Incomplete Information and Investment Inaction*

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Abstract

How do investment friction and information friction interact? We study this question in a stylized continuous time model of heterogeneous firms facing incomplete information and irreversible investment. We analytically characterize how the information friction distorts firms' decision rules and stationary distribution. The two frictions interact in rich and substantial ways. At the firm level, noisier information shrinks a firm's inaction region and reduces the elasticity of investment to productivity. In the aggregate, it increases steady-state capital, increases capital misallocation, and attenuates the effect of productivity shocks on investment. Finally, we test and confirm these predictions using Japanese administrative data that match firms' forecasts to their balance sheets, incomes, and expenditures.

Keywords: Heterogeneous Firms, Incomplete Information, Irreversible Investment, Heterogeneous Beliefs, Misallocation, Investment Volatility

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1 Introduction

Two firm-level frictions are known to substantially distort firm dynamics: investment and information frictions. These frictions are crucial for understanding investment behavior at both the micro and macro levels, which is heterogeneous in the cross-section and much less elastic to aggregate shocks than frictionless models imply. And yet, both frictions have mainly been studied independently, so the natural questions are: Do investment and information frictions interact in economically important ways? If so, does it matter for the aggregate economy?

We explore these questions in two steps. In the first step, we construct a stylized continuous time model of heterogeneous firms that make investment decisions with incomplete information. The first friction is that *capital investment is irreversible.* Irreversibility implies inaction: many firms do not invest over a period of time. Irreversibility distorts firm dynamics in a crucial way (Baley and Blanco, 2022), and irreversibility is close to full for many types of capital.[1] The second friction is that *firms do not know their productivity exactly.* Instead, they receive a noisy signal of their productivity and only learn the true value with some delay. This friction represents that there is a delay of information between when firms purchase new capital and when its effects on revenue become apparent to decision-makers.

The tractability of the continuous time model makes it clear that these two frictions interact in a number of meaningful ways. We start by documenting its micro implications. First, we find that the information friction *reduces* a firm's inaction region. All else equal, uncertainty about a firm's current productivity makes the firm more willing to invest. At first glance, this might be surprising given the well-documented relationship between investment and uncertainty over future productivity. When *future* productivity is more uncertain, firms are less willing to invest because uncertainty increases the option value of delaying irreversible investment, also known as the wait-and-see effect (Leahy and Whited, 1996; Hassler, 1996; Bloom, 2009; Bloom et al., 2018). However, when *current* productivity is uncertain, firms prefer to invest more because the marginal value of capital is convex in log productivity, so Jensen's inequality makes firms act as if they are risk-loving over productivity. This effect also holds in traditional models of future uncertainty, but is always dominated by the wait-and-see effect.

Second, the information friction *reduces* the short-run elasticity of investment to productivity shocks. This standard effect of incomplete information tends to attenuate the effects of shocks by reducing how informative a shock is for forecasting future fundamentals. When firms receive a productivity shock, their noisy signal increases one-for-one, but firms do not know whether

[1] Ramey and Shapiro (2001) find that upon sale, aerospace plants recover less than 30% of the replacement cost of capital; selling imposes additional wind-down costs and takes years to implement. Kermani and Ma (2023) document that the liquidation value of capital for non-financial firms is only 35% of the net book value.

this change was due to productivity or noise, so their productivity nowcast increases less than one-for-one. This attenuation effect is potentially valuable for describing aggregate investment behavior. Koby and Wolf (2020) argue that firm-level investment is relatively inelastic to aggregate shocks, which is necessary to match the empirical aggregate investment dynamics over the business cycle or responses to monetary policy shocks (Winberry, 2021; Fang, 2020). These heterogeneous firm models require high fixed capital adjustment costs to deliver this low elasticity but generate unrealistic non-smooth investment distribution at the firm level; this is a challenge for the literature. In sum, a model needs to generate a realistically low investment elasticity to aggregate shocks and realistic investment inaction simultaneously. But non-convex non-convex adjustment costs cannot deliver this result; House (2014) argues that firms must face other frictions in order to have a realistic investment elasticity. We show that information frictions can help deliver the required low investment elasticity to aggregate shocks, low inaction rates, and low spike rates.

We then document its macro implications. We find that the above micro-mechanisms also have surprising effects on the essential aggregate moments: capital accumulation, capital allocation, and capital fluctuation. First, greater information frictions increase the aggregate capital stock for the same reason that they decrease inaction: uncertainty about current productivity increases firms' willingness to invest. Second, irreversibility introduces capital misallocation, and we show analytically that the information friction increases this misallocation. This effect occurs as firms make investment mistakes when their *expected* productivity is higher or lower than the actual level. Third, we find that the information friction actually reduces the riskiness of capital, as measured by sales volatility. Even though firms make more mistakes, their attenuated response to productivity shocks reduces riskiness on net.

Finally, we also find that introducing information frictions affects the conventional relationship between volatility and investment inaction. In full information investment models, the option value effect implies that firms facing higher productivity volatility will be more inactive *ceteris paribus*. However, more severe information frictions dampen this relationship. If information frictions are severe, then raising productivity volatility has only small effects on investment inaction. This is because increasing productivity volatility raises the variance of nowcast errors, leading to more severe information frictions and shrinking the inaction region. In total, this information effect counteracts the classical option value effect, attenuating the relationship between volatility and investment inaction.

In the second step, we test our central theoretical and quantitative predictions using Japanese administrative data. We construct a merged firm-level dataset that combines the Business Outlook Survey (BOS) and the Financial Statements Statistics of Corporations (FSS) conducted by the Ministry of Finance and the Cabinet Office of Japan. The advantage of this dataset is that firms report both realized and forecasted sales for the past semi-year and the coming semi-year in the BOS. Therefore, we use the predictability of sales forecast errors to estimate the severity of information frictions at the industry level. Firms also report investment and investment plans in the BOS and detailed information on capital stock, employment, and costs in the FSS, which enables us to construct investment-related and productivity-related variables.

We begin our empirical work by measuring the severity of firms' information frictions. First, we group firms into industries based on the industry codes provided by the Ministry of Finance and estimate the friction severity for each industry by regressing the sales forecast error on past productivity growth. Under full information, this coefficient should be zero as past productivity is incorporated into current beliefs immediately. However, our incomplete information model implies that this coefficient is positive, as the incorporation of past information is *incomplete and delayed.* From the data, we find that this "underreaction coefficient" is indeed positive for all industries, albeit with substantial heterogeneity.

Next, we explore how information frictions affects firms' investment behavior. According to our theory, firms with more severe information frictions should be *less* likely to remain inactive, conditioning on firm-level productivity and size. We test this prediction by regressing the binary variable of investment inaction on the industry-level underreaction coefficients. The estimated effect is indeed negative, confirming the model's prediction. Moreover, a one standard deviation increase in this coefficient reduces the average inaction rate at the industry level by more than five percentage points, which is quantitatively large.[2] Then, we test the prediction of the attenuated response of investment to productivity shocks, using an interaction term between industry-level underreaction and the change in firm-level productivity. This interaction regression allows us to control for all industry-level time-varying and firm-level time-invariant factors that affect firm investment. Again, we confirm the model's predictions: firms in industries with more severe information frictions are less sensitive to productivity shocks.

We also test the model's implication that information frictions attenuate the relationship between productivity volatility and investment inaction. The Japanese firms behave as predicted: a rise in productivity volatility across regions in Japan increases the inaction rate, but *less so* when firms face more severe information frictions.

Across all these tests, we directly compare our regression results with analogs generated from simulated data using our model. Even though our model is extremely stylized, it makes clear predictions regarding the signs of certain coefficients. Our empirical results from the firm-level

[2] The average inaction rate is around 36% at the semi-year frequency. Thus, so a one standard deviation increase in the severity of the information friction reduces the conditional inaction rate by nearly 17%.

regressions comport with the model's analytical and quantitative predictions.

Literature. First, our theoretical work is closely related to a small but growing literature on incomplete information in continuous time models featuring inaction. In this existing literature, inaction is due to fixed costs; to the best of our knowledge, we are the first to study the interaction between incomplete information and irreversibility as the source of inaction. Verona (2014) studies inattentive firms who pay fixed costs to update information (as in Reis, 2006), which leads to periodic large investment spikes. Alvarez et al. (2011), Alvarez et al. (2016), and Stevens (2020) study price-setting by firms facing high fixed costs to both changing prices and observing fundamentals. Baley and Blanco (2019) consider a model of menu costs where firms observe noisy signals of their productivity; they predict that firms with higher uncertainty change prices more often and learn more quickly. We also join broader literature studying how investment dynamics are affected by irreversibility and information frictions.[3]

Second, our theoretical work joins a broad literature on irreversibility and investment inaction. Early work (Pindyck, 1991; Abel and Eberly, 1996; Abel et al., 1996) established the option value of irreversibility, and more recent work (Bloom, 2009; Bloom et al., 2018) explored how such option value affects the effects of uncertainty shocks through capital misallocation. We build on a full irreversibility version of Baley and Blanco (2022) and show that introducing incomplete information would further intensify capital misallocation but attenuate capital fluctuations in the short run.

Third, our empirical work is closely related to a burgeoning literature studying information frictions originating from firms' forecast errors of micro and aggregate variables. Seminal work done by Coibion et al. (2018), Tanaka et al. (2020), and Candia et al. (2024) present stylized facts concerning firm-level expectations. Several studies also use data on Japanese firm-level expectations. Using a dataset of multinational firms, Chen et al. (2023b) document heterogeneity in the information frictions firms face that varies by firm size and age. Second, Chen et al. (2023a) document that the degree of information rigidity firms face is higher for aggregate inflation than for firm-specific outcomes. Finally, Charoenwong et al. (2024) show how capital budgeting can alleviate distortions originating from investment frictions and thus improve productivity.[4]

Finally, our paper links to the literature that uses firm-level survey data to document how mi-

[3]Some canonical and more recent examples studying irreversibility include Pindyck (1991), Bertola and Caballero (1994), Abel and Eberly (1996), Veracierto (2002), Ottonello (2017), and Baley and Blanco (2022). Stokey (2008) provides a textbook treatment. Papers studying business cycle models of capital investment with information frictions include Townsend (1983), Angeletos and Pavan (2004), Graham and Wright (2010), Senga (2015), Angeletos et al. (2018), and Atolia and Chahrour (2020), among many more. In particular, Adams (2023) studies a model with investment frictions that does not induce inaction; instead, firms face investment adjustment costs in the style of Christiano et al. (2005).

[4]Other papers that use Japanese firm-level expectations include Charoenwong et al. (2020), Chen et al. (2020), Chen et al. (2022), etc.

cro, industry, and macro shocks affect firms' expectations formation. Andrade et al. (2022) show that industry-level inflation predicts forecast errors about firms' own prices in a survey of French manufacturers. Massenot and Pettinicchi (2018) and Born et al. (2022) use a survey of German manufacturing firms to document how business conditions and news, respectively, predict forecast errors. Other related papers include Bachmann et al. (2013), Bachmann and Elstner (2015), Bachmann et al. (2021), Born et al. (2023a). Born et al. (2023b) survey additional work in this field, while Candia et al. (2023) survey the larger literature studying biases in firms' expectations of the macroeconomy.

Layout. The remainder of the paper is organized as follows. Sections 2 and 3 lay out the model. Section 4 quantitatively explores the model's predictions. Section 5 describes the Japanese firm-level data and estimation results. Finally, Section 6 concludes.

2 The Model

This section describes the economic environment, investment decisions, and information friction. We derive the value function and optimal decisions and demonstrate analytically how investment decisions depend on different parameters, including those controlling the information friction.

2.1 Firm's Problem

Environment There is a unit measure of atomistic competitive firms. Firms produce using capital K, modified by productivity A. Their production function is $F(A,K) = A^{1-\alpha}K^{\alpha}$ where $\alpha \in (0,1)$. Investment I is irreversible. If firms invest, they do so at cost ψ. Accordingly, their instantaneous profit is $\pi = A^{1-\alpha}K^{\alpha} - \psi I$. Lowercase letters denote logs of variables, e.g., $a = \ln A$. Log productivity follows a random walk $da = \sigma_a dW^a$ where W_a is a Wiener process. The law of motion for capital is $dK = I - \delta K dt$ where δ is the depreciation rate.

Optimal firm behavior for this type of problem is characterized by an inaction region: above some level of capital (that depends on other state variables), firms choose not to invest. Firms discount the future at a constant rate r, so inside the inaction region, a firm's Hamilton-Jacobi-Bellman (HJB) equation is

$$rV(K,A) = A^{1-\alpha}K^{\alpha} - \delta K V_K(K,A) + \frac{\sigma_a^2 A^2}{2} V_{AA}(K,A) \tag{1}$$

This is the *full information* HJB. Of course, firms will not have full information when forecasting. However, in the inaction region, the firm's true value still follows this PDE. The wrinkle to this model is that when firms *do* make an action, they will not know A exactly.

Information Structure Firms do not know their productivity A exactly. Instead, they receive a noisy signal s of log productivity:

$$s = a + n \tag{2}$$

where the noise n follows a random walk:

$$dn = \sigma_n dW^n \tag{3}$$

where the Wiener process W^n is independent of W^a.

Additionally, we assume that after τ time, the productivity level is revealed to the firm, i.e., at time t, firms learn the productivity that they had at time $t - \tau$. This structure represents the notion that decision-makers do not know exactly how productive their firm is at any moment but learn ex-post after an accounting period is completed.

Expectation Formation To characterize how firms form expectations, it is useful to temporarily introduce time subscripts, which we have suppressed so far. Productivity growth over τ time is distributed $(a_t - a_{t-\tau}) \sim N(0, \tau\sigma_a^2)$, while $(s_t - s_{t-\tau})$ is distributed $N(0, \tau(\sigma_a^2 + \sigma_n^2))$ due to the independent Wiener processes W^a and W^n.

Proposition 1. *For a firm with information set* $\Omega_t = \{a_{j-\tau}, s_j\}_{j\le t}$*, productivity is conditionally distributed*

$$a_t|\Omega_t \sim N\left(a_{t-\tau} + \gamma\left(s_t - s_{t-\tau}\right), \nu\right)$$

where

$$\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} \qquad \nu \equiv \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2 + \sigma_n^2}$$

Proof: Appendix B.1

Proposition 2. *A firm's expected productivity* $\hat{a} \equiv \mathbb{E}[a|\Omega]$ *and nowcast error* u *follow the diffusions*

$$d\hat{a} = \sigma_a dW^{\hat{a}} \qquad du = \sigma_u dW^u$$

where

$$dW_t^{\hat{a}} = (1-\gamma)dW_{t-\tau}^A + \gamma dW_t^A + \gamma\frac{\sigma_n}{\sigma_a}(dW_t^n - dW_{t-\tau}^n)$$

$$dW_t^u = (1-\gamma)\frac{\sigma_a}{\sigma_u}(dW_t^A - dW_{t-\tau}^A) + \gamma\frac{\sigma_n}{\sigma_u}(dW_t^n - dW_{t-\tau}^n)$$

$$\sigma_u^2 = 2\frac{\sigma_n^2\sigma_a^2}{\sigma_a^2 + \sigma_n^2}$$

Proof: Appendix B.2

Propositions 1 and 2 describe how firms form expectations of their productivity under incomplete information. Two parameters are worth explaining further. First, the signal coefficient $\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}$ measures how elastic firms' expectations are to the noisy signals. Second, the nowcast error variance $\nu \equiv \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ depends on the information delay τ, the noise volatility σ_n, and productivity volatility σ_a. With longer delays and larger noise, firms make larger nowcast errors.

2.2 Solving the Firm's Problem

The full information HJB equation (1) is homogeneous of degree 1 in (K, A) (Stokey, 2008, Ch. 11) so it is possible to rewrite in terms of a single variable $X \equiv \frac{K}{A}$, which we call "normalized capital":

$$rV(X) = X^{\alpha} - \delta X V'(X) + \frac{\sigma_a^2 X^2}{2} V''(X) \tag{4}$$

Moreover, it is convenient to express the HJB in terms of log normalized capital $x = k - a$:

$$rv(x) = e^{\alpha x} - \mu v'(x) + \frac{\sigma_a^2}{2} v''(x) \tag{5}$$

where $\mu \equiv \delta + \frac{\sigma_a^2}{2}$. This conversion to writing the HJB as a function of $x = \log(X)$ follows from $v(x) = V(X)$, $v'(x) = V'(X)X$, and $v''(x) = V''(X)X^2 + V'(X)X$.

These are the usual full information HJB equations. How does incomplete information affect the firm's problem if it does not change the HJB equation? It changes the *boundary conditions*, which are the equations characterizing optimal action. The HJB has many solutions; the correct solution is determined by the appropriate boundary conditions.

