

Work-from-Home and Cities: An Elementary Spatial Model

by

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Abstract

This paper analyzes the urban impacts of hybrid WFH in the simplest possible model, relying on Leontief utility and production functions and other simplifying assumptions. The analysis shows that introduction of WFH raises both the wage and household land consumption (raising worker utility) while shrinking the size of the business district and reducing business land rent. When WFH requires home work-space, the city's overall spatial size increases, with residential rents rising in the suburbs while falling near the center. The decline in business rent and the rotation of the residential rent contour match empirical evidence showing that WFH reduces office-building values and flattens the residential rent gradient.

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1. Introduction

After the shock of the pandemic demonstrated its feasibility, work-from-home (WFH) has surged across the world’s cities. WFH can be hybrid, with work time split between home and office, or (less commonly) fully remote, with the office seldom or never visited by the worker. The emergence of WFH has upended both labor and real-estate markets. Hybrid WFH options make labor contracts much more appealing to workers, inducing many firms to offer them despite concerns about lower productivity. With fewer workers present, employer demands for office space have plummeted, reducing the values of downtown buildings in what Gupta, Mittal and Van Nieuwerburgh (2022) call an “office apocalypse” (see also Grant, 2024). Less-frequent hybrid commutes have made the suburbs more attractive, flattening residential price gradients, and the relocation of fully remote workers has appeared to put downward pressure on housing prices in some cities (Brueckner, Kahn and Lin, 2023; Gupta, Mittal, Peeters and Van Nieuwerburgh, 2022; Bloom and Ramani, 2022). Overall price trends have nevertheless been upward since the pandemic, and some researchers partly attribute this pattern to higher housing demand spurred by the need for work space at home (Mondragon and Wieland, 2022; Stanton and Tiwari, 2021; Gamber, Graham and Yadav, 2023).

WFH has also spurred a theoretical literature designed to portray its vast effects on the spatial structures and other features of cities, and the present paper offers a contribution to this body of work. While the models in this literature (described below) are richly detailed and highly complex, sometimes being tailored to computational exercises, the model in this paper is instead characterized by its extreme simplicity and transparency. The goal is to develop a

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model containing only the essential elements of WFH, dispensing with many details that may be realistic but stand in the way of an elementary analysis that is easily understood, and to see what the model predicts.

The framework is a variant of the standard urban model (Brueckner, 1987; Duranton and Puga, 2015), modified to include a central business district where firms combine land and labor to produce a tradable good. With the office apocalypse being a major empirical consequence of WFH, a model structured to capture its impact on business land rent is desirable. Another of the model’s distinguishing features is the use of Leontief production and utility functions. While use of production inputs in a fixed proportion is familiar, the same feature in consumption is less so. However, the Leontief assumption on preferences is preferable to the more-extreme assumption of fixed land consumption, which is used in many urban models (including two of the WFH papers discussed below). Even though proportions are fixed, land consumption then remains endogenous and potentially impacted by WFH. In addition, the Leontief assumptions yield analytical tractability, which disappears, for example, if preferences take the familiar Cobb-Douglas form (making numerical analysis necessary).

Under a hybrid WFH arrangement, the frequency of commute trips in the model falls by the exogenous factor $\alpha < 1$, which lowers commuting cost per period. Business land usage falls in step, since fewer workers need to be accommodated. In a second version of the model, WFH also creates a need for work space at home. The analysis derives the impact of the α parameter on the endogenous variables of the model: household land consumption, the wage, and the distance to the edge of the city. The wage solution then determines business rent and the wage and land-consumption solutions together determine residential rent.

Two of the equilibrium conditions are standard, requiring that the city fit its fixed population and that residential land rent equals land’s opportunity cost at the city’s edge. The third condition requires equality between residential and business rents at the edge of the business district. Given the simplicity of the model, closed-form solutions emerge. They show that introduction of WFH raises both the wage and land consumption of households (raising utility) while shrinking the size of the business district and reducing business land rent. When WFH requires home work-space, the city’s overall spatial size increases, with residential rents rising

in the suburbs while falling near the center. While the shrinkage of the business district is a long-run effect that has yet to fully play out in US central cities, the decline in business rent and the rotation of the residential rent contour match empirical evidence showing that WFH reduces office-building values and flattens the residential rent gradient.

Previous theoretical papers analyzing the impacts of hybrid WFH include Delventhal, Kwon and Parkhomenko (DKP, 2022), Kyriakopoulou and Picard (KP, 2023), Behrens, Kichko and Thisse (BKT, 2024), Davis, Ghent and Gregory (DGG, 2024), Delventhal and Parkhomenko (DP, 2022), and Gokan, Kichko, Matheson and Thisse (GKT, 2024), with the last two as yet unpublished.¹ DKP, DGG, and DP present computational models, while the remaining papers are analytical. All these papers develop models that are much richer and more detailed than the present framework. For example, all of the models except for KP have two types of workers with different skills, and some (GKT, DGG) have two types of jobs. For example, nonremote unskilled workers in GKT provide services (e.g., restaurants) in the central city, with the model capturing negative impacts in this sector when office workers stay home, a concern frequently discussed in the media. Unlike in the basic version of the current model, the productivity of work onsite and at home differ in all of the models (constituting distinct inputs in some papers), and the share of work time spent at home is chosen endogenously by workers in DKP and DP, while otherwise being exogenous (as in the present model). In some models (DKP, DP, BKT), workers choose the amount of home work-space, which is instead set exogenously in the present model. In DGG’s model, commuters even experience traffic congestion.

