Reconciling Macroeconomics and Finance for the U.S. Corporate Sector: 1929 - Present*

Andrew Atkeson Jonathan Heathcote Fabrizio Perri
UCLA Federal Reserve Bank of Minneapolis

December 2024

Abstract

We examine how to reconcile, quantitatively, the high volatility of market valuations of U.S. corporations with the relative stability of macroeconomic quantities over the period 1929-present. We use a stochastic growth model extended to incorporate factorless income as a measurement framework to investigate this apparent tension. Macroeconomic and financial variables are measured in a consistent fashion using the Integrated Macroeconomic Accounts of the United States, which offer a unified data set for the income statement, cash flows, and balance sheet of the U.S. Corporate Sector. We use our model to conduct two valuation exercises. First we measure the rates of return to investment in physical capital implied by observed capital-output ratios and model-implied cash flows to capital. Second, we conduct a Campbell and Shiller (1987) style valuation exercise using overall Enterprise Value and Free Cash Flow for the U.S. corporate sector. Based on these two valuation exercises, we argue that fluctuations in expected cash flows to firm owners have been the dominant driver of fluctuations in the market value of U.S. corporation from 1929 to 2023, with modest and transitory fluctuations in expected rates of return playing a smaller role. More important, we find similar time-series for expected returns from these two exercises. In this sense, our model offers a reconciliation of volatile market valuations and stable capital output ratios.

JEL Classification Numbers: E44, G12

Key Words: Dividends, Free Cash Flow, Integrated Macroeconomic Accounts, Stock market, Tobin's Q

*Preliminary. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. We thank Ellen McGrattan and Yueran Ma for very helpful comments. The title of this paper is intended as a tribute to the work of the Bureau of Economic Analysis and Federal Reserve Board in producing the Integrated Macroeconomic Accounts which aim to present a unified accounting a macroeconomic flows and asset valuations implied by financial markets.

1 Introduction

How can the volatile market valuations of U.S. corporations manifest in public equity markets be reconciled with the relatively smooth evolution of most macroeconomic variables observed in data from the National Income and Product Accounts (NIPA)? This question has been challenging to answer with conventional macroeconomic models. The most basic stochastic growth model offers the prediction that the market valuation of U.S. corporations should coincide in equilibrium with the quantity of capital owned by those corporations regardless of investors' expectations of future growth rates of the economy or required rates of return for investment in capital. The data on the market valuation of U.S. corporations and their measured holdings of capital are far from this simple model benchmark. How should we modify this basic stochastic growth model to account for both the data on macroeconomic quantities and market valuations of the U.S. corporate sector? That is the question we take up in this paper.

The development of a unified data set known as the Integrated Macroeconomic Accounts (henceforth IMA) offers the opportunity to make new progress towards an answer to this question. This IMA data set is developed as a joint project between the Bureau of Economic Analysis and the Federal Reserve Board. It combines NIPA data on macroeconomic flows and stocks with comprehensive data drawn from the Financial Accounts of the United States on financial flows and balance sheets with equity measured at market value.[1] What results from this accounting exercise is a coherent set of income statements, cash flow statements, and market value balance sheets for major sectors of the U.S. economy. In this paper, we focus on these data for the U.S. corporate sector.

We use these IMA data together with a simple variant of the stochastic growth model to provide a reconciliation of the volatility of market valuations of U.S. corporations with the relative stability of most macroeconomic aggregates over the period 1929 through 2023. The key modification we make relative to the most basic stochastic growth model is that we assume that firms face a time-varying wedge between total revenue and total costs that leads to a pure rent for firm owners that we refer to as *factorless income* following Karabarbounis and Neiman (2019). The valuation of this factorless income drives a gap in the model between the enterprise value of the corporate sector and the stock of measured capital held by firms in that sector.[2]

In particular, we use these IMA data and our model to address two questions.

[1]The Financial Accounts of the United States produced by the Federal Reserve Board were formerly known as the Flow of Funds. See Cagetti et al. (2013) for an introduction to the construction of the IMA data set.

[2]Our model also features an explicit model of corporate taxes which have important time-varying impacts on after-tax cash flows and valuations.

First, do these IMA data offer a picture of the returns to claims to the U.S. corporate sector and the volatility of the valuation of the firms in that sector in line with the data on public firms available from the Center for Research in Security Prices (CRSP) and Compustat databases? In the first part of this paper, we argue that the answer to this question is yes. Based on this comparison, it appears to us that these IMA data are a valuable new resource for those conducting research in macrofinance.

Second, can the volatile market valuations of U.S. corporations observed in the IMA data be reconciled with the data on the ratio of the replacement cost of capital to output in the corporate sector that is observed in the same IMA data? Our answer here is a qualified yes.

To arrive at this answer, we use the IMA data to conduct two valuation exercises, one macro-model based and the other more reduced form in the style of Campbell and Shiller (1987) and Campbell and Kyle (1993), and compare the results.

In our first macro-model based valuation exercise, we use the capital Euler equation of our modified stochastic growth model to measure the sequence of expected returns to investment in physical capital needed to account for observed capital-output ratios in the IMA data. That is, we ask what sequence of expected returns on investment in physical capital are consistent with observed aggregate investment year-by-year for the period 1929-2023.

In our second, finance-style, valuation exercise, we simply use data on the market valuation of U.S. corporations and the cash flows available to owners of these corporations to estimate the sequence of expected returns on the U.S. corporate sector as a whole needed to account for the aggregate market valuation of that sector, again year-by-year for the period 1929-2023.

We find that, at least in the period after World War II, these two approaches to valuation offer similar estimates of the sequence of expected returns needed to account both for observed aggregate capital-output ratios and market valuations of the U.S. corporations. It is in this sense that we argue that macroeconomic and finance-style approaches to modeling both aggregate investment and valuations of the U.S. corporations can be reconciled. We qualify our conclusion given that there remain discrepancies in the estimated expected returns that we obtain from our two different valuation methodologies. But we believe that further work should be able to resolve these discrepancies in a satisfactory manner.

Our paper is organized as follows.

In Section 2, we place our work in context of a large prior literature on this topic.

In Section 3, we use the IMA data to construct measures of aggregate cash flows to firm owners and firm valuation consistent with the definition of these concepts in a standard stochastic growth model. In particular, we take as a baseline a model in which firms are entirely equity financed, have no financial assets, and pay out all of their after-tax gross operating surplus less investment expenditures each period to firm owners.

We refer to our measure of cash flows as *Free Cash Flow* from operations and define it as gross value added less total taxes less compensation of employees less investment expenditures.[3] We refer to our measure of firm value as *Enterprise Value* and measure it from the balance sheet of the corporate sector as the sum of the Market Value of Equity plus Liabilities less Financial Assets. We show the evolution of these series in Figure 1 and display their relationship to alternative cash flow and valuation measures, including alternative aggregates from Compustat data for public firms in further figures in that section and in Appendix B.

We then demonstrate that the realized annual returns for the U.S. corporate sector over the period 1929-present constructed using these measures of free cash flow and enterprise value look remarkably similar to measures of returns obtained from the CRSP Value-Weighted Total Stock Market Index in Table 1 and Figure 3. Based on these observations we argue that the IMA are a useful unified data set for macrofinance.

In Figure 2 we show one striking feature of our data on Enterprise Value and Free Cash Flow, both relative to Corporate Gross Value Added (GVA). Both of these series are quite volatile over time and they show similar movements, at least at low frequencies. This feature of the data motivates one of the key features of our valuation model, which is that fluctuations in investors' expectations of future free cash flow are a key driver of fluctuations in valuations.

In Section 4 we present the modification of the standard stochastic growth model that we use as a quantitative accounting framework to connect standard macroeconomic flows and stock to financial measures of valuations and returns.

We first use our model to guide our division of the data on free cash flow into a portion that is compensation to owners of the measured capital in the corporate sector and the remainder which corresponds to after-tax factorless income. We make this division under the stark assumption that the share of capital in the production function has been constant over the time period that we study. We use our model to divide the data on enterprise value into components reflecting present values of these two sources of Free Cash Flow based on our model's implication that the valuation of the portion of Free Cash Flow that corresponds to compensation to physical capital is equal to the replacement value of the stock of that physical capital.

When we use our model to partition free cash flow between income to capital and factorless income, we see that the decline in Free Cash Flow relative to GVA in the early decades of our sample shown in Figure 1 largely reflects declining free cash flow to capital, while the rise in the post 2000 period largely reflects a rise in Free Cash Flow associated with factorless

[3]This measure is identical to Gross Operating Surplus less taxes on corporate income and wealth less gross fixed capital formation.

income (see Figures 6 and B.7).

We present our model-implied decomposition of the valuation of U.S. corporations into a valuation of their installed capital and of their future factorless income in Figure 7. What is striking here is that, in the time period from 1929 to World War II, the large fluctuations in the value of U.S. corporations in this time period appear to be accounted for primarily by fluctuations in the capital-output ratio, just as one would expect in the simplest stochastic growth model. In contrast, over the long time period from World War II to the present, fluctuations the value of U.S. corporations appear to be accounted for almost entirely by fluctuations in the value of factorless income, with the measured capital to output ratio being remarkably stable.

In Section 5 we conduct our first macro-model based valuation exercise. In this exercise, we use our model's Euler equation governing optimal investment in capital to measure the realized returns to investment in capital and to estimate the expected returns to such investment that would rationalize observed aggregate investment. We show these estimates in Figure 8. In Figure 9 we show the model's implications for the gap between the expected return to investment in physical capital and a short-term riskless rate. We see that, at least in the period after World War II, this estimated expected excess return to investment in measured capital is fairly steady around 5 percentage points, with exceptions for the period in the early 1980's and in 2022.

In Section 6, we conduct our second, finance-style, valuation exercise following the framework laid out in Campbell and Shiller (1987) and Campbell and Kyle (1993). This framework estimates the following decomposition of the dynamics of the price of an asset $\{p_t\}$ based on a model of the dynamics of the dividends on that asset $\{d_t\}$:

$$p_t = p_t^\star + \phi_t \tag{1}$$

where

$$p_t^\star \equiv \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t d_{t+k} \tag{2}$$

and $\beta < 1$.

In this decomposition, $p_t^\star$ is the expected present value of dividends discounted at a constant rate β. We label $p_t^\star$ the "fundamental" component of price. The component ϕ_t is simply the difference between the observed price p_t and the fundamental component $p_t^\star$. We label ϕ_t the "residual" component of price.

This decomposition is of interest because, for any model of the dynamics of dividends, the dynamics of the residual term $\{\phi_t\}$ correspond to variation over time in what we call *quasi-returns*. In particular, it is the observation that

$$\beta \mathbb{E}_t \phi_{t+1} - \phi_t = \beta \mathbb{E}_t \left[p_{t+1} + d_{t+1} \right] - p_t \tag{3}$$

where we refer to the expression on the right side of this equation as a quasi-return. Thus, the extent to which quasi-returns, as observed in the data, are forecastable is an indication of the extent to which the dynamics of price p_t are driven by the dynamics of the residual term ϕ_t rather then the dynamics of expected future dividends as captured by $p_t^\star$.

To implement this model, one assumes that dividends $\{d_t\}$ follow an ARIMA process and thus can be decomposed into a permanent (Martingale) trend component x_t and a transitory component $y_t = d_t - x_t$ following Beveridge and Nelson (1981). With this assumption, estimation of this model amounts to estimating two unobserved components of the process for dividends (x_t and y_t) and three unobserved components of price (adding ϕ_t), where the model of dividends determines the fundamental price from equation 2 and the residual term satisfies equation 1. In this estimation, ϕ_t should forecast quasi-returns as in equation 3 and y_t should forecast future changes in dividends according to the specific ARMA model for the transitory component of dividends specified in the estimation. In this way, this procedure for decomposing the dynamics of an asset price captures the arguments in Cochrane (2008) that one can estimate the importance of time-varying expected returns and time-varying expected growth of dividends in driving an asset price using evidence on the predictability of returns and changes in dividends.

We estimate this model using our IMA data. We take as our measure of price (p_t) the ratio of Enterprise Value to GVA and our measure of dividends (d_t) the ratio of Free Cash Flow to GVA. We present the results from this estimation procedure in Figure 10. In the left panel of that figure, we see that our estimates imply a relatively small role for variation over time in the residual term ϕ_t in driving the ratio of Enterprise Value to GVA and, in the right panel of that figure, we see that the permanent shocks x_t account for most of the movements over time in the ratio of Free Cash Flow to GVA. In Figure 11, we show that the impact of these permanent shocks x_t to the ratio of Free Cash Flow to GVA on the fundamental price $p_t^\star$ account for the large majority of the dynamics of the ratio of Enterprise Value to GVA.

Given our estimated model of the dynamics of the ratios of Enterprise Value and Free Cash Flow to GVA, we are able to compute a sequence of expected returns on Enterprise Value implied by our finance-style valuation exercise. We present these estimated returns to Enterprise Value in excess of the one year risk-free rate in Figure 12. For purposes of comparison, we include our macro-model based estimates of the expected excess returns to investment in physical capital in this same figure. We take this comparison of these two estimates of expected excess returns as the central contribution of our paper. We see that these two estimates of expected excess returns coincide fairly well, with important exceptions during World War II, the early 1980's, the late 1990's, and the period around 2008.

Finally, in Section 7, we conclude.

2 Related Literature

A full reconciliation of macroeconomics and finance would achieve, at a minimum, three objectives:

1. a general equilibrium theory of the pricing kernel consistent with the asset prices confronting households as they make their consumption and savings decisions,
2. an accounting for the volatility of market valuations of U.S. corporations, and
3. an accounting for the observed investment/capital stock decisions of U.S. corporations given those asset prices.

There is a huge literature in macro-finance aimed at the first point. In particular, following Lucas (1978), this literature aims to develop models of the marginal utility of the marginal investor to resolve the equity premium puzzle of Mehra and Prescott (1985), manifest here as the high unconditional average rate of return observed on claims on the corporate sector in both the public firm and IMA data. We do not attempt such an economic model of the marginal investor. We leave such modeling to future work.[4]

Instead, we build a slightly modified stochastic growth model to use as an accounting framework to address the second and third objectives above, leaving unmodeled the forces accounting for the average value of excess returns on claims to US corporations. That is, we aim to simultaneously account for the observed fluctuations in the value of U.S. corporations year-by-year since 1929 and for measured U.S. corporate holdings of capital and investment in capital over this time period.

As discussed in Gomme, Ravikumar, and Rupert (2011) and as implied by the work of Tobin (1969), one of the challenges to accounting for the volatility of market valuations and returns of U.S. corporations together with the relatively smooth data on measured capital-output ratios and accounting returns to capital in a standard stochastic growth model is that

[4] Tallarini (2000) and Kaltenbrunner and Loechster (2010) are important papers showing how to reconcile standard business cycle fluctuations with a high equity premium in standard stochastic growth models with a representative agent with recursive preferences. These models have an advantage of being fairly tractable using standard approximation techniques. We conjecture that incorporation of shocks to factorless income in models such as these might be a fruitful avenue for developing fully equilibrium macrofinance models, albeit with constant risk premia over time.

in such a model the value of firms is always equal to the value of their installed capital. In our model, this link is broken by the introduction of factorless income and considerations of corporate taxation.

In contrast, much of the macro-finance literature aimed at the second and third objectives above has taken a different approach, relying on time-variation in the risk premium on investment in the capital stock together with adjustment costs for that investment to reconcile the high volatility of corporate valuations and the relatively smooth evolution of the stock of measured capital. Jermann (1998), Gourio (2012), Ilut and Schneider (2014), Basu and Bundick (2017), Hall (2017), Cambell, Pflueger, and Viceira (2020), and Basu et al. (2023) are examples of stochastic growth models with time-varying risk premia on capital arising from a variety of different sources. See also Cochrane (1991), Merz and Yashiv (2007), Philippon (2009), and Jermann (2010).

We depart from this literature in accounting for much of the volatility of corporate valuations based on a model of fluctuations in expected cash flows to owners of firms rather than variation in discount rates.[5] To provide such an accounting, our model combines two key ingredients. First, we include a time-varying wedge between corporate revenue and costs associated with measured capital and labor that generates that we call *factorless income* following Karabarbounis and Neiman (2019). Second, we use a model of the impact of rates of return and taxes on the cash flow to owners of capital and the valuation of that cash flow based on the framework of Hall and Jorgenson (1967). We model taxes in a similar way to Gravelle (1994), Gravelle (2006), and Barro and Furman (2018). As in McGrattan and Prescott (2005) and McGrattan (2023) we find that taxes play an important role in shaping our model's implications for the valuation of and marginal returns to measured capital.[6]

In our accounting of the data since World War II, the high volatility in the valuation of U.S. corporations is driven primarily by shifts in investors' expectations of the share of factorless income in corporate output in the long run. Investment in measured capital, on the other hand, is driven by more near-term considerations such as one-year interest rates, growth rates, corporate tax rates, depreciation rates, and changes in the relative price of capital goods. Thus, investment and measures of Tobin's Q are only weakly connected in our model (see Abel and Eberly (2012) for a related argument).[7]

[5]We pursue this theme further in a companion paper Atkeson, Heathcote, and Perri (2024) that models the volatility of stock prices based on fluctuations in expected cash flows using CRSP data on price per share and dividends per share.

[6]As of yet, we have not included consideration of the impact of taxes on corporate distributions on the level of corporate valuations as in McGrattan and Prescott (2005) and McGrattan (2023). We plan to do so going forward.

[7]We note that our findings about the volatility of cash flows to owners of U.S. corporations differ from the findings in Hall (2003). We believe that this is due to our use of free cash flow rather than EBITDA and to the time periods considered.