Information is incomplete, so x is unknown to firms when making investment decisions. The usual optimality conditions of $V(X)$ cannot be applied in this case. Instead, firms have uncertainty; their expected value of $V(X)$ is given by

$$\mathbb{E}[V(X)|\Omega] = \mathbb{E}[V(X)|\hat{X}] \equiv \hat{V}(\hat{X})$$

Because conditional expectations are normally distributed with constant variance (Proposition 1), *expected log normalized capital* $\hat{x} \equiv \mathbb{E}[x|\Omega]$ is a summary statistic for firms' expectations, as is $\hat{X} \equiv e^{\hat{x}}$, which represents the firm's MLE nowcast of X. The firm's goal is thus to maximize $\mathbb{E}[V(X)|\hat{X}]$, which we write as the *expected value function* $\hat{V}(\hat{X})$.[5]

[5] Appendix A describes the general solution to the HJB equation.

Optimal investment behavior is a threshold strategy, as in the case of full information. Except now, a firm invests only if its expected log normalized capital $\hat{x}$ is less than some boundary $\hat{b}$. Solving the firm's problem involves finding the optimal choice of $\hat{B} \equiv e^{\hat{b}}$. Proposition 3 reports the boundary conditions associated with the optimum. They are analogous to the full information case.

Proposition 3. *Under incomplete information, the boundary conditions consist of two value-matching conditions:*

$$\hat{V}'(\hat{B}) = \psi \qquad\qquad \lim_{\hat{X}\to\infty} \hat{V}'(\hat{X}) = 0$$

and two super contact conditions:

$$\hat{V}''(\hat{B}) = 0 \qquad\qquad \lim_{\hat{X}\to\infty} \hat{V}''(\hat{X}) = 0$$

Proof: Appendix B.3

The proof is standard and follows closely the arguments in Dumas (1991). We include the proof in order to show that we can apply the usual full information optimality conditions to the expected value function $\hat{V}(\hat{X})$.

Proposition 4 below characterizes the solution to the firm's problem. The critical value $\hat{b}$ depends on several parameters: the variance of nowcast errors v, the returns to scale α, the cost of investment ψ, as well as ϱ and m defined as:

$$\varrho \equiv \frac{\mu - \sqrt{\mu^2 + 2\sigma_a^2 r}}{\sigma_a^2} \qquad\qquad m \equiv \frac{1}{r + \mu\alpha - \frac{\sigma_a^2}{2}\alpha^2}$$

Proposition 4. *The critical value of expected normalized capital is*

$$\hat{b} = b^{FI} + \frac{\alpha^2 v}{2(1-\alpha)}$$

where b^{FI} is the full information critical value given by:

$$b^{FI} = \frac{1}{(1-\alpha)} \log\left(\frac{m\alpha(\alpha-\varrho)}{\psi(1-\varrho)}\right)$$

Proof: Appendix B.4

Proposition 4 demonstrates how the information friction affects the firm's optimal investment decisions. Conveniently, most of these terms affect the critical values in the same way as in the full information model. The proposition shows that the difference between full and incomplete information critical values depends only on the variance of nowcast errors v, adjusted by the returns to scale α.[6]

2.3 Micro Implications of Incomplete Information

We can now analytically characterize how the investment and information frictions interact to change firms' investment behavior. First, incomplete information reduces the inaction region. Second, incomplete information attenuates the investment response to productivity shocks.

2.3.1 Incomplete Information and Investment Inaction

The information friction has a clear effect on the firm's optimal behavior: stronger information frictions *shift* the inaction region to the *right*. This is because the optimal boundaries increase as the variance of nowcast errors v gets larger. Corollary 1 formalizes this result.

Corollary 1. *The inaction region bounds are increasing in both the noisy signal variance σ_n^2 and the revelation delay τ.*

Proof. The nowcast error variance $v = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is increasing in both τ and σ_n^2. Proposition 4 implies that $\hat{b}_L$ and $\hat{b}_H$ are increasing in v. □

Why does the information friction shift the inaction region rightwards? The most intuitive answer is because the marginal value of capital is convex in log productivity a. This is true whether information is incomplete or not. The effect of productivity is asymmetric: for the marginal value of capital, the upside of an improvement to a outweighs the downside of a symmetric decrease. Thus a mean-preserving spread in a increases the expected marginal value of capital. And a mean-preserving spread in a is equivalent to a mean-preserving spread in x This makes firms risk-loving over normalized capital: if they do not know the true value, the expected marginal value exceeds the certainty equivalent, i.e.

$$\mathbb{E}[V(\exp(x))|\Omega] > V(\exp(\mathbb{E}[x|\Omega]))$$

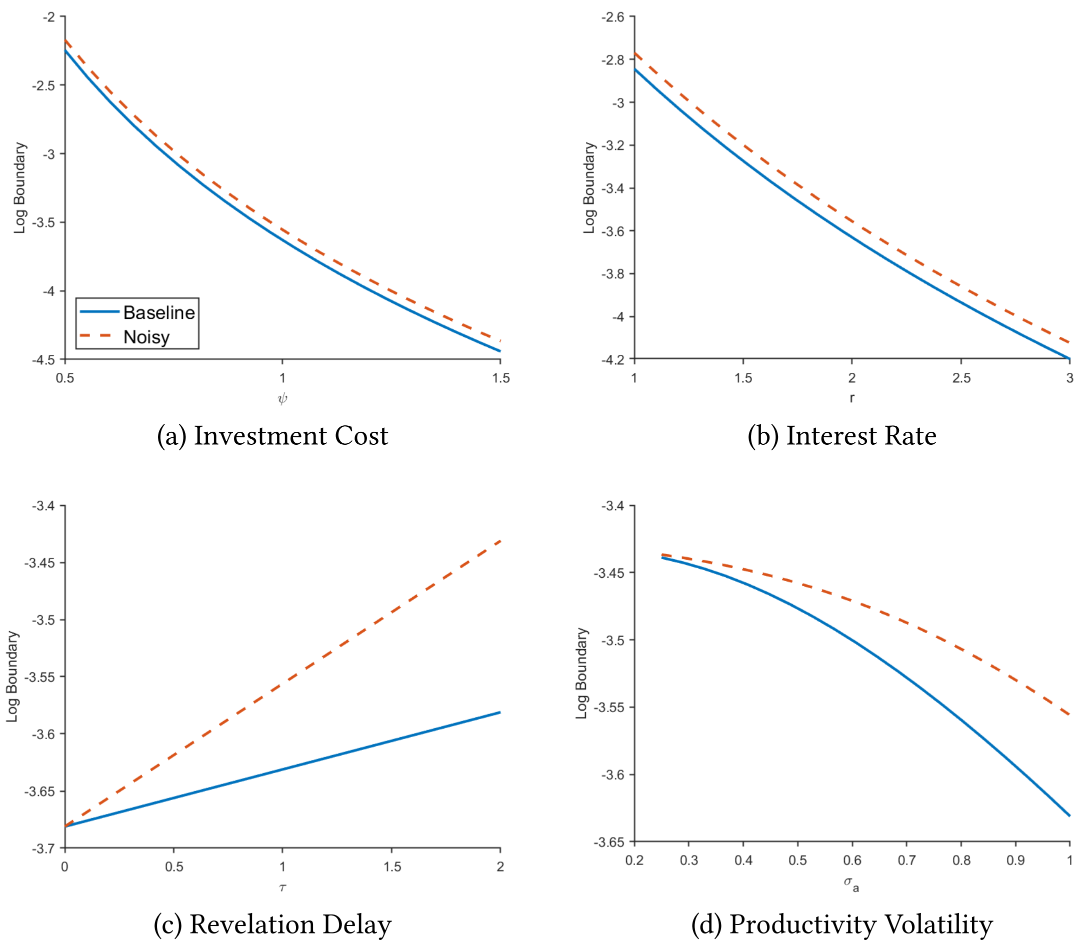
The expected instantaneous return to an additional unit of marginal capital is larger when firms are uncertain about the value of their X. This uncertainty raises a firm's incentive to invest,

[6]This conclusion is not unique to fully irreversible investment. In Appendix C, we extend the model to allow for partial irreversibility, and come to the same conclusion: incomplete information increases inaction region boundaries relative to the full information cast.

thus they are willing to do so at higher levels of expected X, raising the lower bound on their inaction region.

Figure 1 plots how different parameters affect firms' decisions. In all cases, the solid blue line plots how $\hat{b}$ depends on the parameter of interest, with all other parameters matching our baseline calibration. The dashed red line plots the $\hat{b}$ sensitivity for a "noisy" calibration, where the noise standard deviation σ_n is twice that of the baseline calibration. In all cases, the dashed red line is above the solid blue line: when firms have *worse* information, they are *more* willing to invest, as implied by Corollary 1.

Figure 1: How the Boundary $\hat{b}$ Depends on Various Parameters



(a) Investment Cost

(b) Interest Rate

(c) Revelation Delay

(d) Productivity Volatility

The first two panels are standard. Panel 1a shows that $\hat{b}$ is decreasing in the investment cost ψ: when capital is more expensive, firms are less willing to invest and do so only when their normalized capital is lower. Similarly, 1b shows that $\hat{b}$ is decreasing in the interest rate r: when firms discount the future at a higher rate, they are less willing to invest. The information friction raises the inaction boundary for all values of ψ and r, but does not fundamentally change how inaction depends on these parameters.

The next two panels show how signal noise interacts with parameters. Panel 1c demonstrates the other way that the information friction increases willingness to invest. $\hat{b}$ increases in τ, the amount of time before productivity is revealed to the firm. Larger values of τ make firms less certain about their current productivity level, increasing their nowcast error variance (Proposition 1). When τ is larger, the information friction is worse. Like σ_n, exacerbating the information friction increases the incentive to invest and raises $\hat{b}$. Moreover, when the delay is larger, the signal noise volatility has even stronger effects, because firms accumulate disproportionately more noise in their signals. This is why the curves diverge in Panel 1c.

The most unusual result is in Panel 1d, which plots how $\hat{b}$ depends on the productivity standard deviation σ_a. The standard result is higher productivity volatility should make firms less willing to invest because it increases the option value of waiting (Leahy and Whited, 1996; Hassler, 1996). This is the case in our model, too: raising volatility reduces the boundary, i.e., making firms less willing to invest. However, volatility also plays a role in information friction. When volatility increases, firms' nowcasts are less accurate (ν decreases in σ_a). This information effect *attenuates* the standard option-value effect of volatility on inaction. And the attenuating information effect is stronger when signals are noisier. Panel 1d makes this clear: if signals are relatively precise (solid blue line), then volatility sharply reduces the boundary, but if signals are relatively noisy (dashed red line), then volatility has less effect on inaction.

This result demonstrates a new channel by which "uncertainty" affects capital investment. The traditional *wait-and-see channel* is that uncertainty over future productivity reduces the incentive to invest by increasing the option value of delaying new capital. In recent work, uncertainty has been documented to be a major driver of business cycles, and this investment wait-and-see channel is considered a central mechanism (Bloom, 2009; Bloom et al., 2018). The new *information channel* is that uncertainty over current productivity increases the incentive to invest. Moreover, these two channels interact as shown by Panel 1d. When information is incomplete, higher productivity volatility still increases the inaction region of investment. However, information frictions change this relationship. If information frictions are severe, then raising productivity volatility has only small effects on investment inaction. This is because increasing productivity volatility raises the variance of nowcast errors, shrinking the inaction region. This information effect counteracts the classical option value effect, attenuating the relationship between volatility and investment inaction.

2.3.2 Incomplete Information and Investment Sensitivity

The information friction attenuates the short-run impact of shocks by reducing the passthrough from productivity shocks to firms' productivity expectations. With full information, productivity shocks affect forecasts one-for-one because productivity follows a random walk. This is not the case when information is incomplete.

Proposition 5 shows that the short-run attenuation depends on $\gamma = \frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}$, as defined in Proposition 1. When the noise variance σ_n^2 is large, then signals are noisy, and productivity shocks have little effect on firms' immediate nowcasts. But as $\sigma_n^2 \to 0$, firms more accurately nowcast their productivity and approach the full information case.

Proposition 5.

$$\frac{d}{dW^a_{t-h}}\mathbb{E}[a_t|\Omega_t] = \begin{cases} \gamma & 0 \le h < \tau \\ 1 & h \ge \tau \end{cases}$$

Proof: Appendix B.5

Proposition 5 above implies that the immediate passthrough of a productivity shock to the firm's expected log normalized capital ($\hat{x} = k - \hat{a}$) is $-\gamma$. Thus, any quantity that depends on $\hat{x}$ will be similarly attenuated in the short run, e.g., the average time until leaving the inaction region, the expected investment over a time period, and so forth. However, because the true productivity level is eventually revealed to firms, Proposition 5 also says that the long-run passthrough from productivity to nowcasts and forecasts is one-for-one.

What does this result imply for tests of incomplete information? In Section 5, we estimate a standard *underreaction coefficient* (Kohlhas and Walther, 2021). Corollary 2 derives how this coefficient – the effect of productivity shock dW_t^a on forecast error ($a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t]$) – depends on the information friction parameters in the model.

Corollary 2. *A firm's underreaction coefficient is*

$$\frac{d(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])}{dW_t^a} = 1 - \gamma$$

Proof. Log productivity a_t follows a random walk, which implies

$$\frac{d(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])}{dW_t^a} = \frac{da_{t+h}}{dW_t^a} - \frac{d\mathbb{E}[a_t|\Omega_t]}{dW_t^a} = 1 - \frac{d\mathbb{E}[a_t|\Omega_t]}{dW_t^a}$$

with $\frac{d\mathbb{E}[a_t|\Omega_t]}{dW_t^a} = \gamma$ by Proposition 5. $\square$

3 The Macroeconomy

To characterize the aggregate economy, we first must make several assumptions about the distribution of firms. We derive the partial differential equations governing the dynamics of firm distributions and solve explicitly for the stationary distribution. Then, we characterize how the information friction affects macroeconomic aggregates.

3.1 Firm Entry and Exit

We assume that a measure η of firms enter the economy at every moment. Entering firms do not know their productivity. They are as uncertain as existing firms, i.e., their Bayesian prior is that their log productivity is normally distributed with variance ν.

Across entering firms, expected log productivity $\hat{a}$ is distributed by $\hat{a}_{enter} \sim N(0, \varsigma)$. Thus, the entering distribution of actual log productivity is $a_{enter} \sim N(0, \varsigma + \nu)$. Firms' log expected normalized capital $\hat{x}$ enters at the critical value $\hat{b}$: $\hat{x}_{enter} = \hat{b}$. This is a natural assumption for the entering distribution of capital: firms are born with some unknown productivity level and acquire capital until they are no longer willing to invest. Firm exit is random, independent of other variables. We assume that when firms exit, their value is returned to shareholders, so the exit risk does not change the firm's HJB equation. For the probability distribution to integrate into one, firms must exit at the same rate they enter: η.

3.2 The Stationary Distribution of Normalized Capital

Proposition 2 implies that expected normalized capital $\hat{x} = k - \hat{a}$ follows the diffusion

$$\hat{x} = -\delta dt - \sigma_a dW^{\hat{a}}$$

Firms exit at rate η, so the Kolmogorov Forward equation (KFE) for the distribution $h(\hat{x}, t)$ of expected normalized capital in the inaction region is

$$\partial_t h(\hat{x}, t) = \delta \partial_{\hat{x}} h(\hat{x}, t) + D \partial^2_{\hat{x}} h(\hat{x}, t) - \eta h(\hat{x}, t)$$

where $D \equiv \frac{\sigma_a^2}{2}$. PDFs written without time arguments denote stationary distributions. The KFE for the stationary distribution of expected normalized capital is thus

$$0 = \delta h'(\hat{x}) + D h''(\hat{x}) - \eta h(\hat{x}) \tag{6}$$

The boundary condition is that $h(\hat{x})$ must integrate to one on the interval $[\hat{b}, \infty)$. Proposition 6 below gives the solution.

Proposition 6. *The stationary distribution of expected normalized capital $h(\hat{x})$ for $\hat{x} \geq \hat{b}$ is*

$$h(\hat{x}) = \rho e^{-\rho(\hat{x}-\hat{b})}, \quad \textit{where} \quad \rho \equiv \frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$$

Proof: Appendix B.6

Proposition 6 shows that the information friction does not affect the shape of the stationary distribution $h(\hat{x})$. The root ρ is determined from purely economic fundamentals: depreciation δ, productivity volatility σ_a, and exit risk η; ν never appears. The information friction only shifts the stationary distribution left or right by determining the lower bound $\hat{b}$.

The joint distribution $f_{\hat{x},u}(\hat{x}, u)$ of expected normalized capital $\hat{x}$ and productivity nowcast errors $u = a - \hat{a}$ follow straightaway from Proposition 6, because nowcast errors must be independent of nowcasts. Thus, their joint distribution is the product of their marginal distributions:

$$f_{\hat{x},u}(\hat{x}, u) = h(\hat{x})\phi(\frac{u}{\sqrt{\nu}})$$

where $\phi(\cdot)$ is the standard normal pdf. From the joint distribution, it is straightforward to calculate the marginal distribution distribution $f_x(x)$ of actual normalized capital $x = \hat{x} - u$. Proposition 7 below shows the stationary distribution of log normalized capital.

