Implications of Asset Market Data for Equilibrium Models of Exchange Rates*

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Abstract

When investors trade home and foreign currency risk-free bonds without any frictions, it is not possible to match the cyclicality of exchange rates (the Backus-Smith puzzle), the deviations from U.I.P. (the Fama puzzle), and the lack of predictability (the Meese-Rogoff puzzle). The bond Euler equations dictate exchange rates that are conditionally countercyclical and appreciate in bad states of the world for home investors. This prediction can be overturned unconditionally only if the deviations from U.I.P. are implausibly large and exchange rates are highly predictable, in violation of Meese-Rogoff. We characterize the wedges in the bond Euler equations that are needed to resolve these puzzles. The inferred wedges are consistent with a home currency bias, convenience yields on home bonds, and/or financial repression.

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1 Introduction

We consider the following four key facts about exchange rates. First, real exchange rates are only weakly positively correlated with relative aggregate consumption growth [Kollmann, 1991, Backus and Smith, 1993]. The home currency tends to depreciate when the home investors experience adverse macro-economic shocks and thus have high marginal utility growth. In other words, exchange rates are weakly pro-cyclical. Second, exchange rates seem disconnected from the other macro variables that should determine them [Obstfeld and Rogoff, 2000]. Third, as documented by Tryon [1979], Hansen and Hodrick [1980] and Fama [1984], interest rate differences do not predict changes in exchange rates with the right sign to enforce the uncovered interest rate parity (U.I.P.). Instead, currency returns are predictable, but exchange rates themselves are not. In order to explain the negative slope coefficients, risk premia have to be extremely volatile [Fama, 1984]. Fourth, other macro variables also fail to predict exchange rates, as shown by Meese and Rogoff [1983]. It is hard to beat a random walk when predicting exchange rates, especially out of sample and at frequencies lower than daily or weekly [see, e.g. Rossi, 2013, for a survey of the literature].[1] Order flow seems to predict exchange rates only at higher frequencies [Evans and Lyons, 2002b].

In a large class of international real business cycle models, as long as investors can trade home and foreign risk-free bonds frictionlessly, bond investors' Euler equations impose strong restrictions on the relationship between exchange rates and marginal utilities. Let $m_{t,t+1}$ and $m^*_{t,t+1}$ denote the home and foreign SDF in log, let r_t and r^*_t denote the home and foreign risk-free rates in log, and let s_t denote the log spot exchange rate in units of foreign currency per dollar. When s_t increases, the home currency appreciates. Then, the four bond Euler equations are given by:

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[\exp(m_{t,t+1} + r_t)\right], \\
1 &= \mathbb{E}_t\left[\exp(m_{t,t+1} - \Delta s_{t+1} + r^*_t)\right], \\
1 &= \mathbb{E}_t\left[\exp(m^*_{t,t+1} + r^*_t)\right], \\
1 &= \mathbb{E}_t\left[\exp(m^*_{t,t+1} + \Delta s_{t+1} + r_t)\right].
\end{aligned}
$$

The first two equations are the Euler equations for the home investor investing in domes-

[1] Recently, some economists have reported more success using non-standard predictors such as a measure of the net foreign asset position [Gourinchas and Rey, 2007], as well as CIP deviations [Jiang, Krishnamurthy, and Lustig, 2021] and global risk measures (e.g., VIX) as proxies for global safe asset demand when forecasting dollar exchange rates, especially since the GFC [Lilley, Maggiori, Neiman, and Schreger, 2022]. More recently, researchers have found some evidence that survey expectations can predict exchange rates at longer horizons [Kremens, Martin, and Varela, 2023].

tic and foreign currency risk-free bonds. The second set of two Euler equations pertains to the foreign investor.

Consider the implications of these equations for the conditional exchange rate cyclicality, which is defined as the covariance between the exchange rate movement Δs_{t+1} and the SDF differential $m_{t,t+1} - m^*_{t,t+1}$. Building on the work by Lustig and Verdelhan [2019], we obtain a stark result: the conditional covariance is always positive, which means that the exchange rate has to be conditionally counter-cyclical to enforce these bond Euler equations. In complete markets, the reason for this is well understood. When domestic investors have a higher marginal willingness than foreign investors to pay for consumption in some state tomorrow, i.e. to save into that state, then the state-contingent interest rate is correspondingly lower at home, and the real exchange rate has to appreciate in that state to keep arbitrageurs from borrowing domestically and investing abroad in that state of the world. The exchange rate of the home currency has to appreciate to keep the state prices at home and abroad aligned state-by-state. When investors have power utility, this state-contingent version of interest rate parity induces a perfectly negative correlation between the exchange rate and the aggregate consumption growth differential. This result carries over to the incomplete market setting. As long as investors can trade home and foreign risk-free bonds frictionlessly, an "average" version of the complete markets state-contingent interest rate parity prediction survives [Lustig and Verdelhan, 2019].

Next, we consider the unconditional exchange rate cyclicality, which is the evidence in Kollmann [1991] and Backus and Smith [1993].[2] We show that in order to generate unconditionally pro-cyclical exchange rates, stylized fact 1, we need very volatile forward premia or highly predictable exchange rates. The first condition is at odds with the Tryon [1979], Hansen and Hodrick [1980], and Fama [1984] evidence on the violation of U.I.P, stylized fact 3. The second condition is at odds with the Meese and Rogoff [1983] evidence, stylized fact 4.

We thus end up with an impossibility result: when investors can trade domestic and foreign currency bonds, we cannot generate pro-cyclical exchange rates (stylized fact 1) while matching the observed U.I.P. deviations (stylized fact 3) and the lack of exchange rate predictability (stylized fact 4). These financial market facts are tied together through the bond investor Euler equations, implying a close relationship between exchange rate cyclicality and predictability even though they have been largely studied in isolation in the literature.

Turning to a resolution, we analytically characterize the wedges in cross-currency

[2] The unconditional cyclicality is equal to the mean of the conditional exchange rate cyclicality plus a term that captures the extent to which exchange rates can be predicted by the expected SDF differential.

bond Euler equations needed to simultaneously address all of these facts. We offer four interpretations of these wedges. First, investors act as if they face large (perceived or actual) transaction costs associated with buying bonds denominated in a foreign currency, consistent with a large body of empirical evidence [Lewis, 1995a]. Recently, Maggiori, Neiman, and Schreger [2020] report strong evidence of a home currency bias in international mutual fund holdings of corporate bonds.[3]

Second, home investors derive large convenience yields on their home bonds, which can arise for example because home bonds function as private liquidity.[4] Diamond and Van Tassel [2021] show that government bonds around the world carry convenience yields.

Third, home investors are forced to hold home bonds, e.g., due to financial repression. Governments routinely adopt measures to allow themselves to borrow at below-market rates. This is usually referred to as financial repression [see Reinhart, Kirkegaard, and Sbrancia, 2011, Chari, Dovis, and Kehoe, 2020]. An example would be financial regulation that requires some participants to hold home bonds.

Fourth, in models of frictional financial intermediation, the wedges can also be interpreted as the shadow cost of participating via intermediaries in foreign markets. Gabaix and Maggiori [2015], Itskhoki and Mukhin [2021], Fukui, Nakamura, and Steinsson [2023] all consider models in which investors cannot directly access currency markets.[5] Viewed through the lens of these models, our wedges measure the indirect costs of accessing foreign bond markets through intermediaries.

When we calibrate the exchange rate predictability, volatility, the Fama regression coefficient, and the SDF variance to match the data, the cross-currency bond Euler equation wedges have to be at least 38 basis points. Thus, a quantitatively small departure from frictionless markets, either in the form of transaction costs, convenience yields, or intermediation frictions, can account for the facts, thereby ruling in this plausible class of models as a resolution of the exchange rate puzzles. Surprisingly, the key to generating

[3]The only exception is the dollar, the international reserve currency. Investors are willing to buy dollar-denominated bonds. Maggiori et al. [2020] attribute this home currency bias to the costs of currency hedges and behavioral bias.

[4]There is a growing literature on convenience yields in bond markets, starting with Krishnamurthy and Vissing-Jorgensen [2012]

[5]In these models, domestic investors only invest in the domestic bond market. Only the intermediaries can trade foreign currencies. Similarly, Gourinchas, Ray, and Vayanos [2020], Greenwood, Hanson, Stein, and Sunderam [2020] study models with domestic preferred-habitat investors and global arbitrageurs. This class of models keep only two of the Euler equations we consider and add a third Euler equation that captures the global arbitrageurs' long-short portfolio decision over a carry trade return. The approach of these intermediary-centered models are complementary to ours. The approach implies an infinite transaction cost for investing in the foreign currency as domestic investors are disallowed from investing in foreign currency. This is clearly extreme. Our approach provides the minimum bounds needed on investors' Euler equation wedges needed to address exchange rate facts.

pro-cyclical exchange rates is the cross-currency bond Euler equation wedge's level, as opposed to its covariance with the SDF. In fact, a constant Euler equation wedge, which may be interpreted as a constant transaction cost for international investments, is enough to address the exchange rate puzzles. The wedge's first moment directly enters the Euler equations, so that even a constant wedge triggers an endogenous adjustment in the exchange rate cyclicality.[6]

These results clarify the role of frictions in equilibrium models of exchange rates. Lustig and Verdelhan [2019] ask whether market incompleteness helps to resolve outstanding currency puzzles, and they find that it does not. Our paper enriches the results in their paper by additionally considering the implications of the bond Euler equations for the unconditional exchange rate cyclicality and exchange rate predictability. The key ingredient needed to make progress on the exchange rate disconnect and Backus-Smith puzzles is not market incompleteness, but rather frictions that give rise to wedges in the cross-country bond Euler equations.

Likewise, currency risk premium shocks that drive U.I.P. deviations do not resolve the puzzle. In the literature, both risk premium shocks and cross-currency bond Euler equation wedges are often referred to collectively as U.I.P. or financial shocks [see, e.g., Farhi and Werning, 2014, Itskhoki and Mukhin, 2021]. We show that as long as all of the cross-currency bond Euler equations hold without wedges, the Backus-Smith puzzle reappears. Financial shocks that only drive risk premium variations cannot overturn this result unless the Euler equations are violated.

In closely related work, Chernov, Haddad, and Itskhoki [2023b] develop a framework that maps the space of tradable assets to restrictions on the exchange rate moments. They conclude that the exchange rate moments require the asset markets to be segmented or intermediated, and, in this environment, the local financial markets are uninformative about the exchange rate. Consistent with this result, we find that wedges in these Euler equations are required to match the moments of exchange rates, but we show how financial markets can be informative about exchange rates by disciplining the properties of these Euler equation wedges which we ultimately interpret as home bias, domestic convenience yields, or intermediation frictions.

Literature. Chari, Kehoe, and McGrattan [2002] analyze a complete-market model of

[6]The level of the wedges matters because addressing the four facts requires a model to disconnect the relation between relative marginal utility growth ($m_{t,t+1} - m^*_{t,t+1}$) and exchange rate changes Δs_{t+1}. Without wedges, the U.I.P. conditions tie these objects together. With an Euler equation wedge, it is possible to satisfy the U.I.P. conditions while disconnecting the covariance between marginal utility growth and exchange rate changes.

exchange rates with sticky prices and identify the Backus-Smith puzzle as the key failure of their model. Corsetti, Dedola, and Leduc [2008], Pavlova and Rigobon [2012] consider incomplete market models of exchange rates. In their model, domestic and foreign investors only invest in a risk-free bond that pays off in a global numéraire, implicitly dropping all 4 Euler equations. They report progress on the Backus-Smith puzzle.

A different strand of the literature segments the markets and thus introduces wedges into the bond Euler equations. Alvarez, Atkeson, and Kehoe [2002a, 2009a] consider a Baumol-Tobin style model in which investors pay a cost to transact in currency and bond markets. Relatedly, Jiang, Krishnamurthy, and Lustig [2018], Jiang, Krishnamurthy, Lustig, and Sun [2024] explore the dollar exchange rate implications of convenience yields earned on dollar safe assets, another type of the Euler equation wedges. However, these convenience yields on dollar-denominated assets do not help to resolve the bilateral exchange rate puzzles. Finally, a third strand of the literature, starting with the seminal work by Gabaix and Maggiori [2015], Itskhoki and Mukhin [2021], imputes a central role to financial intermediaries, drawing on insights from the literature on intermediary asset pricing.

Other recent work by Hassan [2013], Dou and Verdelhan [2015], Chien, Lustig, and Naknoi [2020], Jiang and Richmond [2023] takes a different tack by introducing heterogeneity in household trading technologies. Active households can freely trade bonds and other state contingent claims, whereas the inactive households have no access to the asset market. In this case, while the four Euler equations we consider in this paper hold for the active households without any wedges, their marginal utilities have different cyclicality than the country-level aggregate marginal utilities. As a result, the model disconnects aggregate consumption from the SDF of the Euler equation to which the model applies.

The paper is organized as follows. We start by discussing the benchmark complete-market case in section 2. Next, section 3 discuss the conditional Backus-Smith puzzle in the incomplete-market case. Section 4 analyzes the unconditional Backus-Smith puzzle in the incomplete-market case. Finally, section 5 inserts bond Euler equation wedges, and characterizes the restrictions these wedges need to satisfy in order to make progress on the unconditional Backus-Smith puzzle. Lastly, the Appendix contains the proof of the propositions.

