Cyclical Inequality in the Cost of Living and Implications for Monetary Policy *

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Abstract

This paper documents that households with higher marginal propensity to consume (MPC) tend to consume goods with more flexible prices. Consequently, this group of households face more cyclical and volatile inflation, and also experience higher inflation after an expansionary monetary policy shock. We embed the relationship between households' MPC and the price stickiness of their consumption basket into a tractable multi-sector Two-Agent New Keynesian (TANK) model. Analytically, we demonstrate that this relationship dampens the effectiveness of monetary policy, with a 15% reduction in efficacy compared to a benchmark model featuring homogeneous consumption baskets. The optimal monetary policy, in the presence of heterogeneous consumption baskets, differs qualitatively from its TANK counterpart. The introduction of such baskets results in an inherently inefficient flexible-price equilibrium and introduces a new type of tradeoff between stabilization and redistribution.

Keywords: TANK, HANK, Monetary transmission, Redistributional channel, Price stickiness, Optimal monetary policy, Inequality, Multi-sector model

JEL Classification: E31, E32, E52, E58

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1 Introduction

Two questions remain central in monetary economics: the impact of monetary policy on aggregate consumption and the optimal design of monetary policy. Recent advances in the Heterogeneous Agent New Keynesian (HANK) literature contribute to the understanding of these questions, emphasizing the critical role of household heterogeneity. A key channel in HANK, known as the redistributional channel (Auclert (2019); Bilbiie (2020)), shows that the potency of monetary policy hinges on the relationship between households' marginal propensity to consume (MPC) and the cyclicality of their *real* income, defined as households' nominal income adjusted by the price index they face.

However, most theories and applications in the HANK literature assume that households consume the same baskets of goods and therefore face the same price index. In doing so, these studies largely overlook the differential cyclicality in households' cost of living (or prices), concentrating instead on the cyclicality of nominal income across households with different MPCs.

In this paper, we argue that it is important to consider the relationship between households' MPC and the cyclicality of their cost of living, both for understanding the monetary transmission and designing the optimal monetary policy. Our argument unfolds in three steps. First, we empirically measure this relationship. We find that households with higher MPCs face more cyclical and volatile inflation, and experience greater inflation following an expansionary monetary policy shock. This is because these households allocate a larger portion of their spending on product categories with more flexible prices. Second, we embed this negative relationship between households' MPC and faced price stickiness in a tractable multi-sector two-agent New Keynesian (TANK) model with heterogeneous consumption baskets, demonstrating analytically that this relationship dampens the general equilibrium effect in monetary transmission. Third, we argue that accounting for cyclical inequality in the cost of living across households is crucial for designing optimal monetary policy. We show that central banks should keep the inflation of the more-flexible-price sector stable to reduce the inflation volatility faced by high-MPC households, in contrast to the conventional wisdom of stabilizing the "core" inflation (e.g. Benigno (2004)).

Our empirical findings build on the use of three data sets: the US Consumer Expenditure Survey (CEX) microdata, the item-level consumer prices data from the Bureau of Labor Statistics (BLS), and the data on frequency of price adjustment constructed by Nakamura and Steinsson (2008). As is well known, households' MPCs are typically not directly observable. We therefore follow the idea in Cloyne et al. (2020) of using housing tenure status as a qualitative proxy for households' MPC. Specifically, we split households in the CEX into two groups based on their housing tenure status: the low-MPC group consisting of outright homeowners, and the high-

MPC group including renters and mortgage payers.

Our empirical analysis offers three main findings. First, high-MPC households spend more on product categories whose prices are on average more flexible. Specifically, 23.4% of goods consumed by high-MPC households change prices in a given month, while this number is 19.7% for low-MPC households. Second, the inflation faced by high-MPC households is more cyclical and more volatile. Third, high-MPC households' Consumer Price Index (CPI) is more responsive to monetary policy shocks. For example, the CPI of high-MPC households responds 10% more than that of low-MPC households 36 months after an expansionary monetary policy shock.

We develop a tractable multi-sector TANK model, incorporating our empirical findings, which permits heterogeneous consumption baskets across households. In our framework, households differ in MPCs, and consume different baskets of goods. There are two types of households: the Keynesians (also known as hand-to-mouth consumers) and the Ricardians (known as savers). Firms adjust prices infrequently à la Calvo, and the degree of price stickiness differs across sectors. Consequently, the Keynesians and the Ricardians consume goods with, on average, different frequency of price adjustment.

Our main finding highlights that the equilibrium responses of aggregate consumption to a sequence of future real interest rate changes, denoted as $\{r_{t+s}\}_{s=0}^{\infty}$, in both TANK and the TANK with heterogeneous consumption baskets (henceforth TANK-HT), can be succinctly summarized through the dampening (amplification) channel in the following two equations:

$$\text{TANK:} \qquad c_t = -\phi_r \mathrm{E}_t \sum_{s=0}^{\infty} r_{t+s}, \tag{1}$$

$$\text{TANK-HT:} \qquad c_t = -\phi_r \mathrm{E}_t \sum_{s=0}^{\infty} r_{t+s} - \phi_p (p_t^K - p_t^R), \tag{2}$$

where $\phi_r > 0$ and $\phi_p > 0$ are functions of model parameters. Compared to a benchmark TANK model, the aggregate consumption response is dampened if Keynesian households face more flexible prices, $p_t^K - p_t^R > 0$; the aggregate consumption response is amplified if Keynesian households face stickier prices, $p_t^K - p_t^R < 0$.

This dampening (amplification) channel works through the general equilibrium effect. In the scenario where Keynesian households face more flexible prices, as suggested by the data, an expansionary monetary policy shock results in higher inflation for these households. Consequently, they receive less real income (consumption). As Keynesian households have a higher MPC, the aggregate Keynesian multiplier weakens, leading to a dampening of the general equilibrium effect.

The dampening (amplification) arises from heterogeneity in inflation responses and is quantitatively substantial. Unlike in the representative agent New Keynesian models (RANK) and

TANK, where price stickiness becomes irrelevant for aggregate demand once the path of real interest rate is fixed, our model introduces a distinctive feature. In our framework, the distribution of price stickiness across agents shapes aggregate demand by dampening (amplifying) the general equilibrium effect, even *conditional* on the path of real interest rates. In the calibrated version of our model, the real effects of monetary policy, measured by the cumulative consumption response, are 15% less compared to a benchmark multi-sector TANK model with homogeneous consumption baskets. Our results suggest that ignoring the negative relationship between households' MPC and price stickiness overstates the real effects of monetary policy.

We finally show that the optimal monetary policy in TANK-HT differs qualitatively from what it would be in its TANK counterpart. The introduction of heterogeneous consumption baskets leads to intrinsically inefficient flexible-price equilibrium. Consequently, stabilizing the output gap, or equivalently "core" inflation, becomes suboptimal. To characterize the optimal monetary policy, we introduce sector-level productivity shocks into our model and approximate the social welfare function to second order. We start with establishing a benchmark result, identifying sufficient and necessary conditions when inequality is irrelevant for designing optimal policy. We show that closing the aggregate output gap simultaneously stabilizes prices, minimizes inequality and achieves the social optimum *if and only if* the following conditions are satisfied: 1) Prices are sticky within at most one sector, and are perfectly flexible in all other sectors, 2) households have same consumption baskets. Under these conditions, the price dispersion between and within sectors, as well as income inequality across households, are exactly proportional to the output gap. Closing the output gap is thus socially optimal. This result is a generalization of the "divine coincidence" in RANK to HANK. In particular, it holds in the one-sector TANK model.

We demonstrate that the introduction of heterogeneous consumption baskets generates an endogenous and time-varying wedge between the flexible-price equilibrium and the efficient one. That is, the flexible-price equilibrium is socially *inefficient*. This inefficiency emerges because households' price indices (or costs of living) are subject to different exposures to sectoral shocks. These exposures cannot be fully insured because the Keynesians lack the ability to trade financial assets. Consequently, the difference in price indices and real wages distorts households' labor supply and consumption decisions, giving rise to misallocation (or inequality). Optimal policy, therefore, necessitates striking a balance between stabilizing inequality and achieving the conventional goal – namely, stabilizing aggregate demand and inflation. As a result, the pursuit of output gap stabilization, as suggested in Aoki (2001), is no longer optimal.

We finally study the optimal inflation index stabilization policy. Our optimal inflation index assigns a greater weight to the flexible-price sector than the classical index in the optimal monetary policy literature, such as Benigno (2004), where output gap stabilization is nearly optimal.[1] Stabilizing the aggregate output gap necessitates assigning larger weights to sectors with stickier prices, ultimately leading to the stabilization of the "core" inflation. However, in our model, the policy to stabilize the aggregate output gap or "core" inflation could generate substantial welfare losses, making it less desirable. To illustrate, consider a negative productivity shock hitting the economy. The aggregate output gap becomes positive, and simultaneously, inequality rises due to a more pronounced increase in the cost of living for the Keynesians. As these households lack the means to insure themselves against such shocks, it becomes optimal for the central bank to tolerate some fluctuations in the aggregate output gap to reduce the volatility of the Keynesians' price indices (or the cost of living). This can be achieved by stabilizing the prices of goods produced in the more-flexible-price sector. When the difference in faced price stickiness between households is large enough, and thus the redistributive motive is sufficiently strong , the result in Benigno (2004) can even be overturned: it becomes more desirable to stabilize the "headline" CPI than the "core" CPI.

Related Literature. This paper mainly contributes to three strands of literature. First, it is related to the recent literature on monetary transmission in HANK models. Recent work includes empirical studies (e.g., Coibion et al. (2017); Cloyne et al. (2020); Holm et al. (2021); Amberg et al. (2022); Andersen et al. (2022); Patterson (2023)), analytical works (e.g. Galí et al. (2007); Bilbiie (2008); Werning (2015); Debortoli and Galí (2017); Auclert (2019); Acharya and Dogra (2020); Bilbiie (2020); Ravn and Sterk (2021); Bilbiie (2021)) and quantitative models (e.g., McKay et al. (2016); Kaplan et al. (2018); Auclert et al. (2020)). Our paper builds upon and contributes to this line of literature in two main aspects. First, we propose a new channel that demonstrates how the relationship between households' MPC and the degree of price stickiness they face affects the monetary transmission to consumption, thereby connecting heterogeneity on the supply side and demand side. Second, we present direct empirical evidence that households with different MPCs consume different baskets of goods, face different degrees of price stickiness, and consequently, experience different cyclicalities in the cost of living.

Second, this paper is also related to the fast-growing literature on designing optimal policies in HANK, both analytically (e.g., Bilbiie (2008); Cúrdia and Woodford (2016); Nistico (2016); Debortoli and Galí (2017); Challe (2020); Bilbiie and Ragot (2021); Bilbiie et al. (2021); Dávila and Schaab (2022); Acharya et al. (2023); Jennifer and Morrison (2023)) and quantitatively (Bhandari et al. (2021); Gornemann et al. (2021); Le Grand et al. (2021); Yang (2022); McKay and Wolf (2023)). Our paper complements these studies through an extended analytical framework. In this framework, we establish both sufficient and necessary conditions under which inequality

[1]The result that output gap stabilization is nearly optimal holds in a variety of New Keynesian models, for recent examples, see La'O and Tahbaz-Salehi (2022) and Rubbo (2023).

becomes irrelevant for optimal policy, so that output gap (or price) stabilization remains optimal. We further demonstrate that the introduction of heterogeneous consumption baskets leads to an intrinsically inefficient flexible-price equilibrium, thereby giving rising to a new type of tradeoff between stabilization and redistribution.

Lastly, our paper is related to research on monetary transmission and optimal monetary policy in multi-sector models. In the existing literature, the primary focus lies in understanding how sectoral heterogeneity shapes the slope of the Phillips curve (e.g., Carvalho (2006); Nakamura and Steinsson (2010);Pasten et al. (2020);Carvalho et al. (2021)). Our paper contributes by demonstrating how mapping the distribution of price stickiness on the supply side to MPC on the demand side provides new insights into monetary transmission. Regarding optimal monetary policy, the conventional wisdom is to stabilize the output gap or, equivalently, prices in sectors with stickier prices (e.g., Mankiw and Reis (2003); Huang and Liu (2005);La'O and Tahbaz-Salehi (2022); Rubbo (2023)). Our paper demonstrates that in the presence of heterogeneous consumption baskets across households, stabilizing the output gap can generate substantial welfare losses due to the redistributive motive arising from stabilizing the cost of living faced by high-MPC households. Our paper offers a new justification on stabilizing the price of flexible-price sectors.

Outline. The paper proceeds as follows. Section 2 describes the data and presents the main empirical findings. Section 3 specifies the multi-sector TANK model. Section 4 illustrates the key mechanism. Section 5 presents the quantitative analysis. Section 6 studies the optimal monetary policy. Section 7 concludes.

2 Empirical Findings

The redistributional channel in HANK (Auclert (2019); Bilbiie (2020)) suggests that, in the wake of a monetary expansion, if households with higher MPC receive disproportionately more *real* income, the general equilibrium (GE) effects become stronger, enhancing the efficacy of monetary policy. This channel can be succinctly summarized in a single equation (as follows), underscoring that the real impact of monetary policy hinges on the covariance between households' MPC and the exposure of their *real* income to changes in real interest rates,

$$\text{cov}\left(\text{MPC}_i, \chi_i\right) = \text{cov}\left(\text{MPC}_i, \frac{\mathrm{d}(E_i/P_i)}{\mathrm{d}R}\frac{R}{E_i/P_i}\right) = \text{cov}\left(\text{MPC}_i, \frac{\mathrm{d}E_i}{\mathrm{d}R}\frac{R}{E_i}\right) - \text{cov}\left(\text{MPC}_i, \frac{\mathrm{d}P_i}{\mathrm{d}R}\frac{R}{P_i}\right), \quad (3)$$

where R is the gross real interest rate, E_i is household i's nominal income, and P_i is the *household-specific* consumer price index (CPI), representing household i's cost of living. Thus,

E_i/P_i quantifies the real income of household i, and $\chi_i = \frac{\mathrm{d}(E_i/P_i)}{\mathrm{d}R}\frac{R}{E_i/P_i}$ denotes the elasticity of household i's real income with respect to the real interest rate.

The covariance in equation (3) can be further decomposed into two terms. The first term involves nominal income E_i and the second term is associated with the cost of living P_i. Existing theories and applications in the HANK literature assume that households consume the same baskets of goods, implying that the price index P_i does not vary across households. Consequently, the second term on the right-hand side of equation (3) becomes zero. In other words, previous studies primarily focus on the first term – the heterogeneity in the cyclicality of *nominal* income of households with different MPCs – while largely ignore the second term: the heterogeneity in households' cost of living.[2]

Motivated by equation (3), our empirical analysis examines whether the cost of living of households with different MPCs responds differently to monetary policy shocks. Specifically, we document the following facts. First, households with different MPCs differ in their consumption baskets. Second and consequently, the goods they purchase are, on average, have different degrees of price stickiness. Finally, household with different MPCs experience different inflation rates (or rises in the cost of living) following an expansionary monetary policy shock.

Additionally, we show that the cost of living for high-MPC households is, on average, more cyclical and volatile. We explore the implications of this finding for social welfare and the design of optimal monetary policy in Section 6.

2.1 Data

To address our empirical question, similar to Cravino et al. (2020), we leverage three data sources: first, the US Consumption Expenditure Survey (CEX) microdata, which provides detailed information on households' expenditure across specific product categories, and importantly, their demographic details, such as, income, gender and housing tenure status; second, the monthly price indices on detailed product categories (item level) from the Bureau of Labor Statistics (BLS); and finally, the frequency of price adjustment (FPA), provided and constructed by Nakamura and Steinsson (2008).

The CEX survey collects households' expenditures through two modules: the Diary and the Interview. The Diary module is designed to collect expenditures on daily and frequent purchases, while the Interview module is focused on measuring large and durable expenditures. These modules survey different households at different frequencies, making it impossible to observe the

[2]The literature actually focuses on $\mathrm{cov}\left(\mathrm{MPC}_i, \frac{\mathrm{d}(E_i/P)}{\mathrm{d}R}\frac{R}{E_i/P}\right)$, which is equal to $\mathrm{cov}\left(\mathrm{MPC}_i, \frac{\mathrm{d}E_i}{\mathrm{d}R}\frac{R}{E_i}\right) - \mathrm{cov}\left(\mathrm{MPC}_i, \frac{\mathrm{d}P}{\mathrm{d}R}\frac{R}{P}\right)$. Since the second term is equal to 0, we have $\mathrm{cov}\left(\mathrm{MPC}_i, \frac{\mathrm{d}(E_i/P)}{\mathrm{d}R}\frac{R}{E_i/P}\right) = \mathrm{cov}\left(\mathrm{MPC}_i, \frac{\mathrm{d}E_i}{\mathrm{d}R}\frac{R}{E_i}\right)$.

full consumption baskets of individual households.[3] Following BLS approach of constructing household consumption basket for CPI, we aggregate households based on demographic characteristics.[4] Specifically, to explore the heterogeneity in consumption baskets across households with different MPCs, we adopt the approach proposed by Cloyne et al. (2020), categorizing households in the CEX into high- and low-MPC groups based on their housing tenure status. Cloyne et al. (2020) demonstrates that housing tenure statuses is the most effective demographic factors for categorizing households' MPC heterogeneity. In particular, they establish that renters and mortgagors are good proxies of the "poor" and "wealthy" hand-to-mouth households, while homeowners capture the permanent-income consumers.[5]

Following their approach, we assign renters and mortgage payers to the high-MPC group and outright homeowners to the low-MPC group. The MPC-group-specific consumption expenditure shares at the detailed product category level are denoted as $\omega^h_{j,t}$, where j is the consumption category, h indicates the household MPC group, and t refers to time period.

The MPC-group-specific frequency of price changes is then computed by combining the the MPC-group-specific expenditure shares with data on the frequency of price changes from Nakamura and Steinsson (2008). Specifically, the average frequency of price changes faced by household-type h is equal to $\alpha^h = (\sum_1^T \sum_j \omega^h_{j,t}\alpha_j)/T$, where T is the length of the sample period and α_j is the average frequency of price changes of goods in product category j.

The MPC-group-specific CPIs, denoted as $p^h_{j,t}$, can then be calculated using the monthly detailed product category level prices $\{p_{j,t}\}$ and the expenditure weights $\omega^h_{j,t}$ constructed above. When calculating these statistics, we closely follow the procedure adopted by the BLS to compute the CPI, including how the weights $\omega^h_{j,t}$ are updated. Section A.1 in the Supplementary Appendix details how we construct the expenditure weights and the group-specific CPIs.

2.2 Relationship between MPC and Price Stickiness

To examine the relationship between households' MPC and the cyclicality of their cost of living, we proceed in three steps. In the first step, we document the relationship between households' MPC and the price stickiness of their consumption baskets. Our findings reveal that the average frequency of price changes for high-MPC households' consumption basket is 23.4%, roughly

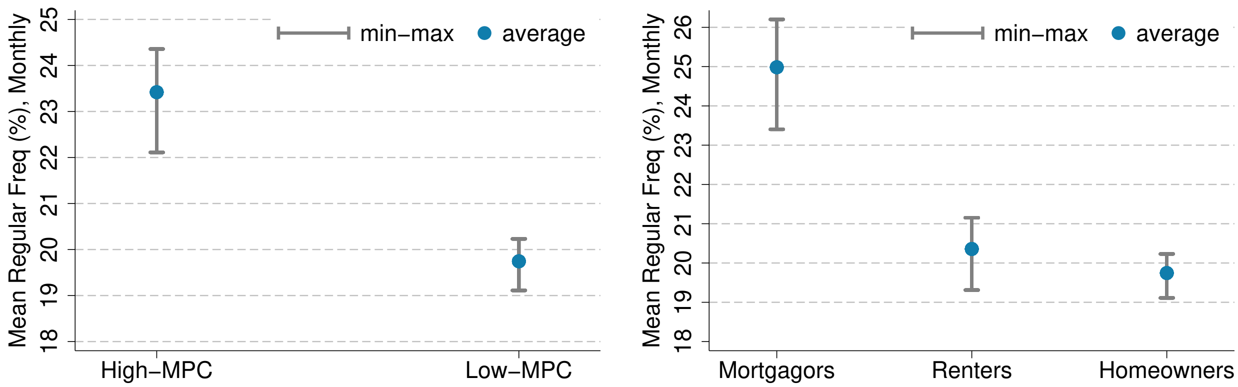
[3] A household in the CEX Diary module does not appear in the Interview module, and vice versa. Please see section A.1 in the Supplementary Appendix for more details.