Despite many differences, the model of Kyriakopoulou and Picard (2023) is perhaps closest to the current one. In the model, a given firm’s output depends on interaction with firms at other locations, which attenuates with distance and rises with the share of workers present in the office. As in the current model, firms also use land in production, although consumer land consumption is fixed at unity (as in GKT) rather than being endogenous, with an incremental amount devoted to home office space. KP focus partly on the land-rent effects of WFH, and

¹ By contrast, instead of focusing on hybrid WFH, Brueckner, Kahn and Lin (2023) and Brueckner and Sayantani (2023) analyze the effect of fully remote WFH, in which workers can relocate to other cities while keeping their original jobs. Multiple cities also play a role in the WFH model of Lee (2024).

they derive results somewhat similar to the present ones, as explained further below. Overall, however, their analytics are considerably more complex than those in the current paper, possibly limiting the accessibility of their work for some readers.

The plan of the paper is as follows. To orient readers, section 2 develops the model in the absence of WFH, when $\alpha = 1$. Section 3 adds WFH to the basic model without including a home work-space requirement, while section 4 includes this requirement. Section 5 presents a robustness analysis, asking how the details of WFH scheduling, a worker-productivity difference between home and office, or unequal Leontief coefficients affect the results of the analysis. Section 6 offers conclusions.

2. The model without WFH

As explained above, the model is based on Leontief utility and production functions.² Utility depends on consumption of land, denoted q , which is a proxy for housing consumption, and nonland consumption, denoted e . The good e is a tradable commodity produced in all cities, and its price is normalized to unity. As in the standard model, all urban residents have the same utility function, and with Leontief preferences, that common function is

$$U(e, q) = \min\{e, q\}. \quad (1)$$

Although, with unitary coefficients on both goods, the corners of the right-angled Leontief indifference curves lie on the 45 degree line, the case of unequal coefficients is considered in section 5 below.

Letting r denote residential rent per unit of land, which accrues to absentee landowners, the consumer budget constraint is given by

$$e + rq = w - td, \quad (2)$$

where w is the wage per period, d is commuting distance, and t is round-trip money cost of commuting per mile traveled. Utility maximization yields $e = q$, with consumption occurring

² For another use of Leontief preferences in an urban model, see Brueckner (2005).

at an indifference-curve corner. Eliminating e from (2) using this equality and then solving for land rent yields

$$r = \frac{w - td}{q} - 1. \quad (3)$$

Eq. (3) gives $\partial r / \partial d = -t/q$, as in the case of general preferences, but the difference with Leontief preferences is that q is spatially constant across the city rather than increasing in d , as in the standard urban model. In addition, note that q under Leontief preferences measures utility, a fact that is crucial below in deriving the welfare impact of WFH.

The city is closed to migration, and for simplicity, it is assumed to be linear with unitary width. The center lies at one end, with distance to the center denoted x , and the other half of the city is suppressed without loss of generality. The business district, where production of the nonland good occurs, occupies the central portion of the city, ranging from $x = 0$ to $x = \hat{x}$, with $\hat{x}$ determined below.

Travel costs within the business district are assumed to be zero, so that the land is spatially undifferentiated, with business rent equal to a constant R . While some of district's output of the tradable nonland good will be consumed locally, any amount exported can then be shipped to the city center (where the export node is located) at zero cost, making the location of production within the district immaterial. Since travel cost within the business district is zero, workers commute to its outer edge, then reaching their nearby worksites at zero cost.³

The inputs into production of the tradable nonland good are labor, denoted H , and land, denoted L . Since constant returns to scale prevails with Leontief production, output can be viewed as coming from a single large firm that behaves competitively. Letting E denote output, the production function can be written as

$$E = \min\{H, L\}, \quad (4)$$

assuming equal labor and land coefficients. The case of unequal coefficients is considered below once the basic results are derived.

³ Since the consumption good e is produced in the business district, workers can be viewed as picking it up during the work trip, for consumption at home.

In the absence of WFH, cost minimization by the central firm leads to choice of a corner on one of the right-angled isoquants, where $H = L$. The firm's zero-profit condition is then given by

$$1 \times E - wH - RL = H - wH - RH = 0, \quad (5)$$

recalling the unitary output price and noting $E = H$ from (4). Cancelling H and using (5) to solve for the business R yields

$$R = 1 - w. \quad (6)$$

Labor supply by the city's workers is assumed to be perfectly inelastic. Therefore, the entire city population N is employed, with $H = N$. The land area L used in production is then equal to N , so that the distance to the edge of the business district is $\hat{x} = N$ (recall the city's unit width). Since workers commute to the outer edge of the business district, their commuting cost from distance x is $t(x - \hat{x}) = t(x - N)$.