In that vein, our accounting for the stability of the ratio of physical capital to output in the face of falling risk free interest rates, changing tax rates, and rising depreciation rates in the past several decades is related to that in Gutiérrez and Philippon (2017). That is, on the one hand, a declining share of income accruing to physical capital coupled with higher average depreciation have been important forces tending to depress investment. But these forces in our accounting have been offset by declining corporate tax rates and declining expected output growth, which in our valuation framework translates to lower required expected returns to all assets. Thus, we see this stable capital to output outcome as largely coincidental. In particular, we note that we do see large changes in the ratio of measured capital to corporate gross value added in the data prior to World War II. Moreover, these large changes in the capital-output ratio accounted for large changes in both the ratios of enterprise value and free cash flow to gross value added in that time period.

Our focus on shocks to current and future factorless income is closely related to the arguments of Lustig and Van Nieuwerburgh (2008) and Greenwald, Lettau, and Ludvigson (2023) that shocks to the distribution of income between workers and owners of firms have been an important driver of fluctuations in the valuation of U.S. corporations. Our principal contribution relative to these papers is to add consideration of physical capital and investment. We follow a large recent literature in macro-finance that builds on these ideas. See, for example, Caballero, Farhi, and Gourinchas (2017), Farhi and Gourio (2018), Crouzet and Eberly (2018), Philippon (2019), Eggertsson, Robbins, and Wold (2021), and Crouzet and Eberly (2023). With the notable exception of Crouzet and Eberly (2023), these papers do not account year-by-year for both corporate valuations and changes in capital investment over a long time period.

In building our accounting model, we make the stark assumption that the production function relating measured capital and labor to aggregate output has remained stable over the past 100 years. It is this assumption that allows us, through the model, to measure the share of factorless income in corporate gross value added year-by-year from data on tax rates and the share of labor compensation in corporate gross value added. Given these estimates of the share of factorless income and our model of the production function, we can then compute the rental rate on measured capital, the share of rental income on measured capital in corporate gross value added, and the pre- and post-tax returns to physical capital implied by the data as interpreted through our model. We evaluate our model's fit to the data on investment and capital stocks based on a comparison of these model-based estimates of the post-tax returns to physical capital to risk free interest rates plus a constant risk premium on measured capital. That is, we ask whether our model's implications for the post-tax returns to measured capital are consistent with observed risk-free interest rates plus a constant risk premium.

In this regard, our work is closely related to recent work by Barkai (2020) and Karabarbounis and Neiman (2019) who both use a Hall and Jorgenson (1967) style measurement framework to estimate the rental rate on measured capital and the corresponding share of rental income on measured capital in gross value added. This prior work differs from ours in that it starts with data on risk free rates and an estimate of the risk premium on capital to estimate the rental rate on measured capital without imposing restrictions on the production function. If the specification of our model is correct (and measurement is without error) then the estimates of rental income on measured capital obtained from our framework and their framework should coincide. It appears that our results do roughly coincide for the period after the late 1980's in that we find that the expected excess returns to investment in physical capital is roughly constant at five percentage points above a risk-free rate, in line with the assumption in Barkai (2020) that the cost of equity financing is five percentage points above a risk free rate.[8]

In the data, as noted by Gomme, Ravikumar, and Rupert (2015), Reis (2022), Harper and Retus (2022), and others, the accounting returns to capital in the corporate sector (measured by the ratio of Net Operating Surplus pre and post tax to installed capital) have remained remarkably constant since at least 1960, even as measures of the risk free interest rates have fallen quite sharply. Our accounting model is consistent with these accounting returns data. And yet we find a falling return to measured capital in recent decades, a fall of similar magnitude to the decline in risk free rates. In fact, we find that in recent years, the return to measured capital is falling close to the threshold for dynamic inefficiency of Abel et al. (1989). In our model, these differential trends between accounting returns and true returns to capital emerge because only a (declining) portion of after-tax Net Operating Surplus is compensation of measured capital, while the remaining (rising) portion contributes to factorless income. Once we differentiate appropriately between free cash flow to physical capital and free cash flow associated with factorless income, we find that expected returns to all assets appear to declining over time, and at similar rates.

Our findings in section 6 that variation over time in expected returns (as captured by the residual term ϕ_t in equation 1) play only a modest role in accounting for fluctuations in the ratio of Enterprise Value to GVA echoes the results of Larraine and Yogo (2008). They used similar measures of the aggregate valuation and aggregate cash flows to the owners of the U.S. corporate sector in a finance-style valuation exercise following Campbell and Shiller (1988) and argued that variation over time in expected cash flows played a more important

[8]For the period prior to 1980, there is a discrepancy between our results and those of Karabarbounis and Neiman (2019) regarding returns to capital and the amount of factorless income. We aim to explore this discrepancy in greater detail in future work.

role in driving fluctuations in value than found in the prior literature. We differ from their paper both in the specific finance-style valuation execise that we conduct and in our use of a macro model to measure the expected returns to investment in physical capital.

In our measurement, we have abstracted from the role of unmeasured intangible capital in accounting for fluctuations in the value of the U.S. corporate sector. Many papers consider the role of unmeasured intangible capital in driving the boom in the market valuation of U.S. firms in recent decades. See, for example, Hall (2001), McGrattan and Prescott (2010) and Crouzet et al. (2022).[9] We see this as a fruitful avenue for future research, but we see two hurdles that should be overcome in developing this hypothesis.

First, the aggregate data on unmeasured capital cited in Corrado et al. (2022) are not favorable to the hypotheses that changes in the stock of unmeasured capital have contributed importantly to fluctuations in the value of the U.S. corporate sector because these data exhibit no trend in the stock of this unmeasured capital relative to value added over the past decade or more.[10]

Second, we suggest that a model of the variability of the market valuation of the U.S. corporate sector over the past century based on fluctuations in the stock of unmeasured capital held by U.S. corporations should also account for observed Free Cash Flow to owners of these corporations, as this measure of cash flow is invariant to failure to measure investment; see Atkeson (2020). Thus, large movements in unmeasured investment in intangible capital over time should correspond to large fluctuations in measured Free Cash Flow. A question for research going forward is whether the fluctuations we see in Free Cash Flow in the data would be consistent with a model with large variation over time in the ratio of the stock of unmeasured intangible capital over measured output.

We now turn to our discussion of the IMA data.

3 Measures of Corporate Value, Cash Flows, and Returns

In this paper, we focus on valuation and cash flow measures in the data from the Integrated Macroeconomic Accounts (IMA) closest to those concepts in a standard macroeconomic stochastic growth model. We refer to the concept of the value of the U.S. corporate sector corresponding to this baseline model as *enterprise value*. We refer to the corresponding

[9] Eisfeldt and Papanikolaou (2014), Belo et al. (2022), Eisfeldt, Kim, and Papanikolaou (2022) and the papers cited therein argue that measured of intangible capital drawn from firms' accounting statements that is not included in the National Income and Product Accounts help account for the valuation of firms in the cross section.

[10] These data are available at `http://www.intaninvest.net/`.

concept of cash flows for the U.S. corporate sector as *free cash flow from operations*, or free cash flow for short. We give a detailed description of the series we compute from the IMA data in Appendix A.

We use the IMA data to construct a measure of enterprise value for the U.S. corporate sector as the sum of the market value of the equity and financial liabilities less the financial assets of U.S. corporations.[11] Our measure of free cash flow in the IMA data is equal to after-tax gross operating surplus less investment expenditures of U.S. corporations. These valuation and cash flow measures are similar to those used in Hall (2001).

We plot our valuation and cash flow measures relative to the gross value added of the U.S. corporate sector in Figure 1. We show enterprise value in the left panel in blue and free cash flow in the right panel in red. We see that both enterprise value and free cash flow are quite volatile relative to the gross value added of the U.S. corporate sector.[12]

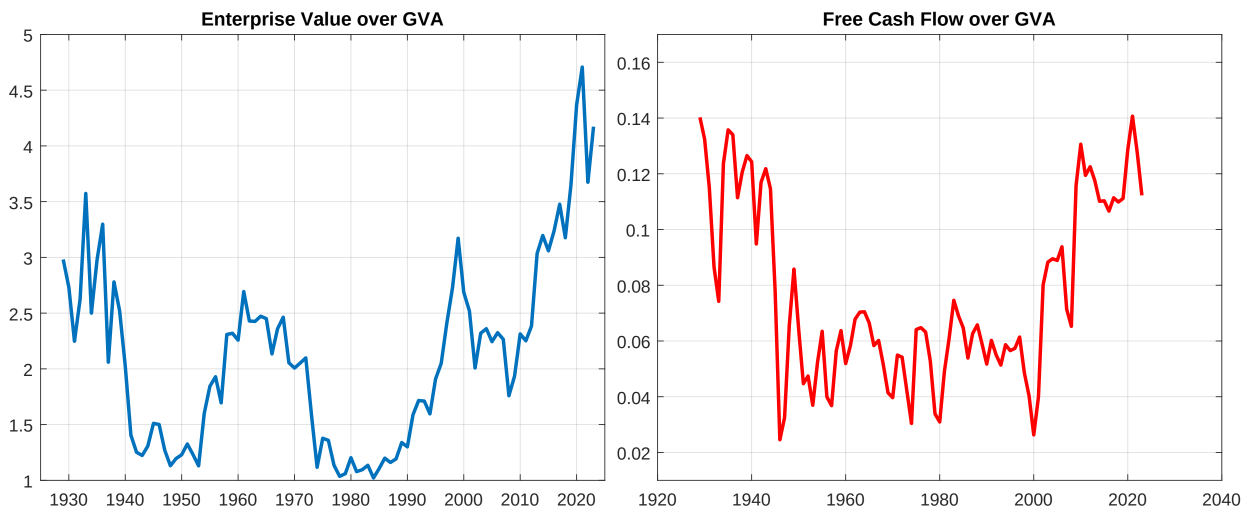

Figure 1: Left Panel: The Enterprise Value of U.S. Corporations over Corporate Gross Value Added. Right Panel: Free Cash Flow from U.S. Corporations over Corporate Gross Value Added. 1929-2023

To what extent do the fluctuations in the ratio of Enterprise Value to GVA and Free Cash Flow to GVA line up with a simple model of valuation? To address this question, in the left panel of Figure 2 the ratio of enterprise value to gross value added in blue and a predicted

[11]This measure of enterprise value for the Financial and Non-Financial corporate sectors is reported on Table B1 "The Derivation of U.S. Net Wealth" of the Financial Accounts of the United States. See https://www.federalreserve.gov/econresdata/notes/feds-notes/2015/us-net-wealth-in-the-financial-accounts-of-the-united-states-20151008.html

[12]In Appendix section B.1, we show the components of Free Cash Flow relative to corporate Gross Value Added (the shares of labor compensation, taxes, and investment in GVA) that have driven these dynamics of Free Cash Flow over corporate GVA.

value of this ratio if enterprise value were a fixed multiple of free cash flow in red.[13] We see in this panel that the low frequency fluctuations in the ratio of enterprise value to gross value added appear to be fairly well accounted for by low frequency fluctuations in the ratio of free cash flow to gross value added, when those are valued at a constant price dividend ratio.

The right panel of Figure 2 shows a valuation statistic suggested by Campbell and Shiller (1987) and Campbell and Kyle (1993) that we refer to as the *price-dividend spread*. This is computed as the difference between the blue line and the red line in the left panel of this figure. That is, it is the difference between the ratio of Enterprise Value to corporate GVA and the predicted value of this ratio if enterprise value were a fixed multiple (31.25) of free cash flow.

We see in the right panel of Figure 2 that the price-dividend spread shows sizable transitory fluctuations, but it does not show any trend over the past century. This observation futher corroborates the view from the left panel of this figure that, at low frequencies, changes in the ratio of Free Cash Flow to GVA at a constant valuation multiple account for much of the fluctuations in Enterprise Value over GVA.[14]

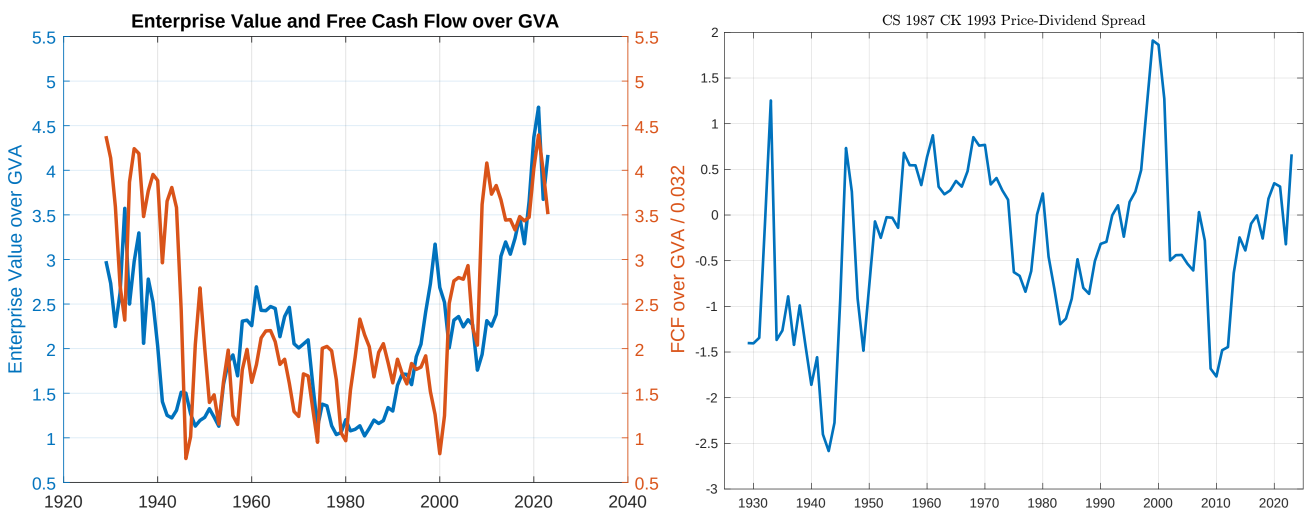

Figure 2: Left Panel: Enterprise Value Actual (in blue left axis) and Predicted (in red right axis) from Corporate Free Cash Flow using a valuation multiple of $1/0.032 = 31.25$ over Gross Value Added. 1929-2023 Right Panel: The price-dividend spread computed as the difference between the blue and red lines as in Campbell and Shiller (1987) and Campbell and Kyle (1993)

[13] We use a valuation multiple of free cash flow of $1/0.032 = 31.25$ in this calculation.

[14] We discuss below that the fluctuations in the price-dividend spread are driven by a combination of fluctuations in expected changes in future values of the ratio of Free Cash Flow to GVA and fluctuations in a residual term corresponding movements in expected returns to Enterprise Value. We use a valuation methodology based on that in Campbell and Shiller (1987) and Campbell and Kyle (1993) to offer a decomposition of the movements in the price-dividend spread into these two components in section 6.

We now consider properties of the annual returns on enterprise value implied by the IMA data. We compute the returns on enterprise value from the perspective of a household in a stochastic growth model that owns the entire corporate sector and receives all cash paid out by that sector. Using that perspective, we denote enterprise value at the end of period t as V_t, free cash flow in period $t+1$ as FCF_{t+1}, and construct realized returns on enterprise value each year as

$$\exp(r_{t+1}^V) = \frac{FCF_{t+1} + V_{t+1}}{V_t}$$

We deflate these and all nominal returns by the growth in the PCE deflator to compute realized real returns.

In Figure ?? we examine the extent to which this measures of realized real returns on Enterprise Value line up with measures of realized real returns on publicly traded equities computed using the CRSP Value-Weighted Total Market portfolio. In this figure, we show a scatter plot of realized annual returns on the CRSP portfolio on the x-axis and returns on enterprise value in the IMA data on the y-axis. The red line is the 45 degree line. The correlation of returns on enterprise value with those on the value-weighted CRSP portfolio is 0.943 for the period 1929-2023.

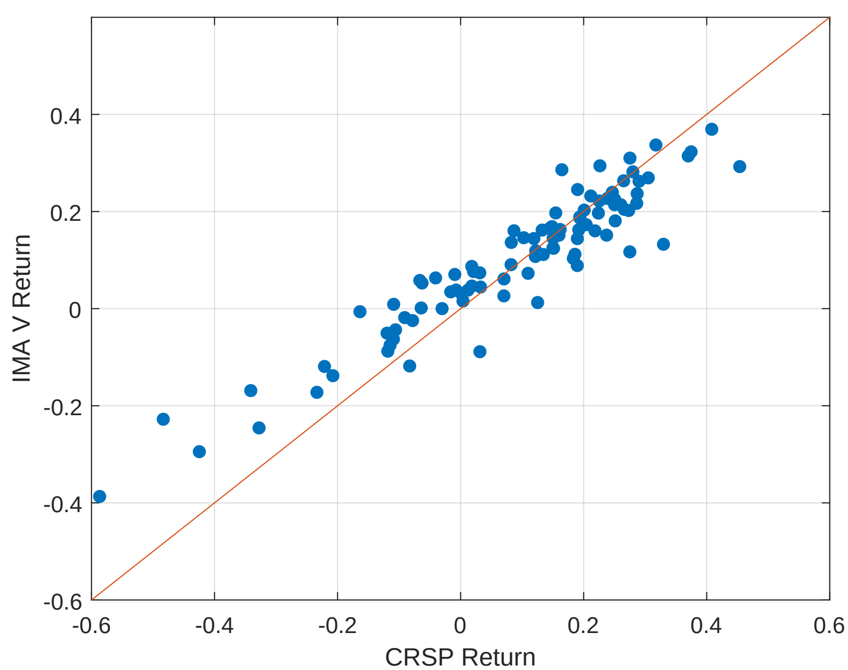

Figure 3: Realized Returns on Enterprise Value vs. CRSP Value-Weighted Total Market Return 1929-2023.

We report some basic statistics of the mean and standard deviations of log real returns using these this IMA return to Enterprise Value concept as well as analogous return and dividend growth statistics computed using CRSP returns on the Total Value-Weighted Market portfolio in Table 1. We see in this table that these two measures of returns have similar means and standard deviations.

