Payments, Reserves, and Financial Fragility*

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October 9, 2024

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Abstract

We propose a dynamic theory of payments and fragility, highlighting a conflict between money's payment and various non-payment functions (e.g., store of value). We show that agents make payments and produce only when money reserves are abundant and when the payment function is relatively more important than non-payment functions. Otherwise, history-dependent equilibria arise in which an agent's payment and production decisions depend on the payment history of other agents within an equilibrium, giving rise to fragilities. The theory explains why payments frequently encounter delays and interruptions even if the reserve is always accepted as means of payment. Improving payment technologies may not eliminate such fragility when reserves remain scarce and valuable for non-payment functions. The theory helps explain the evolution of money and payment systems, encompassing historical metallic and commodity payments, modern bank payments, cross-border payments, and contemporary digital payment systems.

Keywords: payments, reserves, medium of exchange, store of value, fragility, technology

*Goldstein and Zeng are at the Wharton School of the University of Pennsylvania. Yang is at University College London. Goldstein is also with the NBER. We are grateful to Joseph Abadi, Andrew Atkeson, Saki Bigio, Markus Brunnermeier, Darrell Duffie, Vincent Glode, Zhiguo He, Zhengyang Jiang, Narayana Kocherlakota, Nobu Kiyotaki, Arvind Krishnamurthy, Ricardo Lagos, Ye Li, Yiming Ma, Vincent Maurin, Stephen Morris, Guillermo Ordonez, Cecilia Parlatore, Jonathan Payne, Linda Schilling, Hyun Song Shin, Jón Steinsson, Victoria Vanasco, Quentin Vandeweyer, Alex Vardoulakis, Yenan Wang, Wei Xiong, and conference and seminar participants at Bank for International Settlements, Barcelona Summer Forum, Baruch College, Carnegie Mellon University, Chinese University of Hong Kong, EFA, FIRS, Finance Theory Group Toronto Meeting, Georgia State University, INSEAD, Macro Finance Society/Becker Friedman Institute Macro Finance Research Program Workshop, NBER Summer Institute Asset Pricing/Macro, Money and Financial Frictions Workshop, Oxford-Zurich Macro-Finance Conference, SFS Cavalcade, Toulouse School of Economics, University of British Columbia, University of Pennsylvania, University of Rochester, and University of Texas for helpful comments. We thank Ding Ding, Xuning Ding, and Yiqun Wang for excellent research assistance. All errors are ours.

1 Introduction

Money and payments play a crucial role in economic transactions, an idea widely acknowledged since Fisher (1911), Baumol (1952), and Tobin (1956). Throughout history, humans have developed various means of payment, from wheat to shells to metal coins. Modern payments are based on fiat money and intermediated by banks, and the economic scale is huge.[1] Recently, payment systems have undergone significant technological improvement with the emergence of fast payment platforms, cryptocurrencies, and central bank digital currencies (CBDCs) (e.g., Brunnermeier, James and Landau, 2019, Duffie, 2019, Brunnermeier and Payne, 2022). Also, economists have long focused on the ability to maintain stable prices (Friedman and Schwartz, 1963), that is, whether money is accepted (Gorton and Pennacchi, 1990, Kiyotaki and Wright, 1993), when assessing any form of money and payments. Across generations of evolution, it becomes evident that the payment systems that have thrived are indeed the ones that embrace superior technologies while maintaining a stable value of money (Steinsson, 2023a,b).

In this paper, we argue that another simple yet crucial factor shapes the evolution of money and payment systems: whether money is deployed for payments at all, which in turn stems from an overlooked conflict between money's payment and non-payment roles. In reality, payment-making activities are susceptible to delays and interruptions despite technology improvements and even money is known to be accepted as a means of payment. Despite technological advancements, modern banks encounter significant voluntary delays and interruptions in making payments during both normal and crisis times (McAndrews and Potter, 2002, Afonso and Shin, 2011, Afonso, Duffie, Rigon and Shin, 2022), and delays in interbank payments in apparently normal times strongly predict payment market disruptions and liquidity crunches (Copeland, Duffie and Yang, 2024).[2] A notable example is the Herstatt Crisis in 1974, in which major US and global banks delayed paying each other after Herstatt Bank, a relatively small German bank, failed to make payments, which directly led to the creation of the Basel Committee on Banking Supervision (Goodhart, 2011). Another more recent example is the unprecedented spike in US Treasury repo rates by over 1,000 basis points in September 2019 (Afonso, Cipriani, Copeland, Kovner, La Spada and Martin, 2020, Correa, Du and Liao, 2020), where dysfunctional interbank payments played a key role (Afonso, Duffie, Rigon and Shin, 2022, Copeland, Duffie and Yang, 2024).[3]

[1]For example, the Fedwire, the real-time gross settlement funds transfer system for financial institutions operated by the U.S. Federal Reserve Banks, sees a daily volume of more than $4.2 U.S. trillion in 2022. Payments per se also generate huge revenues for the financial institutions that handle them: global payments revenues totaled $2.2 U.S. trillion in 2021, roughly 3% of global GDP.

[2]The interbank lending markets, which rely on short-term credits to facilitate interbank payments, also constantly experience disruptions (e.g., Ashcraft and Duffie, 2007, Afonso, Kovner and Schoar, 2011, Ashcraft, McAndrews and Skeie, 2011, Acharya and Merrouche, 2013, Craig and Ma, 2021).

[3]When testified before the House Financial Services Committee on June 21, 2023, Federal Reserve Chair Jerome

Understanding the root causes of such fragility is vital for comprehending the development and evolution of money and payments.

This paper builds a new framework to understand why people make payments and the financial and real implications. We posit that, economically, making a payment must involve the transfer of a *reserve* good, which has limited supply and holds value for other non-payment functions that may potentially conflict with the reserve's payment function in the short-run. We show that these two natural assumptions of reserve scarcity and multi-functionality have profound implications: they imply that payments are fragile despite consensus on the value of the reserve good, and advancements in payment technologies cannot eliminate this fragility.

The core insight of our framework lies in uncovering a fundamental conflict between two essential functions of the reserve good. On the one hand, the reserve serves as a medium of exchange for payments in that transferring it creates transactional gains, while on the other hand, it carries non-payment functions such as being a store of value, helping avoid regulatory costs or stigmas, or delivering other service flows in that temporarily lacking it incurs costs. This conflict introduces various opportunity costs when the scarce reserve is transferred as a medium of exchange in the economy. This conflict results in a trade-off in payments: while an agent may benefit from successfully transferring the reserve good to another for production and transactions, she risks losing the reserve's non-payment functions if reciprocal payments are not made in the future. The trade-off creates a dynamic coordination motive in agents' asynchronous and reciprocal payment decisions. An agent may cease making payments if it expects other agents to do the same in the future, and past payment histories endogenously arise as a coordination device even without a fundamental shock. Our model thus explains the fragility observed in payments and the prevalence of delays and history-dependent behaviors despite improvements in payment technologies.

The versatility of the reserve good concept allows us to capture many payment contexts:

- The reserve can be historically interpreted as a durable good, such as gold or silver, which is physically scarce. Apart from being used as a medium of exchange, they are also used as jewelry and conductors, as well as a store of value (Jermann, 2022).

- The reserve can be understood as central bank reserves in a modern monetary system, where their short-term supply is constrained by monetary policy implementations. Every interbank payment involves an irrevocable transfer of central bank reserves, but they are also valuable to banks in earning interest on reserves, fulfilling regulatory requirements

Powell admitted that future interest hikes should avoid a repeat of the 2019 repo market and interbank payment crisis. See "Powell Haunted by Repo Crisis as Fed Aims to Cut Balance Sheet," Bloomberg, July 9, 2023.

(e.g., Correa, Du and Liao, 2020), and avoiding daylight overdrafts or discount windows (e.g., Copeland, Duffie and Yang, 2024).

- The reserve can represent a dominant currency like the U.S. dollar due to its extensive use as a means of payment in global trades and financial settlements (e.g., Gopinath and Stein, 2021, Coppola, Krishnamurthy and Xu, 2023). Its supply is limited, and it is also valued as a global safe asset for storing value and currency hedging (e.g., He, Krishnamurthy and Milbradt, 2019, Jiang, Krishnamurthy and Lustig, 2021, Maggiori, Neiman and Schreger, 2021, Brunnermeier, Merkel and Sannikov, 2022) beyond the use in payments.

- The reserve can be understood as commercial bank notes, that is, deposits, whose supply is limited by banks' reserve requirements and the money multiplier (Tobin, 1965). Deposits are used by households and firms as a means of payment and also serve as a store of value.

- The reserve can be also interpreted as a digital currency such as stablecoins or central bank digital currencies (CBDCs), whose supply is constrained by design. Beyond potential payment functions, they deliver other non-payment functions. For example, stablecoins are widely held by investors as collateral for speculating on other crypto-assets (Gorton, Klee, Ross, Ross, and Vardoulakis, 2023).

Our model shows that when the conflict between payment and non-payment functions is small (big), the equilibrium is good (bad) in that agents always make (deny) payments to each other and the production level is high (low), irrespective of past payment histories. These two equilibrium types serve as benchmarks where the payment system functions or freezes. However, when the magnitude of the conflict falls within an intermediate range, payment decisions become history-dependent due to an endogenous asynchronous coordination motive. This can result in fragility even without any fundamental shocks. Agents anticipate reciprocal payments from others based on historical payment patterns within an equilibrium, which emerge as a coordination device. Even well-funded agents may delay or halt payments if they observe delays or halts by others in the past. These history-dependent payment behaviors and potential coordination failures contribute to the fragility widely observed in payment systems. Regarding the Herstatt Crisis in 1974, our framework ascribes it to the history-dependence of payments, implying that observed payment failures in the past may coordinate banks to cease making payments in the future despite sound funding conditions. Regarding interbank payment and repo market disruptions in 2019 and 2020, our framework attributes them to increasing bank balance sheet costs and "quantitative tightening" in recent years, which contributed to reserve scarcity and the conflict between reserves' payment and non-payment functions. This explanation is consistent with recent empirical work such as Copeland, Duffie and Yang (2024) that shows a strong relationship between the amount of central bank reserves and payment market efficiency.

In addition to modern payment disruptions, our framework provides a new perspective for evaluating the historical and future evolution of payments, highlighting a previously overlooked conflict between the reserve good's medium-of-exchange and store-of-value functions and its impact on financial fragility. Along this line, our framework can explain historically "the big problem of small change" and the various payment crises under the metallic system (e.g., Sargent and Velde, 2001, Steinsson, 2023a), which arise precisely due to the reserve good's non-payment functions outweighing its payment function. Similarly, our model provides one complementary explanation for the decline of the gold standard and further suggests that money should possess no intrinsic value to mitigate the conflict between its payment and non-payment functions. By the same token, our model suggests that it is optimal to set payment deposits' interest rate to zero, providing an alternative explanation for why commercial banks exert market power on the deposit markets (e.g., Drechsler, Savov and Schnabl, 2017). Furthermore, our framework suggests that advancements in payment technologies through digitalization may not necessarily reduce payment fragility when the underlying reserves remain scarce and useful for other non-payment functions. Instead, we predict that the winning means of payment for the next generation are likely to be those that not only provide superior technologies to improve the payment function but also reduce the demand of holding reserves for other non-payment functions and mitigate the short-run conflict between reserves' medium-of-exchange and store-of-value functions.

A methodological contribution of our paper is the development of a dynamic framework of payments in which strategic complementarity endogenously arises, and payment fragility happens *within* an equilibrium. In doing so, we extend a technique in the literature of repeated games with imperfect public monitoring (Abreu, Pearce, and Stacchetti, 1990) to stochastic dynamic games. Whenever multiple equilibria arise, we can further characterize the welfare outcomes of *all* equilibria in closed form. Thus, our model and the solution method may also inform future studies that focus on asynchronous coordination in dynamic contexts.

Related Literature. Our paper contributes to several branches of the literature on banking, money, and payments. First, our paper is closely related to the theoretical literature on banking, coordination, and financial stability (Diamond and Dybvig, 1983, Allen and Gale, 2000, Diamond and Rajan, 2005, Goldstein and Pauzner, 2005). The most closely related paper is Diamond and Rajan (2006), who analyze the conflict between money's payment function and its "fiscal" function, that is, its use for paying taxes. They demonstrate how this conflict can lead to fragility in bank lending when the demand for payments is high, as depositors are incentivized to store money in banks for fiscal purposes but also need to withdraw it for payments. Complementary to their insights, we introduce the notion of reserve scarcity arising from the conflict between reserves' payment and all non-payment functions and explore the implications on payment fragility. Also relatedly, Freixas, Parigi and Rochet (2000), Donaldson, Piacentino and Thakor (2018), Bolton, Li, Wang, and Yang (2020), Parlour, Rajan and Walden (2020), and Li and Li (2021) show that payment risks lead to inefficient and unstable bank lending through banks' liquidity management.[4] The empirical literature has also widely documented the coordination and history-dependent natures of interbank payments and explored its consequences on financial fragility (e.g., McAndrews and Potter, 2002, Bech and Garratt, 2003, Afonso and Shin, 2011, Afonso, Duffie, Rigon and Shin, 2022, Copeland, Duffie and Yang, 2024). Our key contribution is to uncover the root of the emergence of fragility in the payment context: reserve scarcity and a conflict between its payment and non-payment functions. Notably, strategic complementarity endogenously arises in our model even if the stage game does not feature coordination, and fragility may arise within an equilibrium (rather than requiring the switch or selection between multiple equilibria). Therefore, our framework differs not only from static coordination problems (e.g., Diamond and Dybvig, 1983, Morris and Shin, 1998, Goldstein and Pauzner, 2005), but also from Frankel and Pauzner (2000) and He and Xiong (2012) in which fundamental shocks serve as a coordination device to select an equilibrium.

Our paper also contributes to the large literature on money and payments. Macroeconomic models have increasingly and explicitly incorporated the payment role of money and payment risks, demonstrating their significant impact on macroeconomic outcomes and optimal policy design (e.g., Lagos and Wright, 2005, Lagos and Zhang, 2020, Bianchi and Bigio, 2021, Piazzesi, Rogers and Schneider, 2021, Piazzesi and Schneider, 2021, Bigio, 2022, Bigio and Sannikov, 2023). This literature typically considers search and bargaining frictions and focuses on equilibria where money is used as a medium of exchange without fragility. Complementing this literature, we abstract away from search and bargaining but instead focus on coordination, showing that the conflict between money's payment and non-payment functions leads to endogenous fragility in payments and production. On the microeconomic side, this literature has also experienced a recent revival thanks to the fast development of new payment technologies in the last decade,[5]

[4]Another related literature focuses on bank liquidity management due to uncertainty, asymmetric information, or counterparty risks (e.g., Caballero and Krishnamurthy, 2008, Allen, Carletti and Gale, 2009, Acharya and Skeie, 2011, Gale and Yorulmazer, 2013, Heider, Hoerova and Holthausen, 2015).

[5]Recent empirical literature shows that new payment technologies improve economic efficiencies in consumption, investment, and lending decisions (e.g. Jack and Suri, 2014, Muralidharan, Niehaus, and Sukhtankar, 2016, Higgins, 2020, Ghosh, Vallee, and Zeng, 2022, Ding, Gonzalez, Norden, van Doornik and Zeng, 2023) without examining the financial stability implications.

and a growing literature has explored the potential of next-generation payment systems including stablecoins and CBDCs (see, e.g., Duffie, 2019, Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022, Brunnermeier and Payne, 2022, for surveys). We stress that improvements in payment technology may not necessarily reduce payment fragility if reserves remain scarce and valuable for other non-payment functions.

Our framework is also inspired by the large literature that highlights the endogenous emergence of money (e.g., Kiyotaki and Wright, 1989, 1993, Kocherlakota, 1998, Lagos, Rocheteau and Wright, 2017) and credits (e.g., Atkeson, 1991, Kehoe and Levine, 1993, Kocherlakota, 1996, Alvarez and Jermann, 2000) in facilitating payments, highlight whether they are accepted as means of payments. Among this literature, the two most closely related papers are Atkeson (1991) and Kocherlakota (1998) which also built on the framework of Abreu, Pearce, and Stacchetti (1990), focusing on the rise of international credits and "memory" as money, respectively, from repeated interactions. Complementing this literature, we focus on the conflict between the payment and non-payment functions of money and how it leads to fragility rather than explain why money or credits may endogenously emerge as means of payment. To focus on the inalienable role of reserves in settling payments, our model deliberately abstracts away from the distinction between money and credits. An interesting question is how the co-existence of reserves and credits (as modeled in Townsend, 1980, 1989, Gu, Mattesini, and Wright, 2016, etc.) may affect payment fragility, which we leave for future research.

Finally, our paper joins a new literature that studies the implications of monetary policy tightening on financial stability. Since Fisher (1933) and Friedman and Schwartz (1963), it has been recognized that aggregate reserve scarcity may lead to deflation and economic depression, and Brunnermeier and Sannikov (2016) formulate that idea in a model with financial intermediation and different forms of money. Instead of focusing on price levels and money's store of value function, we take a complementary view and show that reserve scarcity has direct financial fragility implications when reserves are used as a medium of exchange. Recently, Acharya and Rajan (2022) and Acharya, Chauhan, Rajan, and Steffen (2023) argue that aggregate reserves matter for bank liquidity provision due to banks' endogenous deposit-issuing and reserve-holding behaviors, which our model highlights. Lopez-Salido and Vissing-Jorgensen (2023) study the effects of quantitative tightening and show that the ratio between reserves and deposits plays a key role in determining important financial market rates. Our framework also complements a fast-growing literature that explores the causes and consequences of the disruptions in interbank payments in September 2019.[6]

[6]As discussed above, Correa, Du and Liao (2020), Afonso, Duffie, Rigon and Shin (2022), and Copeland, Duffie and Yang (2024) empirically document and explore how the scarcity in reserves leads to delays in interbank payments

2 The Model

We build a stochastic, dynamic model of payments. We present two versions of the model which generate identical analytical results but admit complementary interpretations of various non-payment functions of money, helping broaden the applications of the model. The baseline model features exogenous production to focus on coordination in payments and exchanging goods, but we endogenize production in Section A.2 to further show how production and price levels can be driven by payments. Appendix C contains all proofs.

2.1 Baseline model setup

Setup. Consider a discrete-time, infinite-horizon economy, $t = 0, 1, 2, ...$, with two risk-neutral agents, 1 and 2, which we sometimes also call "banks" when interpreting the model. We further extend the model to multiple, $N > 2$ agents in Section A.1. The agents have a common discount factor $\delta \in (0, 1)$, which captures their time preference.

There are two types of goods: a single unit of indivisible *reserve* and potentially many *rewards*. At $t = 0$, only one agent is endowed with the reserve while the other is not. No agents have any rewards at $t = 0$, but rewards can be created when agents make *payments* by transferring the reserve to each other, as we detail below.

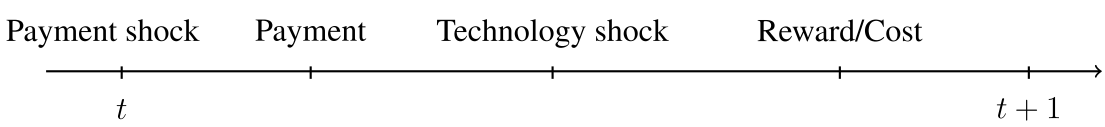

Figure 1: Timeline of the baseline setup

This figure shows the timeline of actions, events, and shocks in the baseline model setup. Both the reward and cost are accrued at the end of time t before the economy continues to time $t + 1$.

The timeline of the economy is illustrated in Figure 1. At any time $t \geq 0$, the agent who holds the reserve, suppose agent i, is subject to a private payment shock: with probability $\lambda \in (0, 1]$ agent i is supposed to transfer the reserve to agent j. Denote by $a \in \{0, 1\}$ agent i's possible private actions: $a = 1$ means she sends the reserve, whereas $a = 0$ means not and she keeps the reserve. When agent i sends the reserve, the potential transfer is subject to another private technology shock: with probability $\mu \in (0, 1]$ the transfer goes through to agent j. With probability $1 - \mu$, however, the transfer fails and the reserve remains with agent i; that is, it gets

as well as the disruptions in repo funding markets. Theoretically, d'Avernas and Vandeweyer (2020) and Yang (2024) build dynamic asset pricing models of repos to explain such empirical patterns, highlighting intraday liquidity management and bank regulations.

bounced back to agent i. Any payment outcome is publicly observable, and we denote by $k = 1$ a successful transfer of the reserve good and $k = 0$ otherwise. If the reserve is successfully transferred to the other agent j, the initially reserve-holding agent i will get $z > 0$ rewards at the end of time t, where z is a parameter. Intuitively, the magnitude of z captures the transaction need or transation gain per transfer, which we take as exogenous in the baseline model but will further endogenize in Section A.2. Rewards are perishable so will have to be consumed immediately by agent i at t, and the consumption value of one unit of rewards is normalized to 1. Finally, at the end of any time $t \geq 0$, the agent who does not hold the reserve suffers a per period cost of c, where $c > 0$ is a parameter. In reality, c parsimoniously captures the various opportunity costs incurred from not holding the reserve, which arise from the reserve's non-payment functions. The economy then moves to $t + 1$ regardless of whether the payment is made or not.

To highlight the conflict between the reserve's payment and non-payment functions, it is important to note that the reserve good has an effective present value of $\kappa = c/(1-\delta)$ if it is never used as a medium of exchange. In other words, permanently losing it would effectively incur a present cost of κ in terms of all non-payment functions to the current reserve-holding agent. Given that the supply of the reserve is normalized to one, κ thus also captures the *scarcity* of the reserve for payment functions: a higher present value of κ for non-payment functions suggests that the reserve good is more scarce for the payment function. Economically, this observation also implies that our notion of reserve scarcity fundamentally arises from the conflict between the reserve good's payment and non-payment functions. Suppose a hypothetical reserve good is completely useless for other non-payment functions but still accepted as a medium of exchange, it is then not scarce under our framework. Formally, we define:

Definition 1. *The reserve good's scarcity for payment functions is captured by the present value of the opportunity costs of transferring it: $\kappa = c/(1-\delta)$. The reserve good is more (less) scarce, or in other words, the conflict between the reserve's payment and non-payment functions is higher (lower), if κ is higher (lower).*

Equilibrium concept. To focus on to what extent current payment decisions depend on past payment histories, we adopt the equilibrium concept of Perfect Public Equilibrium (PPE), in which agents' optimal strategies are allowed to depend on the public history of past outcomes. Once a PPE exists, we can examine under what conditions agents' optimal payment strategies indeed depend on their past payment outcomes, and if yes, to what extent, and whether there are multiple equilibria.

Formally, denote by $s_t \in \{0, 1\}$ the state of the stochastic game at t: $s_t = 1$ means that agent 1 has the reserve and $s_t = 0$ means that agent 2 has it. Since payment outcomes are publicly observable, a generic public history of states is given by $s^t = (s_0, s_1, s_2, \cdots, s_{t-1}) \in \mathcal{S}^t$, where $s^{t+1} = (s^t, s_t)$ and $\mathcal{S}^t \doteq \{0,1\}^t$. The public history at t thus fully summarizes all public signals up to t. Denote by $\mathcal{S}^t$ = all possible public histories at t, and $\mathcal{S} = \bigcup_t \mathcal{S}^t$. A public strategy is then defined by a mapping $\sigma : \mathcal{S} \rightarrow \{0,1\}$. Note that it is sufficient to let σ specify the strategy for the reserve-holding agent but not the other agent because only the reserve-holding agent takes action. For the same reason, it is sufficient to use σ to denote both a public strategy and a public strategy profile. If a public strategy σ exhibits $\sigma(s^t) = 1$ for all $s^t \in \mathcal{S}$ (or $\sigma(s^t) = 0$ for all $s^t \in \mathcal{S}$), then we say σ is history-independent. Otherwise, it is history-dependent.