[4] Please refer to the Bureau of Labor Statistics Handbook of Methods: Consumer Price Index for details, https://www.bls.gov/opub/hom/cpi/home.htm.

[5] Combined with US Survey of Consumer Finance, Cloyne et al. (2020) show that households' housing tenure status predicts well their financial positions: renters typically have little wealth, being younger and poorer, and mortgagors tend to have little liquidity but own sizable illiquid assets. The homeowners own both a significant amount of liquid and illiquid assets. This is consistent with findings in Kaplan et al. (2014). Figure E.1 of the Supplementary Appendix shows the distribution of age, education, and income for renters, mortgagors, and homeowners.

20% higher than that of low-MPC households, which is 19.7%, as shown in the left panel of Figure 1. The right panel, on the other hand, explores the price stickiness of goods purchased by households with different housing status. It shows that prices of goods purchased by mortgagors are the most flexible. On average, 25% of the prices of goods purchased by mortgagors change in a given month, with renters following at 20.4%, while homeowners (without mortgages) face the stickiest prices, with an average frequency of regular price changes falling to 19.7% per month.[6]

Figure 1: Regular Frequency of Price Changes and MPC Groups



Note: This figure plots the average frequency of households with different MPCs (left panel) and different housing tenures (right panel). For each year, we calculate the average frequency of price changes. The gray bar shows the minimum and maximum of these average frequencies of price changes, and the blue dot shows the mean of these averages.

This difference in the price stickiness faced by households across housing tenure groups is primarily driven by their different consumption patterns. Table E.1 in the Supplementary Appendix lists the 10 product categories with the largest differences in expenditure shares between mortgagors and homeowners. The top categories consumed by mortgagors relative to homeowners include goods with the most flexible prices, such as Gasoline, Used cars, and Vehicle leasing. On the flip side, the top categories consumed by homeowners relative to mortgagors are mainly services, especially medical care services such as Hospital services, General medical practice, and Care of elderly in the home, which are among the stickiest-price categories. Overall, the households' MPC and the price stickiness they face exhibit a negative relationship.

2.3 Group-specific Price Responses to Monetary Policy Shocks

In the second step, we assess the responses of household MPC-group-specific prices to monetary policy shocks. We adopt the local projection method of Jordà (2005). In particular, we estimate a

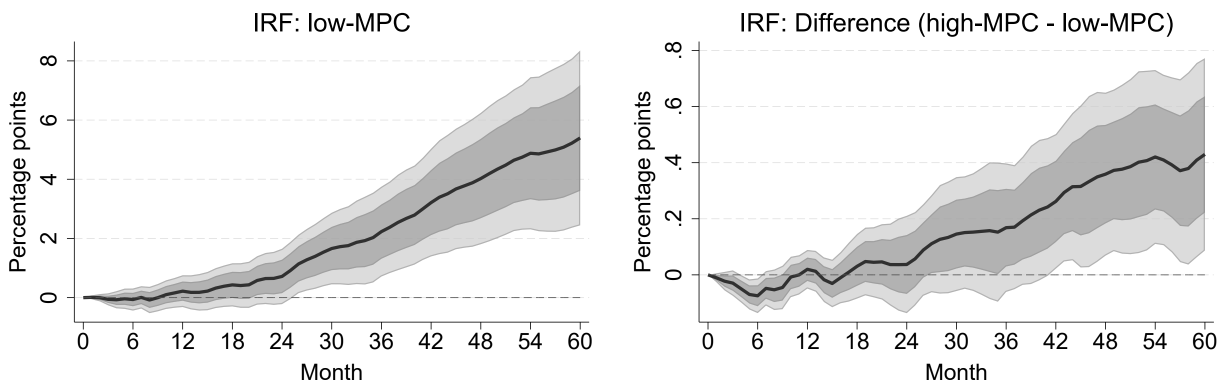
[6]Following the literature, we focus on regular price changes throughout the analysis.

series of regression for each MPC group h over different horizons on period-t monetary policy shocks, controlling for lags of shocks and inflation. Specifically, we run the following regressions:

$$p_{t+s}^h - p_{t-1}^h = \alpha_s^h + \theta_s^h shock_t^{RR} + \sum_{j=1}^{J} \beta_{s,j}^h (p_{t-j}^h - p_{t-j-1}^h) + \sum_{i=1}^{I} \gamma_{s,i}^h shock_{t-i}^{RR} + \epsilon_{t+s}^h, \quad (4)$$

where p_t^h is the log of MPC-group-specific CPIs, and $shock_t^{RR}$ is the Romer and Romer (2004) measure of monetary policy shocks from Coibion et al. (2017). Here, θ_s^h is the coefficient of interest, measuring the percentage (point) change of prices at period $t+s$ relative to period $t-1$ in response to a 100-basis-point monetary policy shock at period t. The control variables contain 24 lags of the shocks (I = 24) and 6 lags of monthly group-specific inflation (J = 6). The sample is monthly from 1969m1 to 2008m12, and the estimated horizon is 60 months.

Figure 2: Impulse responses of prices to monetary policy shocks



Note: The left panel plots the impulse response function (IRF) of (log) CPI of low-MPC households to a 100-basis-point negative interest-rate shock. The right panel plots the difference between the (log) CPI of the high-MPC and low-MPC households.

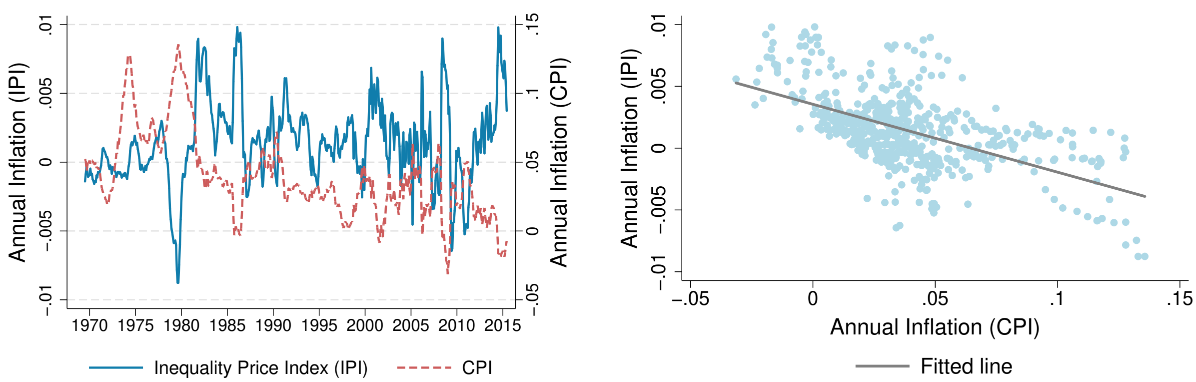
The left panel in Figure 2 plots the impulse responses of low-MPC households' (log) CPI. The CPI increase by roughly 2 percentage points 36 months after an expansionary monetary shock. We further explore the differential response of the CPIs of households with different MPCs by replacing the dependent variable with the difference between the (log) CPI of the high-MPC and low-MPC households in equation (4). The estimated results are plotted in the right panel of Figure 2. The difference in CPI faced by households with different MPCs is approximately 0.2 percentage points 36 months after the shock. That is, the high-MPC households' CPI responds by 10% more than the low-MPC households. All estimates are statistically and economically different from zero at the 1 or 1.65 standard deviation confidence intervals, depicted by the dark and light shaded area, respectively.

2.4 Cyclicality and Volatility of Group-Specific Inflation Rates

In the final step, we show that the inflation faced by high-MPC households is more cyclical and volatile than that of low-MPC households. The left panel in Figure 3 plots the evolution of the annual inflation for the Inequality Price Index (IPI, blue line), defined as $\pi_t^{\text{low-MPC}} - \pi_t^{\text{high-MPC}}$, alongside the inflation of the Consumer Price Index (CPI, red dashed line), for the period from 1970 to 2015. A clear negative comovement is displayed between these two series. Higher aggregate inflation is associated with greater disparity in inflation between high-MPC and low-MPC households — high-MPC households face higher inflation when aggregate inflation is high, and vice versa. This negative correlation is illustrated in the right panel of Figure 3, where each dot represents an annual observation.

Greater cyclicality in the cost of living faced by high-MPC households implies that their inflation is more volatile as well. In fact, inflation volatility faced by high-MPC households is, on average, 8% higher than low-MPC households over the period 1970-2015. This difference has become more salient recently, with high-MPC households' inflation being 15% more volatile since the year 2000.[7] Section 6 highlights the importance of incorporating the difference in cyclicality and volatility of the cost of living across households in evaluating the social welfare and designing the optimal monetary policy.

Figure 3: Evolution of Inequality Price Index (IPI) and CPI



Note: The left panel plots the time series of the Inequality Price Index (IPI), defined as $\pi_t^{\text{low-MPC}} - \pi_t^{\text{high-MPC}}$, alongside the Consumer Price Index (CPI). The right panel plots the relationship between IPI and CPI, with each dot representing an annual observation. The correlation is -0.56 with p-value smaller than 0.001.

[7] Supplementary Appendix B.1 provides additional evidence on the evolution of the inflation and inflation volatility faced by different households over time.

3 Framework: A Multi-Sector TANK Model

Motivated by the empirical evidence, this section develops a theoretical and quantitative framework featuring a multi-sector Two-Agent New Keynesian model with heterogeneous consumption baskets (abbreviated as TANK-HT). The TANK-HT serves as an extension of the TANK model in Bilbiie (2020). In this extension, we incorporate the distinction in consumption baskets between the Ricardian households (or savers) and the Keynesian households (or the hand-to-mouth). These two groups exhibit differences in their expenditure shares across sectors and therefore face different degree of price stickiness.

3.1 Model

3.1.1 Households

There are two types of households in the economy: the Ricardian and the Keynesian, with measures $1-\lambda$ and λ, respectively. The Ricardian households, denoted as R, have unconstrained access to a complete financial market, while the Keynesian households, denoted as K, cannot access financial markets and spend all of their income in each period (i.e., hand-to-mouth). There is a continuum of firms distributed in I sectors. The Ricardian households own all firms in the economy, and each household has an equal share. The utility function for households of type h ($h \in \{R, K\}$) is given by:

$$U(C_t^h, N_t^h) = \frac{(C_t^h)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(N_t^h)^{1+\gamma}}{1+\gamma},$$

where C_t^h denotes the consumption of a composite good, N_t^h is the supply of labor, σ is the elasticity of intertemporal substitution (EIS) and γ is the inverse of Frisch elasticity. We allow households to consume different baskets of goods across product categories, so that the composite good consumed by household type h is aggregated according to

$$C_t^h = \left[\sum_{i=1}^{I} \left(\omega_i^h\right)^{\frac{1}{\eta}} \left(C_{i,t}^h\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where η is the elasticity of substitution between goods produced by different sectors, and ω_i^h is a household-specific taste shifter that can be calibrated using household-specific expenditure shares. Correspondingly, the price index faced by household type h is given by

$$P_t^h = \left(\sum_{i=1}^{I} \omega_i^h P_{i,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}.$$

In sector i, a continuum of monopolistically competitive firms, indexed by j, produce differentiated goods. The goods produced in sector i is defined by

$$C_{i,t} = \left[\int \left[C_{i,t}(j) \right]^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the (price) elasticity of substitution across differentiated goods j within sector i. The price of goods produced in sector i is an aggregation of the prices of the differentiated goods, $P_{i,t} = \left(\int [P_{i,t}(j)]^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$.

According to the Divisia index, the aggregate real value added (or real GDP) in period t is calculated by fixing nominal prices at the base period (see Vom Lehn and Winberry (2022)):

$$C_t = \lambda P^K C_t^K + (1-\lambda) P^R C_t^R,$$

where P^K and P^R are the steady-state prices. The aggregate consumer price index (CPI) in period t is defined as:

$$P_t = (P_t^K)^{\lambda} (P_t^R)^{1-\lambda}.$$

The Ricardian household maximizes its utility by choosing consumption, labor supply and bond holdings.

$$\max_{\{C_t^R, N_t^R, B_{t+1}\}} \; \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^R, N_t^R)$$

$$s.t.\; P_t^R C_t^R + Q_{t,t+1} B_{t+1} \leq W_t N_t^R + B_t + D_t/(1-\lambda) + T_t^R, \quad t = 0, 1, 2, \ldots$$

where β is the discount factor, W_t is nominal wage in the economy, B_{t+1} is the holding of a nominal bond paying at $t+1$, $Q_{t,t+1}$ is its price at period t and D_t denotes total profits of firms. We use T_t^R and T_t^K to denote the nominal amount of lump-sum government transfers per Ricardian capita and per Keynesian capita, respectively. In contrast, the Keynesian household maximizes its utility by choosing the intratemporal consumption and labor supply.

$$\max_{\{C_t^K, N_t^K\}} \; U(C_t^K, N_t^K)$$

$$s.t.\; P_t^K C_t^K = W_t N_t^K + T_t^K, \quad t = 0, 1, 2, \ldots$$

3.1.2 Firms

In each sector i, there is a unit measure of monopolistically competitive firms. Firms set prices as in Calvo (1983), and the frequency of price changes among firms in sector i is $1 - \alpha_i$. The production function of firm j in sector i is given by

$$Y_{i,t}(j) = N_{i,t}(j).$$

Firm j in sector i chooses its pricing strategies to maximize the sum of its discounted profits:

$$\max_{P_{i,t}(j)} \mathrm{E}_t \sum_{s=0}^{\infty} Q_{t,t+s}\alpha_i^s\{[(1+\tau)P_{i,t}(j) - W_{t+s}](\frac{P_{i,t}(j)}{P_{i,t+s}})^{-\epsilon}C_{i,t+s}\},$$

where $Q_{t,t+s} = Q_{t,t+1}Q_{t+1,t+2}...Q_{t+s-1,t+s}$ is the nominal discount factor between period t and $t+s$, and $\tau = 1/(\epsilon-1)$ is the government subsidy rate on revenue, chosen to ensure that firms' profits are zero under steady state. Firms take the nominal wage as given, and this wage rate is uniform across sectors and firms.

3.1.3 Government

The government conducts both fiscal and monetary policies. It levies lump-sum taxes $\sum_{i=1}^{I}\tau\int\left[P_{i,t}(j)C_{i,t}(j)\right]dj$ on firms to generate revenue for the subsidy mentioned earlier to the Keynesian and Ricardian households. Additionally, the government employs a transfer scheme, denoted as, $\{T_t^R, T_t^K\}$. This scheme involves taxing firms' profits at a rate τ^{π} and redistributing the proceeds to the Keynesian households, with the income redistribution determined by $\lambda T_t^K = -(1-\lambda)T^R = \tau_{\pi}D_t$. This type of redistribution plays an important role in shaping the income cyclicality of Keynesian and Ricardian households, and is found to be critical for monetary transmission when agents have heterogeneous MPCs (Bilbiie (2020)). This transfer scheme is simple yet flexible since any redistribution can be achieved by varying one parameter τ^{π}. For example, when $\tau^{\pi} = 1$, all firms' profits are sent to the Keynesian households. We assume that there is no government spending and that the government balances its budget in each period.

Following Coibion and Gorodnichenko (2012), the monetary authority follows an interest rate rule subject to persistent shocks:

$$\exp(i_t) = \exp\left[\rho_i i_{t-1} + (1-\rho_i)\bar{\imath}\right]\left[\left(\frac{P_t^M}{P_{t-1}^M}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{1-\rho_i}\exp(\upsilon_t),$$

where $i_t \equiv -\log Q_{t,t+1}$ is the nominal interest rate, $\bar{\imath}$ is the steady state nominal interest rate, ρ_i is the interest-rate smoothing parameter, ϕ_π and ϕ_y represent central bank's responsiveness to inflation and aggregate output.

The monetary shock is characterized by an AR(1) process, $\upsilon_t = \rho\upsilon_{t-1} + \zeta_t$, where ζ_t is an i.i.d process with zero mean and finite variance. The price index targeted by the central bank is given by $P_t^M = (P_t^K)^{\xi}(P_t^R)^{1-\xi}$. In most sections, we set $\xi = \lambda$, meaning that the targeted inflation index is computed using the aggregate CPI. However, in section 6, we explore the optimization of the central bank's policy by solving for the optimal targeted price index P_t^M.

4 The Inflation Heterogeneity Channel

Armed with the theoretical framework, this section aims to investigate how the relationship between MPC and price stickiness affects households' consumption, the aggregate MPC, and the Keynesian multiplier. In particular, it highlights a novel channel of monetary transmission: the *inflation heterogeneity* channel.

To characterize this channel analytically, we make two simplified assumptions: 1) the number of sectors, denoted as I, is set equal to 2, and 2) the Ricardian and Keynesian households consume completely different sets of goods. Without loss of generality, we assume that the Keynesians only consume composite goods produced in sector 1, and the Ricardians only consume composite goods produced in sector 2, i.e., $\omega_1^R = \omega_2^K = 0$ and $\omega_2^R = \omega_1^K = 1$. With an abuse of notation, we refer to the terms "K good" and "R good" as the corresponding composite goods.[8] The model is essentially a two-sector TANK model with heterogeneous consumption baskets, which we call T-TANK. We intentionally keep the model simple to isolate the emphasized channel, and this simplification allows us to obtain sharp analytical results.

We work with the log-linearized version of the model around a deterministic steady state where the inflation rate is zero. Log-deviations from their steady-state counterparts are denoted by lowercase variables. Supplementary Appendix C provides details on the steady state, log-linearized equilibrium conditions, and conditions for equilibrium determinacy. We consider the behavior of the demand block, and assume that the process of real interest rate $\{r_s\}_{s=t}^{\infty}$ to be exogenous.[9] We characterize the response of the aggregate consumption to the future path of changes in the real interest rate.

4.1 Inflation Heterogeneity and Consumption Response

With the model setup, the aggregate nominal profits in the economy at period t are given by

$$D_t = (1+\tau)\sum_{i\in K,R}\int P_{i,t}(j)Y_{i,t}(j)dj - \sum_{i\in K,R}\int W_t N_{i,t}(j)dj - \tau\sum_{i\in K,R}\int P_{i,t}(j)Y_{i,t}(j)dj.$$

[8] For simplicity, we use "K" to represent "Keynesian" and "R" to represent "Ricardian".

[9] This assumption has been widely used to study the monetary transmission mechanism in the demand block (e.g. Auclert (2019), Bilbiie (2020)). Different from these papers, we do not assume fixed or perfectly sticky prices. Instead, prices in different sectors are subject to heterogeneous degrees of stickiness.

Approximating this expression around the steady state up to first order yields

$$d_t = -(w_t - p_t),$$

where d_t is defined as D_t/Y. The aggregate nominal profits d_t move negatively with the real wage rate $w_t - p_t$. By equating the aggregate labor supply equation $\gamma n_t = w_t - p_t - \sigma^{-1} c_t$ with the labor demand equation $n_t = c_t$, we obtain the expression for the nominal wage rate:

$$w_t = p_t + (\gamma + \sigma^{-1}) c_t. \tag{5}$$

Combining the labor supply equation and budget constraint of the Keynesian household, $p_t^K + c_t^K = w_t + n_t^K + \frac{\tau^D}{\lambda} d_t$, yields the following expression:

$$(\gamma + \sigma^{-1}) c_t^K = (1+\gamma)(w_t - p_t^K) + \frac{\gamma \tau^d}{\lambda} d_t. \tag{6}$$

Replacing expression (5) into expression (6), and recalling that the aggregate profits d_t satisfy $d_t = -(w_t - p_t)$, we demonstrate how the Keynesian household's real income (and consumption) y_t^K moves in tandem with the aggregate income y_t in the following equation,

$$y_t^K = \chi_y y_t - \chi_p \frac{1-\lambda}{\lambda} (p_t^K - p_t^R), \tag{7}$$

where

$$\chi_y \equiv 1 + \gamma (1 - \frac{\tau^d}{\lambda}), \;\; \chi_p \equiv \lambda \frac{\gamma + 1}{\gamma + \sigma^{-1}},$$

Since $y_t = \lambda y_t^K + (1-\lambda) y_t^R$, R's real income (consumption) is given by

$$y_t^R = \frac{1 - \lambda \chi_y}{1-\lambda} y_t + \chi_p (p_t^K - p_t^R). \tag{8}$$

In this expression, $p_t^K - p_t^R$ reflects the cyclical inequality in the cost of living between households, capturing the differential responses of R's and K's price indices (or cost of living). TANK is nested as a special case of T-TANK when the sectoral price responses are equal (i.e., $p_t^K = p_t^R$).