Proposition 7. *The stationary distribution of log normalized capital is*

$$f_x(x) = h(x)e^{\frac{\nu\rho^2}{2}}\Phi\left(\frac{x - (\hat{b} + \nu\rho)}{\sqrt{\nu}}\right)$$

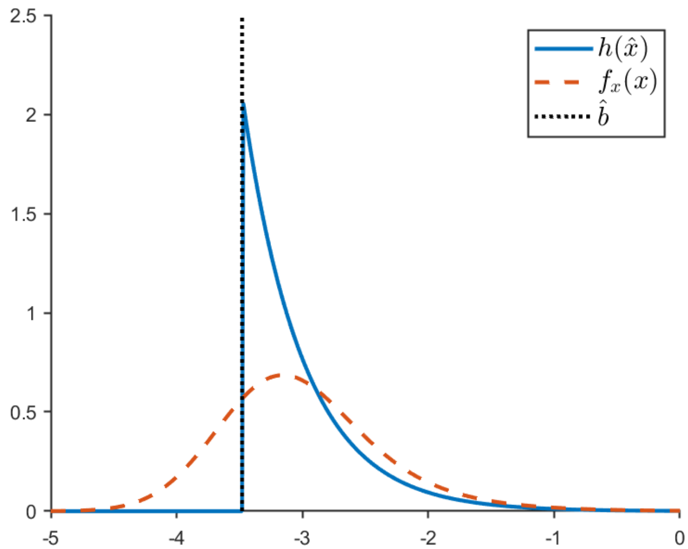
where $\Phi(\cdot)$ is the standard normal CDF.

Proof: Appendix B.7

We illustrate the differences between the two distributions. Figure 2 plots how the stationary distribution of *realized* normalized capital $f_x(x)$ compares to the distribution $h(\hat{x})$ of *expected* normalized capital. $h(\hat{x})$ is monotonic and has a discrete lower bound at the barrier $\hat{x}$. In contrast, normalized capital $x = \hat{x} - u$ is smoothed out because it subtracts an independent Gaussian. Instead of featuring a lower bound, it has an infinite domain. And Proposition 7 implies that the larger the nowcast error variance ν, the more smooth the distribution is.

What is the firm size distribution? We have characterized the distribution of normalized

Figure 2: Stationary Distributions for Expected and Realized Normalized Capital



capital; to answer this question, we need to decompose normalized capital into its capital and productivity components. To do this, we need to solve the KFE in multiple dimensions. In the inaction region, the KFE for the distribution of capital and expected productivity $g(k, \hat{a}, t)$ is

$$\partial_t g(k, \hat{a}, t) = \delta \partial_k g(k, \hat{a}, t) + D\partial_{\hat{a}}^2 g(k, \hat{a}, t) - \eta g(k, \hat{a}, t) \tag{7}$$

With the stationary distribution satisfying the partial differential equation

$$0 = \delta \partial_k g(k, \hat{a}) + D\partial_{\hat{a}}^2 g(k, \hat{a}) - \eta g(k, \hat{a}) \tag{8}$$

This distribution is more challenging to characterize analytically than the univariate normalized capital distribution. Therefore, we solve for this distribution numerically in Section 4 below.

3.3 Macro Implications of Incomplete Information

Having a closed-form solution for the steady-state distribution of expected normalized capital is valuable because it allows us to characterize in the closed form how various macroeconomic aggregates depend on the parameters of the information friction. In this section, we show that information frictions increase both capital misallocation and average normalized capital.

3.3.1 Capital Accumulation

Aggregate normalized capital $\bar{X}$ in the economy is given by

$$\bar{X} \equiv \int_{-\infty}^{\infty} e^x f_x(x) dx$$

where $f_x(x)$ is the stationary distribution of log normalized capital as defined in Proposition 7.

We documented in Section 2.2 that the information friction increases firms' willingness to build capital. Noisier information raised the lower bound on firms' inaction region. This effect increases aggregate normalized capital, as Proposition 8 demonstrates.

Proposition 8. *If $\rho > 1$, then steady state normalized capital is finite and increasing in both the noisy signal variance σ_n^2 and the revelation delay τ.*

Proof: Appendix B.8

3.3.2 Capital Misallocation

We measure misallocation as the variance of the log marginal product of capital:

$$Var\left[\log \frac{\partial F(A, K)}{\partial K}\right] = (1-\alpha)^2 Var[x]$$

The information friction increases capital misallocation in a straightforward way. Misallocation can be decomposed into two components: dispersion in expected capital $Var[\hat{x}]$ and dispersion in nowcast errors. The former is due to endogenous decisions, while the latter is due to mistakes made by firms that do not know their productivity. Proposition 9 shows that the information friction increases misallocation entirely due to mistakes.

Proposition 9. *Steady state capital misallocation is increasing in both the noisy signal variance σ_n^2 and the revelation delay τ.*

Proof. Normalized capital is decomposed into nowcast errors by $x = \hat{x} - u$. $\hat{x}$ and u are independent, so

$$Var[x] = Var[\hat{x}] + \nu$$

where $\nu = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is the nowcast error variance. Proposition 6 implies that $Var[\hat{x}]$ is independent of the information friction parameters. Thus misallocation is increasing in ν, which is increasing in σ_n^2 and τ. $\square$

4 Quantitative Analysis

In this section, we quantitatively solve the model to demonstrate the theoretical properties and predictions. We proceed in two ways. First, we solve the model directly following the KFE in the one-dimensional space of $\hat{x}$. Second, we simulate the joint distribution of firms in the two-dimensional space of $\{\hat{a}, k\}$. Finally, we generate a simulated firm sample with 10,000 firms and 50 quarters according to our calibration of Japanese firm-level data to show model predictions.

4.1 Parameterization

We calibrate our model quantitatively in Table 1 to roughly match our Japanese firm-level data on a quarterly frequency in Table 2. We first fix the interest rate r to be 1% to match an annual rate of 4%, returns to scale α of two-thirds,[7] and a one-period revelation delay before past productivity is revealed, consistent with quarterly accounting reports. We then choose an exit rate η of 2% to match an annual exit rate of 8% and a depreciation rate δ of 0.0136 to match the aggregate quarterly investment rate of 1.36%. Finally, we jointly fit the volatility of productivity and noise shocks to match the investment rate mean and standard deviation.

Table 1: Calibration in Quarterly Frequency

Parameter	Interpretation	Value	Reference
		Fixed Parameters	
r	Real interest rate	0.01	Annual rate of 4%
α	Return to scale	0.67	Investment literature
ψ	Investment cost	1.00	Normalization
τ	Revelation delay	1	Accounting report delay
η	Exit risk	0.02	Japanese firm-level data
δ	Depreciation rate	0.0136	Japanese firm-level data
		Fitted Parameters	
σ_a	S.D. of productivity process	0.20	Japanese firm-level data
σ_n	S.D. of noise process	0.20	Japanese firm-level data

Notes: We choose the parameters to roughly match the moments from the Japanese firm-level data. We do not aim to match all the moments exactly since we only have three fitted parameters. We target the investment rates and volatility, leaving investment inactions and spikes untargeted. We also report the full information model when $\sigma_n = 0$.

We report the moments of the baseline model and their comparison to a full information model in stationary equilibrium in Table 2. Since we aim to keep the baseline model simple,

[7] We choose $\alpha = 0.67$ in the middle of the range in the investment literature [0.50, 0.85] from 0.50 in Baley and Blanco (2022), 0.60 in Baley and Blanco (2021), 0.75 in Abel and Eberly (1999) and Bloom et al. (2018), 0.77 in House (2014), and 0.85 in Winberry (2021). The choice of α will not qualitatively affect the model results.

Table 2: Moments in Quarterly Frequency

Moments	Data	Baseline	Full Info.
Targeted Moments			
Average Exit Rate	2%	2%	2%
Aggregate Investment Rate	1.36%	1.36%	1.36%
Investment Rate Mean	2.10%	2.63%	2.86%
Investment Rate S.D.	7.1%	7.1%	8.7%
Untargeted Moments			
Investment Inaction Rate	57.8%	79.7%	82.9%
Investment Spike Rate	1.4%	4.5%	5.5%

Notes: This table summarizes our moments simulated from the model with 500,000 firms for 500 quarters and their mapping to our data. Data Sources: Economic Outlook Survey and Financial Statement Survey of Corporations of Japan (2004-2019). All statistics are calculated using variables defined at the quarterly frequency. Investment is the sum of equipment/machinery/land investments. Capital is the amount of fixed capital. Investment spike refers to an investment rate $> 20\%$, and investment inaction refers to an investment rate $\leq 1\%$. Forecast error is defined as the log deviation of the realized sales in period t from the sales forecast made in period $t-1$. Top and bottom 1% log sales forecast errors are trimmed (i.e., outliers). The average revenue of firms is around 38 billion JPY (equivalently 330 million USD) per semi-year.

our degrees of freedom are only two parameters: the volatility of productivity shock σ_a and the volatility of noise shock σ_n. Therefore, we cannot jointly target all four micro-level investment moments perfectly. Still, the calibrated model does generate substantial inaction and spikes in firm-level investment rates.[8] Our inaction rate is higher than in the data, partially because of the sample selection of our dataset, which features larger-than-average firms.

4.2 Quantification of the Micro and Macro Implications

In this section, we provide a quantitative analysis of our model properties. We start by analyzing how the firm size distribution and investment decisions are affected when information frictions are present. We then proceed to study how different degrees of information friction severity affect investment sensitivity to productivity shocks and other moments. Finally, we conclude this section by studying the dynamic responses of investment to aggregate productivity shocks.

4.2.1 Simulations with Incomplete Information

We simulate twenty-one industries with different levels of information friction $\{\sigma_{n,s} = 0.01 \times s\}_{s=0}^{20}$ to show the effects of incomplete information on various moments of investment dynamics. Each

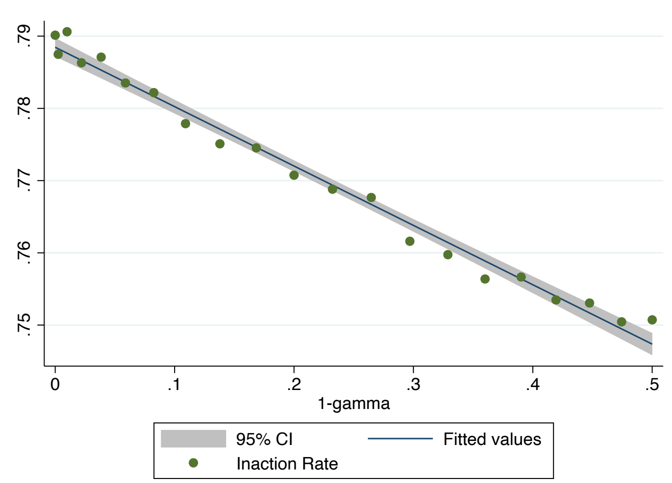
[8]The investment rate is defined as investment this period divided by capital stock last period.

industry has again 10,000 firms for 50 quarters. Our major targets of interest are how investment inaction and elasticity of investment to productivity shocks differ under different degrees of information friction. More specifically, we show how investment inaction and the elasticity of investment to productivity shocks change with the underreaction coefficient $1-\gamma_s$ (Corollary 2). A smaller γ_s denotes more severe information frictions. When $\gamma_s = 1$, the model reduces to the full information case.

4.2.2 Micro Implications of Incomplete Information

Investment Inaction We first quantify how much the information friction *reduces* a firm's inaction region. As suggested by our theoretical analysis, all else equal, uncertainty about a firm's current productivity makes the firm more willing to invest. We show the quantitative results in Figure 3. Quantitatively, when the underreaction coefficient $1-\gamma$ increases from 0 to 0.5, investment inaction reduces by about 4 percentage points. The correlation almost has a linear trend, starting from no information friction $1-\gamma = 0.0$ to the case where the variance of the noise equals the volatility of productivity (i.e., $1-\gamma = 0.5$).

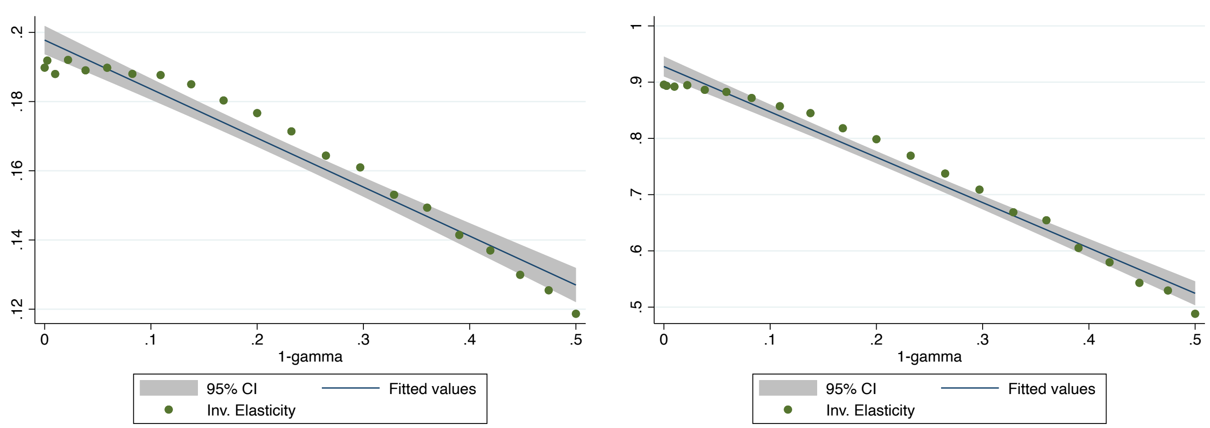
Figure 3: Effects on Investment Inaction: Industry-level Correlation



Note: We show how the industrial average inaction rate changes with the information friction. Each dot represents an industry's average investment inaction rate with underreaction coefficient $1-\gamma$. The result shows that firms in industries with a more severe information friction are less likely to stay in the inaction regions.

Investment Sensitivity We then show how the industrial average investment elasticity to productivity shocks changes with the information friction in Figure 4. Each dot represents the $E_s[i_{i,t+1}/w_{i,t}]$ where $i_{i,t+1}$ is firm i's investment rate at time t, $w_{i,t} = a_{i,t} - a_{i,t-1}$ is firm i's productivity

Figure 4: Effects on Investment Sensitivity: Industry-level Correlation



(a) Including Inaction

(b) Excluding Inaction

Note: We show how the industrial average immediate investment rate response to productivity shocks changes with the information friction. Each dot represents the $E_s[i_{i,t+1}/w_{i,t}]$ where $i_{i,t+1}$ is firm i's investment rate at time t, $w_{i,t} = a_{i,t} - a_{i,t-1}$ is firm i's productivity shock at time t, and s indicates which industry the firm is at. In panel (a), investment inactions $I_{i,t+1} = 0$ are included, while in panel (b), inactions are excluded. The result shows that firms in industries with a more severe information friction are less responsive to productivity shocks, consistent with the theory.

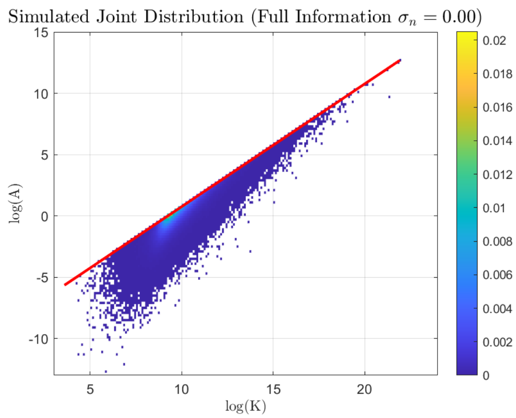
shock at time t, and s indicates which industry the firm is at. In panel (a), investment inactions $I_{i,t+1} = 0$ are included, while in penal (b), inactions are excluded. The result shows that firms in industries with a more severe information friction are less responsive to productivity shocks, consistent with the theory. Quantitatively, when the underreaction coefficient $1 - \gamma$ increases from 0 to 0.5, the elasticity of investment rate to productivity shocks reduces by 30% to 45%, depending on whether inactions are included.

4.2.3 Macro Implications of Incomplete Information

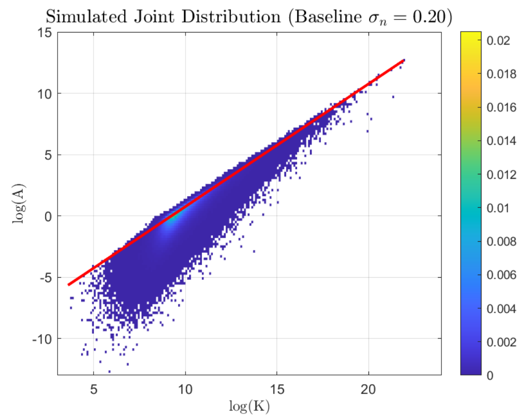
Capital Misallocation To show the role of the information friction on capital misallocation, we compare the baseline model to three alternative models with different levels of information incompleteness (from the case of full information $\sigma_{n,s} = 0$ to the case that the standard deviation of the noise shock is three times relative to the standard deviation of the productivity shock $\sigma_{n,s} = 0.6$ where s denote a specific industry s) to show the effects of incomplete information on various objects of investment dynamics.