Table 1: Mean and Standard Deviation of Real Log Returns and Log Dividend Growth on Enterprise Value, IMA Equity, and CRSP Value-Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std D growth
Enterprise Value	1929-2023	0.073	0.146	0.280
CRSP VW	1929-2023	0.062	0.193	0.138

We now conduct one final comparison of our measures of free cash flow and enterprise value in the IMA data with analogous measures obtained from Compustat data on the financial accounting statements of publicly traded firms.[15] We expect to see differences in these measures of cash flows and valuation from these two data sets for many reasons, two of which stand out.

First, the IMA data are constructed to cover both publicly traded and closely held corporations, while the Compustat data cover only publicly traded corporations.[16] This conceptual distinction between the two data sets should act to make measures of free cash flow and enterprise value larger in the IMA data than corresponding estimates from Compustat data.

Second, the IMA data are constructed to cover only U.S. resident corporations. A U.S. resident corporation is an entity incorporated in the United States. Thus, these corporations include the U.S. subsidiaries of foreign multinational corporations and exclude the foreign subsidiaries of U.S. multinational corporations. In contrast, Compustat data covers the worldwide operations of a list of public companies that are determined to be U.S. corporations in terms of the entity listing equity on U.S. markets. (See Atkeson, Heathcote, and Perri 2023 for further discussion of this point.) To the extent that the foreign subsidiaries of U.S. multinational corporations generate more free cash flow and contribute more to enterprise value than the U.S. subsidiaries of foreign multinationals, this conceptual distinction between these two data sets should act to make measures of free cash flow and enterprise value smaller

[15]In the Compustat data, our measure of free cash flow is computed (following Adame et al. 2023) as Operating Activities–Net Cash Flow (OANCF) minus capital expenditures (CAPX). Enterprise value is computed as Total Market Value (MKVALT) plus Total Liabilities (LT) minus current assets total (ACT), which includes cash and other short term investments, receivables, inventories, and other current assets. Further details are given in Appendix A

[16]For a discussion of the methodology used in the IMA to value closely held corporate equities see https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/corporate-equities-by-issuer-in-the-financial-accounts-of-the-united-states-20160329.html

in the IMA data than corresponding estimates from Compustat data.

In Figure 4 we plot free cash flow (left panel) and enterprise value (right panel) in the IMA and in Compustat, both divided by same denominator, which is gross value added of the corporate sector from the IMA. The left panel shows that in both Compustat and the IMA data the share of free cash flow in GVA roughly doubles from the early 1990s to the late 2000s. The right panel shows that enterprise value relative to IMA GVA increases by a similar amount in both data sets.

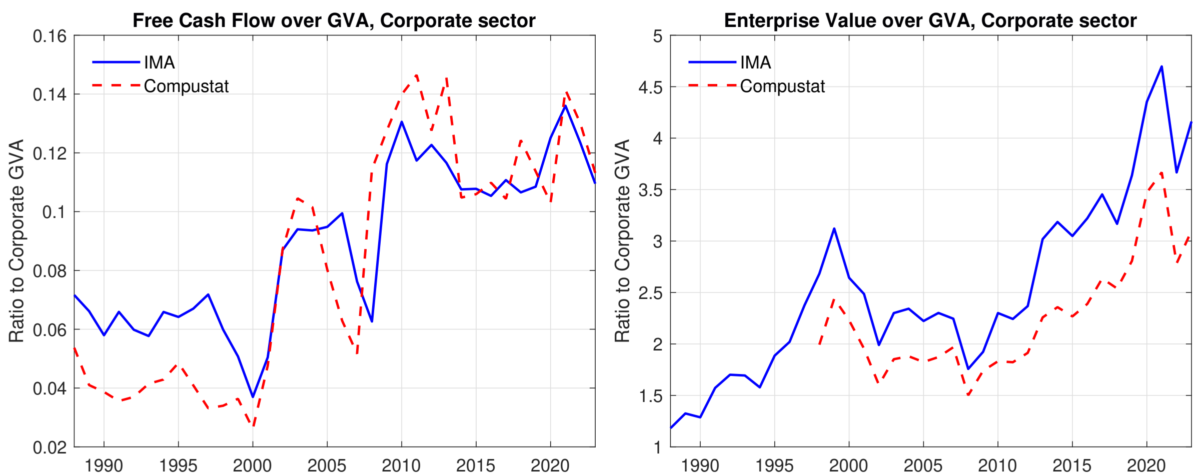

Figure 4: Free Cash Flow and Enterprise Value in the IMA and in Compustat

The close correspondence between measures of value and returns for claims on the U.S. corporate sector from the Integrated Macroeconomic Accounts with measures of value and returns constructed from CRSP and Compustat data on public firms gives us confidence that the Integrated Macroeconomic Accounts are a useful data source for further work in macrofinance aimed at offering an integrated account of aggregate corporate valuations and cash flows.

4 Accounting Model

We now introduce the model we use as our accounting framework for these data.

Our model has two main components.

The first is a model of production and income in the corporate sector that is based on a modified version of the standard stochastic growth model. The main modification we make to the standard model is that we assume that firms' total revenue includes a time-varying wedge relative to the cost of hiring physical capital and labor. With this wedge in our model, a portion of value added corresponds to a pure rent paid to the owners of firms. Following Karabarbounis and Neiman (2019), we refer to this rent as *factorless income*. We also explicitly model corporate taxation, and how it impacts, cash flow, returns, and valuations.

We note that this factorless income can be positive or negative. To the extant that firms have power to charge a markup over the costs of labor and measured capital, factorless income is positive. To the extent that managers of firms fail to earn surplus sufficient to cover the opportunity cost of the measured capital owned by these firms, factorless income is negative.

Our analysis of the IMA data through the lens of our accounting model will proceed in steps.

First, we show that with a minimal set of assumptions, we can use the model structure and the Integrated Macroeconomic Accounts to decompose the series for corporate free cash flow described earlier into a portion of cash flow accruing to owners of measured capital, and a portion accruing to owners of claims to factorless income.

Second, we show how our model can be used to decompose aggregate enterprise value into the market value of cash flows to the owners of the measured capital stock and the market value of claims to factorless income. We show that in the data prior to World War II, much of the fluctuations in enterprise value relative to corporate output correspond fluctuations in the stock of measured capital relative to corporate output, while after World War II, fluctuations in the market value of measured capital account for little of the large observed swings in enterprise value.

Third, we use the capital Euler equation implied by our model to compute a sequence of model-implied realized and expected returns to investment in physical capital year-by-year from 1929 through 2023 given the data on the capital-output ratio, taxes, depreciation, and inflation as measured using the price of investment goods.

We interpret this model-implied sequence of expected returns to investment in physical capital as the sequence of expected returns to capital that would rationalize the observed aggregate investment in the US Corporate Sector. In section 6, we use the valuation framework of Campbell and Shiller (1987) and Campbell and Kyle (1993) to construct estimates of the expected returns to enterprise value needed to rationalize the observed dynamics of free cash flow and enterprise value. To the extent to which these two valuation procedures yield similar estimates of the dynamics of expected returns, we argue that macro and finance perspectives on valuation can be reconciled.

4.1 Technology

We start by describing the production technology.

Aggregate output, corresponding to gross value added of the corporate sector, GVA_t, is given by a Cobb-Douglas production function

$$GVA_t = K_t^{\alpha}(Z_t L)^{1-\alpha}, \tag{4}$$

where K_t is the stock of physical capital in units of capital services, L is labor, which is inelastically supplied, and Z_t is a shock to aggregate productivity. We will assume that the share of capital services in production, denoted by α, is constant over time.

The evolution of the stock of capital services is given by

$$K_{t+1} = (1-\delta_t)K_t + I_t,$$

where δ_t is a time-varying physical depreciation rate for capital services and I_t is investment in new capital services. Note that we assume here that there are no investment adjustment costs.

The terms K_t and I_t are not directly measured in the data. Instead, the IMA report end of period nominal values for the stock of capital at replacement cost, nominal investment expenditures, nominal consumption of fixed capital, and nominal revaluations of the stock of capital carried into the period due to changes in the replacement cost of that capital. We write the nominal end-of-period t replacement cost of capital as $Q_t K_{t+1}$, nominal investment expenditure in period t as $Q_t I_t$, nominal consumption of fixed capital as $\delta_t Q_t K_t$, and nominal revaluations of the replacement value of capital carried into period t as $(Q_t - Q_{t-1})K_t$.[17]

Thus, the accounting for the evolution of the replacement value of the capital stock in the IMA data is

$$\underbrace{Q_t K_{t+1}}_{ReplacementCost_{t+1}} = \underbrace{Q_{t-1} K_t}_{ReplacementCost_t} + \underbrace{(Q_t - Q_{t-1})K_t}_{Reval_t + Other_t} - \underbrace{\delta_t Q_t K_t}_{CFC_t} + \underbrace{Q_t I_t}_{Investment_t}$$

The growth rate for Q_t can be directly inferred by dividing the reported revaluation value by the reported end of t replacement cost. One can then construct a series for $Q_t K_t$ and from that infer a series for δ_t given reported consumption of fixed capital.

For the purposes of interpreting valuations, it is helpful to conceptualize two types of firms operating in the economy. One type, which we call investment firms, holds measured capital, makes investment decisions, and earns income by renting out this capital at a rental rate per unit of capital services R_t^K. The second type of firm, which we call factorless income firms, rent capital and labor, whose wage rate is W_t, and use these inputs to produce the final good according to equation (4). These factorless income firms earn this income by selling output

[17] We include the category *other volume changes* in the IMA in this revaluation category in our model.

with a wedge μ_t between revenue and the cost of measured capital and labor in production. As discussed above, this wedge can be greater or less than one.

We assume that both types of firms are 100 percent equity financed, and that both pay out all the free cash flow they generate as model dividends. We also assume that both types of firms seek to maximize the present value of model dividends payable to shareholders, where these dividends at date $t+k$ are discounted back to date t according to a common pricing kernel, $M_{t,t+k}$.

4.2 Corporate Taxation

To construct measures of free cash flow for these firms we need to specify how they are taxed. We model two sorts of taxes paid by corporations. First, we assume factorless income firms pay a proportional tax at a time-varying rate τ_t^s that applies to their value added GVA_t. This tax in the model corresponds to indirect businss taxes in the data. Thus, we estimate τ_t^s by dividing the sum of "taxes on production and imports less subsidies" plus "business current transfer payments" from NIPA Table 1.14 by corporate gross value-added.

Second, we model corporate income taxes as follows, building on Gravelle (1994) and Barro and Furman (2018). We assume corporate income is taxed at a proportional rate τ_t^c. We assume that investment firms can fully expense economic depreciation and can also expense a constant fraction λ of net new investment. Given these assumptions, the effective tax rate on capital is approximately equal to $\tau_t^c(1-\lambda)/(1-\lambda\tau_t^c)$.[18] Factorless income firms pay the corporate income tax on their factorless income, but are entitled to a time-varying lump-sum tax credit, T_t^L. We use this credit to reconcile marginal tax rates with total corporate income taxes paid in the IMA data.

Given this model, total corporate income taxes paid are given by

$$Taxes_t^c = \tau_t^c\left[(1-\tau_t^s)GVA_t - W_tL - \delta_tQ_tK_t - \lambda Q_t\left(K_{t+1}-K_t\right)\right] - T_t^L \tag{5}$$

We set the value for τ_t^c in each year t equal to corresponding value for the top rate of federal corporate income tax. We set $\lambda = 0.2$. These choices imply a time path for the effective tax rate on capital income similar to the one estimated by Gravelle (2006). Given those choices, we set the time path for T_t^L so that implied total corporate income tax revenue (equation 5) matches the series for "taxes on corporate income" in NIPA Table 1.14. Figure 5 plots corporate income tax revenue as a share of gross value added, and the time paths for the statutory and effective tax rates that we use in our model.

[18] See Appendix C for the derivation and for an exact expression.

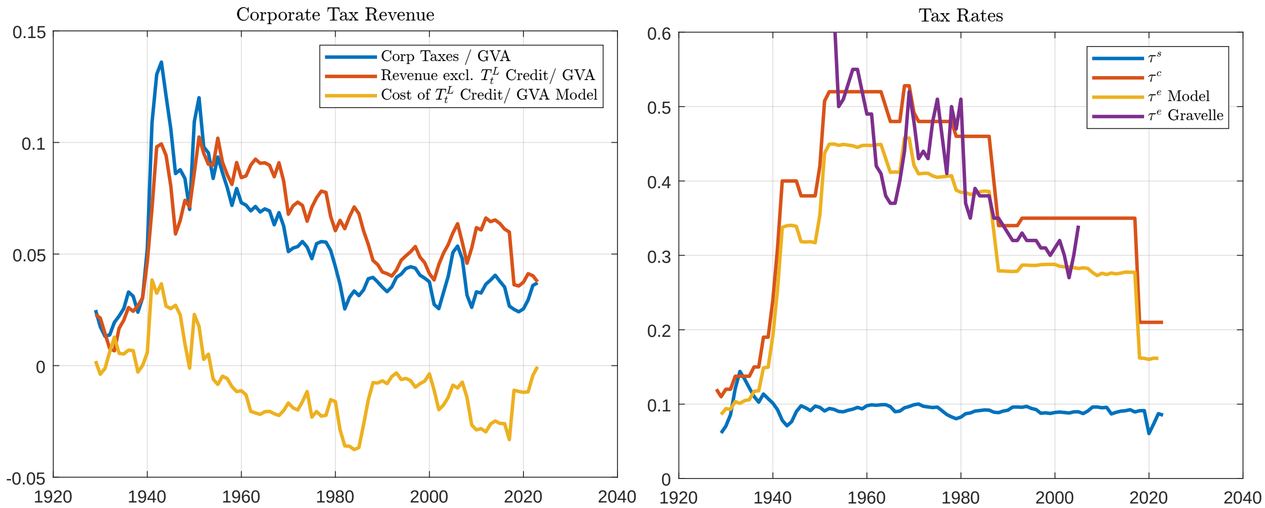

Figure 5: Left Panel: Corporate Income Tax Revenue 1929-2023. Right Panel: Model Tax Rates 1929-2023. τ_t^s is the value-added tax rate, τ_t^c is the statutory corporate income tax rate, and τ_t^e is the marginal effective tax rate on capital. The tax credit T_t^L reconciles tax collections implied by our model tax rates and tax collections in the data.

4.3 Income Shares

We use our model to split total free cash flow into a component going to owners of measured capital, and income to owners of claims to factorless income. After-tax free cash flows from investment-producing and factorless income firms are given, respectively, by

$$FCF_t^K = R_t^K K_t - Q_t I_t - \tau_t^c \left[R_t^K K_t - \delta_t Q_t K_t - \lambda Q_t \left(K_{t+1} - K_t \right) \right] \tag{6}$$

and

$$FCF_t^\Pi = (1 - \tau_t^c)\Pi_t + T_t^L,$$

where

$$\Pi_t = (1 - \tau_t^s) GVA_t - W_t L - R_t^K K_t \tag{7}$$

denotes pre-corporate-tax factorless income.

To construct these free cash flow series requires an estimate of capital rental income $R_t^K K_t$ or equivalently for Π_t. We now use our model structure to construct such a series.

In our model, factorless income firms solve static problems, choosing how much capital and labor to rent each period to minimize costs. Given the Cobb-Douglas production function, the optimal ratio of capital relative to labor services is given by

$$\frac{K_t}{L} = \frac{\alpha}{(1 - \alpha)} \frac{W_t}{R_t^K} \tag{8}$$

These firms set prices net of value-added tax with a time-varying wedge μ_t over unit cost[19]:

$$(1-\tau_t^s)GVA_t = \mu_t(W_tL + R_t^K K_t) \tag{9}$$

Given, equations (8) and (9), our model's implications for the division of gross value added into income shares is as follows. Share τ_t^s, accrues to the government as taxes on production and imports less subsidies. The remainder is divided according to

$$\frac{W_tL}{GVA_t} = (1-\tau_t^s)(1-\alpha)\frac{1}{\mu_t} \tag{10}$$

$$\frac{R_t^K K_t}{GVA_t} = (1-\tau_t^s)\alpha\frac{1}{\mu_t} \tag{11}$$

$$\frac{\Pi_t}{GVA_t} = (1-\tau_t^s)\frac{(\mu_t-1)}{\mu_t} \tag{12}$$

where $\frac{W_tL}{GVA_t}$ is the model equivalent of compensation of employees and the sum $\frac{R_t^K K_t}{GVA_t} + \frac{\Pi_t}{GVA_t}$ is the model equivalent of Gross Operating Surplus. Note that the model equivalent of Taxes on Corporate Income and Wealth is given as in equation 5 and is not considered as an income share in NIPA.

Let κ_t denote free cash flow to owners of factorless income firms at date t, relative to gross-value added:

$$\begin{aligned}\kappa_t &\equiv \frac{FCF_t^{\Pi}}{GVA_t} = (1-\tau_t^c)\frac{\Pi_t}{GVA_t} + \frac{T_t^L}{GVA_t} \\ &= (1-\tau_t^c)(1-\tau_t^s)\frac{\mu_t-1}{\mu_t} + \tau_t^L\end{aligned}$$

where $\tau_t^L = \frac{T_t^L}{GVA_t}$.

Equation (10) implies a tight link between fluctuations in the price-cost wedge μ_t and fluctuations in labor's share of income. Using that relationship one can express free cash flow to factorless income firms κ_t as a function of tax rates and labor's share:

$$\kappa_t = (1-\tau_t^s)(1-\tau_t^c) + \tau_t^L - \frac{(1-\tau_t^c)}{(1-\alpha)}\frac{W_tL_t}{GVA_t} \tag{13}$$

Thus, in this model, given tax parameters and a choice for the share parameter α, the

[19] In Atkeson, Heathcote, and Perri (2023) we show how such wedges can be micro-founded as arising from Bertrand competition between more and less productive potential producers.

path for factorless income as a share of corporate GVA can be identified given a path for compensation to labor as a share of gross value added, which we take straight from the IMA. Of course, given total free cash flow, and free cash flow to factorless income firms, we also have free cash flow to owners of capital.