Note that, while the wage is determined partly by the firm's zero-profit condition (written as (6)), other factors also enter. This dependence can be seen by writing the three equilibrium conditions for the city, using the assumption of absentee land ownership. These conditions require (i) equality between residential rent r and business rent R from (3) and (6) at the edge $\hat{x} = N$ of the business district; (ii) equality between the opportunity cost of land (set at zero) and residential rent at $\bar{x}$, the distance to the edge of the city; (iii) equality between total residential land consumption and the city's residential area, with the latter equal to $\bar{x} - \hat{x} = \bar{x} - N$. These conditions are written, respectively, as

$$\frac{w}{q} - 1 = 1 - w \quad (7)$$

$$\frac{w - t(\bar{x} - N)}{q} - 1 = 0 \quad (8)$$

$$Nq = \bar{x} - N. \quad (9)$$

Note that residential rent at the edge $\hat{x} = N$ of the business district on the LHS of (7) reflects zero commuting cost. In (8), the $r = 0$ condition based on (3) reflects a commuting distance

of $\bar{x} - N$ at the edge of the city. Also, Nq in (9) gives total residential land consumption. Note that the competition for land between the central firm and residences helps determine the wage via (7).

The unknowns in (7)–(9) are w , q , and $\bar{x}$. To solve the system, $\bar{x}$ is eliminated using (8) and (9) to yield $w = (1 + tN)q$. After substituting for w on the LHS of (7), q cancels, yielding

$$w = 1 - tN. \quad (10)$$

Substituting w into the equation prior to (10) and solving for q then yields

$$q = \frac{1 - tN}{1 + tN}. \quad (11)$$

Substituting (11) into (9) to find $\bar{x}$ yields

$$\bar{x} = \frac{2N}{1 + tN}. \quad (12)$$

Business rent R is not an explicit unknown in (7)–(9), but it can be recovered from (6) and (10), yielding $R = tN$, so that w and R sum to the unitary output price, as required.

Simple comparative statics can be checked against those of the standard urban model. As in that model, inspection of (11) shows that land consumption q falls as population N increases. Differentiating (12) shows that $\bar{x}$ rises with N in standard fashion. The wage is usually treated as fixed in the standard model, but (10) shows that w in the present model falls as N increases. Finally, business rent R rises with N .⁴

3. Adding WFH

Under a hybrid WFH arrangement, workers commute to the office for part of the period and work from home for the remaining portion, with the split determined exogenously by

⁴ Other standard comparative-static effects come from an increase in the commuting-cost parameter t . From (12), $\bar{x}$ falls when t rises, as in the standard model, while the spatially uniform q also declines from (11) (changes in q depend on location in the standard model). A higher t also reduces the wage from (10) while raising R .

firms. Importantly, workers are assumed to have the same productivity regardless of where the work occurs, so that labor need not be decomposed into office and home components. This assumption is relaxed in a robustness check in section 5 below. Let the fraction of days that workers are present at their central worksite be denoted $\alpha < 1$. Then, commuting cost from a distance x is reduced to αtx from tx , lowering worker outlays on this important expense.

If α is defined on the basis of a 5-day work week, then it would take the values $4/5$, $3/5$, $2/5$, or $1/5$ depending on whether the number of office days equals 4, 3, 2, or 1. Alternately, α could be defined on an annual basis, in which case it could assume a broad range of fractional values. In line with this view, α is treated as a continuous variable in the ensuing analysis of WFH effects.

In the main part of the analysis, the firm is assumed to set individual WFH schedules so that the number of workers present in the office is the same on each day, with that number then equal to αN . Since fewer workers are present, the central firm requires less land for its operations than before. Land usage falls from $L = N$ to $L = \alpha N$, declining in step with the proportional reduction in workers present. In other words, while fixed proportions ($N/L = 1$) originally applied to all workers, it applies only to those present in the office under WFH ($\alpha N/L = 1$). An alternative assumption is that the firm, seeking to preserve some degree of employee interaction, requires all workers to be present in the office on the same day. In this case, the required land area on that day equals N rather than αN , with the land unused (offices empty) on the remaining days. The robustness analysis in section 5 explores the effects of this alternate assumption. Note finally that the reduction in business land usage under the first assumption reflects the long-run nature of the model, where land usage (more generally, space in office buildings and factories) is fully adjustable.

In working from home, workers use a portion of their living space (instead of business land) in generating output, with no loss in consumption utility from the space. The next section alters this setup by assuming that WFH requires additional home work space, with the amount increasing as α declines.

The firm's zero profit condition is now $N - wN - R(\alpha N) = 0$, indicating that while it continues to pay all its workers the wage w , land cost falls from RN to αRN . Solving for R

yields

$$R = \frac{1 - w}{\alpha}. \quad (13)$$

The city's equilibrium conditions under WFH are

$$\frac{w}{q} - 1 = \frac{1 - w}{\alpha} \quad (14)$$

$$\frac{w - \alpha t(\bar{x} - \alpha N)}{q} - 1 = 0 \quad (15)$$

$$Nq = \bar{x} - \alpha N. \quad (16)$$

In (14), the new R from (13) appears on the RHS. Commuting cost at $\bar{x}$ in the boundary-rent condition (15) is now $\alpha t(\bar{x} - \alpha N)$, and the residential land area in (16) is now $\bar{x} - \alpha N$.