Figure 5 presents the joint stationary distributions for capital and productivity. Panel (a) presents the full information case, while panel (b) presents the baseline incomplete information

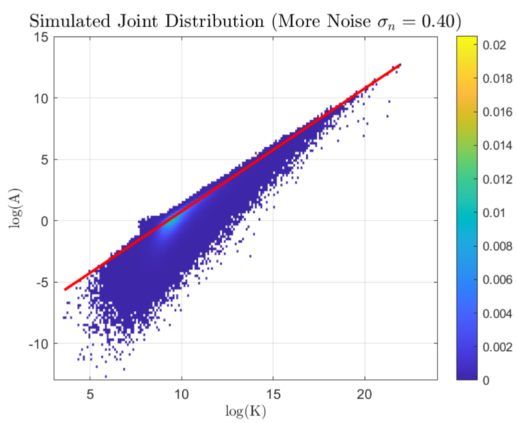
Figure 5: Simulated Joint Stationary Distribution for Capital and Productivity



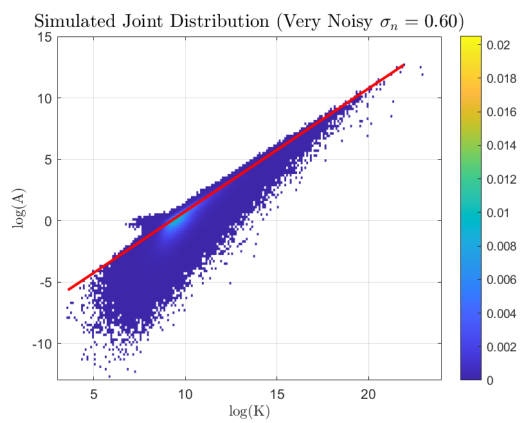
(a) Full Information ($\sigma_n = 0.00$)



(b) Baseline Model ($\sigma_n = 0.20$)



(c) More Noise ($\sigma_n = 0.40$)



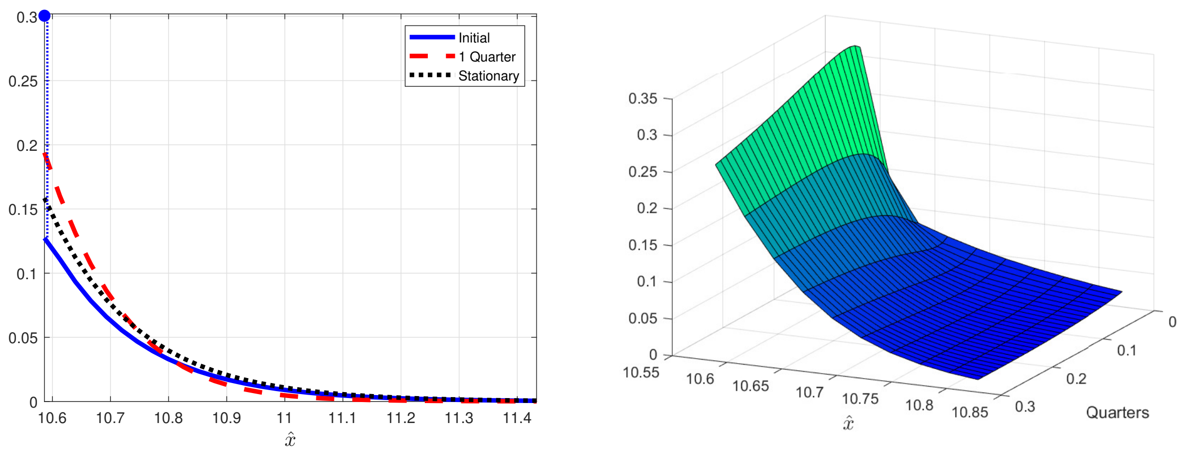
(d) Very Noisy ($\sigma_n = 0.60$)

Note: Histograms generated using simulated data for 500,000 firms. The red solid line denotes the boundary for expected normalized capital $\hat{b}$.

case. Panels (c) and (d) are cases where we have even noisier incomplete information. These plots show the role that information frictions play in terms of how firms adjust their capital stock in noisy scenarios. Specifically, we see that firms tend to remain closer to the boundary relative to incomplete information under full information. Moreover, we see that when information becomes incomplete, firms start making mistakes: there is relatively more mass to the left of the boundary. These two-dimension distributions validate Proposition 9 that more severe information friction leads to more severe capital misallocation.

Aggregate Attenuation of Productivity Shocks Finally, we show how aggregate productivity shocks affect expected log normalized capital $\hat{x}$. Under incomplete information, an aggregate shock to expected productivity differs from one to *actual* productivity.[9] We define an aggregate shock as an exogenous increase of one quarterly standard deviation σ_a in the log productivity of all firms. The aggregate shock induces a shift in the distribution of normalized capital, which is illustrated in Figure 6.

Figure 6: Response of the Normalized Capital Distribution to an Aggregate Productivity Shock



(a) Distribution on Impact and After One Quarter

(b) Distribution Over Time

Figure 6 shows that the aggregate shock immediately pushes many firms to the boundary. If the aggregate shock pushes a firm's normalized capital past the boundary $\hat{b}$, the firm immediately invests in capital to remain at the barrier. Then after the shock, firms start adjusting their expected log normalized capital levels in several directions: we observe firms moving *away* from the boundary where the is excess mass, but we also observe firms moving *toward* the boundary, as the mass on higher values of normalized capital is lost relative to the initial equilibrium.

To analyze the response of average expected normalized capital, we define the impulse response function (IRF) as

$$IRF_{\hat{x}}(t) = \int_{\hat{x}} \hat{x} h(\hat{x}, t) d\hat{x}$$

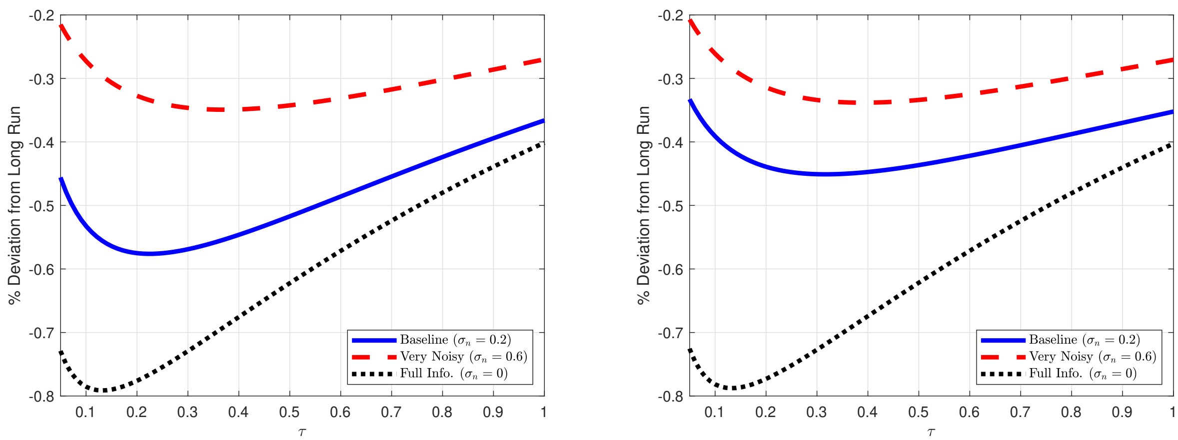
where $h(\hat{x}, 0)$ is the perturbation to the distribution caused by the aggregate shock. Panel (a) in Figure 7 presents the average response of expected normalized capital (solid blue line). We see that the aggregate shock lowers the average expected capital, but after nearly two years, it is roughly back to its long-run average.

To illustrate the role of the information friction, we study two exercises: (1) we consider a noisier signal, characterized by the increased variance of the noise σ_n, and (2) we allow for a longer horizon in terms of the revelation delay τ. In particular, we explore cases where $\sigma_n = 0.6$

[9]Notice that the two responses are equivalent under full information.

(in the baseline case $\sigma_n = 0.2$) and $\tau = 4$ (1 in the baseline scenario). For completeness, we also add the full information case. The red dashed line in panel (a) ((b)) in Figure 7 presents the IRF of average log expected normalized capital $\hat{x}$ for the noisier signal when $\tau = 1$ ($\tau = 4$). The black dotted line corresponds to the average log expected normalized capital response under full information case, for $\tau = 1$ and $\tau = 4$.

Figure 7: Aggregate Impulse Response Function to a Productivity Shock



(a) $\tau = 1$

(b) $\tau = 4$

Figure 7 shows the role of the information friction: higher noise variance attenuates the response of expected normalized capital. Panel (b) shows that the discrepancy mentioned above is exacerbated when the revelation delay is one year rather than a quarter. Intuitively, the response of average expected capital is more severely attenuated the longer it takes the average firm to realize whether the shock it is experiencing is noise or a truly productive one. Lastly, the responses of expected normalized capital are identical across revelation delays when firms have full information; in this case, the information delay τ does not matter for firms' decisions.

5 Empirical Validation with Microdata

This section uses Japanese firm-level data to test our key theoretical predictions. Moreover, we run the same regressions using the microdata and the simulated data and compare the results directly. We show that some key coefficients estimated from our firm-level data are consistent with those obtained using the model's simulated data.

5.1 Japanese Firm-level Data

We use the Business Outlook Survey (BOS) and the Financial Statements Statistics of Corporations (FSS) conducted by the Ministry of Finance and the Cabinet Office of Japan. The two quarterly datasets cover all large firms and a representative sample of small and medium-sized firms. Both datasets cover manufacturing and non-manufacturing enterprises. While both datasets survey large firms every quarter, medium-sized and small firms are randomly sampled regularly.[10]

The sample size of the BOS is about 11,000 (response rate of more than 75%), and the sample size of the FSS is about 21,000 (response rate of about 70%). The FSS covers basic financial statement information in the balance sheet and profit and loss account, while the BOS contains firm-level forecasts of sales, profit (at the semi-year frequency), and firms' investment and investment plans (at the quarterly frequency). Fortunately, both datasets have common time-invariant firm identifiers for firms with registered capital above 1 billion JPY (about 6 million USD in July 2024). As a result, we merge the two datasets and construct a panel dataset that only contains firms with registered capital above 1 billion JPY for 2004-2018.

The unique feature of the BOS is that it provides quantitative forecasts on sales and profits, which allows us to calculate the forecast error. The frequency of reporting both the realized and expected sales and operating profits is semi-annual. In contrast, the frequency of reporting investment plans is quarterly.[11] As the forecasting targets (i.e., sales and profits) are at the semi-year level, we define various variables (e.g., sales, investment, usage of intermediate goods, and productivity) at the semi-year frequency in our analysis. Additionally, the BOS asks firms to report investment directly. Thus, the data allow us to directly measure investment rather than backing it out using information on capital stock.

Table 8 in the appendix presents the summary statistics of observations in our merged and original FSS datasets. First, the number of firms per quarterly is roughly 6,500 in our merged dataset, compared with 21,000 observations in the original FSS dataset. Second, firms are, on average, quite large in both datasets, as the average employment and sales are above 490 and 8500 million JPY (7 million USD) per quarter.

[10] In BOS, all firms with registered capital above 2 billion JPY are sampled every quarter. For firms with registered capital between 0.5 billion JPY and 2 billion JPY, 50% are randomly sampled every quarter. For firms with registered capital between 0.1 billion JPY and 0.5 billion JPY, 10% are randomly sampled every quarter. For firms with registered capital less than 0.1 billion JPY, 1% are randomly sampled every quarter. The random sample is redrawn at the beginning of each fiscal year. Therefore, if a firm is selected in a given fiscal year, it will appear in the survey in all four quarters of that fiscal year. In FSS, all firms with registered capital above 5 billion JPY are sampled quarterly. For firms with registered capital between 1 billion JPY and 5 billion JPY, 50% are randomly sampled every quarter.

[11] For instance, the firm is asked to report realized sales from April to September and its projected sales for the following October to March of next year, when it is surveyed in October. In the same survey, the firm also reports its investment plan for October to December and January to March next year.

5.2 Construction of Variables

In five steps, we construct variables of interest in the closest way possible, as in our model. First, we calculate the percentage and logarithm forecast errors by taking the percentage difference of the realized value from the forecasted value, which is made at the beginning of each semi-year. Second, we calculate the firm-level investment rate and directly define investment inaction/spike at the firm level since the dataset contains information on fixed investment and fixed capital stock.[12] Third, we group firms into 30 industries based on their original industry affiliations reported in the data (47 industries), as more than a dozen industries have too few observations in a given quarter. Fourth, we define labor productivity using revenue per worker as the main productivity measure.[13] In the data, we have information on total sales, the cost of goods sold (including capital appreciation and labor costs), depreciation of capital, and labor costs (wage, salary, and benefits). Thus, we can back out of the purchase of intermediate goods. Finally, we calculate the total amount of fixed investment in a given semi-year by summing up the investment amounts in the two quarters that belong to the same semi-year.

Tables 7 and 9 in the appendix present summary statistics of our constructed (fixed capital) investment-related variables and summary statistics of our constructed (untrimmed) forecast errors, respectively. Statistics from Table 7 confirm that firms in our merged dataset are large, as the average (and aggregate) investment rate and the average investment spike rate are low. Moreover, the average inaction rate is low, and the autocorrelation of the investment rate is high. Finally, from Table 9, we observe that although the median of the two forecast errors is extremely close to zero, the standard deviation is quite substantial. This shows that there is substantial heterogeneity in the constructed forecast errors across firms and over time. We also trim top and bottom 1% sales forecast errors (i.e., outliers) in our empirical regressions.

5.3 Information Incompleteness Estimation

To provide evidence on how the severity of the information friction affects a firm's investment choice, we estimate the industry-specific underreaction coefficient ξ_s, using calculated forecast

[12]"Investment spike" refers to an investment rate $> 20\%$, and "investment inaction" refers to an investment rate $\leq 1\%$. The average revenue of firms is around 38 billion JPY (equivalently 330 million USD) per semi-year.

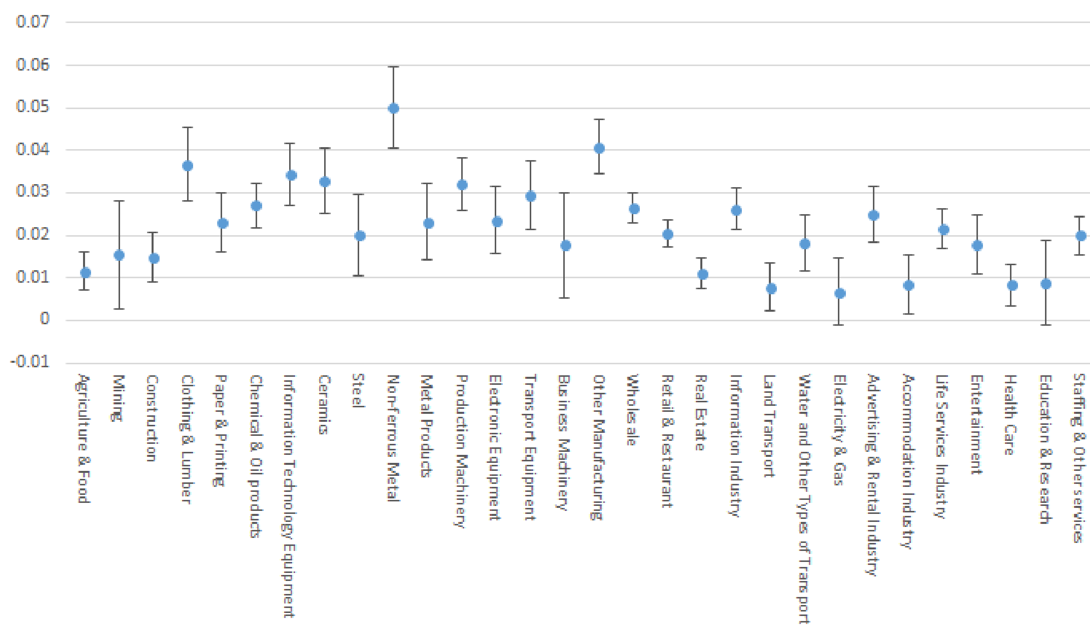
[13]We prefer to use labor productivity instead of the TFP in our baseline estimation for two reasons. First, we need to impose assumptions in order to estimate the production function and, thus, the TFP. Second, as all the estimated coefficients of the production function contain confidence intervals, the TFP that is calculated using these coefficients would be imprecisely estimated rather than directly observed. Thus, revenue per worker is the productivity measure used to measure the regression results reported in this section. However, we also repeat our tests with TFP in Appendix D.5.

errors and the measured past productivity shocks:

$$e_{it+1} = \xi_s w_{it} + \Gamma z_{it} + \gamma_{st} + \gamma_{rt} + \gamma_{gt} + \epsilon_{it+1} \tag{9}$$

where $e_{it+1} = y_{it+1} - \widehat{y_{it+1}}$ denotes the firm's forecast error between the realized y_{it+1} and the forecasted $\widehat{y_{it+1}}$ at time t. $w_{it} = a_{it} - a_{it-1}$ denotes the measured productivity shock where a_{it} is the firm's measured productivity at time t and a_{it-1} is the firm's measured productivity at time $t-1$. Our coefficient of interest is the industry-specific underreaction coefficient ξ_s. We include past measured productivity a_{it-1} in z_{it} as (the only) firm-level control, γ_{st} as industry-time, γ_{rt} as region-time, and γ_{gt} as size-time fixed effects, respectively. The region here refers to the prefecture, and the group of size contains two size groups.