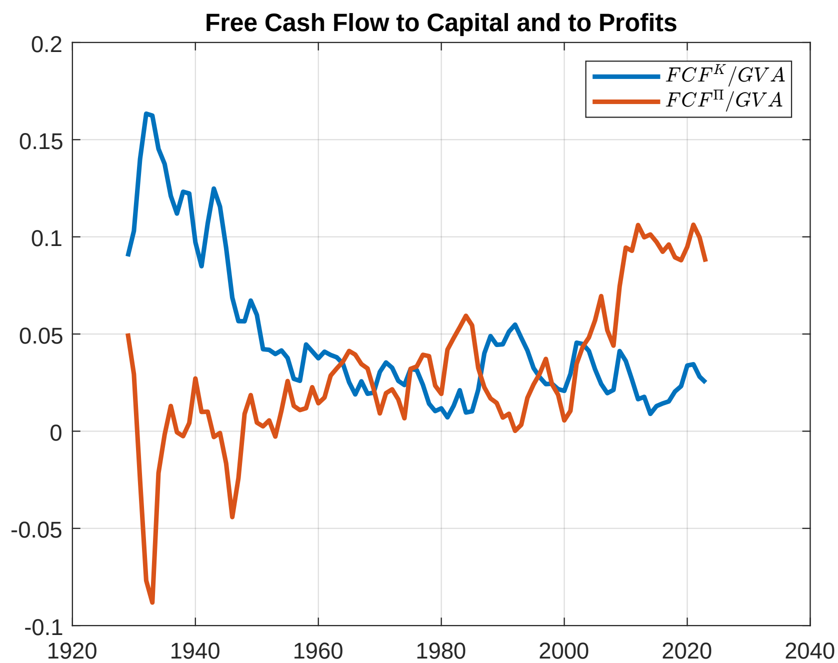

Figure 6: Decomposition of Free Cash Flow into Free Cash Flow to Capital (blue) and Free Cash Flow to Factorless Income (red).

Figure 6 plots factorless income (κ_t) and free cash flow to capital as ratios to corporate GVA, given a choice of $\alpha = 0.29$. We will discuss the logic for this parameter choice below.

The ratio of factorless income to GVA is quite volatile. In addition, it generally appears to trend upward over time, and has risen sharply since 2000, from a share near two percent, to around 10 percent of corporate gross value added. Mechanically, this is largely driven by the decline in labor's share of income over this period: see equation (13) and Figure B.7.

Free cash flow to capital as a share of gross-value added declines quite dramatically in the early decades of our sample, but appears relatively stable from around 1970 onward at a fairly low level. Cash flow to capital was high during the Great Depression because investment and corporate taxes were both very low. Over time, the main driving of declining cash flow to capital has been rising investment as a share of gross value-added (see Figure B.5). This in turn reflects an upward trend in the depreciation rate δ. Rental income from capital $R_t^K K_t$ in the model moves in lockstep with compensation of employees (see equations 10 and 11), so the declining labor share post 2000 also worked to reduce cash flow to capital.

Note that the path for free cash flow to capital that we find reflects a minimum of model structure: our assumption of a Cobb-Douglas production function with a constant capital share in costs α. As for factorless income, alternative values of α simply shift this measure of series up or down without changing the trend.

Note as well that a standard growth model predicts that the share of free cash flow to capital in gross value added on a balanced growth path should be the gap between the expected return to investment in physical capital and expected growth of GVA times the ratio of the replacement value of the capital stock to GVA ($(r-g)QK/GVA$). In the left panel of Figure 7 below, we see that the ratio of the value of the capital stock to GVA after WWII has been steady at roughly 1.5 times GVA. With our valuation multiple of $r - g = 3.2\%$, this gives us the prediction that the ratio of Free Cash Flow to Capital relative to GVA should hover around 4.8% on the balanced growth path. Our measure of Free Cash Flow to Capital shown in Figure 6 is fairly close to this model-implied benchmark.

4.4 Firm Valuation

Enterprise value in our model is the expected discounted present value of free cash flow to owners of firms, with those present values computed using the model's pricing kernel M_{t+1}. Given our division of free cash flow FCF_t into a component that is factorless income $FCF_t^{\Pi} = \kappa_t GVA_t$ and a component that is free cash flow to capital FCF_t^K, it is natural to decompose enterprise value, denoted by V_t as the sum of the values of these two cash flows

$$V_t = V_t^K + V_t^{\Pi} \tag{14}$$

where V_t^K denotes the value of future free cash flow to capital and V_t^{Π} denotes the value of future factorless income.

The firm that owns and manages the physical capital stock takes as given an initial capital stock K_t and chooses future capital $\{K_{t+k}\}$ and after-tax free cash flow payable to owners $\{FCF_{t+k}^K\}$ for $k \geq 1$ to maximize

$$FCF_t^K + V_t^K$$

where

$$V_t^K = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} FCF_{t+k}^K \right]$$

and FCF_t^K is given as in equation 6.

The first-order condition with respect to K_{t+1} is

$$\mathbb{E}_t \left[M_{t,t+1} \left[(1-\tau_{t+1}^c) \left(R_{t+1}^K - Q_{t+1}\delta_{t+1} \right) + (1-\lambda\tau_{t+1}^c) Q_{t+1} \right] \right] = (1-\lambda\tau_t^c) Q_t. \tag{15}$$

If investment firms choose investment according to the capital Euler equation (15) at every date, then one can show that the value of future free cash flow to capital is given by

$$V_t^K = (1 - \lambda \tau_t^c) Q_t K_{t+1} \tag{16}$$

This result is independent of the specification for the pricing kernel $M_{t,t+k}$, but it does rely on the assumption that the production function is constant returns to scale, and that there are no investment adjustment costs.[20] Given this result, we measure the value of factorless income using the IMA data using the difference between enterprise value and the value of the claims to capital:

$$V_t^{\Pi} = V_t - (1 - \lambda \tau_t^c) Q_t K_{t+1}. \tag{17}$$

Note that if $\lambda = 1$ (full expensing of net investment) then a constant corporate tax rate does not distort investment: all the tax terms in equation (15) cancel out. However, firm value is depressed relative to the replacement cost of capital by a factor $(1-\tau_t^c)$. The intuition is that the capital tax depresses income to capital – and thus the market value of capital – which reduces the incentive to invest. But full expensing allowance sufficiently subsidizes the cost of new investment to exactly offset that effect.

Conversely, if $\lambda_t = 0$, then investment and capital are depressed when $\tau_t^c > 0$, but the value of the firm is equal to the replacement cost of its capital. See McGrattan and Prescott (2005) for a related discussion.

We show the breakdown of enterprise value relative to gross value added into these two components in Figure 7. In the left panel of this figure, we show enterprise value (in blue) and the market value of the capital stock (in red). In the right panel of this figure, we again show enterprise value (in blue) and the value of claims to factorless income (in red). We see in the left panel of this figure that between 1929 and World War II (WWII), fluctuations in the value of capital account for much of the fluctuations in enterprise value, but that after WWII, the ratio of the value of capital to value added has remained remarkably stable.

In the right panel of Figure 7, we see that it is fluctuations in the value of claims to factorless income that account for the majority of fluctuations in enterprise value after World War II.

[20]We provide a proof of this result in Appendix D.

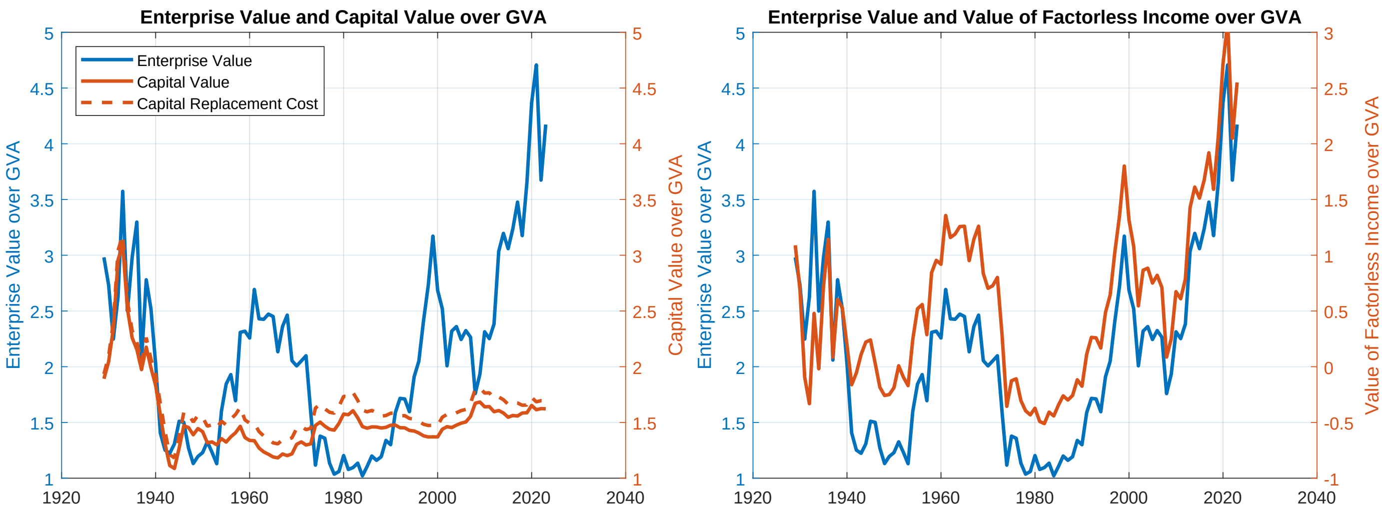

Figure 7: Left Panel: Enterprise Value (left axis) and Value of Capital Stock (right axis) over U.S. Corporate Gross Value Added. 1929-2023 Right Panel: Enterprise Value (left axis) and Value of Factorless Income (right axis) over U.S. Corporate Gross Value Added. 1929-2023

5 Returns to Capital

We now use our model to compute realized and expected returns to investment in physical capital implied by the IMA macroeconomic data.

The gross realized return to capital implied by our model is given by

$$\begin{aligned} 1 + r^K_{t+1} &= \frac{V^K_{t+1} + FCF^K_{t+1}}{V^K_t} \\ &= \frac{1}{(1-\lambda\tau^c_t)} \left[(1-\tau^c_{t+1}) \left(\frac{\frac{R^K_{t+1}K_{t+1}}{GVA_{t+1}} \frac{GVA_{t+1}}{GVA_t}}{\frac{Q_t K_{t+1}}{GVA_t}} - \frac{Q_{t+1}}{Q_t}\delta_{t+1} \right) + (1-\lambda\tau^c_{t+1}) \frac{Q_{t+1}}{Q_t} \right]. \end{aligned}$$

where

$$\frac{R^K_{t+1}K_{t+1}}{GVA_{t+1}} = \alpha \left[(1-\tau^s_{t+1}) - \frac{\left(\kappa_{t+1} - \tau^L_{t+1}\right)}{(1-\tau^c_{t+1})} \right]$$

All of the terms in this expression are available in the IMA data and our series for tax rates τ^c_t, λ except for the parameter α which we have set to 0.29.

Computing the expected value of this expression, $1 + \mathbb{E}_t\left[r^K_{t+1}\right]$, requires specifying a joint stochastic process for $\left(\tau^c_t, \tau^s_t, \tau^L_t, \delta_t, \frac{Q_{t+1}}{Q_t}, \frac{GVA_{t+1}}{GVA_t}\right)$. We assume all these variables are independent, and that for every tax parameter $\mathbb{E}_t\left[\tau_{t+1}\right] = \tau_t$. We assume $\mathbb{E}_t\left[\delta_{t+1}\right] = \delta_t$ and $\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\frac{P_t}{P_{t+1}}\right] = \bar{g}_Q$, where $\bar{g}_Q$ is the sample average gross growth rate for the relative price of investment goods and P_t is the PCE deflator. The expected value for $\mathbb{E}_t \kappa_{t+1}$ is set equal to κ_t.

We allow expected real growth in value added to vary over time, and we set the conditional expectation $\mathbb{E}_t \left[\frac{GVA_{t+1}}{GVA_t} \frac{P_t}{P_{t+1}} \right]$ at each date t such that the expected real return to capital is equal to the expected real return to a GDP bond.[21] We assume in particular that a perpetual claim to future gross value added of the corporate sector has a constant price-dividend ratio of $\beta/(1-\beta) = 31.25$. This is the same valuation multiple that we use in the left panel of Figure 2 for comparing Free Cash Flow and Enterprise Value. This assumption gives us the following equation that determines both the expected return to capital and the expected growth of corporate gross value added in our model.

$$\mathbb{E}_t \left[(1 + r^K_{t+1}) \frac{P_t}{P_{t+1}} \right] = \frac{1}{\beta} \mathbb{E}_t \left[\frac{GVA_{t+1}}{GVA_t} \frac{P_t}{P_{t+1}} \right]. \tag{18}$$

The left panel of Figure 8 plots realized real returns to capital alongside expected returns, computed as just described. The right panel of the figure plots expected real growth in gross value added against realized growth. The series for expected growth inferred from returns tracks actual growth fairly closely.

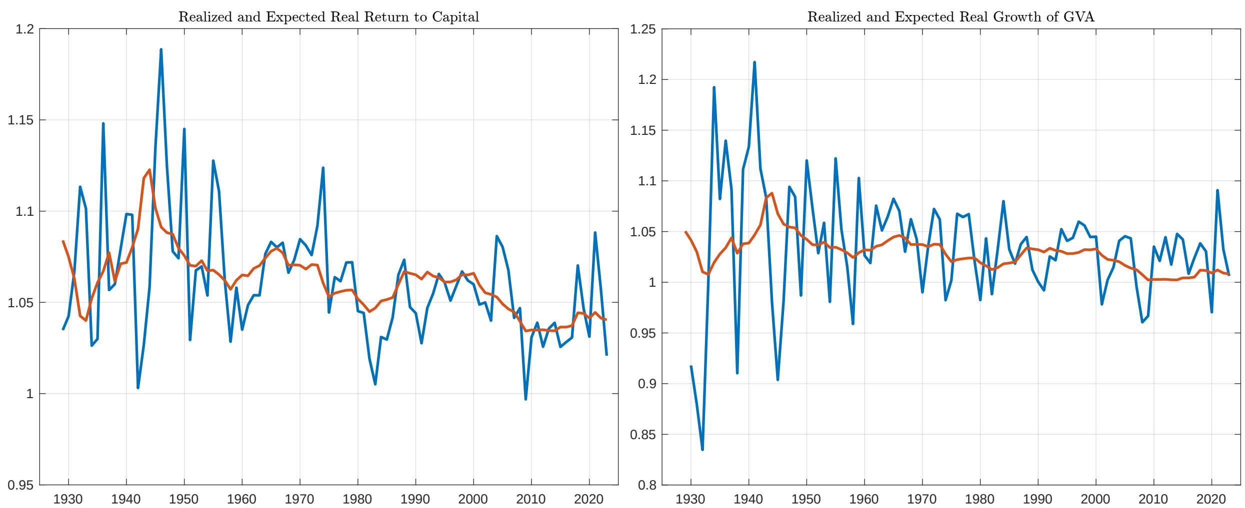

Figure 8: Left Panel: Realized and Expected Real Returns to Capital. Right Panel: Realized and Expected Growth in Real Gross Value Added.

Figure 9 plots the difference between the expected nominal return to capital and the rate on one-year Treasury Securities.[22] Returns to capital greatly exceeded safe returns during the

[21]The logic underlying this choice is that neither GDP bonds nor physical capital are especially risky assets – one could imagine a small relative risk premium in either direction.

[22]We assume that the expected nominal returns to capital are equal to the expected real returns to capital

1940s, but the return differential appears fairly stable at around 5 percentage points during the 1960s and 70s. In the early 1980s the differential fell, as the Federal Reserve pushed up short term rates to combat inflation. But from the 1990s onward, the differential again appears fairly stable at around 5 percent.

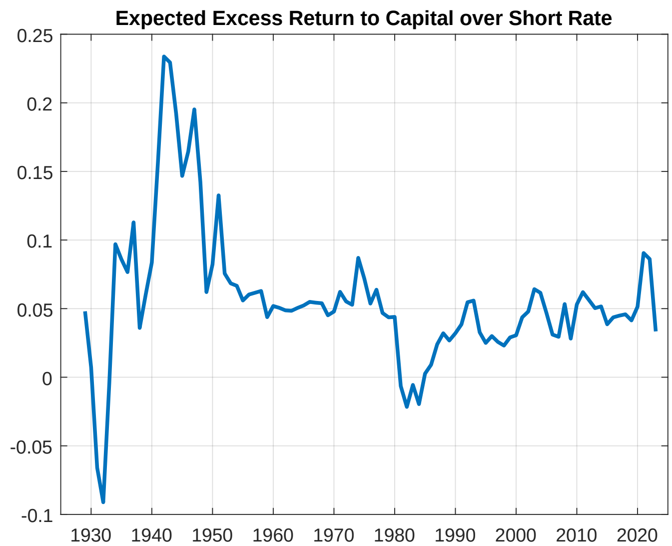

Figure 9: Expected Return to Capital Minus One-Year Treasury Rate. 1929-2023.

We conclude that our decomposition of cash flow between income to capital and factorless income yields a plausible time path for the after-tax return to capital, with a downward trend in expected real returns over time that broadly tracks measured declines in safe rates and expected real growth rates. That is, it appears that for most of the period following WWII, the expected return on investment in measured capital in excess of the one-year Treasury rate has been fairly stable at close to 5 percentage points.

