Cap-and-Apply: Unintended Consequences of College Application Policy in South Korea

Taekyu Eom*

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Abstract

Starting in the 2013 academic year, the South Korean government implemented a policy limiting students to a maximum of six college applications. This paper examines the impact of this application cap on matching quality and the socioeconomic gap in college prestige. Using college-level administrative data, I find that matching quality decreases, as second-tier colleges are able to attract more desirable students following the introduction of the cap. This finding aligns with theoretical predictions from a simplified model in which colleges compete for top applicants, and applicants exhibit ability noise. Moreover, I extend the model to incorporate application constraints influenced by socioeconomic status (SES). The theoretical framework suggests that the cap reduces the socioeconomic gap: after the cap, the number of students from lower socioeconomic backgrounds attending more prestigious colleges increases, as the constraints primarily limit applicants from higher socioeconomic groups. Empirical analysis supports these theoretical predictions. These findings may provide valuable policy insights for the U.S. higher education market.

JEL Codes: C78, D83, I24, I28

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*Department of Economics, University at Buffalo, SUNY, 414 Fronczak Hall, Buffalo, NY, 14260-1520, USA; e-mail: taekyueo@buffalo.edu

1 Introduction

In 2013, the Korean government implemented a cap on the number of applications applicants could submit to undergraduate programs during early admissions. This policy was introduced with the intention of alleviating the burden on households, high schools, and college admissions committees. However, unintended consequences emerged, as the cap influenced the distribution of college applicants (Chen and Kao, 2023; Avery et al., 2014). This is because the inherent uncertainty of admissions, combined with application fees, creates a complex portfolio problem: how many reach, match, and safety schools an applicant should target (Che and Koh, 2016; Fu, 2014). Within this context, financial constraints play a significant role, particularly during early admissions (Avery and Levin, 2010; Hoxby and Avery, 2013).

This study examines the effects of the application cap on two primary outcomes: matching quality and the socioeconomic gap. By extending the theoretical model proposed by Chen and Kao (2023), I derive predictions for these outcomes and validate them with empirical evidence. The results indicate a reduction in matching quality, as fewer desirable students enroll in more prestigious colleges following the cap's implementation. Conversely, the socioeconomic gap narrows, with more students from lower socioeconomic backgrounds enrolling in prestigious institutions post-cap.

The theoretical model considers two colleges of differing prestige and two types of students. It assumes a scarcity of desirable students and high uncertainty regarding national college entrance exam results. Colleges can perfectly screen student types at zero application cost. The model predicts a decrease in the number of desirable students at more prestigious colleges, potentially driven by competition from less selective yet still desirable institutions. To address the socioeconomic gap, the model is extended to include financial constraints, predicting an increase in the enrollment of low socioeconomic status students at prestigious colleges after the cap.

To conduct the empirical analysis, I integrate data from the Korean Council for University

Education (KCUE), which includes college- and department-level information on acceptance rates, attendance, enrollment, graduation, faculty, tuition, and financial aid. Using this data, I construct a panel dataset to code key variables. The top 46 colleges are selected for the sample as the cap primarily affects prestigious institutions, distinguishing their applicant pool from those of other higher education institutions such as community colleges not subject to the cap. Various outcome measures are analyzed to examine the impact of the application cap on the quality and socioeconomic gap. First, I examine the share of graduates from selective high schools, defined as those specializing in science or foreign language instruction. This serves as a proxy for desirable students, as these graduates tend to achieve superior academic and labor market outcomes. In addition, I consider degree completion metrics, including the average duration of study and the probability of graduating on time. Previous research, such as Dillon and Smith (2020), demonstrates that student quality significantly influences these outcomes. Second, the share of student loan debtors for the tuition is used to proxy the share of students with low socioeconomic status. This loan is a government student loan that is distributed based on income brackets.

The key findings are as follows: (1) The share of desirable students at more prestigious colleges decreases post-cap, as measured by the share of graduates from selective high schools and the probability of graduating on time. This aligns with existing literature (Chen and Kao, 2023; Avery et al., 2014). (2) The share of students from lower socioeconomic backgrounds increases at more prestigious colleges, as indicated by the proportion of student loan recipients.

While this study focuses on the Korean higher education market, it offers policy implications for the United States. As Blair and Smetters (2021) highlights, excessive competition among elite colleges can result in significant inefficiencies. Although the coordination of admissions policies and caps may conflict with the Sherman Act's antitrust provisions, I argue that implementing a cap warrants consideration, as it aligns with the public interest.

This paper is organized as follows: Section 2 provides institutional background. Section

3 outlines the theoretical models. Section 4 describes the data and presents descriptive evidence. Section 5 details the identification strategy and empirical results, supplemented by additional evidence. Section 6 concludes. Proofs and other technical details are included in the Appendix.

2 Institutional Background

2.1 College Application

Postsecondary educational institutions in Korea are divided into two: 2-year colleges and 4-year colleges. The former is a vocational institution, and the latter is academic in general. The 4-year college is categorized into the following: general, education, industrial, open, online, and special colleges. The application cap is implemented only in 4-year general and education institutions.

Figure 1 shows the timeline of admission. Students start school days in March and graduate in February. A student in her final year applies to colleges in either two ways: early or regular admission. Early admission starts its process in the middle of August. The applicant submits her high school GPA and proof of extracurricular activities. She may have interviews or college-specific exams around the date of the national college entrance exam, which is held only once a year. The early admission is binding. If the student gets accepted for early admission, she cannot apply for regular admission. After taking the national exam in the middle of November, if she does not apply or is not accepted by any college in the early admission, she applies for regular admission which starts its process in late December. In regular admissions, all general colleges select from three different admission dates. Students can apply to up to three admission units, one for each date. While the rules for regular admissions remained unchanged during the analysis period, early admissions saw significant changes with the introduction of the application cap, which limited the number of programs students could apply to.

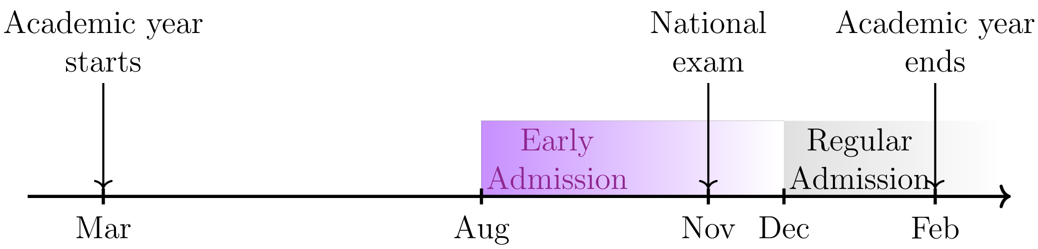

Figure 1: Timeline of Admission

2.2 The Cap in 2013

The application cap was implemented for early admission and 4-year general colleges in the 2013 academic year. That means that the policy change is effective for students who have applied to colleges since 2012. The number 6 is decided by surveying parents and representatives in high schools and colleges several times.

The admissions committee from KCUE announced the policy on December 22nd, 2011. There were exceptions for some 4-year colleges that were founded for special purposes: industrial, open, online, military, science and technology, and music and art. Students can apply up to 6 for general 4-year colleges but they can apply to exceptional institutions with no limit. The policy is implemented with the notification a year before its actual implementation, which implies that there is not enough time to prepare strategic behaviors among applicants.