2 Complete Markets and Exchange Rate Puzzles

In the case of complete markets, exchange rates act as shock absorbers for the shocks to the pricing kernels:

$$\Delta s_{t+1} = m_{t+1} - m^*_{t+1}.$$

This expression for the log change in the real exchange rate has puzzling implications.

Volatility puzzle. As was noted by Brandt, Cochrane, and Santa-Clara [2006b], the implied volatility of the exchange rate will be too high if the market price of risk clears the Hansen-Jagannathan bounds, unless the pricing kernels are highly correlated across countries.

$$var_t(\Delta s_{t+1}) = var_t(m^*_{t+1}) + var(m_{t+1}) - 2corr_t(m_{t+1}, m^*_{t+1}) std_t(m_{t+1}) std_t(m^*_{t+1}).$$

We would need a correlation of the pricing kernels $corr_t(m_{t+1}, m^*_{t+1})$ close to one. In the case of the standard Breeden-Lucas SDF $m_{t+1} = \log\delta - \gamma\Delta c_{t+1}$, this would imply close to perfectly correlated consumption growth across countries: $\rho_t(\Delta c_{t+1}, \Delta c^*_{t+1})$. This prediction is counterfactual [see Backus, Kehoe, and Kydland, 1992].

Counter-cyclicality/Backus-Smith puzzle. When markets are complete, the unconditional exchange rate cyclicality, which we define as the covariance between the exchange rate movement Δs_{t+1} and the SDF differential $m_{t,t+1} - m^*_{t,t+1}$, satisfies

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var(\Delta s_{t+1}) > 0.$$

We obtain a very general result: in any complete-market models, the unconditional exchange rate cyclicality is always positive because a higher marginal utility growth in the home country is associated with a home currency appreciation. The model can only generate exchange rate disconnect by shrinking the variance of the exchange rate to zero. In the Breeden-Lucas case, the implied changes in the log exchange rates are perfectly negatively correlated with consumption growth differences $corr(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta s_{t+1}) = -1$, which is strongly counterfactual [Kollmann, 1991, Backus and Smith, 1993].

These puzzles are partially governed by the specific nature of the pricing kernel. The Breeden-Lucas SDF assumes time-additive utility. Colacito and Croce [2011] impute a preference for early resolution to uncertainty to the stand-in investor in an endowment economy with long-run risks [Bansal and Yaron, 2004a]. In this long-run risks economy, it is feasible to make progress on the volatility puzzle by choosing highly correlated persistent components of consumption growth, while still matching the low correlation of consumption growth observed in the data. The long-run risks can push the correlation of the pricing kernels $corr_t(m_{t+1}, m^*_{t+1})$ to one by choosing perfectly correlated long-run consumption growth $corr_t(x_{t+1}, x^*_{t+1}) = 1$. However, in their benchmark calibration, this mechanism reduces the exchange rate cyclicality to zero, but does not overturn the sign. In closely related work, Verdelhan [2010] explores the habit model's exchange rate implications, and concludes that this model cannot entirely resolve the Backus-Smith puzzle. Next, we examine the exchange rate cyclicality when we shut down some asset markets.

3 Conditional Exchange Rate Cyclicality and Incomplete Markets

We start by assuming that investors can invest in risk-free bonds denominated in domestic and foreign currency. Our analysis is silent on the rest of the market structure.

We assume that the exchange rate and pricing kernel innovations are jointly normally distributed. Then, the four risk-free bond Euler equations imply:

$$
\begin{aligned}
0 &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) + r_t, \\
0 &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m_{t,t+1}, -\Delta s_{t+1}) + r^*_t, \\
0 &= \mathbb{E}_t[m^*_{t,t+1}] + \frac{1}{2}var_t(m^*_{t,t+1}) + r^*_t, \\
0 &= \mathbb{E}_t[m^*_{t,t+1}] + \frac{1}{2}var_t(m^*_{t,t+1}) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m^*_{t,t+1}, \Delta s_{t+1}) + r_t.
\end{aligned}
$$

Reorganizing the terms, we can obtain two expressions that relate the expected excess return of a strategy that goes long in foreign currency and borrows at the domestic risk-free rate to the riskiness of the exchange rate

$$
\begin{aligned}
(r^*_t - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}, -\Delta s_{t+1}), \\
(r_t - r^*_t) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m^*_{t,t+1}, \Delta s_{t+1}).
\end{aligned}
$$

The first expression takes the home investors' perspective. If the foreign currency tends to appreciate (i.e., higher $-\Delta s_{t+1}$) in the home investors' high marginal utility states (i.e., higher $m_{t,t+1}$), then, the foreign currency is a good hedge and the home investors demand a lower expected return to hold it, which leads to a lower $(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1})$ on the left-hand side. Similarly, the second expression takes the foreign investors' perspective, and relates the currency excess return to the covariance between the foreign investors' SDF and the exchange rate movement.

Given the exchange rate variance $var_t(\Delta s_{t+1})$ is positive, these expressions imply

$$\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^* > cov_t(m_{t,t+1}, -\Delta s_{t+1}), \quad (1)$$

$$-(\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^*) > cov_t(m_{t,t+1}^*, \Delta s_{t+1}). \quad (2)$$

We sum (1) and (2) to obtain:

Proposition 1. *In the log-normal case, the conditional exchange rate cyclicality satisfies*

$$cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0. \quad (3)$$

Exchange rate innovations are conditionally positively correlated with marginal utility growth, or equivalently, the home exchange rate depreciates in good states for the home investor. This is a conditional version of the Backus-Smith puzzle. Lustig and Verdelhan [2019] derive a more restrictive version of this result assuming incomplete market wedges that are jointly log-normal with the SDF and the exchange rate. Our derivation does not use incomplete market wedges.[7]

Proposition 1 is a strong restriction, and it is striking that it arises from only four simple bond Euler equations. The pairs of bond Euler equations each define a carry trade return. The expected carry trade return, if it is non-zero, has to be a compensation for covariance risk with SDFs of both home and foreign households. It is this required compensation that underlies the result: if the carry return is compensation for covariance risk, then the covariance of the relative pricing kernels with the exchange rate change has to be positive. We do not need to consider the Euler equations for any more traded assets to reach this stark conclusion, as for example, are considered in Chernov et al. [2023b]. The Euler equations on investments in two short-term bonds are sufficiently informative to yield this strong result.

[7] While Lustig and Verdelhan [2019] find that market incompleteness helps with the Brandt, Cochrane, and Santa-Clara [2006a] puzzle in reducing volatility, it cannot change the sign of the conditional covariance. We have not assumed that markets are complete to derive this result. In the case of complete markets, state-contingent interest rate parity obtains $m_{t,t+1} - m_{t,t+1}^* = \Delta s_{t+1}$ and this covariance result is directly obtained. Our proposition shows that this result is much more general, as long as investors can freely trade home and foreign risk-free bonds. In other words, the risk-free bond Euler equations discipline the joint dynamics of the exchange rates and marginal utility growth to imply counter-cyclical exchange rates.

The result also calls into question the approach of introducing "risk premium shocks" that drive U.I.P. deviations into international finance models. In the literature, both risk premium shocks and wedges that represent deviations from cross-currency bond Euler equations are often referred to collectively as U.I.P. or financial shocks [see, e.g., Farhi and Werning, 2014, Itskhoki and Mukhin, 2021]. However, we see that accounting for risk explicitly and tying risk premia to covarance with the SDF implies a strong restriction such that the exchange rate cyclicality puzzle reappears. Financial shocks that only drive risk premium variations cannot overturn this result

Finally, the SDF and exchange rate dynamics might not be conditionally Gaussian, e.g., as in Rietz [1988], Longstaff and Piazzesi [2004], Barro [2006], Farhi and Gabaix [2016]. We can extend our results to non-Gaussian settings and we present the details in Appendix A.5.

4 Unconditional Exchange Rate Cyclicality and Incomplete Markets

In IRBC models that link the SDFs to aggregate consumption shocks, we are interested in understanding how the exchange rate moves in response to relative consumption growth. Backus and Smith [1993] summarize this relationship by regressing the exchange rate movement on relative consumption growth, and find pro-cyclical exchange rates. To relate our result to this Backus-Smith coefficient, we need to characterize the unconditional exchange rate cyclicality. To do so, we use the law of total covariance:

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = \mathbb{E}[cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})] + cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]).$$

Our previous result shows the conditional exchange rate cyclicality $cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})$ is always positive. To generate a negative unconditional cyclicality $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})$, we need a negative $cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}])$ that is greater in magnitude than the average conditional cyclicality $\mathbb{E}[cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})]$. In other words, the exchange rate movement needs to be strongly predictable by the expected SDF differential $\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}]$. In this way, this equation connects the unconditional exchange rate cyclicality to exchange rate predictability.

Recall the case of complete markets, $\Delta s_{t+1} = m_{t,t+1} - m^*_{t,t+1}$. The exchange rate has to absorb all of the shocks to the pricing kernels in each state of the world. When markets are complete, the unconditional exchange rate cyclicality satisfies

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var(\Delta s_{t+1}) > 0.$$

Thus, in any complete-market models (which by definition allows home and foreign agents to trade risk-free bonds), the unconditional exchange rate cyclicality is always positive: a higher marginal utility growth in the home country is associated with a home currency appreciation. The model can only generate exchange rate disconnect by shrinking the variance of the exchange rate to zero.

Next, we consider a more general case of less than complete markets in which we shut down some trade in non-bond asset markets. We begin by introducing some concepts.

The Fama Regression Coefficient b. We use f_t to denote the log of the one-period forward exchange rate in units of foreign currency per dollar. The log excess return on buying foreign currency forward is

$$rx_{t+1} = f_t - s_{t+1} = -\Delta s_{t+1} + f_t - s_t,$$

where $f_t - s_t$ denotes the forward discount and Δs_{t+1} denotes the appreciation of the home currency. When the Covered Interest Rate Parity holds, we further obtain $f_t - s_t = r^*_t - r_t$ and, as a result, we can restate the log excess return on a long position in foreign currency as $rx_{t+1} = -\Delta s_{t+1} + r^*_t - r_t$.

Now, consider the standard Tryon [1979], Hansen and Hodrick [1980], Fama [1984] time-series regression:

$$\Delta s_{t+1} = a + b(f_t - s_t) + \varepsilon_{t+1}.$$

In the data, the slope coefficient b tends to be negative: a higher-than-usual foreign interest rate predicts further appreciation of the foreign currency. Following Fama [1984], we use

$$p_t = \mathbb{E}_t[rx_{t+1}] = f_t - \mathbb{E}_t[s_{t+1}]$$

to denote the currency risk premium, and

$$q_t = \mathbb{E}_t[\Delta s_{t+1}]$$

to denote the expected exchange rate movement. The forward discount can be decomposed as $f_t - s_t = p_t + q_t$. When the Covered Interest Rate Parity holds, $p_t = r_t^* - r_t - \mathbb{E}_t[\Delta s_{t+1}]$, and $p_t + q_t = r_t^* - r_t$.

As shown by Fama, the slope coefficient in this regression can be restated as:

$$\frac{cov(f_t - s_t, \mathbb{E}_t[\Delta s_{t+1}])}{var(f_t - s_t)} = \frac{cov(p_t + q_t, q_t)}{var(p_t + q_t)} = \frac{cov(p_t, q_t) + var(q_t)}{var(p_t + q_t)}.$$

To get negative slope coefficient b, we need $cov(p_t, q_t)/var(q_t) < -1$. Two necessary conditions have to be satisfied in order to obtain negative slope coefficients: $corr(p_t, q_t) < 0$ and $std(p_t) > std(q_t)$. Risk premia have to be more volatile than the expected change in the spot rate. Backus, Foresi, and Telmer [2001] analyze sufficient conditions for these U.I.P. violations in a large class of affine asset pricing models.

The Meese-Rogoff R^2. The Meese and Rogoff [1983] puzzle states that exchange rates are hard to forecast. Put differently, the $R^2 = var(\mathbb{E}_t[\Delta s_{t+1}])/var(\Delta s_{t+1})$ in a forecasting regression is low. In a recent survey of exchange rate predictability, Rossi [2013] concludes that the Meese-Rogoff findings have not been conclusively overturned.[8] There is some limited evidence of exchange rate predictability but the evidence usually is specific to certain countries, horizons and the predictability is not stable.

When the linear projection yields the best forecast, we can obtain this R^2 from a projection of the exchange rate changes on its predictors. We will assume the linear predictor yields the best forecast. Let R^2 denote the fraction of the predictable variation in the exchange rate:

$$R^2 = \frac{var(\mathbb{E}_t[\Delta s_{t+1}])}{var(\Delta s_{t+1})},$$

and let R^2_{Fama} denote the R^2 of the Fama regression:

$$R^2_{Fama} = \frac{var(b(f_t - s_t))}{var(\Delta s_{t+1})}.$$

Our main result characterizes the unconditional exchange rate cyclicality. Without loss of generality, we assume that the covariance between the home SDF's conditional variance and the exchange rate movement from the home perspective is higher than the

[8] At higher frequencies ranging from one day to one month, order flow seems to predict changes in the spot exchange rate [see Evans and Lyons, 2002a, 2005]. This data is proprietary and may not be available in real-time to all investors. Gourinchas and Rey [2007] report evidence that the net foreign asset position predicts changes in the exchange rate out-of-sample.

covariance between the foreign SDF's conditional variance and the exchange rate movement from the foreign perspective:

$$cov\left(var_t(m_{t,t+1}), \mathbb{E}_t[\Delta s_{t+1}]\right) \geq cov\left(var_t(m^*_{t,t+1}), -\mathbb{E}_t[\Delta s_{t+1}]\right).$$

If this condition is not satisfied, we simply need to swap the labeling of the home and foreign countries.