Definition 2. *A public strategy profile is a perfect public equilibrium (PPE) if for all t and all $s^t \in \mathcal{S}^t$, $\sigma|_{s^t}$ is a Nash equilibrium.*

In words, a PPE specifies a sequential equilibrium that only involves public strategies and which also constitutes a sequential equilibrium for the dynamic game from any date and any history. This definition is a direct counterpart to the standard definition of PPE in a repeated game.

We can immediately define whether a PPE is history-independent (or history-dependent) based on the description of history-independent strategies.

Definition 3. *A perfect public equilibrium (PPE) is history-independent if it involves only history-independent public strategies; otherwise, it is history-dependent.*

2.2 Alternative model setup

Our model can be set alternatively to accommodate aggregate shocks and particularly the store of value function of money while keeping all the equilibrium outcomes the same. Consider a discrete-time, infinite-horizon economy, $t = 0, 1, 2, ...$, with two risk-neutral agents, 1 and 2, which we also call "banks" when interpreting the model. There is no time discount, but starting at the end of $t = 0$ the economy is subject to an aggregate shock every period: with probability $\delta \in (0,1)$ the economy continues to $t+1$, whereas with probability $1-\delta$ the economy discontinues. Note that the continuation probably δ maps to the discount factor in the baseline model discussed above. The two types of goods, reserves, and rewards, are specified the same as in the baseline model.

The timeline of the economy is illustrated in Figure 2. At any time $t \geq 0$, the agent who holds the reserve, suppose agent i, is subject to a private payment shock: with probability $\lambda \in (0,1]$ agent i is supposed to transfer the reserve to agent j. Denote by $a \in \{0,1\}$ agent i's possible private actions: $a = 1$ means she sends the reserve, whereas $a = 0$ means not and she keeps

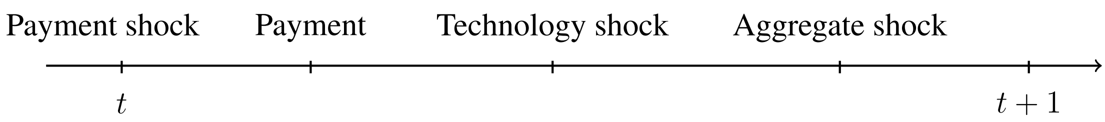

Figure 2: Timeline of the alternative setup

This figure shows the timeline of actions, events, and shocks in the baseline model setup. Both the reward and consumption penalty are accrued at the end of time t before the economy potentially continues to time $t+1$.

the reserve. When agent i sends the reserve, the potential transfer is subject to another private technology shock: with probability $\mu \in (0, 1]$ the transfer goes through to agent j, whereas with probability $1 - \mu$ the transfer fails and the reserve remains with agent i. Any payment outcome is publicly observable. If the reserve is successfully transferred to the other agent j, that is, a payment is successfully made, agent i will get $z > 0$ rewards, where z is a parameter. Rewards are perishable so will have to be consumed immediately by agent i at t, and the consumption value of one unit of rewards is normalized to 1. If the economy continues at the end of time t, then it moves to $t + 1$ regardless of whether the payment is made or not. Rather, if the economy discontinues at the end of time t, the reserve-holding agent consumes the reserve itself. The reserve-holding agent gets a normalized consumption value of 0, and the other who does not hold the reserve gets $-\kappa < 0$, where $\kappa > 0$. Economically, because κ occurs only in the aggregate bad state, it captures the store of value function of the reserve good. In other words, whoever holds the reserve good in the aggregate bad state is protected by the reserve good from a consumption loss. Note that the consumption penalty is analytically the same as the present value of the reserve good in the baseline model. Therefore, κ similarly captures the value and scarcity of the reserve good as in the baseline model.

An important observation is that the alternative economy is observationally equivalent to the baseline economy in terms of equilibrium profiles. In other words, if a strategy profile is a PPE in the baseline economy, it is then also a PPE in the alternative economy, and vice versa. This important feature allows us to flexibly interpret the various non-payment functions of the reserve good and map it to many rich economic applications.

2.3 Mapping the model to realistic payment contexts

Our model is parsimonious but general enough to accommodate many economically relevant applications of payments. Below, we discuss some of the key elements of the model to illustrate how it covers the essence of various payment applications.

Infinite-horizon economy with patient agents in the baseline setup. We set up an infinite-horizon dynamic economy to capture the general notion that payment activities are dynamic and reciprocal, and they involve long-term interactions among agents in most applications. Payment activities create value by solving the lack of double coincidence of wants, captured by the creation of rewards. However, payment activities are also costly and risky because making an outbound payment means a transfer of the reserve good to the other agent. A successful payment means a drawdown on the paying agent's reserve holdings that are potentially useful for other non-payment functions. In our model, it is captured by that the reserve-holding agent will have not maintained the reserve good after making a payment, incurring a cost. It is also uncertain whether other agents will make reciprocal payments back in the future. This implies that the reserve-holding agent may incur prolonged periods of opportunity costs going forward. The reserve-holding agent thus trades off the current benefit against future costs when making the payment decision, which is in turn affected by her time preference.

Infinite-horizon economy with an aggregate shock in the alternative setup. In the alternative model setup, making an outbound payment particularly means transferring a store of value in the aggregate bad state. The aggregate shock of the game ending, naturally representing a bad aggregate state and all agents having to consume in that bad state, thus helps us parsimoniously capture the idea of reserves being not only a medium of exchange but also a store of value in many applications.

Reserves and rewards. We view the essence of any payment as the transfer of a scarce reserve good that is valuable for other non-payment functions. In the model, we take a fixed unit supply with a flexible present value of all non-payment functions to capture the notion of scarcity and its magnitude. That is, a reserve good with a higher present value of non-payment functions is relatively more scarce given the fixed unit supply. This setting of a fixed unit supply greatly helps uncover the economic mechanism at play while keeping the model tractable. In reality, the net supply of means of payment is unlikely to be completely fixed in the long run; it could be adjusted according to a monetary policy rule, for example, the Friedman rule or the Taylor rule. But as long as its supply is relatively inelastic in the short run, it is scarce compared to the transaction needs. Assuming a fixed unit supply of reserves implies that our model does not directly generate quantitative predictions concerning the amount of reserves. However, the ability to use the reserve good's present value to capture the magnitude of scarcity still allows the model to capture rich relationships between reserve scarcity and payment activities. And in exchange for this simplification, we provide a fairly general characterization of the equilibrium without sacrificing the delivery of the underlying economics.

The scarcity of the reserve good naturally implies an imperfect substitutability between it and other consumption goods, which is the reason we separately model the reserve and reward goods.

It is natural that successful payments generate economic value from resolving the lack of double coincidence of wants, which is captured by the creation of rewards. The assumption that rewards are perishable and thus have to be immediately consumed is not crucial. What is crucial in this assumption is that agents cannot generate more reserves by accumulating the rewards in the short run, which, again, fundamentally reflects the scarcity of the reserve.

To be more specific, in what follows we separately describe a number of payment applications and illustrate what are the reserves and rewards, why the reserve has a positive yet limited supply, why it is imperfectly substitutable with the reward, and in what sense the payment and non-payment functions of the reserve good co-exist but can be separated:

- **Metallic payments.** Metallic payments have dominated the payment system for about four millenniums. Silver and gold are the main forms of reserves, whose supply is limited due to physical mint constraints. Around 2000 B.C., the use of metallic coins for payments first appears in ancient Greece and Rome for trading consumption commodities and services – rewards – that are not substitutable with reserves. Greeks employ the Attic silver standard, which becomes the predominant weight standard for coins in the Eastern Mediterranean. The Roman Empire use the Denarius, which has a fixed weight and value, as the basic silver coin. At the same time, high-value trade of consumption goods and services are settled through gold minted in Byzantine or Muslim. The global supply of gold has remained low and stable, only until the 20th century due to vast gold fields being discovered in South Africa and the development of the so-called cyanide process to extract gold from the low-grade ore in these gold fields (Redish, 2000).

  Although silver and gold are used as a medium of exchange, they have high intrinsic value as jewelry and conductors, used on a daily basis. More importantly, they are historically pursued and held by consumers and investors as a store of value in times of low or negative real interest rates and economic crises (Jermann, 2022).

- **Modern interbank payments.** Today, real-time gross settlement (RTGS) interbank payment systems play the leading role in large-size payments. As an example, the Fedwire, the RTGS funds transfer system for financial institutions operated by the U.S. Federal Reserve (Fed) Banks, sees a daily volume of more than $4.2 U.S. trillion in 2022. By construction, Fedwire is open to banks that have accounts at the Fed, and each interbank payment involves the transfer of central bank reserves from one bank to another. Successful payments allow the paying bank to collect fees – rewards – from its clients in the form of lendable cash and deposits, which can be in turn immediately lent out and thus imperfectly substitutable with central bank reserves. Indeed, Diamond, Jiang, and Ma (2022) show that central bank reserves crowd out bank lending, further supporting the notion that lendable funds and reserves are not perfectly substitutable. In addition, the supply of central bank reserves in the U.S. is largely limited and inelastic in the short run, being subject to the Fed's monetary implementation cycles (e.g., Acharya and Rajan, 2022, Acharya, Chauhan, Rajan, and Steffen, 2023, Lopez-Salido and Vissing-Jorgensen, 2023, Copeland, Duffie and Yang, 2024).

Beyond the use in interbank payments, central bank reserves provide other non-payment benefits and functions to banks. First, the Fed may use the interest rate on excess reserves (IOER) as an additional monetary policy tool, allowing banks to directly earn interest income by holding reserves. Second, holding excess reserves helps large banks to more easily meet post-crisis regulatory and liquidity requirements (e.g., Duffie, 2019, Correa, Du and Liao, 2020). Finally, holding excess reserves also saves banks from the stigma of tapping the Fed's discount window in volatile market times (e.g. Afonso, Kovner and Schoar, 2011), thus providing a store of value to banks.

- **Cross-border payments.** U.S. dollar is the dominant currency – reserve – for cross-border payments today. According to the Society for Worldwide Interbank Financial Telecommunication (SWIFT) system, the dollar accounts for 79.5% of payments in international trade between 2010-2020. Additionally, over the period of 1999-2019, the dollar accounted for 96% and 74% of trade invoicing in the Americas and Asia-Pacific regions, respectively (Gopinath and Stein, 2021), and is also predominantly used in settling the payments of global financial contracts (e.g., Coppola, Krishnamurthy and Xu, 2023). The supply of U.S. dollars is limited by the U.S.'s fiscal and monetary capacities, while non-U.S. banks typically collect fees in local currencies after successful payments – rewards – which are not directly substitutable with the U.S. dollar.

  Beyond the use in global payments, the U.S. dollar is widely held by investors, commercial banks, and central banks as a global safe asset for a store of value to hedge against negative economic shocks (e.g., He, Krishnamurthy and Milbradt, 2019, Jiang, Krishnamurthy and Lustig, 2021, Maggiori, Neiman and Schreger, 2021, Brunnermeier, Merkel and Sannikov, 2022). Indeed, long before the U.S. dollar become popular in cross-border payments, it has cemented its role as the global reserve currency since the 1944 Bretton-Woods Agreement.[7] As of 2022, foreign central banks still hold 59% of reserves in U.S. dollars.

[7]Under the Bretton-Wood System, the U.S. dollar was pegged to gold at $35 an ounce, and other countries peg their currencies to the U.S. dollar. The U.S. abandoned this aspect of the agreement as President Nixon abandoned the gold standard in favor of free-floating exchanges in 1971. However, the end of the gold standard had little impact on the U.S. dollar's role as a store of value.

- **Within-bank payments using bank deposits.** Complementary to interbank and cross-border payment systems, modern mid- and small-size payments are also settled by commercial bank notes, that is, deposits, issued by a single fractional reserve bank. Within this single bank, the supply of bank deposits – reserve – is limited by banks' reserve requirements and the money multiplier (Tobin, 1965, Steinsson, 2023b) and thus only inelastic in the short run. Households and firms then use bank deposits to settle the purchases of goods and services – rewards. Indeed, various theories and facts are provided (e.g., Diamond and Rajan, 2006, Gu, Mattesini, Monnet, and Wright, 2013, Donaldson, Piacentino and Thakor, 2018, Parlour, Rajan and Walden, 2020) to justify why bank deposits emerge as a medium of exchange. At the same time, bank deposits also serve as a store of value (e.g., Stein, 2012, Dang, Gorton, Holmström, and Ordoñez, 2017) and may generate interest income for depositors. The payment and non-payment functions of deposits may conflict with each other, as highlighted in Diamond and Rajan (2006). Indeed, depositors who utilize faster payment technologies for more deposit transfers tend to accrue less interest, as empirically documented in Lu, Song and Zeng (2024).

- **Digital and crypto payments.** Stablecoins and CBDCs are widely considered as the next generations of payment methods (see, e.g., Duffie, 2019, Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022, Brunnermeier and Payne, 2022, for surveys). For example, Auer, Frost, Gambacorta, Monnet, Rice and Shin (2022) estimate that the greatest potential for stablecoins is the cross-border remittance markets, on which stablecoins may help reduce the current costs by more than half. In these applications, the transactional gains – rewards – are typically reflected in local currencies, which are not directly substitutable with stablecoins. On the other hand, the supply of stablecoins and CBDCs as reserves is constrained by the various design choices.

  Beyond potential use in payments, stablecoins and CBDCs deliver other non-payment benefits and functions. For example, stablecoins are widely held by investors as collateral for speculating other cryptoassets (Gorton, Klee, Ross, Ross, and Vardoulakis, 2023). CBDCs, regardless of their design choices, are largely perceived to be a store of value accessible to households. Indeed, some recent studies hypothesize that this role of CBDCs may even generate unintended consequences by disintermediating commercial banks, particularly in crisis times (e.g., Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022).

Separation of the roles of medium of exchange and store of value in the short run. Although some non-payment functions such as intrinsic value (e.g., from service flows) and the capacity in fulfilling regulatory requirements can be easily separated from the payment function of the reserve good in the above applications, the store of value function may reinforce the payment function in the long-run so that money is accepted as a medium of exchange without its value being challenged (e.g., Friedman and Schwartz, 1963, Gorton and Pennacchi, 1990). Our separation of them follows from the classic yet somewhat overlooked and under-formulated idea of Hayek (1976) who argues the following. The payment function, that is, the role of reserves as a medium of exchange, stems from them resolving the lack of double coincidence of wants across time. In contrast, the role as a store of value, the leading non-payment function of reserves, originates from them improving the ability to commit to future risk-sharing across different states of the world. Thus, these two functions reduce fundamentally different economic frictions. This separation between the medium-of-exchange and store-of-value functions has also been discussed in the recent literature (e.g. Brunnermeier, James and Landau, 2019).

Payment shock. The payment shock in our model is a parsimonious way to capture uncertain transaction needs and the relative scarcity of reserves compared to transaction needs. In reality, at any given date, an individual may receive goods or services, or may not receive anything. A bank may receive clients' payment requests or not as well. An agent only needs to send a payment if there are transaction needs.

We highlight that only agent i (i.e., the reserve-holding agent) is subject to the payment shock. This setting parsimoniously captures the notion that whoever has to make a payment already has enough reserve funds to do so despite reserves being scarce in the aggregate. Thus, we rule out the mechanical and uninteresting case in which an agent does not make a payment or waits for others to make a payment first simply because she does not have enough reserve funds.

How does the model capture delays? Although the reserve-holding agent only chooses between sending (i.e., $a_t = 1$) and not sending (i.e., $a_t = 0$) upon receiving a payment shock, it precisely captures payment delays in the following sense. If the reserve-holding agent does not make a transfer at time t, she does not enjoy any reward (e.g., the delivery of the consumption good, or payment fees) accordingly and keeps the reserve. At time $t + 1$, she is subject to another payment shock with probability $\lambda > 0$. This can be interpreted as with probability λ the household is tempted to purchase the same good again at $t = 1$ she would have purchased at $t = 0$, or the client's payment request still stays on the bank's payment order book at $t = 1$. If the reserve-holding agent chooses to pay now (i.e., $a_{t+1} = 1$) and the payment goes through, the combined history of $\{s_t = 0, s_{t+1} = 1\}$ suggests that the payment, which could have been made earlier, is delayed for one period. Delays of more periods can be thus captured in the same way.

Technology shock. The technology shock captures the efficiency of the underlying payment technology, highlighting the notion that a payment may fail for reasons that are out of the agents' control. Despite technological improvements, modern, large-scale electronic payment systems are subject to errors and failures. For example, the Fedwire system may occasionally break down; it was disrupted twice in 2019 due to undisclosed technical issues, resulting in significant bounce-backs and time-outs in cross-bank settlements.

The modeling of the payment shock and technology shock allows us to cover many realistic frictions in payment activities. We highlight that both the payment shock and the technology shock are only privately known to the potentially payment-sending agent i, but not j. Those two assumptions imply that seeing no payments, the non-reserve-holding agent j is not sure whether it is because agent i chooses not to make a payment, or if it is just because agent i is simply not requested to send any payment at all, or because the technology fails. This signal-jamming problem implies that past payment outcomes are informative but imperfect signals when an agent tries to look at them to anticipate other agents' future payment patterns. Therefore, we are able to characterize to what extent the past dependence of payments is subject to different types and degrees of shocks in various payment applications.

Private credit markets. To focus on the use of reserves in ultimately settling payments in various contexts, we abstract away from the co-existence of reserves and credits within a given payment system. We regard reserves as the ultimate instrument for payment settlement, characterized by a net positive supply. In contrast, credits, which facilitate payments through borrowing and lending among system participants, inherently have a net zero supply. The literature extensively explores why credit markets may not emerge when outside money is essential for settling payments (e.g., Townsend, 1980). When credit markets emerge endogenously for payment purposes, the supply of credits is usually constrained by scarcity, similar to what we have described for the reserve good, for reasons such as moral hazard (Atkeson, 1991), limited collateralizability (Kiyotaki and Moore, 1997, Holmstrom and Tirole, 1998, Parlatore, 2019) or limited enforceability (Kehoe and Levine, 1993, Kocherlakota, 1996, Alvarez and Jermann, 2000), and thus the economic essence falls into our analysis.

Central bank liquidity provision. Although our model does not explicitly include a central bank or its role as a lender of last resort, the extent of central bank liquidity provision—naturally associated with its own costs—is indirectly represented by c, the per-period cost incurred by the non-reserve-holding agent. If the central bank were to provide an unlimited supply of liquidity to facilitate payments, c would effectively drop to zero, thus eradicating reserve scarcity. From this perspective, a positive c can be interpreted as the private cost of accessing central bank liquidity facilities, such as the discount window, for making payments. A higher c indicates more expensive central bank liquidity provision.

Relationship with repeated games with imperfect public monitoring. As discussed above, the presence of payment and technology shocks introduces a signal-jamming and imperfect monitoring problem. Therefore, the stochastic dynamic economy we set up is similar to a standard repeated game with imperfect public monitoring (e.g., Abreu, Pearce, and Stacchetti, 1990, Fudenberg, Levine and Maskin, 1994). However, some important differences emerge between our model and standard repeated games, which significantly affect the equilibrium analysis. First, our dynamic economy does not involve the repetition of a stage game. The stage game in our economy more closely resembles a decision problem for the reserve-holding agent, taking into consideration the strategic interaction between the two agents only through the continuation value. Second, beyond the public history, which agent holds the reserve good also constructs an important state variable. Despite the two agents' preferences being identical, their continuation values in any given period are thus different depending on who owns the reserve good. We view these features being important because they jointly capture the nature of payments being reciprocal and reserves being scarce.

3 Payment equilibria and financial fragility

We start solving the model by characterizing two important benchmark equilibria. Beyond the formal results, we defer the discussion of various applications of model equilibira to Section 6.

3.1 The "good" payment equilibrium

We characterize the existence of both a "good" and a "bad" payment equilibrium, in which agents' payment decisions are history-independent. In the good (bad) equilibrium, the agent who holds the reserve good always (never) makes a payment, regardless of the past history of the other agent making payments or not. Those two equilibria thus serve as benchmarks and allow us to later uncover the economic conditions under which payment decisions become history-dependent.

Proposition 1. *There exists a good equilibrium in which the reserve-holding agent always makes a payment, if and only if the conflict-adjusted transaction need z/κ satisfies:*

$$\frac{z}{\kappa} \geq \frac{1-\delta}{1-\delta(1-\lambda\mu)}\,. \tag{3.1}$$

The right hand side of (3.1) is strictly smaller than 1, and is decreasing in δ, λ, and μ. The good payment equilibrium is a PPE and is history independent.

The proof is based on Abreu, Pearce, and Stacchetti (1990)'s idea of decomposability, but we give a sketch of the idea here to help build intuition. It is based on the standard one-shot deviation principle to check whether the reserve-holding agent, conditional on being subject to a payment shock and the other agent playing the proposed equilibrium strategy, would find a profitable deviation. Intuitively, $1 - \lambda\mu$ is the per period probability of no transfer and the game continues to the next period conditional on the bank with the reserve transferring the reserve upon request. In other words, $1 - \lambda\mu$ is the probability that the bank keeps its current status in the next period given that it chooses to transfer the reserve this period (which succeeds with probability $\mu < 1$) given the other agent playing the equilibrium strategy. Then, we have

$$z \geq c\left(1 + \delta\left(1 - \lambda\mu\right) + \delta^2\left(1 - \lambda\mu\right)^2 + \ldots\right) , \tag{3.2}$$

meaning that the good payment equilibrium exists if and only if z, the gain from making a payment at time t, is no less than the sum of c, the cost at time t if giving up the reserve, $\delta\left(1 - \lambda\mu\right)c$, the present value of the expected cost at time $t + 1$, $\delta^2\left(1 - \lambda\mu\right)^2 c$, the present value of the expected cost at time $t + 2$, and so on. By the relationship of $\kappa = c/(1 - \delta)$, condition (3.2) immediately yields the equilibrium condition (3.1).

Proposition 1 allows for a number of comparative statics thanks to the explicit characterization of the equilibrium existence region. First, the good payment equilibrium is more likely to happen when the benefit of payment-making is higher (i.e., a larger z). This is intuitive because the benefit from a successful payment can directly compensate for the cost of not having enough reserves in the future, encouraging the reserve-holding agent to make a payment today.

Second, the good payment equilibrium is more likely to happen when the reserve good is less scarce/valuable for non-payment functions (i.e., a smaller κ). Intuitively, when the reserve is more scarce, the reserve-holding agent has a higher incentive to hoard the reserve good because losing it otherwise would incur a larger present loss. By the same token, when the reserve is less scarce, the reserve-holding agent is more encouraged to make a payment. This prediction is directly supported by the recent evidence in, for example, Afonso, Duffie, Rigon and Shin (2022) and Copeland, Duffie and Yang (2024), that interbank payments are more efficient when reserves are more abundant.

Third, the good payment equilibrium is more likely to happen when the agents are more patient (i.e., when δ is larger in the baseline economy), or when the aggregate negative shock is less likely to happen (i.e., when δ is larger in the alternative economy). This result is consistent with the repeated game literature (e.g. Green and Porter, 1984, Fudenberg and Maskin, 1986) that a cooperation equilibrium is more easily sustained when agents are more patient. This analogy points to the endogenous coordination motive embedded in our dynamic payment game despite that the game is not repeated and each stage game does not feature any coordination motives.

Fourth, the good equilibrium is also more likely to happen when the payment technology is better (i.e., when μ is larger). Intuitively, when the technology is better, the reserve-holding agent is more likely to get a reciprocal payment back from its counterpart in the future. This in turn increases the reserve-holding agent's incentives to make a payment, leading to a more likely good payment equilibrium.