Following the same steps as before, the solutions to (14)-(16) are

$$w = 1 - \alpha^2 t N \quad (17)$$

$$q = \frac{1 - \alpha^2 t N}{1 + \alpha t N} \quad (18)$$

$$\bar{x} = \frac{(1 + \alpha)N}{1 + \alpha t N}. \quad (19)$$

Using (13) and (17), business land rent is given by

$$R = \alpha t N. \quad (20)$$

The qualitative comparative-static effects of N and t on the endogenous variables remain the same as in the absence of WFH. More important are the effects of α , which is an inverse measure of the extent of WFH. Introduction of WFH corresponds to a decrease in α from a value of 1 to some smaller value, and its effect can be gauged by evaluating the α derivatives in (17)–(20). Inspection of (17), (18) and (20) shows that the wage w and land consumption q are

decreasing in α and that business rent R is increasing in α . In addition, since the α -derivative of (19) has the sign of $N(1 - tN)$, which is positive from (10), the boundary distance $\bar{x}$ is decreasing in α . Since introduction of WFH corresponds to a decrease in α starting from 1, the following conclusions emerge:

Proposition 1. *With no home work-space requirement, introduction of WFH raises the wage w and land consumption q while reducing business rent R and the boundary distance $\bar{x}$. Given Leontief preferences, the increase in q implies higher worker utility under WFH.*

Note that the decline in $\bar{x}$ shows that the reduction under WFH in the business district's land area (which falls from N to αN) dominates WFH's positive effect on individual land consumption, leading to a smaller overall urban land area. Note also that the decline in business rent matches the predicted drop in office-building values from Gupta, Mittal and Van Nieuwerburgh (2022).

The higher q under WFH arises partly through a higher wage. But changes in residential rent also have an effect, as follows. Since business rent declines, residential rent falls at the edge of the business district ($\hat{x} = \alpha N$), where two rents are equal. After inserting an α before $td = t(x - \alpha N)$ in (3), the slope of the residential rent function ($\partial r / \partial x$) equals $-\alpha t / q$, which can be written using (18) as

$$\frac{\partial r}{\partial x} = -\frac{\alpha t(1 + \alpha tN)}{1 - \alpha^2 tN}. \quad (21)$$

Since (21) becomes less negative as α decreases, it follows that residential rent falls more slowly over distance when WFH is introduced, while starting from a lower value at $\hat{x}$. Empirical evidence in Gupta, Mittal, Peeters and Van Nieuwerburgh (2022) and Brueckner, Kahn and Lin (2023) confirms this predicted decline in the residential rent gradient.

The changes in residential rent farther from the city center are yet to be deduced, but they can be inferred from the decline in $\bar{x}$ under WFH. With $\bar{x}$ falling, it follows that, despite the flatter rent curve, residential rents never rise above their initial levels anywhere in the city, falling everywhere with the introduction of WFH, an effect that may help to spur the increase in q . This pattern can be seen in Figure 1, where the pre- and post-WFH cases are denoted

by the subscripts 0 and WFH on r , R and $\hat{x}$.⁵ The asterisks refer to outcomes analyzed in the next section. Summarizing the land rent impacts of WFH yields

Corollary 1. *With no home work-space requirement, introduction of WFH reduces the residential rent gradient while reducing the level of land rents throughout the city, in both its residential and business areas.*

Proposition 1 is partly altered once a realistic home work-space requirement is introduced, as seen in the next section.

It is important to more fully identify the ultimate sources of lower business rent and the higher wage under WFH. It appears that the root of these changes lies in the flattening of the residential rent gradient. Although this flattening is partly endogenous (being affected by q), the drop in commuting cost per mile from t to αt tends to depress the gradient in an exogenous fashion. But a lower gradient, which reflects the greater attractiveness of the suburbs relative to more-central locations under WFH, in turn tends to put downward pressure on residential rents nearer the city center. That downward pressure then lessens competition for central locations from residences, allowing firms to pay less for land in the business district. With lower business land rent, firms can then compete more vigorously for workers while still earning zero profit, pushing up the wage. This wage effect reinforces the decline in rents in generating a higher q and thus higher utility.

Finally, it is interesting to consider the limiting case of $\alpha = 0$, which corresponds to fully remote work, with a zero fraction of work time spent at the office. Referring to (17)-(20), $\alpha = 0$ yields $w = 1$, $q = 1$ (implying $e = 1$), $\bar{x} = N$, and $R = 0$. Residential land rent is also zero, as can be seen by replacing td in (3) with $\alpha td = 0$ and using $w = q$. In addition, the business district collapses to a point at $x = 0$ since no production land is needed. Given that q is maximal when $\alpha = 0$, workers are best off with fully remote work, which makes sense since commuting cost is then absent. However, this case is unrealistic given most employers' insistence that some work time must be spent in the office.

⁵ Figure 1 is drawn to just illustrate qualitative effects, not being based on particular values of the parameters α , N and t .

4. WFH with a home work-space requirement

Following the evidence of a post-pandemic surge in housing demand (cited in the introduction), suppose now that a home work-space requirement is realistically added to the model. While workers derive utility from land consumption of q , they must now purchase βq worth of land, where $\beta > 1$, to have extra space for working at home. As a result, the previous budget constraint (2) becomes

$$e + r\beta q = w - td. \quad (22)$$

But since utility is still given by $\min\{e, q\}$, q is again substituted in place of e in (22), so that the land-rent expression in (3) changes to

$$r = \frac{1}{\beta} \left(\frac{w - td}{q} - 1 \right). \quad (23)$$

Again substituting $x - \alpha N$ for d in (23), the urban equilibrium conditions now become

$$\frac{1}{\beta} \left(\frac{w}{q} - 1 \right) = \frac{1 - w}{\alpha} \quad (24)$$

$$\frac{w - \alpha t(\bar{x} - \alpha N)}{q} - 1 = 0 \quad (25)$$

$$N\beta q = \bar{x} - \alpha N. \quad (26)$$

Note in (25) that β cancels when residential rent at the edge of the city is set equal to zero, and that residential land consumption in (26) becomes $N\beta q$.