There are pieces of evidence showing that the cap is binding. Figure 2 shows the number of applications per slot by ranking group, and the degree of competition for admission has plummeted for all except the top college (label 1) since 2013. Label 6 implies the lowest ranking group among the top 46. Moreover, the Korean Educational Longitudinal Survey (KELS) in 2011 suggests that around 22% of applicants in early admission applied to more than 6 colleges. With the fact that their average national exam score is higher than the average, the cap would be binding with applicants to the top 46 colleges.

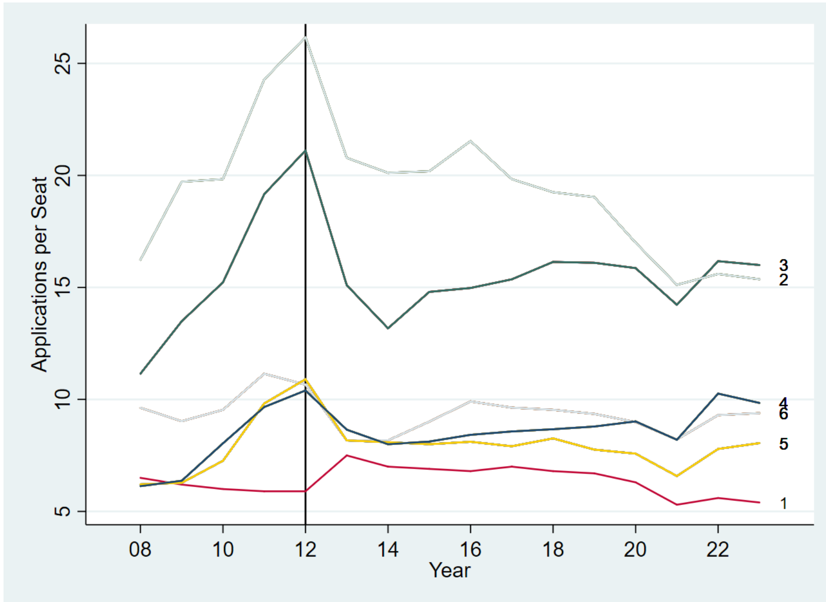

Figure 2: Time Trends of the Applications per a Slot by Ranking Groups

3 Theoretical Model

The model is an adaptation of the Chen and Kao (2023) framework. By adding structure to agent utility functions, The model acquires adequate analytic tractability to provide strong comparative static predictions for behavioral reactions to changes in the degree of competition.

I use a model with two colleges of differing prestige and two types of students with different desirability to demonstrate that matching quality deteriorates while the socioeconomic gap narrows following the implementation of the cap. Regarding matching quality, the model incorporates ability noise and predicts that less prestigious colleges attract more desirable students. Additionally, by extending the model to include application constraints based on students' SES, I show that the socioeconomic gap—measured by the prestige of the colleges students enroll in—decreases after the cap.

Here is the timeline. First, students apply to colleges with signals of their ability from before the national-level college entrance exam: high school GPA, scores on mock tests, and extracurricular activities. Second, after the national exam, colleges screen applicants perfectly and offer admissions. Third, applicants make enrollment decisions based on the offer.

The model is as follows. There are 2 colleges A and B, where capacity k_A and k_B for each. The prestige of each college is a and b, assuming A is more prestigious than B ($a > b$). There is a continuum of students. Student i's utility of enrolling in college A is $V_i(A) = a - e_i$. And $V_i(B) = b + e_i$ if enrolling B. e_i is the relative preference toward B, controlling prestige (e.g., fit, distance from home). This is private information and is independently distributed by the cumulative distribution function $P(e_i \leq y) = F(y)$, which $F(0) = 1/2$. Its probability density function $f(y)$ is supported by $[-\delta_B, \delta_A]$, which is in the range between $[-b, a]$ for ensuring $V_i > 0$. In other words, student i prefers enrolling in either college to not, with this range.

Student i prefers enrolling B if $V_i(B) \geq V_i(A)$, that is, $e_i \geq (a-b)/2$. I denote $(a-b)/2$ as d, the normalized prestige difference. $F(d) = P(e_i \leq d)$ is the normalized number of students who will choose to enroll in college A if they are admitted by both.

3.1 Matching Quality

For simplicity, no application costs are assumed. A continuum of students is categorized into two types: good(μ) and normal($1-\mu$). Colleges can screen applicants perfectly based on the national exam score. I assume $\mu < k_A + k_B < 1$. That is, each college competes to get good students and students have the risk of not being accepted by either college. Students have uncertainty about their true type because of ability noise.[1] Among two types of signal $s_i \in \{h, l\}$, truly good students always get h. Students of normal type get h with probability p and l with probability $1-p$. In that sense, the posterior probability that a student who got $s_i = l$ is always the normal type is 1. And students who got signal $s_i = h$ is high type

[1] For example, they can estimate the probability of being accepted to the college they apply with their high school GPA, scores on national-level mock tests, and extracurricular activities. However, high school GPA is not directly comparable with that of different types of high schools. Even though there are multiple tries of mock tests, there are risks of not being reflected enough because students can take the national exam once a year. Extracurricular activities are meaningful when combined with other criteria.

with $\pi = \mu/((1-\mu)p + \mu)$ and normal type with $1-\pi$.

I will delve into two cases: before and after the implementation of the cap. In the former case, students apply for both A and B since there are no application costs. Let m_A and m_B be the number of good students who enroll in colleges A and B before the cap. In the latter case, they can apply only one because of the cap. Let n_A and n_B be the number of good students enrolled in each college after the cap. If n_B is greater than m_B, this can serve as evidence supporting the hypothesis that matching quality declines after the implementation of the cap.

3.1.1 Before the cap

In this case, applicants do not need to consider the signals.

Proposition 1. Before the cap, $m_A = k_A$ and $m_B = \mu - k_A$ when (i) $\mu F(d) > k_A$. $m_B = k_B$ and $m_A = \mu - k_B$ when (ii) $\mu(1-F(d)) > k_B$ (or equivalently, $\mu F(d) < \mu - k_B$). $m_A = \mu F(d)$ and $m_B = \mu(1-F(d))$ when (iii) $k_A \geq \mu F(d) \geq \mu - k_B$.

With conditions, $\Sigma_j m_j = \mu$, and capacity constraints, $m_j \leq k_j$ for all $j \in \{A, B\}$, I consider all the cases depending on whether competition among applicants exists in either college. First, if the number of applicants who would choose college A among good students is greater than the capacity of A, (i) $\mu F(d) > k_A$, it happens to $m_A = k_A$ and $m_B = \mu - k_A$. Second, if the number of applicants who would choose college B among good students is greater than the capacity of B, (ii) $\mu(1-F(d)) > k_B$, it happens to $m_B = k_B$ and $m_A = \mu - k_B$. I can rewrite the second case as $\mu F(d) < \mu - k_B$. Third, there is no competition among good students in either college, (iii) $k_A \geq \mu F(d) \geq \mu - k_B$. Then $m_A = \mu F(d)$ and $m_B = \mu(1-F(d))$. Additionally, it cannot happen that competition among good students in both colleges. That is, $\mu - k_B > \mu F(d) > k_A$. It is rewritten as $\mu > \mu F(d) + k_B > k_A + k_B$, which contradicts with the assumption $k_A + k_B > \mu$. For the first two cases, m_B weakly decreases in d with capacity constraints. For the third case, m_B is strictly decreasing in d.

Now I know the prestige difference d has a role in the number of good students enrolling in each college.