Finally, the good payment equilibrium is more likely to happen when the other non-reserve-holding agent is more likely to receive a payment request next period (i.e., when λ is larger). To understand this, note that the existence of the good payment equilibrium is guaranteed by an unprofitable one-shot deviation by the reserve-holding agent provided a given payment shock. In other words, the magnitude of the private payment shock is already becoming irrelevant for the reserve-holding agent in question. Rather, when the other non-reserve-holding agent is more likely to receive a payment request going forward, the reserve-holding agent is more likely to get a reciprocal payment back. Just like what a better payment technology implies, this increases the reserve-holding agent's incentives to make a payment and makes a good payment equilibrium more likely to happen.

3.2 The "bad" payment equilibrium

Next, we turn to study the other benchmark equilibrium in which none of the agents makes payments regardless of the payment history.

Proposition 2. *There exists a bad equilibrium in which the reserve-holding agent never makes a payment if and only if the conflict-adjusted transaction need z/κ satisfies:*

$$\frac{z}{\kappa} \leq 1\,. \tag{3.3}$$

The bad payment equilibrium is a PPE and is history independent.

The intuition of Proposition 2 can be also understood by looking at the one-shot deviation of the reserve-holding agent. We have:

$$z \leq c(1 + \delta + \delta^2 + ...)\ , \tag{3.4}$$

In words, the bad payment equilibrium exists if and only if z, the gain from making a payment at time t, is no greater than than the sum of c, the cost at time t if giving up the reserve, δc, the present value of the expected cost at time $t+1$, $\delta^2 c$, the present value of the expected cost at time $t+2$, and so on. By the relationship of $\kappa = c/(1-\delta)$, condition (3.4) immediately yields the equilibrium condition (3.3).

The bad equilibrium is more likely to happen when the benefit of payment-making is smaller (i.e., a smaller z), or when the reserve good is more scarce/valuable for non-payment functions (i.e., a larger κ). This result is similar to its counterpart in Proposition 1.

Finally, different from Proposition 1, an important observation from Proposition 2 is that how likely the bad equilibrium happens does not depend on the agent's time preference (in the baseline economy), the aggregate shock (in the alternative economy), the frequency of payment requests, or the quality of the payment technology. This can be interpreted as a "coordination trap" because the bad equilibrium happens when the reserve-holding agent believes the other agent will never make a returning payment in the future. Intuitively, conditional on the other non-reserve-holding never making a payment in the future regardless of the economic environment, any changes in the various model parameters will not make a one-shot deviation profitable for the reserve-holding agent in question. This result thus points to the importance of trust-building because it suggests that technology improvement, for example, may not necessarily solve the issue of lack of trust.

3.3 Multiple equilibria and payment fragility

Having analyzed the two benchmark equilibria, we immediately have the following result, which directly derives from Propositions 1 and 2:

Proposition 3. *Both the good and bad payment equilibria exist when the conflict-adjusted transaction need z/κ satisfies:*

$$\frac{1-\delta}{1-\delta(1-\lambda\mu)} \leq \frac{z}{\kappa} \leq 1\,. \tag{3.5}$$

Proposition 3 implies that the two payment equilibria may co-exist in the same economy for medium values of z/κ. The existence of multiple equilibria resembles the classic notion of coordination such as that in the bank run models (e.g., Diamond and Dybvig, 1983). There, whether a depositor runs the bank depends on whether other depositors run. Here, whether a reverse-holding agent makes payments depend on whether the non-reserve-holding agent will make a reciprocal payment in the future.

Proposition 3 provides a plausible answer to the motivating question of this paper as to why payments are often fragile. When the economy falls into the parameter region where both equilibria exist, the payment patterns cannot be fully pinned down by fundamentals, implying a potential switch between the two equilibria. For example, when the economy sustains the good payment equilibrium at the lower threshold and an arbitrarily small negative shock hits it, the reserve-holding agent will stop making payments, pushing the economy to switch to a bad payment equilibrium. Such a switch between the two equilibria due to relatively small changes in fundamentals may lead to relatively large changes in equilibrium payment behaviors.

A straightforward but important observation from Proposition 3 is that multiple equilibria and the implied financial fragility would not go away even when there is no uncertainty about the private payment request (i.e., $\lambda = 1$) and when the payment technology is perfect (i.e., $\mu = 1$).

Corollary 1. *When $\lambda \to 1$ and $\mu \to 1$, both the good and bad payment equilibria exist when*

$$1 - \delta \leq \frac{z}{\kappa} \leq 1\,.$$

The importance of Corollary 1 is that it shows what matters most for payment fragility is the scarcity of the reserve good but not the uncertain payment needs or the quality of the payment technology. As long as $\delta < 1$, that is, when the agents are impatient (in the baseline economy) or when the aggregate shock is not zero (in the alternative economy), the reserve good is valuable in the sense that it processes a positive present value $\kappa > 0$. The scarcity of the valuable reserve good thus gives rise to the dynamic coordination motive between the agents in making payments, leading to financial fragility.

Before proceeding, it is important to highlight the scope of payment technology in our model. Currently, we focus solely on one aspect of payment technology: μ, which measures the success rate of a payment. We demonstrate that enhancing this dimension of payment technology does not necessarily bolster financial stability if reserves remain scarce. However, as discussed by Brunnermeier, James and Landau (2019), smart contracts and programmable money could serve as more effective coordination tools, directly steering participants towards a favorable equilibrium when multiple possibilities exist. For instance, the Pix payment system in Brazil is designed to eliminate any bank-induced delays in processing payment requests. When a customer initiates a payment, the system is programmed to handle it instantaneously, thereby precluding any potential coordination failures among banks.[8] Moreover, emerging forms of money, such as central bank digital currencies (CBDCs), could facilitate the implementation of negative nominal interest rates. Enhancing the implementation of such rates could help alleviate conflicts between the reserve's payment and non-payment functions, offering a more integrated solution to financial stability challenges. We leave these important aspects of payment technologies to future research.

[8]In contrast, traditional systems like the Fedwire, even if they operate as a Real-Time Gross Settlement (RTGS) system, do allow banks to retain the discretion to decide when to submit payment orders to Fedwire, which could potentially delay payments until they choose to proceed as we model.

3.4 Extensions of the baseline model

Before proceeding, we discuss three important extensions of the baseline model, which help not only rich the model but also highlight the key economic forces that give rise to the strategic complementarity in payment decisions in the baseline model. We present the formal results in Appendix A while briefly discuss them here to build the intuition.

First, Appendix A.1 extends the baseline model to an economy with more than two agents, maintaining the all the other assumptions and setups. It demonstrates that the good, bad, and multiple equilibria observed in the baseline model persist even as the number of agents increases. The appendix provides insights into how the number of agents influences the likelihood of different equilibria emerging. It shows that as the number of agents grows, the likelihood of a good equilibrium decreases due to increased scarcity of the reserve good for non-payment functions, while the likelihood of a bad equilibrium remains unaffected.

Second, Appendix A.2 modifies the baseline model by endogenizing the transaction gain z, which is interpreted as the level of output or the payoff for the reserve-holding agent. The appendix analyzes how the availability of reserves influences production and payment outcomes joinly in the economy. It introduces the idea that production levels are affected not only by the production function but also by the scarcity of the reserve good. The appendix shows that the balance between the money's payment and non-payment functions is crucial for determining equilibrium outcomes.

Third, Appendix A.3 explores the effects of divisible reserves, while again keeping other elements of the model unchanged. It demonstrates that the strategic complementarity between agents' payment decisions does not depend on the indivisibility of the reserve good as in the baseline model. The appendix shows that the good, bad, and multiple equilibria observed in the baseline model also persist in this extended framework. The results indicate that even when reserves are divisible, the same strategic considerations and equilibrium outcomes apply, reinforcing the robustness of the baseline model's findings.

Taken together, these three extensions demonstrate that the strategic complementarity underlying payment decisions is robust across various scenarios, including different numbers of agents, the incorporation of production, and the consideration of divisible reserves. Consequently, we focus on the baseline model moving forward, as it provides the greatest analytical tractability for analyzing history dependence in payments and the potential amplification effects from payment delays leading to disruptions.

4 The history-dependence of payments

Having analyzed the two benchmark payment equilibria and the existence of multiple payment equilibria, we move forward to analyze how payment fragility may happen *within* an equilibrium. This is important to help further discipline the model and understand the source of fragility without relying on exogenous shocks or sunspots. In other words, we aim to uncover the *endogenous* source that may trigger a payment disruption to better understand the motivating facts, for example, the 2019 repo market crisis. At the same time, we aim to answer why payments often involve delays and history-dependence in the sense that an agent makes outgoing payments only after receiving incoming payments, even if these agents are well funded during normal times (e.g., Afonso, Duffie, Rigon and Shin, 2022, Copeland, Duffie and Yang, 2024). This question cannot be fully answered by Proposition 3 because both equilibria studied there are history-independent.

4.1 The delay-trigger payment equilibrium and history dependence

To illustrate the point of payments being history-dependent and fragile within an equilibrium, we start by introducing the "delay-trigger" strategy, which resembles the classic "grim trigger" strategy first introduced in Friedman (1971) and widely studied in repeated games. The grim trigger strategy is a behavioral strategy in game theory that is used to enforce cooperation among agents in a repeated game. The basic idea behind this strategy is that an agent will initially cooperate with their opponent, but if their opponent ever defects, then the agent will retaliate by defecting in all subsequent rounds of the game. In other words, if one agent cheats or does not cooperate, the other agent will "punish" them by not cooperating in any future rounds, even if it means a worse outcome for both agents. The grim trigger strategy is considered as a form of "tit-for-tat" strategy, where agents mimic the previous action of their opponent. However, the grim trigger is more extreme in that it involves a permanent switch to defecting if the opponent ever defects. The grim trigger strategy is most effective when the game is played repeatedly over a long period of time and when the agents have a long-term perspective. It is also effective when the cost of defecting is higher than the cost of cooperating, as it provides a strong incentive for agents to cooperate in order to avoid the long-term consequences of defection.

Following the idea of grim trigger, we define the delay-trigger strategy in our dynamic economy using a two-state automaton, as follows.

Definition 4. *There is a delay-trigger payment equilibrium if the reserve-holding agent plays the delay-trigger strategy, which is represented by the following automaton. The set of states is* $W = \{w^{(1)}, w^{(0)}\}$*, with the output function* $f\left(w^{(1)}\right) = 1$ *and* $f\left(w^{(0)}\right) = 0$*. The initial state is*

$w^{(1)}$. *The transition function is given by*

$$\tau(w, k) = \begin{cases} w^{(1)}, & \text{if } w = w^{(1)} \text{ and } k = 1\,, \\ w^{(0)}, & \text{otherwise.} \end{cases}$$

A delay-trigger strategy is described by $(W, w^{(1)}, f, \tau)$*, whereas the continuation strategy profile after any history in which a reserve transfer (i.e.,* $k = 1$*) is not publicly observed in at least one period is described by* $(W, w^{(0)}, f, \tau)$*.*

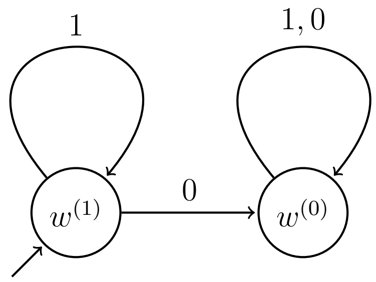

Figure 3: Automaton representation of the delay-trigger equilibrium

This figure shows the automaton that represents the delay-trigger strategy equilibrium. Circles are states and arrows are transitions, labeled by public outcomes that lead to the transitions.

By Definitions 2, 3, and 4, a delay-trigger payment equilibrium is a PPE, and exhibits history dependence. Our next result shows that such an equilibrium exists, and characterizes the conditions under which it exists.

Proposition 4. *There exists a delay-trigger payment equilibrium if and only if*

$$1 - \delta\lambda\mu \leq \frac{z}{\kappa} \leq 1\,. \tag{4.1}$$

Comparing Proposition 4 to Proposition 3 reveals an interesting observation: the region where the delay-trigger equilibrium exists is a strict subset of the region where the good and bad payment equilibria co-exist. To see this, note that

$$\frac{1-\delta}{1-\delta(1-\lambda\mu)} < 1 - \delta\lambda\mu\,.$$

The relationship between the good, bad, and delay-trigger payment equilibrium can be thus illustrated as in Figure 4.

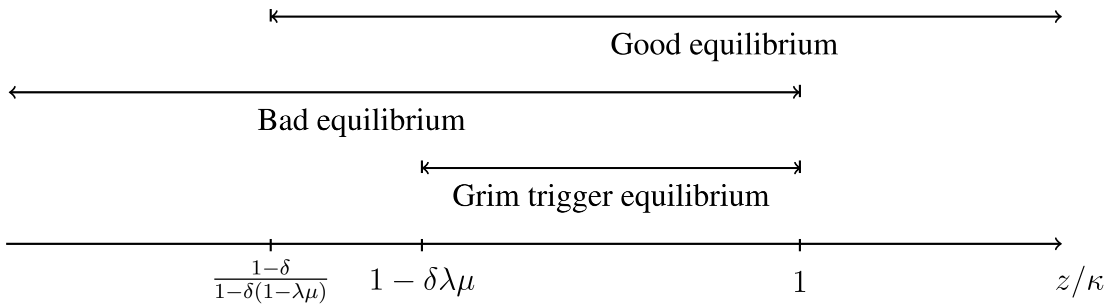

Figure 4: Relationship between the good, bad, and delay-trigger equilibria

This figure shows the relationship of the existence regions of the good, bad, and delay-trigger equilibria. The good and bad equilibria can co-exist in the intermediate region, and the existence region of the delay-trigger equilibrium is a subset of that intermediate region.

The intuition for the relationship between the three equilibria can be understood in two steps. First, notice that the existence of the delay-trigger payment equilibrium relies on the existence of the bad payment equilibrium because the bad payment equilibrium constructs one possible sub-game equilibrium of the delay-trigger equilibrium. This explains that the region where the delay-trigger equilibrium exists is a subset of the region where the bad payment equilibrium exists.

Furthermore, the existence of the delay-trigger payment equilibrium also requires stronger economic fundamentals in terms of z/κ than the good payment equilibrium would require. To understand this more subtle result, it is useful to check the one-shot deviation again. Recall that the good payment equilibrium essentially requires the reserve-holding agent to make a payment conditional on the other agent always making a payment in the future. In contrast, the delay-trigger payment requirement requires even stronger incentives for the reserve-holding agent, because it implies that the reserve-holding agent is still willing to make a payment initially given the existence of some sub-game equilibrium paths along which the other agent will stop making payments. This finally explains why the region where the delay-trigger equilibrium exists is a strict subset of the region where the good and bad payment equilibria co-exist.

Comparative statics of the existence region of the delay-trigger payment equilibrium with respect to the economic environment further reveals a number of interesting economic predictions. Notice that the region $[1-\delta\lambda\mu, 1]$ can only vary with economic parameters at the lower bound, but not the upper bound. Therefore, it suffices to perform comparative statics of the lower bound $1-\delta\lambda\mu$ with respect to δ, λ, and μ, and it is straightforward to see that the lower bound decreases in all of the three. That is, the delay-trigger payment equilibrium is more likely to happen when agents are more patient, when the bad aggregate shock is less likely, when the frequency of payment requests is higher, or when the quality of the payment technology is better.

To understand the intuition, it is useful to be reminded that the delay-trigger payment equilibrium requires the bad payment equilibrium as a sub-game equilibrium, but it is itself an improvement of the bad payment equilibrium in terms of payment efficiency in the dynamic economy because the reserve-holding agent is willing to make payment at the beginning. Hence, taking the scarcity of the reserve good as given, an improvement in other economic conditions (including agents being more patient, the bad aggregate shock being less likely, a higher frequency of payment requests, or a better payment technology) gives the reserve-holding agent a higher incentive to pay a payment initially, supporting a delay-trigger equilibrium.

However, interestingly, these comparative static results also suggest that an improvement in economic conditions may rather increase rather than decrease the likelihood of payments being history-dependent when the conflict between payment and non-payment is large in the sense that $z/\kappa \leq 1$. And the nature of payments being history-dependent would not go away even if there is no uncertainty about the payment needs and when the payment technology is perfect. Similar to Corollary 1, we have the following result:

Corollary 2. *When $\lambda \to 1$ and $\mu \to 1$, the delay-trigger payment equilibrium exists if and only if*

$$1 - \delta \leq \frac{z}{\kappa} \leq 1 .$$

Like Corollary 1, Corollary 2 shows what matters most for the history dependence of payments is the scarcity of the reserve good but not the uncertain payment needs or the quality of the payment technology. This prediction also relates to Proposition 2, which shows that any improvement in economic conditions other than the scarcity of the reserve good cannot change the region where the bad payment equilibrium exists. Thus, even if improvements in economic conditions increase the chances of the reserve-holding agent making an initial payment upon request, they cannot prevent the economy from eventually switching to the bad payment sub-game equilibrium after some unsuccessful history of payments regardless of the reasons. Proposition 4 and Corollary 1 thus again suggest that economic improvements such as a better payment technology may not necessarily eliminate payment delays that fundamentally arise from reserve scarcity, a point we highlight throughout the paper.

To offer another important perspective to understand the history dependence of payments, we consider the time until when the delay-trigger equilibrium "collapses" in that the two agents stop making payments to each other. We can define this time formally and generally:

Definition 5. *For any PPE that admits the bad payment equilibrium as a sub-game equilibrium, the time-to-collapse T is the time when state $w^{(0)}$ is reached, that is, when the bad payment equilibrium is played.*

In our stochastic dynamic game, the time-to-collapse T is itself a random variable in any given equilibrium. Thus, it is useful to consider its distribution and expectation. We have the following result, which immediately follows from Definition 4 and Proposition 4.

Corollary 3. *The time-to-collapse in a delay-trigger payment equilibrium follows a geometric distribution with a parameter $1 - \lambda\mu$, and the expected time-to-collapse is*

$$\mathbb{E}[T] = \frac{1}{1 - \lambda\mu}\,.$$

The intuition behind Corollary 3 directly follows from the definition of the delay-trigger payment equilibrium and its existence. The two agents start by trusting each other and making payments until no reserve transfer is observed for one period. Looking forward from any given period, we know that the probability of a reserve transfer is $\lambda\mu$, that is, if the reserve-holding agent is requested to make a payment, and her payment goes through. Therefore, the delay-trigger payment equilibrium would collapse with probability $1 - \lambda\mu$ in any given period.

Together with Proposition 4 and Corollary 2, Corollary 3 offers a complementary view regarding the history dependence of payments in a delay-trigger payment equilibrium, and particularly, how payment needs and payment technology affect the history dependence. Although, for example, a better payment technology may not necessarily reduce the chance when history dependence arises in terms of economic fundamentals (as illustrated in Proposition Proposition 4 and Corollary 2), it does reduce the time-to-collapse when history dependence actually happens. In other words, a better payment technology may still reduce the sensitivity of payment decisions on past payment histories, despite they being history-dependent in nature.

4.2 Generalized delay-trigger payment equilibria and history dependence

Having studied the delay-trigger equilibrium, we generalize the analysis to a class of equilibria that allows us to further characterize the magnitude of history dependence in payments. Formally, we define an n-delay-trigger payment equilibrium as follows:

Definition 6. *There is an n-delay-trigger payment equilibrium if the reserve-holding agent plays the n-delay-trigger strategy, which is represented by the following automaton. The set of states is $W = \{w^{(l)} | 0 \le l \le n\}$, with the output function $f\left(w^{(l)}\right) = 1$ for $1 \le l \le n$ and $f\left(w^{(0)}\right) = 0$. The initial state is $w^{(n)}$. The transition function is given by*

$$\tau(w, k) = \begin{cases} w^{(l)}, & \textit{if } w = w^{(l)}, 1 \le l \le n, \textit{ and } k = 1\,, \\ w^{(l-1)}, & \textit{if } w = w^{(l)}, 1 \le l \le n, \textit{ and } k = 0\,, \\ w^{(0)}, & \textit{otherwise.} \end{cases}$$

An n-delay-trigger strategy is then described by $(W, w^{(l)}, f, \tau)$.

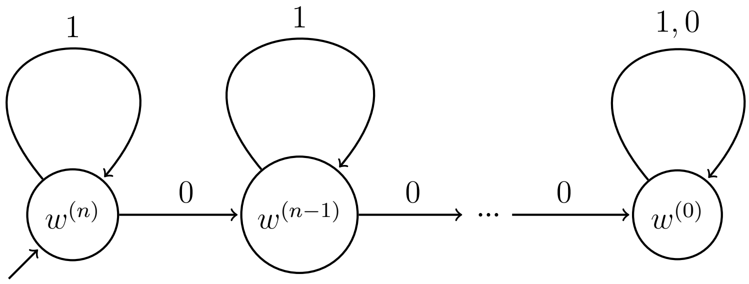

Figure 5: Automaton representation of the n-delay-trigger equilibrium

This figure shows the automaton that represents the n-delay-trigger strategy equilibrium. Circles are states and arrows are transitions, labeled by public outcomes that lead to the transitions.

By Definitions 2, 3, and 6, an n-delay-trigger payment equilibrium is a PPE, and exhibits history dependence. It is straightforward that the delay-trigger payment equilibrium considered in Definition 4 is a special case of the general n-delay-trigger payment equilibrium with $n = 1$. Intuitively, the delay trigger strategy represents the most extreme form of punishment in that the reserve-holding agent never makes any future payments after a reserve transfer has been not publicly observed for just one period. The n-delay-trigger payment equilibrium accommodates the same idea of history dependence but is more general to capture its magnitude: the reserve-holding stop making payments after reserve transfers have been unobserved for a total of n accumulated periods. The parameter n thus naturally captures the magnitude of history dependence in payments; a larger n suggests that payment decisions are less sensitive to past payment histories, and thus a lower magnitude of history dependence.

Proposition 5. *There exists an n-delay-trigger payment equilibrium for all $n \geq 1$ if and only if*

$$1 - \delta\lambda\mu \leq \frac{z}{\kappa} \leq 1\,.$$

Comparing Proposition 5 to Proposition 4 reveals that the region where the n-delay-trigger equilibrium exists is exactly the same as that where the delay-trigger equilibrium exists. In other words, the region where the n-delay-trigger equilibrium exists is independent of n. This somewhat surprising result can be understood from the following two observations.

On the one hand, by definition, the existence of the n-delay-trigger payment equilibrium relies on the existence of the $n-1$-trigger equilibrium because the latter constructs one possible sub-game equilibrium of the former. This explains that the region where the n-delay-trigger equilibrium exists is a subset of that where the $n-1$-trigger equilibrium exists, and by induction, also a subset of that where the delay-trigger equilibrium exists,

On the other hand, the $n+1$-trigger payment equilibrium must exist if the n-trigger payment equilibrium exists, for any $n \geq 1$. This is the key step in the proof of Proposition 5, and its intuition follows from that a larger n represents a lower magnitude of history dependence. To see this, note that the $n+1$-trigger payment equilibrium requires weaker economic fundamentals in terms of z/κ for the reserve-holding agent to make a payment at the initial state compared to the n-delay-trigger payment equilibrium because the former admits one more period of not publicly observing a reserve transfer. In other words, the reserve-holding agent is more encouraged to make a payment in the $n+1$-trigger payment due to a lower threat of experiencing a bad technology shock and the reserve involuntarily failing to be transferred. Hence, the region where the n-delay-trigger equilibrium exists must also be a subset of that where the $n+1$-trigger equilibrium exists. Taken together, these two observations explain why the region where the n-delay-trigger equilibrium exists is independent of n.

Similarly, we can characterize the time-to-collapse for a general n-delay-trigger payment equilibrium, based on Definition 5 and Proposition 5.