The solutions to (24)–(26) are

$$w = 1 - \alpha^2 t N \quad (27)$$

$$q = \frac{1 - \alpha^2 t N}{1 + \alpha \beta t N} \quad (28)$$

$$\bar{x} = \frac{(\alpha + \beta)N}{1 + \alpha \beta t N}. \quad (29)$$

R again equals αtN . For a given $\alpha < 1$, the introduction of a home work-space requirement, with $\beta > 1$ instead $\beta = 1$, leaves the wage and business rent unchanged, reduces q , and (from differentiation of (29)) raises $\bar{x}$.

However, α and β will change together with the introduction of WFH, falling and rising respectively from starting values of 1. To gauge the joint impacts of these changes in α and β , their magnitudes must be tied in some fashion. Although a more-general assumption could be made (see the appendix), one way to tie these magnitudes is to assume that the changes are mirror images of one another, with a decrease in α requiring an equal increase in β .⁶ This assumption, along with requirement that $\beta = 1$ when $\alpha = 1$, implies $\beta = 2 - \alpha$. While (27) is unchanged, substituting this relationship into (28) and (29) reduces these solutions to

$$q = \frac{1 - \alpha^2 tN}{1 + \alpha(2 - \alpha)tN} \quad (30)$$

$$\bar{x} = \frac{2N}{1 + \alpha(2 - \alpha)tN}. \quad (31)$$

Since $\alpha(2 - \alpha)$ is increasing in α (its derivative equals $2(1 - \alpha) \geq 0$), it follows that the denominators of (30) and (31) both decrease as α declines. With the numerators of (30) and (31) rising and constant, respectively, as α declines, it follows that the introduction of WFH raises both q and $\bar{x}$, with the q increase again implying higher utility. Observe, however, that since the denominator of q in (18) is smaller than the denominator in (30) given $\alpha < \alpha(2 - \alpha)$, q is larger without a home-work space requirement than with one, making utility lower in the latter case. Summarizing yields

Proposition 2. *With a home work-space requirement, the introduction of WFH again raises the wage w and land consumption q and reduces business land rent R , but the boundary distance $\bar{x}$ now increases. Worker utility again rises under WFH, but not by as much as in the absence of a home work-space requirement.*

The increase in $\bar{x}$ makes sense given that the need for added work space means that an individual's total land consumption, given by $\beta q = (2 - \alpha)q$ tends to rise more rapidly than before

⁶ Alternatively, WFH could require the addition of a fixed amount of home office space, which is independent of α , to residential space q . However, assuming that office space rises as α falls seems natural, while the fixed-space assumption would in addition prevent comparative-static analysis of α 's effects.

as α declines, now more than offsetting the drop in the area of the business district. But the home work-space requirement makes the increase in q itself smaller than in the absence of this requirement, making the utility gain from WFH smaller.

Residential land rent once again declines in step with business rent at $\hat{x}$, and the rent curve again flattens, with the numerator of (21) becoming $\alpha t(1 + \alpha(2 - \alpha))tN$, which decreases as α falls while the denominator rises. However, since $\bar{x}$ now increases, it follows that residential rents rise in the outer part of the city, as seen in Figure 1, where the r curve and $\bar{x}$ value with WFH subscripts and asterisks are revelant. The rent curve thus rotates around some central point. Summarizing yields⁷

Corollary 2. *With a home work-space requirement, introduction of WFH again reduces the residential rent gradient and business land rent, but residential rent now rises in the outer part of the city while again falling near the business district.*

Kyriakopoulou and Picard (2023) derive related results showing WFH’s land-rent effects.⁸

5. Robustness analysis

This section asks how relaxation of three of the maintained assumptions affects the results of the analysis, focusing on the home work-space case. First, instead of setting WFH schedules to equalize office worker presence on each day, the firm is assumed to schedule workers to be in the office on the same days, leaving the space otherwise vacant. Second, the assumption of equal worker productivities at home and in the office is relaxed. Third, the assumption of unitary and thus equal coefficients in the Leontief utility and production functions is relaxed.

⁷ While evidence supports the land-rent effects in Corollary 2, testing the prediction of a higher wage under WFH poses a challenge since it would be hard to disentangle wage increases induced by the recent inflation from any WFH-related changes.

⁸ They show that residential rent rises everywhere (or rotates as in Figure 1) as $\gamma < (>) 1 - \beta_s$, where γ is the amount of home office space and $1 - \beta_s$ is the savings in firm office space under WFH (β_s is WFH space usage). Business land rent falls if $\beta_t \leq \beta_s$, where β_t is the fractional reduction in commute trips under WFH (in the present model, $\beta_t = \beta_s = \alpha$). While these land-rent results are similar to the present ones, the wage impacts of WFH are not comparable between the models. In KP’s model, the wage paid by a firm depends on its location in the business area (which affects interfirm interactions), and KP’s result is that WFH raises (reduces) the average wage when the city’s firm population is large (small). With consumer land consumption fixed, WFH effects are absent in their model.