3.1.2 After the cap

Since there are 2 colleges, the cap restricts the number of applications to at most one. The application depends on the utility of enrolling in either college and the risk of failing to be accepted. The latter one is added because applicants have only one opportunity. Applicants with signal use their posterior belief to apply. Let's denote $p_j^{s_i}(e_h, e_l)$ as the equilibrium probability of being accepted by college j of an applicant i with signal s_i, given that all other students with either s_i are indifferent to applying A or B. Then I can write down each of the $p_j^{s_i}(e_h, e_l)$ as follows:

$$p_A^l(e_h, e_l) = \min\left[\max\left[0, \frac{k_A - \mu F(e_h)}{(1-\mu)(pF(e_h) + (1-p)F(e_l))}\right], 1\right]$$

$$p_B^l(e_h, e_l) = \min\left[\max\left[0, \frac{k_B - \mu(1 - F(e_h))}{(1-\mu)(p(1-F(e_h)) + (1-p)(1-F(e_l)))}\right], 1\right]$$

$$p_A^h(e_h, e_l) = \pi \min\left[\frac{k_A}{\mu F(e_h)}, 1\right] + (1-\pi)p_A^l(e_h, e_l)$$

$$p_B^h(e_h, e_l) = \pi \min\left[\frac{k_B}{\mu(1 - F(e_h))}, 1\right] + (1-\pi)p_B^l(e_h, e_l)$$

Colleges accept the normal types randomly after filling in good types. The equilibrium probability for $s_i = l$ is zero if the good types applying to the college exceed the capacity. The fraction implies the acceptance rate among normal types. The available seats after filling in good types are divided by normal types who get the signal of h with p or l with $(1-p)$ either. The equilibrium probability for $s_i = h$ implies the weighted average for both cases of good type with $s_i = h$ and normal type with $s_i = h$. Without loss of generality, the first term is just a π if the acceptance rate of either college among good students is larger than 1.

I denote the equation for i who are indifferent between applying to colleges A and B.

$$p_A^{s_i}(e_h, e_l)(a - e_{s_i}) = p_B^{s_i}(e_h, e_l)(b + e_{s_i})$$

For example, $p_A^h(e_h, e_l)(a - e_i) < p_B^h(e_h, e_l)(b + e_i)$ if and only if $e_i > e_h$. The number of good students for each college under the cap: $n_A = \mu F(e_h)$, $n_B = \mu(1 - F(e_h))$. This is based on the equilibrium strategy profile of students, (e_h, e_l).

Before going further, it would be helpful to see the 2 lemmas first. These lemmas will shed light on where should I restrict the circumstances to show the hypothesis is true.

Lemma 1. When $k_A < \mu$, n_B is 0 after the cap for all $p \in [0, 1]$ and for some large d.

Proof. In Appendix.

In words, if there exists a somewhat big enough prestige difference that makes all the good students apply to A under the cap, it is possible that $n_B = 0$. This is because the probability of being accepted by A is small but still positive, and the probability of being accepted by B is 1. However, without the cap, the number of good students enrolling in B is bounded below $\mu - k_A > 0$.

Lemma 2. When $p = 0$, $n_B < m_B$ if $\mu F(d) > k_A$, and $n_B = m_B$ if $\mu F(d) \leq k_A$.

Proof. In Appendix.

In proposition 1, I examined m_B for the following cases: i) competition among good students in A only, ii) in B only, and iii) neither. I will follow these cases to compare m_B and n_B. When there is no uncertainty, all the good students get $s_i = h$ and all the normal students get $s_i = l$. What they are going to do is applying the college they prefer. When there is no competition in both A and B, there is indifference to the number of good students enrolling in both institutes with and without the cap. When there is competition only in B, the number of good students enrolled in B is equivalent in both with and without the cap as the capacity. For A, since some students switch their application decision with the cap, it loses the number of good students enrolled. Symmetrically, when there is competition only in A, B loses the number of good students under the cap.

With the proposition and lemmas above, I know that the college B is hard to get benefits if the prestige difference d is big, $p = 0$, and μ is too large compared to the capacity. I get hints for setting the following environment that college B can benefit from after the cap.

Under the assumption that (i) $k_B > k_A > \mu F(0)$, (ii) $d > 0$, and (iii) $p > 0$, I want to show that the second-best colleges earn more good students comparing to before the cap. Assumptions imply that (i) there is no excessive competition among good students in both A and B, (ii) college A is more prestigious than B (additionally, I will find some $\bar{d} > 0$ for $d < \bar{d}$ to make d not too big), and (iii) some normal type students have the uncertainty about their true type.

Proposition 2. With assumptions above, $n_B > m_B$ if d is sufficiently small.

Proof. In Appendix.

From assumptions (i) and (ii), there is no competition among good students in B. It can happen in A as well if d is small enough. Under the cap, applicants apply considering the probability of being accepted. It makes possible for them to apply to the college less preferred but more likely to be accepted. Since B has more capacities than A, there are more seats available in B for the normal students after fulfilling good students. Since some good students with signal h still have some probability of being normal type, they apply to B even if they prefer A a bit more than B.

3.2 Socioeconomic Gap

My second interest of outcome is the socioeconomic gap. To address this, I added an application constraint to the model. The constraint stems from the fact that students in different SES face different application costs. Low SES students face a greater burden not only from application fees but also costs for preparing college-specific exams, expenses for counseling on application strategies, and the psychic costs associated with application failure compared to high SES students.

Consequently, the application behaviors of low-SES and high-SES students differ. Before the cap, high-SES students apply to both colleges, while low-SES students apply to only one. However, after the cap, both types of students apply to only one college. According to the prediction, after the cap, the number of low-SES students enrolled in more prestigious colleges increased. If I denote v_j and w_j as the number of low-SES students who enroll in college j before and after the cap, my hypothesis is $w_A > v_A$.

For simplicity, I start with the assumption that applicants do not know their true type. Then they apply based only on their preferences.

3.2.1 Before the Cap

Given that High-SES students ($\frac{1}{2}$) apply to both colleges A and B, while $\frac{1}{2}F(d)$ Low-SES students apply to A and $\frac{1}{2}(1-F(d))$ apply to B, the probabilities differ by SES group:

$$p_A^H = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}F(d)}, \quad p_B^H = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}(1-F(d))},$$

$$p_A^L = \frac{\frac{1}{2}F(d)}{\frac{1}{2}+\frac{1}{2}F(d)}, \quad p_B^L = \frac{\frac{1}{2}(1-F(d))}{\frac{1}{2}+\frac{1}{2}(1-F(d))}.$$

In equilibrium, students apply with a cutoff e^*, and colleges accept students probabilistically. High-SES students ($\frac{1}{2}$) and Low-SES students ($\frac{1}{2}F(e^*)$) are accepted under the following scenarios:

High-SES:

A only: $p_A^H(1-p_B^H)$ Cases: (1), (2)

B only: $(1-p_A^H)p_B^H$ Cases: (3), (4)

Both: $p_A^H p_B^H$ Cases: (5), (6)

Neither: $(1-p_A^H)(1-p_B^H)$ Cases: (7), (8)

Low-SES:

$$\text{A only: } p_A^L \quad \text{Case: (9)}$$

$$\text{B only: } p_B^L \quad \text{Case: (10).}$$

For high-SES, each of the two cases implies for the proportion of applicants who prefer A or B. For low-SES, applicants apply which they prefer.