Corollary 4. *The time-to-collapse in an n-delay-trigger payment equilibrium follows a negative binomial distribution with parameter n and $1-\lambda\mu$, and the expected time-to-collapse is*

$$\mathbb{E}[T_n] = \frac{n}{1-\lambda\mu}\,.$$

Corollary 4 nests Corollary 3 and provides a complementary view to illustrate the nature of the n-delay-trigger payment equilibrium. The expected time-to-collapse becomes longer when n is larger. This result naturally follows from a lower magnitude of history dependence: a larger n implies that the payment system is more resilient to potential payment failures despite the nature of payments being history-dependent. All these equilibria can be possibly sustained, allowing our model to capture the rich patterns of history dependence in payments in reality.

5 Full dynamic equilibrium characterization and welfare outcomes

So far, we have focused on multiple equilibria to understand sudden payment halts for non-fundamental reasons and a specific type of history-dependent equilibrium, the delay-trigger payment equilibrium (and its generalized form, the n-delay-trigger payment equilibrium), to understand payments being history-dependent and the resulting payment delays. In this section, we show that the results of payment delays and payments being history-dependent hold much more generally. When the good and bad payment equilibria co-exist, there further exist a set of many history-dependent equilibria, which helps capture the rich pattern of reciprocal payments and their history dependence. As an example, the reserve-holding agent may use a less strict punishment in the sense that it may resume making payments after a number of periods of non-payment. Alternatively, the reserve-holding agent may only stop making payments after having observed multiple periods of non-payment from the other agent.

Given the impossibility to search through a prohibitively immense set of possible equilibria, the repeated game literature has developed an alternative methodology to study the equilibrium outcomes by focusing on the payoffs of agents, that is, the welfare outcomes, rather than imposing any restrictions on the space of the strategy profiles per se. To that end, we adapt the methodology for computing subgame-perfect equilibrium payoffs in repeated games comes from Abreu, Pearce, and Stacchetti (1990),[9] hereafter APS, to our dynamic economy. In this aspect, our analysis is closely related to the two classic studies of Atkeson (1991) and Kocherlakota (1998), which also built on the framework of Abreu, Pearce, and Stacchetti (1990), focusing on the rise of international credits and "memory" as money, respectively, from repeated interactions.[10]

Our main methodological contribution is an extension of the APS methodology to a class of non-repeated stochastic dynamic games, which allows us to characterize the payoff set analytically. In the analogy with dynamic programming, the APS algorithm is identified with value function iteration. We combine this approach with a new form of policy iteration, which is used to solve for equilibrium payoffs when incentive constraints are slack at times. We perform an analysis that yields a recursive characterization of contractual equilibrium payoffs, along the lines of APS, where one relates continuation values that can be achieved from a given period to the continuation values in the next period. The key complication we face here is that the sets of continuation values are different for the two agents depending on their reserve-holding status, and generally differ across periods. Thus, instead of looking for a fixed point set of continuation values, as is the case in the large literature of repeated games, we are looking for a fixed point in

[9] APS show that the set of equilibrium payoffs satisfies a recursive relationship that is analogous to the Bellman equation from dynamic programming. In particular, any equilibrium payoff can be decomposed into a flow payoff from the first period of play plus the expected discounted payoff from the next period onward, which, by subgame perfection, is also an equilibrium payoff. APS call this idea "decomposability." Just as the value function is the fixed point of the Bellman operator, so too is the equilibrium payoff set is the largest fixed point of an operator that produces the set of payoffs that can be generated using continuation values chosen from a given set. Moreover, APS show that iterating this operator on any set that contains all equilibrium payoffs yields a sequence of sets that asymptotically converges to the set of equilibrium payoffs, which they call "self-generation." Although APS focus on repeated games with imperfect monitoring and without a state variable, we show that their methodology can be extended to the class of games studied here, where there is not a repeated stage game and payoffs are generated in each state using continuation payoffs drawn from a received payoff correspondence.

[10] The framework of Abreu, Pearce, and Stacchetti (1990) has been also applied in industrial organization (e.g., Athey, Bagwell and Sanchirico, 2004), contract economics (Levin, 2003, Halac, 2012), and policy designs (Chang, 1998, Phelan and Stacchetti, 2001, Athey, Atkeson, and Kehoe, 2005).

the space of indexed collections of sets of continuation values. The approach also leads to new structural insights about equilibria that generate extreme equilibrium payoffs, namely, that play must be stationary until the first period in which an incentive constraint binds. However, we note that it is generally hard to analytically characterize the equilibrium payoff set in repeated games even with the powerful tool developed by APS. Rather, the literature has largely focused on describing some potential features of the set, for example, whether the set is compact or closed as APS initially focuses on, or imposing restrictions on the discount factor (e.g., Fudenberg, Levine and Maskin, 1994), or imposing restrictions on the strategy space with the notable example of focusing on strongly symmetric PPE in that all agents use the same strategy after every history (e.g., Athey, Bagwell and Sanchirico, 2004). Alternatively, a literature has focused on developing methods to solve for the equilibrium payoff set numerically (e.g., Abreu and Sannikov, 2014, Abreu, Brooks, and Sannikov, 2020). Thanks to the design of our stochastic dynamic game, we are able to further make significant progress by analytically solving for the full equilibrium payoff set without imposing any restrictions on the discount factor or the strategy space.

To fix the intuition, we first identify a boundary of the equilibrium payoff set, V_1, for the two agents, by highlighting some intuitive features of our dynamic payment game without imposing any restrictions on the strategies played.

Lemma 1. *Let $X \subset \mathbb{R}^2$ be a set of pairs of per-period continuation values:*

$$X = \left\{(w_1, w_2) \in \mathbb{R}^2 : -c \leq w_1 + w_2 \leq \lambda\mu z - c,\ w_1 \geq 0 \text{ and } w_2 \geq -c\right\} .$$

Then V_1 must be a subset of X.

The proof of Lemma 1 is constructive; the intuition can be understood in the following four steps. Let $\mathbf{w} = (w_1, w_2) \in V_1$. First, $w_1 + w_2 \leq \lambda\mu z - c$. This is because at any given time t, the maximum expected gain from making a payment is $\lambda\mu z$ (i.e., when the good payment equilibrium is played), and one and only one agent suffers from a (per period) cost for falling short of the reserve good, which is c. This captures the upper bound of the two agents' equilibrium payoff sums.

Second, $w_1 + w_2 \geq -c$. This is because in the worst case, that is, when the bad payment equilibrium is played, the reserve-holding agent does not send it upon the payment shock, and the two agents as a whole enjoy no gain but suffer from a (per period) cost of c. This thus captures the lower bound of the two agents' equilibrium payoff sums.

Third, $w_1 \geq 0$. This is because the reserve-holding agent can guarantee a zero payoff for all future periods by not sending the transfer. This captures the lower bound for the reserve-holding agent's equilibrium payoff.

Fourth, $w_2 \geq -c$. This is because the non-reserve-holding agent will at most suffer from a per period cost of $-c$ in the bad payment equilibrium. This captures the lower bound for the non-reserve-holding agent's equilibrium payoff.

The next step is to further tighten the boundary and analytically characterize V_1. This step is technically involved and reflects the methodological contribution of this paper, which we present in Appendix B in greater detail. Here, we present the following main result of this section, which shows that all the equilibrium-sustainable payoff outcomes for the two agents can be characterized by a triangle. We are able to solve the three extreme points of this triangle in closed form.

Proposition 6. *The equilibrium payoff set V_1 can be characterized by a triangle with the following three extreme points:*

$$\begin{cases} X_1 = (0, -c)', \\ X_2 = \left(\dfrac{\lambda\mu((1-\delta(1-\lambda\mu))z - c)}{1+\delta(2\lambda\mu - 1)}, \dfrac{\delta(\lambda\mu)^2 z - (\delta\lambda\mu + (1-\delta)(1-\lambda\mu))c}{1+\delta(2\lambda\mu-1)} \right)', \\ X_3 = \left(0, -c + \dfrac{\lambda\mu z(2c - (1-\delta)z)}{c + \delta\lambda\mu z} \right)'. \end{cases}$$

The two triangles in Figure 6 illustrate the sets of equilibrium payoffs V_1 for two different sets of model parameters. Specifically, the full set of PPEs can be characterized by a triangle in the space of (w_1, w_2), in which every pair of continuation values are attainable. The equilibrium payoff set V_1 fully captures all the possible welfare outcomes for the two agents in our economy.

It is also worth noting that the two extreme points X_1 and X_2 in V_1, according to Propositions 1 and 2, denote the equilibrium payoffs in the bad and good payment equilibria, respectively, in which payment decisions are history independent. Excluding these two extreme points, the other equilibrium payoffs in V_1 can be achieved by potentially history-dependent payment equilibria, the delay-trigger payment equilibrium being a notable example.

The closed-form solution provided in Proposition 6 allows for easy and explicit comparative statics of the full equilibrium payoff set with respect to important model parameters such as the payment technology and agents' time preferences. Figure 6 shows that the equilibrium payoff set expands in that more equilibrium outcomes can be supported in which both agents enjoy higher payoffs when the quality of the payment technology μ increases. In other words, the equilibrium payoff set for a smaller μ is a subset of that for a larger μ. Intuitively, a better payment technology benefits both agents despite payment decisions potentially being history-dependent. This result is reminiscent of Kandori (1992), which shows that the set of PPE payoffs in repeated games is increasing as the public monitoring technology becomes more precise. We note again, however, an improvement in the payment technology may not necessarily reduce payment fragility, as shown in Corollaries 1 and 2.

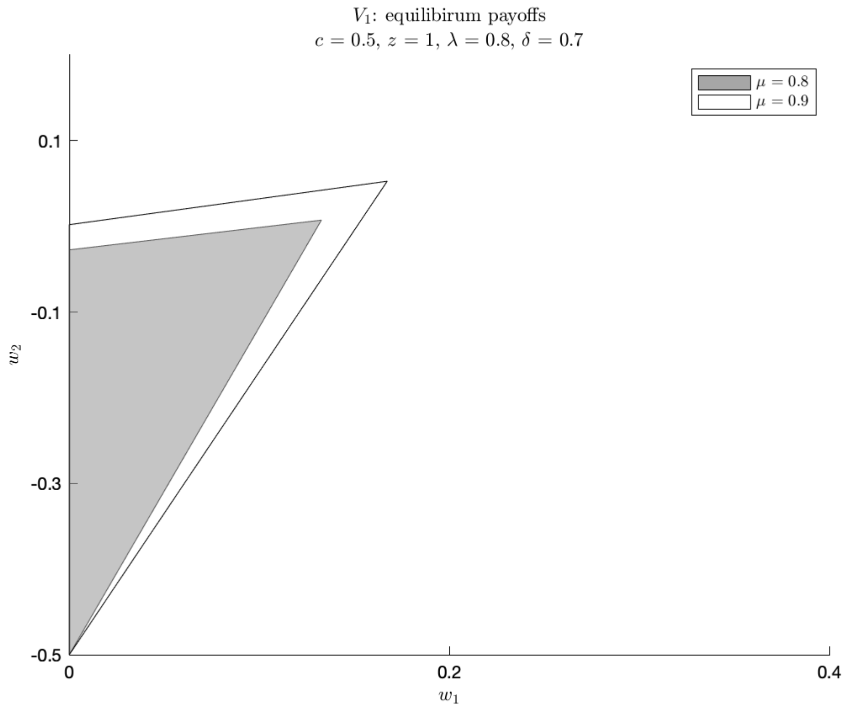

Figure 6: Equilibrium payoffs and comparative statics w.r.t. payment technology

This graph plots the equilibrium set V_1 for two sets of parameters. The shaded area denotes the equilibrium payoffs of equilibria in which payment decisions are history-dependent. Parameters: $c = 0.5$, $z = 1$, $\delta = 0.7$, $\lambda = 0.8$, and μ increases from 0.8 to 0.9.

Similarly, Figure 7 shows the evolution of the equilibrium payoff set when agents become more patient or when the aggregate bad shock is less likely, that is, when δ increases. Interestingly, an increase in δ leads to more equilibrium outcomes that disproportionately benefit the non-reserve-holding agent (i.e., a higher w_2) while hurting the reserve-holding agent (i.e., a lower w_1). This result significantly differs from the standard APS result for a generic repeated game in that under mild conditions the equilibrium PPE set for a larger discount factor is a superset of that for a smaller discount factor. To understand the intuition of Figure 7 and the contrast to existing results in the literature, notice that the two agents' roles are not symmetric in our (non-repeated) dynamic game that captures asynchronous but reciprocal payments. Intuitively, when δ increases, the reserve-holding agent cares more about the future. This implies a higher incentive for her to send the requested payment to the non-reserve-holding agent, everything else equal. The higher incentive thus implies an expected welfare transfer from the reserve-holding agent to the non-reserve-holding agent, which fundamentally arises from the scarcity of the reserve good.

Put differently, a higher δ encourages the reserve-holding agent to give up the scarce reserve good now in exchange for potentially future coordination due to the history-dependence of payments, which disproportionally benefits the currently non-reserve-holding agent.

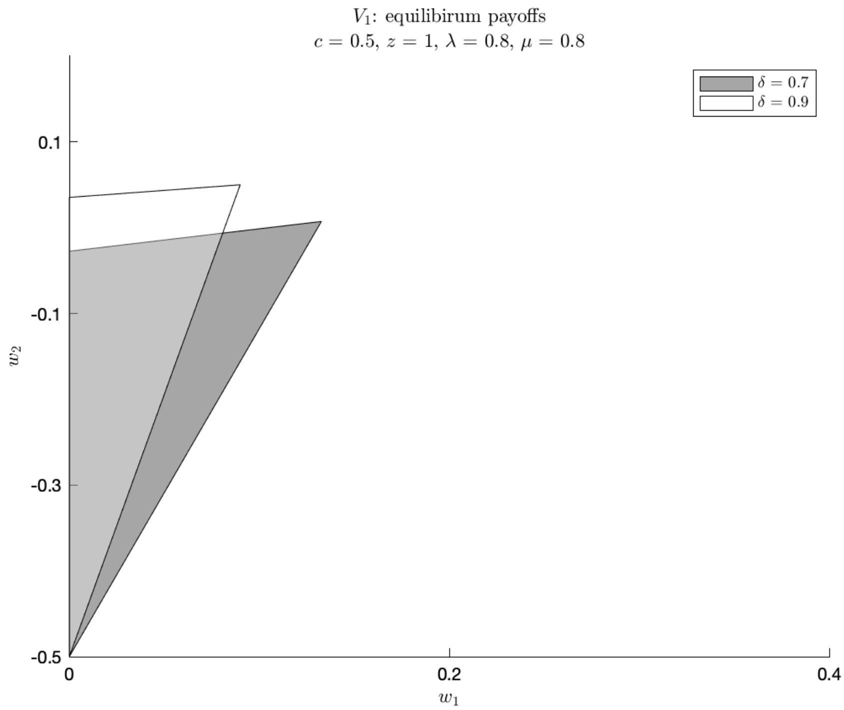

Figure 7: Equilibrium payoffs and comparative statics w.r.t. time preferences

This graph plots the equilibrium set V_1 for two sets of parameters. The shaded area denotes the equilibrium payoffs of equilibria in which payment decisions are history-dependent. Parameters: $c = 0.5$, $z = 1$, $\lambda = 0.8$, $\mu = 0.8$, and δ increases from 0.7 to 0.9.

Furthermore, it is useful to compare Proposition 6 to the results in Abreu, Pearce, and Stacchetti (1986) and Athey, Bagwell and Sanchirico (2004) in which the equilibrium payoff set can be also analytically characterized when only strongly symmetric equilibria are considered. Abreu, Pearce, and Stacchetti (1986) first introduce the concept of symmetric games to focus on PPEs that are strongly symmetric in the sense that each agent uses the same strategy after each history. This is a useful simplification because the equilibrium payoff set of strongly symmetric PPE can be analytically characterized by a compact interval $(\overline{\mathbf{w}}, \underline{\mathbf{w}})$, where $\overline{\mathbf{w}}$ and $\underline{\mathbf{w}}$ are the lowest and highest strongly symmetric PPE payoffs. Specifically, Abreu, Pearce, and Stacchetti (1986) analyze strongly symmetric equilibria in Green and Porter (1984)'s oligopoly game where agents choose quantities and the price is a noisy function of the aggregate quantity. Athey, Bagwell and Sanchirico (2004) study strongly symmetric equilibria in a repeated Bertrand pricing game where firms have private cost information. In contrast, we do not impose any restrictions on the strategy space and show that the full set of equilibrium payoffs in our dynamic economy is captured by a compact area rather than an interval.

Indeed, as Proposition 6 shows, the equilibrium payoff set for strongly symmetric PPEs is captured by the interval $[X_1, X_2]$, where X_1 and X_2 are the equilibrium payoffs for the bad and good payment equilibria that are themselves strongly symmetric PPEs. However, an important observation from Proposition 6 is that the dynamic economy we consider admits many other non-symmetric PPEs. We are able to fully characterize them, which represents significant progress compared to the existing literature.

Finally, it is also useful to compare our methodology to other commonly used equilibrium selection mechanisms in coordination games. In classic static coordination games (e.g., Diamond and Dybvig, 1983), the strategic complementarity lies in agents' simultaneous decisions. The global games technique has been widely adopted to select a unique equilibrium and link it to economic fundamentals (e.g. Morris and Shin, 1998, Goldstein and Pauzner, 2005). However, it is well known that it is difficult to apply the global games technique to general stochastic dynamic games (Angeletos, Hellwig and Pavan, 2007). A related but independent literature considers dynamic games in which random fundamental shocks (rather than past histories of decisions) serve as a coordination device in the presence of dynamic strategic complementarity (e.g., Frankel and Pauzner, 2000, He and Xiong, 2012). This literature can explain financial fragility in dynamic interactions but is not designed to explain endogenous patterns of history-dependence in agents' actions and strategies, which we view as a prevalent feature of payments. Given these challenges, we take an alternative approach that is commonly used in the repeated games literature. Specifically, we choose not to select any specific equilibrium but rather directly characterize the equilibrium outcomes of all the equilibria. This approach helps us capture the rich dynamics of history-dependence in the various applications of payments. Our framework and the solution method may also inform future studies that focus on asynchronous coordination in dynamic contexts.

6 Applications of model equilibria

6.1 The "good" payment equilibrium

Proposition 1 provides a useful benchmark to understand what economic conditions contribute to a good and efficient payment system. It also helps provide complementary answers to a number of important questions regarding the nature and evolution of money and payment systems:

Bank market power on deposit markets. A growing literature (e.g, Drechsler, Savov and Schnabl, 2017) examine commercial banks' market power on the deposit markets. Particularly, checking deposits often entitle their holders to zero interest rates regardless of the policy rate. It is typically understood from this literature that commercial banks process such market power from their franchise value such as the provision of branches, debit cards, and customer services. Proposition 1 provides a complementary explanation from a payment efficiency point of view: zero interest rates, by reducing κ, help to minimize the conflict between checking deposits' payment and non-payment functions because these deposits are indeed designed for payments.

Should CBDCs be interest-bearing? An important debate concerning the design of CBDCs is whether they should be interest-bearing (Auer, Frost, Gambacorta, Monnet, Rice and Shin, 2022). Currently, this debate centers how a potentially interest-bearing CBDC would interact with commercial banks in various aspects. Focusing on improving payment efficiency, which is widely perceived as the top reason to introduce a CBDC, Proposition 1 implies that CBDCs should not bear an interest in order to keep κ low and minimize the conflict between CBDCs' payment and non-payment functions. In fact, CBDCs would allow for the implementation of negative nominal interest rates, which could further reduce CBDCs' store-of-value function compared to the payment function. We acknowledge that to analyze the full implications of CBDCs is beyond the scope of this paper.

Gresham's Law. In monetary economics, Gresham's Law implies that "bad" money that has a lower intrinsic value will drive "good" money out of circulation. That is, ironically, "bad" money appears to be a more popular means of payment compared to "good" money. Historians and economists have offered various explanations for Gresham's Law and explored its implications (see Sargent and Velde, 2001, for a review). Proposition 1 offers a complementary view: everything else equal, a reserve with lower intrinsic value dominates another with higher intrinsic value for payment efficiency thanks to a higher z/κ. Indeed, Proposition 1 implies that any desirable form of reserves for payments should have minimal or zero intrinsic value to mitigate the conflict from other non-payment functions. This idea is reminiscent of Adam Smith's point in the *Wealth of Nations*, who argues from a different perspective that replacing gold and silver coins with bank notes would free up the gold and silver for other use of higher social value.

6.2 The "bad" payment equilibrium

Similarly, Proposition 2 also provides complementary answers to a number of important questions in the evolution of monetary and payment systems:

"Big problem of small change." In monetary economics, the "big problem of small change" refers to the notorious phenomenon over the many years of metallic systems that low-value coins constantly disappear from circulation, and the economy suffers from the lack of means to complete low-value transactions (Sargent and Velde, 2001, Steinsson, 2023a). So far, the leading explanation for the "big problem of small change" is that these low-value coins are so small that they are easily lost or difficult to pick up and count (Redish, 2000), so they are unlikely to be useful in everyday payments. Proposition 2 provides an alternative view: these coins disappear not because their physical size is small but because their z/κ is too small. Specifically, the size of the economic transactions they aim to support is so small that people tend to instead hoard them to enjoy their other non-payment functions. Indeed, Redish (2000) documents anecdotal evidence that some low-value coins were privately destroyed for building other products such as weapons. Combined with Proposition 1, this view also helps understand why the "big problem of small change" is no longer a problem today because even for small z the corresponding κ is significantly reduced by the use of deposits and digital payments.

End of the Gold Standard. The Gold Standard effectively ends in 1971 when President Nixon abandoned the peg of the U.S. dollar to gold. The literature has offered many explanations for the end of the Gold Standard, the leading one being the fear of deflation and devaluation of the U.S. dollar (see Blinder, 2022, for a review). To synthesize these views is beyond the scope of this paper, but Proposition 2 provides a complementary view from the payment evolution angle. Indeed, the U.S. dollar has started to rise as a dominant currency for payments during the same period (Gopinath and Stein, 2021), rendering z/κ to be too low for gold compared to the U.S. dollar and therefore gold to be a dominated reserve good for payments.

6.3 The co-existance of "good" and "bad" payment equilibrium

Proposition 3 maps to several important episodes of disruptions in various payment contexts. Below we describe these contexts and discuss how they connect to our model.

Repo market crisis in 2019. One of the motivating facts of this paper is the repo market crisis in September 2019. Overnight Treasury repo rates spiked by over 1,000 basis points, a level not seen since the 2008 global financial crisis (Afonso, Cipriani, Copeland, Kovner, La Spada and Martin, 2020, Correa, Du and Liao, 2020). Existing empirical literature has ascribed this crisis to dysfunctional interbank payments (Afonso, Duffie, Rigon and Shin, 2022, Copeland, Duffie and Yang, 2024), which was indeed driven by a number of factors including banks' increased demand for reserves to meet regulatory requirements, a reduction in the supply of reserves due to the Fed's balance sheet normalization process, and corporations withdrawing cash from banks to pay quarterly tax obligations. That said, it is less well understood why the repo spike started to happen exactly on September 16, 2019. Proposition 3 provides a fragility perspective to understand it: the Fed's balance sheet normalization process pushes z/κ lower and falling into the intermediate region where multiple equilibra exists, leading to the potential for interbank payment disruptions due to coordination failure.

German interbank crisis in 1931. Blickle, Brunnermeier, and Luck (2022) provide a comprehensive analysis of the run on the German banking system in 1931, which was one of the largest bank runs in history and a key event of the Great Depression. A notable feature of this run episode is the severe disruption in interbank payments and lending. Importantly, the run happened when the Reichsbank, the German central bank at that time, was constrained by the Gold Standard and was mandated to cover 40% of the circulating currency with gold. Proposition 3 thus provides a complementary view to understand the source of fragility in interbank payments and lending: reserve scarcity. The mandate to follow the Gold Standard could be understood as κ falling in the intermediate region so that multiple equilibria exists, leading to the potential for interbank payments and lending disruptions.