5.1. All workers present on the same days

Instead of WFH schedules being set so that αN workers are present each day, suppose instead that all workers go to the office on the same days. As explained in section 3, the number of workers thus equals N on these days, so that the business land area under WFH is the same as in its absence, equal to N rather than αN , with the land unoccupied on the other days. Under this change, the α on the RHS of the equilibrium (24) is replaced by 1, and αN in the conditions (25) and (26) is replaced by N . The solutions are then

$$w = 1 - \alpha t N \quad (32)$$

$$q = \frac{1 - \alpha t N}{1 + \alpha(2 - \alpha)t N} \quad (33)$$

$$\bar{x} = \frac{(3 - \alpha)N}{1 + \alpha(2 - \alpha)t N}, \quad (34)$$

with R again equal to $\alpha t N$. The RHS of (32) and the numerator of (33) again increase as α falls, as does the numerator of (34), with the unchanged denominator expressions rising as before. The following conclusions can then be stated:

Proposition 3. *With a home work-space requirement and all workers present in the office on the same days, the introduction of WFH has the same qualitative effects as when business occupancy is equalized. WFH raises the wage w and land consumption q and reduces business land rent R , and the boundary distance $\bar{x}$ increases. The wage and land consumption do not rise by as much, while $\bar{x}$ increases by more, than when business occupancy is equalized.*

Observe that the proposition's last statement regarding w and q follows from comparing (32) and (33) to (27) and (30) while noting that $\alpha > \alpha^2$ holds given $\alpha > 1$. Also, the $\bar{x}$ claim follows because $3 - \alpha$ in the numerator of (34) exceeds the 2 in (31). Observe also that a larger $\bar{x}$ emerges because the smaller increase in q is more than offset by the lack of shrinkage of the business area. Note finally that, while the decline in business land rent is the same as before, the unchanged business land area means that business competition for residential land is stronger than before, leading to a smaller increase in q and a smaller wage gain.

5.2. The effect of differential office and home productivities

Instead of equal productivities, suppose that worker productivity differs between the home and office, with office productivity equal to 1 and home productivity equal to δ , where δ can be greater or less than 1. Average productivity is then equal to $p(\alpha) \equiv \alpha + (1 - \alpha)\delta = (1 - \delta)\alpha + \delta$, with $p'(\alpha) > (<) 0$ as $\delta < (>) 1$. Thus, average productivity rises as α declines under WFH if home productivity is greater than office productivity (if $\delta > 1$), with the average falling otherwise.⁹ Evidence cited in Barrero, Bloom and Davis (2021) on this issue is mixed, with some studies showing higher home productivity and others showing the reverse.¹⁰

The firm's zero profit condition (in the paragraph just before (13)) is rewritten as $p(\alpha)N - wN - R\alpha N$, with output now equal to $p(\alpha)N$ while $L = \alpha N$ still holds.¹¹ Solving for R and rewriting the first WFH equilibrium condition in (24), the 1 on the RHS is replaced by $p(\alpha)$, while the other equilibrium conditions remain the same. With home work space and $\beta = 2 - \alpha$, the solutions are given by

$$w = p(\alpha) - \alpha^2 t N \quad (35)$$

$$q = \frac{p(\alpha) - \alpha^2 t N}{1 + \alpha(2 - \alpha)t N} \quad (36)$$

$$\bar{x} = \frac{[\alpha + (2 - \alpha)p(\alpha)]N}{1 + \alpha(2 - \alpha)t N}. \quad (37)$$

In order for the results of Proposition 2 to still apply unambiguously, $p' \leq 0$ must hold, indicating equal or higher productivity at home than at the office. The productivity effect

⁹ Agglomeration economies, where worker productivity increases as the size of the workforce or employment density increases, are not explicitly present in the model. But productivity that is higher in the office than at home, perhaps due to face-to-face interaction among workers, can be viewed as capturing an agglomeration effect.

¹⁰ While Bloom, Liang, Roberts and Ying (2015) found higher WFH productivity among call-center workers before the pandemic, Harrington and Emanuel (2024) instead found lower productivity in call centers using post-pandemic data. However, based on recent Chinese survey data, Bloom, Han and Liang (2024) argue that WFH does not impair worker productivity.

¹¹ When productivity per worker changes from 1 to some value $\omega \neq 1$ under Leontief production, the optimal input mix then satisfies $\omega N/L = 1$. In the current case, however, the αN workers that are present in the office still have productivity equal to 1 even though home productivity is now different. Therefore, the factor mix in office production remains the same as before, given by $\alpha N/L = 1$, which again yields land usage of αN .

present in the solutions described above then raises (or leaves unaffected) both w and q as α falls, amplifying the positive α effects operating through the other terms in the solutions. In addition, the numerator of the $\bar{x}$ solution is constant or decreasing in α when $p'(\alpha) \leq 0$.¹² Since the denominator is again increasing in α , a reduction in α raises $\bar{x}$ as before. However, when $p' > 0$, a reduction in α has an ambiguous effect on all the solutions, unless this productivity effect is sufficiently close to zero. Summarizing yields

Proposition 4. *With a home work-space requirement and productivity higher at home than at the office, the results in Proposition 2 continue to hold. When home productivity is lower but close in magnitude to office productivity, this conclusion is unaffected, but otherwise the effects of WFH on the variables of the model are all ambiguous.*

5.3. The effect of unequal Leontief coefficients

Another robustness check is to relax the assumption that the Leontief coefficients are equal in the utility and production functions. This alteration is less significant than the ones just investigated, but it is useful nevertheless to know how it affects the impact of WFH. To address this question, suppose that the utility and production functions are now $U(e, q) = \min\{e, \lambda q\}$ and $E = \min\{H, \theta L\}$. Then, utility and cost minimization yield $e = \lambda q$ and $H = \theta L$, making the assumptions $\lambda, \theta > 1$ natural. With these inequalities, nonland consumption exceeds land consumption and firms employ many workers per unit of land. Despite this view, λ and θ are unrestricted in magnitude.