The key parameter χ_y, which determines the elasticity of the K's real income (and consumption) to aggregate income in TANK and is thoroughly discussed in Bilbiie (2020), governs whether aggregate consumption in TANK is dampened or amplified relative to RANK. The underlying idea is that, following a shock to the real interest rate, if the resulting aggregate real income is disproportionally redistributed to the Keynesian households, given their higher MPC, the general equilibrium effect will be stronger, and the Keynesian multiplier will be larger. The following lemma determines the cyclicality of Keynesians' and Ricardians' real income in T-TANK.

Lemma 1. *The elasticity of the Keynesians' real income to aggregate real income is given by*

$$\frac{dy_t^K}{dy_t} = \chi_y - \chi_p \frac{1-\lambda}{\lambda} \frac{d(p_t^K - p_t^R)}{dy_t},$$

and the elasticity of the Ricardians' real income to aggregate real income is given by

$$\frac{dy_t^R}{dy_t} = \frac{1-\lambda\chi_y}{1-\lambda} + \chi_p \frac{d(p_t^K - p_t^R)}{dy_t}.$$

The cyclicality of K's real income, denoted by $\frac{dy_t^K}{dy_t}$, is no longer exogenously determined, but endogenously determined in equilibrium. For instance, when $p_t^K - p_t^R$ is procyclical or when the K faces greater inflation after a real interest rate shock ($p_t^K > p_t^R$), the K's real income will be less cyclical relative to its counterpart in TANK. On the contrary, R's income elasticity to aggregate income is positively correlated with $p_t^K - p_t^R$

This result is driven by the fact that the relative increase in the cost of living of the Keynesians, or the price of K's consumption baskets, further lowers K's *real* income. This is only possible when $p_t^K \neq p_t^R$, or when consumption baskets are heterogeneous across households. Unlike TANK, price responses in T-TANK are now relevant for the cyclicality of the R's and the K's real income. This feature connects the demand side and the supply side of the economy and will play an important role in the dampening or amplifying result established below.

Combining the R's Euler equation and the intertemporal budget constraint, the R's consumption at period t can be expressed as a function of changes in real interest rates and income,

$$c_t^R = -\sigma\beta \sum_{s=0}^{\infty} \beta^s \mathrm{E}_t r_{t+s} + (1-\beta) \sum_{s=0}^{\infty} \beta^s \mathrm{E}_t y_{t+s}^R.$$

Given the path of real interest rate changes $\{r_{t+s}\}_{s=0}^{\infty}$ and the path of future income $\{y_{t+s}^R\}_{s=1}^{\infty}$, the marginal propensity to consume after a transitory income increase at period t is captured by $1-\beta$. The R's consumption function can be written in the following recursive form,

$$c_t^R = (1-\beta) y_t^R - \sigma\beta r_t + \beta \mathrm{E}_t c_{t+1}^R. \tag{9}$$

Proposition 1 is obtained by 1) replacing (7) into the K's consumption function ($c_t^K = y_t^K$) and plugging (8) into the R's consumption function (9), 2) aggregating these consumption functions across all households, and 3) using the market clearing condition $c_t = y_t$.

Proposition 1. *In response to a path of real interest rate changes* $\{r_{t+s}\}_{s=0}^{\infty}$*, the aggregate consumption at period* t *in RANK, TANK and T-TANK are given by:*

$$\text{RANK:} \qquad c_t = -\sigma \mathrm{E}_t \sum_{s=0}^{\infty} r_{t+s}, \tag{10}$$

$$\text{TANK:} \qquad c_t = -\frac{1-\lambda}{1-\lambda\chi_y}\sigma \mathrm{E}_t \sum_{s=0}^{\infty} r_{t+s}, \tag{11}$$

$$\text{T-TANK:} \qquad c_t = -\frac{1-\lambda}{1-\lambda\chi_y}\sigma \mathrm{E}_t \sum_{s=0}^{\infty} r_{t+s} - \frac{1-\lambda}{1-\lambda\chi_y}\chi_p(p_t^K - p_t^R), \tag{12}$$

Expression (10), (11), and (12) capture the equilibrium consumption responses to a given path of real interest rate changes in RANK, TANK and T-TANK, respectively. Expression (11) echoes the idea in Bilbiie (2020) that the cyclicality of real income inequality determines whether the aggregate consumption response is amplified or dampened in TANK relative to RANK.

One immediate observation of Proposition 1 is that in RANK and TANK, once the path of real interest rates is determined, price responses, and therefore the degrees of price stickiness, play no roles in determining the response of aggregate consumption. In contrast, the aggregate consumption response in T-TANK depends on the heterogeneous price responses across sectors, as indicated by $(p_t^K - p_t^R)$ in expression (12).

Specifically, following a sequence of real interest rate cuts, if the Keynesian households face smaller price responses on impact, the aggregate consumption response is larger in T-TANK than that in TANK. In this case, the aggregate consumption response is *amplified* in T-TANK. Conversely, if the Keynesian households face greater price responses on impact, the aggregate consumption response is *dampened* in T-TANK.

Corollary 1. *Equation* (12) *reduces to* (11) *when* $\omega_i^K = \omega_i^R, \forall i = 1, ..., I$*, and further reduces to* (10) *when* $\lambda = 0$.

More generally, Proposition 1 highlights that the correlation between households' MPC and the cyclicality of their cost of living plays a critical role in monetary transmission. Importantly, Corrolary 1 states that it is *not* the heterogeneous sectoral price responses per se that drive the amplification and dampening result. In fact, the aggregate consumption response collapses to (11) once we assume that the Keynesians and Ricardians consume the same baskets of goods, even if price stickiness is still heterogeneous across sectors.

Proposition 1 implies that the cyclical inequality in the cost of living $p_t^K - p_t^R$ is informative on the efficacy of monetary policy. However, $p_t^K - p_t^R$ is observable only after a policy is implemented, which poses challenges for policy-making. The following lemma establishes the relationship between sectoral price responses and sectoral price stickiness, which are perfectly observable and can be measured by the frequency of price adjustment $\{1-\alpha_i\}_{i=K,R}$.

Lemma 2. *Suppose the following condition holds:*

$$\lambda\kappa_R + (1-\lambda)\kappa_K + \chi_p(\kappa_R - \kappa_K)\frac{1-\lambda}{1-\lambda\chi_y} > 0, \tag{13}$$

where $\kappa_i = (1-\alpha_i)(1-\alpha_i\beta)/\alpha_i$ is the slope of the Phillips curve in sector i. Consider a sequence of unexpected interest rate changes $\{r_s\}_{s=t}^{\infty}$, such that $\phi_t = \mathrm{E}_t \sum_{s=t}^{\infty} r_s$ is bounded. The following result holds,

$$(\alpha_R - \alpha_K)(p_t^K - p_t^R)\phi_t < 0.$$

This result implies that the price of the more flexible-price sector responds more to shocks. Specifically, it increases more in response to an expansionary monetary shock and drops more in response to a tightening monetary shock.

Proposition 2. *Suppose condition (13) holds, and consider a sequence of real interest rate cuts ($r_t < 0$). If the price of goods in the Keynesian sector is more flexible ($\alpha_K < \alpha_R$), aggregate consumption response is dampened in T-TANK, i.e. $c_t^{T-TANK} < c_t^{TANK}$; if the price of goods in the Ricardian sector is more flexible ($\alpha_K > \alpha_R$), aggregate consumption response is amplified, i.e. $c_t^{T-TANK} > c_t^{TANK}$.*

The empirical result in section 2 indicates that the Keynesian households face more flexible prices. Proposition 2 therefore suggests that our proposed channel dampens the effects of monetary policy. The magnitude of this dampening effect is ultimately a quantitative question, which we will explore in a calibrated version of our model in section 5.

4.2 The Keynesian Multiplier and Aggregate MPC

To understand the factors driving our results in Proposition 1, we decompose the aggregate consumption response into the direct effects and the general equilibrium effects. Following Bilbiie (2020), we define c_t^D as the *autonomous spending* , ω as the *aggregate marginal propensity to consume (MPC)* , and Ω as the *Keynesian multiplier* in our model. We consider a sequence of real interest rate cuts with persistence δ, $\{r_{t+s} = -\delta^s r,\ s = 0,1,2,..\}$.

The *autonomous spending* in period t represents the direct response of consumption demand arising from the intertemporal substitution effect of Ricardian households. Combining Ricardians' Euler equation and their consolidated budget constraints, we obtain the autonomous spending in period t in TANK and T-TANK, denoted by $c_t^{D,TANK}$ and $c_t^{D,T-TANK}$,

$$c_t^{D,TANK} = c_t^{D,T-TANK} = -\sigma\beta(1-\lambda)\sum_{s=0}^{\infty} \mathrm{E}_t \beta^s r_{t+s} = \frac{\sigma\beta(1-\lambda)}{1-\beta\delta} r. \tag{14}$$

The autonomous spendings are the same in TANK and T-TANK, as they are both generated by the intertemporal-substitution motives of the Ricardian households.

The *Keynesian multiplier* Ω is the ratio between the aggregate consumption response c_t and the amount of autonomous spending c_t^D, $\Omega = c_t / c_t^D$. The *aggregate marginal propensity to consume (MPC)* ω is defined as $c_t = c_t^D/(1-\omega)$, or equivalently $\omega = 1 - 1/\Omega$.

Proposition 3. *Suppose there is a sequence of real interest rate cuts with persistence* δ, $\{r_{t+s} = -\delta^s r,\ s = 0, 1, 2, ..\}$. *The Keynesian multipliers in period* t *in TANK and T-TANK can be written as,*

$$\textit{TANK:} \quad \Omega = \frac{1-\beta\delta}{\beta(1-\delta)(1-\lambda\chi_y)},$$
$$\textit{T-TANK:} \quad \Omega = \frac{1-\beta\delta}{\beta(1-\delta)(1-\lambda\chi_y)} - \frac{1-\beta\delta}{\sigma\beta(1-\lambda\chi_y)}\chi_p(p_t^K - p_t^R).$$

Proposition 3 is derived by substituting expression (11), (12) and (14) into the definition of the Keynesian multiplier. The aggregate MPC ω is then obtained from $\omega = 1 - 1/\Omega$.

Both the Keynesian multiplier and the aggregate MPC are decreasing functions of the cyclical inequality in the cost of living at period t, $p_t^K - p_t^R$. Therefore, they both depend on the relative degree of price stickiness between sectors, as implied by Lemma 2. This reveals, in the New Keynesian framework, a novel channel through which the supply block interacts with the demand block. In RANK and TANK, price stickiness affects demand by first influencing inflation and, consequently, impacting the magnitude of the real interest rate through the Fisher equation. Once the path of real interest rates is fixed, price stickiness becomes irrelevant. On the contrary, in our model, the distribution of price stickiness across agents shapes aggregate demand by dampening/amplifying the general equilibrium effect, even *conditional* on the path of real interest rates. This dampening or amplification is a consequence of varied inflation responses experienced by different households and we label it as the *inflation heterogeneity channel.*

Discussion. The inflation heterogeneity channel can possibly be generalized to study business cycles and the effects of fiscal policy. Intuitively, by amplifying or dampening the Keynesian multiplier, the inflation heterogeneity channel can amplify or stabilize fluctuations over business cycles.[10] Similarly, government spending exerts uneven inflationary pressure on households' cost of living. The magnitude of the fiscal multiplier can vary depending on the correlation between households' MPC and the relative price stickiness they encounter. We leave the exploration of these applications to future research.

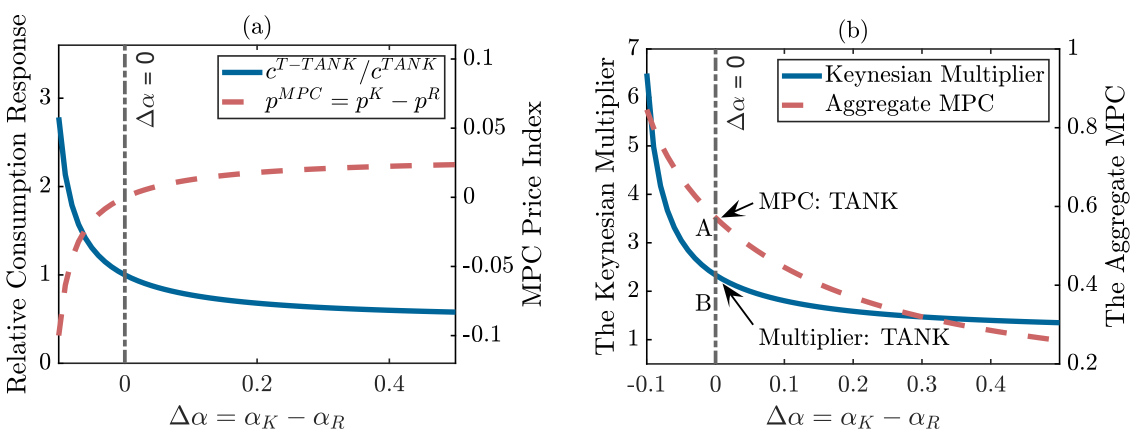
[10] Patterson (2023) shows that the positive covariance between households' MPC and income cyclicality amplifies business-cycle fluctuations. Our proposed channel focuses on the covariance between households' MPC and cyclicality of the cost of living, and can be extended to study the business-cycle fluctuations.

4.3 A Numerical Example

In the remaining part of this section, we illustrate the mechanism of our model through a numerical example. We calibrate $1-\alpha^{TANK}$, the frequency of price changes in TANK, to be 0.208 to match the average frequency of price changes in the economy. The remaining model parameters follow conventional values in the literature.

We begin by solving the TANK model with $\alpha^{TANK} = 0.792$. Subsequently, we solve the T-TANK model, varying the degree of heterogeneity in price stickiness $\Delta\alpha = \alpha_K - \alpha_R$, while keeping the average price stickiness equal to α^{TANK}. Throughout these exercises, the path of real interest rate changes remains unchanged.[11]

Figure 4: Varying Heterogeneity in Price Stickiness in T-TANK



Note: The left panel presents the variation in the relative cumulative consumption responses between T-TANK and TANK (solid blue line, left y-axis) alongside $p^K - p^R$ (dashed red line, right y-axis) concerning the difference in price stickiness between K goods and R goods $\Delta\alpha$. The right panel plots the variations in the Keynesian multiplier (left y-axis) and the variations in aggregate MPC (right y-axis) with respect to $\Delta\alpha$.

In panel (a) of Figure 4, the solid blue line plots $c^{\text{T-TANK}}/c^{\text{TANK}}$, representing the ratio between TANK and T-TANK in the cumulative consumption response to the same path of real interest rate changes, across the degree of heterogeneity in price stickiness $\Delta\alpha = \alpha_K - \alpha_R$.[12] Consistent with our theory, if Keynesians experience more flexible prices ($\alpha^K > \alpha^R$), the aggregate consumption is dampened ($c^{T-TANK} > c^{TANK}$), and vice versa. Additionally, $c^{\text{T-TANK}}/c^{\text{TANK}}$ decreases with $\Delta\alpha$, indicating that a larger difference in sectoral price stickiness results in a higher degree of dampening or amplification. Correspondingly, the dashed red line in panel (a) plots the cumulative cyclical inequality in the cost of living $p^K - p^R$, increasing with $\Delta\alpha$.

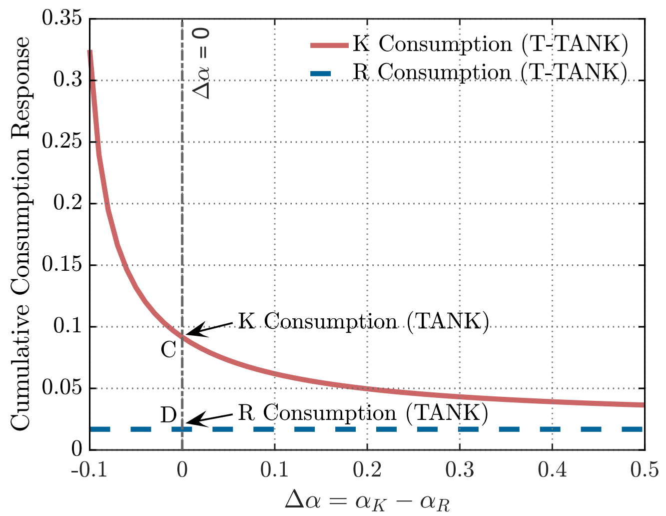
[11] The path of real interest rate changes is plotted in Figure E.2 in the Supplementary Appendix.

[12] Figure E.2 and Figure E.3 in the Supplementary Appendix plot the impulse response functions (IRFs) of the aggregate consumption, the K's consumption, and the R's consumption, respectively, when $\Delta\alpha = 0.1$.

Panel (b) in Figure 4 displays the Keynesian multiplier and the aggregate MPC in T-TANK across varying values of $\Delta\alpha$. Point A and point B represent the multiplier and the aggregate MPC in TANK when $\Delta\alpha$ is equal to zero. Notably, both the Keynesian multiplier and the aggregate MPC decrease as heterogeneity in price stickiness $\Delta\alpha$ increases.

Figure 5 illustrates how Keynesians' and Ricardians' consumption vary with $\Delta\alpha$. Fixing the path of real interest rates, the R's consumption remains constant as $\Delta\alpha$ varies, consistent with the Euler equation. This observation clarifies why the R's consumption in T-TANK is equal to that in TANK (point D). In contrast, K's consumption decreases with $\Delta\alpha$, driven by the general equilibrium effect discussed above. This leads to a decreasing Keynesian multiplier, as shown in Figure 4 panel (b), and consequently, a decreasing aggregate consumption in Figure 4 panel (a).

Figure 5: Consumption Response of K and R in T-TANK and TANK



Note: The left panel plots the variation in cumulative consumption responses for Keynesians (solid red line) and the Ricardians (dashed blue line) with the difference in price stickiness between K goods and R goods $\Delta\alpha$. Point C and D represent the corresponding consumption by Keynesians and Ricardians in TANK, respectively.

5 Quantitative Analysis

In Section 4, we demonstrated how the relationship between MPC and price stickiness influences household consumption, the aggregate MPC, and the Keynesian multiplier through the inflation heterogeneity channel proposed in our framework. In this section, we shift from theoretical analysis to quantitative evaluations. Specifically, we calibrate the multi-sector TANK model outlined in Section 3 using the microdata discussed in Section 2. We aim to quantitatively assess the impact of the inflation heterogeneity channel on the effectiveness of monetary policy.

5.1 Calibration

The model is calibrated at a monthly frequency and the parameters calibrated externally are based on the conventional values in the literature. The discount factor β is calibrated to be 0.9975. The elasticity of intertemporal substitution σ is set to 0.5. To achieve a Frisch labor supply elasticity of 1/3, we select $\gamma = 3$ (e.g. Chetty et al. (2011)). The Taylor rule parameters are determined as $\phi_\pi = 1.24$ and $\phi_y = 0.5/12$. For the monetary shock, we assign a monthly standard deviation and persistence of $\nu = 0.0025$ and $\rho = 0.9$, respectively.

The rest of the parameters are calibrated to match micro moments in Section 2. The number of sectors I are calibrated to be 263 to match the number of Entry Level Items (ELIs) in the data. In both TANK with heterogeneous consumption baskets (TANK-HT) and TANK with homogeneous consumption baskets, the frequency of price changes $1-\alpha_i$ in sector i is set to match the frequency of price changes of goods in ELI i. We assign mortgagors and renters to be the Keynesian households and the outright homeowners to be the Ricardian households. The corresponding consumption weight ω_i^h is set to be the expenditure share consumed on ELI i by household type h, constructed in section 2. The average frequency of price changes faced by K is 0.234, and that faced by R is 0.197. Table 1 presents the calibrated parameters. As the pricing moments documented in Nakamura and Steinsson (2008) are from 1998 to 2005, our calibration uses expenditure shares from the year 2005.[13] In TANK, the consumption weight in sector i is set to $\omega_i = \lambda\omega_i^K + (1-\lambda)\omega_i^R$.

Table 1: Parameters

Externally calibrated		
Discount factor	β	0.9975
EIS	σ	0.5
Frisch elasticity	$\frac{1}{\gamma}$	1/3
Taylor rule coefficient	ϕ_π	1.24
Taylor rule coefficient	ϕ_y	0.5/12
Shock size at impact	ν	0.0025
Shock persistence	ρ	0.9
Interest-rate smoothing	ρ_i	0.9
Internally calibrated		
Number of sectors	K	263
Average freq. of price changes	$1-\overline{\alpha}$	0.208
Average freq. faced by K	$1-\alpha_K$	0.234
Average freq. faced by R	$1-\alpha_R$	0.197

[13] Our results are robust to using expenditure shares in other years.