September 11 attacks. In 2001, the September 11 attacks on the World Trade Center and the Pentagon caused significant disruptions in the U.S. financial system, including the interbank payment systems. The Federal Reserve Bank of New York (New York Fed), which serves as the primary operator of Fedwire, was located just blocks away from the World Trade Center. The attack led to halts and significant delays in the settlement of interbank payments despite major banks being sufficiently funded and the Fedwire operation system itself not being attacked (Afonso and Shin, 2011). In this context, the September 11 attacks could be interpreted as a sudden decrease in δ: agents suddenly become less patient, or the probability of the bad aggregate shock suddenly increases.

SWIFT hacks. SWIFT is a global financial messaging network that facilitates cross-border payments between financial institutions. In recent years, there have been several high-profile hacks of the SWIFT network, including the 2016 Bangladesh Bank heist, in which hackers stole $81 million from the bank's account at the New York Fed (see Hill, 2018, for a comprehensive analysis). Cross-border payment halt after these events, even if the financial institutions in question have not necessarily been subject to these hacks themselves. In this context, the hacks could be interpreted as a sudden quality decrease in the payment technology μ.

6.4 History-dependent payment equilibria

Finally, propositions 4 and 5 helps understand a number of important events and patterns in which payments exhibit history-dependence:

Herstatt bank crisis. The Herstatt Bank, a relatively small bank in Germany, declared bankruptcy on June 26, 1974, an event now widely known as the Herstatt crisis (Goodhart, 2011).

That same day, a number of global large banks had transferred Deutsche Marks (DEM) to Herstatt in Frankfurt in anticipation of receiving US dollars (USD) in New York. However, due to Herstatt's closure at 16:30 German time (10:30 New York time) and the ensuing time zone difference, Herstatt ceased operations during the payment process, leaving the counterparty banks without their USD payments. Although the failure of payments by Herstatt itself only generated modest losses on the counterparties, it triggered a large waves of payment failures and delays among large, global banks even if these large banks remained financially healthy and such delays persisted for several months after the event. These developments eventually prompted central bankers to establish the Basel Committee on Banking Supervision and played a pivotal role in prompting the global adoption of real-time gross settlement (RTGS) systems.

Dependence of outgoing payments on incoming payments in interbank payments. The existence of the delay-trigger payment equilibrium also allows our model to explain why modern interbank payments often involve delays in the sense that an agent makes outgoing payments only after receiving incoming payments, even if these agents are well funded. As described by Afonso and Shin (2011), Afonso, Duffie, Rigon and Shin (2022), and Copeland, Duffie and Yang (2024), banks typically wait until they have received incoming payments to start sending outgoing payments to other banks, resulting in significant delays in intraday payments during both normal and crisis times. This delay pattern tends to be more pronounced when banks face higher capital costs and when aggregate central bank reserves are more scarce. In our model, this pattern can be explained by the agents playing a delay-trigger payment equilibrium when κ, the value of the reserve good is relatively larger, and thus the reserve is more scarce.

7 Conclusion

We present a dynamic theory of payments that establishes a crucial link to the scarcity of reserves. Our proposition posits that all payments entail the transfer of a reserve good, which, apart from serving payment functions, holds value for non-payment purposes and exhibits inelastic supply. The model's insights illuminate the contrasting behaviors of agents in different reserve abundance scenarios: agents make payments when reserves abound, while they cease payments when reserves become scarce relative to the payment technology. When reserve scarcity falls within an intermediate range, the model unfolds multiple equilibria, with agents' payment decisions becoming linked to the payment history of their counterparts within an equilibrium. The model explains why payments frequently encounter delays and halts, even when agents possess adequate funding. The model indicates that advancements in payment technologies might not always be the panacea for fragility reduction. We develop a new methodology to analyze the welfare outcomes of all feasible equilibria, capturing the intricate dynamics governing the history-dependent nature of payments. The model applies to various payment contexts, encompassing metallic payments preceding fiat money, modern bank payments, cross-border transactions, and modern digital payment systems.

Our work extends to illuminating a conflict between price stability and financial stability. In an era grappling with surging inflationary pressures, contemporary economic debate centers around the optimal intensity of monetary policy tightening, reminiscent of the spirited contest between Arthur Burns and Paul Volcker (Blinder, 2022). Yet, we argue that this debate should be understood at a deeper level. As we commemorate the 150th anniversary of Walter Bagehot's seminal publication, *Lombard Street*, a treatise that reshaped the central banking history, our work sheds light on the role of modern central banks. Bagehot's work laid the foundation for central banks' role as lenders of last resort for the money market and the payment function of money. In that spirit, our work highlights that any monetary policies that involve an increase in interest rates or a reduction in the supply of the monetary base may inevitably result in increased reserve scarcity for the payment function, potentially leading to payment fragility. As such, our work advocates for a judicious calibration of monetary policies, navigating the trade-off between price stability and the financial stability.

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Appendix for

Payments, Reserves, and Financial Fragility

Itay Goldstein   Ming Yang   Yao Zeng

A Extensions of the baseline model

A.1 Equilibria with multiple agents

In this appendix, we extend our baseline model to include $N > 2$ agents, while keeping other model primitives unchanged. We demonstrate that the results pertaining to good, bad, and multiple equilibria persist in this extended framework. Additionally, this extension provides further insights into how the number of agents influences the emergence of these equilibria.

To that end, we assume that, similar to the baseline model, only one agent is endowed with the reserve, while the others are not. At any time $t \geq 0$, the reserve-holding agent i experiences a private, recipient-specific payment shock: she is expected to transfer the reserve to another agent $j \in N \setminus i$ with probability $\lambda/(N-1)$. Consequently, the probability of agent i being required to transfer the reserve to any other agent is $\sum_{j \in N \setminus i} \lambda/(N-1) = \lambda$, the same as in the baseline model. If agent i privately sends the reserve, the potential transfer is subject to the same technology shock: with probability μ, the transfer is successful to the specific agent j, and agent i receives z rewards at the end of time t if the transfer succeeds. Note that the extended model encompasses the baseline model when $N = 2$.

In the extended economy, we can show the following results, which extend the baseline Propositions 1, 2, and 3:

Proposition 7. *There exists a good equilibrium in which the reserve-holding agent always makes a payment if and only if the conflict-adjusted transaction need z/κ satisfies:*

$$\frac{z}{\kappa} \geq \frac{1-\delta}{1-\delta(1-\frac{\lambda\mu}{N-1})} \,. \tag{A.1}$$

There exists a bad equilibrium in which the reserve-holding agent never makes a payment if and only if the conflict-adjusted transaction need z/κ satisfies:

$$\frac{z}{\kappa} \leq 1 \,. \tag{A.2}$$

Both the good and bad payment equilibria exist when (A.1) and (A.2) both hold.

Proposition 7 sheds light on how the number of agents in the economy shapes payments. First, the minimal conflict-adjusted transaction needed to support the good equilibrium, represented by the right-hand side of (A.1), becomes higher as N increases. Indeed, comparing (A.1) to its counterpart (3.1) in Proposition 1 in the baseline model reveals that (A.1) reduces to (3.1) when $N = 2$. Intuitively, as the number of agents increases, the likelihood of an agent receiving the reserve good from others decreases, given the fixed total demand for payment. Hence, the reserve good becomes effectively scarcer for its non-payment functions from the perspective of any given agent, making the good equilibrium less likely.

Secondly, the likelihood of the bad equilibrium occurring does not depend on the number of agents in the economy. This result resembles the concept of the "coordination trap" underlying Proposition 2: the bad equilibrium persists when the reserve-holding agent believes that others will never make a reciprocal payment in the future, regardless of the number of participants in the economy.

Finally, it is worth noting that the right-hand side of (A.1) converges to 1 as N increases:

$$\lim_{N\to\infty} \frac{1-\delta}{1-\delta(1-\frac{\lambda\mu}{N-1})} = 1\,.$$

A comparison to the right-hand side of (A.2) reveals that the region where multiple equilibria exist shrinks as N increases and vanishes as $N \to \infty$. This is because the probability of any reserve-holding agentafter successfully sending a payment, becoming a potential recipient and receiving the reserve good again approaches zero as $N \to \infty$, effectively eliminating any strategic interactions among agents. To put it differently, the strategic complementarity in payment decisions is stronger when the number of agents is smaller because the likelihood of the reserve good being circulated back to any given agent is higher, and thus one agent's decision is more dependent on others'. Because interbank payments, our leading application, are typically dominated by a few large banks (e.g., Afonso, Duffie, Rigon and Shin, 2022, Copeland, Duffie and Yang, 2024), we consider our baseline model with $N = 2$ a reasonable benchmark to capture such strategic complementarity and its implications, and we return to that benchmark in the rest of the paper.

A.2 Payments-driven production: Endogenizing transaction gain z

In this appendix, we extend the baseline model to endogenize z, which represents the transaction gain when the reserve good is successfully transferred. In our framework, z is effectively interpreted as the level of output, representing the private payoff gain for the reserve-holding agent, acting as the consumer, upon making a successful payment. It also signifies the quantity of output generated by the non-reserve-holding agent, acting as the producer, following a successful payment. The extended model allows us to solve the game analytically to the point of demonstrating the existence of multiple equilibria and illustrating the economic insights. Particularly, we show that the level of production in an economy is determined by not only the production function but also the scacity of the reserve good, highlighting the role of medium of exchanges in facilitate transactions and in turn productions.

We introduce a modification to the baseline model as follows: At any given time $t \geq 0$, the non-reserve-holding agent, acting as the producer, offers z, which is the reward amount to be produced for the reserve-holding agent, acting as the consumer, upon a successful payment. Upon seeing the producer's offer z, the consumer decides whether to pay or not. This arrangement represents a take-it-or-leave-it offer from the non-reserve-holding agent to the reserve-holding agent, embodying the principle commonly recognized in payment literature that the receiver possesses the bargaining power to accept or reject the payment instrument.[11] Additionally, this model draws inspiration from standard macroeconomic frameworks that incorporate price-setting frictions, such as menu cost models.

Specifically, we assume the cost function for the non-reserve-holding agent to be $\alpha h(z)$, where $h'(\cdot) > 0$, $h''(\cdot) \geq 0$, and $\alpha \geq 0$. Thus, the social surplus generated per transfer is $z - \alpha h(z)$. Define

$$f(z) \equiv \left(1 + \frac{\delta\lambda\mu}{1-\delta}\right) z - \frac{\delta\lambda\mu}{1-\delta}\alpha h(z)\,, \tag{A.3}$$

which can be interpreted as a notion of payment-adjusted social surplus due to the presence of the two payment-related parameters λ and μ in the weights.

We further define

$$\bar{f} \equiv \max_{z\geq 0} f(z)\,, \tag{A.4}$$

which gives the maximum of $f(z)$ over all $z \geq 0$. Intuitively, $\bar{f}$ can be viewed as a measure of reserve-adjusted production capacity, capturing the maximum achievable social surplus, taking into account the payment environment. A higher $\bar{f}$ indicates that the economy has greater production potential. As we will demonstrate, $\bar{f}$ is crucial in determining the economy's output.

We focus on two types of history-independent equilibria: the good and the bad payment equilibria. In the context of a good payment equilibrium, we conjecture, which we subsequently confirm, that the reserve-holding agent employs a monotone strategy. Specifically, there exists a

[11]This approach also mirrors the approach often employed to endogenize the output level in bilateral trade setting, typically by introducing a bargaining game between the sender and receiver (e.g., Bianchi and Bigio, 2021). However, integrating a comprehensive bargaining game would make our dynamic game analytically intractable.

threshold $\hat{z}$, prompting the agent to make a payment if and only if $z \geq \hat{z}$, independent of past payment histories. The non-reserve-holding agent consistently offers $z = \hat{z}$, also disregarding the history.

Proposition 8. *There exists a good equilibrium in which the reserve-holding agent always makes a payment if and only if $\kappa \leq \bar{f}$, that is, when the reserve is abundant enough, or equivalently, when the reserve's non-payment value is small enough compared to the production capacity of the economy. In this equilibrium, the reserve-holding agent makes a payment if and only if $z \geq \hat{z}$ and the non-reserve-holding agent posts $\hat{z}$, where the equilibrium output level $\hat{z}$ is given by $f(\hat{z}) = \kappa$.*

In the extended model, the scarcity of reserves κ, when considered alongside the economy's production capacity captured by $f(\cdot)$, crucially impacts the equilibrium production output. Proposition 8 implies that ample reserves in relation to production capacity, that is, a sufficiently small κ satisfying $\kappa \leq \bar{f}$, lead to smooth payments and a positive output level. This scenario prompts a "payment-driven boom," underscoring the influence of payments in determining producers' production decisions.

A particularly interesting aspect of Proposition 8 is how it reconciles the contradictory yet complementary roles of money as a medium of exchange and a store of value. Past literature has often highlighted the complementary nature of these roles (e.g., Friedman and Schwartz, 1963, Gorton and Pennacchi, 1990, Kiyotaki and Wright, 1993), while this paper highlights their conflicting aspects. On the one hand, the good payment equilibrium in Proposition 8 occurs when the conflict between the two roles is small, and hence, the reserve is not scarce for non-payment functions relative to the production capacity. This observation suggests that the utilization (rather than hoarding) of money for payments by whoever holds money is crucial for production outcomes, echoing the findings of the baseline model. On the other hand, however, conditional on a good equilibrium, the output level is in fact higher when the reserve asset is more highly valued for store-of-value and other non-payment uses, as indicated by the equilibrium condition $f(\hat{z}) = \kappa$. The proof of Proposition 8 reveals that this equilibrium condition arises from the non-reserve-holding agent's incentive compatibility condition, which is the key deviation of the extended model from the baseline model. This observation indicates a need for balance in the relationship between the money supply and production: money must be sufficiently abundant compared to production capacities to encourage its transfer as a medium of exchange rather than merely being hoarded, yet it must also be scarce enough that producers are motivated to accept it in transactions, as opposed to questioning its value. The latter economic force is not present in the baseline model, where it is assumed that the reserve-holding agent always accepts the reserve good for payments, irrespective of κ.

We next examine the bad payment equilibrium. We conjecture that, there exists a threshold $\hat{z}$, wherein the reserve-holding agent decides to make a payment if and only if $z \geq \hat{z}$, without consideration of past interactions. However, in such an equilibrium, the non-reserve-holding agent sets a value z' that is less than $\hat{z}$, also irrespective the history of past payments, implying that no payment will be made in equilibrium. We show that such an equilibrium exists.

Proposition 9. *There exists a bad equilibrium in which the reserve-holding agent never makes a payment if and only if $\alpha h(\kappa) \geq \kappa$, that is, when the reserve is scarce enough, or equivalently, when the reserve's non-payment value is large enough compared to the production capacity of the economy. In this equilibrium, the reserve-holding agent makes a payment if and only if $z \geq \hat{z} = \kappa$ and the non-reserve-holding agent posts some $z' < \hat{z}$, where the equilibrium output level is 0.*

Proposition 9 demonstrates that if reserves are excessively scarce relative to production capacity, such that $\alpha h(\kappa) \geq \kappa$, payments come to a halt. Intuitively, when κ is excessively high, the reserve good becomes so valuable for non-payment functions that the reserve-holding agent, as the consumer, only uses it for payments if the production level is equally high to offset the significant opportunity cost. Concurrently, production incurs costs for the non-reserve-holding agent, as the producer, with these costs escalating at higher production levels. Thus, the non-reserve-holding agent is disinclined to post such a high production level. Consequently, the actually posted production level is unable to justify the high opportunity cost of making payments borne by the reserve-holding agent, who ends up not using the reserve good for payments. In this scenario, the output level drops to zero, leading to a "payment-driven bust."

Finally, combining Propositions 8 and 9 yields the following result:

Proposition 10. *Both the good and bad payment equilibria exist when $\kappa \leq \min(\bar{f}, \alpha h(\kappa))$.*

Proposition 10 directly follows from Propositions 8 and 9 and examines scenarios where reserve scarcity is moderate—neither excessively low nor high but within an intermediate range. In such cases, payment fragility arises, potentially leading to significant fluctuations in output. This scenario characterizes a "payment-driven economic crisis." Complementing recent literature that explicitly considers frictions in payment markets affecting macroeconomic outcomes like consumption and investment (e.g., Bianchi and Bigio, 2021, Piazzesi and Schneider, 2021, Piazzesi, Rogers and Schneider, 2021), Propositions 8, 9, and 10 shed new light on the relationship between payment systems, reserve scarcity, and production outcomes by distinguishing money's dual roles as a medium of exchange and a store of value.

A.3 Equilibria with divisible reserves

In this appendix, we extend our baseline model to accomodate divisible reserves and proportional rewards, while keeping other model primitives unchanged. We demonstrate that the results pertaining to good, bad, and multiple equilibria also persist in this extended framework, suggesting that the strategic complementarity between the two agents' payment decisions is not driven by the setup of an indivisible reserve in the baseline model.

To start, we first denote the distribution of reserves across the two agents $(q, 1-q)$ under the extended model with divisible reserves, where $q \in [0, 1]$. At any time $t \geq 0$, with probability $2\lambda \in [0, 1]$, one of the agents receives a request of sending a payment to the other with equal chance. That is, each period, with probability $\lambda \in [0, 1/2]$, one of the two agents is chosen by nature to send a payment to the other. In what follows, we call this agent the requested sender. Without loss of generality, let the amount of reserve held by the requested sender be q. Further, the size of the requested payment is $x = \alpha q$, where $\alpha \in [0, 1]$ is a random variable uniformly distributed over $[0, 1]$.[12] At any given t, whether an agent is chosen as a requested sender is the agent's private information. If the requested sender decides to send the payment, the transfer succeeds with probability $\mu \in [0, 1]$, capturing the same technology shock as in the baseline model. We also allow for arbitrarily partial payments, meaning the sender can decide what fraction of the requested amount x to send, choosing any $y \in [0, x]$. After a successful payment, the sender receives a reward $zy \geq 0$. At the end of each period, an agent holding q units of the reserve suffers a loss of $c\,(1-q)$, again, capturing the same opportunity cost of payments as in the baseline model. Note that the extended model encompasses the baseline model with a singleton distribution with all probability mass loaded on $\alpha = 1$.

In the extended economy, we can show the following results, which extend the baseline Propositions 1, 2, and 3:

Proposition 11. *There exists a good equilibrium in which the requested sender always makes a full payment, that is, $y = x$, if and only if the conflict-adjusted transaction need z/κ satisfies:*

$$\frac{z}{\kappa} \geq \frac{1-\delta}{1-\delta\left(1-\frac{\lambda\mu}{2}\right)}\,. \tag{A.5}$$

There exists a bad equilibrium in which the requested sender never makes a payment, that is, $y = 0$, if and only if the conflict-adjusted transaction need z/κ satisfies:

$$\frac{z}{\kappa} \leq 1\,. \tag{A.6}$$

[12] The results do not depend on the specific distributional assumption of a uniform distribution.

Both the good and bad payment equilibria exist when (A.5) *and* (A.6) *both hold.*

Proposition 11 demonstrates that the strategic complementarity underlying payment decisions does not depend on the assumption of the reserve good being indivisible in the baseline model. Notably, this result holds even when the distribution of reserves across the two agents $(q, 1 - q)$, the requested payment amount x, and the potentially partial payment amount y are all specified in a general manner to reflect the complete divisibility of the reserve good.

As shown in the proof of Proposition 11, the key to this irrelevance result lies in the fact that the rewards the sender receives after a successful transfer are proportional to the amount of payment the sender chooses to send. Theoretically, this setup can be easily micro-founded through a bargaining protocol that results in a proportional division of unit gains from trade between the two agents. Empirically, this specification aligns with many real-world applications of payments that motivate our study, such as interbank payments, where the payment fee is pre-determined rather than negotiated in real time, while the primary decision for the requested bank is whether to send the payment and when to do so (Copeland, Duffie and Yang, 2024). Under this assumption, the trade-off facing the requested sender remains the same as in the baseline model, regardless of the size of the transfer, within a framework of complete divisibility of the reserve good.

Similarly, the likelihood of the bad equilibrium occurring does not depend on whether the reserve good is divisible either. This result again mirrors the concept of the "coordination trap" underlying Proposition 2: the bad equilibrium persists when the sender believes that others will never make a reciprocal payment in the future, regardless of how much of the reserve good she is requested to send when the reserve good is divisible.

B Characterizing the full equilibrium outcome set V_1

This appedix provides details of the technical analysis underlying Section 5, which also served as the formal proof for Proposition 6.

We first provide some primitive analysis to illustrate the structure of a generic PPE under our framework. In our two-agent stochastic dynamic game, a PPE can be fully characterized by a pair of both agents' continuation values, where the two components in the pair separately capture the continuation values depending on whether a payment is successfully made, that is, whether the reserve good is transferred from one agent to the other. This is because, different from a classic repeated game, the stochastic dynamic game we consider requires an additional state variable of who owns the reserve good, which is scarce. We note that this is different from APS and the subsequent work on repeated games in which the stage game is repeated and does not change over time. This also implies that our methodology involves significant differences from the original APS framework. This methodological contribution provides a useful tool to study long-term interactions in non-repeated stochastic dynamic games and can inform future work in related areas.

Formally, consider any time $t \geq 0$. Suppose the reserve-holding agent holds the reserve good at the beginning of date t while the non-reserve-holding agent does not. Let w_i be agent i's equilibrium payoff, that is, per-period continuation value at the beginning of date t, $i \in \{1, 2\}$. Let w_i^k, where $k \in \{0, 1\}$, be agent i's per period continuation value at the beginning of date $t + 1$, with $k = 1$ meaning that a transfer of the reserve good successfully occurs at time t and $k = 0$ meaning not.

In what follows, we extend the APS framework of equilibrium payoff construction to our stochastic dynamic game in several steps.

Decomposability. First, following the idea of decomposability in APS, we can decompose the two agents' equilibrium payoff into the current period's payoff and the expected continuation payoff. Note that, any meaningful payment equilibrium profile other than the bad payment equilibrium (described in Proposition 2) must involve the reverse-holding agent choosing to make a payment (upon receiving the private payment shock) at least at some history; otherwise it is the bad payment equilibrium. Thus, without loss of generality, we start from the initial state that the reserve-holding agent chooses to make a payment, and we can write:

$$\begin{aligned} w_1 &= (1-\delta)\,\lambda\mu\,(z-c) + \delta\left(\lambda\mu w_1^1 + (1-\lambda\mu)\,w_1^0\right) \\ &= \lambda\mu\left(\delta w_1^1 + (1-\delta)\,(z-c)\right) + (1-\lambda\mu)\,\delta w_1^0 \end{aligned} \tag{B.7}$$

and

$$\begin{aligned} w_2 &= (1-\delta)\,(1-\lambda\mu)\,(-c) + \delta\left(\lambda\mu w_2^1 + (1-\lambda\mu)\,w_2^0\right) \\ &= \lambda\mu\delta w_2^1 + (1-\lambda\mu)\left(\delta w_2^0 - (1-\delta)\,c\right)\ , \end{aligned} \tag{B.8}$$

which can be summarized by the following vector operation:

$$\mathbf{w} \triangleq \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \left(\delta\cdot\mathbf{w}^1 + \begin{pmatrix} (1-\delta)\,(z-c) \\ 0 \end{pmatrix}, \delta\cdot\mathbf{w}^0 - \begin{pmatrix} 0 \\ (1-\delta)\,c \end{pmatrix}\right)\begin{pmatrix} \lambda\mu \\ 1-\lambda\mu \end{pmatrix}, \tag{B.9}$$

where the bold letters denote vectors, and $\mathbf{w}^1$ ($\mathbf{w}^0$) is the pair of the two agents' date-$t+1$ per period continuation values if a transfer occurs (does not occur) at date t.