Steps similar to the previous ones yield a new set of equilibrium conditions under WFH. Land usage in the business district becomes $L = (\alpha/\theta)N$, an expression that replaces αN in the equilibrium conditions (24) and (25). In addition, $-\lambda$ replaces the -1 in (24) while multiplying q on the RHS of (25). Finally, θ multiplies the RHS of (23). The new set of solutions is then

$$w = 1 - (\alpha^2/\theta)tN \quad (38)$$

$$q = \frac{1 - (\alpha^2/\theta)tN}{\lambda + \alpha(2 - \alpha)tN} \quad (39)$$

¹² The derivative is $1 - p + (2 - \alpha)p'$. Since $p' \leq 0$ implies $1 - p \leq 0$, this expression is zero or negative.

$$\bar{x} = \frac{[2 + \alpha(\lambda/\theta - 1)]N}{1 + \alpha(2 - \alpha)tN}. \quad (40)$$

From (38 and (39), the previous conclusion that w and q are decreasing in α (thus rising with the introduction of WFH) remain unchanged, while R again increases with α .¹³ However, the effect of α on the numerator of (40) is unclear, making the effect on $\bar{x}$ ambiguous in general. But if $\lambda < \theta$, then the factor multiplying α is negative and the numerator is decreasing in α . With the denominator increasing as before, $\bar{x}$ is then decreasing in α , yielding

Proposition 5. *With a home work-space requirement and unequal Leontief coefficients, the introduction of WFH again raises the wage w and land consumption q and reduces business land rent R . The boundary distance $\bar{x}$ increases if $\lambda < \theta$, or if a household's nonland consumption per unit of land is less than employment per unit of land, but changes in an ambiguous direction otherwise.*

Recall that the negativity of WFH's $\bar{x}$ effect was overturned once a home work-space requirement was added to the model, in which case the expansion of the city's residential area was sufficient to offset the shrinkage of the business district. In order for this offset to persist with unequal Leontief coefficients, the land intensity of the consumption bundle (the reciprocal of λ) must be larger than the land intensity of production (the reciprocal of θ), or $\lambda < \theta$, which is a natural conclusion.¹⁴

6. Conclusion

This paper has analyzed the urban impacts of hybrid WFH in the simplest possible model, relying on Leontief utility and production functions and other simplifying assumptions. The

¹³ Note that since an increase in λ , the nonland/land ratio in consumption, reduces q , it steepens the rent gradient, using (3).

¹⁴ In other robustness checks, WFH effects on w and q are ambiguous when $\bar{x}$ is fixed (a possible short-run assumption, even though q and business land-use remain flexible), while a non-zero opportunity cost of land (r_a) leaves the results of Proposition 2 unchanged. In this case, $-\alpha r_a/\beta$ is subtracted from the w solution in (27) and from the numerator of (28), while r_a is added to the denominators of (28) and (29). Another possible robustness check would involve replacement of absentee landownership with internal ownership, where total urban rent is equally redistributed among the residents (the city is then fully closed). To carry out such an analysis, an expression for total rent per capita, denoted I , is required. However, I itself also appears on the RHS of the equality defining I , since rent depends on income, now equal to $w + I$. But with Leontief preferences, I cancels from both sides of this equation, so that a fully closed city cannot be analyzed, which in turn prevents an analysis of the efficiency of the WFH equilibrium using this approach.

analysis shows that introduction of WFH raises both the wage and household land consumption (raising utility) while shrinking the size of the business district and reducing business land rent. When WFH requires home work-space, the city’s overall spatial size increases, with residential rents rising in the suburbs while falling near the center. The model’s predicted rent impacts match empirical evidence from the WFH literature.

The decline in business rent and residential rents near the business district would generalize, in a more fully developed model, to a drop in both office and residential property values in the central part of the city. If this central-city area constitutes a separate taxing jurisdiction that provides its own public goods, with the suburbs being a separate jurisdiction, then the drop in values will reduce central-city property-tax revenue, creating a fiscal deficit when the tax rate and public-good levels are held constant. As a result, either the tax rate must rise or public spending must fall, both of which reduce the central city’s attractiveness to businesses and residents. This scenario corresponds to the “urban doom loop” sketched by Gupta, Mittal and Van Nieuwerburgh (2022) in their analysis of the impact of WFH on the office market. Proper analysis of this doom loop lies beyond the scope of this paper (public goods must then be added to production and utility functions), but the topic is an important one that has so far escaped formal treatment in the WFH literature.

The analysis in this paper complements previous theoretical WFH research, which relies on rich, complex models that may be inaccessible to some readers interested in the WFH topic. It is hoped that this paper, through its simplicity and therefore its pedagogical value, will provide additional illumination for a wider audience on how WFH affects cities.