For equilibrium, college A receives $\frac{1}{2} + \frac{1}{2}F(e^*)$ applications, while college B receives $\frac{1}{2} + \frac{1}{2}(1 - F(e^*))$ applications. Using these values, we calculate v_A and v_B under different scenarios. For example:

If $\frac{1}{2}+\frac{1}{2}F(e^*) > k_A$ and $\frac{1}{2}+\frac{1}{2}(1-F(e^*)) > k_B, v_A = \frac{(9)}{(1)+(2)+(5)+(9)}k_A = \frac{p_A^L}{p_A^H(1-p_B^H)+p_A^H p_B^H F+p_A^L}k_A = \frac{2F-F^2}{1+2F-F^2}k_A$. $v_B = \frac{(10)}{(3)+(4)+(6)+(10)}k_B = \frac{1-F^2}{1+2F-F^2}k_B$.

If $\frac{1}{2} + \frac{1}{2}F(e^*) > k_A$ and $\frac{1}{2} + \frac{1}{2}(1 - F(e^*)) < k_B, p_A = \frac{k_A}{\frac{1}{2}+\frac{1}{2}F(e^*)}, p_B = 1$. Then $v_A = \frac{(5)}{(5)+(9)}k_A = \frac{1}{2}k_A, v_B = \frac{1}{(3)+(4)+(6)+1}k_B = \frac{1+F}{2+F}k_B$.

If $\frac{1}{2} + \frac{1}{2}F(e^*) < k_A$ and $\frac{1}{2} + \frac{1}{2}(1 - F(e^*)) > k_B, v_A = \frac{1}{(1)+(2)+(5)+1}k_A = \frac{2-F}{3-F}k_A, v_B = \frac{(10)}{(6)+(10)}k_B = \frac{1}{2}k_B$.

The case that $\frac{1}{2} + \frac{1}{2}F(e^*) < k_A$ and $\frac{1}{2} + \frac{1}{2}(1 - F(e^*)) < k_B$ conflicts with $k_A + k_B < 1$.

The cutoff e^* is determined such that the expected payoff of enrolling in either college is equal for High-SES or Low-SES students:

High-SES:

$$p_A^H(e^*)(a - e^*) = p_B^H(e^*)(b + e^*).$$

There are four cases.

- **Case 1**: $(p_A^H, p_B^H) = (1, 1)$ conflicts with the assumption $k_A + k_B < 1$.
- **Case 2**:$(p_A^H, p_B^H) = \left(\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}F(e^*)}, \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}(1-F(e^*))}\right)$ if $\frac{1}{2} + \frac{1}{2}F(e^*) > k_A$ and $\frac{1}{2} + \frac{1}{2}(1 - F(e^*)) > k_B$. Rearranging gives:

$$\frac{1}{1 + F(e^*)}(a - e^*) = \frac{1}{2 - F(e^*)}(b + e^*) \to aF(e^*) + bF(e^*) + 3e^* = 2a - b$$

- **Case 3**:$(p_A^H, p_B^H) = \left(1, \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}(1-F(e^*))}\right)$ if $\frac{1}{2} + \frac{1}{2}F(e^*) < k_A$ and $\frac{1}{2} + \frac{1}{2}(1 - F(e^*)) > k_B$.

  Rearranging gives:

$$(2 - F(e^*))(a - e^*) = b + e^* \to e^* = \frac{(1 + F(e^*))a - b}{2 + F(e^*)}$$

- **Case 4**: $(p_A^H, p_B^H) = \left(\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}F(e^*)}, 1\right)$ if $\frac{1}{2} + \frac{1}{2}F(e^*) > k_A$ and $\frac{1}{2} + \frac{1}{2}(1 - F(e^*)) < k_B$

  Rearranging gives:

$$a - e^* = (1 + F(e^*))(b + e^*) \to e^* = \frac{a - (1 + F(e^*))b}{2 + F(e^*)}$$

  All the cases are not the closed form. It might need numerical methods, for example, Newton-Raphson method.

Low-SES:

$$p_A^L(e^*)(a - e^*) = p_B^L(e^*)(b + e^*).$$

There are four cases:

- **Case 1**: $(p_A^L, p_B^L) = (1, 1)$

  This conflicts with the assumption $k_A + k_B < 1$.

- **Case 2**: $(p_A^L, p_B^L) = \left(\frac{\frac{1}{2}F(e^*)}{\frac{1}{2}+\frac{1}{2}F(e^*)}, \frac{\frac{1}{2}(1-F(e^*))}{\frac{1}{2}+\frac{1}{2}(1-F(e^*))}\right)$

  This holds if $\frac{1}{2} + \frac{1}{2}F(e^*) > k_A$ and $\frac{1}{2} + \frac{1}{2}(1 - F(e^*)) > k_B$. Rearranging gives:

$$(2F(e^*) - F^2(e^*))a - e^* = (1 - F^2(e^*))(b + e^*) \to e^* = \frac{(2F(e^*) - F^2(e^*))a - (2 - F^2(e^*))b}{1 + 2F(e^*) - 2F^2(e^*)}.$$

- **Case 3**: $(p_A^L, p_B^L) = \left(1, \frac{\frac{1}{2}(1-F(e^*))}{\frac{1}{2}+\frac{1}{2}(1-F(e^*))}\right)$

This holds if $\frac{1}{2}+\frac{1}{2}F(e^*) < k_A$ and $\frac{1}{2}+\frac{1}{2}(1-F(e^*)) > k_B$. Rearranging gives:

$$(2-F(e^*))(a-e^*) = b+e^* \to e^* = \frac{(2-F(e^*))a-b}{3-2F(e^*)}.$$

- **Case 4**: $(p_A^L, p_B^L) = \left(\frac{\frac{1}{2}F(e^*)}{\frac{1}{2}+\frac{1}{2}F(e^*)}, 1\right)$
  This holds if $\frac{1}{2}+\frac{1}{2}F(e^*) > k_A$ and $\frac{1}{2}+\frac{1}{2}(1-F(e^*)) < k_B$. Rearranging gives:

$$\frac{F(e^*)}{1-2F(e^*)}(a-e^*) = b+e^* \to e^* = \frac{aF(e^*)-b(2-F(e^*))}{2}.$$

All the cases are not the closed form. It might need numerical methods, for example, Newton-Raphson method.

3.2.2 After the Cap

- The case that $F(e^*) < k_A$ and $1-F(e^*) < k_B$ cnflicts with $k_A+k_B<1$.
- If $F(e^*) > k_A$ and $1-F(e^*) > k_B, w_A = \frac{1}{2}k_A, w_B = \frac{1}{2}k_B$.
- If $F(e^*) > k_A$ and $1-F(e^*) < k_B$, $w_A = \frac{1}{2}k_A, w_B = \frac{1}{2}k_B$.
- If $F(e^*) < k_A$ and $1-F(e^*) > k_B$, $w_A = \frac{1}{2}k_A, w_B = \frac{1}{2}k_B$ in equilibrium.

Comparing before and after the cap cases, w_A is bigger than v_A when we assume that college A is more prestigious than B (the third case is excluded.).

To find equilibrium e^*, e^* s.t. $p_A(e^*)(a-e^*) = p_B(e^*)(b+e^*)$ for all H and L. e_i prefers B if $e_i > e^*$

- $p_A = 1$ if $k_A > F(e^*)$, $p_A = \frac{1}{2}$ if $k_A < F(e^*)$
- $p_B = 1$ if $k_B > 1-F(e^*)$, $p_B = \frac{1}{2}$ if $k_B < 1-F(e^*)$
- $(p_A, p_B) = (1,1)$: Conflict with $k_A+k_B<1$
- $(p_A, p_B) = (\frac{1}{2}, \frac{1}{2}) : \frac{1}{2}(a-e^*) = \frac{1}{2}(b+e^*) \to e^* = \frac{a-b}{2}$.