We provide some intuition to help understand conditions (B.7) and (B.8). In (B.7), the reserve-holding agent's current period payoff is $\lambda\mu(z-c)$ because she gets a net payoff of $z-c$ if and only if a successful payment is made, which happens with probability $\lambda\mu$. Otherwise, she does not enjoy any rewards from making a payment but does not suffer from any cost either, resulting in a net payoff of 0. Next period, there are two possible states. If the reserve good is successfully transferred (with probability $\lambda\mu$), her continuation value is w_1^1, while it is w_1^0 if not. Taking expectations of these two continuation values and combining them with the current period payoff gives condition (B.7).

Similarly, (B.8) decomposes the non-reserve-holding agent's equilibrium payoff. Her current period payoff is $(1-\lambda\mu)(-c)$ because she suffers from a cost of lacking the reserve good if and only if she has not received any payment from the other reserve-holding agent, which happens with probability $1-\lambda\mu$. Otherwise, she gets the reserve good and avoids the cost, getting a net payoff of 0. Next period, there are still two possible states. If the reserve good is successfully transferred (with probability $\lambda\mu$), her continuation value is w_2^1, while it is w_2^0 if not. Taking expectations of these two continuation values and combining them with the current period payoff gives condition (B.8).

Based on the construction above, we can define the following operator for any $\mu \in (0,1)$, $\mathbf{w}^1, \mathbf{w}^0 \in \mathbb{R}^2$,

$$\phi\left(\mu, \mathbf{w}^1, \mathbf{w}^0\right) = \left(\delta \cdot \mathbf{w}^1 + \begin{pmatrix} (1-\delta)(z-c) \\ 0 \end{pmatrix}, \delta \cdot \mathbf{w}^0 - \begin{pmatrix} 0 \\ (1-\delta)\, c \end{pmatrix}\right) \begin{pmatrix} \lambda\mu \\ 1-\lambda\mu \end{pmatrix}. \tag{B.10}$$

Then, (B.9) can be written as

$$\mathbf{w} = \phi\left(\mu, \mathbf{w}^1, \mathbf{w}^0\right).$$

Enforceability (incentive compatibility). The second step is to incorporate the two agents' incentive compatibility conditions, which APS call "enforceability." Recall that, due to the imperfect payment technology, a payment will go through with probability μ when the reserve-holding agent chooses to send it. Facing future continuation values $\mathbf{w}^1$ and $\mathbf{w}^0$, therefore, the reserve-holding agent effectively chooses the probability at which a payment is successfully made:

$$\mu = \mu\left(\mathbf{w}^1, \mathbf{w}^0\right) \triangleq \begin{cases} \mu & \text{if } (1-\delta)(z-c) + \delta w_1^1 \geq \delta w_1^0 \\ 0 & \text{if } (1-\delta)(z-c) + \delta w_1^1 < \delta w_1^0, \end{cases} \tag{B.11}$$

where we break the tie by assuming that any agent makes a payment when she is indifferent.

The intuition of the enforceability condition (B.11) is straightforward: conditional on receiving the private payment shock, the reserve-holding agent chooses to make a payment if and only if this action gives her a higher total payoff (including current period's payoff and next period's expected payoff) compared to not making a payment.

Generating function and evolution of the state variable. Third, we construct the generating function as in APS. Notably, we develop an approach to accommodate the evolution of the state variable (i.e., the ownership of the scarce reserve good), which is not present in APS. The change of reserve ownership differentiates our stochastic dynamic game from standard repeated games in which the stage game is repeated and the space of actions in a stage game does not change over time.

For any payoff set $W \subset \mathbb{R}^2$, define its transpose as

$$\mathbb{T}(W) = \{(w_1, w_2) : (w_2, w_1) \in W\} \ .$$

With a little abuse of notation but no confusion, for any $\mathbf{w} = (w_1, w_2) \in \mathbb{R}^2$, we also write $\mathbb{T}(\mathbf{w}) = (w_2, w_1) \in \mathbb{R}^2$.

Let $V_s \subset \mathbb{R}^2$ denote the set of per-period continuation value pairs that can be supported in a PPE when the state is $s \in \{0, 1\}$, that is, the equilibrium payoff set for the two agents, where s is the same as defined in Section 2. Specifically, V_1 is set of the equilibrium value pairs when the reserve-holding agent has the reserve good, while V_0 is that when the reserve-holding agent does not have the reserve good.

Definition 4. *Let $B(\cdot)$ be a set operator such that for any $W \subset \mathbb{R}^2$,*

$$B(W) = \left\{\mathbf{w} \in \mathbb{R}^2 : \exists \mathbf{w}^0 \in W \text{ and } \mathbf{w}^1 \in \mathbb{T}(W), \text{ such that } \mathbf{w} = \phi\left(\mu\left(\mathbf{w}^1, \mathbf{w}^0\right), \mathbf{w}^1, \mathbf{w}^0\right)\right\} \ .$$

Following APS, we call the operator $B(\cdot)$ the generating function. As discussed before, however, the structure of this generating function is significantly different from its original version in APS and the large literature of repeated games following APS. Specifically, $B(W)$ in our non-repeated dynamic stochastic game consists of pairs of continuation values that can be supported by future continuation value pairs $\mathbf{w}^1$ and $\mathbf{w}^0$ chosen from $\mathbb{T}(W)$ and W, respectively, contingent on whether there is a successful transfer of the reserve good or not. Note that $\mathbf{w}^1$, the pair of future continuation values after a transfer, is chosen from $\mathbb{T}(W)$ instead of W, because after a transfer of the reserve good the state of the game switches from s to $1 - s$, and $V_{1-s} = \mathbb{T}(V_s)$. Fortunately, due to the symmetry between V_0 and V_1, we need to characterize one of them only. We will focus on V_1 below, a generic element of which is (w_1, w_2), with w_1 being the continuation value of the reserve-holding agent, and w_2 being the continuation value of the non-reserve-holding agent.

We provide another perspective to understand why we can handle the evolution of the reserve good ownership as the state variable in the generating function without actually tracking it. In our economy, this stems from the fact that the two agents jointly own one indivisible reserve good. Whenever a successful transfer of the reserve good takes place, the initially reserve-holding (non-reserve-holding) agent becomes the non-reserve-holding (reserve-holding) agent. The resulting new stage game, albeit different from the initial stage game due to the change of ownership, mirrors the initial game by switching the roles of the two agents. Mathematically, this can be thus handled by the transpose of the initial vector that consists of the two agents' payoffs. This feature plays an important role in the analysis of our economy. As discussed above, this feature requires a setup that prevents us from directly making quantitative predictions regarding the amount of the reserve goods. However, it does sufficiently capture the scarcity of the reserve good, which is the key. Furthermore, we also gain the analytical tractability to make significant progress in characterizing the full equilibrium payoff set.

Self-generation and equilibrium payoff set. Having developed an approach to handle the evolution of the state variable, we extend the notion of self-generation in APS to our framework and present an analytical procedure to characterize the equilibrium payoff set V_1, as the last step.

Lemma 2. *The equilibrium payoff set is self-generating in the sense that* $V_1 \subset B(V_1)$.

Lemma 2 holds by definition of PPE: any sub-game equilibrium of a PPE is itself a PPE. Economically, this means that the set of equilibrium payoffs V_1 should be self-generating in the sense that it is possible to sustain average payoffs in V_1 by promising different continuation values in V_1.

Lemma 3. *If* $W \subset \mathbb{R}^2$ *is bounded and* $W \subset B(W)$*, then* $B(W) \subset V_1$.

Lemma 3 then gives us a criterion for identifying subsets of the equilibrium payoff set V_1, because any self-generating set is such a subset. Interestingly, other than boundedness, Lemma 3 does not impose any restrictions on the payoff set W. One might expect that applying the generating function $B(\cdot)$ on W would generate payoffs that are not attainable in our dynamic economy. Indeed, the real requirement for Lemma 3 is that W must be able to generate a superset of itself. Intuitively, the generating function given by Definition 4 implies that any $B(W)$ must be itself enforceable in our dynamic economy. Because any enforceable payoff constructs an equilibrium payoff by the definition of PPE, we have the desired result.

Proposition 12. *The equilibrium payoff set satisfies* $V_1 = B(V_1)$.

Proposition 12 is important and directly follows from Lemmas 2 and 3. It states that the equilibrium payoff set V_1 is a fixed point of the generating function $B(\cdot)$. Following APS, we thus call that the equilibrium payoff set V_1 can be factorized. Economically, this implies that V_1 can be found by characterizing the largest fixed point of $B(\cdot)$.

Lemma 4. *If $W \subset \widetilde{W}$, then $B(W) \subset B\left(\widetilde{W}\right)$.*

Lemma 4 further states that the generating function $B(\cdot)$ is monotone with respect to the partial order induced by set inclusion "$\subset$". In addition, since all subsets of X form a complete lattice with respect to "$\cap$" and "$\cup$" and the partial order, according to Tarski's fixed point theorem, the fixed points of $B(\cdot)$ form a complete lattice and thus there is a maximal one, which is V_1. In particular, by definition, $B^n(X)$ forms a non-increasing set sequence and $V_1 = \lim_{n\to\infty} B^n(X)$. Combining Proposition 12 and Lemmas 1 and 4 yields Proposition 6, the main result of Section 5.

C Proofs

Proof of Proposition 1. The proof follows the idea of APS's decomposability and is based on the formal setup we lay out in Section 5. Let $\mathbf{w} = (w_1, w_2)$ be the pair of per-period continuation values, that is, the per-period equilibrium payoffs of the two agents in the good payment equilibrium. Note that if a good payment equilibrium exists, (w_1, w_2) must be unique since the agents' actions are history independent. By the nature of the good payment equilibrium, the currently reserve-holding agent always has a per-period continuation value w_1 and the non-reserve-holding agent always has a per-period continuation value w_2 regardless of the public history. Plugging $\mathbf{w}^1 = \mathbb{T}(\mathbf{w})$ and $\mathbf{w}^0 = \mathbf{w}$ into equation (B.9), we obtain

$$w_1 = (1-\delta)\lambda\mu(z-c) + \delta(\lambda\mu w_2 + (1-\lambda\mu)w_1) \tag{C.12}$$

and

$$w_2 = (1-\delta)(1-\lambda\mu)(-c) + \delta(\lambda\mu w_1 + (1-\lambda\mu)w_2)\ . \tag{C.13}$$

Taking the difference of (C.12) and (C.13) yields

$$w_1 - w_2 = \frac{(1-\delta)(\lambda\mu z - (2\lambda\mu - 1)c)}{1 + \delta(2\lambda\mu - 1)}\ . \tag{C.14}$$

Taking the sum of (C.12) and (C.13) yields

$$w_1 + w_2 = \lambda\mu z - c\,, \tag{C.15}$$

which is intuitive. It is straightforward to see that the good payment equilibrium, if exists, is the only equilibrium that attains the boundary $\{(w_1, w_2) : w_1 + w_2 = \lambda\mu z - c\}$. According to (B.11), this equilibrium exists if and only if

$$\delta w_2 + (1-\delta)(z-c) \geq \delta w_1 \, ,$$

that is,

$$\delta (w_1 - w_2) \leq (1-\delta)(z-c) \ .$$

By (C.14), this is equivalent to

$$z/c \geq (1 - \delta(1-\lambda\mu))^{-1} \ . \tag{C.16}$$

By the relationship of $\kappa = c/(1-\delta)$, this immediately yields the equilibrium condition (3.1).

We can further solve the two continuation values from (C.14) and (C.15), which are given by

$$w_1 = \frac{\lambda\mu}{1 + \delta(2\lambda\mu - 1)} \left((1 - \delta(1-\lambda\mu)) z - c \right) \tag{C.17}$$

and

$$w_2 = \frac{\delta(\lambda\mu)^2 z - (\delta\lambda\mu + (1-\delta)(1-\lambda\mu)) c}{1 + \delta(2\lambda\mu - 1)} \, . \tag{C.18}$$

Note that (C.16) implies that w_1, the per-period continuation value of the agent with the reserve in this good equilibrium as given by (C.17), is non-negative. This concludes the proof. □

Proof of Proposition 2. The bad payment equilibrium consists of the strategy that the agent with the reserve never transfer the reserve upon request. Without loss of generality, let the reserve-holding agent have the reserve at the beginning of the period. Then, if both agents follow the bad strategy, the reserve-holding agent's per-period continuation value is 0 and the non-reserve-holding agent's is $-c$. This strategy is an equilibrium if and only if one-shot deviation is not profitable. No profitable one-shot deviation is equivalent to

$$\mu \left(z - (1-\delta) \cdot \frac{c}{1-\delta} + \delta \cdot \frac{-c}{1-\delta} \right) \leq 0 \, ,$$

i.e.,

$$z \leq \frac{c}{1-\delta} \, .$$

By the relationship of $\kappa = c/(1-\delta)$, this immediately yields the equilibrium condition (3.3).

Similarly, we can easily calculate the continuation values for the two agents in the bad payment equilibrium as $w_1 = 0$ and $w_2 = -c$. This concludes the proof. □

Proof of Proposition 4. The proof proceeds in two steps. These two steps give the upper and lower bounds of the region where the delay-trigger payment equilibrium exists.

STEP 1. Note that the delay-trigger payment equilibrium admits the bad payment equilibrium as a sub-game equilibrium. By Definition 2, the existence of the delay-trigger payment equilibrium thus requires the existence of the bad payment equilibrium. That is, condition (3.3) must hold.

STEP 2. Following Definition 4 and APS's decomposability conditions, we can write:

$$w_1^{(1)} = \lambda\mu \left(\delta w_2^{(1)} + (1-\delta)(z-c)\right) + (1-\lambda\mu)\,\delta w_1^{(0)} \tag{C.19}$$

and

$$w_2^{(1)} = \lambda\mu\delta w_1^{(1)} + (1-\lambda\mu)\left(\delta w_2^{(0)} - (1-\delta)c\right). \tag{C.20}$$

Note that, different from the counterparts in the proofs of Propositions 1 and 2, the pair of continuation values as well as agents' payment decisions are no longer history independent in the delay-trigger equilibrium. Rather, they follow the automaton as the state of the game evolves.

By Proposition 2, we easily have $w_1^{(0)} = 0$ and $w_2^{(0)} = -c$. Plugging them into (C.19) and (C.20) yields:

$$w_1^{(1)} = \lambda\mu \left(\delta w_2^{(1)} + (1-\delta)(z-c)\right) \tag{C.21}$$

and

$$w_2^{(1)} = \lambda\mu\delta w_1^{(1)} - (1-\lambda\mu)c\,. \tag{C.22}$$

Further plugging (C.22) into (C.21) yields:

$$w_1^{(1)} = \frac{\lambda\mu\left(\delta(1-\lambda\mu)(-c) + (1-\delta)(z-c)\right)}{1-\delta^2\lambda^2\mu^2} \tag{C.23}$$

On the other hand, the existence of the delay-trigger payment equilibrium requires incentive compatibility for the reserve-holding agent in state $w^{(1)}$:

$$\delta w_2^{(1)} + (1-\delta)(z-c) \geq \delta w_1^{(0)}\,. \tag{C.24}$$

Plugging (C.22) and $w_1^{(0)} = 0$ into (C.24) simplies the incentive compatability condition to:

$$w_1^{(1)} \geq 0\,. \tag{C.25}$$

Combining (C.23) and (C.25) finally yields:

$$\frac{z}{c} \geq \frac{1-\delta\lambda\mu}{1-\delta}.$$

We can now close the proof by combining results from the two steps above:

$$\frac{1-\delta\lambda\mu}{1-\delta} \leq \frac{z}{c} \leq \frac{1}{1-\delta}.$$

By the relationship of $\kappa = c/(1-\delta)$, this immediately yields the equilibrium condition (4.1). This concludes the proof. □

Proof of Proposition 5. The proof uses mathematical induction and builds upon the proof of Proposition 4. It proceeds in three steps.

STEP 1. Based on the proof of Proposition 4, it is known that the delay-trigger strategy, that is, the 1-trigger strategy constitutes a PPE when

$$\frac{1-\delta\lambda\mu}{1-\delta} \leq \frac{z}{c} \leq \frac{1}{1-\delta}.$$

STEP 2. We show that the $n+1$-trigger strategy must constitute a PPE if the n-delay-trigger strategy constitutes a PPE, for all $n \geq 1$. Following Definition 4 and APS's decomposability conditions, we start from:

$$w_1^{(n+1)} = \lambda\mu\left(\delta w_2^{(n+1)} + (1-\delta)(z-c)\right) + (1-\lambda\mu)\,\delta w_1^{(n)}$$

and

$$w_2^{(n+1)} = \lambda\mu\delta w_1^{(n+1)} + (1-\lambda\mu)\left(\delta w_2^{(n)} - (1-\delta)c\right).$$

that is,

$$\mathbf{w}^{(n+1)} \triangleq \begin{pmatrix} w_1^{(n+1)} \\ w_2^{(n+1)} \end{pmatrix} = \left(\delta \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{w}^{(n+1)} + \begin{pmatrix} (1-\delta)\,(z-c) \\ 0 \end{pmatrix}, \delta \cdot \mathbf{w}^{(n)} - \begin{pmatrix} 0 \\ (1-\delta)\,c \end{pmatrix}\right) \begin{pmatrix} \lambda\mu \\ 1-\lambda\mu \end{pmatrix}$$

where we have $w_1^0 = 0$ and $w_2^0 = -c$ by Proposition 2. Recast the equation above to get:

$$\left(\mathbf{I} - \lambda\mu\delta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right) \mathbf{w}^{(n+1)} = (1-\lambda\mu)\delta\mathbf{w}^{(n)} + (1-\delta)\begin{pmatrix} \lambda\mu(z-c) \\ -(1-\lambda\mu)c \end{pmatrix},$$

which implies

$$\mathbf{w}^{(n+1)} = (1-\lambda\mu)\delta\mathbf{A}^{-1}\mathbf{w}^{(n)} + (1-\delta)\mathbf{A}^{-1}\begin{pmatrix} \lambda\mu(z-c) \\ -(1-\lambda\mu)c \end{pmatrix},$$

where $\mathbf{A} \triangleq \mathbf{I} - \lambda\mu\delta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

On the other hand, incentive compatibility requires that

$$(0,\delta)\mathbf{w}^{(n+1)} \geq (\delta,0)\mathbf{w}^{(n)} - (1-\delta)(z-c)\,. \tag{C.26}$$

In order to further characterize $\mathbf{w}^{(n)}$ and the incentive compatibility condition, let $\mathbf{y}^{(n)} = \mathbf{w}^{(n)} + \mathbf{x}$ for some $\mathbf{x}$, such that $\mathbf{y}^{(n)}$ takes the form

$$\mathbf{y}^{(n+1)} = (1-\lambda\mu)\delta\mathbf{A}^{-1}\mathbf{y}^{(n)}\,, \tag{C.27}$$

that is,

$$\begin{aligned} \mathbf{w}^{(n+1)} + \mathbf{x} &= (1-\lambda\mu)\delta\mathbf{A}^{-1}\left(\mathbf{w}^{(n)} + \mathbf{x}\right)\,, \\ \Rightarrow \mathbf{x} &= (1-\delta)\left((1-\lambda\mu)\delta\mathbf{I} - \mathbf{A}\right)^{-1}\begin{pmatrix} \lambda\mu(z-c) \\ -(1-\lambda\mu)c \end{pmatrix}. \end{aligned}$$

Now we can characterize $\mathbf{y}^{(n)}$ and equilibirum payoff $\mathbf{w}^{(n)}$ with boundary conditions. Note that (C.27) implies contraction, hence $\mathbf{y}^{(\infty)} = 0$ is the unique fixed point and $\mathbf{w}^{(\infty)} = -\mathbf{x}$. The incentive compatibility condition (C.26) now becomes

$$(\delta,0)\left(\mathbf{y}^{(n)} - \mathbf{x}\right) - (0,\delta)\left(\mathbf{y}^{(n+1)} - \mathbf{x}\right) \leq (1-\delta)(z-c)\,.$$

Plugging in (C.27) and that $\mathbf{w}^{(\infty)} = -\mathbf{x}$ yields

$$\begin{aligned} \left((\delta,0) - (0,\delta)\mathbf{A}^{-1}(1-\lambda\mu)\delta\right)\mathbf{y}^{(n+1)} &\leq \delta(1,-1)\mathbf{x} + (1-\delta)(z-c)\,, \\ \Rightarrow \frac{1}{1-\lambda\mu}(1,-\delta)\mathbf{y}^{(n+1)} &\leq \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,. \end{aligned} \tag{C.28}$$

To show that the $n+1$-trigger strategy must constitute a PPE if the n-delay-trigger strategy constitutes a PPE, it now suffices to show that (C.28) must hold if $\mathbf{y}^{(n)}$ satisfies

$$\frac{1}{1-\lambda\mu}(1,-\delta)\mathbf{y}^{(n)} \le \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,. \tag{C.29}$$

By definition, $\mathbf{w}^{(\infty)}$ is the payoff of the good payment equilibrium, thus the RHS of (C.28) must be positive. This is because the RHS is exactly the expected payoff from the good payment equilibrium minus the expected payoff from one-shot deviation, hence the RHS is non-negative if and only if the good payment equilibrium exists.

It is straightforward to see that the eigenvalues and the corresponding normalized eigenvectors of $(1-\lambda\mu)\delta\mathbf{A}^{-1}$ are

$$m_1 = \frac{(1-\lambda\mu)\delta}{1-\lambda\mu\delta}, \quad \mathbf{e_1} = 2^{-\frac{1}{2}}\begin{pmatrix}1\\1\end{pmatrix},$$
$$m_2 = \frac{(1-\lambda\mu)\delta}{1+\lambda\mu\delta}, \quad \mathbf{e_2} = 2^{-\frac{1}{2}}\begin{pmatrix}1\\-1\end{pmatrix},$$

where $\mathbf{y}^{(0)}$ is given by

$$\mathbf{y}^{(0)} = \mathbf{w}^{(0)} - \mathbf{w}^{(\infty)} = \begin{pmatrix}-w_1^{(\infty)}\\ -c-w_2^{(\infty)}\end{pmatrix}.$$

Note that $w_1^{(\infty)} \ge 0$, $w_2^{(\infty)} \le -c$ implies $y_1^{(0)} \le 0$, $y_2^{(0)} \le 0$. Let $\mathbf{y}^{(0)} = (\mathbf{e_1}, \mathbf{e_2})(\beta_1, \beta_2)^T$, where β_1, β_2 are given by

$$\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} = 2^{-\frac{1}{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}y_1^{(0)}\\ y_2^{(0)}\end{pmatrix},$$

and hence $\beta_1 \le 0$. By (C.27), $\mathbf{y}^{(n)} = ((1-\lambda\mu)\delta\mathbf{A}^{-1})^n\,\mathbf{y}^{(0)} = (m_1^n\mathbf{e_1}, m_2^n\mathbf{e_2})(\beta_1,\beta_2)^T$. Since the n-delay-trigger strategy constitutes a PPE, plugging $\mathbf{y}^{(0)}$ into (C.29) to get

$$\begin{aligned}
&\frac{1}{1-\lambda\mu}(1,-\delta)(m_1^n\mathbf{e_1}, m_2^n\mathbf{e_2})\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} \le \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,,\\
\Rightarrow\ &\frac{2^{-\frac{1}{2}}}{1-\lambda\mu}(1-\delta, 1+\delta)\begin{pmatrix}\mathbf{e_1}^T\\ \mathbf{e_2}^T\end{pmatrix}(m_1^n\mathbf{e_1}, m_2^n\mathbf{e_2})\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} \le \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,,\\
\Rightarrow\ &\frac{2^{-\frac{1}{2}}}{1-\lambda\mu}(m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2) \le \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,.
\end{aligned} \tag{C.30}$$

To show (C.28), it suffices to show (C.30) holds for $n+1$. To see this, note that

$$m_1^{n+1}(1-\delta)\beta_1+m_2^{n+1}(1+\delta)\beta_2 = m_2\left(m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right)+(m_1-m_2)m_1^n(1-\delta)\beta_1\,,$$

where $0 < m_2 < m_1 < 1$. As shown above, $\beta_1 \leq 0$, hence $(m_1 - m_2)m_1^n(1-\delta)\beta_1 \leq 0$. Recall that $\delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c) \geq 0$. If $m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2 \leq 0$, we have

$$\begin{aligned}
& \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left(m_1^{n+1}(1-\delta)\beta_1 + m_2^{n+1}(1+\delta)\beta_2\right) \\
= & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left(m_2\left(m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right) + (m_1-m_2)m_1^n(1-\delta)\beta_1\right) \\
\leq & \ 0 \\
\leq & \ \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,,
\end{aligned}$$

otherwise,

$$\begin{aligned}
& \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left(m_1^{n+1}(1-\delta)\beta_1 + m_2^{n+1}(1+\delta)\beta_2\right) \\
= & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left(m_2\left(m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right) + (m_1-m_2)m_1^n(1-\delta)\beta_1\right) \\
\leq & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left(m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2 + (m_1-m_2)m_1^n(1-\delta)\beta_1\right) \\
\leq & \frac{2^{-\frac{1}{2}}}{1-\lambda\mu}\left(m_1^n(1-\delta)\beta_1 + m_2^n(1+\delta)\beta_2\right) \\
\leq & \ \delta(1,-1)\mathbf{w}^{(\infty)} + (1-\delta)(z-c)\,,
\end{aligned}$$

where the last inequality follows from (C.30) for n. This thus concludes the second step of induction. It implies that the region where the n-delay-trigger payment equilibrium must be a superset of that where the 1-trigger payment equilibrium exists.