Proposition 2. *Each of the following is a necessary condition for a negative unconditional exchange rate cyclicality, i.e.,* $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$*:*

(a)

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}} + \sqrt{R^2}\left(\frac{1}{b}\frac{R^2_{Fama}}{R^2} - 1\right). \quad (4)$$

If the Fama regression yields the best predictor of the exchange rate movement, then, we can simplify this formula to

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}} + \sqrt{R^2}\left(\frac{1}{b} - 1\right) = \frac{1}{\sqrt{R^2}} - \sqrt{R^2} + sign(b)\frac{std(f_t - s_t)}{std(\Delta s_{t+1})}. \quad (5)$$

(b)

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t+1}])}{std(\mathbb{E}_t[\Delta s_{t+1}])} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{std(\Delta s_{t+1})}{std(\mathbb{E}_t[\Delta s_{t+1}])} = \frac{1}{\sqrt{R^2}}.$$

Between these conditions,

- *(a)* $\Rightarrow$ *(b).*
- *If Fama regression yields the best predictor and* $b \notin (0,1)$*, (b)* $\Rightarrow$ *(a); otherwise (b) is a weaker condition.*

Conditions (a) and (b) are necessary, but not sufficient conditions for a negative unconditional exchange rate cyclicality. The bounds tighten as the R^2 decreases: as exchange rates become less predictable, we need more variations in the conditional risk premia and the conditional price of risk to generate a negative unconditional exchange rate cyclicality. In the limit, as we approach the Meese and Rogoff [1983]'s benchmark random walk case in which exchange rate movements are not predictable, $1/\sqrt{R^2}$ on the right-hand side approaches infinity. The model simply cannot deliver pro-cyclical exchange rates. As such, these bounds deliver an impossibility result: if we take Meese and Rogoff [1983] random walk result regarding *exchange rate predictability* at face value, then we cannot make progress on the Backus and Smith [1993] puzzle regarding *exchange rate cyclicality*.

On the other hand, while it is enticing to conclude that, holding R^2 constant, a small but negative Fama coefficient b can lower the right-hand side of Eq. (4) and make this condition more likely to hold, we note that $\sqrt{R^2}$ and b are closely related. When the b shrinks towards zero, the R^2 shrinks towards zero as well. In fact, Eq. (5) shows that, holding R^2 constant, a small but negative b means a high forward premium volatility $std(f_t - s_t)$ relative to the exchange rate volatility $std(\Delta s_{t+1})$, which is also rejected by the data.

Before we turn to more realistic cases, we make two more observations. First, in the case of U.I.P., $b = 1$ and Eq. (5) becomes:

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}}.$$

A natural case to consider under the U.I.P. is the case of constant market prices of risk. Then, we get an impossibility result: $0 \geq 1/\sqrt{R^2}$, so the unconditional exchange rate cyclicality cannot be negative.

Second, in the case of a fully predictable exchange rate movement, the R^2 tends to one and Eq. (5) becomes:

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{b}.$$

As long as the slope coefficient b is negative, then the bound is trivially satisfied, even when the R^2 is very high but not equal to 1.

4.1 Calibrated Examples

Let us compute bounds using some empirically plausible values. In the data, the exchange rate movements are only moderately predictable. Suppose the Fama regression yields the best predictor of the exchange rate movement, $R^2 = 5\%$ at the one-year horizon, the Fama regression coefficient is $b = -1$, and the annualized exchange rate volatility is $std(\Delta s_{t+1}) = 10\%$. Then, Eq. (5) implies that the log SDF needs to have a very high variability in its conditional variance:

$$std\left(var_t(m_{t,t+1})\right) \geq 0.4. \tag{6}$$

For comparison, the unconditional standard deviation of the log SDF variance is only

0.067 in the long-run risk model in Bansal and Yaron [2004b], and 0.26 in the external habit model in Campbell and Cochrane [1999], even though both models manage to generate empirically plausible levels of the equity risk premium. Figure (1) plots the distributions of the conditional SDF volatility in these two models, which are related to the maximum Sharpe ratio in the economy according to the Hansen and Jagannathan [1991] bound. In the long-run risk model, the conditional log SDF volatility is between 0.3 and 0.7, a narrow range that gives rise to a small $std\left(var_t(m_{t,t+1})\right)$. In the external habit model, the conditional log SDF volatility is more variable, at the expense of having some states in which the SDF is not very volatile and the maximum Sharpe ratio is very small, and some states in which the SDF is extremely volatile and the maximum Sharpe ratio is very high.

If we take either model as a quantitatively accurate representation of the SDF, then, Eq. (6) implies that the model cannot generate a negative unconditional exchange rate cyclicality. In other words, the home and foreign investors' Euler equations governing their risk-free bond holdings impose important restrictions on the equilibrium exchange rate dynamics, such that a negative unconditional exchange rate cyclicality requires a very volatile SDF conditional variance that is not in line with standard asset pricing models.

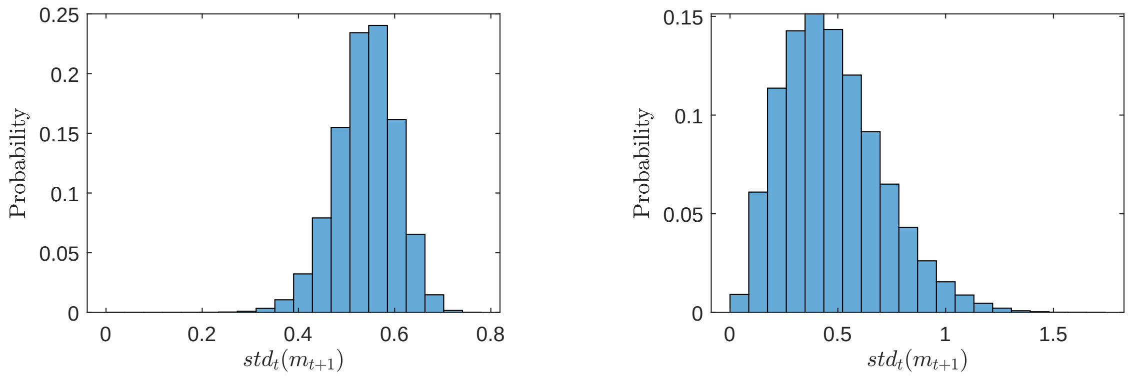
Alternatively, suppose that the forward premium has no predictive power for the exchange rate movement, which implies $R^2_{Fama} = 0$ and $b = 0$. Suppose some other predictor generates an $R^2 = 5\%$ and the annualized exchange rate volatility is $std(\Delta s_{t+1}) = 10\%$. Then, Eq. (4) implies

$$std\left(var_t(m_{t,t+1})\right) \geq std(\Delta s_{t+1})\left(\frac{1}{\sqrt{R^2}} - \sqrt{R^2}\right) = 0.42,$$

which provides a similar bound as in the previous case.

We can also generalize the bound in Eq. (6) to a wider range of exchange rate predictability. In Figure (2), we vary the R^2 from the Fama regression and report the bound on the volatility of the conditional log SDF variance. As the exchange rate predictability increases, the lower bound on the unconditional volatility of the conditional log SDF variance declines. When R^2 is 20% in the Fama regression, the lower bound becomes 0.13, compared to 0.40 when R^2 is 5%. In other words, as the exchange rate becomes more predictable, the Euler equations can rationalize a negative unconditional exchange rate cyclicality without requiring a higher degree of variability in the conditional SDF variance.

Finally, Proposition 2 offers a loose bound in the sense that we used the property $corr\left(var_t(m_{t,t+1}), \mathbb{E}_t[\Delta s_{t+1}]\right) \leq 1$ to derive the inequality, just like Hansen and Jagannathan [1991], which makes the bounds in this proposition necessary but not sufficient



(A) Long-Run Risk Model (B) External Habit Model

Figure 1. Distributions of the Conditional Log SDF Volatility in Different Asset Pricing Models

Notes: The figure plots the distributions of the conditional log SDF volatility in the long-run risk model in Bansal and Yaron [2004b] and the external habit model in Campbell and Cochrane [1999]. We simulate 1,000,000 monthly periods in each model and report the histogram of the annualized log SDF volatility.

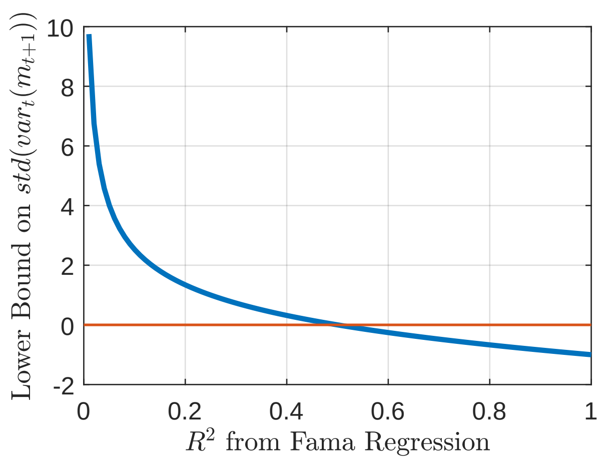


Figure 2. Lower Bound on the Variability of the Conditional Log SDF Variance as a Function of Exchange Rate Predictability

Notes: The figure plots the lower bound on $std\left(var_t(m_{t,t+1})\right)$ as a function of the Fama regression R^2 as implied by Eq. (5). We take $b = -1$ and $std(\Delta s_{t+1}) = 10\%$.

conditions. The bounds are only sufficient conditions when $corr\,(var_t(m_{t,t+1}), \mathbb{E}_t[\Delta s_{t+1}]) = 1$. If, instead, the correlation is 0.5, Eq. (6) in the numerical example becomes $std\,(var_t(m_{t,t+1})) \geq 0.4/(1/2) = 0.8$, which further sharpens the result by doubling the lower bound on $std\,(var_t(m_{t,t+1}))$.

4.2 Role of the Horizon

We consider a Fama regression with horizon k periods:

$$\Delta s_{t,t+k} = a_k + b_k(f_t^k - s_t) + \varepsilon_{t+k}.$$

Similarly, we use $R^2_{Fama,k}$ to denote the R^2 of this regression, and we define

$$R_k^2 = \frac{var(\mathbb{E}_t[\Delta s_{t,t+k}])}{var(\Delta s_{t,t+k})}.$$

Proposition 3. *Each of the following is a necessary condition for a negative unconditional exchange rate cyclicality, i.e., $cov(m_{t,t+k} - m^*_{t,t+k}, \Delta s_{t,t+k}) < 0$:*

(a)

$$\frac{std\,(var_t(m_{t,t+k}))}{std(\Delta s_{t,t+k})} \geq \frac{1}{\sqrt{R_k^2}} + \sqrt{R_k^2}\left(\frac{1}{b_k}\frac{R^2_{Fama,k}}{R_k^2} - 1\right).$$

If the Fama regression yields the best predictor of the exchange rate movement, then, we can simplify this formula to

$$\frac{std\,(var_t(m_{t,t+k}))}{std(\Delta s_{t,t+k})} \geq \frac{1}{\sqrt{R_k^2}} + \sqrt{R_k^2}\left(\frac{1}{b_k} - 1\right) = \frac{1}{\sqrt{R_k^2}} - \sqrt{R_k^2} + sign(b_k)\frac{std(f_t^k - s_t)}{std(\Delta s_{t,t+k})}$$

(b)

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t,t+k}])}{std(\mathbb{E}_t[\Delta s_{t,t+k}])} + \frac{std\,(var_t(m_{t,t+k}))}{std(\mathbb{E}_t[\Delta s_{t,t+k}])}} \geq \frac{std(\Delta s_{t,t+k})}{std(\mathbb{E}_t[\Delta s_{t,t+k}])} = \frac{1}{\sqrt{R_k^2}}.$$

Among these conditions,

- *(a) $\Rightarrow$ (b).*
- *If Fama regression yields the best predictor and $b \notin (0,1)$, (b) $\Rightarrow$ (a); otherwise (b) is a weaker condition.*

As we increase the horizon, $std\left(var_t(m_{t,t+k})\right)$ increases faster than $\sqrt{k}$, while $std(\mathbb{E}_t[\Delta s_{t,t+k}])$ converges to zero if a long-run version of PPP holds and the real exchange rate is stationary.[9] On the other hands, as $k \to \infty$, a long-run version of U.I.P kicks in and $b_k \to 1$. Lustig, Stathopoulos, and Verdelhan [2019] show that long-run U.I.P. is implied by no arbitrage when real exchange rates are stationary. Finally, there is evidence that exchange rates are more predictable as k rises, indicating that R_k^2 rises. Thus, as the horizon increases, the bound is easier to satisfy. We return to this observation in the next section on wedges.