6 A Finance-Style Valuation Exercise

We now conduct a finance-style valuation exercise that is not based on a particular macroeconomic model. Instead, it is based on the valuation framework laid out in Campbell and Shiller (1987) and Campbell and Kyle (1993). Our goal in this second valuation exercise is to estimate the extent to which fluctuations in the price-dividend spread measured using the ratio of Enterprise Value to corporate GVA and Free Cash Flow to corporate GVA as shown in the right panel of Figure 2 are due to changes in the rate of return investors expect to

shown in Figure 8 times expected PCE inflation $\mathbb{E}_t \frac{P_{t+1}}{P_t}$. We assume that expected PCE inflation is given by lagged PCE inflation, or $\mathbb{E}_t \frac{P_{t+1}}{P_t} = \frac{P_t}{P_{t-1}}$.

earn on Enterprise Value versus changes in the expected ratio of future Free Cash Flow to corporate gross value added. We use this second valuation model to construct an estimated series for expected returns on Enterprise Value every year from 1929-2023 and compare those expected returns to those that we have obtained in Figures 8 and 9 from our macro-style measurement of the expected returns to investment in physical capital. To the extent that the series for expected returns obtained from these two different valuation methods coincide, we argue that macro and finance valuation approaches offer a consistent view of the factors driving the volatility of Enterprise Value and the relatively stable observed ratio of measured capital to output.

Our finance-style valuation model is based on the following identity. For any observed sequence of prices $\{p_t\}$ for an asset with observed dividends (cash flows) $\{d_t\}$, we have the following valuation identity

$$p_t = \underbrace{p_t^\star}_{fundamental\ price} + \underbrace{\phi_t}_{residual}$$

where we define the "fundamental price" $p_t^\star$ as the discounted expected value of dividends at constant discount rate β:

$$p_t^\star \equiv \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t d_{t+k}$$

By definition, the residual ϕ_t represents all other influences on price. Note that the value of the parameter β and the stochastic process for dividends that defines conditional expectations $\mathbb{E}_t d_{t+k}$ used to construct a series for the fundamental price $\{p_t^\star\}$ are inputs into the model.

In the general specification of this valuation framework, we assume that dividends follow an ARIMA process, that is, they are integrated of order one. Under this assumption, following Beveridge and Nelson (1981), we can decompose the dynamics of dividends d_t into a trend component x_t and a transitory component y_t. We define the trend component as the expected value of dividends in the long run

$$x_t \equiv \lim_{k\to\infty} \mathbb{E}_t d_{t+k}$$

By construction, we have that

$$\mathbb{E}_t x_{t+s} = x_t$$

The transitory component of dividends is then given by $y_t = d_t - x_t$.

An estimation of this model is then an estimate of the two unobserved components of dividends $\{x_t, y_t\}$ and the third unobserved residual term $\{\phi_t\}$ influencing price. We describe the specific assumptions we use to estimate these three unobserved components of our valuation model below. Before doing do, we first describe the restrictions that the general specification of the model imposes on the dynamics of these three unobserved components given as follows.

First, the trend component of dividends should satisfy $\mathbb{E}_t x_{t+s} - x_t = 0$. That is, $\{x_t\}$ should be a Martingale.

Second, we have that the dynamics of the residual term ϕ_t are related to what we call *quasi-returns* by

$$r_{t,t+1} \equiv \beta \mathbb{E}_t \left(p_{t+1} + d_{t+1} \right) - p_t = \beta \mathbb{E}_t \phi_{t+1} - \phi_t \tag{19}$$

This equation holds regardless of the dynamics of dividends. To see this result, observe that

$$\beta p^\star_{t+1} - p^\star_t = \sum_{k=2}^{\infty} \beta^k \left[\mathbb{E}_{t+1} d_{t+k} - \mathbb{E}_t d_{t+k} \right] - \beta \mathbb{E}_t d_{t+1}$$

Since, by the Law of Iterated Expectations,

$$\mathbb{E}_t \mathbb{E}_{t+1} d_{t+k} = \mathbb{E}_t d_{t+k}$$

we have

$$\mathbb{E}_t \beta p^\star_{t+1} - p^\star_t = -\beta \mathbb{E}_t d_{t+1}$$

Thus, since,

$$\beta p_{t+1} - p_t = \beta p^\star_{t+1} - p^\star_t + \beta \phi_{t+1} - \phi_t$$

we have equation 19.

We define longer horizon quasi-returns by $r_{t,t+s} \equiv \sum_{k=0}^{s-1} \beta^k r_{t+k,t+k+1}$ and equation 19 then implies

$$\mathbb{E}_t r_{t,t+s} = \beta^s \mathbb{E}_t \phi_{t+s} - \phi_t \tag{20}$$

Third, we have that the difference between the price-dividend spread and the residual ϕ_t given by

$$p^T_t - \phi_t = p^\star_t - \frac{\beta}{1-\beta} d_t$$

should forecast future changes in dividends at a rate predicted by the specific ARMA dynamics of y_t. To see this result, observe that since x_t is a Martingale

$$p^\star_t - \frac{\beta}{1-\beta} d_t = \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t \left(d_{t+k} - d_t \right) = \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t \left(y_{t+k} - y_t \right) \tag{21}$$

The specific ARMA model one chooses for the transitory component of dividends y_t determines how y_t influences both the price dividend spread p^T_t and the future expected dynamics of y_t in equation 21.

We apply this valuation framework to our data on Enterprise Value and Free Cash Flow as follows. We let p_t denote Enterprise Value over corporate GVA (V_t/GVA_t). We let d_t correspond to Free Cash Flow over corporate GVA (FCF_t/GVA_t). We choose $\beta/(1-\beta) = 31.25$ so that the model price-dividend spread p_t^T corresponds to the series shown in the right panel of Figure 2.

We note that in our data on Enterprise Value and Free Cash Flow, this price-dividend spread p_t^T forecasts both quasi-returns on Enterprise Value (relative to GVA) and changes in future ratios of Free Cash Flow to GVA in regressions of the form

$$r_{t,t+s} = \alpha_{r,s} + \gamma_{r,s} p_t^T + \epsilon_{r,t+s} \tag{22}$$

$$d_{t+s} - d_t = \alpha_{d,s} + \gamma_{d,s} p_t^T + \epsilon_{d,t+s} \tag{23}$$

This observation indicates from equation 19 that fluctuations in the residual term ϕ_t play a role in driving fluctuations in the price-dividend spread (and hence in the ratio of Enterprise Value of GVA). Likewise, from equation 21, we have the fluctuations in the transitory component of dividends y_t also play a role in driving fluctuations in the price-dividend spread. We seek to estimate how large a role each of these play in these fluctuations in the price-dividend spread as follows.

We further observe that, in our data on Enterprise Value and Free Cash Flow, the price-dividend spread itself is well modeled as an AR1 with persistence parameter $\rho = 0.75$. (See Table B.3).

We then estimate our valuation model under the restrictions that both ϕ_t and y_t are AR1 processes with the same persistence $\rho = 0.75$ with means $\bar{\phi} = \text{mean}(p_t^T)$ and $\text{mean}(y_t) = 0$. We further impose that these series are perfectly correlated. In particular, we choose a parameter ψ such that

$$\phi_t - \bar{\phi} = \psi\left(p_t^T - \text{mean}(p_t^T)\right) \tag{24}$$

and

$$y_t = -\frac{1-\psi}{\Gamma}\left(p_t^T - \text{mean}(p_t^T)\right) \tag{25}$$

where

$$\Gamma = \frac{\beta}{1-\beta} - \frac{\beta\rho}{1-\beta\rho}$$

Note that under our assumption that y_t is an AR1 with persistence ρ, we have

$$\sum_{k=1}^{\infty} \beta^k \mathbb{E}_t\left(y_{t+k} - y_t\right) = -\Gamma y_t$$

With these choices for ϕ_t and y_t implied by the parameters ψ, β and ρ, the series for x_t implied by our model is given as $x_t = d_t - y_t$.

With these restrictions, our model predicts that the estimated slope coefficients $\hat{\gamma}_{r,s}$ in regressions 22 should be given by $(\beta^s \rho^s - 1)\psi$ and the estimate slope coefficients $\hat{\gamma}_{d,s}$ in regressions 23 should be given by $(1 - \rho^s)(1 - \psi)/\Gamma$. That is, the extent to which the price-dividend spread corresponds to fluctuations in the residual term ϕ_t as parameterized by ψ should be revealed by these quasi-return and dividend change forecasting regressions.

We report results from running regressions 22 and 23 in Tables 2 and 3. In these tables, we report the theoretical value of these regression coefficients with $\beta/(1-\beta) = 31.25$, $\rho = 0.75$, and $\psi = 0.25$.[23] We see in these tables that the theoretical regression coefficients implied by our restricted valuation model match the empirical regression coefficients well.

horizon	$s = 1$	$s = 2$	$s = 3$	$s = 5$	$s = 10$	$s = 15$
$(\beta^s \rho^s - 1)\psi$	-0.0683	-0.1180	-0.1540	-0.1993	-0.2397	-0.2479
$\hat{\gamma}_{r,s}$	-0.0815	-0.1110	-0.1125	-0.1740	-0.3464	-0.2856
S.E.	(0.0413)	(0.0512)	(0.0531)	(0.0690)	(0.0903)	(0.1048)
t-Stat	-1.974	-2.168	-2.117	-2.522	-3.835	-2.725
R^2	0.0406	0.0491	0.0474	0.0674	0.1510	0.0869

Table 2: Estimates from quasi-return forecasting regressions 22 with $\beta/(1-\beta) = 31.25$. The theoretical values of these coefficients are computed with $\rho = 0.75$ and $\psi = 0.25$.

horizon	$s = 1$	$s = 2$	$s = 3$	$s = 5$	$s = 10$	$s = 15$
$(1 - \rho^s)(1 - \psi)/\Gamma$	0.0066	0.0115	0.0152	0.0200	0.0248	0.0259
$\hat{\gamma}_{d,s}$	0.0056	0.0135	0.0183	0.0181	0.0245	0.0307
S.E.	(0.0018)	(0.0026)	(0.0027)	(0.0024)	(0.0032)	(0.0039)
t-Stat	3.073	5.219	6.885	7.489	7.754	7.975
R^2	0.0931	0.2300	0.3450	0.3890	0.4200	0.449

Table 3: Estimates from dividend forecasting regressions 23 with $\beta/(1-\beta) = 31.25$. The theoretical values of these coefficients are computed with $\rho = 0.75$ and $\psi = 0.25$.

To check that the series for $\{x_t\}$ implied by our model is indeed a Martingale, we run regressions to predict changes in x_t of the form

$$x_{t+s} - x_t = \alpha_{x,s} + \gamma_{x,s} p_t^T + \epsilon_{x,t+s}$$

[23] We have $\bar{\phi} = \text{mean}(p_t^T) = -0.2985$ in our data with this choice of β.

and

$$x_{t+s} - x_t = \alpha_{dx,s} + \gamma_{dx,s}(x_t - x_{t-1}) + \epsilon_{dx,t+s}$$

for $s = 1, 2, 3, 5, 10, 15$. We find that these estimated coefficients are not statistically difference from zero.

This estimation procedure yields the following decomposition of the dynamics of the ratio of Enterprise Value to corporate GVA into components driven by the residual and components driven by the dynamics of dividends as captured by $p_t^\star$ shown in the left panel of Figure 10. The blue line in that figure is the data on Enterprise Value over corporate GVA. The red line in this figure is our estimate of ϕ_t constructed so its deviations from its mean $\bar{\phi}$ are $\psi = 0.25$ times the deviation of the price-dividend spread p_t^T from its mean. By construction, the remaining fluctuations in Enterprise Value over GVA (the difference between the blue line and the red line) are due to the dynamics of expected future ratios of Free Cash Flow to GVA as captured by $p_t^\star$.

The trend and transitory components of the ratio of Free Cash Flow to corporate GVA implied by our estimation procedure are shown in the right panel of Figure 10. The data on the ratio of Free Cash Flow to corporate GVA are shown in blue, with the trend component x_t shown in red and the transitory component y_t shown in yellow. We see in this figure that our estimation finds large movements in the trend component of Free Cash Flow to GVA, and these movements in x_t contribute much to the volatility in the ratio of Enterprise Value to GVA.

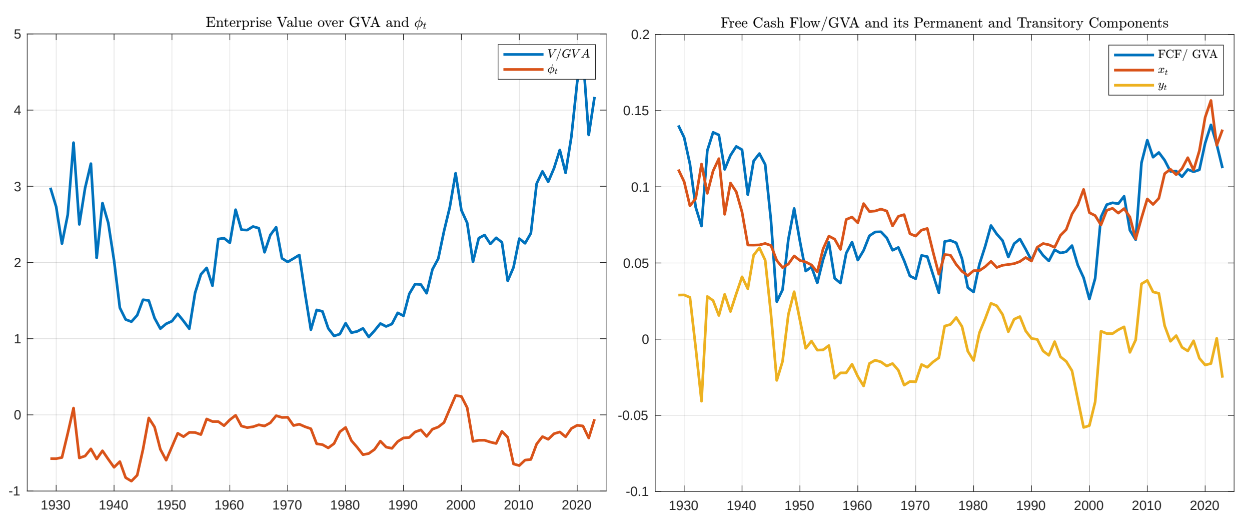

Figure 10: Left Panel: Enterprise Value over GVA in blue and our estimate of the residual ϕ_t in red. Right Panel: Free Cash Flow over GVA in blue and our estimates of the trend component of this ratio x_t in red and the transitory component y_t in yellow.

In our estimated model, the ratio of Enterprise Value to GVA is given as a function of our unobserved components ϕ_t, y_t and x_t by

$$\frac{V_t}{GVA_t} = \phi_t + \frac{\beta\rho}{1-\beta\rho} y_t + \frac{\beta}{1-\beta} x_t$$

Innovations to these unobserved components are correlated so that we cannot offer an unambigious decomposition of the drivers of the ratio of Enterprise Value to GVA. We show, however, in Figure 11, that the impact of movements in x_t alone, as captured by the term $\bar{\phi} + \frac{\beta}{1-\beta} x_t$, account for the overwhelming majority of the fluctuations in the ratio of Enterprise Value to GVA. To understand this finding, observe that

$$\frac{V_t}{GVA_t} = \bar{\phi} + p_t^T - \text{mean}(p_t^T) + \frac{\beta}{1-\beta} y_t + \frac{\beta}{1-\beta} x_t$$

From equation 25, given our parameter estimates, we have that the influence of the price dividend spread p_t^T and the transitory component of dividends y_t on the ratio of Enterprise Value to GVA roughly cancel out.

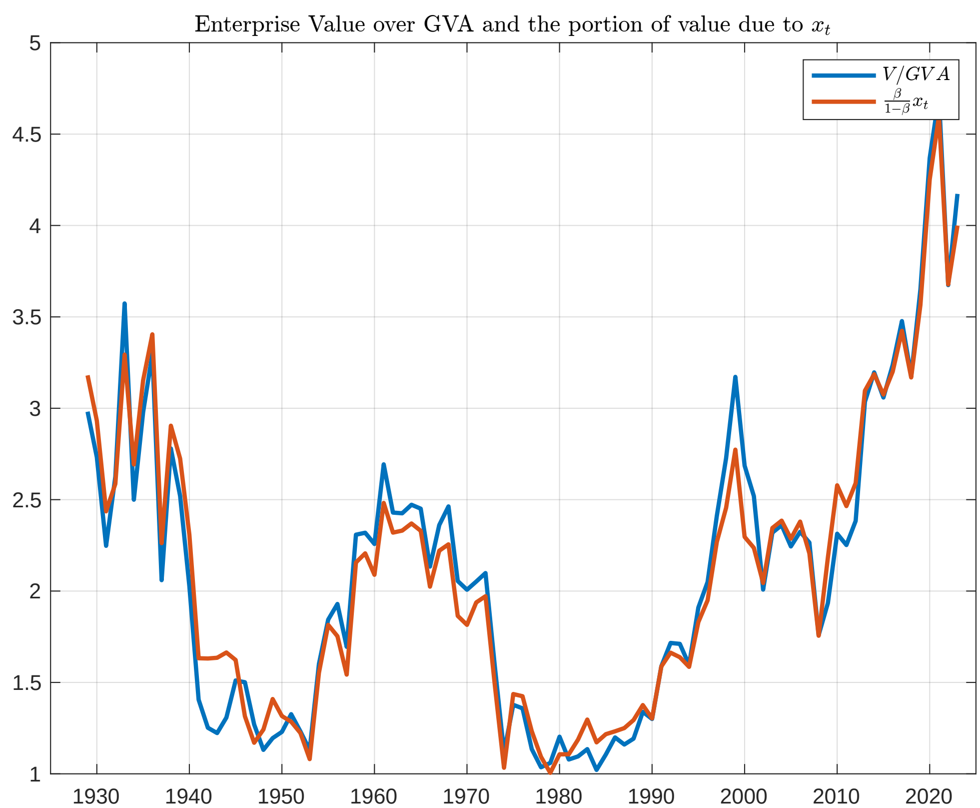

Figure 11: Enterprise Value over GVA in blue and the level of this ratio accounted for by x_t as given by $\bar{\phi} + \frac{\beta}{1-\beta} x_t$

To compute the expected returns to enterprise value, we combine the results from this estimated model of the dynamics of the ratio of Enterprise Value to GVA and of Free Cash Flow to GVA with our estimated series for the expected growth of corporate GVA derived from our macro-valuation exercise. Specifically, we have

$$\mathbb{E}_t \frac{FCF_{t+1}}{GVA_{t+1}} = x_t + \rho y_t$$

and

$$\mathbb{E}_t \frac{V_{t+1}}{GVA_{t+1}} = \bar{\phi} + \rho(\phi_t - \bar{\phi}) + \frac{\beta}{1-\beta} x_t + \frac{\beta\rho}{1-\beta\rho} \rho y_t$$

We compute the expected real returns to Enterprise Value at each date t as

$$\mathbb{E}_t \left[1 + r^V_{t+1}\right] \frac{P_t}{P_{t+1}} = \left[\frac{GVA_t}{V_t}\right] \mathbb{E}_t \left[\frac{FCF_{t+1}}{GVA_{t+1}} + \frac{V_{t+1}}{GVA_{t+1}}\right] \mathbb{E}_t \left[\frac{GVA_{t+1}}{GVA_t} \frac{P_t}{P_{t+1}}\right].$$

Note that we take the first term in square brackets on the right side of this equation (the inverse of the ratio of Enterprise Value to GVA in period t) directly from the data. We take the second term in square brackets reflecting the expected values of the ratio of Free Cash Flow and Enterprise Value to GVA in period $t+1$ from the formulas above. Finally, we take the values for expected real growth of GVA in the third term on the right side of this equation from the estimates shown in the right panel of Figure 8.