STEP 3. Note that, by definition, the n-delay-trigger payment equilibrium admits the 1-trigger payment equilibrium as a possible sub-game equilibrium. Hence, the region where the n-delay-trigger payment equilibrium must also be a subset of that where the 1-trigger payment equilibrium exists. This concludes the proof. □

Proof of Proposition 6. This proof is given in Appendix B. □

Proof of Proposition 7. We first consider the good equilibrium. Following the idea of APS's decomposability as outlined in the proof of Proposition 1, we can express the reserve-holding agent i's per-period continuation value as

$$w_1 = (1-\delta)\,\lambda\mu\,(z-c) + \delta\,(\lambda\mu w_{-1} + (1-\lambda\mu)\,w_1)\ , \tag{C.31}$$

and each non-reserve-holding agent j's per-period continuation value as

$$w_{-1} = (1-\delta)\left(1-\frac{\lambda\mu}{N-1}\right)(-c) + \delta\left(\frac{\lambda\mu}{N-1}w_1 + \left(1-\frac{\lambda\mu}{N-1}\right)w_{-1}\right)\ . \tag{C.32}$$

Solving the system of (C.31) and (C.32) with respect to $\mathbf{w}= (w_1, w_{-1})$, we get:

$$w_1 = \lambda\mu\left(\frac{1-\delta+\delta\frac{\lambda\mu}{N-1}}{1-\delta+\delta\lambda\mu\frac{N}{N-1}}z - \frac{1}{1-\delta+\delta\lambda\mu\frac{N}{N-1}}c\right)\ , \tag{C.33}$$

and

$$w_{-1} = \frac{\delta\lambda\mu\frac{\lambda\mu}{N-1}}{1-\delta\,(1-\lambda\mu)+\delta\frac{\lambda\mu}{N-1}}z - \left(1-\frac{\frac{\lambda\mu}{N-1}}{1-\delta\,(1-\lambda\mu)+\delta\frac{\lambda\mu}{N-1}}\right)c\,. \tag{C.34}$$

At the same time, the incentive compatibility constraint for agent i is given by

$$(1-\delta)\,(z-c) \geq \delta\,(w_1 - w_{-1})\ . \tag{C.35}$$

Plugging the solutions (C.33) and (C.34) into condition (C.35) yields

$$\delta^{-1}(z-c)\left(1-\delta\,(1-\lambda\mu)+\delta\frac{\lambda\mu}{N-1}\right) \geq \lambda\mu z - \left(-(1-\lambda\mu)+\frac{\lambda\mu}{N-1}\right)c\,,$$

that is,

$$z \geq \frac{c}{1-\delta\left(1-\frac{\lambda\mu}{N-1}\right)}\,, \tag{C.36}$$

which immediately implies condition (A.1) by the fact that $c = \kappa(1-\delta)$. Thus, (A.1) holds as the sufficient and necessary condition for the existence of the good equilibrium in the extended model with $N \geq 2$.

We then consider the bad equilibrium, which inherits the same equilibrium conditions from the baseline model. To see this, notice that in the bad equilibrium, the reserve-holding agent i's per-period continuation value is always $w_1 = 0$, whereas each non-reserve-holding agent j's per-period continuation value is always $w_{-1} = -c$. The incentive compatibility constraint for agent i is

$$(1-\delta)\,(z-c) \leq \delta\,(w_1 - w_{-1})\ ,$$

that is

$$z \leq \frac{c}{1-\delta}\,, \tag{C.37}$$

which is the same as its counterpart in the baseline model. Thus, again by the fact that $c = \kappa(1-\delta)$, (A.2) holds as the sufficient and necessary condition for the existence of the bad equilibrium in the extended model with $N \geq 2$.

Finally, it is straightforward that for any $N \geq 2$, both the good and the bad equilibria co-exist when conditions (C.36) and (C.37), and consequently, conditions (A.1) and (A.2) both hold. This concludes the proof. □

Proof of Propositions 8 and 9. Again, let w_1 and w_2 denote the reserve-holding and non-reserve-holding agents' per-period continuation values, respectively. In the proposed good payment equilibrium, the reserve-holding agent's strategy would be sending the reserve if $z \geq \hat{z}$, regardless of the history, while the non-reserve-holding agent's strategy would be to always propose $z = \hat{z}$ regardless of the history. As a result, two value functions w_1 and w_2 must satisfy:

$$\begin{cases} w_1 = (1-\lambda\mu)\delta w_1 + \lambda\mu[(1-\delta)(\hat{z}-c)] + \delta w_2, \\ w_2 = (1-\lambda\mu)[\delta w_2 - (1-\delta)c] + \lambda\mu[-(1-\delta)\alpha h(\hat{z}) + \delta w_1]\,. \end{cases} \tag{C.38}$$

Similarly, the incentive compatability (IC) constraint for the reserve-holding agent is

$$(1-\delta)(\hat{z}-c) \geq \delta(w_1 - w_2)\,. \tag{C.39}$$

As opposed to the baseline model without production, we now also consider the non-reserve-holding agent's IC constraint. Note that given the reserve-holding agent's strategy, the non-reserve-holding agent does not want to choose any $z \geq \hat{z}$, since it only incurs a greater production cost $\alpha h(z)$ but does not change the probability of transfer. Thus, the IC constraint should guarantee that the non-reserve-holding agent does not benefit from choosing $z < \hat{z}$, which results in a lack of transfer during the current period. That is,

$$(1-\lambda\mu)[-(1-\delta)c + \delta w_2] + \lambda\mu[-(1-\delta)\alpha h(\hat{z}) + \delta w_1] \geq -(1-\delta)c + \delta w_2,$$

that is,

$$\delta(w_1 - w_2) \geq (1-\delta)(\alpha h(\hat{z}) - c)\,. \tag{C.40}$$

Note that by letting $\alpha = 0$, (C.40) becomes

$$\delta(w_1 - w_2) \geq -(1-\delta)c, \tag{C.41}$$

which can be viewed as the situation that the per-period reward to successful transfer is exogenously given as $\hat{z}$, so that the non-reserve-holding agent doesn't need to choose z and incur a cost of production; however, he can choose to reject the transfer so that we need (C.41) to guarantee his incentive of acceptance.

Now, solving the system of w_1 and w_2 as in (C.38) yields:

$$\begin{cases} w_1 = \dfrac{\lambda\mu[\delta\lambda\mu(\hat{z} - \alpha h(\hat{z})) + (1-\delta)\hat{z} - c]}{1 - \delta + 2\delta\lambda\mu}, \\ w_2 = \dfrac{\mu\lambda[\delta(\hat{z} - 2h(\hat{z})) - (1-\delta)2h(\hat{z})] - [(1-\delta)^2 - \delta^2]c}{1 - \delta + 2\delta\lambda\mu}, \end{cases} \tag{C.42}$$

which combined with (C.39) implies that the IC constraint for the reserve-holding agent is

$$\hat{z} - c \geq \frac{\delta[\lambda\mu(\hat{z} - c + \alpha h(\hat{z})) + (1-\lambda\mu)c]}{1 - \delta + 2\delta\lambda\mu},$$

which further reduces to

$$c \leq (1 - \delta + \delta\lambda\mu)\hat{z} - \delta\lambda\mu\alpha h(\hat{z}). \tag{C.43}$$

Similarly, we combine (C.42) with (C.40) to further consider the IC constraint for the non-reserve-holding banking sector, that is,

$$\alpha h(\hat{z}) - c \leq \frac{\delta[\lambda\mu(\hat{z} - c + \alpha h(\hat{z})) + (1-\lambda\mu)c]}{1 - \delta + 2\delta\lambda\mu},$$

which further reduces to

$$(\hat{z} - c)(1 - \delta + \delta\lambda\mu) \leq \delta[\lambda\mu\alpha h(\hat{z}) + (1 - \delta\mu)c] + (1 - \delta + 2\delta\lambda\mu)(\hat{z} - \alpha h(\hat{z})). \tag{C.44}$$

Combining both agents' IC constraints (C.43) and (C.44), a necessary condition for them to hold is thus:

$$\hat{z} - \alpha h(\hat{z}) \geq 0.$$

To verify the good payment equilibrium with production, note that $f(0) = 0$ since $h(0) = 0$. Let

$$\bar{f} = \max_{z \geq 0} f(z),$$

then if $\kappa \leq \bar{f}$, by continuity, $\exists \hat{z} \geq 0$ s.t. $\kappa = f(\hat{z})$, i.e. both IC constraints hold. This concludes the proof of Proposition 8.

To verify the bad payment equilibrium with production, note that the reserve-holding agent makes transfer iff $z \geq \hat{z}$ but $\hat{z}$ is so high that the non-reserve-holding agent always offers $z < \hat{z}$, so there is no transfer.

$$\begin{cases} w_1 = 0\,, \\ w_2 = -c\,. \end{cases}$$

Here, the IC constraint for the reserve-holding agent should be:

$$\mu\left(z - c - \frac{\delta c}{1-\delta}\right) \geq 0 \text{ iff } z \geq \hat{z}\,,$$

while the IC constraint for the non-reserve-holding agent should be that proposing $\hat{z}$ is weakly dominated by proposing $z < \hat{z}$, which results in no transfer, that is,

$$(1-\lambda\mu)\frac{-c}{1-\delta} + \lambda\mu[-\alpha h(\hat{z})] + \delta \cdot 0 \leq \frac{-c}{1-\delta}\,,$$

which leads to $\alpha h(\kappa) > \kappa$. In this equilibrium, the reserve-holding agent makes transfer iff $z \geq \hat{z} = \kappa$, and the non-reserve-holding agent always proposes $z < \hat{z}$. This concludes the proof of Proposition 9. □

Proof of Proposition 11. We first consider the good equilibrium. In this equilibrium, transfer always occurs regardless of the history or reserve holdings. Given $q_i \in [0,1]$ and $q_j = 1 - q_i$, Agent i's equilibrium value function is

$$\begin{aligned}
U_i(q_i) &= (1-2\lambda)\left[-(1-\delta)\kappa(1-q_i) + \delta U_i(q_i)\right] \\
&\quad + \lambda\left[\int_0^1 \mu \cdot [zq_i\alpha - (1-\delta)\kappa(1-q_i+\alpha q_i) + \delta U_i(q_i - \alpha q_i)]\, d\alpha + (1-\mu)\left[-(1-\delta)\kappa(1-q_i)\right.\right. \\
&\quad + \lambda\left[\int_0^1 \mu \cdot [-(1-\delta)\kappa(1-q_i-\alpha q_j) + \delta U_i(q_i+\alpha q_j)]\, d\alpha + (1-\mu)\left[-(1-\delta)\kappa(1-q_i) + \delta U\right.\right. \\
&= (1-2\lambda+\lambda(1-\mu)+\lambda(1-\mu))\left[-(1-\delta)\kappa(1-q_i) + \delta U_i(q_i)\right] \\
&\quad + \lambda\mu\int_0^1 [zq_i\alpha - (1-\delta)\kappa(1-q_i+\alpha q_i) + \delta U_i(q_i-\alpha q_i)]\, d\alpha \\
&\quad + \lambda\mu\int_0^1 [-(1-\delta)\kappa(1-q_i-\alpha q_j) + \delta U_i(q_i+\alpha q_j)]\, d\alpha \\
&= (1-2\lambda\mu)\left[-(1-q_i)c + \delta U_i(q_i)\right] \\
&\quad + \lambda\mu\int_0^1 [zq_i\alpha - c[1-q_i(1-\alpha)] + \delta U_i(q_i(1-\alpha))]\, d\alpha \\
&\quad + \lambda\mu\int_0^1 [-(1-q_i)(1-\alpha)c + \delta U_i(q_i+\alpha(1-q_i))]\, d\alpha\,.
\end{aligned}$$

Note that (C.45) uniquely determines $U_i(\cdot)$, since the interation is a contraction by the Blackwell's conditions.

Conjecture that

$$U_i(q) = kq + b \tag{C.46}$$

for some coefficients k and b to be determined. Then, (C.45) becomes

$$\begin{aligned}
kq_i + b &= [1-2\lambda\mu]\left[-(1-q_i)c + \delta k q_i + \delta b\right] \\
&\quad + \lambda\mu \int_0^1 \left[z q_i \alpha - c + c q_i (1-\alpha) + \delta k q_i (1-\alpha) + \delta b\right] d\alpha \\
&\quad + \lambda\mu \int_0^1 \left[-(1-q_i)(1-\alpha)c + \delta k q_i + \delta k \alpha (1-q_i) + \delta b\right] d\alpha \\
&= [1-2\lambda\mu]\left[-(1-q_i)c + \delta k q_i + \delta b\right] \\
&\quad + \lambda\mu \left[-c + \frac{z q_i + c q_i + \delta k q_i}{2} + \delta b\right] \\
&\quad + \lambda\mu \left[\frac{-(1-q_i)c + 2\delta k q_i + \delta k (1-q_i)}{2} + \delta b\right] \\
&= \left[1 - 2\lambda\mu + \frac{\lambda\mu}{2} + \frac{\lambda\mu}{2}\right](c + \delta k) q_i \\
&\quad + [1-2\lambda\mu][-c + \delta b] + \lambda\mu\left[\frac{z}{2} q_i - c + \delta b\right] + \lambda\mu\left[\frac{-c+\delta k}{2} + \delta b\right] \\
&= \left[(1-\lambda\mu)(c+\delta k) + \frac{1}{2}\lambda\mu z\right] q_i \\
&\quad + [1-2\lambda\mu][-c+\delta b] + \lambda\mu[-c+\delta b] + \lambda\mu\left[\frac{-c+\delta k}{2} + c - c + \delta b\right] \\
&= \left[(1-\lambda\mu)(c+\delta k) + \frac{1}{2}\lambda\mu z\right] q_i - c + \delta b + \frac{\lambda\mu}{2}[c + \delta k].
\end{aligned}$$

Hence, to match the coefficients, we obtain

$$\begin{cases} k = (1-\lambda\mu)(c+\delta k) + \frac{1}{2}\lambda\mu z \\ b = -c + \delta b + \frac{\lambda\mu}{2}[c+\delta k] \end{cases},$$

i.e.,

$$\begin{cases} k = \frac{(1-\lambda\mu)c + \frac{1}{2}\lambda\mu z}{1-(1-\lambda\mu)\delta} \\ (1-\delta) b = -\left(1 - \frac{\lambda\mu}{2}\right)c + \frac{\delta\lambda\mu}{2} k \end{cases},$$

i.e.,

$$\begin{cases} k = \frac{(1-\lambda\mu)c + \frac{1}{2}\lambda\mu z}{1-(1-\lambda\mu)\delta} \\ (1-\delta) b = -\left(1 - \frac{\lambda\mu}{2}\right)c + \frac{\delta\lambda\mu}{2}\frac{(1-\lambda\mu)c + \frac{1}{2}\lambda\mu z}{1-(1-\lambda\mu)\delta} \\ \quad = -c + \lambda\mu\left[\frac{c}{2} + \frac{\delta}{2}\frac{(1-\lambda\mu)c + \frac{1}{2}\lambda\mu z}{1-(1-\lambda\mu)\delta}\right] \\ \quad = -c + \frac{\lambda\mu}{2}\frac{c + \frac{1}{2}\delta\lambda\mu z}{1-(1-\lambda\mu)\delta} \end{cases},$$

i.e.,

$$\begin{cases} k = \frac{(1-\lambda\mu)c+\frac{1}{2}\lambda\mu z}{1-(1-\lambda\mu)\delta} \\ b = (1-\delta)^{-1}\left(-c + \frac{\lambda\mu}{2}\frac{c+\frac{1}{2}\delta\lambda\mu z}{1-(1-\lambda\mu)\delta}\right) \end{cases} . \tag{C.47}$$

Note that the per-period continuation value for the agent with zero reserve holdings is

$$\begin{aligned} (1-\delta) U_i(0) &= (1-\delta) b \\ &= -c + \frac{\lambda\mu}{2}\frac{c+\frac{1}{2}\delta\lambda\mu z}{1-(1-\lambda\mu)\delta} > -c . \end{aligned}$$

For the good equilibrium, we need to further verify that the players do prefer transfering the reserve when chosen by nature. Consider player i with reserve holdings $q_i \in [0,1]$ who has a request to send payment $x_i \in [0, q_i]$. If he chooses to send, his expected payoff is

$$\mu[zx_i - (1-q_i+x_i)c + \delta U_i(q_i - x_i)] + (1-\mu)[-(1-q_i)c + \delta U_i(q_i)] ,$$

if not, his expected payoff is

$$-(1-q_i)c + \delta U_i(q_i) .$$

Hence, to support this good equilibrium, the IC is

$$zx_i - (1-q_i+x_i)c + \delta U_i(q_i - x_i) \geq -(1-q_i)c + \delta U_i(q_i)$$

for all $q_i \in [0,1]$ and $x_i \in [0, q_i]$. That is,

$$(z-c)x_i \geq \delta U_i(q_i) - \delta U_i(q_i - x_i) . \tag{C.48}$$

By (C.46), this is equivalent to

$$(z-c)x_i \geq \delta k x_i ,$$

i.e.,

$$z - c \geq \delta k ,$$

i.e.,

$$\begin{aligned} z &\geq \delta\frac{(1-\lambda\mu)c+\frac{1}{2}\lambda\mu z}{1-(1-\lambda\mu)\delta} + c \\ &= \frac{c+\frac{1}{2}\delta\lambda\mu z}{1-(1-\lambda\mu)\delta} , \end{aligned}$$

i.e.,

$$\frac{z}{c} \geq \frac{1}{1-\delta+\frac{1}{2}\lambda\mu\delta}\,, \tag{C.49}$$

or equivalently,

$$\frac{z}{\kappa} \geq \frac{1-\delta}{1-\delta+\frac{1}{2}\lambda\mu\delta}\,. \tag{C.50}$$

We then consider the bad equilibrium, which inherits the same equilibrium conditions from the baseline model. In this equilibrium, for agent i with reserve holdings q_i, the equilibrium value function is

$$\begin{aligned} U_i\left(q_i\right) &= -\left(1-\delta\right)\kappa\left(1-q_i\right)+\delta U_i\left(q_i\right) \\ &= -\left(1-q_i\right)c+\delta U_i\left(q_i\right)\ , \end{aligned}$$

i.e.,

$$U_i\left(q_i\right) = \frac{-\left(1-q_i\right)c}{1-\delta}. \tag{C.51}$$

To support this equilibrium, similar to (C.48), for all $q_i \in [0,1]$ and $x_i \in [0,q_i]$, we need

$$\left(z-c\right)x_i \leq \delta U_i\left(q_i\right)-\delta U_i\left(q_i-x_i\right)\ ,$$

by (C.51), i.e.,

$$\left(z-c\right)x_i \leq \frac{\delta c x_i}{1-\delta}\,,$$

i.e.,

$$\frac{z}{c} \leq \frac{1}{1-\delta}\,, \tag{C.52}$$

i.e.,

$$\frac{z}{\kappa} \leq 1\,, \tag{C.53}$$

which is the same as in the baseline model. This concludes the proof. □

Proof of Lemma 3. For any w $\in B\left(W\right)$, by the definition of $B\left(\cdot\right)$, there exist $\mathbf{w}^0 \in W$ and $\mathbf{w}^1 \in \mathbb{T}\left(W\right)$ such that $\mathbf{w} = \phi\left(\mu\left(\mathbf{w}^1,\mathbf{w}^0\right),\mathbf{w}^1,\mathbf{w}^0\right)$. Since $W \subset B\left(W\right)$, we can find for $\mathbf{w}^0$ two pairs of continuation values $\mathbf{w}^{00} \in W$ and $\mathbf{w}^{01} \in \mathbb{T}\left(W\right)$ such that $\mathbf{w}^0 = \phi\left(\mu\left(\mathbf{w}^{01},\mathbf{w}^{00}\right),\mathbf{w}^{01},\mathbf{w}^{00}\right)$, where the right superscript of each pair of continuation values denote the associated history of public signals (e.g., $\mathbf{w}^{01} = \left(w_1^{01},w_2^{01}\right)$ is the pair of continuation values for the two agents after "no transfer" in period 1 and "transfer" in period 2). Similarly, since $\mathbb{T}\left(\mathbf{w}^1\right) \in W \subset B\left(W\right)$, we can find for $\mathbf{w}^1$ two pairs of continuation values $\mathbf{w}^{10} \in W$ and $\mathbf{w}^{01} \in \mathbb{T}\left(W\right)$ such that $\mathbb{T}\left(\mathbf{w}^1\right) = \phi\left(\mu\left(\mathbb{T}\left(\mathbf{w}^{11}\right),\mathbb{T}\left(\mathbf{w}^{10}\right)\right),\mathbb{T}\left(\mathbf{w}^{11}\right),\mathbb{T}\left(\mathbf{w}^{10}\right)\right)$. In this way, we can find for every public history $h^t = (h_1, h_2, \cdots, h_{t-1}) \in \{0,1\}^{t-1}$ (where h^0 denotes the null history right before period 1 with the initial pair of continuation values $\mathbf{w}$, and write $h^{t+1} = (h^t, h_t)$), a pair of continuation values

$$\mathbf{w}^{(h^t,0)} \in \begin{cases} W & \text{if } s_t \oplus 0 = 1 \\ \mathbb{T}(W) & \text{if } s_t \oplus 0 = 0\,, \end{cases}$$

and a pair of continuation values

$$\mathbf{w}^{(h^t,1)} \in \begin{cases} W & \text{if } s_t \oplus 1 = 1 \\ \mathbb{T}(W) & \text{if } s_t \oplus 1 = 0\,, \end{cases}$$

where $s_t \in \{0,1\}$ is the state of the game (specifying which agent has the reserve) after history h^t and $s_{t+1} = (s_t + h_t) \bmod 2 \triangleq s_t \oplus h_t$, such that i) if $s_t = 1$, then $\mathbf{w}^{h^t} = \phi\left(\mu\left(\mathbf{w}^{(h^t,1)}, \mathbf{w}^{(h^t,0)}\right), \mathbf{w}^{(h^t,1)}, \mathbf{w}^{(h^t,0)}\right)$ and the reserve-holding agent's action after h^t is $\mu\left(\mathbf{w}^{(h^t,1)}, \mathbf{w}^{(h^t,0)}\right)$; ii) if $s_t = 0$, then $\mathbb{T}\left(\mathbf{w}^{h^t}\right) = \phi\left(\mu\left(\mathbb{T}\left(\mathbf{w}^{(h^t,1)}\right), \mathbb{T}\left(\mathbf{w}^{(h^t,0)}\right)\right), \mathbb{T}\left(\mathbf{w}^{(h^t,1)}\right), \mathbb{T}\left(\mathbf{w}^{(h^t,0)}\right)\right)$ and the non-reserve-holding agent's action after h^t is $\mu\left(\mathbb{T}\left(\mathbf{w}^{(h^t,1)}\right), \mathbb{T}\left(\mathbf{w}^{(h^t,0)}\right)\right)$.