5 Bond Euler Equation Wedges

Proposition 1 shows that IRBC models with the four bond Euler equations cannot simultaneously generate a negative Backus-Smith coefficient and replicate the Fama regression coefficient and the Meese-Rogoff puzzle. As a result, we need to entertain models that break these four Euler equations in one way or another.

One approach to doing so is in Corsetti et al. [2008], Pavlova and Rigobon [2012]. These papers consider incomplete-market settings in which only one type of bond is traded. When the bond is denominated in a given country's numéraire, this set-up drops two of the four Euler equations we consider. When the bond is denominated in a basket of country-level numéraires, this set-up drops all of our four Euler equations and replaces them with two new ones. Investors around the world clearly do trade bonds in different currencies, so that we will not entertain this possibility as a resolution to the puzzle.

We instead outline three other approaches that can work: introducing convenience yields on home bonds, introducing transaction costs when investing in foreign bonds, and introducing financial intermediary frictions. We study each of these cases next and explain how they implicitly insert cross-currency Euler equation wedges into the model we have considered.

5.1 Home Currency Bias, Convenience Yields, and Financial Repression

We break the four Euler equations by considering the case where investors have an extra preference towards purchasing the bonds in their own currency. We denote these home wedges for the domestic and foreign investor respectively as (ϕ_t, ϕ_t^*). Then, the four Euler

[9] As noted by Rogoff [1996], the real exchange rate's rate of convergence to its long-run mean is slow.

equations can be expressed as follows:

$$\begin{aligned}\exp(\phi_t) &= \mathbb{E}_t\left[\exp(m_{t,t+1} + r_t)\right], \\ 1 &= \mathbb{E}_t\left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right], \\ \exp(\phi_t^*) &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + r_t^*)\right], \\ 1 &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + \Delta s_{t+1} + r_t)\right].\end{aligned} \tag{7}$$

Assuming log-normality, we obtain two expressions that relate the expected currency excess return to the perceived risks from the home and foreign perspectives as well as the wedges:

$$\begin{aligned}(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}, -\Delta s_{t+1}) - \phi_t, \\ (r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}^*, \Delta s_{t+1}) - \phi_t^*.\end{aligned}$$

Combining these equations yields the following characterization of the conditional exchange rate cyclicality:

Proposition 4. *In the presence of Euler equation wedges, the conditional exchange rate cyclicality is given by:*

$$cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) + (\phi_t + \phi_t^*),$$

and the conditional correlation is given by:

$$corr_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = \frac{std_t(\Delta s_{t+1})}{std_t(m_{t,t+1} - m_{t,t+1}^*)}\left(1 + \frac{(\phi_t + \phi_t^*)}{var_t(\Delta s_{t+1})}\right).$$

In order to obtain a pro-cyclical exchange rate with $cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) < 0$, we need

$$var_t(\Delta s_{t+1}) < -(\phi_t + \phi_t^*). \tag{8}$$

In other words, there is no need to shrink the conditional exchange rate variance to zero in order to generate an exchange rate disconnect from SDF innovations. Negative wedges ϕ and ϕ^* can mitigate the conditional version of the Backus-Smith puzzle. These wedges reduce the need for the exchange rate to respond to SDF shocks in order to enforce the bond Euler equations.

Importantly, the wedges enter the determination of the exchange rate cyclicality directly in their first moments, as opposed to their variances or covariances with other variables. In fact, the wedges can even be constant and still matter for exchange rate cyclicality. This is because the wedges directly enter the investors' Euler equations (10), which require endogenous adjustments in the covariances between the exchange rate and the SDFs. In other words, these wedges are substitutes for covariances between the exchange rate and the SDFs.

The negative bond wedges ϕ and ϕ^* can be interpreted in a number of ways. First, they can reflect a home bias by investors, which has been documented extensively in the literature [see, e.g. Lewis, 1995b], and particularly a home currency bias in bond holdings as documented by Maggiori et al. [2020]. In this case, investors are willing to accept lower expected returns on their home bonds.

Second, they can be a symptom of high convenience yields that home investors receive on their home bond holdings. Krishnamurthy and Vissing-Jorgensen [2012] present such evidence for U.S. Treasury bonds, while Diamond and Van Tassel [2021] present evidence for risk-free government bonds around the world. Note that the negative ϕ and ϕ^* can capture home investors' preference for home bonds, and not foreign investors' preferences for U.S. dollar bonds as argued for in Jiang et al. [2018]. We return to this latter case below.

Third, the negative ϕ and ϕ^* can be seen as a symptom of financial repression. Governments routinely adopt measures to allow themselves to borrow at below-market rates. This is usually referred to as financial repression [see Reinhart et al., 2011, Chari et al., 2020]. During the Great Financial Crisis, banks were induced by their national governments to buy the sovereign debt of their countries [Acharya and Steffen, 2015, De Marco and Macchiavelli, 2016, Ongena, Popov, and Van Horen, 2019]. Since the 2008 GFC, central banks in advanced economies have increased the size of their balance sheets to purchase government bonds, a new wave of financial repression [see Hall and Sargent, 2022, for a comparison of the pandemic and two World Wars]. Financial repression come in other forms, including macro-prudential regulation that favors government bonds, direct lending to the government by domestic pension funds and banks, moral suasion used to increase domestic bank holdings of government bonds [see Acharya and Steffen, 2015, De Marco and Macchiavelli, 2016, Ongena et al., 2019, for examples from Europe during the GFC]. Japan, and its yield curve control policy, is a textbook example of financial repression. In these situations, financial institutions also behave as if they have a preference for holding government bonds.

5.2 Cross-Currency Wedges and Costs of Foreign Bond Investment

We next consider wedges only in the Euler equations of investors buying foreign currency risk-free bonds. These wedges for the home and foreign investors respectively are denoted as ξ_t and ξ_t^*. Then, the four Euler equations can be expressed as follows:

$$\begin{aligned} 1 &= \mathbb{E}_t\left[\exp(m_{t,t+1} + r_t)\right], \\ \exp(\xi_t) &= \mathbb{E}_t\left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right], \\ 1 &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + r_t^*)\right], \\ \exp(\xi_t^*) &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + \Delta s_{t+1} + r_t)\right]. \end{aligned}$$

These wedges are security-specific: they only apply to the bonds denominated in a currency different from the domestic currency. If the wedges are positive, investors effectively apply a higher discount rate to the foreign-currency bond payoffs and therefore require a higher expected return.

Reorganizing the terms, we obtain two expressions that relate the expected currency excess return to the perceived risks from the home and foreign perspectives as well as the wedges:

$$\begin{aligned} (r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}, -\Delta s_{t+1}) + \xi_t, \\ (r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}^*, \Delta s_{t+1}) + \xi_t^*. \end{aligned}$$

Combining these expressions, we directly obtain the following characterization of the conditional exchange rate cyclicality in the presence of Euler equation wedges.

Proposition 5. *In the presence of Euler equation wedges, the conditional exchange rate cyclicality is given by:*

$$cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) - (\xi_t^* + \xi_t),$$

and the conditional correlation is given by:

$$corr_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = \frac{std_t(\Delta s_{t+1})}{std_t(m_{t,t+1} - m_{t,t+1}^*)}\left(1 - \frac{(\xi_t^* + \xi_t)}{var_t(\Delta s_{t+1})}\right).$$

In order to obtain conditionally pro-cyclical exchange rates with $cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) <$

0, we need positive wedges that exceed the exchange rate variance:

$$var_t(\Delta s_{t+1}) < (\xi_t^* + \xi_t). \tag{9}$$

Consider an equilibrium in which agents hold a positive quantity of foreign bonds, as is the case in the data, then the ξ's are effectively a tax on the return obtained by investors on the foreign bond holdings. Thus, this case can reflect home currency bias as well as the transaction cost models of Alvarez, Atkeson, and Kehoe [2002b, 2009b] in which agents need to pay a cost to access foreign securities and currency markets.[10]

In Proposition 3, we showed that the k-horizon bound is easier to satisfy as k rises, in part because there is stronger exchange rate predictability in the long run than in the short run. The wedge counterpart of this result is that the wedges required for making long-horizon bond investments are lower than the wedges required for making short-horizon investments. When the wedges are interpreted in terms of transaction costs, this result is plausible, as a long-horizon investor will effectively pay a smaller per-period cost for cross-currency investments.

5.3 Four Wedges and Foreign Demand for Dollar Bonds

Finally, we consider the case with all four wedges:

$$\begin{aligned} \exp(\phi_t) &= \mathbb{E}_t \left[\exp(m_{t,t+1} + r_t)\right], \\ \exp(\xi_t) &= \mathbb{E}_t \left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right], \\ \exp(\phi_t^*) &= \mathbb{E}_t \left[\exp(m_{t,t+1}^* + r_t^*)\right], \\ \exp(\xi_t^*) &= \mathbb{E}_t \left[\exp(m_{t,t+1}^* + \Delta s_{t+1} + r_t)\right]. \end{aligned} \tag{10}$$

Assuming log-normality, we obtain two expressions that relate the expected excess return of a strategy that goes long the foreign bond to the perceived risks from the home and foreign perspectives as well as the wedges:

$$(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) = -cov_t(m_{t,t+1}, -\Delta s_{t+1}) + \xi_t - \phi_t,$$

[10] These cross-country wedges can also be interpreted as the product of subjective belief mistakes in the joint dynamics of the foreign bond return and the exchange rate. In this case, while the Euler equations hold without wedges under investors' subjective expectations, their biased beliefs implimply Euler equation wedges for cross-country bond holdings under the econometrician's information set — for example, if investors are systematically pessimistic about the foreign bond returns or they overestimate the risks, their belief bias can lead to positive ξ_t and ξ_t^* in the Euler equations.

$$(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) = -cov_t(m_{t,t+1}^*, \Delta s_{t+1}) + \xi_t^* - \phi_t^*.$$

Combining these equations yields the following characterization of the conditional exchange rate cyclicality.

Proposition 6. *In the presence of Euler equation wedges, the conditional exchange rate cyclicality is given by:*

$$cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) - (\xi_t^* + \xi_t) + (\phi_t + \phi_t^*),$$

and the conditional correlation is given by:

$$corr_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = \frac{std_t(\Delta s_{t+1})}{std_t(m_{t,t+1} - m_{t,t+1}^*)} \left(1 + \frac{(\phi_t + \phi_t^*) - (\xi_t^* + \xi_t)}{var_t(\Delta s_{t+1})}\right).$$

In order to obtain pro-cyclical exchange rates $cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) < 0$, we need

$$var_t(\Delta s_{t+1}) < (\xi_t^* + \xi_t) - (\phi_t + \phi_t^*).$$

Next, we derive restrictions on the wedges that are needed to change the sign of the unconditional exchange rate cyclicality.

Proposition 7. *Recall that* $q_t = \mathbb{E}_t[\Delta s_{t+1}]$. *Let*

$$\omega = \mathbb{E}[-(\xi_t^* + \xi_t) + (\phi_t + \phi_t^*)] - cov\left(\phi_t^*, q_t\right) + cov\left(\phi_t, q_t\right).$$

In the presence of Euler equation wedges, each of the following is a necessary condition for a negative unconditional exchange rate cyclicality, i.e., $cov(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) < 0$*:*

(a)

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}} \left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) + \sqrt{R^2} \left(\frac{1}{b} \frac{R_{Fama}^2}{R^2} - 1\right).$$

If the Fama regression yields the best predictor of the exchange rate movement, then, we can simplify the formula to

$$\begin{aligned} \frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} &\geq \frac{1}{\sqrt{R^2}} \left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) + \sqrt{R^2} \left(\frac{1}{b} - 1\right) \\ &= \frac{1}{\sqrt{R^2}} \left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) - \sqrt{R^2} + sign(b) \frac{std(f_t - s_t)}{std(\Delta s_{t+1})}. \end{aligned}$$

(b)

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t+1}])}{std(\mathbb{E}_t[\Delta s_{t+1}])}+\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{1}{R^2}\left(1+\frac{\omega}{var(\Delta s_{t+1})}\right).$$

Among these conditions,

- *(a) $\Rightarrow$ (b).*
- *If Fama regression yields the best predictor and $b \notin (0,1)$, (b) $\Rightarrow$ (a); otherwise (b) is a weaker condition.*

In the presence of the Euler equation wedges, these conditions can now be satisfied even if the exchange rate is close to a random walk with small R^2. If ω is negative enough to flip the sign on the right-hand side, then, we do not need to rely on a highly volatile market price of risk and/or large U.I.P. deviations. This being the case, the necessary condition of a negative unconditional exchange rate cyclicality can be satisfied if $b \leq 0$, even with constant market prices of risk.

Jiang et al. [2021] present evidence that foreign investors obtain a higher convenience yield on dollar bonds than U.S. investors. They show that this possibility helps to explain why the dollar appreciates during global recessions. This case is one where $\xi_t^* < 0$ and $\xi_t^* < \phi_t$, or $-\xi_t^* + \phi_t > 0$. Note that this case makes it harder to satisfy the restriction in Proposition 7. That is, with,

$$\omega = \mathbb{E}[(-\xi_t^* + \phi_t) + (-\xi_t + \phi_t^*)] - cov\left(\phi_t^*, q_t\right) + cov\left(\phi_t, q_t\right),$$

if $(-\xi_t^* + \phi_t) > 0$, the rest of the terms need to be sufficiently negative to offset this positive term. This can occur if $\xi_t > 0$ and U.S. investors face transaction costs of investing in foreign bonds, as well as if $\phi_t^* < 0$ and foreign investors derive a convenience yield on their own domestic currency holdings. As such, while foreign investors' bond convenience yield can generate the dollar appreciation during global recessions, a different type of the Euler equation wedge, which is closer to home bias, is needed to generate a negative unconditional exchange rate cyclicality.