- $(p_A, p_B) = (1, \frac{1}{2}) : e^* = \frac{2a-b}{3}$. The higher admission probability to A, the more preference to A.

- $(p_A, p_B) = (\frac{1}{2}, 1) : e^* = \frac{a-2b}{3}$. The higher admission probability to B, the more preference to B.

4 Data and Sample

I construct two types of panel datasets by combining administrative data from the KCUE: college-level and department-level. Since both have some commonalities and differences in their variables including the outcomes of interest, I merge department-level data into college-level data for the outcome variables which the latter does not include. The KCUE assembles records related to following (bi)annually from colleges: rules, curriculum, admission, enrollment, graduation, faculty, research, accounting, tuition, accordance with the Higher Education Act, development plan, industry-academic cooperation, library, etc.[2] [3]

Using data from 2008 to 2023, I restrict the sample to the top 46 colleges the cap covers. This is because the cap is binding for the applicants of prestigious colleges.[4] Moreover, in terms of the application pool, applicants in the top 46 are separated from the students in other higher educational institutions such as community colleges which are not treated by the cap. I exclude the top college (Seoul National University) which is affected little by the cap and has noise in outcomes[5], and 5 science-and-technology-focused colleges in the top 50, which are not covered by the cap.

[2]https://www.academyinfo.go.kr/intro/intro0320/intro.do

[3]Some higher-educational institutions subject to national defense are excluded according to *Act on Special Cases Concerning the Disclosure of Information by Education-related Institutions §2(5).*

[4]There were 188 colleges in 2013 (KEDI).

[5]I check robustness including Seoul National University.

4.1 Outcomes of Interest

For the matching quality, I use 3 different outcome variables for the desirable students (or high-achieving students). One of the outcomes is the share of freshmen from selective high schools. In the KCUE data, high schools are categorized into academic, science, foreign languages and international, art and physical education, meister, vocational, autonomous, talented, etc.[6] I categorized science, foreign languages and international high schools, and talented academies as selective high schools. These types of high schools are selective in terms of both input and output. Desirable students apply to these high schools with GPA, extracurricular activities, and interviews. 24 out of the top 30 for the share of those who are in the 89th percentile of the average score of Korean, Math, and English in CSAT are these types of high schools (29 out of 30 if autonomous private high schools are included).[7]

For the second and third ones, the average duration and the probability of graduating on time, I reorganize the graduation data. Since the data in each year reveals not the duration of freshmen but the number of graduates who completed the degree in that year, I append all the years, resort by the entering year, and calculate the weighted average of the proxies. I also restrict the sample to those who graduate within 8 years to avoid issues that earlier cohorts have a longer duration. The last year of the sample is 2015. For the probability of graduating on time, I calculate the ratio of graduates who complete the degree within 6 years[8] among freshmen in the year to the total number of entrants in the year. The last year of the sample is 2017.

For college ranking, I use JoonAng (2010). Korea JoongAng Daily publishes the ranking of colleges in Korea annually since 1995[9]. The institute quantifies the educational environ-

[6]In 2013, there were 2,322 high schools. 1,359(58.5%) of high schools were academic. 669(28.8%) for vocational and meister, 165(7.1%) for autonomous, 63(2.7%) for science, foreign languages, international, and talented, and 42(1.8%) for arts and physical education high schools, 24(1.0%) for alternatives (KEDI).

[7]Autonomous private high schools are also selective. However, the data offers the discerned autonomous high schools with private and public since 2021. I investigate the outcome with autonomous high schools in the robustness check section.

[8]I exclude graduates with less than 4 years because it can include the case of the transfer, which has a duration of around 2 years.

[9]Not published in 2020 because of COVID-19.

ment, internationalization, research, and reputation. The ranking can be a proxy for the selectiveness of colleges by comparing it with admission data from a cram school (Table 1). Daesung, one of the big cram schools, annually publishes tentative scores for applying to colleges. This is made from the following resources: the predicted distribution of current-year SAT scores, admitted records from previous years, and two mock exam results in the current year (June and September).

Table 2 and Table 3 show summary statistics of the sample from the college and department levels. Table 2 is from the college-level data. The outcome variable in the data is the share of freshmen from selective high schools with a unit of percentage. The top 46 colleges have around 5% of their freshmen from selective high schools. Yonsei University recorded a high of 25.9% in 2010. Seoul National University recorded 21%, and Korea University 20.5% in the same year. The number of observations depends on the data availability in each year. colleges set their number of slots (projected number of students) within the extent that meets government criteria of the size of a school building, a school site, and faculty, and the value of for-profit assets[10]. The application per slot among the top 46 is 11.2, which is the inverse of the acceptance rate (8.9%). Chung-Ang University recorded the highest 35.2 and Jeju National University had the lowest, 2.7. College-level data discerns the number of slots and enrollment with early and regular admission. 59% of slots are for early admission. Since early admission is not the last chance, the enrollment rate is lower than the regular case. Some enrollment rates in regular admission are over 100 because the unfilled slots for early admission caused by not being enrolled and not being qualified are carried over to regular admission.[11] For the early admission, there are 2 cases which are 105.91 and 111.19. In the case of affirmative action, exceptionals are conditionally acceptable.

Table 3 is from the department-level data. Two outcome variables in the data are the average duration in years and the probability of graduation on time. The male group spends more than 1.2 years to graduate because of military service. The systemic difference applies

[10] *Enforcement Decree Of The Higher Education Act §28(1)*

[11] Among 91 cases, 2 are around 150, 6 are between 110 and 120, and the rest are under 110.

Table 1: Daesung (2012) and JEDI (2010)

Daesung (2012)[a]		JEDI (2010)[b]		
CSAT Score	Rank	Rank	Rank Group	College
384.21	1	1	1	Seoul National University
380.96	2	4	2	Yonsei University
377.03	4	5	2	Korea University
373.04	6	6	2	Sungkyunkwan University
364.76	12	7	2	Kyung Hee University
376.00	5	8	2	Sogang University
369.42	7	9	2	Hanyang University
367.30	9	13	3	Ewha Women's University
351.80	26	14	3	Inha University
363.20	14	15	3	Chung-Ang University
347.93	29	16	3	Ajou University
351.65	27	17	3	Konkuk University
365.58	11	18	3	Hankuk University Of Foreign Studies
364.05	13	19	3	University Of Seoul
347.65	30	20	3	Dongguk University
335.64	35	21	4	Kyungpook National University
358.63	15	22	4	Sookmyung Women's University
332.71	37	23	4	Pusan National University
334.68	36	24	4	The Catholic University Of Korea
.	.	25	4	Jeonbuk National University
308.58	53	26	4	Chungnam National University
.	.	27	4	Yeungnam University
326.23	44	28	4	Chonnam National University
.	.	29	4	University Of Ulsan
352.83	25	30	4	Hongik University
295.98	57	31	5	Chungbuk National University
.	.	32	5	Korea University of Technology and Education
355.00	18	33	5	Handong Global University
.	.	34	5	Korea Aerospace University
344.40	33	35	5	Soongsil University
.	.	36	5	Kangwon National University
.	.	37	5	Hallym University
.	.	38	5	Inje University
291.07	58	39	5	Gyeongsang National University
346.70	32	40	5	Kookmin University
300.97	56	41	6	Pukyong National University
331.54	39	42	6	Dankook University
329.99	40	43	6	Myongji University
320.58	47	44	6	Kwangwoon University
.	.	45	6	National Korea Maritime & Ocean University
.	.	46	6	Soonchunhyang University
318.95	50	47	6	Incheon National University
331.80	38	48	6	Sejong University
.	.	49	6	Jeju National University
.	.	50	6	Dong-A University
.	.	51	6	Dongguk University - Campus

[a] I restricted the sample to colleges that evaluate all 4 subjects in CSAT: Korean, Math, English, and Social Studies or Science. [b] All the science and technology-focused colleges are excluded.