Figure 8: Estimated Attenuation Coefficients across Industries: Labor Productivity



Note: This figure shows how the coefficient governs the impulse response of the (log) sales forecast error made in period $t+1$ with respect to the realized (log) productivity innovation in period t. Each dot denotes the estimate for an industry (with the 95% confidence interval), and there are 30 industries in total. Top and bottom 1% observations are trimmed out (i.e., outliers). The data frequency is semi-year, and the productivity measure measures labor productivity.

Figure 8 shows the estimated coefficients and their 95% confidence intervals. It is clear from the figure that the estimated industry-specific coefficient ξ_s is always positive and statistically significant for most industries. This substantiates the existence of information frictions even among the largest (and oldest) firms in the economy.[14] Moreover, this coefficient varies substan-

[14]Evidence from Chen et al. (2023a) suggest small and young firms face much higher degrees of information

tially across industries, ranging from 0.006 to 0.055. Interestingly, manufacturing industries seem to face higher degrees of information frictions, as they are more likely to be exposed to both domestic and foreign shocks through international trade.

5.4 Information Frictions and Micro Investment Behaviors

Investment Inaction First, we ran regressions to test whether the industry-level coefficient ξ_s negatively affects the probability of **not** investing in fixed capital at the firm level, specifically firm-level investment inaction:

$$y_{it} = \alpha \xi_s + \Gamma z_{it} + \gamma_{semi} + \epsilon_{it} \tag{10}$$

where $y_{it} = inaction_{it}$ is a binary variable of investment inaction which equals one if investment rate $\leq 1\%$ and zero otherwise. z_{it} includes various firm-level variables such as lagged (log) capital stock k_{it-1}, (log) labor productivity a_{it}, and the usage of intermediate goods per worker m_{it}. γ_{semi} is the semi year (i.e., time) fixed effects. We standardize ξ_s to facilitate the interpretation; more specifically, the mean and standard deviation of the variable are normalized to zero and one, respectively. Since our time-invariant attenuation coefficient only varies at the industry level, we cannot include firm- or industry-fixed effects in the regressions. We cluster the standard error at the industry level, as the variable of interest, ξ_s, varies at this level. We use the same simulated firm sample with 21 industries as in Section 4.2.1 to run the same model-based regression.

Table 3: Incomplete Information and Investment Inaction

	Data			Model	
	investment inaction = 1			*investment inaction* = 1	
ξ_s	-0.076**	-0.079***	-0.0544**	-0.085	-0.087
	(0.028)	(0.026)	(0.025)		
$a_{i,t}$	0.039	0.059*	0.104***	-0.028	-0.035
	(0.034)	(0.031)	(0.038)		
$k_{i,t-1}$		-0.050***	-0.049***		-0.007
		(0.009)	(0.008)		
$m_{i,t}$			-0.026		
			(0.021)		
Time FE	Yes	Yes	Yes	Yes	Yes
N	99027	99027	86294		
adj. R^2	0.038	0.069	0.063	0.052	0.053

Table 3 presents the estimation results.[15] Specifically, the first three columns present the

frictions.

[15] Table 10 in Appendix D.3 presents more detailed regression results.

regression results using our firm-level data, while the last two present the results using the simulated data. Consistent with what the theory predicts, the industry-specific attenuation coefficient negatively affects investment inaction at the firm level. Moreover, one standard deviation increase in this measure reduces the inaction probability by 6%, which is quantitatively substantial both in the data and model.[16]

Many other factors at the industry level probably can affect the average inaction rate. For instance, firms in rising industries (e.g., high-tech) tend to invest more aggressively than those in declining industries. Moreover, more volatile industries should have lower investment rates, as higher uncertainty dampens firms' incentives to invest, ceteris paribus. Therefore, although the above regression provides supporting evidence for our model's prediction, it can suffer from serious econometric issues such as omitted variable bias.

Productivity Shocks and Investment Inaction To address the above issue and further explore investment behaviors, we run an interaction regression by investigating how a *realized and unexpected* productivity innovation $w_{i,t}$ affects the firm's investment inaction and investment rate *differently* in industries with different degrees of information frictions. The key variable of interest is the interaction term, $w_{it} \times \xi_s$. Specifically, we run our regressions at the extensive (whether to invest or not) and intensive margins (how much to invest in investing) as follows:

$$y_{it} = \beta(w_{it} \times \xi_s) + \gamma w_{it} + \Gamma z_{it} + \gamma_i + \gamma_{semi} + \epsilon_{it} \tag{11}$$

where $y_{it} = inaction_{it}$ denotes the extensive-margin regression. z_{it} includes various firm-level variables such as the productivity innovation w_{it}, lag (log) capital stock k_{it-1}, lagged (log) labor productivity a_{it-1}, and the usage of intermediate goods per worker m_{it}. γ_s is the firm fixed effect, and γ_{semi} is the semi year (i.e., time) fixed effects. Again, we standardize ξ_s to facilitate the interpretation. When we run the intensive margin regression, we restrict our sample to firms active in investment. The benefit of running this interaction regression is that we can now include the firm-fixed effects to tease out the effect of all firm-level time-invariant factors that affect investment. We use the same simulated firm sample with 21 industries as in Section 4.2.1 to run the same regression.

Table 4 presents the regression results.[17] The first four columns present the regression results using our firm-level data, while the last one presents the results using the simulated data. We are flexible in the estimation, as we allow for the inclusion of various sets of fixed effects. We also back out the productivity innovations from both a random walk process (which matches the model) and from an AR(1) process. Consistently, the coefficient β is positively significant for

[16] Average inaction rate is roughly 36.6% at the semi-year frequency and 20.7% at the annual frequency.

[17] Table 11 in Appendix D.3 presents detailed results of the estimation.

Table 4: Incomplete Information and Investment Sensitivity

	Data				Model
	investment inaction = 1				*investment inaction* = 1
$\xi_s \times w_{i,t}$	0.009*	0.010**	0.010**	0.009*	0.136
	(0.005)	(0.005)	(0.005)	(0.005)	
w_{it}	-0.003	0.003	-0.005	0.002	-0.212
	(0.009)	(0.009)	(0.009)	(0.009)	
a_{it-1}	-0.028**	-0.015	-0.029**	-0.016	-0.022
	(0.012)	(0.012)	(0.011)	(0.011)	
Productivity	rand. walk	rand. walk	AR(1)	AR(1)	rand. walk
Firm FE	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y
Industry-year FE	N	Y	N	Y	-
N	84656	84656	84313	84313	
adj. R^2	0.446	0.451	0.446	0.451	0.240

Note: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. The degree of information friction is estimated at the industry level. Top and bottom 1% productivity observations are trimmed out as outliers. Lagged capital stock and the usage of intermediate goods (per worker) are included as the independent variables.

the investment inaction in the first four columns, which qualitatively matches the model, albeit with a smaller magnitude. Moreover, the quantitative magnitude of this estimate is substantial compared with the coefficient in front of the productivity shock w_{it}. In summary, the data do suggest that firms in industries with more severe information frictions (compared to those facing less severe information frictions) are more likely to remain inactive in terms of investment after a positive productivity shock.

Productivity Volatility and Investment Inaction Finally, we investigate the interaction between information friction severity and productivity volatility in determining investment inaction. Our model predicts that a rise in the volatility of fundamental productivity, σ_a, increases the inaction rate of investment, but *less* so when firms face a higher degree of information friction. Now, we test this prediction. Since our predictions are cross-sectional predictions and the information friction is measured at the industry level, we intend to measure fundamental productivity volatility σ_a differences at the regional level to be linearly independent of industry-specific factors. We calculate the volatility (i.e., standard deviation) of sales growth and average sales growth for each prefecture.[18] We do not have enough observations to reliably measure either quantity at

[18]Prefectures in Japan are analogous to states in the U.S., and there are 47 prefectures in Japan. The measure we use to calculate firm-level volatility is the firm's sales growth. To account for firm entry/exit, we use the mid-point growth rate calculation from Davis et al. (1998), which is formally $sg_{it} = \frac{sales_{i,t} - sales_{i,t-1}}{(sales_{i,t} + sales_{i,t-1})/2}$, where i indexes the firms, t refers to time, and the denominator is the average sales in semi-years t and $t-1$. Note that the sales growth rate

the further disaggregated industry-region level.

Table 5: Incomplete Information, Productivity Volatility, and Investment Inaction

	Data	Model
	inaction	
$vol_r \times \xi_s$	-0.00549**	-0.009***
	(0.00253)	(0.001)
ξ_s	-0.0551**	-0.145***
	(0.0231)	(0.001)
vol_r	0.00612	0.041***
	(0.00524)	(0.000)
Time FE	Y	Y
N	85920	4178503
adj. R^2	0.067	0.016

Note: Standard errors are clustered at the industry and the prefecture levels. * 0.10 ** 0.05 *** 0.01. The degree of information frictions is estimated at the industry level. Sales growth volatility is estimated at the prefecture level. Top and bottom 1% productivity observations are trimmed out as outliers. Both the industry-level and prefecture-level independent variables are standardized. Lagged capital stock, labor productivity, the usage of intermediate goods (per worker), and prefecture-level mean of firms' sales growth (i.e., the first moment) are included as the independent variables.

Specifically, we run the following regression:

$$y_{it} = \beta(vol_r \times \xi_s) + \gamma_1 \xi_s + \gamma_2 vol_r + \Gamma z_{i,t} + \bar{sg}_r + \gamma_{semi} + \epsilon_{it} \tag{12}$$

where the regression specifications are similar to the specifications above. $y_{it} = inaction_{it}$ denotes investment inaction dummy. z_{it} includes various firm-level variables such as lagged (log) capital stock k_{it-1}, log labor productivity a_{it} and the usage of intermediate goods (per worker) m_{it}. γ_{semi} is the semi year (i.e., time) fixed effects. We also control the mean of firms' sales growth $\bar{sg}_r$ at the regional level (i.e., the first moment). Our variable of interest is the interaction of regional volatility and industrial information friction ($vol_r \times \xi_s$). In addition to the 21 industries with different degrees of information friction ξ_s ranging from full information to baseline noise in the simulated firm sample in Section 4.2.1, we assume the 10,000 firms in each industry are equally distributed in 20 regions with σ_a^r ranging from half to double of the baseline σ_a.

Table 5 presents the results supporting our model's prediction. First, as expected, though not significant, sales growth volatility negatively affects the investment inaction rate. Second and

defined in this way is bounded by −200% and 200%. Moreover, a firm entry (exit) leads to a 200% (and −200%) sales growth. We then calculate volatility based on this sales growth measure.

importantly, the interaction term between regional sales growth volatility and industrial attenuation is estimated to be negatively significant, implying that a rise in regional-level sales growth volatility increases the investment inaction rate in industries with low information frictions. This is exactly what our model predicts. Finally, the industry-level attenuation measure is still estimated to significantly negatively affect investment inaction, even when we control for the first and second moments of regional-level sales growth volatility. Overall, the evidence confirms our model's cross-sectional prediction on how information frictions affect the relationship between fundamental productivity volatility and investment inaction.

5.5 Robustness Checks

In Appendix D.3, D.4, and D.5, we provide additional results and various robustness checks. Specifically, we present the full regression results of our main specifications in Appendix D.3. In Appendix D.4, we first show that our regression result of investment sensitivity presented in Table 4 is robust to the inclusion of the industry-year fixed effects, which absorb industry-level overall productivity change/inflation. In fact, the variable of interest becomes large and statistically more significant, as shown by Table 13. Next, we show that our regression result presented in Table 4 is robust to using the statistical innovation of labor productivity, which is estimated from an AR(1) process. See Tables 14 and 15 for details. Finally, we present our regression result of equation (11) using estimated total factor productivity (TFP). The results presented in Appendix D.5 are qualitatively similar to the results obtained using labor productivity. In summary, our above main results hold in various alternative specifications, including different estimation methods of productivity, different productivity processes, and different regression specifications.

6 Conclusion

This paper provides a framework to better understand the role of two important frictions that affect investment dynamics: irreversibility and information frictions. Our framework is tractable and generates testable theoretical predictions, many of which can be derived analytically. This stylized model can be a building block for further studies with further features, additional realism, and more complicated information problems.

We learned that investment irreversibility and information frictions interact in important ways. Two results stand out. First, information frictions introduce a new type of uncertainty that raises firms' willingness to invest, in contrast to the current effects of uncertainty in the literature; this effect reduces inaction and increases capital. Second, information frictions reduce the elasticity of investment to aggregate shocks, a valuable property for investment frictions that have macroeconomic effects in larger models.

We disciplined our model with rich Japanese firm-level data. Firm heterogeneity allowed us to test the model's predictions. We found that firms facing worse information frictions are less inactive and less elastic to productivity shocks. This confirms the theory's characteristic prediction: firms should be more elastic on the extensive margin than on the intensive margin.

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Appendix A The General Solution to the HJB

This intermediate result is used in multiple proofs that follow.

Lemma 1. *The normalized HJB (5) is solved by*

$$v(x) = me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

for some c_1 and c_2.

Proof. The normalized HJB (5) has a particular solution

$$v_p(x) = me^{\alpha x}$$

with m solved by

$$rme^{\alpha x} = e^{\alpha x} - \mu\alpha me^{\alpha x} + \frac{\sigma^2}{2}\alpha^2 me^{\alpha x}$$

$$rm = 1 - \mu\alpha m + \frac{\sigma^2}{2}\alpha^2 m$$

$$m = \frac{1}{r + \mu\alpha - \frac{\sigma^2}{2}\alpha^2}$$

The homogeneous solution is

$$v_h(x) = c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

where ϱ_1 and ϱ_2 are roots of the polynomial $-\frac{\sigma^2}{2}\varrho_j^2 + \mu\varrho_j + r = 0$. Thus the value function is

$$v(x) = me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

□

One result of Lemma 1 is that the expected value function $\hat{v}(\hat{x})$ has a similar form.

Corollary 3. *The expected value function $\hat{v}(\hat{x})$ satisfies*

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c_1 e^{\varrho_1 \hat{x}} e^{\frac{\varrho_1^2 \nu}{2}} + c_2 e^{\varrho_2 \hat{x}} e^{\frac{\varrho_2^2 \nu}{2}} \tag{13}$$

for some c.