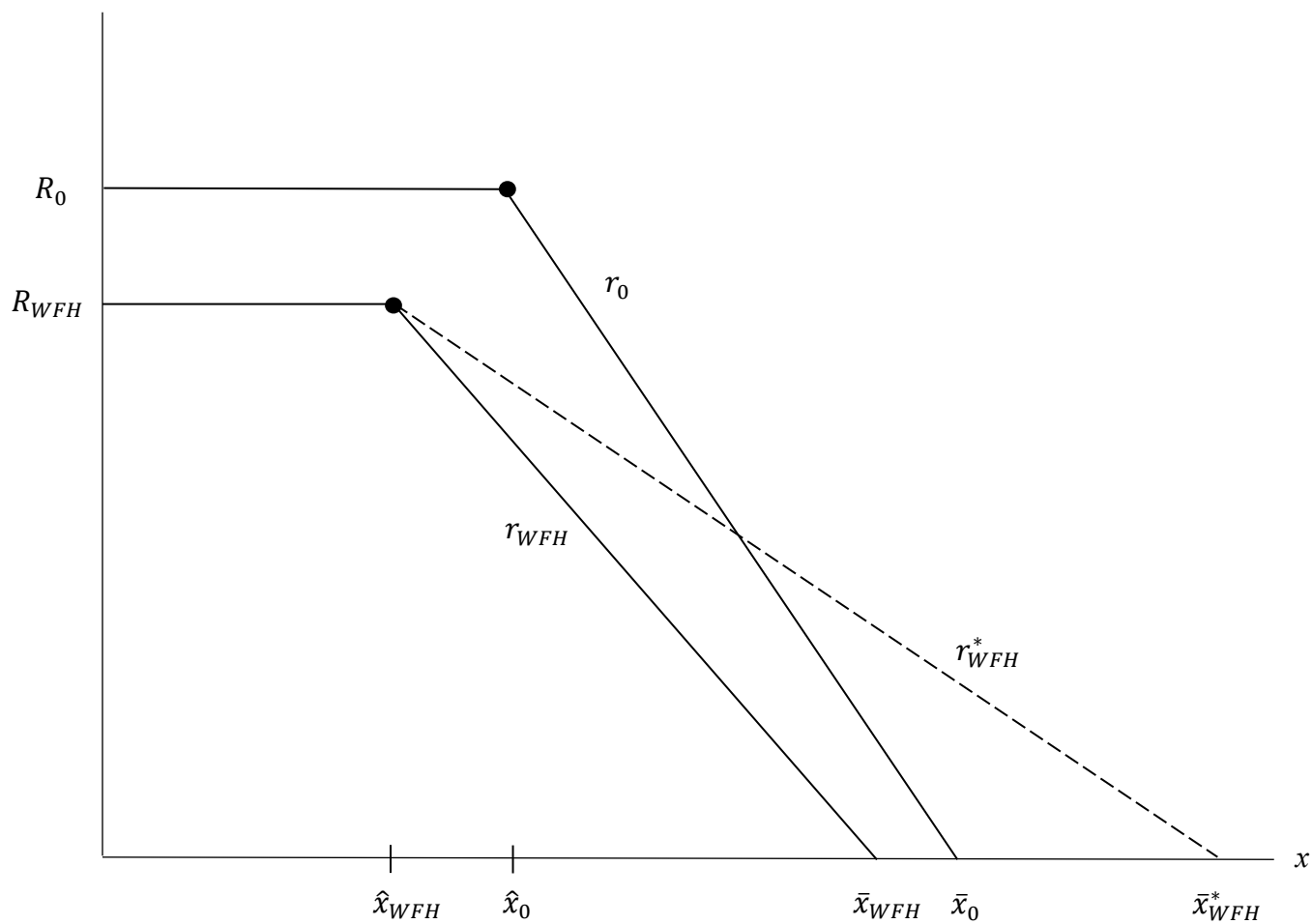


Figure 1: Spatial-structure effects of WFH

Appendix

If a general function $\beta(\alpha)$ with $\beta' < 0$ and $\beta(1) = 1$ is used instead of $\beta = 2 - \alpha$ in the home work-space analysis, Proposition 2 becomes narrower. The α -derivative of the denominator of the q solution in (28) is proportional to $\beta + \alpha\beta'$, which is generally ambiguous in sign but negative or zero evaluated at $\alpha = \beta = 1$ provided $1 + \beta'(1) \leq 0$, or $-\beta'(1) \geq 1$. When this inequality holds, a marginal decrease in α starting at 1 leads to a larger or equal increase in β . Under this condition, the decrease in α reduces or leaves unchanged the denominator of (28) while increasing the numerator, thus raising q . In addition, the derivative of the $\bar{x}$ expression in (29) is proportional to $1 - \beta^2 tN + \beta'(1 - \alpha^2 tN)$. Evaluating at $\alpha = \beta = 1$ and factoring out $1 - tN$ (which is positive from (17)), the derivative is zero or negative when $1 + \beta'(1) \leq 0$ or $-\beta'(1) \geq 1$, the same condition as above. Therefore, when this condition holds, a marginal introduction of WFH (a marginal decrease in α starting at 1) raises q and raises $\bar{x}$ or leaves it unchanged. WFH again raises the wage w unconditionally.

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