Table 2: Descriptive Statistics for the Top 46 colleges (from college-level data)

	Mean	SD	N
From Selective High Schools (%)	4.94	6.19	598[a]
Student Loan Debtor (%)	9.28	4.80	679[b]
Seoul Capital Area	0.57	0.50	782[c]
Public	0.30	0.46	782[c]
Faculties	741	425	736
Number of Slots (early)	1,968	848	734
Number of Slots (regular)	1,359	644	734
Applications	30,215	19,019	734
Applications per slot	11.18	5.79	734
Enrollment (early)	1,698	819	734
Enrollment (regular)	1,327	639	734
Enrollment rate (early)	85.29	12.43	734
Enrollment rate (regular)	97.49	5.98	734
Attendance	13,304	5,165	777[c]

[a] Outcome variable has been available since 2010.
[b] Variables have been available since 2008 and not available for 2023. [c] Variables have been available since 2007.

to the probability of graduation on time, either. The difference in the number of observations is from the two women's colleges. Regarding the number of departments, Korea Aerospace University has the lowest number because of its distinctive establishment. There are other cases of bigger admission units: schools instead of departments.

5 Empirical testing

5.1 Identification Strategy

I use the two-way fixed effects model to estimate the impact of the application cap in 2013, assuming that the intensity of exposure from the cap varies by prestige. In other words, among desirable colleges, less prestigious institutes benefited more from the cap.

Table 3: Descriptive Statistics for the Top 46 colleges (from department-level data)

	Mean	SD	N
Avg Duration[a]	5.65	0.27	368
Avg Duration (male)	6.27[b]	0.24	352
Avg Duration (female)	5.02	0.30	368
Pr(Grad. in 4 yrs)	16.66	5.82	460
Pr(Grad. in 4 yrs), male	8.93	4.58	440
Pr(Grad. in 4 yrs), female	25.66	9.85	460
Pr(Grad. in 6 yrs)	55.58	8.50	460
Pr(Grad. in 6 yrs), male	35.36	8.86	440
Pr(Grad. in 6 yrs), female	79.92	5.62	460
Number of Dept.	79	46	736
Faculties	506	230	736
Number of Slots	2,975	1,103	736
Applications	30,854	19,126	736
Applications per slot	10.30	4.98	736
Enrollment	2,944	1,099	736
Enrollment (male)	1,553	667	736
Enrollment (female)	1,391	651	736
Enrollment rate	98.94	3.20	736
Attendance	12,904	5,034	736
Attendance (male)	6,871	3,105	736
Attendance (female)	6,033	2,907	736

[*] Differences with the college-level data are because of merge, division, and name changes among departments.

[a] Average duration is measured with those who graduate within 8 years. Since the last year in the data is 2023, the year 2015 is the last year for the variable. And there are 2 women's-only colleges.

[b] Military service is mandatory for males (18 to 24 months).

$$Y_{it} = \alpha + \beta Rank_i \times Post_t + X'_{it}\gamma + \theta_i + \delta_t + \epsilon_{it} \tag{1}$$

Y_{it} is a set of outcomes for college i at year t. For the matching quality, I use the share of freshmen who graduate from selective high schools as a proxy for the share of desirable students. As an alternative, I use the average duration of graduation and the share of graduates graduating in 6 years. If the application cap weakens the matching quality, all but the duration would decrease in more prestigious colleges. For the socioeconomic gap, I use the share of student loan debtors as a proxy for the share of low-SES students. If the cap reduces the socioeconomic gap, the outcome would increase in more prestigious colleges. $Rank_i$ is the inversed ranking of college i. The top college, SNU, is ranked number 46, and the second-ranked is number 45, etc. I use the ranking in the year 2010 since that is the first year where the main outcome variable is available. I should check robustness with ranking in the years before 2013. $Post_t$ is a dummy variable indicating the post-period, year 2013 and later. In the vector of controls, X'_{it}, I include the number of departments, faculty members, and slots[12]. θ_i is college-specific fixed effect and δ_t is year fixed effect. ϵ_{it} is the idiosyncratic error term. β represents the impact of the cap on less selective but still desirable colleges.

For the credibility of the results, I should satisfy following conditions: parallel trends between the colleges with different level of prestige, no anticipatory effects

For the parallel trend, figure 3 shows the change in the share of desirable students for each ranking group.[13] There happens an increase in the share for ranking groups 2 and 3 right after the cap. Decreases after 2013 in these groups, apparently in group 2, might happen because science and technology-focused universities attract desirable students. I will show the parallel trends for the duration and time to graduate.

[12]While public colleges are under the direct maximum student number controls of the government, private colleges are free to set their slots, except for the Seoul Capital Area (SCA) and special majors such as medicine and education. *Enforcement Decree Of The Higher Education Act §28*

[13]The top school, SNU is noted as group 1. There is a bump in 2014. One explanation is that graduates from the science and the talented high schools the SNU accepted early graduation from these institutes increases. Moreover, it seems to be affected by the weighting more in extracurricular activities than GPA for other selective high schools.

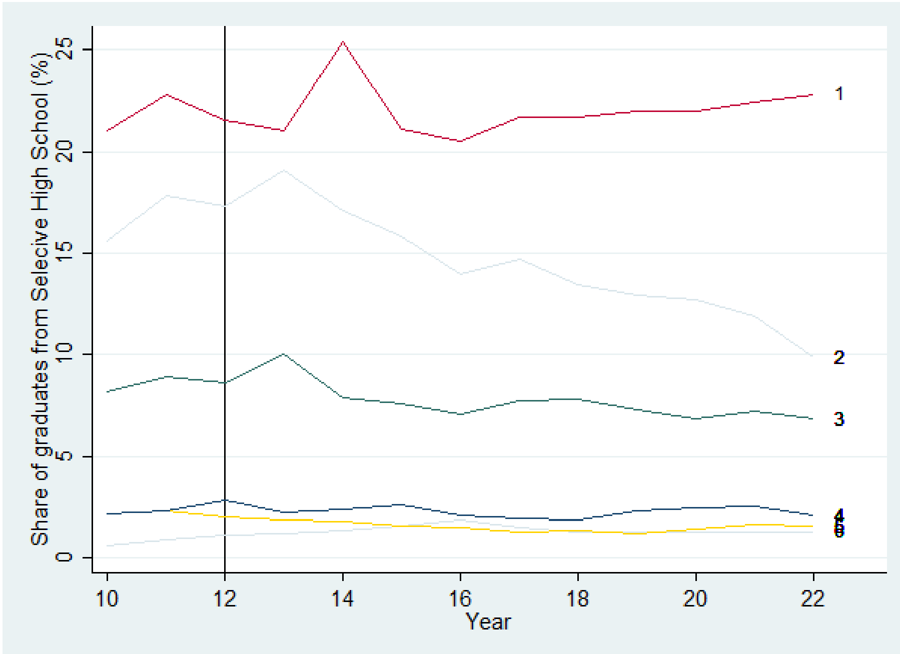

Figure 3: Pre-trends of the Share of Desirable Students by Ranking Group

Second, there is little room to respond to the cap for both the applicants and universities. It would be difficult to change applicants' strategy to apply for either early admission or regular. For universities, they have much less time to react to be strategic because they should announce the admission criteria to students in around April.