We now compare the expected returns to Enterprise Value that we obtain from our finance-style valuation model with the expected returns to capital that we obtained from our macro-style valuation model for capital. We do so using our two measures of expected returns in excess of the one-year Treasury rate.[24] Figure 12 shows the expected excess returns over the one year nominal interest rate implied by these two valuation exercises. Our series for expected excess returns on Enterprise Value is in blue and our series for expected excess returns on capital is in red.

[24] As we did with returns on capital, we estimate the expected nominal returns to Enterprise Value as the product of the expected real returns given above times expected PCE inflation. We assume that $\mathbb{E}_t \frac{P_{t+1}}{P_t} = \frac{P_t}{P_{t-1}}$.

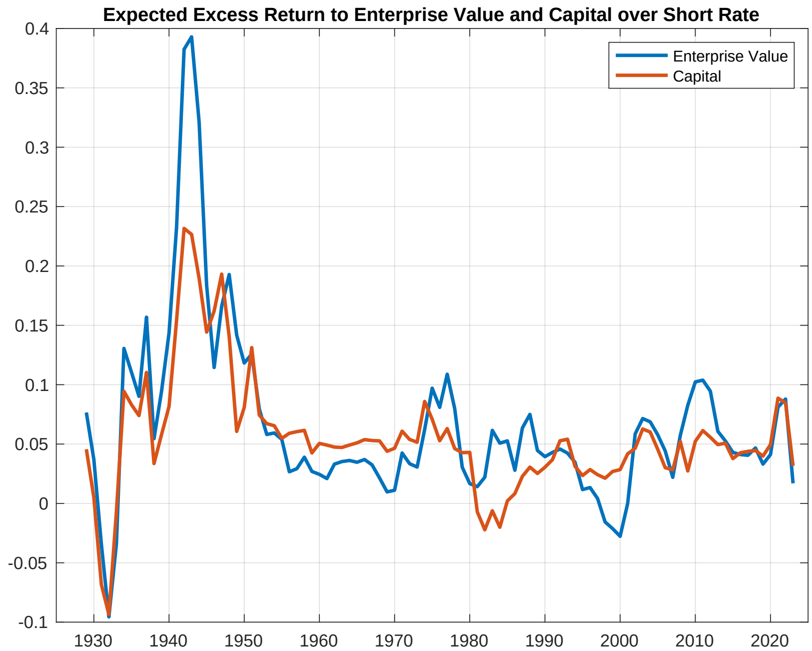

Figure 12: Expected Returns to Enterprises and to Capital in excess of the one year Treasury rate.

We see in this figure that our estimates of the expected excess returns to investment in physical capital obtained from our macro models and our estimates of the expected excess returns to Enterprise Value obtained from our finance-style valuation exercise correspond fairly well. What features of our two estimation procedures explain this correspondence?

To address this question, consider the following counterfactual estimation outcome as a benchmark. If in our finance valuation exercise we had found that the residual term $\phi_t = 0$ for all dates, then our two estimates for expected excess returns on capital and enterprise value would coincide exactly.

To see this result, first observe from equation 18 that in estimating the expected real return to capital, we imposed that these expected returns are equal to a constant $1/\beta$ times expected real growth of corporate GVA. This assumption led us to the estimates for expected real growth of corporate GVA shown in the right panel of Figure 8 given data on the capital-output ratio, corporate taxes, depreciation, and expected changes in the relative price of investment goods and consumption. We see the results in this figure as indicating that this assumption regarding expected returns to capital is not contradicted by the data.

Next observe from equation 19 that in our finance-style valuation exercise, expected returns to Enterprise Value in excess of real growth in GVA are given by

$$\beta \mathbb{E}_t (1 + r^V_{t+1}) \left[\frac{GVA_{t+1}}{GVA_t} \frac{P_t}{P_{t+1}} \right] - 1 = \frac{r_{t,t+1}}{p_t} = \frac{1}{p_t} \left[\beta \mathbb{E}_t \phi_{t+1} - \phi_t \right] \tag{26}$$

We see from this equation that if $\phi_t = 0$ for all dates t, then these expected returns to Enterprise value in excess of real growth of GVA are constant at $1/\beta$, which, by the assumption above, is the same expected returns to capital in excess of real growth of GVA.

We thus see the discrepancy between the estimated expected excess returns on Enterprise Value and capital shown in Figure 12 as a second metric for understanding the importance of the residual term ϕ_t in driving the ratio of Enterprise Value to GVA that is alternative to the results shown in the left panel of Figure 10. That is, Figure 12 reveals the extent to which the residual term shown in the left panel of Figure 10 corresponds to variation over time in expected returns to Enterprise Value in excess of real growth of corporate GVA.

Note that there are two reasons that the residual term ϕ_t accounts for variation over time in the expected return to Enterprise Value in excess of real growth in GVA. We can see these two reasons if we write the term on the right side of 26 as

$$\frac{1}{p_t}(\beta - 1)\bar{\phi} + \frac{1}{p_t}\left[\beta \mathbb{E}_t(\phi_{t+1} - \bar{\phi}) - (\phi_t - \bar{\phi})\right]$$

We thus see that our valuation model implies that expected returns to Enterprise Value in excess of growth in real GVA can vary over time if the ratio of Enterprise Value to GVA (p_t) varies relative to the mean value of the residual term as well as if there are dynamics of the residual term relative to its mean. In our estimated model, most of the discrepancy between the expected excess returns to Enterprise Value and those to capital shown in Figure 12 are due to our estimated dynamics of ϕ_t. To illustrate this point, in Appendix Figure B.6 we show our model's implications for the expected excess returns to Enterprise Value and capital if $\psi = 0$ so that the residual term ϕ_t is constant at its mean value $\bar{\phi}$. We see in this figure that in this case, the discrepancy between estimated expected excess returns on Enterprise Value and capital are very small.

7 Conclusion

We interpret the results shown in Figure 12 as indicating that the estimated series for expected returns obtained from our two different valuation methods, one macro-model based and one purely finance-based, coincide fairly well outside of the period of World War II. Certainly, the correspondence is not perfect, but, given the difficulties of measurement, we argue that macro and finance valuation approaches we have followed in this paper offer, quantitatively, a reconciliation of the observed volatility of the ratio of Enterprise Value to GVA and the relatively stable observed ratio of measured capital to GVA, at least for the time period following World War II.

This reconciliation is based on the hypothesis that the volatility of the ratio of Enterprise Value to GVA after World War II is driven largely by news that alters investors' expectations of the ratio of Free Cash Flow to GVA in the distant future. Our two valuation methods put a substantially smaller weight on variation over time in the expected returns demanded by investors to hold claims on the Enterprise Value of the U.S. corporate sector.

According to our macro-model based valuation exercise, most of the volatility in the ratio of Enterprise Value to GVA and of Free Cash Flow to GVA for the US corporate sector in the period following World War II has been due to fluctuations in investors' expectations of the share of factorless income in GVA in the long run. These fluctuations in long-run expectations of the share of factorless income in GVA have been consistent with firms' maintaining a fairly stable ratio of measured capital to GVA for two main reasons.

First, it is only near-term expectations of the share of factorless income to GVA that enter into the capital Euler equation describing firms' optimal choice of capital. Thus, news that alters investors' long-run expectations of the share of factorless income in GVA do not have an immediate impact on firms' incentives to invest in measured capital.

Second, over the long-term, the sustained rise in the share of factorless income in GVA that we have seen over the past two decades has been accompanied by a decline in expected rates of return and expected growth rates as well as a decline in the effective tax rate on investment in capital. We see it as largely coincidental that these two influences on investment in capital over the long-run appear to have largely offset each other.

We take as evidence for this interpretation the observation that the ratio of measured capital to GVA did, in fact, move by large amounts in the period prior to World War II. In fact, the observed movements in the stock of measured capital relative to GVA in this early period were so large that it appears that they account for much of the observed variation in the ratio of Enterprise Value to GVA, as would be indicated by standard theory.

However, at the same time, we note that our two estimates of expected excess returns during the period before the end of World War II are both quite volatile. Is this finding due to estimation error in the turbulent periods of the Great Depression and World War II? Or did expected excess returns actually vary a great deal during this time period? We do not know. We see the resolution of these questions as a subject for future research.

References

Abel, Andrew, N. Gregory Mankiw, Lawrence H. Summers, and Richard J. Zeckhauser. 1989. “Assessing Dynamic Efficiency: Theory and Evidence.” *Review of Economic Studies* 56 (1):1–20.

Abel, Andrew B. and Janice C. Eberly. 2012. “Investment, Valuation, and Growth Options.” *The Quarterly Journal of Finance* 1 (1).

Adame, Katherine, Jennifer L. Koski, Katie Lem, and Sarah McVay. 2023. “Free Cash Flow Disclosure in Earnings Announcements.” Working Paper 3134922, SSRN.

Atkeson, Andrew. 2020. “Alternative Facts Regarding the Labor Share.” *Review of Economic Dynamics* 37 (Supplement 1):S167–S180.

Atkeson, Andrew, Jonathan Heathcote, and Fabrizio Perri. 2023. “The End of Privilege: A Reexamination of the Net Foreign Asset Position of the United States.” Working Paper 29771, NBER.

———. 2024. “There is No Excess Volatility Puzzle.” Working Paper 32481, NBER.

Barkai, Simcha. 2020. “Declining Capital and Labor Shares.” *Journal of Finance* 75 (5):2421–2463.

Barro, Robert and Jason Furman. 2018. “Macroeconomic effects of the 2017 tax reform.” *Brookings Papers on Economic Activity* :257–345.

Basu, Susanto and Brent Bundick. 2017. “Uncertainty Shocks in a Model of Effective Demand.” *Econometrica* 85 (3):937–958.

Basu, Susanto, Giacomo Candian, Ryan Chadrour, and Rosen Valchev. 2023. “Risky Business Cycles.”

Belo, Federico, Vito Gala, Juliana Salomao, and Maria Ana Vitorino. 2022. “Decomposing Firm Value.” *Journal of Financial Economics* 143 (2):619–639.

Beveridge, Stephen and Charles R. Nelson. 1981. *Journal of Monetary Economics* 7 (2):151–174.

Caballero, Ricardo, Emmanuel Farhi, and Pierre-Olivier Gourinchas. 2017. “Rents, Technical Change, and Risk Premia: Accounting for Secular Trends in Interest Rates, Returns on Capital, Earning Yields, and Factor Shares.” *American Economic Review* 107 (5):614–620.

Cagetti, Marco, Elizabeth Ball Holmquist, Lisa Lynn, Susan Hume McIntosh, and David Wasshousen. 2013. “The Integrated Macroeconomic Accounts of the United States.” Finance and Economics Discussion Series 2012-81, Federal Reserve Board of Governors.

Cambell, John Y., Carolin Pflueger, and Luis M. Viceira. 2020. “Macroeconomic Drivers of Bond and Equity Risks.” *Journal of Political Economy* 128 (8):3148–3185.

Campbell, John Y. and Albert S. Kyle. 1993. *Review of Economic Studies* 60 (1):1–34.

Campbell, John Y. and Robert J. Shiller. 1987. "Cointegration and tests of present value models." *Journal of Political Economy* 95 (1062-1088).

———. 1988. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *The Review of Financial Studies* 1 (3):195–228.

Cochrane, John. 1991. "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations." *Journal of Finance* 46 (1):209–237.

———. 2008. "The Dog that Did Not Bark: A Defense of Return Predictability." *The Review of Financial Studies* 21 (4):1533–1575.

Corrado, Carol, Jonathan Haskel, Cecilia Jona-Lasino, and Massimiliano Iommi. 2022. "Intangible Capital and Modern Economies." *Journal of Economic Perspectives* 36 (3):3–28.

Crouzet, Nicolas and Janice Eberly. 2018. "Understanding Weak Capital Investment: The Role of Market Concentration and Intagibles." *Proceedings of the 2018 Jackson Hole Symposium* :87–148.

———. 2023. "Rents and Intangible Capital: A Q+ Framework." *Journal of Finance* 78 (4):1873–1916.

Crouzet, Nicolas, Andrea Eisfeldt, Janice C. Eberly, and Dimitris Papanikolaou. 2022. "A Model of Intangible Capital." Working Paper 30376, NBER.

Eggertsson, Gauti B., Jakob Robbins, and Ella Getz Wold. 2021. "Kaldor and Piketty's Facts: The Rise of Monopoly Power in the United States." *Journal of Monetary Economics* 124:S19-S38.

Eisfeldt, Andrea, Edward T. Kim, and Dimitris Papanikolaou. 2022. "Intangible Value." *Critical Finance Review* 11 (2):299–332.

Eisfeldt, Andrea and Dimitris Papanikolaou. 2014. "The Value and Ownership of Intangible Capital." *American Economic Review* 104 (5):189–194.

Farhi, Emmanuel and François Gourio. 2018. "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia." *Brookings Papers on Economic Activity* (2):147–250.

Fullerton, Don. 1983. "Which Effective Tax Rate?" Working Paper 1123, NBER.

Gomme, Paul, B. Ravikumar, and Peter Rupert. 2011. "The return to capital and the business cycle." *Review of Economic Dynamics* 14 (2):262–278.

———. 2015. "Secular Stagnation and Returns on Capital." Economic Synopses 19, Federal Reserve Bank of St. Louis.

Gourio, François. 2012. "Disaster Risk and Business Cycles." *American Economic Review* 102 (6):2734–66.

Gravelle, Jane G. 1994. *The Economic Effects of Taxing Capital Income*. MIT Press.

———. 2006. "Historical Effective Marginal Tax Rates on Capital Income." CRS Report RS21706, Congressional Research Service.

Greenwald, Daniel, Martin Lettau, and Sydney Ludvigson. 2023. "How the Wealth Was Won: Factor Shares as Market Fundamentals." Working Paper 25769, NBER.

Gutiérrez, Germán and Thomas Philippon. 2017. "Investmentless Growth: An Empirical Investigation." *Brookings Papers on Economic Activity* (2):89–169.

Hall, Robert E. 2001. "The Stock Market and Capital Accumulation." *American Economic Review* 91 (5):1185–1202.

———. 2003. "Corporate Earnings Track the Competitive Benchmark." Working Paper 10150, NBER.

———. 2017. "High Discounts and High Unemployment." *American Economic Review* 107 (2):305–30.

Hall, Robert E. and Dale W. Jorgenson. 1967. "Tax Policy and Investment Behavior." *American Econmic Review* 57 (3):391–414.

Harper, Justin and Bonnie A. Retus. 2022. "Returns for Domestic Nonfinancial Business." *Survey of Current Business* 102 (3):online.

Ilut, Cosmin and Martin Schneider. 2014. "Amigious Business Cycles." *American Economic Review* 104 (8):2368–99.

Jermann, Urban J. 1998. "Asset Pricing in Production Economies." *Journal of Monetary Economics* 41:257–275.

———. 2010. "The Equity Premium Implied By Production." *Journal of Financial Econmics* 98:279–296.

Kaltenbrunner, Georg and Lars A. Loechster. 2010. "Long Run Risk Through Consumption Smoothing." *Review of Financial Studies* 23 (8):3190–3224.

Karabarbounis, Loukas and Brent Neiman. 2019. "Accounting for Factorless Income." *NBER Macroeconomics Annual* 33 (1):167–228.

Larraine, Borja and Motohiro Yogo. 2008. "Does firm value move too much to be justified by subsequent changes in cash flow?" *Journal of Financial Economics* 87 (1):200–226.

Lucas, Jr., Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica* 46 (6):1429–1445.

Lustig, Hanno and Stijn Van Nieuwerburgh. 2008. "The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street." *Review of Financial Studies* 21 (5):2097–2137.

McGrattan, Ellen. 2023. "Taxes, regulations, and the value of U.S. corporations: A reassessment." *Review of Economic Dynamics* 50:131–145.

McGrattan, Ellen R. and Edward C. Prescott. 2005. "Taxes, Regulations, and the Value of US and UK Corporations." *Review of Economic Studies* 72 (3):767–796.

———. 2010. "Unmeasured Investment and the Puzzling US Boom in the 1990's." *American Economic Journal: Macroeconomics* 2 (4):88–123.

Mehra, Rajnish and Edward C. Prescott. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (2):145–161.

Merz, Monika and Eran Yashiv. 2007. "Labor and the Market Value of the Firm." *American Economic Review* 97 (4):1419–1431.

Philippon, Thomas. 2009. "The Bond Market's Q." *Quarterly Journal of Economics* 124 (3):1011–1056.