Calibrating λ and χ_y. Two key parameters determine the effects of monetary policy: the fraction of K households denoted by λ, and the real income cyclicality of K households in TANK denoted by χ_y. Bilbiie (2020) demonstrates that by adjusting these two parameters, the TANK model can accurately replicate the aggregate effects of monetary policy shocks in the existing quantitative-HANK studies, including Kaplan et al. (2018), Gornemann et al. (2021), Debortoli and Galí (2017), Hagedorn et al. (2019), and Auclert et al. (2018).[14] For the sake of robustness, we follow the calibration outlined in Bilbiie (2020), allowing for different values of λ and χ_y as detailed in Table 2. The first column lists names of the studies, and the second and third column presents the calibrated values of χ_y and λ for each study. For our baseline calibration, we pick the median value from these studies, setting $\lambda = 0.3$ and $\chi_y = 2.16$.[15]

Table 2: Alternative Calibrations and Results

Studies	Parameters		Dampening effect
	χ_y	λ	
Kaplan et al	1.48	0.41	15%
Gornemann et al	2.16	0.3	15%
Debortoli Gali	2.55	0.21	10%
Hagedorn et al	3.1	0.24	17%
Auclert et al	1.51	0.36	13%
Baseline (This paper)	2.16	0.3	15%

5.2 The Effects of Monetary Policy

With the calibrated economy as our laboratory, we proceed to assess the real effects of monetary policy in TANK-HT. As discussed earlier, we set a fixed path for real interest rate changes and measure monetary non-neutrality by calculating the cumulative consumption responses.

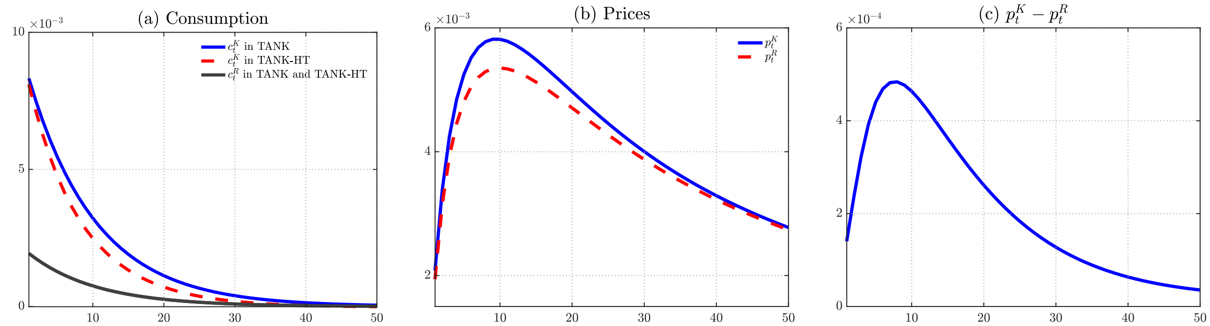
In our baseline calibration, the cumulative aggregate consumption response is 0.0427. Of this, 58.6% is attributed to the Keynesian households, with a response of 0.0251, while the remaining 41.4% totaling 0.0177, is consumed by the Ricardian households. In TANK, the cumulative aggregate consumption response is 0.05. The Keynesian households consume 0.0323 (64.6%),

[14] Specifically, Bilbiie (2020) set χ and λ in TANK to match the Keynesian multiplier Ω and aggregate MPC ω in the quantitative-HANK studies listed in Table 2.

[15] These values are, coincidentally, also the mean of the first and second column.

and the Ricardian households consume 0.0177 (34.8%). By this metric, the aggregate effects of monetary policy are 14.6% smaller in TANK-HT than those in TANK.

Figure 6: Impulse Responses to the Real Interest Rate Shock



Note: Panel (a) illustrates the impulse responses of the consumption of the Keynesians and the Ricardians in TANK-HT and TANK, respectively. Panel (b) plots the impulse responses of the prices faced by the Keynesians and the Ricardians. Panel (c) shows the difference between the two price indices, $p_t^K - p_t^R$.

As shown in panel (a) of Figure 6, the difference in consumption response is entirely driven by the reduction in consumption of the Keynesian households. Consequently, the Keynesian multipliers, as defined in section 4, are 1.69 in TANK-HT and 2 in TANK. The corresponding aggregate MPCs are 0.41 and 0.5, respectively. Panel (b) and panel (c) plot the responses of p_t^R and p_t^K as well as their difference $p_t^K - p_t^R$. The peak response of p_t^K is approximately 9.1% higher than that of p_t^R, consistent with its empirical counterpart in Figure 2, which is about 10%.

In addition, we assess the robustness of our results across different calibrations of λ and χ_y listed in Table 2. The last column in Table 2 quantifies the extend to which aggregate consumption in TANK-HT is dampened relative to TANK. The dampening effect ranges from 10% calibrated to Debortoli and Galí (2017), to 17% calibrated to Hagedorn et al. (2019).

Heterogeneous Consumption Baskets or Heterogeneous Price Stickiness. To demonstrate the necessity of both heterogeneous consumption baskets across households and heterogeneous price stickiness across sectors for our results, we systematically eliminate each assumption in TANK-HT. Specifically, we investigate two scenarios: one where consumption baskets are homogeneous across households while price stickiness across sectors is heterogeneous and the other where price stickiness is homogeneous but the consumption baskets are heterogeneous. In both scenarios, we find that the IRFs are identical to those in TANK. This finding suggests that it is crucial to connect heterogeneity in the demand side with heterogeneity in the supply side to accurately evaluate the efficacy of monetary policy.

6 Optimal Monetary Policy

In models with heterogeneous agents, such as TANK, the presence of imperfect insurance leads to cyclical income inequality between Ricardians and Keynesians. Our empirical evidence suggests that differences in households' cost-of-living cyclicality introduce a new form of cyclical inequality. Does this particular type of inequality affect the design of optimal policy? If so, what constitutes the optimal monetary policy? This section delves into these questions.

We begin by establishing a benchmark result in Section 6.1 — identifying the conditions under which inequality does not impact the design of optimal monetary policy. Under these conditions, stabilizing the output gap simultaneously stabilizes prices and minimizes inequality, achieving the social optimum. We refer to this result as the "triple divine coincidence" and show that it holds in a standard one-sector TANK model. In Section 6.2, we explore the impact of introducing heterogeneous consumption baskets to TANK. This new feature generates a time-varying inefficient wedge between the flexible-price equilibrium level of output and the efficient one, rendering the flexible-price equilibrium socially inefficient. This inefficiency arises from cyclical inequality in the cost of living across households, making inequality relevant for optimal monetary policy. Lastly, in Section 6.3, we show that the policy proposed by Aoki (2001) and Benigno (2004) to stabilize the output gap, or equivalently the "core" inflation, becomes suboptimal. This is because central banks find it optimal to tolerate some fluctuations in the output gap to reduce the volatility of inflation faced by Keynesian households, thereby lowering fluctuations in the cost-of-living inequality.

In the remaining part of this section, unless otherwise specified, we assume that household h's utility function in period t is represented by

$$W_t^h = U(C_t^h) - V(N_t^h).$$

If the central bank assigns equal weights to each household, the social welfare in period t is expressed as

$$W_t = \lambda U(C_t^K) + (1-\lambda)U(C_t^R) - \left[\lambda V(N_t^K) + (1-\lambda)V(N_t^R)\right]. \tag{15}$$

Fluctuations in the economy are driven by sectoral productivity shocks, and the monetary authority decides how to respond to these shocks. Specifically, the production function of firm j in sector i is given by

$$Y_{i,t}(j) = A_{i,t}N_{i,t}(j),$$

where $\{A_{i,t}\}$ are productivity shocks in sector i,independent across sectors and periods.

In line with earlier sections, we restrict our analysis to the cases where there is no inequality in the steady state, i.e., $C^K = C^R$ and $N^K = N^R$. This assumption not only facilitates tractability but also enables us to focus on the cyclical component of inequality driven by short-run fluctuations. By adopting this assumption, we refrain from exploring the use of monetary policy to address steady-state inequality, as such tasks are more appropriately managed by the fiscal authority.

6.1 Benchmark: The Triple Divine Coincidence

A classical result in RANK is the "divine coincidence": stabilizing the aggregate output gap simultaneously stabilizes prices and, therefore, achieves the social optimum (Galí (2015), Woodford (2003)). We revisit this result in our multi-sector TANK framework, which allows for household heterogeneity (two types of households) with imperfect insurance, heterogeneous price stickiness across sectors, and heterogeneous consumption baskets across households. In this flexible framework, we explore the conditions under which the divine coincidence result holds, specifically, when stabilizing the aggregate output gap leads to efficient allocations. The following proposition provides the answer and serves as a benchmark for our subsequent discussion on optimal monetary policies. Additionally, it provides an irrelevance result to a question central to the recent policy debate: when should central banks care about inequality?

Proposition 4 (Triple Divine Coincidence). *Consider the model outlined in Section 3 with sectoral productivity shocks. Closing the aggregate output gap minimizes price dispersion, eliminates inequality, and therefore achieves social optimum* ***if and only if*** *the following two conditions hold:*

1. *Prices are sticky within at most one sector, and are perfectly flexible in all other sectors.*
2. *Households have the same consumption baskets.*

The proof of Proposition 4 is available in Supplementary Appendix D.3. This proposition, referred to as the "triple divine coincidence" result, extends the "divine coincidence" result in one-sector RANK and the two-sector RANK in Aoki (2001) to a heterogeneous-agent setting.[16] To establish this proposition, we first demonstrate its validity in the one-sector TANK model, as presented in Supplementary Appendix D.1. Subsequently, we extend this result to the TANK version of Aoki (2001), as outlined in Supplementary Appendix D.2.

With heterogeneous agents, central banks not only aim to stabilize output gaps and price dispersion, but also incorporate minimizing inequality as one of their objectives. This becomes evident once we approximate the welfare function (15) to the second order[17],

[16] In a two-sector model where one sector features sticky prices and the other has perfectly flexible prices, Aoki (2001) demonstrates that the optimal policy involves stabilizing prices in sector with sticky prices, which is equivalent to stabilizing the aggregate output gap.

[17] Lowercase letters denote variables that are log-linearized around the steady state.

$$W_t = -\frac{V'(\overline{Y})\overline{Y}}{2}\Big[\underbrace{(\gamma+\sigma^{-1})[1+\frac{\omega_F}{\omega_S}\eta(\gamma+\sigma^{-1})](y_t-y_t^n)^2+\frac{\omega_S\theta\alpha_S}{(1-\alpha_S)^2}\pi_{S,t}^2}_{\text{Conventional term}}$$
$$+\underbrace{(\sigma^{-1}-1)\lambda(1-\lambda)(c_{K,t}-c_{R,t})^2+\gamma\lambda(1-\lambda)(n_{K,t}-n_{R,t})^2}_{\text{Inequality term}}\Big]. \quad (16)$$

Expression (16) represents the social welfare function of a model that satisfies the conditions outlined in Proposition 4. We consider a scenario with two sectors: one featuring sticky prices (denoted as S) and the other with perfectly flexible prices (denoted as F). The natural rate of output is denoted by y_t^n and inflation in the sticky-price sector is denoted by $\pi_{S,t}$. All households have identical consumption baskets, denoted by ω_S and ω_F, which represent the consumption weights on goods produced in sector S and F, respectively.[18] The other parameters remain the same as those specified in Section 3.

This welfare function consists of two components, the conventional term and the inequality term. The "conventional term" in expression (16) captures the traditional welfare-loss components caused by the deviations from the efficient output level and the positive price dispersion due to non-zero inflation. The "Inequality term" arises from imperfect insurance between households, capturing the welfare loss due to cyclical consumption and labor-supply inequality.

In TANK, stabilizing output gap minimizes both the conventional term and the inequality term because consumption and labor-supply inequality across households are proportional to the output gap, as demonstrated in the following equations,

$$c_t^K - c_t^R = \frac{\chi_y - 1}{1-\lambda}\left(y_t - y_t^n\right), \quad (17)$$

$$n_t^K - n_t^R = \frac{\sigma^{-1}(1-\chi_y)}{\gamma(1-\lambda)}(y_t - y_t^n). \quad (18)$$

The intuition behind these two equations is as follows. Household h's consumption gap — the difference between consumption c_t^h and the "natural rate" of consumption $(c_t^h)^n$, represented by $c_t^h - (c_t^h)^n$ — is a linear combination of the real wage gap and the profits gap, both of which are proportional to the aggregate output gap $y_t - y_t^n$. With flexible prices, the "natural rate" of consumption inequality $(c_t^K)^n - (c_t^R)^n$ is equal to zero. Consequently, the consumption inequality $c_t^K - c_t^R$ is equal to the consumption inequality gap $c_t^K - (c_t^K)^n - \left(c_t^R - (c_t^R)^n\right)$, and therefore, it is proportional to the output gap $y_t - y_t^n$. The same reasoning extends to labor supply

[18] The welfare function of the one-sector TANK model is obtained when we set $\omega_F = 0$ and $\omega_S = 1$, as shown in Supplementary Appendix D.1.

inequality $n_t^K - n_t^R$.

It is important to note that the presence of non-zero consumption and labor-supply inequality arises from the assumption that Keynesians and Ricardians have different (real) income cyclicality.[19] In fact, there is no consumption or labor-supply inequality when households share the same income cyclicality, i.e., $\chi_y = 1$.

Turning to the supply side, the Phillips Curve in the sticky-price sector is given by

$$\pi_{S,t} = \kappa_S (y_t - y_t^n) + \beta \mathrm{E}_t \pi_{S,t+1}, \tag{19}$$

where

$$\kappa_S = \frac{(1-\alpha_S)(1-\alpha_S\beta)}{\alpha_S}\frac{\sigma^{-1}+\gamma}{\omega_S}.$$

It indicates output gap stabilization not only stabilizes inequality but also minimizes price dispersion. This is why we refer to the result in Proposition 4 as the "triple divine coincidence".

Proposition 4 suggests that breaking the irrelevance result, or in other words, making inequality relevant for monetary policy, can be achieved through two avenues. The first one involves assuming sticky prices in more than one sectors. This is known as a result of the "lack of instrument" argument. Essentially, with only one instrument at their disposal, monetary policymakers face limitations in implementing efficient allocation in a multi-sector economy. Further elaboration on this aspect is provided in Section 6.3. Alternatively, the second approach involves assuming heterogeneity in consumption baskets across households.

Our primary focus in this paper is on the assumption of heterogeneous consumption baskets — a novel assumption in the existing literature. In the subsequent section, we will delve into the consequence of introducing heterogeneous consumption baskets. It's noteworthy that, when the "triple divine coincidence" result holds, the optimal policy aligns with replicating the flexible-price equilibrium, which is inherently efficient. However, the introduction of heterogeneous consumption baskets gives rise to a a time-varying inefficient wedge in the output level between the flexible-price equilibrium and the efficient one. The flexible-price equilibrium, while traditionally considered efficient, now exhibits social inefficiency, and this inefficiency itself is time-varying. This dynamics introduces a fundamental shift in the nature of optimal monetary policy.

[19]This assumption is supported by recent empirical evidence (Patterson (2023)). The results in proposition 4 also hinge on the assumption that Keynesians' income cyclicality χ_y is linear and constant. Models that deviate from these assumptions fall beyond of the scope of this paper, and we leave them for future research. Jennifer and Morrison (2023) allows income distribution to vary with business cycles, essentially violating the linear assumption.

6.2 The Inefficiency of Flexible-Price Equilibrium: a Time-Varying Wedge

This section illustrates the inefficiency of the flexible-price equilibrium when households have different consumption baskets. This inefficiency arises because households' price indices have different exposures to sectoral shocks, leading to cyclical inequality in their cost of living, even under flexible prices. The inability of Keynesians to trade financial assests prevents households from fully sharing such risks.

As a benchmark, we first establish conditions under which the first-best (or efficient) allocations are satisfied, as summarized in the following lemma.[20]

Lemma 3. *Consider the TANK-HT model specified in section 3 with sectoral productivity shocks* $\{A_{i,t}\}$. *The first-best allocation* $\{\widetilde{C}^h_{i,t}, \widetilde{C}^h_t, \widetilde{N}^h_t\}$ *satisfies the following conditions:*

$$u'(\widetilde{C}^h_t)\frac{d\widetilde{C}^h_t}{d\widetilde{C}^h_{i,t}}A_{i,t} = V'(\widetilde{N}^h_t), \tag{20}$$

$$\frac{V'(\widetilde{N}^K_t)}{V'(\widetilde{N}^R_t)} = 1, \tag{21}$$

$$\frac{U'(\widetilde{C}^K_t)}{U'(\widetilde{C}^R_t)} = \left(\frac{A^K_t}{A^R_t}\right)^{-1}, \tag{22}$$

for households $h \in \{R, K\}$ *and all sectors* $i = 1, .., S$, *where* $A^h_t = (\sum_i \omega^h_i A^{\eta-1}_{i,t})^{1/(\eta-1)}$.

Condition (20) indicates that it is optimal to equate households' marginal disutility of labor on the right-hand side of (20) with the marginal benefit of labor on the left-hand side. Condition (21) shows that given any amount of total labor supply, it is optimal for the social planner to equate N^K_t with N^R_t, thereby minimizing labor supply inequality. Condition (22) states that the social planner chooses to allocate more consumption to the baskets with, on average, higher productivity.

Subsequently, our focus shifts to the set of conditions characterizing the flexible-price equilibrium, as presented in the following proposition.[21]

[20] Supplementary Appendix D.4 proves this lemma.
[21] Supplementary Appendix D.5 proves this proposition.

Proposition 5. *The flexible-price equilibrium in TANK-HT satisfies the following conditions:*

$$u'(C_t^h)\frac{dC_t^h}{dC_{i,t}^h}A_{i,t} = V'(N_t^h), \tag{23}$$

$$\frac{V'(N_t^K)}{V'(N_t^R)} = \varepsilon_t \times \frac{V'(\widetilde{N}_t^K)}{V'(\widetilde{N}_t^R)}, \tag{24}$$

$$\frac{U'(C_t^K)}{U'(C_t^R)} = \varepsilon_t^{-1} \times \frac{U'(\widetilde{C}_t^K)}{U'(\widetilde{C}_t^R)}, \tag{25}$$

where

$$\varepsilon_t = \left(\frac{P_t^K}{P_t^R}\right)^{\frac{(\sigma-1)\gamma}{1+\sigma\gamma}}, \tag{26}$$

$$P_t^K = (A_t^K)^{-1}; \; P_t^R = (A_t^R)^{-1},$$

for households $h \in \{R, K\}$ and all sectors $i = 1, .., I$, where A_t^K and A_t^R are defined in Lemma 3, and $\{\widetilde{C}_{i,t}^h, \widetilde{C}_t^h, \widetilde{N}_t^h\}$ denotes the first-best allocation. The flexible-equilibrium is inefficient except in the case when $\sigma = 1$. In other words, the flexible-price equilibrium is generically inefficient.

Equation (23) in Proposition 5 is identical to equation (20) in Lemma 3, indicating that, given the aggregate consumption C_t^h and labor supply N_t^h, household h's consumption of sectoral goods $\{C_{i,t}^h\}$ is efficient. However, in contrast to conditions (21) and (22) in Lemma 3, conditions (24) and (25) introduce a new wedge ε_t.

The time-varying wedge ε_t in equation (26), determined by the ratio between two price indices P_t^K/P_t^R, implies that the allocation of consumption and labor between Keynesian and Ricardian households are *not* efficient, except when $\sigma = 1$. This infers a generic inefficiency in the flexible-price equilibrium. The wedge arises from the different exposures to sectoral shocks, driven by the presence of different consumption baskets, leading to different price indices P_t^h and consequently different real wages W_t/P_t^h for Keynesian and Ricardian households. The difference in the cost of living and real wages distorts households' labor supply and consumption decisions, resulting in inequality and misallocation.

Unlike exogenous inefficient wedges commonly assumed in the literature, the wedge ε_t is endogenous to monetary policy. This implies, everything else equal, monetary policymakers should strive to minimize the difference between the cost of living for Ricardians and Keynesians, thereby reducing fluctuations in inequality.

Example: Revisiting Aoki (2001) in T-TANK. In a seminal paper, Aoki (2001) investigates optimal monetary policy in a RANK model featuring two sectors — one with perfectly flexible prices and the other with sticky prices. It shows that the optimal policy involves stabilizing the price index of the sticky-price sector (often referred to as the "core" CPI), thereby simultaneously stabilizing the aggregate output gap. Proposition 4 in the previous section shows that this policy remains optimal in its TANK counterpart.