5.2 Results

Table 4 shows that the share of desirable students in the more prestigious colleges decreases after the cap with controlling time-invariant unit characteristics. I add college characteristics such as the number of departments, faculty members, and the number of projected slots as controls in columns 2 and 4. Moreover, I add dummies for each year in the last two columns. Column (2) shows that a college ranked one unit higher loses 0.06 percentage points (pp) less in its share of desirable students than a college ranked one unit lower, with all controls.

Moreover, I estimate the effect of the cap on other academic outcomes related to graduation (Table 5). The average duration, years to complete undergraduate education, seems to be affected by the cap for all estimations. However, the probability of graduation in a time (6 years) shows a decrease of .07 pp among less selective colleges after the cap only for the last column.

For the socioeconomic gap, table 6 shows that the share of tuition loan borrowers increases in more prestigious colleges. With controls, a college ranked one unit higher gains 0.09 pp more in its share of low socioeconomic status students than a college ranked one unit lower. Table 7 shows similar but less significant results with alternative measures, loans for both tuition and living costs.

Table 4: Estimated Effects of the Cap on the Share of Desirable Students (SDS)

	(1)	(2)
	SDS	
Rank × Post	-0.065***	-0.061***
	(0.020)	(0.021)
Number of Departments		0.003
		(0.004)
Number of Faculties		0.001
		(0.002)
Number of Slots		-0.001
		(0.001)
Constant	4.651***	5.656**
	(0.252)	(2.264)
College FE	X	X
Year FE	X	X
R sq.	0.166	0.172
Obs.	585	584

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Estimated Effects of the Cap on the Graduation

	(1)	(2)	(3)	(4)
	Avg. Duration		P(Grad in 6yr)	
Rank × Post	0.002*	0.002**	-0.066	-0.066*
	(0.001)	(0.001)	(0.041)	(0.037)
Number of Departments		-0.001**		0.005
		(0.000)		(0.005)
Number of Faculties		0.000		-0.002
		(0.000)		(0.007)
Number of Slots		-0.000		-0.001
		(0.000)		(0.003)
Costants	5.746***	5.760***	48.979***	52.087***
	(0.011)	(0.098)	(0.516)	(10.042)
College FE	X	X	X	X
Year FE	X	X	X	X
R sq.	0.420	0.434	0.599	0.601
Obs.	352	352	440	440

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Estimated Effects of the Cap on the Share of Low SES Students

	(1)	(2)
	Share of Loan Borrowers (Tuition)	
Rank × Post	0.089***	0.092***
	(0.023)	(0.026)
Number of Departments		0.001
		(0.005)
Number of Faculties		-0.001
		(0.002)
Number of Slots		-0.000
		(0.001)
Constant	12.883***	13.646***
	(0.385)	(1.143)
College FE	X	X
Year FE	X	X
R sq.	0.701	0.699
Obs.	664	659

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Estimated Effects of the Cap on the Share of Low SES Students 2

	(1)	(2)
	Share of Loan Borrowers (Tuition and Living Costs)	
Rank × Post	0.044*	0.047*
	(0.022)	(0.024)
Number of Departments		0.000
		(0.006)
Number of Faculties		-0.001
		(0.002)
Number of Slots		-0.000
		(0.000)
Constant	13.117***	13.922***
	(0.293)	(1.066)
College FE	X	X
Year FE	X	X
R sq.	0.162	0.164
Obs.	664	659

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

5.3 Robustness Check

I check the results with the top school. Table 8 shows its results in the share of desirable students, average duration, probability of graduating in 6 years, and the share of low socioeconomic status students. The magnitude of the effects decreases when the top school is included, which implies the SNU benefits as well in terms of desirable students. There is a small increase in the average duration. However, the probability of graduation in time turns significant with a size increase. In figure 4, time trends by ranking groups show that the probability converges among groups and SNU shows a decreasing trend. In the sense that the probability of the SNU is still higher than others, it would be caused by a technical issue.

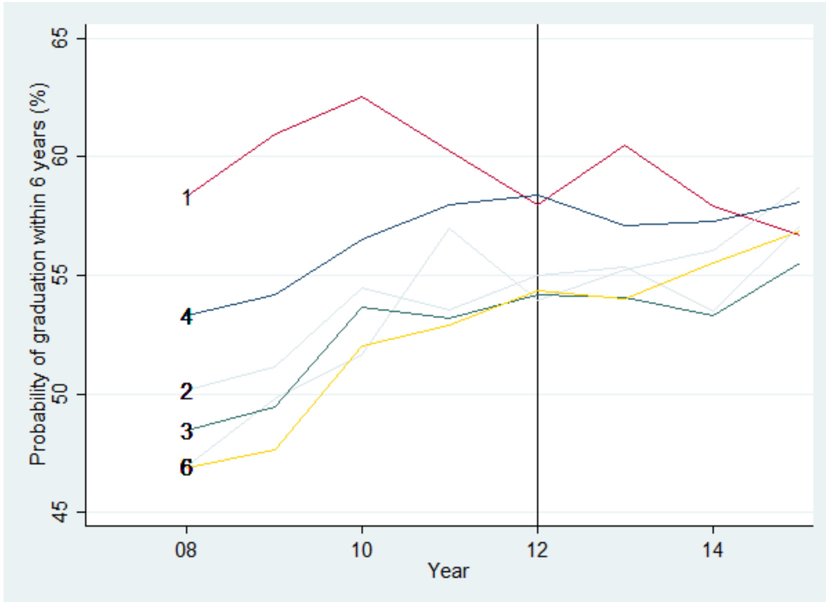

Figure 4: Time trends of the Probability of Graduating in 6 years by Ranking Groups, including Top School

Table 8: Estimated Effects of the Cap on Outcome Variables, Including the Top School

	(1)	(2)	(3)	(4)
	SDS	AvgD	PG6	%L-SES
Rank × Post	-0.052**	0.003***	-0.081**	0.094***
	(0.021)	(0.001)	(0.037)	(0.023)
Obs.	597	360	450	674

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

5.4 Heterogeneous Effects

I examine the effect of the cap by gender because of the possibility of systematic differences in graduation. In the summary statistics, the average duration differs by around 1.2 years between males and females. Dividing by gender, I can find consistent change from the cap in the male group (Table 9). Duration increases by 1.5 days (= 0.004×365) and the probability of graduation in time decreases by 0.13 pp. These results can connect with the evidence that the share of desirable students decreases in more prestigious colleges.

I examine all the outcomes with the effect of region and whether the institution is public or not. This is because some evidence of heterogeneity in these factors might suggest policy implications at a point of geographical inequality or accountability. However, Table 10 shows little heterogeneous effect for the share of desirable students by region or public institution. Table 11 gives results that low-SES students increases with public colleges.