5.4 Calibrated Examples

Let us return to the numerical example we considered in Section 4.1. If we assume that the Fama regression yields the best predictor, we can rearrange the condition in Proposition

7 to obtain

$$-\omega \geq var(\Delta s_{t+1})\left[1 + R^2\left(\frac{1}{b} - 1\right)\right] - \sqrt{R^2} std\left(var_t(m_{t,t+1})\right) std(\Delta s_{t+1}),$$

which provides a lower bound on the sum of the wedges. If we assume $var(\Delta s_{t+1}) \leq 0.1^2$, $R^2 \leq 0.05$ and $b = -1$ from data, and $std\left(var_t(m_{t,t+1})\right) = 0.067$ from the long-run risk model, then, we obtain

$$-\omega \geq 0.75\%.$$

For example, if we attribute ω all to the wedges ξ_t and ξ_t^* associated with the foreign transaction costs, then, the average cost $\mathbb{E}[\xi_t^* + \xi_t]/2$ that is necessary to generate a negative unconditional exchange rate cyclicality is about 38 basis points.

Moreover, given a negative Fama coefficient, i.e., $b < 0$, this bound becomes even tighter if the exchange rate is less predictable. In the limit, $R^2 = 0$ and the bound becomes $-\omega \geq var(\Delta s_{t+1}) = 1\%$, which is in line with our characterization of the conditional exchange rate cyclicality in Proposition 5 and Eq. (9) in particular. The bound is also tighter if $std\left(var_t(m_{t,t+1})\right)$ is smaller, i.e., if the conditional log SDF variance has a lower volatility.

We can also vary the value of $std\left(var_t(m_{t,t+1})\right)$ and plot the lower bound on the wedge $-\omega$. Figure (3) plots this relationship. Consistent with our results in Section 4.1, $std\left(var_t(m_{t,t+1})\right)$ needs to be about 0.40 in order to generate a negative unconditional exchange rate cyclicality without the Euler equation wedges. This value, as we showed above, is much higher than the moments from standard asset pricing models such as the long-run risk model and the external habit model.

Alternatively, if we assume that the forward premium has no predictive power for the exchange rate movement with $b = 0$, while maintaining the other assumptions of $R^2 = 5\%$, $std(\Delta s_{t+1}) = 10\%$, and $std\left(var_t(m_{t,t+1})\right) = 0.067$, we obtain a slightly tighter bound

$$-\omega \geq 0.80\%.$$

Finally, in more recent research, new variables have been shown to predict exchange rate movements with corresponding increases in R^2. See Stavrakeva and Tang [2020], Chahrour, Cormun, De Leo, Guerrón-Quintana, and Valchev [2021], Dahlquist and Pénasse [2022], Chernov, Dahlquist, and Lochstoer [2023a]. Consider Kremens and Martin [2019], Kremens et al. [2023] who show that a combination of the quanto risk premium and inter-

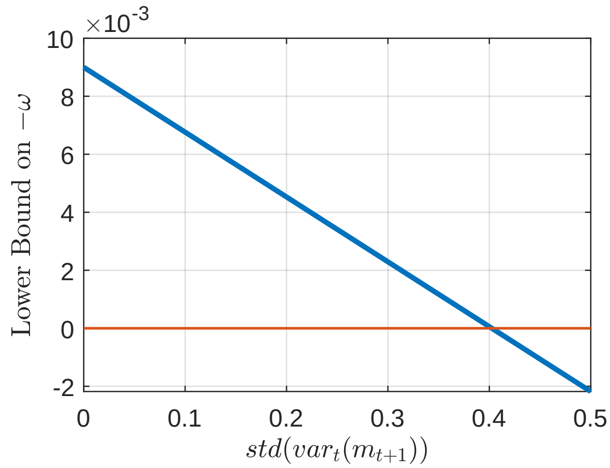


FIGURE 3. LOWER BOUND ON THE EULER EQUATION WEDGE AS A FUNCTION OF THE VARIABILITY OF THE CONDITIONAL LOG SDF VARIANCE

Notes: The figure plots the lower bound on $-\omega$ as a function of $std\left(var_t(m_{t,t+1})\right)$ as implied by Proposition 7. We take $b = -1$, $R^2 = 5\%$, and $std(\Delta s_{t+1}) = 10\%$.

est rate differential can capture 16% of the exchange rate variation at the two-year horizon. If we assume that a similar explanatory power can be attained at the one-year horizon, then, Proposition 7(a) implies that, using the same calibration targets $var(\Delta s_{t+1}) \leq 0.1^2$, $R^2_{Fama} \leq 0.05$ and $b = -1$, and $std\left(var_t(m_{t,t+1})\right) = 0.067$, we obtain a lower bound of $-\omega$ at 0.52%. Compared to the bound of 0.75% we obtained above, this example shows that greater exchange rate predictability lowers the magnitude of the wedges required to generate a pro-cyclical exchange rate.

5.5 Intermediation Models

Gabaix and Maggiori [2015], Itskhoki and Mukhin [2021] consider models in which the bond markets are segmented between countries and financial intermediation is required for cross-country investments. Since the households cannot directly trade the foreign bond, their Euler equations for the foreign bonds do not hold. That is, this model is consistent with our characterization by eliminating two of the four Euler equations we consider. The intermediation models include a third set of agents ("arbitrageurs") that partly integrate markets and whose Euler equations price currency returns. In particular, these models consider an agent that chooses a portfolio to maximize wealth w_{t+1}, where this wealth can be measured in either home or foreign currency without any substantive change in results. The arbitrageur trades in both home and foreign bonds and prices the carry trade return:

$$(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) = -cov_t(w_{t+1}, -\Delta s_{t+1}) + \chi_t. \tag{11}$$

The covariance term here is the risk premium required for bearing carry trade risk by the intermediary, as in intermediary pricing models [He and Krishnamurthy, 2013]. The additional wedge χ_t can reflect a Lagrange multiplier on a leverage constraint as in Gabaix and Maggiori [2015]. The signs of both terms can depend on which currency the arbitrageur invests and which it borrows.

These models fall within the set of models that in our characterization can address exchange rate disconnect. They do so by adding wedges in two of the Euler equations. To understand the connection in further detail, we consider the Gabaix and Maggiori [2015] model and map the key equations of that model to the wedges we have considered.

Specifically, we consider an extension of the Gabaix and Maggiori [2015] model in which households receive non-tradable endowment shocks which drive variations in their marginal utilities. The model can be presented in a two-period setting, with periods $t = 0$ and 1. We leave the details in Appendix B.

In this model, the households can freely trade domestic risk-free bonds. So, the Euler equations for domestic bonds holds:

$$1 = \mathbb{E}_0[\exp(m_1 + r_0)],$$
$$1 = \mathbb{E}_0[\exp(m_1^* + r_0^*)].$$

However, due to market segmentation, the households cannot directly trade foreign bonds. The Euler equations for foreign bonds have endogenously determined cross-currency wedges:

$$\exp(\xi_0) = \mathbb{E}_t\left[\exp(m_1 - \Delta s_1 + r_0^*)\right],$$
$$\exp(\xi_0^*) = \mathbb{E}_t\left[\exp(m_1^* + \Delta s_1 + r_0)\right].$$

These wedges can be interpreted as the reductions in the expected returns on foreign bonds that would make the households indifferent between holding domestic and foreign bonds. If we open the foreign bond markets but keep the risk-free rates as they are, the households have incentives to either buy or short-sell the foreign bonds if the wedges are zero.

The model has a date 1 non-traded endowment shock (which affects the SDF) and an import demand shock which must be financed via international capital flows. Since households cannot directly access international financial markets, the currency imbalance is accommodated by an arbitrageur. The exchange rate and the expected currency return adjust to satisfy the pricing condition for the arbitrageur. The details of the model are in Appendix B.

Figure (4) present the equilibrium magnitude of the Euler equation wedges $\xi_0 + \xi_0^*$ and exchange rate cyclicality $corr(m_1 - m_1^*, \Delta s_1)$, as we vary correlation between the non-traded endowment shock (which affects the SDF) and the import demand shock (which affects the currency imbalance that the arbitrageur needs to absorb). Our calibration details are also presented in Appendix B. In the presence of market segmentation, the wedges are always non-zero and lower the term $1 - (\xi_0 + \xi_0^*)/var(\Delta s_1)$ below 1. When the home SDF becomes more correlated with the import demand shock which increases the demand for foreign goods and depreciates the home currency, the exchange rate becomes more pro-cyclical (i.e., more negative $corr(m_1 - m_1^*, \Delta s_1)$) while the wedges increase (i.e., more negative $1 - (\xi_0 + \xi_0^*)/var(\Delta s_1)$).

This result supports the exchange rate cyclicality result in Proposition 5, reproduced below:

$$corr_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = \frac{std_t(\Delta s_{t+1})}{std_t(m_{t,t+1} - m_{t,t+1}^*)} \left(1 - \frac{(\xi_t^* + \xi_t)}{var_t(\Delta s_{t+1})}\right),$$

which shows that the Euler equation wedges $\xi_t + \xi_t^*$ are closely tied to the exchange rate

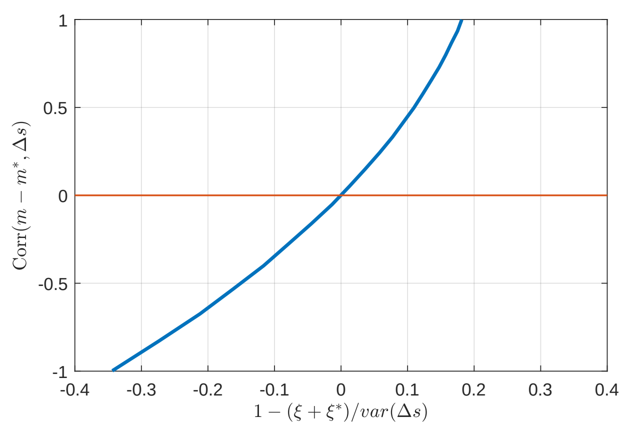


FIGURE 4. EULER EQUATION WEDGE AND EXCHANGE RATE CYCLICALITY IN THE GABAIX AND MAGGIORI [2015] MODEL

cyclicality, even when the wedges are not directly measurable transaction costs or convenience yields, but instead arise as the "shadow prices" in a segmented market setting. In this sense, the wedge accounting provides a framework to understand the exchange rate cyclicality in intermediation models.

5.6 Discussion of Other Specific Models

Lastly, we discuss other specific models that nest our Euler equations and clarify the wedges they imply.

Bond Convenience Yields. The wedge equation wedges can also arise from bond convenience yields. For simplicity, we follow Jiang et al. [2024] and assume convenience yields arise from a bond-in-utility set-up, which can be microfounded by more structural models.

Specifically, the home households' utility is derived over consumption and the market value of home bond holdings:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\delta^t(u(c_t)+v(b_{H,t}))\right],$$

and their budget can be expressed as

$$y_t + b_{H,t-1}\exp(r_{t-1}) + b_{F,t-1}\exp(r^*_{t-1}-s_t) = c_t + b_{H,t} + b_{F,t}\exp(-s_t) + \bar{b}_{t-1}\exp(r_{t-1}).$$

Similarly, the foreign households also derive utility from holding the home bond, which leads to the following set of Euler equations:

$$\begin{aligned}
1-\frac{v'(b_{H,t})}{u'(c_t)} &= \mathbb{E}_t\left[\exp(m_{t+1}+r_t)\right],\\
1 &= \mathbb{E}_t\left[\exp(m_{t+1}-\Delta e_{t+1}+r^*_t)\right],\\
1 &= \mathbb{E}_t\left[\exp(m^*_{t+1}+r^*_t)\right],\\
1-\frac{v'(b^*_{H,t})}{u'(c^*_t)} &= \mathbb{E}_t\left[\exp(m^*_{t+1}+\Delta e_{t+1}+r_t)\right].
\end{aligned}$$

In this setting, due to the non-pecuniary utility derived from holding the home bond, home and foreign households are willing to accept a lower return on the home bond, which is determined by the marginal utility of bond holding scaled by the marginal utility of consumption. The Euler equation wedges can be expressed as

$$\phi_t = \log\left(1 - \frac{v'(b_{H,t})}{u'(c_t)}\right), \qquad \phi_t^* = 0,$$
$$\xi_t^* = \log\left(1 - \frac{v'(b_{H,t}^*)}{u'(c_t^*)}\right), \qquad \xi_t = 0.$$

This specification also has a simple interpretation: for example, when the foreign households derive a higher marginal utility $v'(b_{H,t}^*)$ from holding the home bond, they accept a lower risk-adjusted return on the home bond, which is associated with a lower wedge ξ_t^*.