Proof. The firm's expectation of the value function derived in Lemma 1 is

$$\hat{v}(\hat{x}) = E[v(x)|\hat{x}] = E[me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}|\hat{x}]$$

The firm's conditional expectation of x is $x \sim N(\hat{x}, v)$:

$$= \int_{-\infty}^{\infty} (me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x})\, \phi(\frac{x-\hat{x}}{\sqrt{v}})dx = \int_{-\infty}^{\infty} \left(me^{\alpha(x-\hat{x})}e^{\alpha\hat{x}} + c_1 e^{\varrho_1(x-\hat{x})}e^{\varrho_1\hat{x}} + c_2 e^{\varrho_2(x-\hat{x})}e^{\varrho_2\hat{x}}\right) \phi(\frac{x-\hat{x}}{\sqrt{v}})dx$$

Then use that $e^{\alpha(x-\hat{x})}$, $e^{\varrho_1(x-\hat{x})}$ and $e^{\varrho_2(x-\hat{x})}$ are log-normal, where the associated normal distributions have zero mean and variance $\alpha^2 v$, $\varrho_1^2 v$ and $\varrho_2^2 v$ respectively:

$$= me^{\alpha\hat{x}}e^{\frac{\alpha^2 v}{2}} + c_1 e^{\varrho_1\hat{x}}e^{\frac{\varrho_1^2 v}{2}} + c_2 e^{\varrho_2\hat{x}}e^{\frac{\varrho_2^2 v}{2}}$$

□

Appendix B Proofs

B.1 Proof of Proposition 1

Proof. The firm's conditional expectation of a_t is

$$E[a_t|\Omega_t] = a_{t-\tau} + E[a_t - a_{t-\tau}|\Omega_t]$$

From the firm's perspective, $s_t - s_{t-\tau}$ is a noisy signal of $a_t - a_{t-\tau}$:

$$s_t - s_{t-\tau} = a_t - a_{t-\tau} + n_t - n_{t-\tau}$$

the noise $n_t - n_{t-\tau}$ is independent of productivity and distributed $N(0, \tau\sigma_n^2)$, while $a_t - a_{t-\tau}$ is distributed $N(0, \tau\sigma_a^2)$. Therefore:

$$E[a_t - a_{t-\tau}|\Omega_t] = \frac{Cov(a_t - a_{t-\tau}, s_t - s_{t-\tau})}{Var(s_t - s_{t-\tau})}(s_t - s_{t-\tau}) = \frac{\tau\sigma_a^2}{\tau\sigma_a^2 + \tau\sigma_n^2}(s_t - s_{t-\tau})$$

and the definition $\gamma = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2}$ implies

$$E[a_t|\Omega_t] = a_{t-\tau} + \gamma\,(s_t - s_{t-\tau})$$

The nowcast errors $u_t = a_t - E[a_t|\Omega_t]$ are normally distributed and have variance

$$Var\,(a_t - E[a_t|\Omega_t]) = Var\,(a_t - a_{t-\tau} - \gamma\,(s_t - s_{t-\tau})) = Var\,((1-\gamma)(a_t - a_{t-\tau}) - \gamma\,(n_t - n_{t-\tau}))$$

$$= Var\,((1-\gamma)(a_t - a_{t-\tau})) + Var\,(\gamma\,(n_t - n_{t-\tau})) = (1-\gamma)^2\tau\sigma_a^2 + \gamma^2\tau\sigma_n^2$$

$$= \left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2 \tau\sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \tau\sigma_n^2 = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2} = \nu$$

□

B.2 Proof of Proposition 2

Proof. Proposition 1 implies that the diffusion for $\hat{a}_t$ is given by

$$d\hat{a}_t = da_{t-\tau} + \gamma(ds_t - ds_{t-\tau}) = (1-\gamma)da_{t-\tau} + \gamma da_t + \gamma dn_t - \gamma dn_{t-\tau}$$

$$= (1-\gamma)\sigma_a dW_{t-\tau}^A + \gamma\sigma_a dW_t^A + \gamma\sigma_n dW_t^n - \gamma\sigma_n dW_{t-\tau}^n$$

The right-hand side is the sum of independent innovations, so they can be recomposed as innovations to a single Wiener process:

$$d\hat{a}_t = \sigma_{\hat{a}} dW^{\hat{a}}$$

It remains to show that $\sigma_{\hat{a}} = \sigma_A$. The independence of the innovations imply

$$\sigma_{\hat{a}}^2 = (1-\gamma)^2\sigma_a^2 + \gamma^2\sigma_a^2 + 2\gamma^2\sigma_n^2$$

$$= \left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_a^2 + 2\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_n^2$$

$$= \frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2} + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_n^2$$

$$= \frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2} + \frac{\sigma_a^4}{\sigma_a^2+\sigma_n^2} = \sigma_a^2$$

As a result, innovations to nowcast errors follow

$$du_t = da_t - d\hat{a}_t = \sigma_a dW_t^A - ((1-\gamma)\sigma_a dW_{t-\tau}^A + \gamma\sigma_a dW_t^A + \gamma\sigma_n dW_t^n - \gamma\sigma_n dW_{t-\tau}^n)$$

$$= (1-\gamma)(\sigma_a dW_t^A - \sigma_a dW_{t-\tau}^A) - \gamma(\sigma_n dW_t^n - \sigma_n dW_{t-\tau}^n) = \sigma_u dW_t^u$$

Again, independence of the innovations implies

$$\sigma_u^2 = 2(1-\gamma)^2\sigma_a^2 + 2\gamma^2\sigma_n^2$$

$$= 2\left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_a^2 + 2\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_n^2 = 2\frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2}$$

$\square$

B.3 Proof of Proposition 3

Proof. The value-matching and super contact conditions at infinity are standard.

Corollary 3 gives the firm's expectation of the value function in terms of two roots ϱ_1 and ϱ_2. The conditions at infinity imply that the coefficient on the positive root is zero. We write the remaining negative root ϱ and coefficient c without subscripts; the expected value function becomes

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}}e^{\frac{\alpha^2 v}{2}} + ce^{\varrho\hat{x}}e^{\frac{\varrho^2 v}{2}}$$

which in levels is

$$\hat{V}(\hat{X};\hat{B}) = m\hat{X}^{\alpha}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\hat{X}^{\varrho}e^{\frac{\varrho^2 v}{2}} \tag{14}$$

This solution is written as a function of the boundary $\hat{B}$, to be clear about how the choice of $\hat{B}$ determines which solution to the HJB is the true value function.

To derive the value-matching condition at the boundary, use that firms are indifferent between applying the infinitesimal regulator dI at the boundary $\hat{B}$:

$$\hat{V}(\hat{B}) = \hat{V}(\hat{B} + dI) - \psi dI$$

$$\hat{V}(\hat{B}) = \hat{V}(\hat{B}) + \hat{V}'(\hat{B})dI - \psi dI$$

$$\implies \psi = \hat{V}'(\hat{B})$$

To derive the super-contact condition at the boundary, first consider the problem of a firm: their only decision is to select the critical value $\hat{B}$ that maximizes their value (14). The first order condition for this problem is

$$c'(\hat{B})\hat{X}^{\varrho}e^{\frac{v}{2}} = 0 \tag{15}$$

Next, take the derivative of the value matching condition $\psi = \hat{V}'(\hat{B})$ with respect to $\hat{B}$:

$$0 = m\alpha\hat{B}^{\alpha-1}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\varrho\hat{B}^{\varrho-1}e^{\frac{v}{2}} + c'(\hat{B})\hat{B}^{\varrho}e^{\frac{\varrho^2 v}{2}}$$

then substitute using (15) to find the super contact condition:

$$0 = m\alpha\hat{B}^{\alpha-1}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\varrho\hat{B}^{\varrho-1}e^{\frac{\varrho^2 v}{2}} = \hat{V}''(\hat{B})$$

$\square$

B.4 Proof of Proposition 4

Proof. Per Corollary 3 , the first derivative of the value function in expected log normalized capital $\hat{v}(\hat{x})$ is

$$\hat{v}'(\hat{x}) = m\alpha e^{\alpha\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho e^{\varrho\hat{x}} e^{\frac{\varrho^2 \nu}{2}}$$

Apply this to the value-matching condition from Proposition 3 (using $\hat{V}'(\hat{X}) = \frac{d\hat{V}(\hat{X})}{d\hat{X}} = \frac{d\hat{V}(\hat{X})}{d\hat{x}} \frac{d\hat{x}}{d\hat{X}} = \hat{v}'(\hat{x})e^{-\hat{x}}$):

$$\psi = \hat{v}'(\hat{b})e^{-\hat{b}} = m\alpha e^{(\alpha-1)\hat{b}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho e^{(\varrho-1)\hat{b}} e^{\frac{\varrho^2 \nu}{2}} \tag{16}$$

Before evaluating the super contact condition, it is helpful to rewrite $\hat{V}''(\hat{X})$ in terms of $\hat{x}$:

$$\hat{V}''(\hat{X}) = \frac{d\hat{V}'(\hat{X})}{d\hat{X}} = \frac{d\hat{V}'(\hat{X})}{d\hat{x}} \frac{d\hat{x}}{d\hat{X}} = \frac{d\hat{v}'(\hat{x})e^{-\hat{x}}}{d\hat{x}} \frac{1}{\hat{X}} = (\hat{v}''(\hat{x})e^{-\hat{x}} - \hat{v}'(\hat{x})e^{-\hat{x}})\frac{1}{\hat{X}}$$

$$= (\hat{v}''(\hat{x}) - \hat{v}'(\hat{x}))e^{-2\hat{x}} = m\alpha(\alpha-1)e^{(\alpha-2)\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho(\varrho-1)e^{(\varrho-2)\hat{x}} e^{\frac{\varrho^2 \nu}{2}}$$

The super contact condition from Proposition 3 becomes

$$0 = \hat{V}''(\hat{b}) = m\alpha(\alpha-1)e^{(\alpha-2)\hat{b}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho(\varrho-1)e^{(\varrho-2)\hat{b}} e^{\frac{\varrho^2 \nu}{2}} \tag{17}$$

Equations (16) and (17) imply

$$\psi(1-\varrho) = m\alpha(\alpha-\varrho)e^{(\alpha-1)\hat{b}} e^{\frac{\alpha^2 \nu}{2}}$$

$$\implies \hat{b} = \frac{1}{1-\alpha} \log\left(\frac{m\alpha(\alpha-\varrho)}{\psi(1-\varrho)}\right) + \frac{\alpha^2 \nu}{2(1-\alpha)}$$

□

B.5 Proof of Proposition 5

Proof. Per Proposition 1:

$$\frac{d}{dW^a_{t-h}} \mathbb{E}[a_t|\Omega_t] = \frac{d}{dW^a_{t-h}} (a_{t-\tau} + \gamma(s_t - s_{t-\tau}))$$

There are two cases. In both, $\frac{ds_t}{dW^a_{t-h}} = 1$. But if $0 \leq h < \tau$, then $\frac{da_{t-\tau}}{dW^a_{t-h}} = \frac{ds_{t-\tau}}{dW^a_{t-h}} = 0$:

$$[0 \leq h < \tau] : \qquad \frac{d}{dW^a_{t-h}} \mathbb{E}[a_t|\Omega_t] = \gamma \frac{d}{dW^a_{t-h}} s_t = \gamma$$

If $h \geq \tau$, then $\frac{da_{t-\tau}}{dW^a_{t-h}} = \frac{ds_{t-\tau}}{dW^a_{t-h}} = 1$:

$$[h \geq \tau] : \qquad \frac{d}{dW^a_{t-h}} \mathbb{E}[a_t | \Omega_t] = 1 + \gamma - \gamma = 1$$

□

B.6 Proof of Proposition 6

Proof. The general solution to the ODE (6) is

$$h(\hat{x}) = c_{h1} e^{-\rho_1 \hat{x}} + c_{h2} e^{-\rho_2 \hat{x}}$$

where ρ_1 and ρ_2 are roots of the equation

$$0 = D\rho_j^2 - \delta\rho_j - \eta$$

Using $D = \frac{\sigma_a^2}{2}$, the roots are given by

$$\rho_j = \frac{\delta}{\sigma_a^2} \pm \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$$

To satisfy the boundary condition at infinity, only the positive root can have a non-zero coefficient. Therefore, the solution simplifies to

$$h(\hat{x}) = c_h e^{-\rho \hat{x}}$$

where ρ (without subscript) denotes the positive root $\frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$. The coefficient c_h is yet to be found.

The remaining boundary condition is that $h(\hat{x})$ integrates to one:

$$1 = \int_{\hat{b}}^{\infty} c_h e^{-\rho \hat{x}} d\hat{x} = \frac{c_h}{\rho} e^{-\rho \hat{b}}$$

which implies

$$c_h = \rho e^{\rho \hat{b}}$$

□

B.7 Proof of Proposition 7

Proof. The joint distribution $f_{\hat{x},u}(\hat{x},u)$ implies

$$f_x(x) = h(x) * \phi(-\frac{x}{\sqrt{v}})$$

$$= \int_{-\infty}^{\infty} h(\hat{x})\phi(\frac{\hat{x}-x}{\sqrt{v}})d\hat{x}$$

$h(\hat{x}) = 0$ for $\hat{x} < \hat{b}$, so the convolution becomes

$$= \int_{\hat{b}}^{\infty} h(\hat{x})\phi(\frac{\hat{x}-x}{\sqrt{v}})d\hat{x} = \int_{\hat{b}}^{\infty} \rho e^{-\rho(\hat{x}-\hat{b})}\frac{1}{\sqrt{2\pi v}}e^{-\frac{(\hat{x}-x)^2}{2v}}d\hat{x}$$

$$= e^{-\frac{x^2}{2v}}\int_{\hat{b}}^{\infty} \rho e^{\rho\hat{b}}\frac{1}{\sqrt{2\pi v}}e^{-\frac{\hat{x}^2-2(x-v\rho)\hat{x}}{2v}}d\hat{x} = e^{-\frac{x^2-(x-v\rho)^2}{2v}}\int_{\hat{b}}^{\infty} \rho e^{\rho\hat{b}}\frac{1}{\sqrt{2\pi v}}e^{-\frac{\hat{x}^2-2(x-v\rho)\hat{x}+(x-v\rho)^2}{2v}}d\hat{x}$$

$$= \rho e^{-\rho(x-\hat{b})}e^{\frac{v\rho^2}{2}}\int_{\hat{b}}^{\infty} \frac{1}{\sqrt{2\pi v}}e^{-\frac{(\hat{x}-(x-v\rho))^2}{2v}}d\hat{x} = \rho e^{-\rho(x-\hat{b})}e^{\frac{v\rho^2}{2}}\int_{\hat{b}}^{\infty}\phi(\frac{\hat{x}-(x-v\rho)}{\sqrt{v}})d\hat{x}$$

$$= \rho e^{-\rho(x-\hat{b})}e^{\frac{v\rho^2}{2}}\left(1-\Phi\left(\frac{\hat{b}+v\rho-x}{\sqrt{v}}\right)\right) = h(x)e^{\frac{v\rho^2}{2}}\Phi\left(\frac{x-(\hat{b}+v\rho)}{\sqrt{v}}\right)$$

□

B.8 Proof of Proposition 8

Proof. Decompose normalized capital into the independent nowcasts and errors by $x = \hat{x} - u$:

$$\int_{-\infty}^{\infty} e^x f_x(x)dx = \int_{\hat{b}}^{\infty}\int_{-\infty}^{\infty} e^{\hat{x}-u} f_{\hat{x},u}(\hat{x},u)dud\hat{x}$$

$$= \int_{\hat{b}}^{\infty}\int_{-\infty}^{\infty} e^{\hat{x}-u}h(\hat{x})\phi(\frac{u}{\sqrt{v}})dud\hat{x} = \int_{\hat{b}}^{\infty} e^{\hat{x}}h(\hat{x})\int_{-\infty}^{\infty} e^{-u}\phi(\frac{u}{\sqrt{v}})dud\hat{x}$$

Use that $\int_{-\infty}^{\infty} e^{-u}\phi(\frac{u}{\sqrt{v}})du = e^{\frac{v}{2}}$ is the mean of a log-normal distribution:

$$= e^{\frac{v}{2}}\int_{\hat{b}}^{\infty} e^{\hat{x}}h(\hat{x})d\hat{x} = e^{\frac{v}{2}}\int_{\hat{b}}^{\infty} e^{\hat{x}}\rho e^{-\rho(\hat{x}-\hat{b})}d\hat{x}$$

$$= \frac{e^{\frac{v}{2}+\hat{b}}\rho}{\rho-1}$$

which is increasing in v, per Proposition 1, and $v = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is increasing in σ_n^2 and τ. □

B.9 Proof of Proposition 10

Proof. Per Corollary 3, the firm's expectation of the value function derived in Lemma 1 is

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}}e^{\frac{\alpha^2\nu}{2}} + c_1e^{\varrho_1\hat{x}}e^{\frac{\varrho_1^2\nu}{2}} + c_2e^{\varrho_2\hat{x}}e^{\frac{\varrho_2^2\nu}{2}}$$

which in levels is

$$\hat{V}(\hat{X};\hat{B}) = m\hat{X}^{\alpha}e^{\frac{\alpha^2\nu}{2}} + c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} \tag{18}$$

This solution is written as a function of the boundaries $(\hat{B}_L, \hat{B}_U)$, to be clear about how the boundary choice determines which solution to the HJB is the true value function.