To examine the impact of introducing heterogeneous consumption baskets, we revisit Aoki (2001)'s analysis of optimal monetary policy in T-TANK, as specified in section 4. We maintain the assumption from Section 4 with the modification that the Keynesians consume goods with completely flexible prices, while the Ricardians consume goods with sticky prices. This exercise serves two purposes: (1) it involves minimal deviation from assumptions in Proposition 4, where the flexible-price equilibrium is efficient and output gap stabilization is optimal, and therefore distills the role of heterogeneous consumption baskets on optimal monetary policy in a tractable manner, (2) it offers intuition that, with heterogeneous consumption baskets, central banks should assign a larger weight to the flexible-price sector when designing the optimal inflation index for the inflation targeting policy. This intuition extends to a more realistic setting discussed in Section 6.3.

Lemma 4 derives the second-order approximation of the period-t social welfare function.[22]

Lemma 4. *The social welfare loss function at period t is approximated by the following expression*

$$W_t = -\frac{V'(\overline{Y})\overline{Y}}{2}\Big[\underbrace{\left(\gamma+\sigma^{-1}+\omega_s\omega_F\frac{1+\gamma-\chi_y}{1-\lambda}\right)(y_t-y_t^n)^2+\frac{\omega_S\theta\alpha_S}{(1-\alpha_S)^2}\pi_{S,t}^2}_{\text{Conventional term}} + \underbrace{(1-\sigma^{-1})\lambda(1-\lambda)(c_{K,t}-c_{R,t})^2+\gamma\lambda(1-\lambda)(n_{K,t}-n_{R,t})^2}_{\text{Inequality term}}\Big] \quad (27)$$

up to the second order, where consumption inequality $c_{K,t}-c_{R,t}$ and labor inequality $n_{K,t}-n_{R,t}$ are given by equation (28) *and* (29)*, respectively:*

$$c_{K,t}-c_{R,t} = \frac{\chi_y-1}{1-\lambda}\left(y_t-y_t^n\right)-\frac{\gamma+1}{\gamma+\sigma^{-1}}(p_t^K-p_t^R), \quad (28)$$

$$n_{K,t}-n_{R,t} = -\frac{\sigma^{-1}(\chi_y-1)}{\gamma(1-\lambda)}(y_t-y_t^n)+\frac{\sigma^{-1}}{\gamma}\frac{\gamma+1}{\gamma+\sigma^{-1}}(p_t^K-p_t^R). \quad (29)$$

Fluctuations in the aggregate output gap and inflation in the sticky-price sector constitute the conventional component of the social welfare loss function. The inequality component includes consumption and labor-supply inequality. However, compared to equations (17) and (18), there exists non-zero inequality even if the aggregate output is equal to its natural rate. This inequality is due to the time-varying wedge discussed above, reflecting the cyclical inequality in

[22] Please refer to Supplementary Appendix D.6 for detailed proofs.

the cost of living.

The Phillips Curve is given by equation (19). The next proposition states that the optimal policy outlined in Aoki (2001) is no longer optimal in current framework.

Proposition 6. *Stabilizing the aggregate output gap (or prices in the sector with sticky prices) is no longer the optimal policy in T-TANK. There exists a tradeoff between minimizing inequality and achieving the conventional goal of stabilizing the output gap and minimizing price dispersion. Furthermore, monetary policy cannot achieve the socially efficient (first-best) allocations.*

To build intuition, we examine the static case with the discount factor $\beta = 0$. When a central bank aims to solely minimize the inequality term in expression (27), the optimal policy is to set the aggregate output gap to be proportional to $y_t^n - y_{F,t}^n$, that is,

$$y_t - y_t^n = \psi(y_t^n - y_{F,t}^n),$$

where

$$\psi = \frac{1}{\chi_y - \gamma - 2},$$

and the natural rates of the aggregate output and the flexible-price-sector output are given by

$$y_t^n = \frac{\gamma + 1}{\gamma + \sigma^{-1}} a_t,\ y_{F,t}^n = \frac{\gamma + 1}{\gamma + \sigma^{-1}} a_{F,t}.$$

In contrast, a monetary policymaker who assigns no weight to the inequality term will choose to stabilize $\pi_{S,t}$, thereby simultaneously stabilizing the aggregate output gap.

To sum up, when households have heterogeneous consumption baskets, the central bank encounters a tradeoff between achieving conventional goals and minimizing inequality. Consequently, stabilizing the aggregate output gap is no longer optimal and the optimal monetary policy cannot restore the efficient allocation. In the next section, we extend our analysis to the more realistic case where prices are sticky in both sectors.

6.3 Revisiting Benigno (2004) in TANK-HT

The "triple divine coincidence" result in Proposition 4, where output gap stabilization is socially optimal, critically depends on the assumption that prices in all but one sectors are perfectly flexible. In a two-sector representative-agent model with sticky prices in both sectors, Benigno (2004) shows that monetary policy cannot achieve the socially efficient outcome due to a lack of instruments. While output gap stabilization is no longer optimal, it remains close to optimal.[23] Addi-

[23] Chapter 4.3 in Woodford (2003) provides a more detailed and comprehensive discussion. The welfare loss between the output gap stabilization policy and the optimal monetary policy is minimal.

tionally, Benigno (2004) studies the optimal inflation index among the set of inflation-targeting policies and concludes that the central bank should assign a larger weight to the sector with more sticky prices.

In our TANK-HT framework specified in section 3, We revisit this result and demonstrate that stabilizing the aggregate output gap can lead to substantial welfare loss relative to the optimal policy. Consequently, the optimal inflation index should assign a larger weight to the flexible-price sector than in Benigno (2004).

In this framework, households allocate consumption between two sectors, denoted as sector 1 and sector 2, with different frequencies of price changes represented by $1-\alpha_1$ and $1-\alpha_2$, respectively. The consumption weights on sector 1 and sector 2 are denoted by ω_1^K and ω_2^K for Keynesian households, and ω_1^R and ω_2^R for Ricardian households. The following proposition gives the approximated welfare function in period t.

Proposition 7 (Welfare Function)**.**

$$W_t = -\frac{V'(\overline{Y})\overline{Y}}{2}\Big[\underbrace{(\gamma+\sigma^{-1})(y_t-y_t^n)^2+\sum_{i=1,2}\frac{\omega_i\theta\alpha_i}{(1-\alpha_i)^2}\pi_{i,t}^2+\omega_1\omega_2\left[(y_{1,t}-y_{2,t})-(a_{1,t}-a_{2,t})\right]^2}_{\text{Conventional term}}$$
$$+\underbrace{(\sigma^{-1}-1)\lambda(1-\lambda)(c_{K,t}-c_{R,t})^2+\gamma\lambda(1-\lambda)(n_{K,t}-n_{R,t})^2}_{\text{Inequality term}}\Big], \quad (30)$$

where $\omega_i = \lambda\omega_i^K + (1-\lambda)\omega_i^R$, and the consumption inequality and labor supply inequality are given by (28) *and* (29)*, respectively.*

The conventional term in expression (30) consists of three components. The first component is proportional to the volatility of the output gap. The second component quantifies the welfare loss due to price dispersion within sectors, capturing the misallocation arising from nonzero inflation in the textbook New Keynesian models. The third component arises from the misallocation of outputs between sectors, deviating from their efficient levels. Consistent with our earlier findings , the inequality term is a function of the output gap and the difference in price indices. This introduces a tradeoff between output gap stabilization and inequality stabilization.

In the subsequent discussion of this section, we explore three sets of policies: 1) the optimal monetary policy, 2) the policy to completely stabilize the aggregate output gap, and 3) inflation-targeting policies. Among the inflation targeting policies, the central bank chooses an inflation index, denoted as $\pi_t^O = \phi_1\pi_{1,t}+\phi_2\pi_{2,t}$ and satisfying the constraint $\{\phi_1,\phi_2 : \ \phi_1+\phi_2 = 1\}$, to maximize the social welfare

$$U = \sum_{t=0}^{\infty}\beta^t W_t, \quad (31)$$

subject to the following zero-inflation constraint:

$$\phi_1 \pi_{1,t} + \phi_2 \pi_{2,t} = 0.$$

Our TANK-HT framework departs from Benigno (2004) in two assumptions: 1) the presence of two distinct household types (K and R) instead of a representative household, and 2) the introduction of heterogeneity in the consumption baskets of K and R households. We investigate how these two assumptions reshape the optimal inflation-targeting policy in Benigno (2004). We reveal that the second assumption qualitatively changes the optimal monetary policy, while the first assumption does not. To delve into the details, We initiate our analysis by temporarily disregarding the second assumption to isolate the effect of the first assumption.

The Role of Heterogeneous Agents. All households have identical consumption baskets, implying $\omega_1^K = \omega_1^R$ and $\omega_2^K = \omega_2^R$. Essentially, We are comparing a two-sector TANK model with a two-sector RANK model, assuming heterogeneous price stickiness between sectors.

Introducing heterogeneous agents, as opposed to its RANK counterpart, introduces an inequality term to the welfare loss function. This term turns out to be proportional to the output gap, as shown by the following corollary:

Corollary 2. *The approximated welfare loss function when households have homogeneous consumption baskets is given by*

$$W_t = -\frac{V'(\overline{Y})\overline{Y}}{2}\Big[\underbrace{(\gamma+\sigma^{-1})(y_t - y_t^n)^2 + \frac{\omega_i \theta \alpha_i}{(1-\alpha_i)^2}\pi_{i,t}^2 + \sum_{i,j}\omega_i\omega_j\left[(y_{i,t}-y_{j,t})-(a_{i,t}-a_{j,t})\right]^2}_{\text{Conventional term}}$$
$$+ \underbrace{(\sigma^{-1}+\gamma-\gamma/\sigma^{-1})\lambda(1-\lambda)\frac{\sigma^{-1}}{\gamma}\left(\frac{\chi_y - 1}{1-\lambda}\right)^2 (y_t - y_t^n)^2}_{\text{Inequality term}}\Big], \quad (32)$$

The expression (32) implies that, compared to the RANK model in Benigno (2004), the central bank finds it optimal to put a greater weight on stabilizing the aggregate output gap. However, given that output gap stabilization is already deemed nearly optimal in Benigno (2004),[24] introducing heterogeneous agents does not qualitatively change the optimal monetary policy.

In the context of inflation-targeting policies, the redistributive motive suggests that the central bank benefits from adopting a more aggressive stance in stabilizing the inflation of the sector with stickier prices. This is due to the fact that an equivalent amount of variation in the sectoral inflation results in a smaller sectoral output gap when a sector has relatively stickier prices.

[24] Recent papers studying optimal monetary policy in RANK have similar findings (e.g., Rubbo (2023); La'O and Tahbaz-Salehi (2022))

We illustrate these points with a numerical example in the Supplementary Appendix D.7. The results demonstrate that stabilizing the aggregate output gap is nearly optimal in both RANK and TANK. However, compared to RANK, the optimal inflation index in TANK assigns more weight to the sector with stickier prices.

The Role of Heterogeneous Consumption Baskets. In the subsequent analysis, we relax the assumption on homogeneous consumption baskets. As demonstrated earlier, introducing heterogeneous consumption baskets generates a new type of tradeoff. Importantly, output gap stabilization leads to substantial welfare loss when the difference in consumption baskets between households is sufficiently large.

Given the inherent complexity in this class of models, obtaining analytical expressions for the optimal policy is typically infeasible. We employ a numerical example to illustrate our findings. In particular, we set $\alpha_1 = 0.75$, $\alpha_2 = 0.85$, $\lambda = 0.5$, and use $\Delta\omega = \omega_1^K - \omega_2^K = \omega_2^R - \omega_1^R$ to measure the degree of heterogeneity in households' consumption baskets.

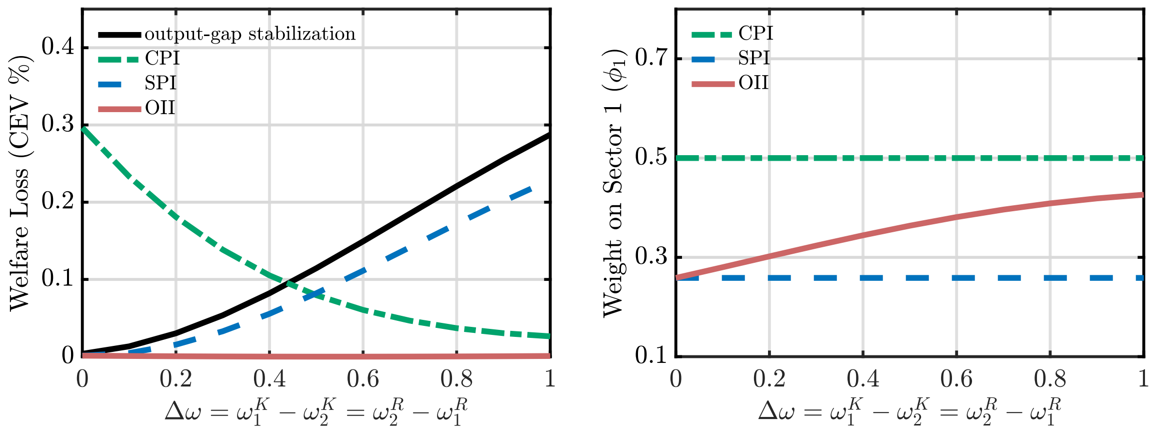
We examine the output-gap stabilization policy along with three sets of inflation targeting policies: stabilizing the Consumer Price Index (CPI), stabilizing the Sticky Price Index (SPI), and stabilizing the Optimal Inflation Index (OII). We intentionally choose a symmetric structure to keep the CPI and SPI unchanged as $\Delta\omega$ varies. For each $\Delta\omega \in [0,1]$, the weight of the CPI is given by $\{0.5, 0.5\}$, and the SPI is obtained by calculating the optimal inflation index, assuming households have the same consumption baskets with weights given by $\{\lambda\omega_1^K + (1-\lambda)\omega_1^R, \lambda\omega_2^K + (1-\lambda)\omega_2^R\}$.[25] The OII is defined as the optimal inflation index with heterogeneous consumption baskets under TANK-HT.

Panel (a) in Figure 7 plots the welfare loss (relative to that of the optimal monetary policy) under different policies. As the difference in expenditure share $\Delta\omega$ increases, the welfare loss of stabilizing the output gap becomes more substantial. This contrasts with RANK and TANK, where output gap stabilization is nearly optimal. Stabilizing the SPI results in a similar magnitude of welfare loss. However, stabilizing OII implements the optimal monetary policy almost perfectly. Interestingly, when $\Delta\omega$ is sufficiently large, stabilizing CPI becomes more desirable than stabilizing the output gap and SPI. For example, when $\Delta\omega = 1$, or the Keynesians only consume goods produced in sector 1, output-gap stabilization leads to welfare loss that is an order of magnitude larger than stabilizing CPI.

Panel (b) in figure 7 illustrates how the weight on the more-flexible-price sector 1 , denoted as ϕ_1, varies with the difference in expenditure share, $\Delta\omega$, when the central bank aims to stabilize one of the three inflation indices. In line with Benigno (2004), the SPI allocates more weight to

[25] It is essentially the optimal price index in TANK with homogeneous consumption baskets, as in Benigno (2004).We refer to it as sticky price index.

Figure 7: Monetary Policies in TANK-HT



Note: This figure plots the variations in welfare loss (relative to optimal policy) and the weight on the flexible-price sector 1 with the difference in expenditure shares, $\Delta\omega = \omega_1^K - \omega_2^K$, under the output-gap stabilization policy and three different inflation-targeting policies — stabilizing the CPI, stabilizing the optimal inflation index in Benigno (2004) under TANK (SPI), and stabilizing the optimal inflation index under TANK-HT (OII). We set $\gamma = 0.5$ and $\sigma = 3$ to generate reasonable degree of strategic complementarity and slope of Phillips curves, as suggested by the empirical evidence.

the sticky-price sector compared to the CPI ($\phi_1^{SPI} < \phi_1^{CPI}$). When $\Delta\omega$ equals zero ($\omega_1^K = \omega_2^K = \omega_2^R = \omega_1^R = 0.5$), the OII and SPI are essentially identical in terms of inflation weights and welfare losses. However, as Keynsians spend more on goods from the more-flexible-price sector ($\Delta\omega > 0$, with $\omega_1^K > \omega_2^K$ and $\omega_1^R < \omega_2^R$), ϕ_1^{OII} becomes greater than ϕ_1^{SPI} due to a stronger redistributive motive. To reduce inequality, the central bank should assign more weight to the flexible-price sector, disproportionally consumed by the Keynesians, to stabilize its inflation.

Furthermore, ϕ_1^{OII} increases with $\Delta\omega$. This is because as the degree of heterogeneity in consumption baskets increases, the difference in price flexibility faced by different households becomes larger. Consequently, the inequality in consumption and labor supply, driven by heterogeneous price indices, also increases. Therefore, the central bank places more weight on stabilizing the inflation of the flexible-price sector to mitigate inequality.

7 Conclusion

This paper documents the existence of cyclical inequality in the cost of living, as well as a negative relationship between households' marginal propensity to consume (MPC) and the price stickiness of goods they consume. We argue that this negative relationship is essential for understanding the monetary transmission mechanism and optimal monetary policy in HANK.

Our framework intentionally abstracts from (the cyclicality of) idiosyncratic risks faced by households when studying the monetary transmission mechanism. This deliberate simplification is motivated by our focus on the redistributional channel, and the Two-Agent framework is chosen for its parsimony and tractability in examining this specific channel. It's worth noting that our results can potentially be extended to models with idiosyncratic risks (e.g., Werning (2015) and Acharya and Dogra (2020)), as these models often rely on the cyclicality of real spending. However, we leave the exploration of this extension to future research.

Our analysis underscores the importance of considering the cyclical inequality in the cost of living when central banks conducting monetary policy. First, ignoring this statistic may lead to overestimating the effectiveness of monetary policy. Second, this statistic can also serve as a measure for assessing the degree of inefficiency inherent in the flexible-price equilibrium.

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Supplementary Appendix

(For Online Publication Only)

Cyclical Inequality in the Cost of Living and

Implications for Monetary Policy

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A Data Appendix

A.1 Constructing Group-Specific CPIs

A.1.1 Consumer Expenditure Survey

The Consumer Expenditure Survey (CEX), conducted by the U.S. Census Bureau, is the major source of constructing the weights for the U.S. Consumer Price Index, due to its extensive information on households' expenditures.

The CEX contains two modules, the Diary and the Interview. The Diary is designed to measure households' non-durable consumption and services, such as groceries and other frequent purchases. So it is surveyed weekly and therefore contains weekly expenditures. The Interview is designed to measure households' durable consumptions, such as vehicles and other large infrequent purchases. It records expenditures over the previous three months. The Diary and Interview modules together collect households' expenditures on overall approximately 600 Universal Classification Code (UCC) categories, 250 UCCs in the Diary module and 350 UCCs in the Interview module.

The Diary and Interview modules survey different households, so it is impossible to observe the full consumption baskets of an individual household. We instead split households' into different groups and compute the group- specific expenditure shares across consumption categories, as we do next.

A.1.2 Constructing group-specific expenditure weights

The CEX also contains information on households' tenure status. To construct CPIs for households with different housing tenure status (or MPCs), we first combine the information on housing tenure with the Diary and Interview Survey to obtain the group-specific expenditure weights.

However, the item-level prices to construct the CPI are provided by the BLS using a different classification system, with 8 major groups, 70 expenditure classes, 211 item strata (item level) and 303 entry level items (ELI). Hence, before constructing the expenditure weights across consumption categories, we follow Cravino et al. (2020) to build a concordance between UCC categories, item strata and ELIs. The concordance between UCCs and item strata is used to compute the group-specific CPIs. The concordance between UCCs and ELIs is used to compute the group-specific average frequency of price adjustment.

Armed with the concordance, we are able to compute the group-specific expenditure weights. To do so, we follow closely the procedure in the BLS document "*CPI requirements for CE*". In particular, we first make adjustments on housing, medical care and transportation, to meet BLS's requirements for constructing CPI expenditure weights. We then follow the BLS manual to calculate the annualized average expenditure for each UCC category for high- and low-MPC households respectively, denoted by $X^h_{i,t}$, where i is UCC category and h is the household type.

We then aggregate the expenditures to the item strata and ELI level using the concordance above, denoted by $X^h_{j,t}$. The corresponding expenditure weights are given by $\omega^h_{j,t} = \frac{X^h_{j,t}}{\sum_j X^h_{j,t}}$.