Financial Repression. We consider a very stylized setting in which agents find domestic bonds more desirable due to regulatory reasons. This can be modeled as a shadow value $h(\cdot)$ from holding the domestic bond, which similarly enter the households' utility functions. This implies the following Euler equations:

$$\begin{aligned}
1 - \frac{h'(b_{H,t})}{u'(c_t)} &= \mathbb{E}_t\left[\exp(m_{t+1} + r_t)\right], \\
1 &= \mathbb{E}_t\left[\exp(m_{t+1} - \Delta e_{t+1} + r_t^*)\right], \\
1 - \frac{h'(b_{F,t}^*)}{u'(c_t^*)} &= \mathbb{E}_t\left[\exp(m_{t+1}^* + r_t^*)\right], \\
1 &= \mathbb{E}_t\left[\exp(m_{t+1}^* + \Delta e_{t+1} + r_t)\right].
\end{aligned}$$

In this case, the Euler equation wedges can be expressed as

$$\phi_t = \log\left(1 - \frac{h'(b_{H,t})}{u'(c_t)}\right), \qquad \phi_t^* = \log\left(1 - \frac{h'(b_{F,t}^*)}{u'(c_t^*)}\right),$$
$$\xi_t^* = 0, \qquad \xi_t = 0,$$

which implies that the regulatory constraints effectively lower the required return on the domestic bond, which gives rise to home bias in bond holdings for both home and foreign households.

As long as $\phi_t < 0$ and $\phi_t^* < 0$, the ω term in the unconditional exchange rate cyclicality bound can be negative, which will make it easier to generate a pro-cyclical exchange rate.

Holding Cost. Moreover, if the agents face a holding cost or capital gain tax from holding foreign assets, the Euler equations also imply wedges. For example, if the home and foreign households both have positive positions on the cross-country bond holdings, the Euler equations can be expressed as

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[\exp(m_{t+1} + r_t)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t+1} - \Delta e_{t+1} + r_t^* - \tau_t)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t+1}^* + r_t^*)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t+1}^* + \Delta e_{t+1} + r_t - \tau_t^*)\right],
\end{aligned}
$$

where τ_t and τ_t^* are positive numbers denoting the reduction in returns from the cross-country positions. These wedges can be motivated by the capital gain tax on the returns r_t and r_t^* from foreign bond holdings, or by the management fee for international portfolios. In this case,

$$
\begin{aligned}
\phi_t = 0, &\qquad \phi_t^* = 0, \\
\xi_t^* = \tau_t^*, &\qquad \xi_t = \tau_t.
\end{aligned}
$$

As long as the transaction costs ξ_t and ξ_t^* are positive, the ω term in the unconditional exchange rate cyclicality bound can be negative, which will make it easier to generate a pro-cyclical exchange rate.[11]

6 Conclusion

We derive a simple and general characterization of the bilateral exchange rate's cyclicality with respect to the differential between home and foreign SDFs. If investors can freely trade the risk-free bonds in both countries, their optimality conditions impose strong restrictions on the relation between exchange rates and macro fundamentals that is difficult to reconcile with the data. In order to break this relation, models need to produce wedges in the cross-currency Euler equations of investors.

We regard our results as ruling in a class of models involving home bias/transaction costs, convenience yields, or intermediation frictions as resolutions to the exchange rate puzzles. In particular, the wedge of 38 basis points in each direction is plausible and in the range of frictions measured in the literature, as for example in the literature on

[11] Having said this, if the households hold negative positions on the foreign bond, the Euler equation wedges τ_t and τ_t^* will be negative in order to reflect the reductions in their short positions.

CIP deviations [Du, Tepper, and Verdelhan, 2018]. Alternatively, at a broader level, our approach offers a model-free diagnosis for the necessary conditions needed to explain exchange rate disconnect and predictability.

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Appendix

A Proof

A.1 Proposition 1

Proof. Combining

$$\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^* = cov_t(m_{t,t+1}, -\Delta s_{t+1}) + \frac{1}{2}var_t(\Delta s_{t+1}),$$
$$-(\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^*) = cov_t(m^*_{t,t+1}, \Delta s_{t+1}) + \frac{1}{2}var_t(\Delta s_{t+1}),$$

we directly obtain

$$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0.$$

□

A.2 Proposition 2

Proof. Using the definition of p_t and q_t, we can restate the covariance as follows:

$$cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) = cov(p_t + q_t, q_t) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right).$$

Note $cov(p_t + q_t, q_t) = b \times var(p_t + q_t)$ by the construction of the Fama regression. Then,

$$cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) = b \times var(p_t + q_t) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right).$$

A negative unconditional exchange rate cyclicality then implies

$$\begin{aligned} cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = {} & \mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(p_t + q_t) \\ & + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) \leq 0 \end{aligned}$$

Rearranging terms,

$$\frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), -q_t\right) \geq \mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(p_t + q_t)$$

Without loss of generality,

$$\begin{aligned}cov\left(var_t(m_{t,t+1}), q_t\right) &\geq \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), -q_t\right)\\ &\geq \mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(p_t + q_t)\end{aligned}$$

We note $corr\left(var_t(m_{t,t+1}), q_t\right) \leq 1$. Hence, a necessary (but not sufficient) condition is given by:

$$\begin{aligned}std\left(var_t(m_{t,t+1})\right) &\geq \frac{\mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])}\\ std\left(var_t(m_{t,t+1})\right) &\geq \frac{var(\Delta s_{t+1}) - var(\mathbb{E}_t[\Delta s_{t+1}]) + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])}\\ std\left(var_t(m_{t,t+1})\right) + std(\mathbb{E}_t[\Delta s_{t+1}]) &\geq \frac{var(\Delta s_{t+1}) + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])}\\ \frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 &\geq \frac{var(\Delta s_{t+1}) + b \times var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])}\end{aligned}$$

which implies

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])} \geq \frac{var(\Delta s_{t+1})}{var(\mathbb{E}_t[\Delta s_{t+1}])} = \frac{1}{R^2} \tag{A.1}$$

Now, notice that

$$b = \frac{cov(\Delta s_{t+1}, f_t - s_t)}{var(f_t - s_t)} = \frac{std(\Delta s_{t+1})}{std(f_t - s_t)}corr(\Delta s_{t+1}, f_t - s_t).$$

We obtain

$$\begin{aligned}\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})\sqrt{R^2}} + 1 - b \times \frac{var(f_t - s_t)}{var(\Delta s_{t+1})R^2} &\geq \frac{1}{R^2}\\ \frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} + \sqrt{R^2} - b \times \frac{corr(\Delta s_{t+1}, f_t - s_t)^2}{b^2\sqrt{R^2}} &\geq \frac{1}{\sqrt{R^2}}\\ \frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} &\geq \frac{1}{\sqrt{R^2}} - \sqrt{R^2} + \frac{R^2_{Fama}}{b\sqrt{R^2}}\end{aligned}$$

When Fama Regression yields the best predictor, $R^2_{Fama} = R^2$, the formula is simplified to

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}} - \sqrt{R^2} + \frac{1}{b}\sqrt{R^2},$$

where

$$\frac{1}{b}\sqrt{R^2} = \frac{1}{b}\frac{|b|std(f_t - s_t)}{std(\Delta s_{t+1})} = sign(b)\frac{std(f_t - s_t)}{\Delta s_{t+1}}.$$

Hence, we arrive at condition (a).

Next, we show condition (b). By using the definition of covariance and imposing symmetry:

$$\begin{aligned} cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) &= cov(p_t + q_t, q_t) - cov\left(var_t(m_{t,t+1}), q_t\right) \\ &= var(q_t) + cov(p_t, q_t) - cov\left(var_t(m_{t,t+1}), q_t\right). \end{aligned}$$

Using the definition of the covariance, this covariance expression on the left-hand side can be bounded below as follows:

$$cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \geq var(q_t) - std(p_t)std(q_t) - std(q_t)std\left(var_t(m_{t,t+1})\right).$$

This lower bound can be restated as:

$$cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \geq var(q_t)\left(1 - \frac{std(p_t)}{std(q_t)} - \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)}\right).$$

To get a negative unconditional Backus Smith coefficient, we need the following condition:

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = \mathbb{E}[var_t(\Delta s_{t+1})] + cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \leq 0.$$

This can be restated as follows:

$$-\mathbb{E}[var_t(\Delta s_{t+1})] \geq cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \geq var(q_t)\left(1 - \frac{std(p_t)}{std(q_t)} - \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)}\right).$$

Rearranging terms, we obtain the following result:

$$\frac{std(p_t)}{std(q_t)} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)} \geq 1 + \frac{\mathbb{E}[var_t(\Delta s_{t+1})]}{var(q_t)}.$$

Note that

$$1 + \frac{\mathbb{E}[var_t(\Delta s_{t+1})]}{var(q_t)} = \frac{var(\mathbb{E}_t[\Delta s_{t+1}]) + \mathbb{E}[var_t(\Delta s_{t+1})]}{var(\mathbb{E}_t[\Delta s_{t+1}])} = \frac{var(\Delta s_{t+1})}{var(\mathbb{E}_t[\Delta s_{t+1}])}.$$

Hence, we obtain the necessary condition (b) by using the definition of the unconditional variance:

$$\sqrt{\frac{std(p_t)}{std(q_t)} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)}} \geq \frac{std(\Delta s_{t+1})}{std(q_t)} = \frac{1}{\sqrt{R^2}}.$$

To compare conditions (a) and (b), note that condition (a) can be written as Eq. (A.1), reproduced below,

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])} \geq \frac{1}{R^2},$$

it suffices to compare the term $std(\mathbb{E}_t[rx_{t+1}])/std(\mathbb{E}_t[\Delta s_{t+1}]) = std(p_t)/std(q_t)$ in (b) with $1 - bvar(f_t - s_t)/var(\mathbb{E}_t[\Delta s_{t+1}]) = 1 - bvar(p_t + q_t)/var(q_t)$ in (a). Consider the general case, when Fama regression does not necessarily yield the best predictor. Take conditional expectation on both sides of the regression yields

$$q_t = a + b(p_t + q_t) + x_t$$

where $x_t = \mathbb{E}_t[\varepsilon_{t+1}]$ satisfies that

$$\begin{aligned} cov(x_t, p_t + q_t) &= cov(\varepsilon_{t+1}, p_t + q_t) - cov(\varepsilon_{t+1} - x_t, p_t + q_t) = 0 \\ cov(x_t, q_t) &= cov(x_t, b(p_t + q_t)) + var(x_t) = var(x_t) \end{aligned}$$

Hence, $var(q_t) = b^2 var(p_t + q_t) + var(x_t)$, and

$$\begin{aligned} 1 - b\frac{var(p_t + q_t)}{var(q_t)} &= 1 - \frac{1}{b}\left(1 - \frac{var(x_t)}{var(q_t)}\right) \\ &= \left(1 - \frac{1}{b}\right) + \frac{1}{b}\frac{var(x_t)}{var(q_t)}. \end{aligned}$$

On the other hand, $p_t = -a/b + (1/b - 1)q_t - x_t/b$, which implies

$$\begin{aligned} \frac{std(p_t)}{std(q_t)} &= \frac{1}{std(q_t)}\sqrt{\left(\frac{1}{b} - 1\right)^2 var(q_t) + \frac{1}{b^2}var(x_t) - \frac{2}{b}\left(\frac{1}{b} - 1\right)var(x_t)} \\ &= \sqrt{\left(\frac{1}{b} - 1\right)^2 + \left(\frac{2}{b} - \frac{1}{b^2}\right)\frac{var(x_t)}{var(q_t)}}. \end{aligned}$$

Note that

$$\left(1-b\frac{var(p_t+q_t)}{var(q_t)}\right)^2=\left(\frac{1}{b}-1\right)^2+\left(\frac{2}{b}-\frac{2}{b^2}\right)\frac{var(x_t)}{var(q_t)}+\frac{1}{b^2}\left(\frac{var(x_t)}{var(q_t)}\right)^2$$
$$=\left(\frac{std(p_t)}{std(q_t)}\right)^2-\frac{1}{b^2}\left(1-\frac{var(x_t)}{var(q_t)}\right)\frac{var(x_t)}{var(q_t)}$$

which implies that, when $var(x_t)>0$, i.e., the Fama regression does not yield the best predictor, condition (a) is always tighter.

When $var(x_t)=0$, i.e., Fama regression yields the best predictor, we obtain

$$\left(1-b\frac{var(p_t+q_t)}{var(q_t)}\right)^2=\left(\frac{std(p_t)}{std(q_t)}\right)^2,$$

and

$$1-b\frac{var(p_t+q_t)}{var(q_t)}=\left(1-\frac{1}{b}\right).$$

When $1-1/b>0$, i.e., $b<0$ or $b>1$, condition (a) and (b) are equivalent. Otherwise, $1-b\frac{var(p_t+q_t)}{var(q_t)}<0<\frac{std(p_t)}{std(q_t)}$, and condition (a) is tighter.

□

A.3 Proposition 3

The proof is identical to the proof of Proposition 2. Just replace the one-period objects (e.g., $m_{t,t+1}$) with the multi-period objects (e.g., $m_{t,t+k}$).