Table 9: Subgroup Analysis by Gender

	(1) AvgD(male)	(2) AvgD(female)	(3) PG6(male)	(4) PG4(female)
Rank × Post	0.004***	-0.001	-0.133***	0.011
	(0.001)	(0.002)	(0.035)	(0.037)
Obs.	344	360	430	540

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Heterogeneity in Cap Effects (SDS) by Region and Public college

	(1)	(2)	(3)
	SDS	SDS	SDS
Rank × Post	-0.061***	-0.063***	-0.056**
	(0.021)	(0.022)	(0.021)
× Seoul Capital Area (SCA)		-0.006	
		(0.009)	
× Public			0.015*
			(0.008)
Obs.	584	584	584

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 11: Heterogeneity in Cap Effects (Share of Low SES Students) by Region and Public College

	(1)	(2)	(3)
	Share of Loan Borrowers (Tuition)		
Rank× Post	0.092***	0.106***	0.067**
	(0.026)	(0.027)	(0.025)
× Seoul Capital Area (SCA)		-0.043	
		(0.021)	
× Public			0.057***
			(0.018)
Obs.	659	659	659

Notes: Standard errors in parentheses are clustered at the college level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

6 Conclusion

I investigate the unintended distributional impact of the cap on college applications in the Korean setting. The application cap affects to matching quality Empirical evidence has been found that the application cap weakens matching quality and reduces the socioeconomic gap. The examined empirical results are based on the theoretical analysis. On average, less prestigious colleges benefit from this policy in terms of desirable students. Moreover, students with less affluent background enrolled in more prestigious colleges more after the cap. Without regional differences, the effect is heterogeneous among gender groups because of the systematic difference. For the policymaker, the study might give some insights from the empirical evidence.

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Appendix. Proofs

Proof of Lemma 1

Because $k_A < \mu$, there exists $d_1 \in (-\delta_B, \delta_A)$ which satisfies $\mu F(d_1) = k_A$. Without the cap, $m_A = k_A$ regardless of whether $\mu F(d_1)$ is greater or less than k_A. This is by proposition 1. So does $m_B = \mu - k_A$. m_A and m_B have the same value for $d > d_1$. With the cap, the posterior probability should be taken into account. Since π is at least μ and the probability that good students are admitted by the college A is at least k_A/μ, $p_A^h(e_h, e_l) > k_A$. Back to the indifference equation, $n_B = 0$ happens when $p_A^h(e_h, e_l)(a - e_h) > p_B^h(e_h, e_l)(b + e_h)$. In other words, all applicants with $s_i = h$ apply to the college A if their expected utility is larger in A. Then $n_A = k_A$, $n_B = 0$, and $p_B^h(e_h, e_l) = 1$.

Additionally, I have the range of d which I noted as somewhat big. Using k_A which is the lower bound of $p_A^h(e_h, e_l)$, I can rewrite the case that the expected utility is larger in A as follows. $k_A(a - \delta_A) > b + \delta_A$. The e_h is substituted by δ_A because now I show $n_B = 0$ for some large $d > d_1$. Since the upper bound of d_1 is δ_A, I set $e_h(= d) = \delta_A$. When I rewrite the inequality with $d = \frac{a-b}{2}$, $d > \frac{(1+k_A)\delta_A + (1-k_A)a}{2}$.

Proof of Lemma 2

Since $p = 0$, $\pi = 1$. I can rewrite the indifference equation for applicants with $s_i = h$ as

$$\min\left[\frac{k_A}{\mu F(e_h)}, 1\right](a - e_h) = \min\left[\frac{k_B}{\mu(1 - F(e_h))}, 1\right](b + e_h) \qquad (2)$$

First, when there is no competition ($k_A \geq \mu F(d) \geq \mu - k_B$), $\min\left[1, \frac{k_A}{\mu F(d)}\right] = 1 = \min\left[1, \frac{k_B}{\mu(1-F(d))}\right]$. It implies $e_h = d$. Therefore, $m_B = n_B = \mu(1 - F(d))$ ($m_A = n_A = \mu F(d)$). Second, when there is competition only in B ($\mu(1 - F(d)) > k_B \leftrightarrow \mu F(d) < \mu - k_B$), $\min\left[1, \frac{k_A}{\mu F(d)}\right] = 1 > \min\left[1, \frac{k_B}{\mu(1-F(d))}\right]$. It implies $\mu(1 - F(d)) > \mu(1 - F(e_h))$. It is equivalent to $e_h > d$. Using $d = \frac{a-b}{2}$, it implies $b + e_h > a - e_h$. Then I need

$\min\left[1, \frac{k_A}{\mu F(e_h)}\right] > \min\left[1, \frac{k_B}{\mu(1-F(e_h))}\right]$, which implies $k_B < \mu(1-F(e_h))$ by the range I started with ($\mu(1-F(d)) > k_B$) and $e_h > d$. By the *Proposition 1*, $m_B = n_B = k_B$. $m_A = \mu - k_B = \mu - m_B > n_A = \mu F(e_h)$ by $k_B < \mu(1-F(e_h))$[14]. Third, when there is competition only in A ($\mu F(d) > k_A$), the results of $n_B = \mu(1-F(e_h)) < m_B = \mu - k_A = \mu - m_A (n_A = m_A = k_A)$ can be derived by symmetrically with the second case.

Proof of Proposition 2

I can find some $\bar{d} > 0$ such that $n_B > m_B$ for $d < \bar{d}$. For doing that, I claim that $e_h < d$ whenever (i) $\mu \leq k_A$ or (ii) $\mu > k_A \geq \mu F(d)$.

I show $e_h < d$ by contradiction. Suppose $e_h \geq d$. From (i) and assumptions, $k_B > k_A > \mu$. Then $k_B > \mu(1-F(d))$. From (ii) and assumptions, $k_B > k_A \geq \mu F(d)$. Then $k_B > \mu(1-F(d))$. Since I suppose $e_h \geq d$, $k_B > \mu(1-F(e_h))$. Then $p_B^h(e_h, e_l)$ can be rewritten as $\pi + (1-\pi)p_B^l(e_h, e_l)$. And $p_B^l(e_h, e_l) = \min\left[\frac{k_B - \mu(1-F(e_h))}{(1-\mu)(p(1-F(e_h)) + (1-p)(1-F(e_l)))}, 1\right]$. However, from $e_h \geq d$, $p_A^h > p_B^h$ and it implies $p_A^l > p_B^l$ (*). It requires $e_l \geq d > 0$. From $F(d) > 0.5$, the denominator of p_A^l is greater than that of p_B^l. And from $k_B > k_A$, the numerator of p_B^l is greater than that of p_A^l. By these, I can conclude $p_A^l < p_B^l$. This contradicts (*).

When I rewrite the claim, $e_h < d$ for all $d > 0$ when $\mu \leq k_A$. Or, $e_h < d$ when $\mu > k_A \geq \mu F(d)$. For the former one, $k_A > \mu F(d)$ when $\mu \leq k_A$. Then $m_B = \mu(1-F(d))$. However, since $e_h < d$, $n_B = \mu(1-F(e_h)) > m_B$. For the latter one, I can rewrite the range as follows: for all $d \in (0, d_1]$ such that $\mu F(d_1) = k_A$. In this case, $m_B = \mu(1-F(d))$ for $d < d_1$ and $\mu - k_A$ for $d \geq d_1$. This is because $k_A \geq \mu F(d)$ for $d \geq d_1$. For both cases, $n_B > m_B$. By continuity, there exists $\bar{d} > d_1$ such that $n_B > m_B$ for $d \in (d, \bar{d})$.

[14] If $\mu(1-F(d)) > k_B, k_A > \mu F(d)$ by $\mu - k_B > \mu F(d)$ and $k_A + k_B > \mu$