A.1.3 Constructing group-specific CPIs

The BLS releases item-level consumer price data every month. Among their releases. we use the seasonally adjusted data for all urban consumers. We follow the formula from the BLS manual "*Chapter 17. The Consumer Price Index*" to construct the group-specific CPIs:

$$PIX^h_t = PIX^h_v \cdot \sum_j (\omega^h_{j,\phi} \times \frac{P_{j,t}}{P_{j,t}}),$$

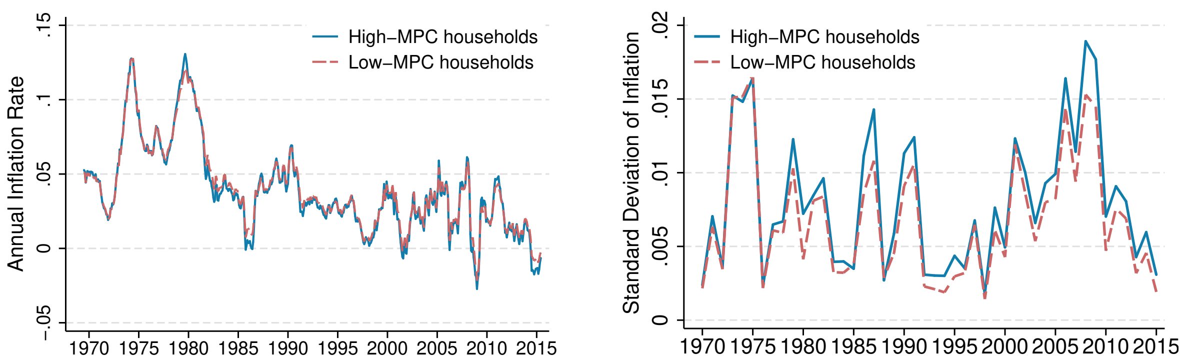
where PIX^h_t is the CPI for household type h in month t, v is the pivot year and month (usually December), α is the expenditure weight reference period determined by the BLS, $P_{j,t}$ is the price of item j at month t and $\omega^h_{j,\phi}$ is the expenditure weight of household type h for item j during the expenditure weight reference period ϕ.

B Additional Empirical Results

B.1 Volatility of Group-specific Inflation Rates

The left panel in Figure B.1 plots the annual inflation of group-specific CPIs. As illustrated in the figure, the annual inflation is more volatile for households with high MPCs. We next compute the annual standard deviation of monthly inflation. The right panel in Figure B.1 plots this standard deviation of monthly inflation. The volatility of high-MPC households is greater than low-MPC households in every year. On average, the standard deviation of annualized monthly inflation is 0.464% for high-MPC households, 9% greater than that of low-MPC households, whose is 0.426%.

Figure B.1: Group-specific Inflation and Inflation Volatility



C Model Details in Section 4

C.1 Steady state

Under zero inflation steady state, there are no price changes and prices are constant. Firm j in sector i sets price:

$$
\begin{aligned}
P_i(j) &= \frac{\epsilon}{\epsilon-1}\frac{1}{1+\tau}W \\
&= W
\end{aligned}
$$

where $\tau = \frac{1}{\epsilon-1}$. Government imposes lump-sum taxes on profits to provide subsidy. The equilibrium profits are zero. Thus we have $W = P^K = P^R$. Combining first order condition $\frac{W}{P^i} = (C^i)^{\gamma^{-1}}(N^i)^{\gamma}$and households' budget constraint $WN^i = P^iC^i$ yields:

$$
C = C^K = C^R = 1.
$$

Where C^K and C^R are consumption per capita. We also notice that, this is the only steady state equilibrium where profits are zero and the amount of subsidies equals the lump-sum taxes on profits.

C.2 Log-linearized equations

Because steady state profits are zero, before listing all the log-linearized equations, I denote $\pi_t = \ln(\Pi_t/C)$

Euler equation:

$$c_t^R = \mathrm{E}_t c_{t+1}^R - \sigma r_t \tag{C.1}$$

Labor supply:

$$\gamma n_t^K = w_t - p_t^K - \sigma^{-1} c_t^K \tag{C.2}$$

$$\gamma n_t^R = w_t - p_t^R - \sigma^{-1} c_t^R \tag{C.3}$$

Labor demand:

$$n_t = \lambda y_t^K + (1-\lambda) y_t^R$$

Labor market clearing condition:

$$\lambda n_t^K + (1-\lambda) n_t^R = n_t$$

Goods market clearing condition:

$$c_t^K = y_t^K;\ c_t^R = y_t^R$$

Keynesian households' budget constraint:

$$p_t^K + c_t^K = w_t + n_t^K + \frac{\tau^\pi \pi_t}{\lambda}$$

Phillips curves:

$$\beta \mathrm{E}_t \pi_{t+1}^R = \pi_t^R + \kappa_R[-\lambda p_t^r - (\gamma + \sigma^{-1}) c_t],$$

$$\beta \mathrm{E}_t \pi_{t+1}^K = \pi_t^K + \kappa_K[(1-\lambda) p_t^r - (\gamma + \sigma^{-1}) c_t],$$

$$\text{where } p_t^r = p_t^K - p_t^R \text{ and } \kappa_i = \frac{(1-\alpha_i)(1-\alpha_i\beta)}{\alpha_i}.$$

Interest rates:

$$r_t = i_t - \mathrm{E}_t \pi_t^R$$

$$i_t = \phi_\pi \pi_t^M + v_t$$

C.3 Equilibrium and Determinancy

An equilibrium in this economy is a set of endogenous price and quantity variables defined above, such that given the prices, households maximize their utilities, firms maximize their profits and all markets clear. To keep the analysis tractable, we approximate the economy around its deterministic zero inflation steady state up to first order. The following lemma shows under what conditions the equilibrium exists and is locally unique.[26]

Lemma 5. (Determinancy) *Given any set of parameters, to first order, there exists a locally unique equilibrium if and only if the number of eigenvalues of $A^{-1}B$ outside of the unit circle is equal to 3, where*

$$A = \begin{bmatrix} 1 & \frac{1-\lambda}{1-\lambda\chi_y}[1-\frac{\lambda(1+\gamma)}{\gamma+\sigma^{-1}}] & \frac{\lambda(1-\lambda)(1+\gamma)}{(1-\lambda\gamma)(\gamma+\sigma^{-1})} & 0 \\ 0 & \beta & 0 & \lambda\kappa_R \\ 0 & 0 & \beta & -(1-\lambda)\kappa_K \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & \frac{(1-\xi)(1-\lambda)\sigma\phi_\pi}{1-\lambda\chi_y} & \frac{(1-\lambda)\sigma\phi_\pi\xi}{1-\lambda\chi_y} & 0 \\ -\kappa_R(\gamma+\sigma^{-1}) & 1 & 0 & 0 \\ -\kappa_K(\gamma+\sigma^{-1}) & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

Proof. Combining equations in section C.2 we get the following linear systems which solve the equilibrium:

$$r_t = \phi_\pi[\xi\pi_t^K + (1-\xi)\pi_t^R] - \mathrm{E}_t\pi_{t+1}^R + v_t$$

$$\mathrm{E}_t c_{t+1} = c_t + \frac{1-\lambda}{1-\lambda\chi_y}[\sigma r_t + \lambda\frac{1+\gamma}{\gamma+\sigma^{-1}}(\mathrm{E}_t\pi_{t+1}^R - \mathrm{E}_t\pi_{t+1}^K)]$$

$$\beta\mathrm{E}_t\pi_{t+1}^R = \pi_t^R - \kappa_R[\lambda p_t^r + (\gamma+\sigma^{-1})c_t] \tag{C.4}$$

$$\beta\mathrm{E}_t\pi_{t+1}^K = \pi_t^K - \kappa_K[-(1-\lambda)p_t^r + (\gamma+\sigma^{-1})c_t] \tag{C.5}$$

$$p_t^r = p_{t-1}^r + \pi_t^K - \pi_t^R$$

$$v_t = \rho v_{t-1} + \zeta_t$$

where

$$p_t^r = p_t^K - p_t^R,$$

$$\kappa_i = \frac{(1-\alpha_i)(1-\alpha_i\beta)}{\alpha_i}$$

[26] Since $p_{t-1}^r = p_t^K - p_t^R$ is a state variable at t, we need to solve a quartic (4th degree polynomial) difference equation to obtain the condition for determinancy. Although quartic equation is the equation which has a solution formula, with non-numerical parameters, it is almost impossible to get analytical solution.

Rearrange and write the system of equations into AR(1) representation:

$$A\begin{bmatrix} \mathrm{E}_t c_{t+1} \\ \mathrm{E}_t \pi^R_{t+1} \\ \mathrm{E}_t \pi^K_{t+1} \\ p^r_t \end{bmatrix} = B\begin{bmatrix} c_t \\ \pi^R_t \\ \pi^K_t \\ p^r_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{\sigma(1-\lambda)}{1-\lambda\chi_y} \\ 0 \\ 0 \\ 0 \end{bmatrix} v_t$$

where

$$A = \begin{bmatrix} 1 & \frac{1-\lambda}{1-\lambda\chi_y}[1-\frac{\lambda(1+\gamma)}{\gamma+\sigma^{-1}}] & \frac{\lambda(1-\lambda)(1+\gamma)}{(1-\lambda\gamma)(\gamma+\sigma^{-1})} & 0 \\ 0 & \beta & 0 & \lambda\kappa_R \\ 0 & 0 & \beta & -(1-\lambda)\kappa_K \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & \frac{(1-\eta)(1-\lambda)\sigma\phi_\pi}{1-\lambda\chi_y} & \frac{(1-\lambda)\sigma\phi_\pi\xi}{1-\lambda\chi_y} & 0 \\ -\kappa_R(\gamma+\sigma^{-1}) & 1 & 0 & 0 \\ -\kappa_K(\gamma+\sigma^{-1}) & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

Matrix A is an upper triangle matrix and thus is invertible. We get a first order system of difference equations:

$$\begin{bmatrix} \mathrm{E}_t c_{t+1} \\ \mathrm{E}_t \pi^R_{t+1} \\ \mathrm{E}_t \pi^K_{t+1} \\ p^r_t \\ v_t \end{bmatrix} = A^{-1}B\begin{bmatrix} c_t \\ \pi^R_t \\ \pi^K_t \\ p^r_{t-1} \end{bmatrix} + A^{-1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\zeta_t \tag{C.6}$$

In the system of equations, there are three jump variables $\{c_t, \pi^R_t, \pi^K_t\}$ and two pre-determined variables p^r_t, v_t. According to Blanchard-Kahn condition (Blanchard and Kahn (1980)), the system has a unique stable solution if and only if the number of eigenvalues of $A^{-1}B$ outside of the unit circle is equal to 3. □

C.4 Proof of Proposition 1

Replacing (7) into K's consumption function $c^K_t = y^K_t$ and (8) into R's consumption function (9), and aggregating these consumption functions across all households yields the following lemma.

Lemma 6. *The aggregate consumption functions at period t in RANK, TANK and T-TANK are:*

$$\text{RANK:} \qquad c_t^{RANK} = (1-\beta)y_t - \beta\sigma r_t + \beta \mathrm{E}_t c_{t+1}, \tag{C.7}$$

$$\text{TANK:} \qquad c_t^{TANK} = \left[1-\beta(1-\lambda\chi_y)\right] y_t - (1-\lambda)\beta\sigma r_t + \beta(1-\lambda\chi_y)\mathrm{E}_t c_{t+1}, \tag{C.8}$$

$$\text{T-TANK:} \qquad c_t^{T-TANK} = \left[1-\beta(1-\lambda\chi_y)\right] y_t - (1-\lambda)\beta\sigma r_t + \beta(1-\lambda\chi_y)\mathrm{E}_t c_{t+1} - \beta\chi_p(1-\lambda)\mathrm{E}_t(\pi_{t+1}^R - \pi_{t+1}^K). \tag{C.9}$$

Since in equilibrium the aggregate consumption c_t in equation (C.9) is equal to the aggregate income y_t, we obtain the aggregate Euler equation in T-TANK:

$$c_t = \mathrm{E}_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi_y}\sigma r_t - \frac{1-\lambda}{1-\lambda\chi_y}\chi_p(\mathrm{E}_t\pi_{t+1}^R - \mathrm{E}_t\pi_{t+1}^K), \tag{C.10}$$

Iterating equation (C.10) forward, we prove Proposition 1.

C.5 Proof of Lemma 2

Combining the sectoral Phillips curves (C.4), (C.5) and the expression for aggregate consumption (12) delivers

$$p_{t+1}^r - \left[\lambda\kappa_R + (1-\lambda)\kappa_K + \chi_p(\kappa_R-\kappa_K)\frac{1-\lambda}{1-\lambda\chi_y} + 2\right] p_t^r + p_{t-1}^r = z_t, \tag{C.11}$$

where

$$z_t = (\kappa_R - \kappa_K)(\gamma\sigma+1)\frac{1-\lambda}{1-\lambda\chi_y}\mathrm{E}_t\sum_{s=0}^{\infty} r_{t+s}.$$

The corresponding characteristic function is

$$g(x) = x^2 - \left[\lambda\kappa_R + (1-\lambda)\kappa_K + (\kappa_R-\kappa_K)\chi_p\frac{1-\lambda}{1-\lambda\chi_y} + 2\right]x + 1 = 0$$

Denote the two roots of the characteristic function as x_1 and x_2. Denote $A = \lambda\kappa_R + (1-\lambda)\kappa_K + (\kappa_R-\kappa_K)\chi_p\frac{1-\lambda}{1-\lambda\chi_y} + 2$, and consider the following two cases:

Case 1 ($\boldsymbol{\kappa_R > \kappa_K}$): Let us first consider the case where $\kappa_R > \kappa_K$. Determinacy requires that $g(1) < 0$, which is always satisfied with assumption (13). Without loss of generality, let us assume $x_1 < 1$ and $x_2 > 1$. Equation (C.11) can be written as

$$(L^{-1} - x_1)(L^{-1} - x_2)p_{t-1}^r = z_t$$

It follows that

$$(L^{-1} - x_1)p_{t-1}^r = -\frac{1}{x_2(1-x_2^{-1}L^{-1})}z_t$$

We then obtain

$$p_t^r = x_1 p_{t-1}^r - \frac{1}{x_2}\sum_{j=0} x_2^{-j} z_{t+j}$$

When $\kappa_R > \kappa_K$, it follows that $z_t p_t^r < 0$ for all t, So that $(\alpha_R - \alpha_K) p_t^r \phi_t < 0$ holds, where $\phi_t = \mathrm{E}_t \sum_{s=t}^{\infty} r_s$.

Case 2 ($\boldsymbol{\kappa_R < \kappa_K}$): Consider the second case where $\kappa_R < \kappa_K$. Determinacy requires that $g(1) < 0$ which is satisfied by assumption. Similar to the proof of the first case, it is straightforward to prove that $(\alpha_R - \alpha_K) p_t^r \phi_t < 0$ holds.

C.6 Aggregate MPC Decomposition in Partial Equilibrium

To begin with, consider an economy with heterogeneous agents in partial equilibrium, where the aggregate real output is exogenously increased by $\mathrm{d}Y$, and prices are adjusted infrequently in response to this shock. The aggregate MPC of this economy can be decomposed into three distinct terms:

$$MPC = \sum_i \frac{\mathrm{d}(MPC_i E_i / P_i)}{\mathrm{d}Y} = \sum_i \underbrace{\left(MPC_i \frac{1}{P_i}\frac{\mathrm{d}E_i}{\mathrm{d}Y}\right)}_{\text{Agg. MPC with prices fixed}} - \underbrace{\frac{\mathrm{d}\log P}{\mathrm{d}\log Y}}_{\text{Agg. inflation}} - \underbrace{\mathrm{Cov}\left(MPC_i, \frac{E_i}{P_i Y}\frac{\mathrm{d}\log P_i}{\mathrm{d}\log Y}\right)}_{\text{Covariance}},$$

where E_i is household i's nominal expenditure, P_i is the household-specific price index. In this decomposition, the first term is the aggregate MPC if prices are fixed at their pre-shock value. The second term captures the negative effect of aggregate inflation on aggregate MPC if nominal expenditure E_i remains unchanged. This term is equal to zero if prices are perfectly sticky. The sum of the first two terms is equal to the aggregate MPC in the corresponding TANK model with homogeneous consumption baskets. The last term captures the covariance between households' MPC and the inflation response of the household-specific price index. This term arises from the heterogeneity in price stickiness faced by households with different MPCs.

While it is generally difficult to extend this analytical result to general equilibrium, thanks to our tractable framework, in the main text we show that in general equilibrium the effect of the covariance term on the aggregate consumption is simply summarized by the inflation of of the relative price faced by the Keynesian households, expressed as $p_t^K - p_t^R$.

D Appendix for the Optimal Monetary Policy

This section contains the derivations and proofs in Section 6. The first two sections cover models satisfying conditions in Proposition 4. Specifically, Section D.1 shows how we obtain the welfare loss function and optimal monetary policy in TANK, and Section D.2 revisits Aoki (2001) in TANK with homogeneous consumption baskets. Section D.3 proves Proposition 4 (Triple Divine Coincidence). Section D.4 and D.5 prove Lemma 3 and Proposition 5 respectively.

All the log-linear and log-quadratic approximations below are expanded around a deterministic steady state with $\overline{Y}_K = \overline{Y}_R = \overline{Y}$, $\overline{P}(j) = \overline{P}_R = \overline{P}_K = \overline{P}$, $\overline{A}_K = \overline{A}_R = \overline{A} = 1$. In the steady state, allocation is efficient which implies that

$$\frac{V'(\overline{Y})}{U'(\overline{Y})} = 1.$$

D.1 Optimal Monetary Policy in TANK

We introduce the aggregate productivity shock to the one-sector TANK model. Firm j's production function is

$$Y_t(j) = A_t N_t(j),$$

where A_t is the aggregate productivity shock with finite variance and independent across periods.

