\ -(1-\varepsilon)(1-\gamma)d\mu_H - \left(1-(1-\varepsilon)(1-\gamma)\right) d\mu_F \\ -(1-\varepsilon)(1-\gamma)d\mu_F^* - \left(1-(1-\varepsilon)(1-\gamma)\right) d\mu_H^* \\ -\left(1-(1-\theta)\phi\right) d\mu_{HX} - (1-\theta)\phi d\mu_{FX} \\ -\left(1-(1-\theta)\phi^*\right) d\mu_{FX}^* - (1-\theta)\phi^* d\mu_{HX}^* \\ -(1-\theta)(1-\phi)d\mu_{HX} - \left(1-(1-\theta)(1-\phi)\right) d\mu_{FX} \\ -(1-\theta)(1-\phi^*)d\mu_{FX}^* - \left(1-(1-\theta)(1-\phi^*)\right) d\mu_{HX}^* \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ dy \\ dy^* \\ dy \\ dy^* \end{bmatrix}$$

$$+ \begin{bmatrix} -\left(1-(1-\varepsilon)\gamma\right) & -(1-\varepsilon)\gamma \\ -(1-\varepsilon)\gamma & -\left(1-(1-\varepsilon)\gamma\right) \\ -(1-\varepsilon)(1-\gamma) & -\left(1-(1-\varepsilon)(1-\gamma)\right) \\ -\left(1-(1-\varepsilon)(1-\gamma)\right) & -(1-\varepsilon)(1-\gamma) \\ (1-\theta)\phi & -(1-\theta)\phi \\ -(1-\theta)\phi^* & (1-\theta)\phi^* \\ \left(1-(1-\theta)(1-\phi)\right) & -\left(1-(1-\theta)(1-\phi)\right) \\ -\left(1-(1-\theta)(1-\phi^*)\right) & \left(1-(1-\theta)(1-\phi^*)\right) \end{bmatrix} \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} \end{bmatrix} \begin{bmatrix} d\upsilon - da + \frac{\alpha\varphi}{1+\varphi} dy \\ d\upsilon^* - da^* + \frac{\alpha^*\varphi}{1+\varphi} dy^* \end{bmatrix}$$

Plugging this into the log-linearized market clearing conditions, I get the desired result.

## Proposition 1

From Lemma 1 and Lemma 4,

$$\frac{dmc}{d\mathcal{M}} + \frac{dy}{d\mathcal{M}} = \psi_{11}\frac{du}{d\mathcal{M}} + \psi_{21}\frac{\bar{Y}^*}{\bar{Y}}\frac{du^*}{d\mathcal{M}} = \begin{bmatrix} -\frac{1-\gamma}{\bar{Y}}\psi_{11} \\ -\frac{1-\gamma}{\bar{Y}}\psi_{21} \\ -\frac{\gamma}{\bar{Y}}\psi_{21} \\ -\frac{\gamma}{\bar{Y}}\psi_{11} \\ -(1-\alpha)(1-\phi)\psi_{11} \\ -(1-\alpha^*)(1-\phi^*)\frac{\bar{Y}^*}{\bar{Y}}\psi_{21} \\ -(1-\alpha)\phi\psi_{21} \\ -(1-\alpha^*)\phi^*\frac{\bar{Y}^*}{\bar{Y}}\psi_{11} \end{bmatrix}$$

$$\frac{dmc^*}{d\mathcal{M}} + \frac{dy^*}{d\mathcal{M}} = \psi_{12}\frac{\bar{Y}}{\bar{Y}^*}\frac{du}{d\mathcal{M}} + \psi_{22}\frac{du^*}{d\mathcal{M}} = \begin{bmatrix} -\frac{1-\gamma}{\bar{Y}^*}\psi_{12} \\ -\frac{1-\gamma}{\bar{Y}^*}\psi_{22} \\ -\frac{\gamma}{\bar{Y}^*}\psi_{22} \\ -\frac{\gamma}{\bar{Y}^*}\psi_{12} \\ -(1-\alpha)(1-\phi)\frac{\bar{Y}}{\bar{Y}^*}\psi_{12} \\ -(1-\alpha^*)(1-\phi^*)\psi_{22} \\ -(1-\alpha)\phi\frac{\bar{Y}}{\bar{Y}^*}\psi_{22} \\ -(1-\alpha^*)\phi^*\psi_{12} \end{bmatrix}$$

Therefore,

$$\frac{dx_R}{d\mathcal{M}} = w\left(\frac{dmc}{d\mathcal{M}} + \frac{dy}{d\mathcal{M}}\right) + (1-w)\left(\frac{dmc^*}{d\mathcal{M}} + \frac{dy^*}{d\mathcal{M}}\right) = \begin{bmatrix} -(1-\gamma)\left(\frac{w}{\bar{Y}}\psi_{11} + \frac{1-w}{\bar{Y}^*}\psi_{12}\right) \\ -(1-\gamma)\left(\frac{w}{\bar{Y}}\psi_{21} + \frac{1-w}{\bar{Y}^*}\psi_{22}\right) \\ -\gamma\left(\frac{w}{\bar{Y}}\psi_{21} + \frac{1-w}{\bar{Y}^*}\psi_{22}\right) \\ -\gamma\left(\frac{w}{\bar{Y}}\psi_{11} + \frac{1-w}{\bar{Y}^*}\psi_{12}\right) \\ -(1-\alpha)(1-\phi)\left(w\psi_{11} + (1-w)\frac{\bar{Y}}{\bar{Y}^*}\psi_{12}\right) \\ -(1-\alpha^*)(1-\phi^*)\left(w\frac{\bar{Y}^*}{\bar{Y}}\psi_{21} + (1-w)\psi_{22}\right) \\ -(1-\alpha)\phi\left(w\psi_{21} + (1-w)\frac{\bar{Y}}{\bar{Y}^*}\psi_{22}\right) \\ -(1-\alpha^*)\phi^*\left(w\frac{\bar{Y}^*}{\bar{Y}}\psi_{11} + (1-w)\psi_{12}\right) \end{bmatrix}$$

The desired result is derived by the chain rule when combining the above result with Lemma 3 as below.

$$\frac{dx_R}{dm^\$} = \frac{dx_R}{d\mathcal{M}}\frac{d\mathcal{M}}{dm^\$}$$

where

$$\frac{d\mathcal{M}}{dm^{\$}} = \begin{bmatrix} (1-\alpha)\phi\theta^{X*}_{DCP} \\ (1-\alpha^*)\phi^*\theta^{X}_{DCP} \\ -\theta^{C*}_{DCP} + (1-\alpha^*)\phi^*\theta^{X}_{DCP} \\ -\theta^{C}_{DCP} + (1-\alpha)\phi\theta^{X*}_{DCP} \\ (1-\alpha)\phi\theta^{X*}_{DCP} \\ (1-\alpha^*)\phi^*\theta^{X}_{DCP} \\ -\theta^{X*}_{DCP} + (1-\alpha^*)\phi^*\theta^{X}_{DCP} \\ -\theta^{X}_{DCP} + (1-\alpha)\phi\theta^{X*}_{DCP} \end{bmatrix}$$