To derive the value-matching condition at the lower boundary, use that firms are indifferent between applying the infinitesimal regulator dI at the boundary $\hat{B}_L$:

$$\hat{V}(\hat{B}_L) = \hat{V}(\hat{B}_L + dI) - \psi_+ dI$$

$$\hat{V}(\hat{B}_L) = \hat{V}(\hat{B}_L) + \hat{V}'(\hat{B}_L)dI - \psi_+ dI$$
$$\implies \psi_+ = \hat{V}'(\hat{B}_L)$$

and a similar argument gives the value-matching condition at the upper boundary:

$$\psi_- = \hat{V}'(\hat{B}_U)$$

To derive the super-contact condition at the boundary, first consider the problem of a firm: their only decision is to select the critical values $\hat{B}_L$ and $\hat{B}_U$ that maximize their value (14). The first order conditions for this problem are

$$\partial_{\hat{B}_L}c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_L}c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} = 0 \tag{19}$$

$$\partial_{\hat{B}_U}c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_U}c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} = 0 \tag{20}$$

Next, take the derivative of the value matching condition $\psi_+ = \hat{V}'(\hat{B}_L)$ with respect to $\hat{B}_L$:

$$0 = m\alpha\hat{B}_L^{\alpha-1}e^{\frac{\alpha^2\nu}{2}} + c_1(\hat{B}_L,\hat{B}_U)\varrho_1\hat{B}_L^{\varrho_1-1}e^{\frac{\varrho_1^2\nu}{2}} + c_2(\hat{B}_L,\hat{B}_U)\varrho_2\hat{B}_L^{\varrho_2-1}e^{\frac{\varrho_2^2\nu}{2}} + \partial_{\hat{B}_L}c_1(\hat{B}_L,\hat{B}_U)\hat{B}_L^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_L}c_2(\hat{B}_L,\hat{B}_U)\hat{B}_L^{\varrho_1}e^{\frac{\varrho_2^2\nu}{2}}$$

then substitute using (19) to find the super contact condition:

$$0 = m\alpha \hat{B}_L^{\alpha-1} e^{\frac{\alpha^2 \nu}{2}} + c_1(\hat{B}_L, \hat{B}_U)\varrho_1 \hat{B}_L^{\varrho_1 - 1} e^{\frac{\varrho_1^2 \nu}{2}} + c_2(\hat{B}_L, \hat{B}_U)\varrho_2 \hat{B}_L^{\varrho_2 - 1} e^{\frac{\varrho_2^2 \nu}{2}} = \hat{V}''(\hat{B}_L)$$

Again, a similar argument taking the derivative of the value matching condition $\psi_- = \hat{V}'(\hat{B}_U)$ with respect to $\hat{B}_U$ gives the super contact condition at the upper boundary:

$$0 = \hat{V}''(\hat{B}_U)$$

□

B.10 Proof of Proposition 11

Before the proof, we prove a lemma that is independently useful for computing the model:

Lemma 2. *The difference between the upper and lower log bounds of the inaction region $\Delta \equiv \hat{b}_H - \hat{b}_L$ solves the implicit equation*

$$\frac{\psi_- e^{(1-\varrho_2)\Delta} - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+}{\psi_- e^{(1-\varrho_2)\Delta} - \varrho_1 \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \varrho_2 \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+} = \frac{\left(e^{(\alpha-\varrho_2)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-\varrho_2)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}{\left(\alpha e^{(\alpha-\varrho_2)\Delta} - \varrho_1 \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-\varrho_2)\Delta} + \varrho_2 \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)} \tag{21}$$

and the lower bound $\hat{b}_L$ is given in terms of Δ by

$$\hat{b}_L = \frac{\alpha^2 \nu}{2(1-\alpha)} - \frac{1}{1-\alpha} \log\left(\frac{1}{m\alpha} \frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta}} \right) \tag{22}$$

Proof. Per Corollary 3, the first derivative of the value function in expected log normalized capital $\hat{v}(\hat{x})$ is

$$\hat{v}'(\hat{x}) = m\alpha e^{\alpha \hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c_1 \varrho_1 e^{\varrho_1 \hat{x}} e^{\frac{\varrho_1^2 \nu}{2}} + c_2 \varrho_2 e^{\varrho_2 \hat{x}} e^{\frac{\varrho_2^2 \nu}{2}}$$

Apply this to the value-matching conditions (using $\hat{V}'(\hat{X}) = \frac{d\hat{V}(\hat{X})}{d\hat{X}} = \frac{d\hat{V}(\hat{X})}{d\hat{x}} \frac{d\hat{x}}{d\hat{X}} = \hat{v}'(\hat{x})e^{-\hat{x}}$):

$$\psi_+ e^{\hat{b}_L} = \hat{v}'(\hat{b}_L) = m\alpha e^{\alpha \hat{b}_L} e^{\frac{\alpha^2 \nu}{2}} + c_1 \varrho_1 e^{\varrho_1 \hat{b}_L} e^{\frac{\varrho_1^2 \nu}{2}} + c_2 \varrho_2 e^{\varrho_2 \hat{b}_L} e^{\frac{\varrho_2^2 \nu}{2}} \tag{23}$$

$$\psi_- e^{\hat{b}_H} = \hat{v}'(\hat{b}_H) = m\alpha e^{\alpha \hat{b}_H} e^{\frac{\alpha^2 \nu}{2}} + c_1 \varrho_1 e^{\varrho_1 \hat{b}_H} e^{\frac{\varrho_1^2 \nu}{2}} + c_2 \varrho_2 e^{\varrho_2 \hat{b}_H} e^{\frac{\varrho_2^2 \nu}{2}} \tag{24}$$

Before evaluating the super contact conditions, it is helpful to rewrite $\hat{V}''(\hat{X})$ in terms of $\hat{x}$:

$$\hat{V}''(\hat{X}) = \frac{d\hat{V}'(\hat{X})}{d\hat{X}} = \frac{d\hat{V}'(\hat{X})}{d\hat{x}}\frac{d\hat{x}}{d\hat{X}} = \frac{d\hat{v}'(\hat{x})e^{-\hat{x}}}{d\hat{x}}\frac{1}{\hat{X}} = (\hat{v}''(\hat{x})e^{-\hat{x}} - \hat{v}'(\hat{x})e^{-\hat{x}})\frac{1}{\hat{X}}$$

$$= (\hat{v}''(\hat{x}) - \hat{v}'(\hat{x}))e^{-2\hat{x}}$$

Thus the super contact conditions $0 = \hat{V}''(\hat{B}_L)$ and $0 = \hat{V}''(\hat{B}_H)$ imply $\hat{v}''(\hat{b}_L) = \hat{v}'(\hat{b}_L)$ and $\hat{v}''(\hat{b}_H) = \hat{v}'(\hat{b}_H)$ respectively. These conditions become

$$\psi_+ e^{\hat{b}_L} = \hat{v}''(\hat{b}_L) = m\alpha e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{b}_L}e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L}e^{\frac{\varrho_2^2\nu}{2}} \tag{25}$$

$$\psi_- e^{\hat{b}_H} = \hat{v}''(\hat{b}_H) = m\alpha^2 e^{\alpha\hat{b}_H}e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1^2 e^{\varrho_1\hat{b}_H}e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2^2 e^{\varrho_2\hat{b}_H}e^{\frac{\varrho_2^2\nu}{2}} \tag{26}$$

Combining the b_L value-matching condition (23) and super contact condition (25) can be used to solve for c_1 and c_2 in terms of b_L. First, difference out the c_2 term:

$$(1-\varrho_2)\psi_+ e^{\hat{b}_L} = m\alpha(\alpha-\varrho_2)e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1(\varrho_1-\varrho_2)e^{\varrho_1\hat{b}_L}e^{\frac{\varrho_1^2\nu}{2}}$$

$$\implies c_1\varrho_1 e^{\frac{\varrho_1^2\nu}{2}} = \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_1)\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_1)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}$$

Plug back into the value matching condition (23):

$$\psi_+ e^{\hat{b}_L} = m\alpha e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L}e^{\frac{\varrho_2^2\nu}{2}}$$

$$\implies c_2\varrho_2 e^{\frac{\varrho_2^2\nu}{2}} = \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_2)\hat{b}_L} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_2)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}$$

Use these expressions to substitute for c_1 and c_2 in the b_H value matching condition (24):

$$\begin{aligned}\psi_- e^{\hat{b}_H} = {} & m\alpha e^{\alpha\hat{b}_H}e^{\frac{\alpha^2\nu}{2}} \\ & + \left(\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_1)\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_1)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}\right)e^{\varrho_1\hat{b}_H} \\ & + \left(\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_2)\hat{b}_L} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_2)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}\right)e^{\varrho_2\hat{b}_H}\end{aligned}$$

Express in terms of the difference $\Delta \equiv \hat{b}_H - \hat{b}_L$:

$$\psi_- = m\alpha e^{(\alpha-1)(\hat{b}_L+\Delta)} e^{\frac{\alpha^2 \nu}{2}} + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_1-1)\Delta} - m\alpha \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_1-1)\Delta} e^{\frac{\alpha^2 \nu}{2}} \\ + \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_2-1)\Delta} + m\alpha \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_2-1)\Delta} e^{\frac{\alpha^2 \nu}{2}} \tag{27}$$

and do the same for the super contact condition (26):

$$\psi_- = m\alpha^2 e^{(\alpha-1)(\hat{b}_L+\Delta)} e^{\frac{\alpha^2 \nu}{2}} + \varrho_1 \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_1-1)\Delta} - m\alpha\varrho_1 \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_1-1)\Delta} e^{\frac{\alpha^2 \nu}{2}} \\ + \varrho_2 \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_2-1)\Delta} + m\alpha\varrho_2 \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_2-1)\Delta} e^{\frac{\alpha^2 \nu}{2}} \tag{28}$$

Collect terms in equation (27):

$$\psi_- = m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 \nu}{2}} \left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right) \\ + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_1-1)\Delta} + \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_2-1)\Delta}$$

and equation (28):

$$\psi_- = m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 \nu}{2}} \left(\alpha e^{(\alpha-1)\Delta} - \varrho_1 \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \varrho_2 \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right) \\ + \varrho_2 \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_2-1)\Delta} + \varrho_1 \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_1-1)\Delta}$$

Rearrange both to isolate $\hat{b}_L$:

$$m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 \nu}{2}} = \frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_2-1)\Delta}}{\left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)}$$

$$m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 \nu}{2}} = \frac{\psi_- - \varrho_1 \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_1-1)\Delta} - \varrho_2 \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)} \psi_+ e^{(\varrho_2-1)\Delta}}{\left(\alpha e^{(\alpha-1)\Delta} - \varrho_1 \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \varrho_2 \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)}$$

either of which give $\hat{b}_L$ in terms of Δ.

Combining the two equations yields an implicit equation determining Δ:

$$\frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{\psi_- - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}} = \frac{\left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}\right)}{\left(\alpha e^{(\alpha-1)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}\right)}$$

which can be rearranged as

$$\frac{\psi_- e^{(1-\varrho_2)\Delta} - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+}{\psi_- e^{(1-\varrho_2)\Delta} - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+} = \frac{\left(e^{(\alpha-\varrho_2)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}{\left(\alpha e^{(\alpha-\varrho_2)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}$$

□

With Lemma 2 in hand, proving Proposition 11 is straightforward:

Proof of Proposition 11. ν does not appear in equation (21), so Δ is unaffected by the information friction. b_L^{FI} denotes the solution for $\nu = 0$. Equation (22) implies

$$\hat{b}_L = b_L^{FI} + \frac{\alpha^2\nu}{2(1-\alpha)}$$

Finally, $\Delta = \hat{b}_H - \hat{b}_L = b_H^{FI} - b_L^{FI}$ implies

$$\hat{b}_H = b_H^{FI} + \frac{\alpha^2\nu}{2(1-\alpha)}$$

□

Appendix C Partial Irreversibility

In this appendix, we modify the baseline model to relax the assumption of full irreversibility.

Investment I is now partially irreversible. If firms invest, they do so at cost $\Psi(I)$:

$$\Psi(I) = \begin{cases} \psi_+ I & I \geq 0 \\ \psi_- I & I < 0 \end{cases}$$

with $\psi_+ > \psi_- > 0$. Accordingly, their instantaneous profit is $\pi = A^{1-\alpha}K^{\alpha} - \Psi(I)$.

Optimal firm behavior for this type of problem is characterized by an inaction region: for a range of capital values (that depends on other state variables), firms choose to neither invest nor divest. Firms with partial irreversibility face the usual full information HJB equation (1) in the inaction region.

Optimal investment behavior is a threshold strategy, as in the full information case. Except now, a firm invests only if its expected normalized capital $\hat{X}$ is less than than some critical lower value $\hat{B}_L$, and divests only if above some upper value $\hat{B}_U$. So solving the firm's problem comes down to finding the optimal choice of $\hat{B}_L$ and $\hat{B}_U$. Proposition 3 reports the boundary conditions associated with the optimum. They are analogous to the full information case.

Proposition 10. *Under incomplete information, the boundary conditions consist of two value-matching conditions:*

$$\hat{V}'(\hat{B}_L) = \psi_+ \qquad \hat{V}'(\hat{B}_U) = \psi_-$$

and two super contact conditions:

$$\hat{V}''(\hat{B}_L) = 0 \qquad \hat{V}''(\hat{B}_U) = 0$$

Proof: Appendix B.9

Proposition 11 summarizes the solution to the firm's problem. The log critical values $\hat{b}_L \equiv \log \hat{B}_L$ and $\hat{b}_U \equiv \log \hat{B}_U$ depend on several parameters: the interest rate r, depreciation rate δ, time series properties of the productivity process, the investment and divestment costs ψ_+ and ψ_-, and so forth. But conveniently, most of these terms affect the critical values in the same way that they would in the full information model. The proposition shows that the difference between full and incomplete information critical values depend only on the variance of nowcast errors v, and the returns to scale α.

Proposition 11. *The critical values of expected normalized capital are*

$$\hat{b}_L = b_L^{FI} + \frac{\alpha^2 v}{2(1-\alpha)} \qquad \hat{b}_H = b_H^{FI} + \frac{\alpha^2 v}{2(1-\alpha)}$$

where b_L^{FI} and b_H^{FI} denote the full information solutions such that $v = 0$.

Proof: Appendix B.10

Appendix D Quantitative and Empirical Analyses

D.1 Calibration and Data Statistics

We lay out additional data moments for our calibration here. Table 6 shows our moments in semi-year frequency since the Japanese forecast data is only available in semi-year frequency. Table 7 shows the data moments in semi-year frequency that only use fixed investment as the firms' total investment rates. We find the moments are not dramatically different from Table 6. Therefore, we calibrate our model mainly following the investment rates using firms' total investment rates.

Table 6: Baseline Moments (Semi-Year Frequency)

Moments	Model	Data
Investment Moments		
Aggregate Investment Rate	2.73%	2.64%
Investment Rate Mean	4.29%	4.00%
Investment Rate S.D.	10.1%	18.7%
Investment Inaction Rate	72.8%	36.6%
Investment Spike Rate	8.4%	2.45%
Other Moments (semi-year)		
(Log) Forecast Error Autocorrelation (for sales)	-	0.251

Notes: This table summarizes our baseline moments from the model and their mapping in our data. Since our data is in semi-year frequency, we also report the moments in semi-year. Data Sources: Economic Outlook Survey and Financial Statement Survey of Corporations of Japan (2004-2019). All statistics are calculated using variables defined at the semi-year frequency. Investment is the sum of equipment/machinery/land investments and purchases of software. Capital is the amount of fixed capital (including the software). Investment spike refers to investment rate $>$ 20%, and investment inaction refers to investment rate $\leq$ 1%. Forecast error is defined as the log deviation of the realized sales in period t from the sales forecast made in period $t-1$. Top and bottom 1% log sales forecast errors are trimmed (i.e., outliers). The average revenue of firms is around 38 billion JPY (equivalently 330 million USD) per semi-year.

Finally, we want to emphasize that we cannot hit all the moments in our sample partially because our Japanese firm-level data is highly skewed toward large firms, and we need to match the expected survey data. The data truncates the sample by utilizing firms with more than 100 million JPY registered capital only, while the original dataset contains both large and small firms. Table 8 compares the entire sample and the sample used for our analysis. Our sample is significantly larger in firm sizes.

Table 7: Moments using Fixed Investment Only (Semi-Year Frequency)

Moments	Data
Investment Moments	
Aggregate Investment Rate	2.64%
Investment Rate Mean	4.00%
Investment Rate S.D.	18.7%
Investment Inaction Rate	36.6%
Investment Spike Rate	2.45%
Other Moments	
(Log) Forecast Error Autocorrelation (for sales)	0.251

Data Sources: Economic Outlook Survey and Financial Statement Survey of Corporations of Japan (2004-2019). All statistics are calculated using variables defined at the semi-year frequency. Investment is the sum of equipment/machinery/land investments. Capital is the amount of fixed capital. Investment spike refers to investment rate $> 20\%$, and investment inaction refers to investment rate $\leq 1\%$. Forecast error is defined as the log deviation of the realized sales in period t from the sales forecast made in period $t-1$. Top and bottom 1% log sales forecast errors are trimmed (i.e., outliers). The average revenue of firms is around 38 billion JPY (equivalently 330 million USD) per semi-year.

Table 8: Sample Comparison (Quarterly Frequency)

Moments	Merged dataset	Entire Sample (FSS)
The number of obs. (non-missing sales)	392,158	1,260,836
Average employment	1040.582	491.6123
Average sales (million JPY)	19991.75	8541.767
Average fixed capital stock	59919.34	24842.79

Notes: The time span is 2004-2018 (15 years and 60 quarters)

Table 9: Summary Statistics of Untrimmed Forecast Errors (Semi-Year Frequency)

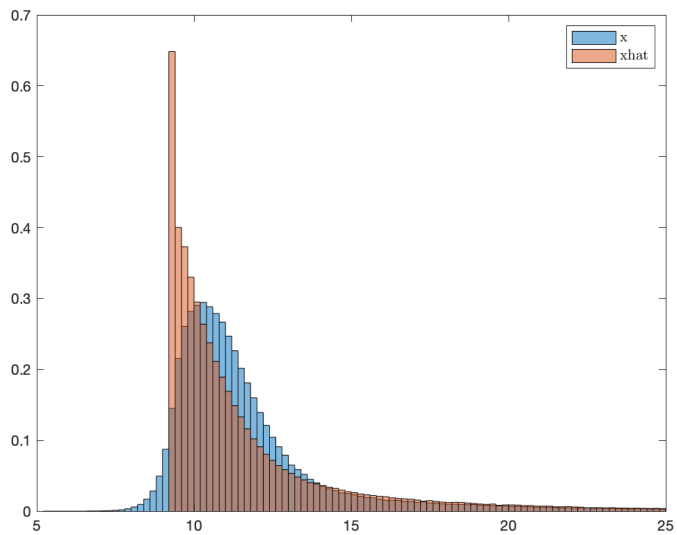
Variable	Obs.	mean	median	standard deviation	min.	max.
log forecast error of sales	119,335	-.0106	-.0005	0.199	-8.472	5.759
percentage forecast error of sales	119,359	.0198	-.0005	1.556	1	316

Notes: The time span is 2004-2018 (15 years and 29 semi-years). The forecast error is defined as the deviation of realized sales from the forecasted sales (made at the beginning of each semi-year).