D.1.1 Natural Rate of Output

Substituting $n_t = y_t - a_t$ into the labor supply equation we obtain the expression for the nominal wage rate:

$$w_t = (\gamma + \sigma^{-1}) y_t + p_t - \gamma a_t.$$

With flexible prices, firms set their prices equal to the marginal costs $p_t^n = w_t^n - a_t$, which delivers the expression for the natural rate of output:

$$y_t^n = \frac{\gamma + 1}{\gamma + \sigma^{-1}} a_t. \tag{D.1}$$

D.1.2 Cyclical Consumption and Labor Supply

With aggregate productivity shock a_t, the aggregate profit is

$$d_t = p_t + y_t - w_t - n_t = -(w_t - p_t) + a_t, \tag{D.2}$$

Substituting equation (D.1) and (D.2) into equation (6) we have

$$c_{i,t} = \chi_y y_t - \frac{\gamma + 1}{\gamma + \sigma^{-1}} (\chi_y - 1) a_t.$$

Ricardians' consumption is given by

$$c_{R,t} = \frac{1 - \lambda \chi_y}{1 - \lambda} y_t + \frac{\lambda}{1 - \lambda} \frac{\gamma + 1}{\gamma + \sigma^{-1}} (\chi_y - 1) a_t$$

The labor supply of the Keynesians and the Ricardians are given by

$$n_{i,t} = \left[1 - \frac{\sigma^{-1}}{\gamma}(\chi_y - 1)\right] y_t - \left[1 - \frac{\sigma^{-1}(\gamma+1)}{\gamma(\gamma+\sigma^{-1})}(\chi_y - 1)\right] a_t,$$

$$n_{R,t} = \left[1 + \frac{\sigma^{-1}}{\gamma}\frac{\lambda}{1-\lambda}(\chi_y - 1)\right] y_t - \left[1 + \frac{\sigma^{-1}(\gamma+1)}{\gamma(\gamma+\sigma^{-1})}\frac{\lambda}{1-\lambda}(\chi_y - 1)\right] a_t.$$

Hence, the consumption difference between the Keynesians and Ricardians is given by

$$\begin{aligned} c_t^K - c_t^R &= \frac{\chi_y - 1}{1-\lambda}\left(y_t - \frac{\gamma+1}{\gamma+\sigma^{-1}} a_t\right) \\ &= \frac{\chi_y - 1}{1-\lambda}\left(y_t - y_t^n\right). \end{aligned} \tag{D.3}$$

The second equality is due to the fact that the natural rate of aggregate output y_t^n is equal to $\frac{\gamma+1}{\gamma+\sigma^{-1}} a_t$. Similarly, the difference of labor supply is given by

$$n_t^K - n_t^R = \frac{\sigma^{-1}(1-\chi_y)}{\gamma(1-\lambda)}(y_t - y_t^n). \tag{D.4}$$

D.1.3 Deriving the Social Welfare Loss Function

Household type i's utility function is denoted by

$$U(Y_{i,t}) + V(N_{i,t})$$

The second-order Taylor expansion of the firs term of the utility around the steady state is

$$\begin{aligned} U(Y_{i,t}) &= U(\overline{Y}) + U'(\overline{Y})(Y_{i,t} - \overline{Y}) + \tfrac{1}{2}U''(\overline{Y})(Y_{i,t} - \overline{Y})^2 + o(2) \\ &= U(\overline{Y}) + \overline{Y}U'(\overline{Y})y_{i,t} + \tfrac{1}{2}\left(U'(\overline{Y})\overline{Y} + U''(\overline{Y})\overline{Y}^2\right)y_{i,t}^2 + o(2) \end{aligned} \tag{D.5}$$

The second equality follows from the Taylor expansion

$$Y_{i,t}/\overline{Y} = 1 + y_{i,t} + \tfrac{1}{2}y_{i,t}^2 + o(2).$$

Summing across agents delivers

$$\begin{aligned} &\lambda U(Y_{K,t}) + (1-\lambda)U(Y_{R,t}) \\ &= U(\overline{Y}) + \overline{Y}U'(\overline{Y})y_t + \tfrac{1}{2}\left(U'(\overline{Y})\overline{Y} + U''(\overline{Y})\overline{Y}^2\right)\left[\lambda y_{K,t}^2 + (1-\lambda)y_{R,t}^2\right] + o(2) \\ &= U(\overline{Y}) + U'(\overline{Y})\overline{Y}\left[y_t + \frac{1}{2}(1-\sigma^{-1})y_t^2 + \frac{1}{2}(1-\sigma^{-1})\lambda(1-\lambda)(y_{K,t} - y_{R,t})^2\right] \end{aligned} \tag{D.6}$$

where $\sigma^{-1} = -\overline{Y}\frac{U''(\overline{Y})}{U'(\overline{Y})}$, and the second equality uses the fact that

$$\lambda y_{K,t}^2 + (1-\lambda) y_{R,t}^2 = \lambda(1-\lambda)(y_{K,t} - y_{R,t})^2 + y_t^2.$$

The second term of the utility can be approximated around the steady state to second order expressed as

$$\begin{aligned}
&\lambda V(N_{i,t}) + (1-\lambda) V(N_{R,t}) \\
&= V(\overline{N}) + V'(\overline{N})\left[\lambda(N_K - \overline{N}) + (1-\lambda)(N_R - \overline{N})\right] + \frac{1}{2}V''(\overline{N})\left[(\lambda(N_K - \overline{N})^2 + (1-\lambda)(N_R - \overline{N})^2\right] \\
&= V(\overline{N}) + V'(\overline{N})\overline{N}\left[[\lambda n_K + (1-\lambda) n_R] + \frac{1}{2}\left[\lambda n_K^2 + (1-\lambda) n_R^2\right]\right] + \frac{1}{2}V''(\overline{N})\overline{N}^2\left[\lambda n_K^2 + (1-\lambda) n_R^2\right] \\
&= V(\overline{N}) + V'(\overline{N})\overline{N}\left[\lambda n_K + (1-\lambda) n_R + \frac{1}{2}\left[\lambda n_K^2 + (1-\lambda) n_R^2\right]\right] + \frac{1}{2}V'(\overline{N})\overline{N}\gamma\left[\lambda(1-\lambda)(n_K - n_R)^2 + n^2\right] \qquad \text{(D.7)}
\end{aligned}$$

where $\gamma = \overline{N}\frac{V''(\overline{N})}{V'(\overline{N})}$. Denote $\widetilde{Y}_t = \int_{j\in[0,1]} y_t(j)dj$, and approximate the labor market clearing condition

$$\lambda N_{K,t} + (1-\lambda) N_{R,t} = \widetilde{Y}_{S,t}/A_{S,t} + \widetilde{Y}_{F,t}/A_{F,t}$$

to second order

$$\begin{aligned}
&\overline{N}\left[\lambda n_{K,t} + (1-\lambda) n_{R,t}\right] + \frac{1}{2}\overline{N}\left[\lambda n_{K,t}^2 + (1-\lambda) n_{R,t}^2\right] \\
&= \overline{Y}\left(\widetilde{y}_t + \frac{1}{2}\widetilde{y}_t^2 - \widetilde{y}_t a_t\right) + \text{t.i.p} \\
&= \overline{Y}\left(y_t + \frac{1}{2\theta}\text{var} y_t(j) + \frac{1}{2}y_t^2 - y_t a_t\right) + \text{t.i.p} \\
&= \overline{Y}\left(y_t + \frac{\theta\alpha}{2(1-\alpha)^2}\pi_t^2 + \frac{1}{2}y_t^2 - y_t a_t\right) + \text{t.i.p} \qquad \text{(D.8)}
\end{aligned}$$

where t.i.p represents terms independent of policy, the second equality follows from the relationship between $\widetilde{y}_t$ and y_t to second order:

$$\widetilde{y}_t = y_t + \frac{1}{2\theta}\text{var} y_t(j),$$

and the third equality follows from the next two equations:

$$\begin{aligned}
\text{var} y_t(j) &= \theta^2 \text{var} p_t(j), \\
\text{var} p_t(j) &= \frac{\alpha}{(1-\alpha)^2}\pi_t^2.
\end{aligned}$$

Substituting equation (D.8) into equation (D.7) yields

$$\begin{aligned}
&\lambda V(N_{K,t}) + (1-\lambda) V(N_{R,t}) \\
&= V(\overline{Y}) + V'(\overline{Y})\overline{Y}\left[y_t - (1+\gamma) y_t a_t + \frac{1+\gamma}{2}y_t^2 + \frac{\theta\alpha}{2(1-\alpha)^2}\pi_t^2 + \frac{\gamma}{2}\lambda(1-\lambda)(n_{K,t} - n_{R,t})^2\right] + \text{t.i.p} \qquad \text{(D.9)}
\end{aligned}$$

Subtracting expression (D.9) from (D.6), substituting equation (D.3) and equation (D.4) for $c_{K,t} - c_{R,t}$ and $n_{K,t} - n_{R,t}$ and collecting like terms, we finally obtain the expression of the welfare loss function:

$$W_t = -\frac{V'(\overline{Y})\overline{Y}}{2}\Big[\underbrace{(\sigma^{-1}+\gamma)(y_t - y_t^n)^2 + \frac{\theta\alpha}{(1-\alpha)^2}\pi_t^2}_{\text{RANK term}} + \underbrace{(\sigma^{-1}+\gamma-\gamma/\sigma^{-1})\lambda(1-\lambda)\frac{\sigma^{-1}}{\gamma}\left(\frac{\chi_y - 1}{1-\lambda}\right)^2 (y_t - y_t^n)^2}_{\text{Inequality term}}\Big]. \tag{D.10}$$

It is straightforward to show that expression (D.10) is a just special case of expression (16) when $\omega_S = 1$. The optimal monetary policy is to stabilize the aggregate output gap, and it achieves the efficient allocation.

D.2 Revisiting Aoki (2001) in TANK

We assume that the Keynesians and Ricardians consume the *same* baskets of goods across two sectors. Prices in sector F are perfectly flexible but in section S are sticky, adjusted with probability $1-\alpha_S$ in every period. The production function of firm j in sector i is

$$Y_{i,t}(j) = A_{i,t}N_{i,t}(j),$$

where $A_{i,t}$ is the productivity shock in sector i at period t. The consumption weight for goods in sector S and F are ω_S and ω_F respectively, with $\omega_S + \omega_F = 1$. The remaining specification is identical to that in section 4.

With flexible prices, the sectoral prices are equal to the marginal cost, which leads to the following log-linearized equations on sectoral and aggregate natural rate of output:

$$(\sigma^{-1}+\gamma)y_t^n - \gamma a_t - a_{F,t} = -\eta^{-1}(y_{F,t}^n - y_t^n),$$
$$(\sigma^{-1}+\gamma)y_t^n - \gamma a_t - a_{S,t} = -\eta^{-1}(y_{S,t}^n - y_t^n).$$

It follows that

$$y_t^n = \frac{\gamma+1}{\gamma+\sigma^{-1}}a_t,$$
$$y_t^n - y_{F,t}^n = \eta(a_t - a_{F,t}),$$
$$y_t^n - y_{S,t}^n = \eta(a_t - a_{S,t}).$$

We define the relative price in sector i as $x_{j,t} = p_{j,t} - p_t$. The real price in the flexible-price sector are again equal to the real marginal cost, so that

$$\begin{aligned} x_{F,t} &= w_t - p_t - a_{F,t}, \\ &= (\gamma+\sigma^{-1})(y_t - y_t^n) + \eta^{-1}(y_t^n - y_{F,t}^n). \end{aligned} \tag{D.11}$$

The expressions for the wage rate and total profits remain the same as in TANK, which are given by $w_t - p_t = (\gamma+\sigma^{-1})y_t - \gamma a_t$ and $d_t = -(w_t - p_t) + a_t$. It follows that the consumption and labor inequality are also given by equation (17) and (18).

D.2.1 The Phillips Curve in the Sticky-price Sector

Having the chance to reset its prices, firm j in sector S chooses $p^*_{i,t}(j)$ to maximize its discounted sum of future profits:

$$\mathrm{E}\sum_{t=1}^{\infty}\left\{Q_{t,t+1}\left[(1+\tau)P_{S,t}(j)-\frac{W_t}{A_{S,t}}\right]Y_{S,t}(j)\right\}$$

Following the standard practice in the New Keynesian literature, the inflation in the sticky-price sector is given by

$$\pi_{S,t}=\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left[\frac{1-\omega_S}{\omega_S}x_{F,t}+(\gamma+\sigma^{-1})(y_t-y_t^n)+\eta^{-1}(y_t^n-y_{F,t}^n)\right]+\beta\mathrm{E}_t\pi_{S,t+1}$$

Considering that $y_t^n-y_{F,t}^n=-\frac{\omega_S}{1-\omega_S}(y_t^n-y_{S,t}^n)$, and substituting the expression (D.11) into the above expression delivers the Phillips curve in the sticky price sector:

$$\pi_{S,t}=\kappa_S(y_t-y_t^n)+\beta\mathrm{E}_t\pi_{S,t+1}, \tag{D.12}$$

where

$$\kappa_S=\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}+\gamma}{\omega_S}.$$

D.2.2 Deriving the Social Welfare Loss Function

Similar to the previous section, the consumption utility and labor disutility are again approximated by equation (D.6) and (D.7) respectively. We approximate the labor market clearing condition

$$\lambda N_{K,t}+(1-\lambda)N_{R,t}=\widetilde{Y}_{S,t}/A_{S,t}+\widetilde{Y}_{F,t}/A_{F,t}$$

to second order

$$\begin{aligned}
&\overline{N}\left[\lambda n_{K,t}+(1-\lambda)n_{R,t}\right]+\frac{1}{2}\overline{N}\left[\lambda n_{K,t}^2+(1-\lambda)n_{R,t}^2\right]\\
&=\overline{Y}\left(\omega_s\widetilde{y}_{S,t}+\omega_F\widetilde{y}_{F,t}+\frac{1}{2}\omega_s\widetilde{y}_{S,t}^2+\frac{1}{2}\omega_F\widetilde{y}_{F,t}^2-\omega_S\widetilde{y}_{S,t}a_{S,t}-\omega_F\widetilde{y}_{F,t}a_{F,t}\right)+\text{t.i.p}\\
&=\overline{Y}\left(\omega_S y_{S,t}+\omega_F y_{F,t}+\sum_{i\in\{S,F\}}\frac{\omega_i}{2\theta}\mathrm{var}\,y_{i,t}(j)+\frac{1}{2}\omega_s y_{S,t}^2+\frac{1}{2}\omega_F y_{F,t}^2-\omega_S y_{S,t}a_{S,t}-\omega_F y_{F,t}a_{F,t}\right)+\text{t.i.p}\\
&=\overline{Y}\left(y_t+\frac{1}{2}\omega_S\omega_F\eta^{-1}\left[(y_{S,t}-y_{F,t})-(y_{S,t}^n-y_{F,t}^n)\right]^2-ya+\sum_{i\in\{S,F\}}\frac{\omega_i}{2\theta}\mathrm{var}\,y_{i,t}(j)+\frac{y_t^2}{2}\right)+\text{t.i.p}
\end{aligned} \tag{D.13}$$

The second equality follows from the expression below

$$\widetilde{y}_{i,t}=y_{i,t}+\frac{1}{2\theta}\mathrm{var}\,y_{i,t}(j)+o(2).$$

The third equality makes use of the following two equations:

$$\omega_S y_{S,t} + \omega_F y_{F,t} = y_t - \frac{\omega_S \omega_F (1-\eta^{-1})}{2}(y_{F,t} - y_{S,t})^2,$$
$$\omega_S y_{S,t}^2 + \omega y_{F,t}^2 = \omega_S \omega_F (y_{F,t} - y_{S,t})^2 + y_t^2,$$

and the following factoring technique:

$$\omega_S a_{S,t} y_{S,t} + \omega_F a_{F,t} y_{F,t} = y_t a_t + \omega_S \omega_F \eta^{-1}(y_{S,t} - y_{F,t})(y_{S,t}^n - y_{F,t}^n)$$

Plugging equation (D.13) into (D.7), and using similar techniques in the previous section yields

$$\lambda V(N_{K,t}) + (1-\lambda)V(N_{R,t}) = V(\overline{N}) + V'(\overline{N})\overline{N}\Big[\frac{1}{2}(1+\gamma)y_t^2 + y_t + \frac{1}{2}\omega_S\omega_F\eta^{-1}\big[(y_{S,t} - y_{F,t}) - (y_{S,t}^n - y_{F,t}^n)\big]^2 + \sum_{i\in\{S,F\}} \frac{\omega_i}{2\theta}\text{var}\, y_{i,t}(j) - (1+\gamma)y_t a_t + \frac{1}{2}\gamma\lambda(1-\lambda)(n_{K,t} - n_{R,t})^2\Big]. \quad \text{(D.14)}$$

Note that the following results hold

$$(y_{S,t} - y_{F,t}) = \frac{\eta}{\omega_S} x_{F,t} = \frac{\eta(\gamma + \sigma^{-1})}{\omega_S}(y_t - y_t^n) + (y_{S,t}^n - y_{F,t}^n)$$
$$\text{var}\, y_{S,t}(j) = \frac{\alpha\theta^2}{(1-\alpha)^2}\pi_{S,t}^2$$

Substitute these expressions into (D.14) we obtain

$$\lambda V(N_{K,t}) + (1-\lambda)V(N_{R,t}) = V(\overline{N}) + V'(\overline{N})\overline{N}\Big[\frac{1}{2}(1+\gamma)y_t^2 + y_t + \frac{1}{2}\frac{\omega_F}{\omega_S}\eta(\gamma+\sigma^{-1})^2(y_t - y_t^n)^2 + \frac{\omega_S\theta\alpha}{2(1-\alpha)^2}\pi_{S,t}^2 - (1+\gamma)y_t a_t + \frac{1}{2}\gamma\lambda(1-\lambda)(n_{K,t} - n_{R,t})^2\Big]. \quad \text{(D.15)}$$

Subtracting expression (D.15) from (D.6), substituting equation (17) and equation (18) for $c_{K,t} - c_{R,t}$ and $n_{K,t} - n_{R,t}$, and collecting like terms we obtain the expression (16).

The optimal monetary policy is to stabilize the aggregate output gap, or equivalently, inflation in the sticky-price sector π_t^S.

D.3 Proof of Proposition 4

The previous sections D.1 and D.2 already show that aggregate output gap stabilization is the optimal policy in one-sector TANK and two-sector TANK with the setup in Aoki (2001). The proof for the N-sector case (N>1) where there is only one sticky-price sector is very similar to the two-sector case. We only need to show that if the triple divine coincidence holds, these scenarios are the only possible cases.

We first consider the case where households have homogeneous consumption baskets. Existing literature has shown that in a multi-sector model, when there are more than one sectors with sticky prices, stabilizing aggregate output is not the optimal policy (e.g. Benigno (2004), Rubbo (2023)).

When households have heterogeneous consumption baskets, Section 6.2 and Section 6.3 show that stabilizing aggregate output is not the optimal monetary policy. Proposition 4 is therefore proved.

D.4 Proof of Lemma 3

The social planner maximizes the social welfare subject to the resource constraints:

$$\max_{\{N_t^K, N_t^R, C_{i,t}^K, C_{i,t}^R, N_{i,t}\}} \lambda\left[u(C_t^K) - V(N_t^K)\right] + (1-\lambda)\left[U(C_t^R) - V(N_t^R)\right]$$

subject to

$$[\psi]: \ \ \lambda N_t^K + (1-\lambda) N_t^R = \sum_i N_{i,t}$$

$$[\mu_i]: \ \ \lambda C_{i,t}^K + (1-\lambda) C_{i,t}^R = A_{i,t} N_{i,t}$$

$$C_t^R = \left(\sum_i (\omega_i^R)^{\frac{1}{\eta}} (C_{i,t}^R)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} ; \ C_t^K = \left(\sum_i (\omega_i^K)^{\frac{1}{\eta}} (C_{i,t}^K)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

The first order conditions give the following equations:

$$\begin{aligned}
\lambda U'(C_t^K)\frac{dC_t^K}{dC_{i,t}^K} - \lambda \mu_i &= 0 \\
(1-\lambda)U'(C_t^R)\frac{dC_t^R}{dC_{i,t}^R} - (1-\lambda)\mu_i &= 0 \\
\lambda V'(N_t^K) + \psi\lambda &= 0 \\
(1-\lambda)V'(N_t^R) + \psi(1-\lambda) &= 0 \\
\psi + \mu_i A_{i,t} &= 0
\end{aligned}$$

Combining and rearranging equations yields equation (20) and (21). To obtain equation (22), substituting the expression of C_t^R and C_t^K into (20) gives

$$C_{i,t}^h = A_{i,t}^{\eta} (C_t^K)^{1-\eta/\sigma} \omega_i^h (N_{i,t})^{-\eta\gamma} \tag{D.16}$$

Combining this expression with the definition of C_t^h we have

$$\left[\sum_i (\omega_i^h)^{\frac{1}{\eta}} (C_{i,t}^h)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} = C_t^h = (C_t^h)^{(1-\frac{\eta}{\sigma})} (N_t^h)^{-\gamma\eta} (\sum_i \omega_i^h A_{i,t}^{\eta-1})^{\frac{\eta}{\eta-1}}. \tag{D.17}$$

Simplifying,

$$(C_t^h)^{-\sigma^{-1}} = (N_t^h)^{\gamma} (\sum_i \omega_i^h A_{i,t}^{\eta-1})^{\frac{1}{1-\eta}},$$

so that we obtain equation (22)

$$\left(\frac{C_{K,t}}{C_{R,t}}\right)^{-\frac{1}{\sigma}} = \left(\frac{\sum_i \omega_i^K A_{i,t}^{\eta-1}}{\sum_i \omega_i^R A_{i,t}^{\eta-1}}\right)^{\frac{1}{1-\eta}}.$$

Since $N_t^K = N_t^R$ in the efficient allocation, we denote $N_t = N_t^K = N_t^R$. In fact, we can solve for the labor supply N_t using the goods market clearing condition.

$$N_{i,t} A_{i,t} = (\lambda C_{i,t}^K + (1-\lambda) C_{i,t}^R)$$

Aggregating across sectors yields

$$N_t = \sum_i \left(\lambda A_{i,t}^{-1} C_{i,t}^K + (1-\lambda) A_{i,t}^{-1} C_{i,t}^R\right)$$

Substituting equation (D.16) into this expression:

$$N_t = \sum_i \left(\lambda \omega_i^K A_{i,t}^{\eta-1} (C_t^K)^{-\eta\sigma^{-1}+1} N_t^{-\eta\gamma} + (1-\lambda) \omega_i^R A_{i,t}^{\eta-1} (C_t^R)^{-\eta\sigma^{-1}+1} N_t^{-\eta\gamma}\right)$$

Substituting C_h^t using equation (D.4) we solve for N_t:

$$N_t = \left[\lambda (A_t^K)^{\sigma-1} + (1-\lambda)(A_t^R)^{\sigma-1}\right]^{\frac{1}{1+\sigma\gamma}},$$

where $A_t^K = (\sum_i \omega_i^K A_{i,t}^{\eta-1})^{1/(\eta-1)}$ and $A_t^R = (\sum_i \omega_i^R A_{i,t}^{\eta-1})^{1/(\eta-1)}$.