———. 2019. *The Great Reversal: How America Gave Up on Free Markets*. Harvard University Press.

Reis, Ricardo. 2022. "Which r-star, public bonds or private investment? Measurement and policy implications."

Tallarini, Thomas. 2000. "Risk-sensitive real business cycles." *Journal of Monetary Economics* 45 (3):507–532.

Tobin, James. 1969. "A General Equilibrium Approach to Monetary Theory." *Journal of Money, Credit, and Banking* 1:15–29.

Appendices

A Data Appendix

In this appendix we list the sources for the data used in this paper. We make reference to four main sources of information. There is often considerable overlap between these data sources.

A.1 Aggregate data

- Integrated Macroeconomic Accounts (IMA) Tables S5 and S6 for the Nonfinancial Corporate and Financial Sectors respectively. These tables present flows from 1946-2022 and end of year balance sheets from 1945 to 2022. These tables can be found towards the back of the publication Z1 *Financial Accounts of the United States* (previously known as the Flow of Funds). They can also be found on the website of the Bureau of Economic Analysis.

- National Income and Product Accounts (NIPA) Table 1.14. This table offers annual data on gross value added and the breakdown of income into components for the corporate sector from 1929 through 2022. We also refer to other tables from the NIPA and refer to them with this abbreviation.

- Fixed Assets (FA) Tables 6.1, 6.4, and 6.7 which offer data on investment, consumption of fixed capital (depreciation), and year-end capital stocks for the non-financial corporate and financial sectors from 1929 to the present.

- Various tables from the *Financial Accounts of the United States* which we refer to by the abbreviation FOF and the table number.

We now describe our specific data sources.

The following series for the corporate sector 1929-2023 are taken from NIPA Table 1.14. These data series are also available broken down for the non-financial corporate and financial sectors separately on NIPA Table 1.14 and on IMA Tables S5 and S6. These tables are updated on different schedules, so the source with the most up to date data depends on the time of year. Small differences between these two data sources may exist due to different accounting standards for the NIPA and the IMA. We list the line and table numbers for these series below.

- Gross Value Added. NIPA Table 1.14 Line 1. See also IMA Tables S5 FA106902501.A and S6 FA796902505.A

- Tax Payments are the sum of three lines from NIPA Table 1.14. These are line 7, Taxes on production and imports less subsidies, line 10, Business current transfer payments (net), and line 12, Taxes on corporate income. See also IMA Tables S5 FA106240101.A, FA106403001.A, FA106220001.A and S6 FA796240101.A, FA796403005.A, FA796220001.A

- Compensation of Employees. NIPA Table 1.14 Line 4. See also IMA Tables S5 FA106025005.A, FA796025005.A

- Consumption of Fixed Capital. NIPA Table 1.14 Line 2. See also IMA Tables S5 FA106300003.A, FA796330081.A

We obtain data on investment expenditures (gross fixed capital formation) by the corporate sector from two sources listed below. Small differences between the data on Fixed Assets Table 6.7 and IMA Tables S5 and S6 for the period for which they overlap are due to different accounting standards for the two accounts.

- Investment 1929 - 1945. Fixed Assets Table 6.7 line 2

- Investment 1946 - 2022. The sum of IMA Tables S5 line FA105019085.A and S6 line FA795013005.A

We obtain data on the reproduction value of the capital stock in the corporate sector from two sources listed below. It is important to note that the value of nonfinancial assets listed on the balance sheets of Tables S5 and S6 include measures of the value of land, which we exclude from our model. Thus, we do not use those measures. Instead we use the following sources that are restricted to fixed assets.

- Capital 1929 - 1944. Fixed Assets Table 6.1 line 2

- Capital 1945 - 2023. FOF Table L4. Sum of lines FL105015085.A and FL795013865.A

We obtain data on enterprise value of the corporate sector from two sources.

- For the period 1945 - 2023, we use balance sheet data from IMA Tables S5 and S6. These series are constructed for the Nonfinancial corporate sector as the difference between the line Total Liabilities and Net Worth minus the line titled Financial Assets. The construction is the same for the Financial Sector using Table S6. These series for enterprise value are reported on FOF Table B1 in lines LM102010405.A and LM792010405.A. We use these series from B1.

- For the period 1929 - 1944 we use data from the 1945 Statistics of Income Part 2 available here https://www.irs.gov/pub/irs-soi/45soireppt2ar.pdf. We use data from Table 20 on page 420 of this document (page 425 of the PDF). For financial assets, we use Total Assets on line 9 less Capital Assets on line 7. For liabilities, we use Total Liabilities on line 21 less Capital Stock Common on line 17. For the market value of equity, we use the total market capitalization for the CRSP Value Weighted Index.

We use two sources of data on the market value of corporate equities. These are

- 1929-1944: CRSP Value Weighted Index Total Market Capitalization

- 1929-1944: FOF Table L224 Nonfinancial Corporate Equities LM103164105.A for both public and closely held and LM103164115.A for public alone plus Financial Sector Corporate Equities LM793164105.A less equities issued by Closed End Funds from FOF Table L123 LM554090005.A and by ETFs from FOF Table L124 FA564090005.A.

We use two sources of data on dividend payments. We pay particular attention to distinguishing between dividends as reported in the IMA (and other places) and monetary dividends paid. One of the big distinctions between these two concepts concerns the treatment of dividends on foreign direct investment which are an accounting entry and not a measure of dividends paid. Note, however, that our measure of dividends does include cash dividends paid on foreign direct investment in the U.S. Our sources are

- 1929 - 1945. NIPA Table 7.16 line 31 (Dividends paid in cash or assets, IRS), plus line 32 (Post tabulation amendments and revisions), less line 33 (Dividends paid by Federal Reserve Banks).

- 1946 - 2022. NIPA Table 7.10 line 2 (Monetary Dividends Paid Domestic Corporate Business) less line 4 (Paid by Federal Reserve Banks).

We use two sources of data for returns on corporate equities.

- 1929-1945. Returns without dividends are set equal to returns without dividends on the CRSP Value Weighted Total Market Index. The dividend return is computed as the ratio of dividends paid in year $t + 1$ to the value of corporate equities in year t.

- 1946 - 2022. Returns without dividends are set equal to the ratio of the sum from Table S5 of revaluations of nonfinancial corporate equities FR103164105.A and non financial foreign direct investment in the U.S. FR103192105.A and from Table S6 revaluations of financial corporate equities FR793164105.A and financial foreign direct investment in the U.S. FR793192105.A to the sum of the corresponding levels of these variables at the end of the previous year: from S5 LM103164105.A + FL103192105.A and from S6 LM793164105.A + LM793192105.A. Returns with dividends adds to this return the ratio of dividends in year t to the value of corporate equities plus FDI equity in the U.S. in year $t - 1$.

A.2 Compustat data

For producing the Compustat lines in figures 4 and **??** we restrict the Compustat sample as follows. First we select only firms incorporated in the United States, we then drop observations for firms which report their 10k in financial services format, observations for which there is no year information or for which a firm year is duplicated. That leaves us with a sample of 316562 firm/year observations over the 1988-2023 period. Free cash flow is computed (following Adame et al. 2023) as Operating Activities–Net Cash Flow (OANCF) minus capital expenditures (CAPX). Enterprise value is computed as Total Market Value (MKVALT) plus Total Liabilities (LT) minus current assets total (ACT), which includes cash and other short term investments, receivables, inventories, and other current assets.

B Additional Data Plots

In this appendix, we include additional data plots.

We can use the IMA to construct a market valuation of the equity of U.S. corporations (both publicly traded and closely held corporations) and a corresponding cash flow measure of monetary dividends paid to the owners of these corporations. We include these alternative measures of valuation and cash flow to facilitate comparisons between the IMA data and work using data from CRSP and Compustat for publicly traded firms. We show the IMA measures for the value of equity and for dividends relative to gross value added for the U.S. corporate sector in red in the left and right panels of Figure B.1. Our measures of enterprise value and free cash flow are in blue.

We see in the left panel of Figure B.1 that the fluctuations in the market value of equity and enterprise value for U.S. corporations are tightly linked. By comparing the different scales for enterprise value (left axis) and equity (right axis), we see that enterprise value is consistently about 50 percentage points of gross value added larger than the market value of equity. This difference between enterprise value and equity value corresponds to net debt of the U.S. corporate sector.

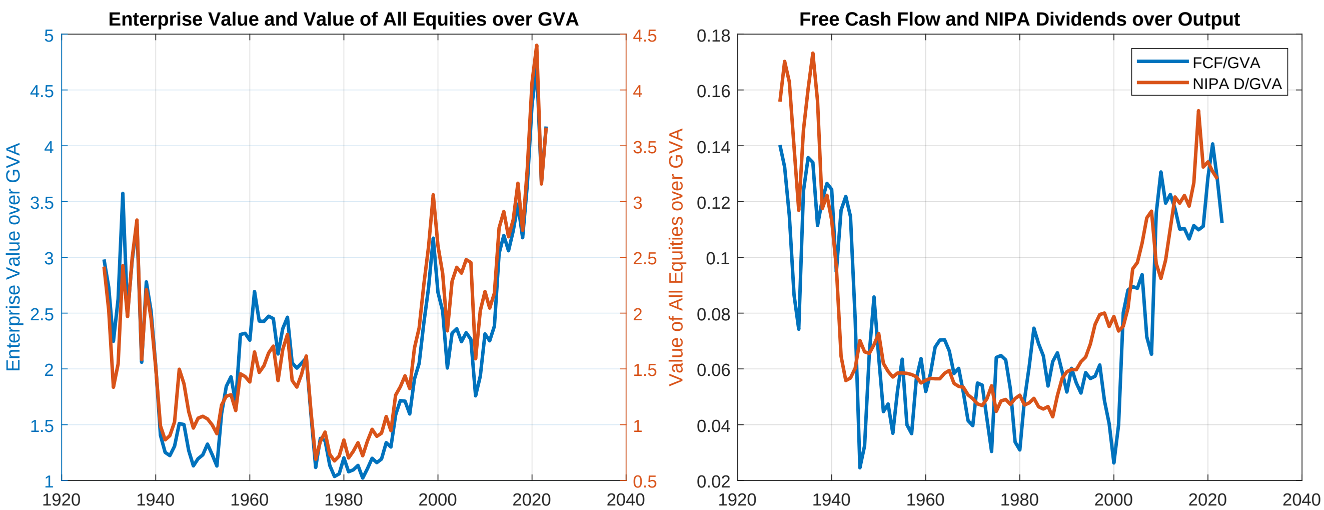

Figure B.1: Left Panel: Enterprise Value (left axis) and Equity Value (right axis) of U.S. Corporations over Corporate Gross Value Added. Right Panel: Free Cash Flow and NIPA Monetary Dividends Paid over Corporate Gross Value Added. 1929-2023. NIPA Dividends Paid data are not yet available for 2023

The IMA imputes the market value of closely held equity from data on the market value of public equities. In Figure B.2, we show the IMA measures of Enterprise Value over corporate GVA and the Market Value of Public Equities over corporate GVA.

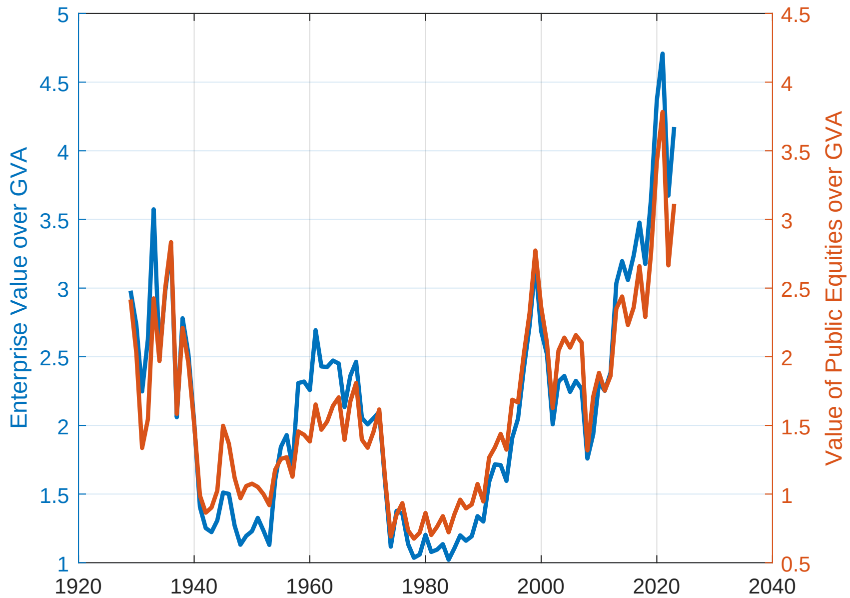

Figure B.2: Enterprise Value (left axis) and the Market Value of Corporate Public Equities (right axis) over Gross Value Added. 1929-2023

We can use the IMA data to compute realized returns on equity from the perspective of a household that purchases equity at the end of period t at price V_t^E, collects dividend payments in year $t+1$, D_{t+1}^{IMA}, and sells that equity realizing a capital gain corresponding to the IMA reported revaluation of outstanding equity at $t+1$, $REVAL_{t+1}^E$. We compute this realized return as

$$\exp(r_{t+1}^E) = \frac{D_{t+1}^{IMA} + REVAL_{t+1}^E + V_t^E}{V_t^E}$$

This calculation of returns is closer to what is done in CRSP or Standard and Poors' data for public firms. We report basic statistics for these IMA returns to equity in Table B.1

In Figure B.3 we examine the extent to which these measures of realized real returns on enterprise value and on IMA equity line up with measures of realized real returns computed using the CRSP value-weighted portfolio. In the left panel, we show a scatter plot of realized annual returns on the CRSP portfolio on the x-axis and returns on enterprise value on the y-axis. The red line is the 45 degree line. We show the corresponding scatter plot for CRSP returns and realized returns on IMA equity in the right panel. We show the same scatter plots using data from the 1946-2022 time period in Appendix Figure B.4. The correlation of returns on enterprise value with those on the value-weighted CRSP portfolio is 0.943 for the period 1929-2023. The corresponding correlation for IMA equity returns with CRSP returns is 0.981 for 1929-2022. Note that one would expect some deviation of returns on enterprise value from returns on equity given the presence of net debt documented in the left panel of Figure B.1.

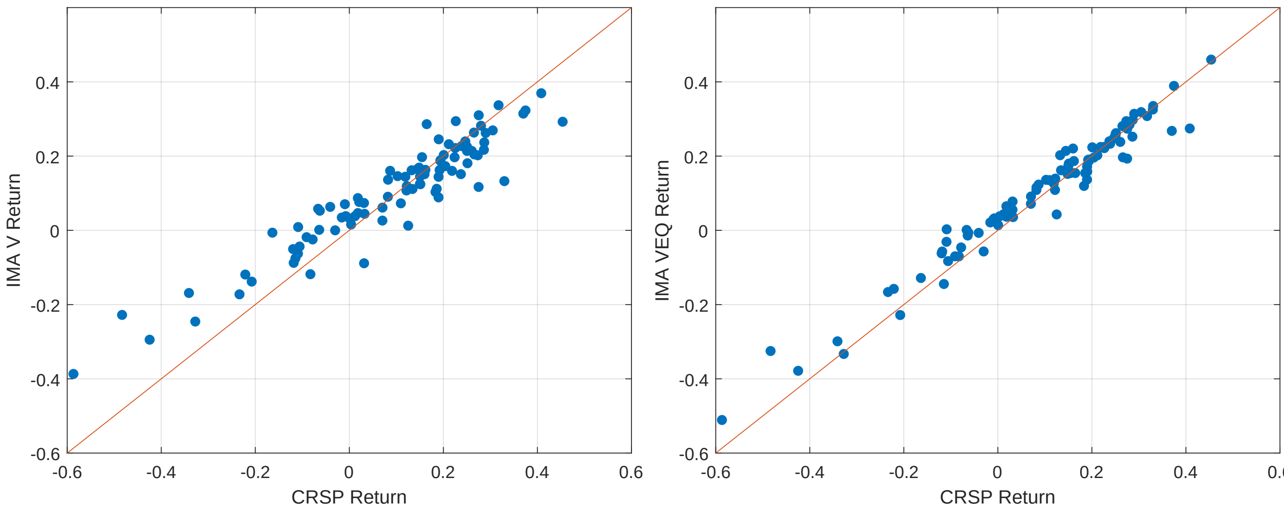

Figure B.3: Left Panel: Realized Returns on Enterprise Value vs. CRSP Value-Weighted Return 1929-2023. Right Panel: Realized Returns on IMA Equity Value vs. CRSP Value-Weighted Return: 1929-2022

Table B.1: Mean and Standard Deviation of Real Log Returns and Log Dividend Growth on Enterprise Value, IMA Equity, and CRSP Value-Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std D growth
Enterprise Value	1929-2023	0.073	0.146	0.280
IMA Equity	1929-2022	0.076	0.173	0.073
CRSP VW	1929-2023	0.062	0.193	0.138

The Integrated Macroeconomic Accounts start with measures of end of year balance sheet items in 1945. In this section, we report statistics computed using only the data from these accounts.