D.2 Simulation

We lay out the details for our simulation here. For the distributions in Figure 5, we simulate 500,000 firms for 500 quarters. For the industry-level correlations, we simulate 10,000 firms for 50 quarters for each industry. Since our simulation is in discrete time (with a period interval denoting one quarter), we would like to provide validation that when the interval shrinks, the simulation would be consistent with the continuous time model. Figure 9 shows that our simulated distribution of x and $\hat{x}$ is consistent with the theoretical predictions.

Figure 9: Simulated Stationary Distribution for Expected and Realized Normalized Capital



Notes: These figures show the simulation counterparts of the theoretical normalized capital distribution in Figure 2. The blue histogram is the distribution of x, and the orange histogram is the $\hat{x}$ distribution. We exclude the large fixed mess of entry firms at the boundary so the simulated stationary distribution is consistent with the theoretical one.

D.3 Regression Results using Labor Productivity

We present the full regression results of our main specifications in this subsection of appendix. See Tables 10-12 below for more details.

D.4 Main Robustness Checks

First, regression results reported in Table 11 are robust to including the industry-year fixed effects, which absorb industry-level overall productivity change/inflation. In fact, the variable of interest becomes large and statistically more significant, as shown by Table 13. Next, although log productivity is assumed to follow a random walk in the model, it is possibly true that the log productivity follows an AR(1) process in the data. As a result, we need to reconstruct our (log) productivity innovation in such a case. To tackle this problem, we first run an AR(1) regression using the current (and lagged) log productivity and obtain the statistical innovations of the log

Table 10: Incomplete Information and Investment Inaction: Labor Productivity

	investment inaction = 1		
ξ_s	-0.0762**	-0.0786***	-0.0544**
	(0.0282)	(0.0259)	(0.0252)
$a_{i,t}$	0.0386	0.0586*	0.104***
	(0.0340)	(0.0312)	(0.0375)
$k_{i,t-1}$		-0.0496***	-0.0489***
		(0.00859)	(0.00838)
$m_{i,t}$			-0.0255
			(0.0210)
Time FE	Yes	Yes	Yes
N	99027	99027	86294
adj. R^2	0.038	0.069	0.063

Note: Standard errors are clustered at the industry level. * 0.10 ** 0.05 *** 0.01. The degree of information friction is estimated at the industry level. Top and bottom 1% productivity obs. are trimmed out as outliers.

Table 11: Incomplete Information and Investment Sensitivity: Labor Productivity

	(1)	(2)	(3)	(4)
	inv. inaction = 1		*inv. rate* (*inv. inaction* $\neq$ 1)	
$\xi_s \times w_{i,t}$	0.00848*	0.00885*	-0.0400	-0.0408
	(0.00466)	(0.00465)	(0.0386)	(0.0388)
$w_{i,t}$	0.00213	-0.00344	0.0170	0.0325
	(0.00931)	(0.00931)	(0.0179)	(0.0279)
$a_{i,t}$	-0.0204*	-0.0281**	-0.0259	-0.00409
	(0.0119)	(0.0120)	(0.0299)	(0.0158)
$m_{i,t}$	-0.00891	-0.00523	0.00593	-0.00534
	(0.00551)	(0.00552)	(0.00411)	(0.00795)
$k_{i,t-1}$		0.0771***		-0.152
		(0.00857)		(0.103)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
N	84656	84656	57143	57143
adj. R^2	0.444	0.446	0.045	0.059

Note: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are trimmed out (i.e., outliers).

Table 12: Investment Inaction and Region-level Volatility: Labor Productivity

	inv. inaction = 1		
$vol_r \times \xi_s$	-0.0113**	-0.00927**	-0.00549**
	(0.00434)	(0.00368)	(0.00253)
ξ_s	-0.0769***	-0.0796***	-0.0551**
	(0.0260)	(0.0238)	(0.0231)
vol_r	0.00684	0.00636	0.00612
	(0.00513)	(0.00529)	(0.00524)
$mean_r$	-0.0199**	-0.0318***	-0.0365***
	(0.00873)	(0.0107)	(0.00817)
$a_{i,t}$	0.0375	0.0565**	0.101***
	(0.0291)	(0.0264)	(0.0320)
$k_{i,t-1}$		-0.0512***	-0.0507***
		(0.00748)	(0.00727)
$m_{i,t}$			-0.0249
			(0.0195)
Time FE	Yes	Yes	Yes
N	98515	98515	85920
adj. R^2	0.039	0.072	0.067

Note: Standard errors are clustered at the industry level. * 0.10 ** 0.05 *** 0.01. The degree of information friction is estimated at the industry level. Top and bottom 1% productivity obs. are trimmed out as outliers.

productivity (i.e., the residuals). The estimated serial correlation is around 0.91, close to the random walk case. Next, we use statistical innovations to rerun the investment sensitivity regression specified in equation (11). The regression results are reported in Tables 14 and 15. It is clear that the estimated interaction effect (around 0.009) is barely changed.

D.5 Robustness Checks using Total Factor Productivity

So far, we have utilized labor productivity (i.e., revenue per worker) and its innovation. The labor productivity measure is transparent but imperfect, as it does not exclude the impact of other factors, such as capital and usage of intermediate goods, on firm-level productivity. In the literature, the total factor productivity (TFP) is an often-used productivity measure. In this subsection, we implement robustness checks for our empirical findings using this alternative productivity measure. The conclusion is that all empirical findings we have documented are qualitatively unchanged.

We construct our TFP measure by using the standard approach from the IO literature (e.g., Olley and Pakes (1996) and Levinsohn and Petrin (2003)). Specifically, we follow Olley and Pakes (1996) to use investment as the proxy for the TFP to do the inversion from investment to the TFP (conditioning on the capital stock). We believe that this approach is appropriate for our analysis

Table 13: Incomplete Information and Investment Sensitivity:
Labor Productivity and Industry-year Fixed Effects

	(1)	(2)	(3)	(4)
	inv. inaction = 1		*inv. rate* (*inv. inaction* ≠ 1)	
$\xi_s \times w_{i,t}$	0.00991**	0.01000**	-0.0424	-0.0424
	(0.00473)	(0.00472)	(0.0406)	(0.0401)
$a_{i,t-1}$	-0.00857	-0.0146	-0.0313	-0.0127
	(0.0120)	(0.0119)	(0.0326)	(0.0202)
$w_{i,t}$	0.00735	0.00302	0.0146	0.0276
	(0.00930)	(0.00919)	(0.0170)	(0.0253)
$m_{i,t}$	-0.00879*	-0.00633	0.00705	-0.00133
	(0.00524)	(0.00514)	(0.00443)	(0.00585)
$k_{i,t-1}$		0.0824***		-0.168
		(0.00823)		(0.114)
Industry-year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Time (semi-year) FE	Yes	Yes	Yes	Yes
N	84656	84656	57137	57137
adj. R^2	0.448	0.451	0.044	0.059

Note: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are trimmed out (i.e., outliers).

Table 14: Incomplete Information and Investment Sensitivity:
AR(1) Process of Labor Productivity

	(1)	(2)	(3)	(4)
	inv. inaction = 1		*inv. rate* (*inv. inaction* ≠ 1)	
$\xi_s \times w_{i,t}$	0.00952*	0.00976**	-0.0408	-0.0414
	(0.00500)	(0.00499)	(0.0396)	(0.0396)
$a_{i,t-1}$	-0.0218*	-0.0288**	-0.0266	-0.00653
	(0.0112)	(0.0113)	(0.0305)	(0.0173)
$w_{i,t}$	0.000772	-0.00456	0.0182	0.0334
	(0.00934)	(0.00935)	(0.0189)	(0.0287)
$m_{i,t}$	-0.00890	-0.00536	0.00548	-0.00552
	(0.00547)	(0.00549)	(0.00403)	(0.00807)
$k_{i,t-1}$		0.0764***		-0.153
		(0.00861)		(0.103)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
N	84313	84313	56911	56911
adj. R^2	0.444	0.446	0.045	0.059

Note: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are trimmed out (i.e., outliers).

Table 15: Incomplete Information and Investment Sensitivity:
AR(1) Process of Labor Productivity and Industry-year Fixed Effects

	(1)	(2)	(3)	(4)
	inv. inaction = 1		*inv. rate* (*inv. inaction* ≠ 1)	
$\xi_s \times w_{i,t}$	0.00905*	0.00922*	-0.0416	-0.0419
	(0.00498)	(0.00496)	(0.0402)	(0.0399)
$a_{i,t-1}$	-0.0104	-0.0158	-0.0324	-0.0157
	(0.0114)	(0.0113)	(0.0334)	(0.0222)
$w_{i,t}$	0.00561	0.00161	0.0148	0.0270
	(0.00931)	(0.00921)	(0.0173)	(0.0250)
$m_{i,t}$	-0.00863	-0.00631	0.00704	-0.000940
	(0.00525)	(0.00515)	(0.00442)	(0.00566)
$k_{i,t-1}$		0.0818***		-0.169
		(0.00827)		(0.114)
Industry-year fixed effects	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes
Time (semi-year) fixed effects	Yes	Yes	Yes	Yes
N	84313	84313	56906	56906
adj. R^2	0.449	0.451	0.043	0.059

Note: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are trimmed out (i.e., outliers).

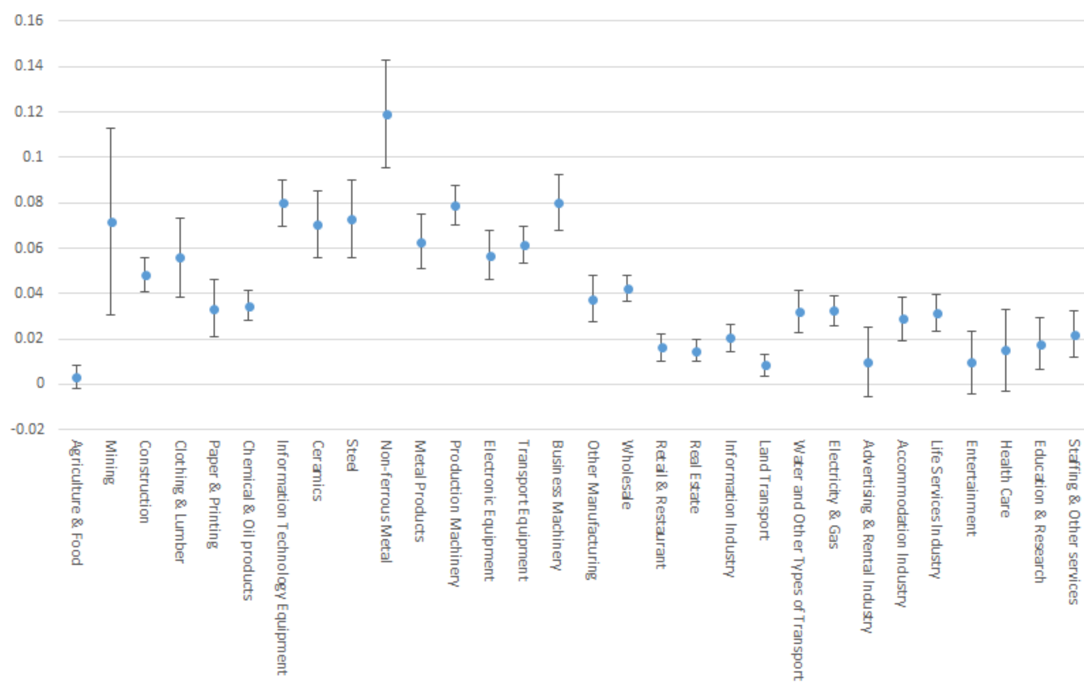
for two reasons. First, the firms in our sample are large firms. Thus, the usual zero investment issue that constrains the use of the Olley and Pakes approach is less of a concern in our case. Second, the usual collinearity problem (between the usage of the intermediate goods and the labor choice) is less of a concern in our context as well, as we use firm investment as the proxy. Thus, we estimate a firm-level Cobb-Douglas production function that inputs labor, capital, and intermediate goods. The estimates of the production are reported as follows:

$$\beta_l = 0.219; \quad \beta_k = 0.116; \quad \beta_m = 0.585,$$

which shows that the uncovered (revenue) production function displays slightly decreasing returns to scale. We recover the TFP for each firm-year observation in our data based on these estimates.

Next, we re-estimate the industry-specific coefficient that governs the impulse response of the (log) sales forecast error made in period $t + 1$ with respect to the realized (log) productivity innovation in period t for the 30 industries. The industry-level attenuation coefficients estimated using the labor productivity and the TFP are highly correlated (correlation coefficient: 0.66), lending us the confidence to use these two measures to gauge the degree of industry-level information frictions.

Figure 10: Estimated Attenuation Coefficients across Industries: TFP



Note: This figure shows how the coefficient governs the impulse response of the (log) sales forecast error made in period $t + 1$ with respect to the realized (log) productivity innovation in period t. Each dot denotes the estimate for an industry (with the 95% confidence interval), and there are 30 industries in total. Top and bottom 1% observations are trimmed out (i.e., outliers).

We rerun the regressions specified in equations (10) and (11) using firm-level TFP measures and the attenuation coefficients obtained using the TFP measures. Table 16 presents the regression result for equation (10). Both the magnitudes and the statistic stanificance levels barely change from the main text. Table 17 presents the regression results. Although the coefficient of interest ($\xi_s \times w_{i,t}$) becomes smaller for investment inaction, the estimate is still positive and statistically significant, consistent with what we found using the labor productivity measure. The specification using the industry-year fixed effects yields the results that are reported in Table 18, which are similar to the result reported in Table 17. In summary, our documented empirical findings are robust to various TFP measures we use to construct the attenuation coefficients.

Table 16: Investment Inaction and the Degree of Information Friction (semi year frequency): TFP

	investment inaction = 1		
$attenuation_{TFP}^{industry}$	-0.0445*	-0.0401	-0.0461**
	(0.0245)	(0.0242)	(0.0231)
$log(TFP)$	-0.0377	-0.00736	-0.0289
	(0.0683)	(0.0698)	(0.0386)
$lagged\ log(capital)$		-0.0367***	-0.0421***
		(0.00836)	(0.00903)
$log(inter_{per})$			0.0481*
			(0.0245)
Year × quarter fixed effects	Yes	Yes	Yes
N	84987	84987	84987
adj. R^2	0.016	0.033	0.051

The degree of information friction is estimated at the industry level.
Standard errors are clustered at the industry level. * 0.10 ** 0.05 *** 0.01
Top and bottom 1% productivity obs. are trimmed out (i.e., outliers).

Table 17: Incomplete Information and Investment Sensitivity: TFP

	(1)	(2)	(3)	(4)
	inv. inaction = 1		*inv. rate* (*inv. inaction* ≠ 1)	
$\xi_s \times w_{i,t}$	0.00581*	0.00600*	0.000342	0.000204
	(0.00340)	(0.00339)	(0.00105)	(0.00105)
$a_{i,t-1}$	-0.112***	-0.103***	0.0235***	0.0208***
	(0.0132)	(0.0134)	(0.00516)	(0.00527)
$w_{i,t}$	-0.0404***	-0.0350***	0.0117***	0.00976***
	(0.00848)	(0.00854)	(0.00282)	(0.00284)
$m_{i,t}$	-0.00566	-0.00418	0.00476***	0.00360*
	(0.00634)	(0.00637)	(0.00184)	(0.00193)
$k_{i,t-1}$		0.0727***		-0.0408***
		(0.00873)		(0.00498)
Firm FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
N	80508	80508	54747	54747
adj. R^2	0.445	0.447	0.303	0.312

Note: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are trimmed out (i.e., outliers).

Table 18: Incomplete Information and Investment Sensitivity: TFP and using Industry-year Fixed Effects

	(1)	(2)	(3)	(4)
	inv. inaction $= 1$		*inv. rate* (*inv. inaction* $\neq 1$)	
$\xi_s \times w_{i,t}$	0.00471	0.00492	0.000664	0.000518
	(0.00340)	(0.00339)	(0.00105)	(0.00105)
$a_{i,t-1}$	-0.0989***	-0.0848***	0.0225***	0.0165***
	(0.0130)	(0.0131)	(0.00565)	(0.00561)
$w_{i,t}$	-0.0325***	-0.0249***	0.0106***	0.00739**
	(0.00845)	(0.00849)	(0.00295)	(0.00293)
$m_{i,t}$	-0.00121	-0.000539	0.00354*	0.00299
	(0.00615)	(0.00615)	(0.00183)	(0.00185)
$k_{i,t-1}$		0.0807***		-0.0457***
		(0.00856)		(0.00535)
Firm FE	Y	Y	Y	Y
Industry-time FE	Y	Y	Y	Y
N	80508	80508	54741	54741
adj. R^2	0.449	0.451	0.316	0.327

Note: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are trimmed out (i.e., outliers).