D.5 Proof of Proposition 5

Denote ϕ_t^K and ϕ_t^R as the Lagrangian multiplier of the budget constraint of Keynesians and Ricardians at period t under the flexible-price equilibrium. The first-order conditions of household-type h are:

$$U'(C_t^h) \frac{dC_t^h}{dC_{i,t}} = \phi_t^h P_{i,t}$$
$$V'(N_t^h) = \phi_t^h W_t$$

Combined with the firms' price setting function $P_{i,t} = W_t / A_{i,t}$ we obtain equation (23). Combining the intratemporal condition of households $U'(C_t^h)/V'(N_t^h) = P_t^h / W_t$ and the budget constraint $P_t^h C_t^h = W_t N_t^h$ we can solve for N_t^h:

$$N_t^h = (A_t^h)^{\frac{\sigma-1}{1+\sigma\gamma}}$$

which yields equation (24). Using the intratemporal condition we obtain C_t^h:

$$C_t^h = (A_t^h)^{\frac{\sigma(1+\gamma)}{1+\sigma\gamma}}.$$

D.6 Monetary Policy in T-TANK

When adding sectoral productivity shocks, the consumption and labor supply of the Keynesians and Ricardians are given by

$$
\begin{aligned}
c_{K,t} &= \chi_y y_t - \frac{\gamma+1}{\gamma+\sigma^{-1}}(\chi_y-1)a_t - \frac{\gamma+1}{\gamma+\sigma^{-1}}x_{F,t},\\
c_{R,t} &= \frac{1-\lambda\chi_y}{1-\lambda}y_t + \frac{\lambda}{1-\lambda}\frac{\gamma+1}{\gamma+\sigma^{-1}}(\chi_y-1)a_t + \frac{\lambda}{1-\lambda}\frac{\gamma+1}{\gamma+\sigma^{-1}}x_{F,t},\\
n_{K,t} &= \left[1-\frac{\sigma^{-1}}{\gamma}(\chi_y-1)\right]y_t - \left[1-\frac{\sigma^{-1}(\gamma+1)}{\gamma(\gamma+\sigma^{-1})}(\chi_y-1)\right]a_t + \frac{\sigma^{-1}}{\gamma}\frac{\gamma+1}{\gamma+\sigma^{-1}}x_{F,t}\\
n_{R,t} &= \left[1+\frac{\sigma^{-1}}{\gamma}\frac{\lambda}{1-\lambda}(\chi_y-1)\right]y_t - \left[1+\frac{\sigma^{-1}(\gamma+1)}{\gamma(\gamma+\sigma^{-1})}\frac{\lambda}{1-\lambda}(\chi_y-1)\right]a_t - \frac{\sigma^{-1}}{\gamma}\frac{\lambda}{1-\lambda}\frac{\gamma+1}{\gamma+\sigma^{-1}}x_{F,t}.
\end{aligned}
$$

It then follows that the natural rate of sectoral and aggregate output are given by

$$
y_t^n = \frac{\gamma+1}{\gamma+\sigma^{-1}}a_t,\ y_{i,t}^n = \frac{\gamma+1}{\gamma+\sigma^{-1}}a_{i,t},
$$

and the consumption and labor supply inequality are expressed as

$$
\begin{aligned}
c_{K,t}-c_{R,t} &= \frac{\chi_y-1}{1-\lambda}\left(y_t-y_t^n\right) - \frac{1}{1-\lambda}\frac{\gamma+1}{\sigma^{-1}+\gamma}x_{F,t},\\
n_{K,t}-n_{R,t} &= \frac{\sigma^{-1}(1-\chi_y)}{\gamma(1-\lambda)}(y_t-y_t^n) + \frac{\sigma^{-1}}{\gamma}\frac{1}{1-\lambda}\frac{\gamma+1}{\gamma+\sigma^{-1}}x_{F,t}.
\end{aligned}
$$

Again, define the relative price in sector i as $x_{j,t} = p_{j,t} - p_t$. The real price in the flexible-price sector are again equal to the real marginal cost, so that

$$
\begin{aligned}
x_{F,t} &= w_t - p_t - a_{F,t},\\
&= (\gamma+\sigma^{-1})(y_t-y_t^n) + \frac{\gamma+\sigma^{-1}}{\gamma+1}(y_t^n - y_{F,t}^n).
\end{aligned} \tag{D.18}
$$

Similar to the previous section, the Phillips curve in the sticky-price (or Keynesian) sector is given by

$$
\pi_{S,t} = \kappa_S(y_t-y_t^n) + \beta \mathrm{E}_t\pi_{S,t+1}, \tag{D.19}
$$

where

$$
\kappa_S = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}+\gamma}{\omega_S}.
$$

D.6.1 Deriving the Social Welfare Loss Function

In T-TANK, the consumption utility and labor disutility are again approximated by equation (D.5) and (D.7) respectively. We approximate the labor market clearing condition

$$
\lambda N_{K,t} + (1-\lambda)N_{R,t} = \widetilde{Y}_{S,t}/A_{S,t} + \widetilde{Y}_{F,t}/A_{F,t}
$$

to second order

$$\overline{N}\left[\lambda n_{K,t}+(1-\lambda)n_{R,t}\right]+\frac{1}{2}\overline{N}\left[\lambda n_{K,t}^2+(1-\lambda)n_{R,t}^2\right]$$
$$=\overline{Y}\left(y_t+\frac{\omega_S\omega_F}{2}\frac{2+\gamma-\chi_y}{1-\lambda}(y_t-y_t^n)^2-y_ta_t+\sum_{i\in\{S,F\}}\frac{\omega_i}{2\theta}\text{var}y_{i,t}(j)+\frac{y_t^2}{2}\right)+\text{t.i.p} \tag{D.20}$$

Plugging equation (D.20) into (D.7), and using similar techniques in the previous section yields

$$\lambda V(N_{K,t})+(1-\lambda)V(N_{R,t})=V(\overline{N})+V'(\overline{N})\overline{N}\Big[\frac{1}{2}(1+\gamma)y_t^2+y_t+\frac{\omega_S\omega_F}{2}\frac{2+\gamma-\chi_y}{1-\lambda}(y_t-y_t^n)^2$$
$$+\sum_{i\in\{S,F\}}\frac{\omega_i}{2\theta}\text{var}y_{i,t}(j)-(1+\gamma)y_ta_t+\frac{1}{2}\gamma\lambda(1-\lambda)(n_{K,t}-n_{R,t})^2\Big]. \tag{D.21}$$

Summing up equation (D.6) and (D.21), and collecting like terms we obtain the expression (27).

D.7 A Numerical Example: Role of Heterogeneous Agents

We use a numerical example to illustrate the role of heterogeneous agents in Section 6.3. The model is calibrated as follows. We follow Woodford (2003) Chapter 4.3 to generate a similar degree of strategic complementarity and the slope of sectoral Phillips curves, so that we set the elasticity of substitution σ to be 5 and the inverse of Frisch elasticity γ to be 0.2. We set the elasticity of substitution across differentiated goods θ to be 4, so that the average markup is 66%.

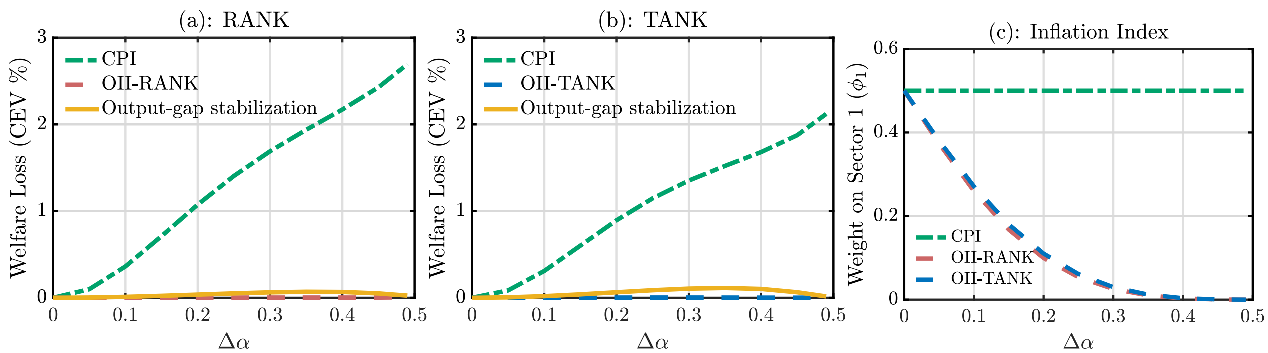
For illustrational purpose, we set consumption weights $\omega_1^K=\omega_1^R=\omega_2^K=\omega_2^R=0.5$. We set $\alpha_1=0.5-\Delta\alpha$ and $\alpha_2=0.5+\Delta\alpha$, and vary $\Delta\alpha$ from 0 to 0.5.[27] We consider and compare two models: RANK when $\lambda=0$ and TANK when $\lambda=0.5$.

Panel (a) and panel (b) of Figure D.1 show the welfare loss of (1) CPI stabilization policy (2) stabilizing the optimal inflation index (OII) and (3) the output-gap stabilization policy in RANK and TANK respectively.[28] In both cases, the welfare difference between stabilizing the OII and the optimal policy is negligible. Output-gap stabilization is also nearly optimal, leading to slightly greater welfare loss than OII. Stabilizing CPI is not desirable when the difference in price stickiness between the two sectors is large.

Panel (c) of Figure D.1 plots ϕ_1: the weight on sector 1 (the flexible-price sector) of the optimal inflation index. Compared to RANK, the optimal inflation index (OII) in TANK assigns more weight to sector 2, the sector with more sticky prices. However, the difference between OII-RANK and OII-TANK is small, consistent with the prediction of our theory.

[27] Note that the frequency of price changes in sector i is $1-\alpha_i$. So prices in sector 2 are more sticky.
[28] All welfare losses are relative to the welfare loss under the optimal monetary policy.

Figure D.1: Monetary Policies: RANK vs TANK



Note: Panel (a) and panel (b) plot how the the welfare loss (relative to optimal policy) vary with difference in price stickiness $\Delta\alpha = (\alpha_2 - \alpha_1)/2$ under different monetary policies: stabilizing the output gap, stabilizing the CPI, stabilizing the optimal inflation index in RANK (OII-RANK) and in TANK (OII-TANK) respectively. Panel (c) plots how the weight on the flexible-price sector 1 varies with $\Delta\alpha$.

E Figures and Tables

Figure E.1: Housing Tenure Status: Household Demographics

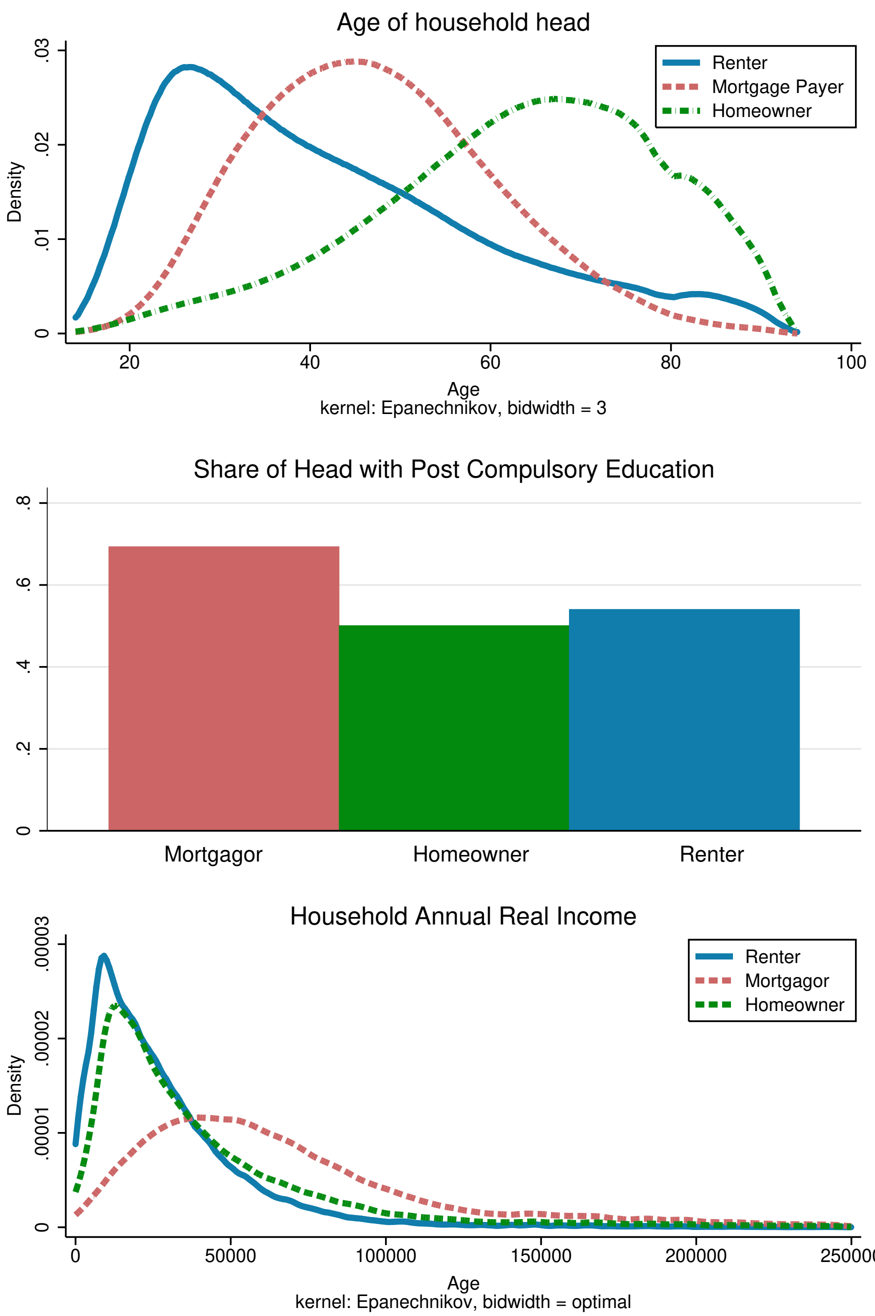
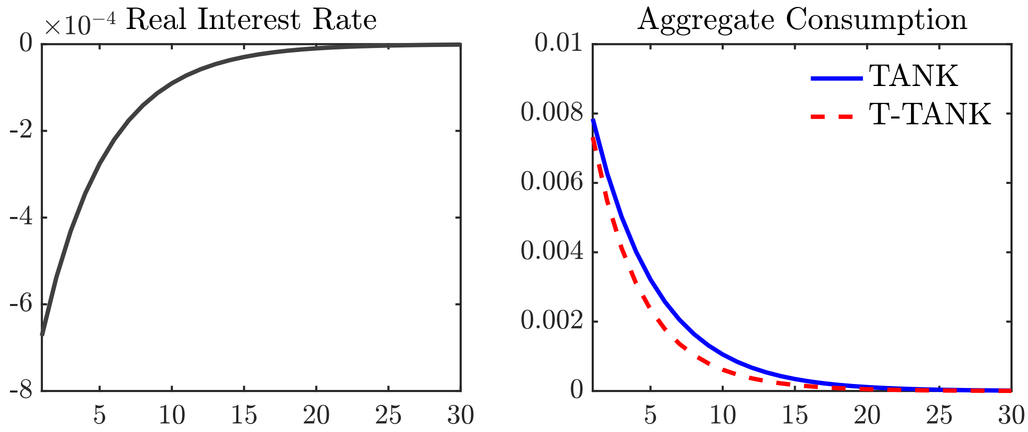


Table E.1: Expenditure Share Differences and Frequency of Price Changes

Category	Expenditure Shares			Freq of Regular
	Morgagors	Homeowners	Differences	Price Changes (%)
		Top 10 larger expenditure shares by mortgagors		
Gasoline(all types)	0.090	0.035	0.054	87.7
Day care and nursery school	0.010	0.002	0.008	6.9
Limited Service meals and snacks	0.025	0.019	0.005	6.1
Used cars	0.019	0.015	0.004	100
Cellular Telephones	0.017	0.014	0.054	13.0
Elementary/high school tuition and fixed fees	0.005	0.003	0.003	6.2
Vehicle leasing	0.006	0.004	0.002	42.4
Full college tuition and fixed fees	0.016	0.014	0.002	5.8
Fees for lessons or instructions	0.003	0.001	0.002	3.3
Food at employee sites and schools	0.003	0.002	0.001	2.9
		Top 10 larger expenditure shares by homeowners		
Hospital services	0.113	0.126	-0.013	6.3
Prescription drugs	0.044	0.056	-0.012	15.0
General medical practice	0.085	0.096	-0.010	3.4
Electricity	0.031	0.037	-0.007	38.1
Motor vehicle insurance	0.025	0.030	-0.005	8.1
Prosthodontics and implants	0.023	0.027	-0.005	4.5
Funeral expenses	0.001	0.004	-0.003	8.9
Community antenna or cable TV	0.014	0.017	-0.003	12.4
Care of elderly in the home	0.0004	0.003	-0.003	2.8
Internal and respiratory drugs	0.004	0.006	-0.002	7.9

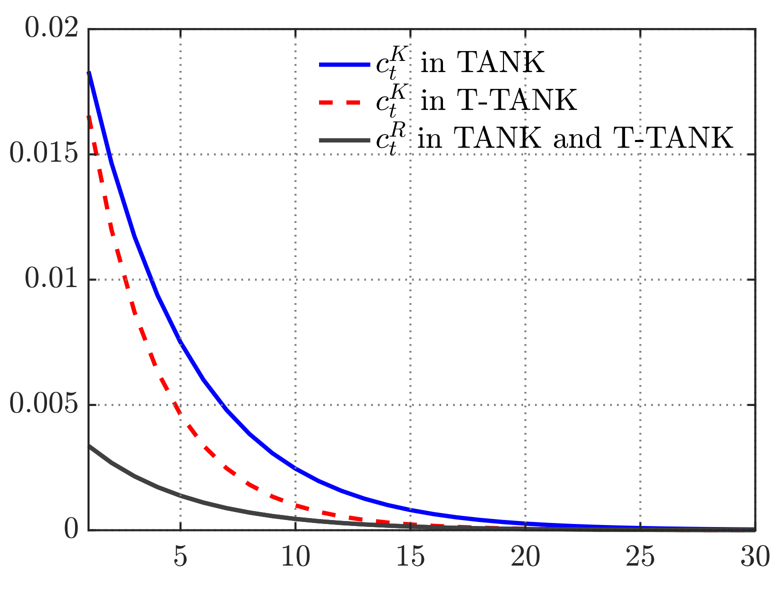
Note: This table lists the top-10 product categories with the largest difference in expenditure shares between morgagors and homeowners. It also reports the frequency of regular price changes of these categories. All values are calculated using data in 2005.

Figure E.2: IRFs: Real Interest Rates and Aggregate Consumption



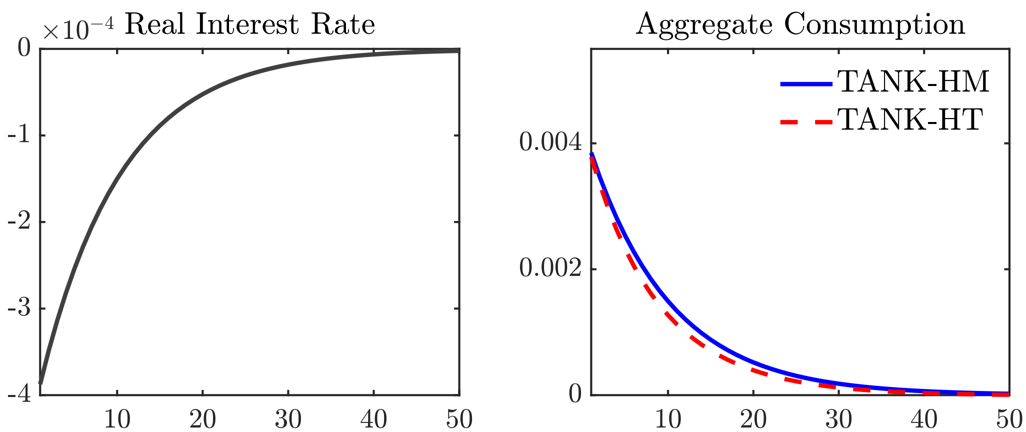
Note: This figure plots the path of real interest rate and the IRFs of aggregate consumption in the numerical example in Section 4.3 when $\Delta_\alpha = 0.1$.

Figure E.3: IRFs: K's and R's Consumption



Note: This figure plots the IRFs of Keynesians' and Ricardians' consumption in TANK and T-TANK in the numerical example in Section 4.3 when $\Delta_\alpha = 0.1$.

Figure E.4: IRFs: Real Interest Rates and Aggregate Consumption



Note: This figure plots the path of the future real interest rate, and the IRFs of aggregate consumption to this real interest rate shock in TANK-HT and TANK-HM in our calibrated model in Section 5.2.