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Proof. From

$$\phi_t=\mathbb{E}_t[m_{t,t+1}]+\frac{1}{2}var_t(m_{t,t+1})+r_t,$$
$$\xi_t=\mathbb{E}_t[m_{t,t+1}]+\frac{1}{2}var_t(m_{t,t+1})-\mathbb{E}_t[\Delta s_{t+1}]+\frac{1}{2}var_t(\Delta s_{t+1})+cov_t(m_{t,t+1},-\Delta s_{t+1})+r_t^*,$$
$$\phi_t^*=\mathbb{E}_t[m_{t,t+1}^*]+\frac{1}{2}var_t(m_{t,t+1}^*)+r_t^*,$$
$$\xi_t^*=\mathbb{E}_t[m_{t,t+1}^*]+\frac{1}{2}var_t(m_{t,t+1}^*)+\mathbb{E}_t[\Delta s_{t+1}]+\frac{1}{2}var_t(\Delta s_{t+1})+cov_t(m_{t,t+1}^*,\Delta s_{t+1})+r_t,$$

we obtain

$$\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}] = \frac{1}{2}var_t(m^*_{t,t+1}) + r^*_t - \phi^*_t - \frac{1}{2}var_t(m_{t,t+1}) - r_t + \phi_t$$

Then

$$\begin{aligned} cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) &= cov(p_t + q_t, q_t) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) \\ &- cov\left(\phi^*_t, q_t\right) + cov\left(\phi_t, q_t\right) \\ &= b \times var(p_t + q_t) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) \\ &- cov\left(\phi^*_t, q_t\right) + cov\left(\phi_t, q_t\right) \end{aligned}$$

A negative unconditional exchange rate cyclicality then implies

$$\begin{aligned} &cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) \\ =&\mathbb{E}[var_t(\Delta s_{t+1}) - (\xi^*_t + \xi_t) + (\phi_t + \phi^*_t)] + b \times var(p_t + q_t) \\ +&\frac{1}{2}cov\left(var_t(m^*_{t,t+1}), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) - cov\left(\phi^*_t, q_t\right) + cov\left(\phi_t, q_t\right) \leq 0 \end{aligned}$$

Rearranging terms,

$$\begin{aligned} &\frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), -q_t\right) - cov\left(\phi_t, q_t\right) + cov\left(\phi^*_t, q_t\right) \\ \geq&\mathbb{E}[var_t(\Delta s_{t+1}) - (\xi^*_t + \xi_t) + (\phi_t + \phi^*_t)] + b \times var(p_t + q_t) \end{aligned}$$

By assumption,

$$\begin{aligned} &cov\left(var_t(m_{t,t+1}), q_t\right) - cov\left(\phi_t, q_t\right) + cov\left(\phi^*_t, q_t\right) \\ \geq&\mathbb{E}[var_t(\Delta s_{t+1}) - (\xi^*_t + \xi_t) + (\phi_t + \phi^*_t)] + b \times var(p_t + q_t) \end{aligned}$$

Let $\omega = \mathbb{E}[-(\xi^*_t + \xi_t) + (\phi_t + \phi^*_t)] - cov\left(\phi^*_t, q_t\right) + cov\left(\phi_t, q_t\right)$ denote the new adjustment term that arises from the wedges. Then, a necessary (but not sufficient) condition is given by:

$$\begin{aligned} std\left(var_t(m_{t,t+1})\right) &\geq \frac{\mathbb{E}[var_t(\Delta s_{t+1})] + \omega + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\ std\left(var_t(m_{t,t+1})\right) &\geq \frac{var(\Delta s_{t+1}) - var(\mathbb{E}_t[\Delta s_{t+1}]) + \omega + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \end{aligned}$$

$$
\begin{aligned}
std\left(var_t(m_{t,t+1})\right) + std(\mathbb{E}_t[\Delta s_{t+1}]) &\geq \frac{var(\Delta s_{t+1}) + \omega + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 &\geq \frac{var(\Delta s_{t+1}) + \omega + b \times var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])}
\end{aligned}
$$

which implies

$$
\begin{aligned}
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])} &\geq \frac{var(\Delta s_{t+1}) + \omega}{var(\mathbb{E}_t[\Delta s_{t+1}])} \\
&= \frac{1}{R^2}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right)
\end{aligned}
$$

Recall that $R^2_{Fama} = b^2 var(f_t - s_t)/var(\Delta s_{t+1})$. Hence,

$$
\begin{aligned}
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})\sqrt{R^2}} + 1 - b \times \frac{var(f_t - s_t)}{var(\Delta s_{t+1})\sqrt{R^2}} &\geq \frac{1}{R^2}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} + \sqrt{R^2} - \frac{R^2_{Fama}}{b} &\geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} &\geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) - \sqrt{R^2} + \frac{R^2_{Fama}}{b\sqrt{R^2}}
\end{aligned}
$$

When Fama Regression yields the best predictor, $R^2_{Fama} = R^2$, the formula is simplified to

$$
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) - \sqrt{R^2} + \frac{\sqrt{R^2}}{b}
$$

where

$$
\frac{1}{b}\sqrt{R^2} = \frac{1}{b}\frac{|b| std(f_t - s_t)}{std(\Delta s_{t+1})} = sign(b)\frac{std(f_t - s_t)}{\Delta s_{t+1}}.
$$

Hence, we arrive at condition (a).

Similarly, condition (b) can be derived as

$$
\sqrt{\frac{std(\mathbb{E}_t[rx_{t+1}])}{std(\mathbb{E}_t[\Delta s_{t+1}])} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{1}{R^2}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right).
$$

the relation between condition (a) and (b) depends only on the Fama regression but not the Euler equations, thus are identical to that in Proposition 2. □

A.5 Non-Gaussian Case

We define conditional entropy as follows:

$$L_t(X_{t+1}) = (\log \mathbb{E}_t[X_{t+1}] - \mathbb{E}_t[x_{t+1}]) .$$

We can use μ_{it} to denote the i-th central conditional moment of $\log X$. Then we can state:

$$\log \mathbb{E}_t \exp(s x_{t+1}) = \sum_{j=1}^{\infty} s^j \kappa_{j,t}/j! = k_t(x_{t+1}; s)$$

where $\kappa_{1t} = \mu_{1t}, \kappa_{2t} = \mu_{2t}, \kappa_{3t} = \mu_{3t}, \kappa_{4t} = \mu_{4t} - 3\mu_{2t}^2$. This implies that the conditional entropy can be stated as the sum of the higher order *cumulants*:

$$L_t(X_{t+1}) = \sum_{j=2}^{\infty} \kappa_{j,t}/j! = k_t(x_{t+1}; 1) - \kappa_1.$$

The log of the currency risk premium (in levels) earned by domestic investors can be stated as:

$$(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + L_t\left[\frac{S_t}{S_{t+1}}\right] = -C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right),$$

where co-entropy is defined as $C_t(x_{t+1}, y_{t+1}) = L_t(x_{t+1}y_{t+1}) - L_t(x_{t+1}) - L_t(y_{t+1})$ [Backus, Boyarchenko, and Chernov, 2018]. If x_{t+1} and y_{t+1} are independent, then $C_t(x_{t+1}, y_{t+1}) = 0$. If we define the cumulant generating function,

$$\log \mathbb{E}_t \exp(s_1 x_{t+1} + s_2 y_{t+1}) = k_t(s_1, s_2)$$

then $C_t(x_{t+1}, y_{t+1}) = k_t(1,1) - k_t(1,0) - k_t(0,1)$. A long position in foreign currency is risky for the domestic investor when the foreign currency tends to depreciate (and the home currency appreciates) in worse states for the domestic investor, i.e. when

$$C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right) < 0$$

is more negative. Similarly, the log of the currency risk premium (in levels) earned by domestic investors can be stated as the co-entropy of the domestic SDF with the domestic currency's rate of appreciation:

$$(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + L_t\left[\frac{S_{t+1}}{S_t}\right] = -C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t}).$$

Proposition 8. *In the non-normal case, the conditional exchange rate cyclicality satisfies*

$$-C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t}) - C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right) = L_t\left[\frac{S_t}{S_{t+1}}\right] + L_t\left[\frac{S_{t+1}}{S_t}\right] > 0. \tag{A.2}$$

In the normal case, we recover the covariance result in Proposition 1. When the domestic currency appreciates in worse states for the domestic investor, we have $C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right) < 0$. Similarly, when the foreign currency appreciates in worse states for the foreign investor, we have $C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t}) < 0$.

Because the right-hand side is positive, as entropy is non-negative, we know that exchange rates will have to be counter-cyclical for at least one of the countries, and possibly both.

Proof. The U.S. Euler equations are given by

$$\begin{aligned}\mathbb{E}_t[\exp(m_{t+1} + r_t)] &= 1 \\ \mathbb{E}_t[\exp(m_{t+1} + r_t^* - \Delta s_{t+1})] &= 1\end{aligned}$$

where $m_{t+1} = \log M_{t+1}$. By the definition of entropy and co-entropy, we recast the equations as follows

$$\begin{aligned}0 &= \log \mathbb{E}_t[\exp(m_{t+1} + r_t)] \\ &= \log \mathbb{E}_t[\exp(m_{t+1})] + r_t \\ &= \log \mathbb{E}_t[m_{t+1}] + L_t(M_{t+1}) + r_t \\ 0 &= \log \mathbb{E}_t[\exp(m_{t+1} + r_t^* - \Delta s_{t+1})] \\ &= \log \mathbb{E}_t[\exp(m_{t+1} - \Delta s_{t+1})] + r_t^* \\ &= \mathbb{E}_t[m_{t+1} - \Delta s_{t+1}] + L_t(M_{t+1}\frac{S_t}{S_{t+1}}) + r_t^* \\ &= \mathbb{E}_t[m_{t+1} - \Delta s_{t+1}] + r_t^* + C_t(M_{t+1}, \frac{S_t}{S_{t+1}}) + L_t(M_{t+1}) + L_t(\frac{S_t}{S_{t+1}})\end{aligned}$$

Subtract the first equation from the second to get

$$r_t^* - r_t + \mathbb{E}_t[\Delta s_{t+1}] + L_t(\frac{S_t}{S_{t+1}}) = -C_t(M_{t+1}, \frac{S_t}{S_{t+1}}).$$

Similarly, from the foreign Euler equations we obtain

$$r_t - r_t^* - \mathbb{E}_t[\Delta s_{t+1}] + L_t(\frac{S_{t+1}}{S_t}) = -C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t})$$

Add up the two equations to get the proposition. Note that entropy is always greater than zero, which ensures the inequality in the proposition.

□

B Supplemental Appendix: Details of the Gabaix and Maggiori [2015] Model

We present details of the Gabaix and Maggiori [2015] model. Extending their benchmark two-period model, we introduce a stochastic χ / Y_{NT} ratio between goods preference and the endowment of non-traded goods instead of assuming that it is a constant 1. We do this because in the benchmark model, the SDF is a constant in the numéraire in which bond returns are denominated. With our extension, the SDF is stochastic and could be correlated with financial shocks.

B.1 Environment

Time is discrete and there are two periods: $t = 0$ and 1. There are two countries, the US (H) and the foreign country (F). Each country is populated by a unit mass of households, and has a non-tradable good and a tradable good. For the convenience of exposition, we use the word currency interchangeably with the numéraire of the economy. The *dollar* value and the *foreign currency* value mean values expressed in units of the non-tradable good in each country. To make the results comparable to those in Gabaix and Maggiori [2015], we adopt their exchange rate convention: $\mathcal{E}_t$ is defined as the quantity of the dollar per unit of the foreign currency, which measures the strength of the foreign currency. We use $e_t = \log \mathcal{E}_t$ to denote the log exchange rate.

Households The US households derive utility from consumption in both periods:

$$\theta_0 \log C_0 + \beta \mathbb{E}_0[\theta_1 \log C_1],$$

where C_t is a consumption basket consisting of $C_{NT,t}$ units of the local non-tradable good, $C_{H,t}$ units of the US tradable good, and $C_{F,t}$ units of the foreign tradable good:

$$C_t = \left((C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t} \right)^{1/\theta_t}.$$

The preference parameters χ_t, a_t and ι_t are non-negative and stochastic. we define $\theta_t = \chi_t + a_t + \iota_t$. we use the local non-tradable good as the numéraire in each country, so that its price in local currency is $p_{NT,t} = 1$ in both periods.

Non-tradable goods can only be traded domestically, whereas tradable goods can be traded internationally without any friction. In each period, the US households receive an endowment of $Y_{NT,t}$ units of the local non-tradable good and $Y_{H,t}$ units of the US tradable good. The household owns the claim to this endowment and this claim cannot be sold.

By the first-order conditions,

$$\frac{\chi_t}{C_{NT,t}} = \lambda_t$$
$$\frac{a_t}{C_{H,t}} = \lambda_t p_{H,t}$$
$$\frac{\iota_t}{C_{F,t}} = \lambda_t p_{F,t}$$

For simplicity, we fix χ_t to be a constant between 0 and 1, and allow $C_{NT,t} = Y_{NT,t}$ to vary, which creates variations in λ_t, which we treat as an exogenous variable.