Define a public strategy profile σ as

$$\sigma\left(h^t\right) = \begin{cases} \text{agent 1 chooses } \mu\left(\mathbf{w}^{(h^t,1)}, \mathbf{w}^{(h^t,0)}\right), \text{ the non-reserve-holding agent no action} & \text{if } s_t = 1 \\ \text{agent 2 chooses } \mu\left(\mathbb{T}\left(\mathbf{w}^{(h^t,1)}\right), \mathbb{T}\left(\mathbf{w}^{(h^t,0)}\right)\right), \text{ the reserve-holding agent no action} & \text{if } s_t = 0\,. \end{cases}$$

Then the original $\mathbf{w} \in B(W)$ is attained, and $\mathbf{w}^{h^t}$ is also attained after every public history h^t because W is bounded. Moreover, by construction, there is no profitable one-shot deviation and thus σ is a PPE. Therefore, $B(W) \subset V_1$. This concludes the proof. □

Proof of Proposition 12. The proof takes several steps to explicitly construct the equilibrium payoff set V_1. These steps are developed and presented as lemmas below.

For any closed convex set $V \in \mathbb{R}^2$, define

$$\mathbf{Ext}(V) = \left\{ \begin{array}{c} \mathbf{w} \in V: \text{ there do not exist } \mathbf{w}' \text{ and } \mathbf{w}'' \text{ in } V \text{ such that} \\ \mathbf{w} = \alpha \cdot \mathbf{w}' + (1-\alpha) \cdot \mathbf{w}'' \text{ for some } \alpha \in (0,1) \end{array} \right\}$$

as the set of extreme points of V. We will allow public randomization so that V_1 is convex. It is also straightforward to see that V_1 is closed. Since a closed and convex set can be characterized by its extreme points, we next study $\mathbf{Ext}(V_1)$.

Note that the good equilibrium is the only equilibrium that attains the boundary

$\{(w_1, w_2) \in \mathbb{R}^2 : w_1 + w_2 = \lambda\mu z - c\}$. Hence, $\mathbf{w}= (w_1, w_2)$ is an extreme point of V_1. Let $\underline{\mathbf{w}}= (0, -c)$. By Lemma 2, $\underline{\mathbf{w}}$ is the pair of continuation values of the equilibrium in which agents never transfer the reserve. Since $\underline{\mathbf{w}}\in V_1 \subset X$ and $\underline{\mathbf{w}}$ is an extreme point of X, it is also an extreme point of V_1.

Lemma 5. *If* $(0, -c) \in V_1$*, then* $\forall \mathbf{w} \in \mathbf{Ext}(V_1)\backslash\{(0, -c)\}$*, the associated current-period equilibrium decision for the reserve-holding agent is* $\sigma = 1$*.*

Proof. We prove by contradiction. First, if $\sigma = 0$, (B.9) implies that $\mathbf{w} = \delta \mathbf{w}^0 + (1-\delta)(0, -c)^T$ for some $\mathbf{w}^0 \in V_1$. Since $\mathbf{w} \neq (0, -c)^T$, we also have $\mathbf{w}^0 \neq (0, -c)^T$. Then $\mathbf{w}$ is a strict convex combination of $\mathbf{w}^0$ and $(0, -c)^T$, which are two different points in V_1, contradicting to the assumption that $\mathbf{w} \in \mathbf{Ext}(V_1)$.

Second, if $\sigma \in (0, 1)$, that is, the reserve-holding agent is indifferent between making a payment or not, there must exist $\mathbf{w}^0 \in V_1$ and $\mathbf{w}^1 \in \mathbb{T}(V_1)$ such that $\mathbf{w} = \phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$. Since the reserve-holding agent is indifferent, both $\phi(0, \mathbf{w}^1, \mathbf{w}^0)$ and $\phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$ belong to V_1 as they should be both supported in a PPE. However, by Lemma 2, for $\mu \in (0, \mu_1)$, $\phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$ is a strict convex combination of $\phi(0, \mathbf{w}^1, \mathbf{w}^0)$ and $\phi(\mu, \mathbf{w}^1, \mathbf{w}^0)$, contradicting to the assumption that $\mathbf{w} \in \mathbf{Ext}(V_1)$.

Therefore, it must be that $\sigma = 1$. This concludes the proof. □

Lemma 5 states the current-period equilibrium decision associated with any extreme point of V_1 other than that of the bad payment equilibrium is to transfer the reserve good.

For any $W \subset \mathbb{R}^2$, let ∂W denote the set of boundary points of W.

Lemma 6. *For any* $\mathbf{w} \in \mathbf{Ext}\,(V_1)$ *and* $\mathbf{w} \neq (0, -c)$*, let* $\mathbf{w} = \phi\,(\mu, \mathbf{w}^1, \mathbf{w}^0)$ *for some* $\mathbf{w}^0 \in V_1$ *and* $\mathbf{w}^1 \in \mathbb{T}\,(V_1)$*. Then* $\mathbf{w}^0 \in \partial V_1$ *and* $\mathbf{w}^1 \in \partial\mathbb{T}\,(V_1)$*. Moreover, at least one of* $\mathbf{w}^0$ *and* $\mathbf{w}^1$ *is an extreme point of* V_1*.*

Proof. Denote the line segment connecting $\underline{\mathbf{w}}$ and $\overline{\mathbf{w}}$ by $L\,(\underline{\mathbf{w}}, \mathbf{w})$, i.e.,

$$L\,(\underline{\mathbf{w}}, \mathbf{w}) = \left\{\mathbf{w} \in \mathbb{R}^2 : \mathbf{w} = \alpha \cdot \mathbf{w} + (1-\alpha) \cdot \underline{\mathbf{w}} \text{ for some } \alpha \in (0, 1)\right\} .$$

Then $L\,(\underline{\mathbf{w}}, \mathbf{w}) \subset V_1$. Thus, $V_1 = \lim_{n\to\infty} B^n\,(L\,(\underline{\mathbf{w}}, \mathbf{w}))$. This concludes the proof. □

Lemma 6 states that there exist non-extreme equilibria in which agents stop making transfers after observing histories of non-transfers, capturing the idea that anticipating that the other agent may not transfer the reserve back in the future, the reserve-holding agent is reluctant to transfer the reserve today.

Recall from Propositions 1 and 2 that the good payment equilibrium exists if and only if $\frac{z}{c} \geq \frac{1}{1-\delta(1-\lambda\mu_1)}$ and the bad payment equilibrium exists if and only if $\frac{z}{c} \leq \frac{1}{1-\delta}$. Also note that $\frac{1}{1-\delta(1-\lambda\mu_1)} < \frac{1}{1-\delta}$. For any $\mathbf{x} \in \mathbb{R}^2, \alpha \in \mathbb{R}, W \subset \mathbb{R}^2$, define

$$\alpha \cdot W \triangleq \{\alpha \cdot \mathbf{w} : \mathbf{w} \in W\}$$
$$\mathbf{x} + W \triangleq \{\mathbf{x} + \mathbf{w} : \mathbf{w} \in W\}$$

Denote the stage game payoffs as

$$g(0) = \begin{pmatrix} 0 \\ -c \end{pmatrix}, \quad g(1) = \begin{pmatrix} \lambda\mu_1(z-c) \\ -(1-\lambda\mu_1)c \end{pmatrix}$$

For any $\mathbf{w}^0 \in \mathbb{R}^2$ and $W \subset \mathbb{R}^2$, define

$$Q_a^0(\mathbf{w}^0, W) = \begin{cases} \{\mathbf{w}^1 \in \mathbb{T}(W) : \delta w_1^1 + (1-\delta)(z-c) \leq \delta w_1^0\} & \text{if } a = 0, \\ \{\mathbf{w}^1 \in \mathbb{T}(W) : \delta w_1^1 + (1-\delta)(z-c) \geq \delta w_1^0\} & \text{if } a = 1. \end{cases}$$

Given $\mathbf{w}^0$, the continuation value when no transfer occurs, $Q_a^0(\mathbf{w}^0, W)$ is the set of $\mathbf{w}^1$ in W such that action $a \in 0, 1$ is incentive compatible.

Analogously, define

$$Q_a^1(\mathbf{w}^1, W) = \begin{cases} \{\mathbf{w}^0 \in \mathbb{T}(W) : \delta w_1^1 + (1-\delta)(z-c) \leq \delta w_1^0\} & \text{if } a = 0, \\ \{\mathbf{w}^0 \in \mathbb{T}(W) : \delta w_1^1 + (1-\delta)(z-c) \geq \delta w_1^0\} & \text{if } a = 1. \end{cases}$$

For any $W \subset \mathbb{R}^2$, let $Co(W)$ denotes the convex hull of W, i.e.,

$$Co(W) = \{\mathbf{w} : \exists \mathbf{x}, \mathbf{y} \in W \text{and } \alpha \in [0,1], \text{s.t. } \mathbf{w} = \alpha\mathbf{x} + (1-\alpha)\mathbf{y}\}$$

Modify the definition of $\mu(\mathbf{w}^1, \mathbf{w}^0)$ to allow mixed actions when the reserve-holding agent is indifferent, i.e.,

$$\mu(\mathbf{w}^1, \mathbf{w}^0) = \begin{cases} \{\mu_1\} & \text{if } \delta w_1^1 + (1-\delta)(z-c) > \delta w_1^0, \\ (0, \mu_1) & \text{if } \delta w_1^1 + (1-\delta)(z-c) = \delta w_1^0, \\ \{0\} & \text{if } \delta w_1^1 + (1-\delta)(z-c) < \delta w_1^0. \end{cases}$$

Accordingly, $\phi(\mu(\mathbf{w}^1, \mathbf{w}^0), \mathbf{w}^1, \mathbf{w}^0)$ should be understood as a set, especially when $\mu(\mathbf{w}^1, \mathbf{w}^0)$ is a set. We can now generalize the definition of $B(.)$:

Definition 5. *Let B(.) be a set operator such that for any $W \subset \mathbb{R}^2$,*

$$B(W) = \left\{ \mathbf{w} \in \mathbb{R}^2 : \exists \mathbf{w}^0 \in W \text{ and } \mathbf{w}^1 \in \mathbb{T}(W), \text{ s.t. } \mathbf{w} \in \phi(\mu(\mathbf{w}^1, \mathbf{w}^0), \mathbf{w}^1, \mathbf{w}^0) \right\}$$

Since public randomization is allowed, following the approach in Abreu and Sannikov (2014), we can write $B(W)$ as

$$B(W) = Co(B_0(W) \cup B_1(W)), \tag{C.54}$$

where

$$\begin{aligned} B_a(W) = &(1-\delta)g(a) \\ &+ \delta\left(\left(\bigcup_{\mathbf{w}^0 \in W} ((1-\lambda\mu_a)\mathbf{w}^0 + \lambda\mu_a Q_a^0(\mathbf{w}^0, W))\right)\right. \\ &\left.\bigcup \left(\bigcup_{\mathbf{w}^1 \in \mathbb{T}(W)} (\lambda\mu_a \mathbf{w}^1 + (1-\lambda\mu_a)Q_a^1(\mathbf{w}^1, W))\right)\right) \end{aligned} \tag{C.55}$$

Note that $a \in \{0, 1\}, \mu_0 = 0, \mu_1 \in [0, 1]$. Also, note that any closed convex set can be identified as the convex hull of its extreme points. We will also characterize V_1, by its extreme points. First, by (C.54), for any $W \subset \mathbb{R}^2$, e can characterize $B(W)$ by the extreme points of $B_a(W)$. Since public randomization is allowed, without loss of generality, we focus on closed convex set.

Lemma 7. *Suppose $W \subset \mathbb{R}^2$ is closed and convex. Let $\mathbf{w} \in \mathbf{Ext}(B_a(W))$ and $\mathbf{w} = \psi(\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ for some $\mathbf{w}^1 \in \mathbb{T}(W)$ and $\mathbf{w}^0 \in W$. Then,*

1. *$\mathbf{w}^0 \in \mathbf{Ext}(Q_a^1(\mathbf{w}^1, W))$, $\mathbf{w}^1 \in \mathbf{Ext}(Q_a^0(\mathbf{w}^0, W))$;*

2. *At least one of the following two statements is true: (a) $\mathbf{w}^0 \in \mathbf{Ext}(W)$, (b) $\mathbf{w}^1 \in \mathbf{Ext}(\mathbb{T}(W))$.*

Proof. Since $\mathbf{w}^1$ and $\mathbf{w}^0$ induce action a, $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$ and $\mathbf{w}^0 \in Q_a^1(\mathbf{w}^1, W)$.

1. Suppose $\mathbf{w}^1 \notin \mathbf{Ext}(Q_a^0(\mathbf{w}^0, w))$, then there exist $\mathbf{w}^{1x}, \mathbf{w}^{1y} \in Q_a^0(\mathbf{w}^0, w)$ s.t. $\mathbf{w}^1$ is a strict convex combination of $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$. Then $\mathbf{w} = \phi(\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ is also a strict convex combination of $\phi(\mu_a, \mathbf{w}^{1x}, \mathbf{w}^0)$ and $\phi(\mu_a, \mathbf{w}^{1y}, \mathbf{w}^0)$, which are both in $B_a(W)$, contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$. Thus $\mathbf{w}^1 \in \mathbf{Ext}(Q_a^0(\mathbf{w}^0, W))$. A similar argument can be shown for $\mathbf{w}^0$.

2. Suppose $\mathbf{w}^0 \notin \mathbf{Ext}(W)$ and $\mathbf{w}^1 \notin \mathbf{Ext}\mathbb{T}(W)$.
   Case (1): At least one of $\mathbf{w}^0$ and $\mathbf{w}^1$ is on a vertical boundary of W and $\mathbb{T}(W)$, respectively.

If $\mathbf{w}^0$ is on a vertical boundary of W, then since $\mathbf{w}^0 \notin \mathbf{Ext}(W)$, we can find $\mathbf{w}^{0x}$ and $\mathbf{w}^{0y}$ on that vertical boundary of W, s.t. $\mathbf{w}^{0x} \neq \mathbf{w}^{0y}$ and $\mathbf{w}^0$ is a strict convex combination of $\mathbf{w}^{0x}$ and $\mathbf{w}^{0y}$. Since $\mathbf{w}_1^{0x} = \mathbf{w}_1^{0y} = \mathbf{w}_1^0$ and $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$, we have $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^{0x}, W) \cap Q_a^0(\mathbf{w}^{0y}, W)$. Then $\phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^{0x})$ and $\phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^{0y})$ are both in $B_a(W)$, and $\mathbf{w} = \phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ is a strict convex combination of them, contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$.

If $\mathbf{w}^1$ is on a vertical boundary of $\mathbb{T}(W)$, then since $\mathbf{w}^0 \notin \mathbf{Ext}(\mathbb{T}(W))$, we can find $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$ on that vertical boundary of $\mathbb{T}(W)$, s.t. $\mathbf{w}^{1x} \neq \mathbf{w}^{1y}$ and $\mathbf{w}^1$ is a strict convex combination of $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$. Since $\mathbf{w}_1^{1x} = \mathbf{w}_1^{1y} = \mathbf{w}_1^1$ and $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$, we have $\{\mathbf{w}^{1x}, \mathbf{w}^{1y}\} \subset Q_a^0(\mathbf{w}^{0y}, W)$. Then $\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^0)$ and $\phi(\lambda\mu_a, \mathbf{w}^{1y}, \mathbf{w}^0)$ are both in $B_a(W)$, and $\mathbf{w} = \phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^0)$ is a strict convex combination of them, contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$.

Case (2): Neither $\mathbf{w}^0$ nor $\mathbf{w}^1$ is on a vertical boundary of W and $\mathbb{T}$ respectively. Then, there exist $\mathbf{w}^{0x} \neq \mathbf{w}^{0y}$ in W and $\mathbf{w}^{1x} \neq \mathbf{w}^{1y}$ in $\mathbb{T}(W)$, s.t. $\mathbf{w}^0$ is a strict convex combination of $\mathbf{w}^{0x}$ and $\mathbf{w}^{0y}$ and $\mathbf{w}^1$ is a strict convex combination of $\mathbf{w}^{1x}$ and $\mathbf{w}^{1y}$, and $w_1^{0x} = w_1^0 = w_1^{1x} - w_1^1$, $w_1^{0y} - w_1^0 = w_1^{1y} - w_1^1$. Note that these two equations ensure that (a)$\exists \alpha \in [0,1]$, s.t. $\mathbf{w}^0 = \alpha\mathbf{w}^{0x} + (1-\alpha)\mathbf{w}^{0y}$, $\mathbf{w}^1 = \alpha\mathbf{w}^{1x} + (1-\alpha)\mathbf{w}^{1y}$. Note that this α is common for the combination of both $\mathbf{w}^0$ and $\mathbf{w}^1$; (b) $\mathbf{w}^{1x} \in Q_a^0(\mathbf{w}^{0x}, W)$, $\mathbf{w}^{1y} \in Q_a^0(\mathbf{w}^{0y}, W)$, because $\mathbf{w}^1 \in Q_a^0(\mathbf{w}^0, W)$. Then both $\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^{0x})$ and $\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^{0y})$ belong to $B_a(W)$. Then,

$$\begin{aligned}\mathbf{w} &= \phi(\lambda\mu_a, \mathbf{w}^1, \mathbf{w}^0) \\ &= \alpha\phi(\lambda\mu_a, \mathbf{w}^{1x}, \mathbf{w}^{0x}) + (1-\alpha)\phi(\lambda\mu_a, \mathbf{w}^{1y}, \mathbf{w}^{0y})\end{aligned}$$

contradicting to $\mathbf{w} \in \mathbf{Ext}(B_a(W))$.

This completes the proof. □

Lemma 7 allow us to rewrite (C.55) as

$$\begin{aligned}B_a(W) = &(1-\delta)g(a) \\ &+ \delta Co\left(\bigcup_{\mathbf{w}^0 \in \mathbf{Ext}(W)} \left((1-\lambda\mu_a)\mathbf{w}^0 + \lambda\mu_a \mathbf{Ext}(Q_a^0(\mathbf{w}^0, W))\right)\right. \\ &\cup \left.\left(\bigcup_{\mathbf{w}^1 \in \mathbf{Ext}(\mathbb{T}(W))} \left(\lambda\mu_a \mathbf{w}^1 + (1-\lambda\mu_a)\mathbf{Ext}(Q_a^1(\mathbf{w}^1, W))\right)\right)\right)\end{aligned}$$

which simplifies the characterization below. Define

$$\begin{cases} \mathbf{X}^1 = (0, -c)', \\ \mathbf{X}^2 = \left(\frac{\lambda\mu((1-\delta(1-\lambda\mu))z - c)}{1+\delta(2\lambda\mu - 1)}, \frac{\delta(\lambda\mu)^2 z - (\delta\lambda\mu + (1-\delta)(1-\lambda\mu))c}{1+\delta(2\lambda\mu-1)} \right)', \\ \mathbf{X}^3 = \left(0, -c + \frac{\lambda\mu z(2c - (1-\delta)z)}{c + \delta\lambda\mu z}\right)'. \end{cases}$$

Note that $\mathbf{X}^2$ is the average continuation payoff vector in the good payment equilibrium. If such a equilibrium does not exist, $\mathbf{X}^2$ can be understood as the average continuation payoff vector resulted from the history-independent strategy that the two agents always make transfers upon receive a request. Define $W^* = Co(\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3)$.

Lemma 8. *When* $\frac{1}{1-\delta(1-\lambda\mu_1)} < \frac{z}{c} < \frac{1}{1-\delta}$, $W^* = B(W^*)$.

Proof. We need to show $W^* \subset B(W^*)$ and $B(W^*) \subset W^*$. To show $W^* \subset B(W^*)$, it is sufficient to show $\{\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3\} \subset B(W^*)$. Since $\frac{1}{1-\delta(1-\lambda\mu_1)} < \frac{z}{c} < \frac{1}{1-\delta}$, both $\mathbf{X}^1$ and $\mathbf{X}^2$ are equilibrium payoff vectors. In particular, $\mathbf{X}^1 = \phi(0, \mathbb{T}(\mathbf{X}^1), \mathbf{X}^1)$, $\mathbf{X}^2 = \phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^2)$. Thus $\{\mathbf{X}^1, \mathbf{X}^2\} \subset B(W^*)$.

For $\mathbf{X}^3$, it is straightforward to verify that

$$\begin{pmatrix} -(\delta^{-1}-1)(z-c) \\ \frac{\mathbf{X}^3_2 - \mathbf{X}^2_2}{\mathbf{X}^3_1 - \mathbf{X}^2_1}(c - (\delta^{-1}-1)(z-c)) \end{pmatrix} \in Q^0_1(\mathbf{X}^3, W^*)$$

and

$$\mathbf{X}^3 = \phi\left(\mu_1, \begin{pmatrix} -(\delta^{-1}-1)(z-c) \\ \frac{\mathbf{X}^3_2 - \mathbf{X}^2_2}{\mathbf{X}^3_1 - \mathbf{X}^2_1}(c - (\delta^{-1}-1)(z-c)) \end{pmatrix}, \mathbf{X}^3\right).$$

Thus, $\mathbf{X}^3 \in B(W^*)$.

To show $B(W^*) \subset W^*$, it suffices to show $\mathbf{Ext}(B_1(W^*)) \cup \mathbf{Ext}(B_0(W^*)) \subset W^*$. By Lemma 7, we know that any extreme point of $B(W^)$ can be generated by some $\mathbf{w}^0 \in \mathbf{Ext}(W^*)$ and $\mathbf{w}^1 \in \mathbf{Ext}(Q^0_a(\mathbf{w}^0, W^*))$, or some $\mathbf{w}^1 \in \mathbf{Ext}(\mathbb{T}(W^*))$ and $\mathbf{w}^0 \in \mathbf{Ext}(Q^1_a(\mathbf{w}^1, W^*))$, for some $a \in \{0, 1\}$.

Therefore, we can check all points generated in this way to show that they are all in W^*. We give an example below: let $\mathbf{w}^1 = \mathbb{T}(\mathbf{X}^2)$, then $\mathbf{Ext}(Q^1_1(\mathbf{w}^1, W^*)) = \{\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3\}$, $Q^1_0(\mathbf{w}^1, W^*) = \varnothing$. Then we have three potential extreme points:

1. $\phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^1)$, which equals to $\delta(1-\lambda\mu_1)\mathbf{X}^1 + [1-\delta(1-\lambda\mu_1)]\mathbf{X}^2 \in W^*$.

2. $\phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^2) = \mathbf{X}^2 \in W^*$, because this corresponds to the good payment equilibrium.

3. $\phi(\mu_1, \mathbb{T}(\mathbf{X}^2), \mathbf{X}^3)$, which equals to $[1 - \delta(1 - \lambda\mu_1)]\mathbf{X}^2 + \delta(1 - \lambda\mu_1)\mathbf{X}^3 \in W^*$.

Other potential extreme points of $B(W^*)$ can be checked accordingly. This concludes the proof. $\square$

This concludes the proof of Proposition 12. $\square$