Table B.2: Mean and Standard Deviation of Real Log Returns and Log Dividend Growth on Enterprise Value, IMA Equity, and CRSP Value Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std D growth
Enterprise Value	1946-2022	0.08	0.132	0.279
IMA Equity	1946-2022	0.082	0.15	0.15
CRSP VW	1946-2022	0.069	0.172	0.132

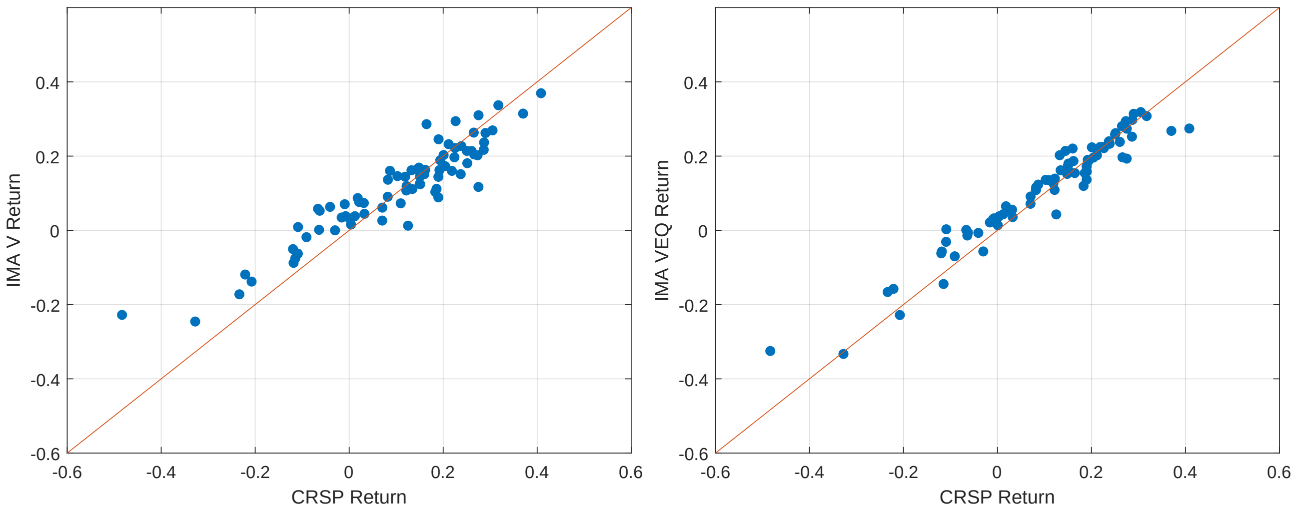

Figure B.4: Left Panel: Realized Returns on Enterprise Value vs. CRSP Value Weighted Return 1946-2023. Right Panel: Realized Returns on IMA Equity Value vs. CRSP Value Weighted Return: 1946-2022

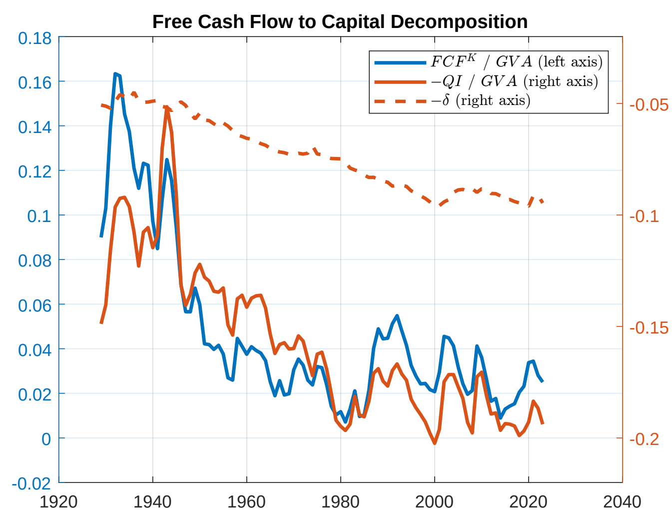

Figure B.5: Free Cash Flow to Capital and the Contribution of Investment.

Modeling the dynamics of the price-dividend spread

$$p^T_{t+s} - p^T_t = \alpha_{p,s} + \gamma_{p,s} p^T_t + \epsilon_{d,t+s} \tag{27}$$

horizon	$s=1$	$s=2$	$s=3$	$s=5$	$s=10$	$s=15$
$\rho^s - 1$	-0.2500	-0.4375	-0.5781	-0.7627	-0.9437	-0.9866
$\hat{\gamma}_{p,s}$	-0.23265	-0.49403	-0.63464	-0.67903	-1.0733	-1.2295
S.E.	(0.066527)	(0.089121)	(0.096301)	(0.097801)	(0.10288)	(0.092135)
t-Stat	-3.497	-5.5433	-6.5902	-6.9429	-10.433	-13.344
R^2	0.117	0.252	0.325	0.354	0.567	0.695

Table B.3: Estimates from price-dividend spread forecasting regressions 27 with $\beta/(1-\beta) = 31.25$. The theoretical values of these coefficients are computed with $\rho = 0.75$.

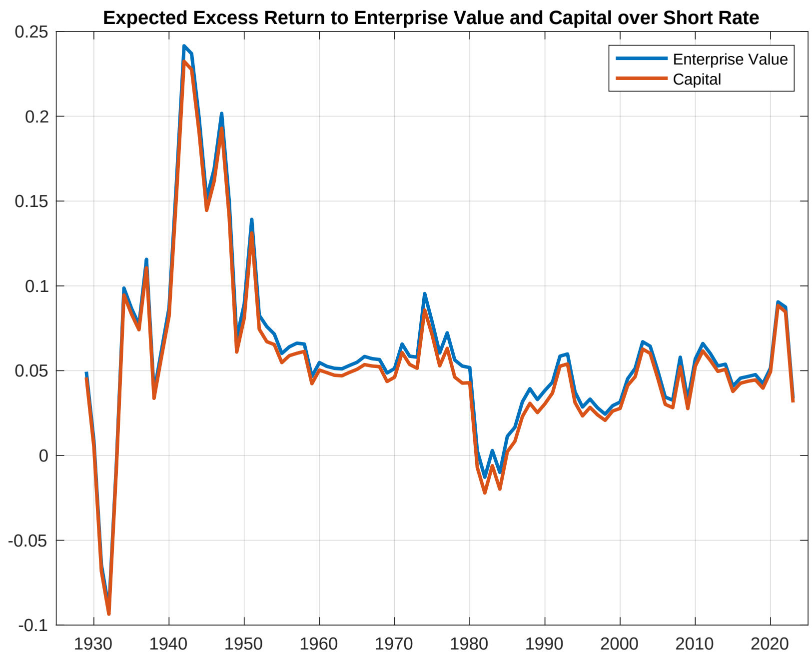

Figure B.6: Expected Returns to Enterprises and to Capital in excess of the one year Treasury rate.

B.1 Drivers of Free Cash Flow

Given our assumed tax structure, total corporate free cash flow is

$$FCF_t = (1-\tau_t^s)GVA_t - W_tL - Q_tI_t - Taxes_t^c. \tag{28}$$

Measuring total free cash flow for the corporate sector using equation (28) is straightforward given the IMA series for gross-value added, compensation, investment, and corporate taxes. Figure B.7 documents the contributions of these different components in accounting for observed changes in total free cash flow. To make the plot easier to read, we measure deviations of each component from their sample average, and filter the series to remove high frequency fluctuations. The message of this figure is that changes in labor's share of income, changes in investment, and changes in corporate taxes are all important drivers of total corporate free cash flow. Note, first, that the well-documented decline in labor's share of value-added in the post 2000 period accounts for essentially all of the observed increase in free cash flow over that period. At the start of the sample period, the key reason cash flow declines is that corporate taxes paid rise. Between the end of World War II and 2000, labor's share of income is relatively stable. Investment rises steadily as a share of value added, reducing free cash flow, but this trend is largely offset by a declining corporate tax burden.

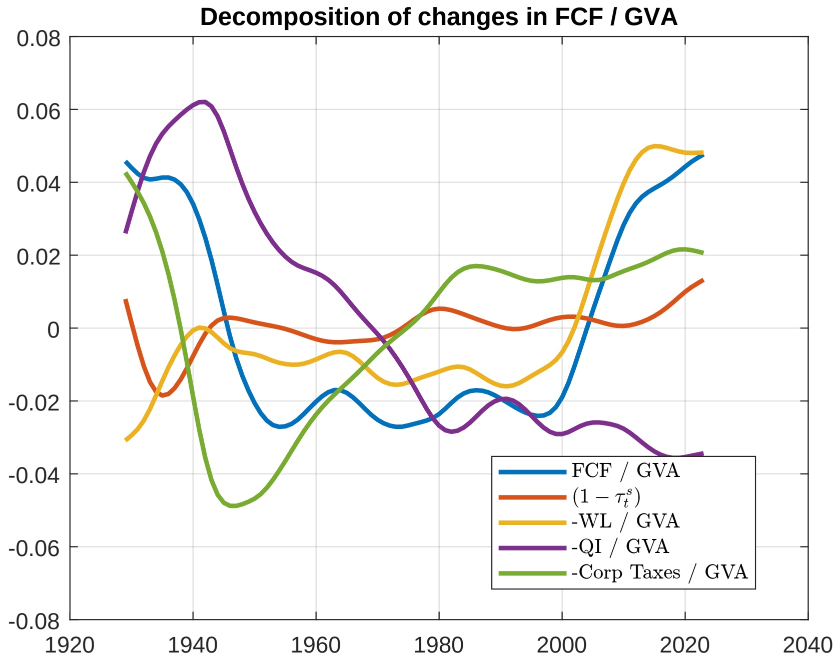

Figure B.7: Decomposition of Total Corporate Free Cash Flow. For each component, we plot the Hodrick-Prescott trend value (smoothing parameter = 100) for the difference of the component from its sample mean.

C Appendix on Effective Tax Rates

Let r_{t+1}^{net} and r_{t+1}^{pretax} denote net real returns to capital between t and $t+1$ including taxes and before taxes:

$$r_{t+1}^{net} = \frac{(1-\tau_{t+1}^c)}{(1-\lambda_t\tau_t^c)}\frac{R_{t+1}^K - Q_{t+1}\delta_{t+1}}{Q_t} + \frac{(1-\lambda_{t+1}\tau_{t+1}^c)}{(1-\lambda_t\tau_t^c)}\frac{Q_{t+1}}{Q_t} - 1,$$

$$r_{t+1}^{pretax} = \frac{R_{t+1}^K - Q_{t+1}\delta_{t+1}}{Q_t} + \frac{Q_{t+1}}{Q_t} - 1.$$

Define the effective tax rate on capital at t as the value for τ_t^e that satisfies

$$\mathbb{E}_t\left[M_{t,t+1}\left(1+(1-\tau_t^e)r_{t+1}^{gross}\right)\right] = \mathbb{E}_t\left[M_{t,t+1}\left(1+r_{t+1}^{net}\right)\right].$$

If we assume zero correlation between returns and the pricing kernel, we have

$$(1-\tau_t^e)\mathbb{E}_t\left[r_{t+1}^{pretax}\right]=\mathbb{E}_t\left[r_{t+1}^{net}\right]$$

and thus

$$\tau_t^e=\frac{\mathbb{E}_t\left[r_{t+1}^{pretax}\right]-\mathbb{E}_t\left[r_{t+1}^{net}\right]}{\mathbb{E}_t\left[r_{t+1}^{pretax}\right]},$$

which is the conventional way the marginal effective tax rate is defined (see, e.g., line 4 in Table 1 of Fullerton 1983).

In computing expected returns, we assume that all the tax parameters (τ_t^s, τ_t^c, τ_t^L and λ_t) are expected to remain unchanged between t and $t+1$.

Then

$$\mathbb{E}_t\left[r_{t+1}^{net}\right] = \frac{(1-\tau_t^c)}{(1-\lambda_t\tau_t^c)}\left(\frac{\mathbb{E}_t\left[\frac{R_{t+1}^K K_{t+1}}{GVA_{t+1}}\frac{GVA_{t+1}}{GVA_t}\right]}{\frac{Q_tK_{t+1}}{GVA_t}}-\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\delta_{t+1}\right]\right)+\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]-1,$$

$$\mathbb{E}_t\left[r_{t+1}^{pretax}\right] = \left(\frac{\mathbb{E}_t\left[\frac{R_{t+1}^K K_{t+1}}{GVA_{t+1}}\frac{GVA_{t+1}}{GVA_t}\right]}{\frac{Q_tK_{t+1}}{GVA_t}}-\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\delta_{t+1}\right]\right)+\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]-1.$$

Plugging these expressions into the formula for the effective tax rate gives

$$\tau_t^e=\frac{\left(\frac{\tau_t^c-\lambda_t\tau_t^c}{1-\lambda_t\tau_t^c}\right)}{1+\frac{\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]-1}{\mathbb{E}_t\left[r_{t+1}^{pretax}\right]+1-\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]}}$$

When the expected gross growth rate for the relative price of investment is equal to one, this expression simplifies to

$$\tau_t^e=\frac{\tau_t^c-\lambda_t\tau_t^c}{1-\lambda_t\tau_t^c}.$$

Note that when $\lambda=0$ (no expensing for net investment), the effective tax rate is equal to the statutory one: $\tau_t^e=\tau_t^c$.

When $\lambda=1$ (full expensing for net investment), the effective tax is zero.

The series for τ_t^e plotted in Figure 5 is constructed assuming that $\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]$ is constant and equal to the average observed value over our sample period. The expected net pre-tax interest rate $\mathbb{E}_t\left[r_{t+1}^{pretax}\right]$ is computed as described in the text.

D Appendix on Value of Capital

The firm that owns and manages the physical capital stock takes as given an initial capital stock K_t and chooses future capital $\{K_{t+k}\}$ and after-tax free cash flow payable to owners $\{FCF_{t+k}^K\}$ for $k\geq 1$ to maximize

$$FCF_t^K+V_t^K$$

where

$$V_t^K = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} FCF_{t+k}^K \right]$$

and

$$FCF_{t+k}^K = (1 - \tau_{t+k}^c) \left(R_{t+k}^K - Q_{t+k}\delta_{t+k} \right) K_{t+k} - (1 - \lambda_{t+k}\tau_{t+k}^c) Q_{t+k} \left(K_{t+k+1} - K_{t+k} \right).$$

The first-order condition with respect to K_{t+k+1} is

$$\begin{aligned} &\mathbb{E}_{t+k} \left[M_{t,t+k+1} \left[(1 - \tau_{t+k+1}^c) \left(R_{t+k+1}^K - Q_{t+k+1}\delta_{t+k+1} \right) + (1 - \lambda_{t+k+1}\tau_{t+k+1}^c) Q_{t+k+1} \right] \right] \\ &= M_{t,t+k} (1 - \lambda_{t+k}\tau_{t+k}^c) Q_{t+k}. \end{aligned} \tag{29}$$

Multiplying through by K_{t+k+1} gives

$$\begin{aligned} &\mathbb{E}_{t+k} \left[M_{t,t+k+1} \left[(1 - \tau_{t+k+1}^c) \left(R_{t+k}^K - Q_{t+k+1}\delta_{t+k+1} \right) K_{t+k+1} + (1 - \lambda_{t+k+1}\tau_{t+k+1}^c) Q_{t+k+1} K_{t+k+1} \right] \right] \\ &= M_{t,t+k} (1 - \lambda_{t+k}\tau_{t+k}^c) Q_{t+k} K_{t+k+1}. \end{aligned} \tag{30}$$

The value of the firm managing the capital stock is

$$V_t^K = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} \left[(1 - \tau_{t+k}^c) \left(R_{t+k}^K - Q_{t+k}\delta_{t+k} \right) K_{t+k} + (1 - \lambda_{t+k}\tau_{t+k}^c) Q_{t+k} K_{t+k} - (1 - \lambda_{t+k}\tau_{t+k}^c) Q_{t+k} K_{t+}\right.\right.$$

Using the equation (30), we can see that the term $-M_{t,t+k}(1-\lambda_{t+k}\tau_{t+k}^c)Q_{t+k}K_{t+k+1}$ cancels with $\mathbb{E}_t \left[M_{t,t+k+1} \left[(1 - \tau_{t+k+1}^c) R_{t+k+1}^K K_{t+k+1} + (1 - \lambda_{t+k+1}\tau_{t+k+1}^c) Q_{t+k+1} (1 - \delta_{t+k+1}) K_{t+k+1} \right] \right]$ and so on moving up through time. The logic is that the value of free cash flow at $t+k$ that is sacrificed to increase next period capital must equal the expected value of the income plus resale value of that capital at $t+k+1$.

The only term that is left is

$$V_t^K = \mathbb{E}_t \left[M_{t,t+1} \left[(1 - \tau_{t+1}^c) \left(R_{t+1}^K - Q_{t+1}\delta_{t+1} \right) K_{t+1} + (1 - \lambda_{t+1}\tau_{t+1}^c) Q_{t+1} K_{t+1} \right] \right]$$

or, using equation (30) again, we have

$$V_t^K = (1 - \lambda_t \tau_t^c) Q_t K_{t+1} \tag{31}$$

Note that if $\lambda_t = 1$ (full expensing) then investment is not distorted, but firm value is depressed by $(1 - \tau_t^c)$.

If $\lambda_t = 0$, then investment and capital depressed when $\tau_t^c > 0$, but the value of the firm is the replacement cost of its capital.

Note that one way to check that this is the right expression for valuation is to show that it gives the correct expression for the after-tax return to capital:

$$\frac{FCF^K_{t+1}+V^K_{t+1}}{V^K_t} = \frac{(1-\tau^c_{t+1})\left(R^K_{t+1}-Q_{t+1}\delta_{t+1}\right)K_{t+1}-(1-\lambda_{t+1}\tau^c_{t+1})Q_{t+1}\left(K_{t+2}-K_{t+1}\right)+(1-\lambda_{t+1}\tau^c_{t+1})Q_{t+1}}{(1-\lambda_t\tau^c_t)Q_tK_{t+1}}$$

$$= \frac{(1-\tau^c_{t+1})\left(R^K_{t+1}-Q_{t+1}\delta_{t+1}\right)K_{t+1}+(1-\lambda_{t+1}\tau^c_{t+1})Q_{t+1}K_{t+1}}{(1-\lambda_t\tau^c_t)Q_tK_{t+1}}$$

$$= \frac{(1-\tau^c_{t+1})\left(\frac{R^K_{t+1}}{Q_t}-\frac{Q_{t+1}}{Q_t}\delta_{t+1}\right)+(1-\lambda_{t+1}\tau^c_{t+1})\frac{Q_{t+1}}{Q_t}}{(1-\lambda_t\tau^c_t)}$$

which is the same as the return expression we derived earlier.