The setup for the foreign households is similar. The foreign households' utility is

$$\theta_0^* \log C_0^* + \beta^* \mathbb{E}_0[\theta_1^* \log C_1^*], \quad \text{where } C_t^* = \left((C_{NT,t}^*)^{\chi_t^*} (C_{F,t}^*)^{a_t^*} (C_{H,t}^*)^{\xi_t} \right)^{1/\theta_t^*}.$$

The net export in dollar is

$$NX_t = \mathcal{E}_t p_{H,t}^* C_{H,t}^* - p_{F,t} C_{F,t} = \frac{\xi_t \mathcal{E}_t}{\lambda_t^*} - \frac{\iota_t}{\lambda_t}$$

Domestic bond market The bond is denominated in the domestic non-tradable goods. Households can freely trade the domestic bond. For the U.S. households, the Euler equation is

$$1 = \mathbb{E}_t \left[\beta \exp(r_0) \frac{U'_{NT,t+1}}{U'_{NT,t}} \right] = \mathbb{E}_t \left[\beta \exp(r_0) \frac{\lambda_{t+1}}{\lambda_t} \right]$$

For simplicity, we assume that the distribution of λ_t is such that $\mathbb{E}_t[\lambda_{t+1}/\lambda_t] = 1$, which implies $\exp(r_0) = 1/\beta$. Similarly, we assume $\mathbb{E}_t[\lambda_{t+1}^*/\lambda_t^*] = 1$, which implies $\exp(r_0^*) = 1/\beta^*$.

The financial market is segmented. The US households can only trade the dollar bond, and the foreign households can only trade the foreign bond. Let $b_{H,0}$ denote the quantity of the dollar bond held by the US households in period 0, which is redeemed in period 1, and let r denote the log dollar risk-free rate. The US households' budget constraints in the two periods are

$$Y_{NT,0} + p_{H,0} Y_{H,0} = C_{NT,0} + p_{H,0} C_{H,0} + p_{F,0} C_{F,0} + b_{H,0},$$
$$Y_{NT,1} + p_{H,1} Y_{H,1} = C_{NT,1} + p_{H,1} C_{H,1} + p_{F,1} C_{F,1} - \exp(r_0) b_{H,0}.$$

Financiers In period 0, risk-neutral financiers are born, who intermediate currency flows subject to a financial friction. In period 1, all financiers unwind their portfolios. To simplify the expression, we follow Gabaix and Maggiori [2015] and assume their profits or losses are paid to the foreign households.

Each financier manages a financial intermediary that can hold bonds in both currencies. In period 0, the financier has zero net worth, and its balance sheet consists of Q_0 units of the dollar and $-Q_0/\mathcal{E}_0$ units of the foreign currency. When $Q_0 > 0$, the financier takes a long position on the dollar bond, and when $Q_0 < 0$, it takes a short position. The expected profit in dollar terms is

$$V_0 = \mathbb{E}_0[\beta(\exp(r_0) - \exp(r_0^*)\mathcal{E}_1/\mathcal{E}_0)Q_0].$$

Although the financiers are risk-neutral, they have limited risk-bearing capacity due to the agency friction considered by Gabaix and Maggiori [2015]. In period 0, each financier can divert a portion of the funds under its management. If it chooses to do so, it unwinds its portfolio and receives the portion it diverts, and its lenders receive the rest of the funds. As the lenders rationally anticipate the financier's incentive and diversion is a less desirable outcome, they supply funding to the financier only when the financier has no incentive to divert the fund, which leads to the following credit constraint:

$$\underbrace{\frac{V_0}{\mathcal{E}_0}}_{\text{Intermediary Value in Foreign Currency}} \geq \underbrace{\left|\frac{Q_0}{\mathcal{E}_0}\right|}_{\text{Portfolio Size}} \cdot \underbrace{\Gamma\left|\frac{Q_0}{\mathcal{E}_0}\right|}_{\text{Divertible Portion}}, \tag{B.1}$$

where the divertible portion is increasing in the size of the balance sheet $|Q_0/\mathcal{E}_0|$ and in the parameter $\Gamma > 0$ proxying for the severity of the financial constraint.

Since the value function V_0 is linear in the financier's currency position Q_0 while the constraint is convex, the constraint is always binding. Let

$$\rho_0 = \mathbb{E}_0[\beta(\exp(r_0) - \exp(r_0^*)\mathcal{E}_1/\mathcal{E}_0)]$$

denote the expected excess return of the dollar against the foreign currency. Then, the equilibrium portfolio allocation on the dollar bond is

$$Q_0 = \frac{\rho_0\mathcal{E}_0}{\Gamma} = \frac{1}{\Gamma}\mathbb{E}_0[\beta(\exp(r_0)\mathcal{E}_0 - \exp(r_0^*)\mathcal{E}_1)].$$

Intuitively, the portfolio position on the dollar bond Q_0 is increasing in the dollar bond's expected excess return ρ_0, and decreasing in the severity of the financial friction as captured by the parameter Γ. A higher Γ means the manager has a stronger incentive to divert the funds, leading to less risk taking in equilibrium. In this way, the agency friction gives rise to a well-defined currency position and rules out pricing inconsistency despite the currency financiers being risk-neutral.

B.2 Equilibrium

In equilibrium

$$\frac{\xi_0 \mathcal{E}_0}{\lambda_0^*} - \frac{\iota_0}{\lambda_0} + Q_0 = 0$$
$$\frac{\xi_1 \mathcal{E}_1}{\lambda_1^*} - \frac{\iota_1}{\lambda_1} - Q_0 \exp(r_0) = 0$$

To streamline the algebra and concentrate on the key economic content, we follow Gabaix and Maggiori [2015] to impose the following simplifying assumption: $\beta = \beta^* = \exp(r_0) = \exp(r_0^*) = 1$, $\xi_t = 1$, $\lambda_t^* = 1$, and $\iota_t > 0$ for $t = 0, 1$. We also assume $\lambda_0 = 1$ and $\iota_0 = 1$. Then, the equilibrium conditions simplify to

$$\mathcal{E}_0 - 1 + \frac{1}{\Gamma}\mathbb{E}_0[\mathcal{E}_0 - \mathcal{E}_1] = 0$$
$$\mathcal{E}_1 - \iota_1 \lambda_1^{-1} - \frac{1}{\Gamma}\mathbb{E}_0[\mathcal{E}_0 - \mathcal{E}_1] = 0$$

We have

$$\mathcal{E}_0 = \frac{(1+\Gamma) + \mathbb{E}_0[\iota_1]}{2+\Gamma}$$
$$\mathcal{E}_1 = \iota_1 \lambda_1^{-1} - \mathbb{E}_0[\iota_1 \lambda_1^{-1}] + \frac{1 + (1+\Gamma)\mathbb{E}_0[\iota_1]}{2+\Gamma}$$

Denote the home and foreign SDFs as

$$\exp(m_1) = \beta \frac{\lambda_1}{\lambda_0}$$
$$\exp(m_1^*) = \beta \frac{\lambda_1^*}{\lambda_0^*}$$

The four Euler equations are

$$\mathbb{E}_0[\exp(m_1 + r_0)] = 1$$
$$\mathbb{E}_0[\exp(m_1^* + r_0^*)] = 1$$

$$\mathbb{E}_0\left[\exp(m_1+r_0^*)\frac{\mathcal{E}_1}{\mathcal{E}_0}\right]=\mathbb{E}_0\left[\lambda_1\frac{\iota_1\lambda_1^{-1}-\mathbb{E}_0[\iota_1\lambda_1^{-1}]+\frac{1+(1+\Gamma)\mathbb{E}_0[\iota_1]}{2+\Gamma}}{\frac{(1+\Gamma)+\mathbb{E}_0[\iota_1]}{2+\Gamma}}\right]$$

$$\mathbb{E}_0\left[\exp(m_1^*+r_0)\frac{\mathcal{E}_0}{\mathcal{E}_1}\right]=\mathbb{E}_0\left[\frac{\frac{(1+\Gamma)+\mathbb{E}_0[\iota_1]}{2+\Gamma}}{\iota_1\lambda_1^{-1}-\mathbb{E}_0[\iota_1\lambda_1^{-1}]+\frac{1+(1+\Gamma)\mathbb{E}_0[\iota_1]}{2+\Gamma}}\right]$$

In the language of our wedge algebra,

$$\phi_t=0,\qquad \phi_t^*=0,$$

$$\xi_t=\log\mathbb{E}_0\left[\lambda_1\frac{\iota_1\lambda_1^{-1}-\mathbb{E}_0[\iota_1\lambda_1^{-1}]+\frac{1+(1+\Gamma)\mathbb{E}_0[\iota_1]}{2+\Gamma}}{\frac{(1+\Gamma)+\mathbb{E}_0[\iota_1]}{2+\Gamma}}\right],\qquad \xi_t^*=\log\mathbb{E}_0\left[\frac{\frac{(1+\Gamma)+\mathbb{E}_0[\iota_1]}{2+\Gamma}}{\iota_1\lambda_1^{-1}-\mathbb{E}_0[\iota_1\lambda_1^{-1}]+\frac{1+(1+\Gamma)\mathbb{E}_0[\iota_1]}{2+\Gamma}}\right]$$

The exchange rate cyclicality is

$$cov_0(m_1^*-m_1,e_1-e_0)=cov_0(-\log\lambda_1,e_1-e_0)$$

If we mute the variability of ι_1, then, the foreign currency strength $\mathcal{E}_1$ is decreasing in the home households' marginal utility λ_1, which generates a pro-cyclical exchange rate. The intuition is that the higher marginal utility in the home country is created by a shortage of the non-tradable goods, which also lowers the demand for the imported foreign goods. To equilibrate the trade balance, the foreign currency must depreciate, which is consistent with the standard frictionless setting that leads to the Backus-Smith puzzle.

To overturn this result, we need to introduce a negative correlation between ι_1 and λ_1^{-1}, so that a higher marginal utility in the home country is associated with a higher home demand for the foreign goods, which appreciates the foreign currency. The variation in the correlation between ι_1 and λ_1^{-1} also affects the exchange rate wedges ξ_t and ξ_t^*. As such, the levels of these wedges can be interpreted as covariances under this segmented market model.

B.3 Numerical Example

To make this point more concretely, we consider a very stylized numerical example. We set $\Gamma=1$, and $(\log\lambda_1,\log\iota_1)$ is drawn from a multivariate normal distribution. To emphasize the role played by ι_1, we assume that $\log\lambda_1$ has a standard deviation of 0.1, and $\log\iota_1$ has a standard deviation of 0.4. We set the mean so that $\mathbb{E}_0[\lambda_1]=1$ and $\mathbb{E}_0[\iota_1]=1$.

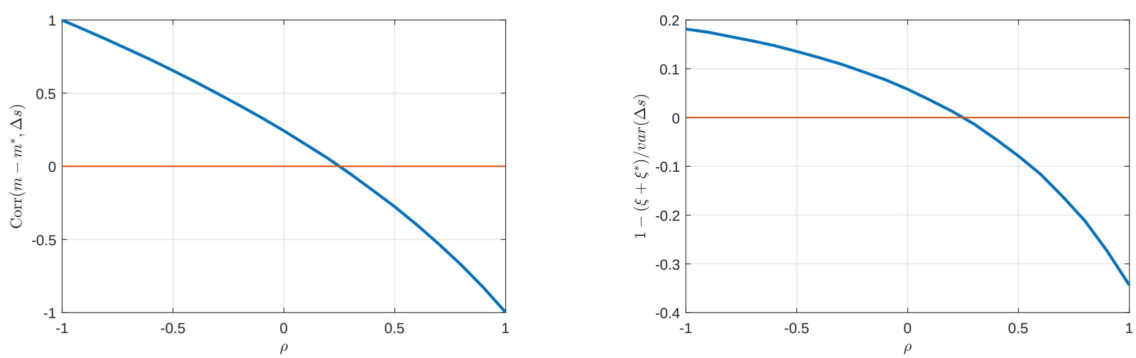
We vary the correlation between $\log\lambda_1$ and $\log\iota_1$ from -1 to 1. We simulate this model by drawing 1,000,000 random samples and compute the exchange rate moments.

To return to our notation in the main text, we define $s_t = -e_t$ as the home currency strength. Figure (B.1)(a) reports the exchange rate cyclicality, defined as $corr_0(m_1 - m_1^*, \Delta s_1)$. Consistent with our theoretical argument, as we increase the correlation ρ between $\log \lambda_1$ and $\log \iota_1$, we introduce a negative correlation between ι_1 and λ_1^{-1}, which depreciates the foreign currency when the home marginal utility is high. The exchange rate becomes pro-cyclical.

Consistent with this exchange rate behavior, the exchange rate wedge also adjusts as we vary the correlation ρ. By Proposition 5, the conditional correlation is given by:

$$corr_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = \frac{std_t(\Delta s_{t+1})}{std_t(m_{t,t+1} - m_{t,t+1}^*)} \left(1 - \frac{(\xi_t^* + \xi_t)}{var_t(\Delta s_{t+1})}\right).$$

Figure (B.1)(b) reports the wedge $1 - (\xi_0^* + \xi_0)/var_0(\Delta s_1)$, which is decreasing and eventually negative as we increase the correlation ρ. This result makes it clear that the exchange rate cyclicality in this segmented market model is consistent with our characterization based on the Euler equation wedges.



(A) FX CYCLICALITY $corr_0(m_1 - m_1^*, \Delta s_1)$ (B) WEDGE $1 - (\xi_0^* + \xi_0)/var_0(\Delta s_1)$

FIGURE B.1. THE EXCHANGE RATE CYCLICALITY AND WEDGE IN GABAIX-MAGGIORI MODEL